Additional Material

Advanced Algorithms - Master DSC/MLDM

Note on Master theorems

*Simple formulation 1

Theorem 1 (Master Theorem 1). *If $T(n) \leq aT(n/b) + O(n^d)$ for some positive constants $a > 0, b > 0, d \geq 0$ then*

1. $T(n) \in O(n^d)$ *if* $a < b^d$
2. $T(n) \in O(n^d \log n)$ *if* $a = b^d$
3. $T(n) \in O(n^{\log_b a})$ *if* $a > b^d$.

*More complex formulation 2

Theorem 2 (Master theorem 2). *Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function and let $T(n)$ be defined on the non negative integers by the recurrence*

$$T(n) = aT(n/b) + f(n)$$

where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$, then $T(n)$ can be bounded asymptotically as follows:

1. *if* $f(n) \in O(n^{\log_b a - \epsilon})$ *for some constant* $\epsilon > 0$, *then* $T(n) \in \theta(n^{\log_b a})$
2. *if* $f(n) \in \theta(n^{\log_b a} \log^k n)$ *with with* $k \geq 0$ *a constant, then* $T(n) \in \theta(n^{\log_b a} \log^{k+1} n)$
3. *if* $f(n) \in \Omega(n^{\log_b a + \epsilon})$ *for some constant* $\epsilon > 0$, *and if* $af(n/b) \leq cf(n)$ *for some constant* $c < 1$ *and all sufficiently large* n, *then* $T(n) \in \theta(f(n))$.