Phys 177 Lab Assignment

Week 02

1) Create your own functions to compute integrals using the trapezoidal rule and Simpson's rule. These functions should take as input an array with the x values and y values of the function to be integrated. **Hint**: notice that the book has an example program to execute the trapezoidal rule already, Sec. 5.1, Example 5.1

2) Download from the github repository of the course (https://github.com/lsales/rep_Phys177), the file called "velocities.txt" in the "extra_info_labs" folder (just make "git pull" if you have already cloned the repository before).
This file has 2 columns, time *t* in seconds (1st column), and the *x*-velocity velocity in meters per second of a particle, measured once every second from time t=0 to t=100.

Write a program to do the following:

- Read in the data and calculate the approximate distance traveled by the particle in the *x* direction as a function of time. Use your functions to compute this using the trapezoidal rule and the Simpson's rule. Print results to a file in both cases.
- Extend your program to make a graph that shows both, the original velocity curve and the distance curve, in two separate panels of the same figure. Hint: Use plt.subplot(2, 1, j), where j = 1 or 2 for upper/lower panels.

3) Write a program to calculate the integral of the function: f(x) = x^4 - 2x + 1, in the x-interval [0,2] using <u>your</u> functions for trapezoidal rule and Simpson's rule (from Ex. 1). Use N=20 slices in both cases. Print the result to screen.
Now, compute the same integral using numpy or scipy intrinsic functions for trapezoidal and Simpson's rule. Print both results to screen again.
Use your function or the numpy/scipy to evaluate the error of the integrals for trapezoidal and Simpson's rule case using the "practical estimation of errors". For this you will need to re-compute the integrals with N=10 and apply the formulas we saw in class (Eq. 5.28 and 5.29 in the book). Make the program compare with the true result (solve the integral by hand and simply enter the result as a constant).

4) Consider the integral of the function f(x) = $\sin^2$ * sqrt(100x) in the range x=[0,1]

Write a program that uses the adaptive trapezoidal rule method (Eq. 5.34) to calculate the value of this integral to an approximate accuracy of e=10^{-6} (i.e. correct to six digits after the decimal point). Start with a single integration slice and work up from there to two, four, eight and so forth. Have the program print out the number of slices, its estimate of the integral, and its estimate of the error on the integral, for each value of the number of slices N, until the target accuracy is reached. (Hint: you should find the result around I=0.45)

Extra credit (optional exercise):

Write a second program to evaluate the same integral than Exercise 4 above, but using the Romberg integration technique (use implicit functions from Scipy or specific libraries like numerical recipes for C/fortran). Have your program print out a triangular table of values (as the one in page 161 of the book) of all the Romberg estimates of the integral. Calculate the error on your estimates using Eq. 5.49 and again continue the calculation until you reach an accuracy of $e=10^{-6}$. You should find that the Romberg method reaches the required accuracy considerably faster than the trapezoidal rule alone.