NON-PARAMETRIC INFERENCE | MTH516A

# Non-parametric Inference on Global $CO_2$ Emission from Cement Production

**Soumya Paul (211391), Sankhadeep Mitra (211369), Sampriti Dutta (211366)**

*Under the guidance of*

Prof. Sharmishtha Mitra

Department of Mathematics and Statistics,

Indian Institute of Technology, Kanpur

0.0 Acknowledgement

We are delighted to give a project report on "*Non-parametric Inference on Global $CO_2$ Emission from Cement Production*". Every accomplishment has received ongoing encouragement and counsel from excellent and noble minds to aid us in directing our efforts in the appropriate direction to bring the project to completion. We would like to thank our professor, **Prof. Sharmishtha Mitra**, for her consistent assistance and support during the project's completion. It would have been practically impossible to work on this project as a team and comprehend the practical component of the course *MTH516A: NON-PARAMETRIC INFERENCE* without her invaluable instruction and enthusiasm. We are also grateful to all faculty members and seniors who provided assistance at various phases of the project. Finally, our heartfelt gratitude goes to our friends, who were always there for us when we needed them, no matter what.

Sampriti Dutta

Sankhadeep Mitra

Soumya Paul

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## 0.1 Introduction

A hypothesis test is a statistical test that is used to determine whether or not there is sufficient evidence to back a hypothesis. For example, there is a variation in average height between men and women. Non-parametric hypothesis tests do not make the assumptions like normality, equal variance. etc. They are common alternatives to parametric tests because they make few or no assumptions about the data or population distribution. Non-parametric tests are frequently dependent on the initial numerical data's ranks. Non-parametric tests are generally thought to be relatively simpler to conduct, but some issues can arise. When dealing with significant amounts of data, such tests can be time-consuming. Many fields of study, such as psychology, have very narrow ranges of scores, which can result in the same value showing multiple times in a collection of data. With more tied results, rank-based tests can become more complicated.

The different types of non-parametric hypothesis tests are:

- Sign Test,
- Wilcoxon Rank-Sum (Mann-Whitney U) Test,
- Mood's Test,
- Kruskal-Wallis Test,
- Kendall's Tau Test

## 0.2 Objective

In this project, we have studied the nature of global $CO_2$ emission from cement production. To do so, we have validated the following statements on $CO_2$ emission from cement production with some non-parametric tests.

- For different countries
  - The $CO_2$ emissions differ in terms of location
  - The $CO_2$ emissions differ in terms of scale
  - There is association among countries in terms of $CO_2$ emissions
- Globally, after every decade
  - The $CO_2$ emissions differ in terms of location
  - The $CO_2$ emissions differ in terms of scale

# 0.3 Exploratory Data Analysis

## 0.3.1 Data Description

Here is the link of the dataset we are going to use Cement Emissions. We have subsetted the data from 1931 onward and dropped any columns with all *NAs* or 0s and also the column named *Global*, which actually represents the total $CO_2$ emissions for different time stamps. The table below shows the data comprising $CO_2$ emissions of 24 different countries from 1931-2021.

| Year | Argentina | Australia | Belgium | Brazil | Canada | Chile | China | Democratic Republic of the Congo | Denmark | Egypt | Finland | Israel | Italy | Japan | Mozambique | Norway | Peru | Portugal | Romania | Spain | Sweden | Syria | Turkey | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1931 | 265.3 | 196.3 | 1218.0 | 83.59 | 799.5 | 50.88 | 97.93 | 21.81 | 250.8 | 119.9 | 79.95 | 39.98 | 1519.0 | 1788.0 | 10.90 | 109.70 | 14.54 | 47.25 | 98.14 | 806.8 | 257.9 | 3.63 | 50.83 | 10810 |
| 1932 | 247.1 | 123.6 | 1039.0 | 72.69 | 363.4 | 54.51 | 79.57 | 7.27 | 203.6 | 119.9 | 76.23 | 50.88 | 1545.0 | 1843.0 | 10.90 | 117.30 | 10.90 | 58.15 | 105.40 | 705.1 | 241.0 | 10.90 | 54.46 | 6656 |
| 1933 | 254.5 | 159.9 | 963.1 | 109.00 | 189.0 | 69.05 | 113.20 | 3.63 | 272.6 | 141.7 | 80.05 | 65.42 | 1756.0 | 2366.0 | 10.18 | 110.80 | 14.53 | 79.95 | 109.00 | 694.1 | 200.7 | 14.54 | 58.15 | 5587 |
| 1934 | 279.8 | 207.2 | 937.6 | 159.90 | 272.6 | 101.70 | 97.93 | 3.63 | 381.6 | 145.4 | 112.70 | 69.05 | 2013.0 | 2304.0 | 7.27 | 123.60 | 21.82 | 90.94 | 156.30 | 672.3 | 287.1 | 10.90 | 83.59 | 6900 |
| 1935 | 356.3 | 276.2 | 1087.0 | 181.60 | 272.6 | 141.60 | 156.10 | 3.63 | 374.3 | 189.0 | 134.30 | 90.86 | 2086.0 | 2904.0 | 7.27 | 130.80 | 29.07 | 105.40 | 189.00 | 668.7 | 367.1 | 14.54 | 65.42 | 6684 |
| 1936 | 410.7 | 323.5 | 1163.0 | 239.70 | 388.9 | 123.50 | 428.50 | 3.63 | 392.4 | 167.2 | 163.50 | 76.32 | 1890.0 | 3082.0 | 7.27 | 149.00 | 36.34 | 119.90 | 185.30 | 297.9 | 392.5 | 29.07 | 69.05 | 9950 |
| 1937 | 512.3 | 363.4 | 1486.0 | 283.50 | 483.4 | 156.30 | 437.70 | 3.77 | 334.4 | 163.5 | 203.40 | 79.95 | 2155.0 | 2984.0 | 7.27 | 159.90 | 39.96 | 127.10 | 225.30 | 189.0 | 432.5 | 61.78 | 105.40 | 10370 |
| 1938 | 614.3 | 178.1 | 1508.0 | 305.50 | 432.5 | 181.70 | 9.18 | 7.50 | 316.2 | 185.3 | 236.20 | 47.25 | 2279.0 | 2729.0 | 11.99 | 163.60 | 50.86 | 130.80 | 221.70 | 294.4 | 490.6 | 61.78 | 130.90 | 9239 |
| 1939 | 556.0 | 338.0 | 1261.0 | 345.50 | 450.7 | 167.20 | 223.40 | 17.44 | 345.3 | 185.0 | 279.60 | 54.51 | 2526.0 | 2508.0 | 14.54 | 192.60 | 58.18 | 145.40 | 261.70 | 588.8 | 585.1 | 58.15 | 141.70 | 10790 |
| 1940 | 534.4 | 348.8 | 105.4 | 367.10 | 592.4 | 189.00 | 272.40 | 11.45 | 218.1 | 178.1 | 149.00 | 72.69 | 2373.0 | 2101.0 | 14.54 | 167.20 | 61.78 | 134.50 | 196.30 | 770.5 | 345.3 | 21.81 | 130.90 | 11550 |
| 1941 | 577.9 | 341.6 | 312.5 | 378.00 | 657.8 | 178.10 | 495.80 | 21.81 | 207.2 | 207.2 | 152.60 | 58.15 | 1381.0 | 2882.0 | 14.54 | 156.30 | 83.59 | 134.50 | 305.20 | 810.4 | 319.7 | 21.81 | 134.60 | 14520 |
| 1942 | 530.7 | 279.8 | 250.8 | 370.50 | 716.0 | 181.60 | 654.90 | 32.71 | 214.4 | 207.2 | 87.22 | 109.00 | 1108.0 | 2151.0 | 10.90 | 185.50 | 94.49 | 119.90 | 181.70 | 814.1 | 407.0 | 54.51 | 105.40 | 16090 |
| 1943 | 472.3 | 196.2 | 229.0 | 370.50 | 574.2 | 185.30 | 639.60 | 32.71 | 316.2 | 159.9 | 116.40 | 83.59 | 846.4 | 1861.0 | 15.66 | 152.60 | 101.80 | 123.40 | 225.30 | 839.5 | 457.8 | 36.34 | 83.59 | 12040 |
| 1944 | 534.2 | 218.1 | 298.0 | 399.80 | 563.3 | 178.10 | 489.70 | 43.61 | 319.8 | 210.6 | 87.22 | 87.22 | 668.0 | 1465.0 | 13.63 | 159.80 | 123.60 | 120.10 | 159.90 | 912.1 | 523.3 | 18.17 | 141.70 | 8032 |
| 1945 | 537.9 | 348.9 | 319.8 | 381.60 | 665.1 | 203.50 | 18.36 | 37.13 | 109.0 | 214.4 | 137.30 | 72.69 | 563.8 | 577.9 | 16.87 | 69.05 | 130.80 | 130.80 | 123.60 | 952.2 | 600.1 | 18.17 | 141.70 | 9114 |
| 1946 | 563.3 | 363.4 | 934.0 | 406.80 | 908.5 | 287.10 | 85.69 | 39.98 | 247.1 | 290.7 | 163.50 | 130.80 | 999.4 | 457.9 | 13.63 | 214.40 | 127.20 | 159.90 | 156.20 | 1061.0 | 723.2 | 21.81 | 159.90 | 14650 |
| 1947 | 672.5 | 443.4 | 1290.0 | 450.70 | 937.6 | 298.00 | 174.40 | 57.50 | 312.5 | 319.8 | 207.20 | 163.50 | 1363.0 | 610.6 | 17.10 | 232.60 | 127.20 | 210.80 | 207.20 | 1079.0 | 766.8 | 25.44 | 174.40 | 16650 |
| 1948 | 617.8 | 508.8 | 1646.0 | 548.80 | 1108.0 | 265.50 | 82.64 | 61.78 | 381.5 | 378.0 | 276.20 | 79.95 | 1555.0 | 919.5 | 18.17 | 261.60 | 138.20 | 247.10 | 221.70 | 1152.0 | 734.1 | 25.44 | 170.80 | 18420 |
| 1949 | 716.0 | 530.6 | 1446.0 | 632.10 | 1250.0 | 243.50 | 274.70 | 72.69 | 410.8 | 439.7 | 323.50 | 119.90 | 1995.0 | 1621.0 | 23.26 | 294.40 | 141.70 | 258.00 | 276.20 | 1112.0 | 839.5 | 29.07 | 185.30 | 18790 |
| 1950 | 777.7 | 632.4 | 1759.0 | 686.90 | 1312.0 | 254.40 | 586.90 | 87.22 | 432.5 | 505.2 | 367.30 | 189.00 | 2475.0 | 2206.0 | 25.34 | 287.10 | 163.40 | 283.50 | 319.80 | 1247.0 | 963.0 | 33.00 | 196.30 | 20120 |
| 1951 | 770.5 | 610.6 | 2173.0 | 719.60 | 1334.0 | 345.30 | 1036.00 | 101.80 | 487.0 | 559.7 | 410.90 | 218.10 | 2758.0 | 3238.0 | 39.98 | 345.10 | 178.10 | 316.40 | 563.40 | 1356.0 | 1007.0 | 32.71 | 196.20 | 22040 |
| 1952 | 766.8 | 672.3 | 2032.0 | 799.40 | 1454.0 | 403.40 | 1203.00 | 119.70 | 599.8 | 468.8 | 385.10 | 221.70 | 3289.0 | 3518.0 | 39.98 | 348.80 | 181.60 | 359.80 | 741.40 | 1465.0 | 1047.0 | 76.32 | 225.30 | 22260 |
| 1953 | 817.7 | 788.7 | 2286.0 | 1003.00 | 1744.0 | 378.00 | 1615.00 | 123.40 | 621.5 | 541.5 | 461.80 | 229.00 | 3871.0 | 4332.0 | 43.61 | 374.50 | 221.70 | 381.60 | 937.80 | 1610.0 | 1163.0 | 109.00 | 261.70 | 23520 |
| 1954 | 843.1 | 944.9 | 2162.0 | 1236.00 | 1759.0 | 381.60 | 1915.00 | 170.80 | 603.3 | 661.6 | 512.20 | 279.80 | 4328.0 | 5277.0 | 50.72 | 381.60 | 239.70 | 385.20 | 792.30 | 1883.0 | 1217.0 | 124.40 | 349.00 | 24130 |
| 1955 | 923.1 | 984.9 | 2319.0 | 1370.00 | 1973.0 | 396.10 | 1873.00 | 199.90 | 621.5 | 676.0 | 516.10 | 327.10 | 5270.0 | 5222.0 | 69.24 | 396.10 | 268.90 | 385.00 | 955.90 | 2141.0 | 1261.0 | 130.80 | 407.10 | 26580 |
| 1956 | 1021.0 | 1058.0 | 2304.0 | 1617.00 | 2253.0 | 381.70 | 2660.00 | 225.30 | 585.1 | 668.7 | 476.10 | 301.60 | 5604.0 | 6436.0 | 76.50 | 443.40 | 272.60 | 505.20 | 1036.00 | 2337.0 | 1232.0 | 159.90 | 479.70 | 28390 |
| 1957 | 1170.0 | 1148.0 | 2326.0 | 1679.00 | 2711.0 | 359.80 | 2855.00 | 229.00 | 574.2 | 723.2 | 468.80 | 356.20 | 5906.0 | 7501.0 | 83.59 | 501.50 | 268.90 | 483.40 | 1163.00 | 2457.0 | 1210.0 | 156.30 | 625.10 | 27010 |
| 1958 | 1221.0 | 1214.0 | 2006.0 | 1872.00 | 2758.0 | 359.80 | 3871.00 | 192.40 | 527.0 | 748.7 | 457.90 | 352.50 | 6338.0 | 7407.0 | 87.22 | 508.80 | 298.00 | 505.20 | 1272.00 | 2631.0 | 1239.0 | 192.60 | 748.80 | 27760 |
| 1959 | 1170.0 | 1294.0 | 2195.0 | 1897.00 | 2817.0 | 414.30 | 5107.00 | 170.80 | 687.0 | 883.1 | 577.90 | 385.20 | 7120.0 | 8533.0 | 105.40 | 559.70 | 287.10 | 508.80 | 1410.00 | 2831.0 | 1396.0 | 222.10 | 857.50 | 30230 |
| 1960 | 1305.0 | 1381.0 | 2170.0 | 2210.00 | 2595.0 | 414.30 | 6514.00 | 98.13 | 712.2 | 1010.0 | 621.50 | 399.80 | 7915.0 | 11140.0 | 109.00 | 570.60 | 297.90 | 592.40 | 1508.00 | 2835.0 | 1388.0 | 239.90 | 1007.00 | 28800 |
| 1961 | 1436.0 | 1414.0 | 2351.0 | 2309.00 | 2784.0 | 436.10 | 2585.00 | 68.94 | 781.4 | 1018.0 | 665.20 | 417.90 | 8911.0 | 12180.0 | 105.40 | 628.70 | 294.40 | 614.20 | 1635.00 | 3275.0 | 1504.0 | 265.30 | 1003.00 | 28700 |
| 1962 | 1446.0 | 1450.0 | 2366.0 | 2462.00 | 3086.0 | 505.30 | 2497.00 | 98.13 | 806.7 | 1105.0 | 672.20 | 472.50 | 9969.0 | 14230.0 | 87.38 | 697.80 | 345.30 | 694.10 | 1723.00 | 3605.0 | 1519.0 | 301.30 | 1148.00 | 29880 |
| 1963 | 1254.0 | 1541.0 | 2326.0 | 2493.00 | 3144.0 | 577.90 | 3355.00 | 120.10 | 752.3 | 1239.0 | 705.10 | 504.90 | 10920.0 | 14800.0 | 83.59 | 712.30 | 374.20 | 708.70 | 2159.00 | 3831.0 | 1632.0 | 338.00 | 1334.00 | 31440 |
| 1964 | 1439.0 | 1792.0 | 2889.0 | 2679.00 | 3547.0 | 625.10 | 5032.00 | 112.90 | 937.6 | 1247.0 | 777.70 | 541.30 | 11290.0 | 16300.0 | 90.86 | 763.20 | 403.40 | 802.90 | 2348.00 | 4201.0 | 1792.0 | 312.50 | 1454.00 | 33030 |
| 1965 | 1632.0 | 1879.0 | 2918.0 | 2651.00 | 3747.0 | 585.10 | 6801.00 | 123.40 | 984.9 | 1196.0 | 875.90 | 621.50 | 10230.0 | 16160.0 | 109.00 | 792.30 | 501.70 | 828.60 | 2671.00 | 4790.0 | 1864.0 | 334.40 | 1646.00 | 33290 |
| 1966 | 1723.0 | 1817.0 | 2864.0 | 2825.00 | 4001.0 | 675.80 | 8387.00 | 130.80 | 1036.0 | 1207.0 | 777.70 | 578.00 | 11080.0 | 18910.0 | 112.70 | 901.30 | 526.80 | 850.40 | 2907.00 | 5967.0 | 1857.0 | 305.30 | 1912.00 | 34570 |
| 1967 | 1756.0 | 1890.0 | 2875.0 | 2965.00 | 3583.0 | 610.40 | 6085.00 | 145.40 | 1087.0 | 1359.0 | 748.70 | 396.10 | 12980.0 | 21240.0 | 123.60 | 1065.00 | 537.90 | 901.30 | 3129.00 | 6658.0 | 1926.0 | 298.00 | 2100.00 | 33590 |
| 1968 | 2064.0 | 1941.0 | 2838.0 | 3337.00 | 3660.0 | 610.70 | 5253.00 | 145.40 | 1127.0 | 1555.0 | 730.40 | 545.00 | 14560.0 | 23560.0 | 141.70 | 1134.00 | 548.80 | 919.50 | 3474.00 | 7392.0 | 1933.0 | 454.30 | 2337.00 | 35320 |
| 1969 | 2148.0 | 2130.0 | 3100.0 | 3552.00 | 3700.0 | 708.70 | 7613.00 | 159.90 | 1290.0 | 1785.0 | 886.80 | 646.70 | 15490.0 | 25400.0 | 149.00 | 1232.00 | 563.20 | 1007.00 | 3714.00 | 8064.0 | 1955.0 | 461.60 | 2864.00 | 35700 |
| 1970 | 2344.0 | 2228.0 | 3325.0 | 4047.00 | 3562.0 | 1010.00 | 10720.00 | 207.20 | 1287.0 | 1820.0 | 926.80 | 683.40 | 16350.0 | 28270.0 | 196.30 | 1290.00 | 559.80 | 1152.00 | 4016.00 | 8253.0 | 2013.0 | 476.40 | 3151.00 | 34920 |
| 1971 | 2736.0 | 2314.0 | 3397.0 | 4407.00 | 4068.0 | 663.50 | 13120.00 | 225.30 | 1357.0 | 1937.0 | 903.30 | 683.60 | 15560.0 | 29270.0 | 207.20 | 1357.00 | 714.10 | 1207.00 | 4201.00 | 8433.0 | 1955.0 | 450.70 | 3703.00 | 35510 |
| 1972 | 2699.0 | 2436.0 | 3442.0 | 5117.00 | 4477.0 | 658.00 | 14720.00 | 236.20 | 1432.0 | 1890.0 | 969.80 | 740.00 | 16210.0 | 32530.0 | 232.60 | 1351.00 | 798.90 | 1358.00 | 4531.00 | 9562.0 | 1922.0 | 498.10 | 4107.00 | 36520 |
| 1973 | 2570.0 | 2591.0 | 3390.0 | 6023.00 | 4995.0 | 641.80 | 15460.00 | 265.30 | 1444.0 | 1788.0 | 1018.00 | 593.20 | 17410.0 | 38190.0 | 301.60 | 1352.00 | 1157.00 | 1603.00 | 4830.00 | 10860.0 | 2094.0 | 417.70 | 4332.00 | 36920 |
| 1974 | 2675.0 | 2568.0 | 3563.0 | 6707.00 | 5135.0 | 652.70 | 15340.00 | 308.90 | 1251.0 | 1614.0 | 1065.00 | 833.00 | 17230.0 | 35610.0 | 229.00 | 1309.00 | 931.50 | 1636.00 | 5476.00 | 10740.0 | 1648.0 | 476.10 | 4294.00 | 36810 |
| 1975 | 2715.0 | 2473.0 | 3252.0 | 7525.00 | 4933.0 | 455.00 | 19100.00 | 308.90 | 1126.0 | 1770.0 | 991.50 | 1000.00 | 16080.0 | 31800.0 | 138.10 | 1347.00 | 928.80 | 1655.00 | 5623.00 | 11560.0 | 1556.0 | 490.60 | 5103.00 | 30460 |
| 1976 | 2842.0 | 2494.0 | 3515.0 | 8608.00 | 4767.0 | 423.20 | 19250.00 | 319.80 | 1189.0 | 1658.0 | 869.80 | 898.30 | 16890.0 | 33230.0 | 104.40 | 1334.00 | 955.00 | 1771.00 | 6371.00 | 12020.0 | 1433.0 | 542.90 | 5850.00 | 32370 |
| 1977 | 2987.0 | 2474.0 | 3602.0 | 9496.00 | 4776.0 | 483.90 | 22910.00 | 238.00 | 1171.0 | 1604.0 | 812.20 | 867.00 | 17570.0 | 35240.0 | 156.30 | 1152.00 | 934.40 | 2036.00 | 6737.00 | 13270.0 | 1311.0 | 676.00 | 6507.00 | 33980 |
| 1978 | 3148.0 | 2459.0 | 3480.0 | 10800.00 | 5113.0 | 495.20 | 26810.00 | 224.70 | 1337.0 | 1474.0 | 803.80 | 865.00 | 17400.0 | 40740.0 | 154.50 | 1113.00 | 978.50 | 2411.00 | 7115.00 | 14250.0 | 1180.0 | 686.60 | 7167.00 | 35620 |
| 1979 | 3328.0 | 2581.0 | 3508.0 | 11340.00 | 5833.0 | 557.10 | 30320.00 | 211.70 | 1233.0 | 1450.0 | 819.50 | 818.80 | 17680.0 | 41980.0 | 128.40 | 1095.00 | 1157.00 | 2406.00 | 7534.00 | 13080.0 | 1201.0 | 874.20 | 6389.00 | 35940 |
| 1980 | 3562.0 | 2652.0 | 3372.0 | 12080.00 | 5208.0 | 636.40 | 32710.00 | 205.70 | 984.3 | 1483.0 | 845.30 | 770.30 | 18600.0 | 41900.0 | 109.60 | 1100.00 | 1044.00 | 2675.00 | 7522.00 | 13040.0 | 1233.0 | 933.80 | 5926.00 | 32920 |
| 1981 | 3323.0 | 2954.0 | 2988.0 | 11930.00 | 5034.0 | 732.00 | 33900.00 | 227.00 | 825.1 | 1710.0 | 840.10 | 847.30 | 18290.0 | 40260.0 | 108.30 | 917.60 | 1478.00 | 2635.00 | 7085.00 | 13300.0 | 1168.0 | 1073.00 | 6873.00 | 31960 |
| 1982 | 2815.0 | 2823.0 | 2796.0 | 11560.00 | 4182.0 | 435.10 | 38870.00 | 247.70 | 915.6 | 2076.0 | 855.10 | 885.40 | 17300.0 | 38150.0 | 160.70 | 892.60 | 1239.00 | 2740.00 | 7187.00 | 13600.0 | 1163.0 | 1190.00 | 7156.00 | 28440 |
| 1983 | 2818.0 | 2377.0 | 2505.0 | 9252.00 | 3906.0 | 471.60 | 44130.00 | 231.40 | 857.8 | 2676.0 | 869.90 | 816.20 | 16880.0 | 38100.0 | 188.40 | 834.50 | 937.20 | 2770.00 | 6679.00 | 14000.0 | 1136.0 | 1643.00 | 6119.00 | 30440 |
| 1984 | 2619.0 | 2683.0 | 2473.0 | 8246.00 | 4276.0 | 511.20 | 50070.00 | 238.30 | 868.6 | 3157.0 | 747.70 | 803.80 | 16080.0 | 37000.0 | 199.20 | 772.70 | 927.50 | 2512.00 | 6683.00 | 11550.0 | 1212.0 | 1917.00 | 7028.00 | 32910 |
| 1985 | 2326.0 | 2891.0 | 2376.0 | 8759.00 | 5063.0 | 509.00 | 59300.00 | 193.60 | 910.0 | 2787.0 | 726.30 | 771.20 | 15430.0 | 34050.0 | 87.04 | 803.00 | 834.00 | 2420.00 | 5819.00 | 10920.0 | 1080.0 | 1902.00 | 7793.00 | 31710 |
| 1986 | 2790.0 | 2908.0 | 2446.0 | 10500.00 | 5270.0 | 498.50 | 67360.00 | 191.00 | 1064.0 | 3681.0 | 636.00 | 769.90 | 14940.0 | 33190.0 | 31.83 | 877.40 | 1044.00 | 2437.00 | 6742.00 | 10780.0 | 1044.0 | 1838.00 | 8798.00 | 32850 |
| 1987 | 3170.0 | 2878.0 | 2393.0 | 10660.00 | 6259.0 | 540.00 | 75430.00 | 210.50 | 993.8 | 4221.0 | 635.20 | 816.40 | 15310.0 | 33190.0 | 31.30 | 822.90 | 1222.00 | 2604.00 | 6763.00 | 10250.0 | 1148.0 | 1673.00 | 9593.00 | 32980 |
| 1988 | 3048.0 | 3136.0 | 2683.0 | 10500.00 | 5986.0 | 619.80 | 84960.00 | 207.70 | 888.7 | 4715.0 | 666.80 | 833.80 | 15380.0 | 35840.0 | 27.90 | 716.40 | 1184.00 | 2607.00 | 6818.00 | 11250.0 | 1150.0 | 1424.00 | 9821.00 | 33250 |
| 1989 | 2254.0 | 3380.0 | 2768.0 | 10510.00 | 6263.0 | 642.50 | 84890.00 | 189.70 | 1063.0 | 6000.0 | 701.20 | 803.50 | 15790.0 | 36700.0 | 30.59 | 691.20 | 987.20 | 2629.00 | 6243.00 | 12040.0 | 1246.0 | 1478.00 | 10220.00 | 33260 |
| 1990 | 1822.0 | 3463.0 | 2824.0 | 11060.00 | 5823.0 | 659.20 | 84510.00 | 189.30 | 882.4 | 6767.0 | 729.20 | 986.90 | 15850.0 | 38700.0 | 32.43 | 634.00 | 1025.00 | 3176.00 | 4445.00 | 12280.0 | 1272.0 | 1461.00 | 10440.00 | 33480 |
| 1991 | 2196.0 | 3183.0 | 2880.0 | 11780.00 | 4771.0 | 773.10 | 101300.00 | 102.50 | 1088.0 | 7855.0 | 568.90 | 1194.00 | 15690.0 | 40350.0 | 32.79 | 592.00 | 1027.00 | 3271.00 | 3201.00 | 11700.0 | 1137.0 | 1450.00 | 12020.00 | 32740 |
| 1992 | 2495.0 | 2923.0 | 3089.0 | 9770.00 | 4804.0 | 849.00 | 122400.00 | 71.22 | 1193.0 | 8100.0 | 473.50 | 1302.00 | 16430.0 | 41670.0 | 12.28 | 653.00 | 973.00 | 3136.00 | 2906.00 | 10440.0 | 1089.0 | 1521.00 | 13270.00 | 32990 |
| 1993 | 2761.0 | 3005.0 | 3082.0 | 10160.00 | 4893.0 | 860.10 | 142800.00 | 60.89 | 1206.0 | 7596.0 | 361.30 | 1457.00 | 14070.0 | 41220.0 | 8.17 | 838.00 | 1161.00 | 3195.00 | 2833.00 | 9914.0 | 1092.0 | 1836.00 | 13050.00 | 34840 |
| 1994 | 3050.0 | 3484.0 | 3179.0 | 10090.00 | 5769.0 | 935.30 | 164400.00 | 67.73 | 1192.0 | 7615.0 | 363.60 | 1506.00 | 13790.0 | 42300.0 | 24.48 | 845.00 | 1471.00 | 3292.00 | 2771.00 | 11500.0 | 1109.0 | 1823.00 | 13490.00 | 36310 |
| 1995 | 2621.0 | 3358.0 | 3255.0 | 11530.00 | 6529.0 | 991.80 | 184600.00 | 95.74 | 1204.0 | 8325.0 | 377.90 | 1900.00 | 15310.0 | 42140.0 | 24.44 | 875.00 | 1751.00 | 3462.00 | 3146.00 | 12360.0 | 1296.0 | 1794.00 | 14130.00 | 37080 |
| 1996 | 2422.0 | 3244.0 | 3009.0 | 13880.00 | 6185.0 | 961.20 | 192900.00 | 98.04 | 1282.0 | 8780.0 | 381.40 | 1673.00 | 13990.0 | 42560.0 | 73.22 | 863.00 | 1771.00 | 3388.00 | 3200.00 | 12110.0 | 1225.0 | 1795.00 | 14660.00 | 37310 |
| 1997 | 3169.0 | 3185.0 | 3162.0 | 15270.00 | 6657.0 | 945.90 | 202900.00 | 50.77 | 1343.0 | 9215.0 | 450.00 | 1719.00 | 14230.0 | 39930.0 | 89.36 | 922.00 | 1974.00 | 3561.00 | 3005.00 | 12750.0 | 1105.0 | 1916.00 | 15100.00 | 38560 |
| 1998 | 3283.0 | 3475.0 | 3175.0 | 16180.00 | 6828.0 | 1131.00 | 211400.00 | 54.34 | 1390.0 | 9787.0 | 447.80 | 2189.00 | 14540.0 | 35360.0 | 85.98 | 900.00 | 1987.00 | 3537.00 | 2916.00 | 13810.0 | 1133.0 | 1809.00 | 15290.00 | 39460 |
| 1999 | 3290.0 | 3520.0 | 3113.0 | 16440.00 | 7118.0 | 950.80 | 228300.00 | 64.38 | 1355.0 | 10820.0 | 478.60 | 2273.00 | 15280.0 | 35010.0 | 87.46 | 892.00 | 1734.00 | 3827.00 | 2645.00 | 14430.0 | 1139.0 | 2000.00 | 14590.00 | 40240 |
| 2000 | 2771.0 | 3621.0 | 3270.0 | 16050.00 | 7232.0 | 1010.00 | 244400.00 | 68.33 | 1385.0 | 11170.0 | 504.80 | 2152.00 | 15860.0 | 35090.0 | 109.20 | 890.00 | 1778.00 | 3809.00 | 2656.00 | 14730.0 | 1288.0 | 1790.00 | 15180.00 | 41450 |
| 2001 | 2481.0 | 3541.0 | 2957.0 | 15230.00 | 6983.0 | 1013.00 | 270800.00 | 80.94 | 1388.0 | 11750.0 | 503.60 | 1820.00 | 16390.0 | 34370.0 | 106.70 | 853.00 | 1806.00 | 3627.00 | 2768.00 | 15010.0 | 1332.0 | 1897.00 | 15090.00 | 41610 |
| 2002 | 1729.0 | 3488.0 | 2993.0 | 14390.00 | 7148.0 | 1079.00 | 296600.00 | 106.30 | 1416.0 | 12720.0 | 484.90 | 1821.00 | 16290.0 | 32420.0 | 196.50 | 875.00 | 1827.00 | 3910.00 | 2642.00 | 15530.0 | 1280.0 | 1737.00 | 15510.00 | 43160 |
| 2003 | 2280.0 | 3584.0 | 2933.0 | 13100.00 | 7248.0 | 1113.00 | 345200.00 | 132.20 | 1330.0 | 11890.0 | 466.60 | 1885.00 | 16940.0 | 31940.0 | 239.70 | 904.00 | 1845.00 | 3616.00 | 2650.00 | 16040.0 | 1206.0 | 1754.00 | 16020.00 | 43350 |
| 2004 | 2701.0 | 3555.0 | 2837.0 | 13270.00 | 7520.0 | 1194.00 | 380000.00 | 160.10 | 1459.0 | 12690.0 | 528.80 | 1946.00 | 17410.0 | 31280.0 | 218.80 | 716.00 | 2132.00 | 4004.00 | 2992.00 | 16290.0 | 1284.0 | 1694.00 | 17210.00 | 45890 |
| 2005 | 3241.0 | 3664.0 | 2934.0 | 14350.00 | 7614.0 | 1152.00 | 411600.00 | 206.60 | 1363.0 | 14150.0 | 542.20 | 2158.00 | 17400.0 | 32280.0 | 194.20 | 780.00 | 2376.00 | 4088.00 | 3175.00 | 16790.0 | 1315.0 | 1638.00 | 19120.00 | 46190 |
| 2006 | 3764.0 | 3888.0 | 3112.0 | 15440.00 | 7728.0 | 1236.00 | 470100.00 | 203.80 | 1395.0 | 15570.0 | 573.90 | 2265.00 | 17470.0 | 31990.0 | 237.50 | 805.00 | 2612.00 | 4000.00 | 3656.00 | 16740.0 | 1442.0 | 1696.00 | 20840.00 | 46850 |
| 2007 | 3998.0 | 3972.0 | 3087.0 | 17200.00 | 7731.0 | 1316.00 | 515000.00 | 205.30 | 1407.0 | 17100.0 | 605.50 | 2271.00 | 17910.0 | 30660.0 | 257.50 | 869.00 | 2730.00 | 4166.00 | 4054.00 | 16820.0 | 1339.0 | 1848.00 | 22780.00 | 45510 |
| 2008 | 3990.0 | 3863.0 | 3033.0 | 18880.00 | 6988.0 | 1307.00 | 525900.00 | 158.10 | 1155.0 | 16190.0 | 619.80 | 2171.00 | 16130.0 | 28550.0 | 286.20 | 833.00 | 2939.00 | 4012.00 | 4143.00 | 14390.0 | 1396.0 | 1910.00 | 24850.00 | 41420 |
| 2009 | 3810.0 | 3829.0 | 2795.0 | 19030.00 | 5364.0 | 1081.00 | 583600.00 | 177.20 | 764.4 | 19970.0 | 385.30 | 1815.00 | 13340.0 | 25310.0 | 299.20 | 842.00 | 3330.00 | 3223.00 | 3093.00 | 11400.0 | 1263.0 | 1861.00 | 27040.00 | 29610 |
| 2010 | 4178.0 | 3549.0 | 2582.0 | 21290.00 | 6005.0 | 1046.00 | 639600.00 | 189.00 | 672.2 | 20510.0 | 525.70 | 2023.00 | 13280.0 | 24320.0 | 340.90 | 754.00 | 3338.00 | 3376.00 | 2778.00 | 11200.0 | 1324.0 | 2151.00 | 29980.00 | 31450 |
| 2011 | 4586.0 | 3496.0 | 2762.0 | 22840.00 | 6020.0 | 1080.00 | 708600.00 | 175.20 | 861.8 | 20100.0 | 557.80 | 2251.00 | 12580.0 | 24980.0 | 373.30 | 749.00 | 3300.00 | 2813.00 | 3089.00 | 9523.0 | 1361.0 | 1769.00 | 31450.00 | 32210 |
| 2012 | 4184.0 | 3518.0 | 2643.0 | 25000.00 | 6532.0 | 1128.00 | 714800.00 | 157.90 | 871.1 | 20970.0 | 497.20 | 2252.00 | 10070.0 | 25620.0 | 452.70 | 725.00 | 3731.00 | 2550.00 | 3150.00 | 8754.0 | 1479.0 | 2087.00 | 31370.00 | 35270 |
| 2013 | 4581.0 | 3294.0 | 2541.0 | 26650.00 | 5973.0 | 1086.00 | 748300.00 | 174.00 | 867.1 | 20210.0 | 481.80 | 2170.00 | 8877.0 | 26810.0 | 505.60 | 731.00 | 4257.00 | 2814.00 | 2695.00 | 7642.0 | 1402.0 | 1393.00 | 33910.00 | 36370 |
| 2014 | 4336.0 | 3138.0 | 2643.0 | 26910.00 | 5912.0 | 1022.00 | 778600.00 | 127.90 | 887.3 | 20760.0 | 468.80 | 2292.00 | 8339.0 | 26560.0 | 585.90 | 727.00 | 4590.00 | 3096.00 | 2944.00 | 8897.0 | 1399.0 | 646.10 | 34500.00 | 39440 |
| 2015 | 4571.0 | 3076.0 | 2348.0 | 25080.00 | 6185.0 | 1033.00 | 722000.00 | 154.60 | 931.5 | 21650.0 | 462.10 | 2233.00 | 8196.0 | 25940.0 | 614.20 | 672.00 | 4476.00 | 2921.00 | 3337.00 | 9216.0 | 1537.0 | 646.10 | 34440.00 | 39910 |
| 2016 | 4029.0 | 2931.0 | 2436.0 | 22420.00 | 6114.0 | 1120.00 | 743000.00 | 98.04 | 1095.0 | 22820.0 | 553.20 | 1983.00 | 7680.0 | 25970.0 | 947.80 | 684.00 | 4340.00 | 2297.00 | 3181.00 | 9414.0 | 1554.0 | 855.70 | 37530.00 | 39440 |
| 2017 | 4362.0 | 3019.0 | 2291.0 | 19080.00 | 6827.0 | 865.90 | 758200.00 | 348.70 | 1194.0 | 21770.0 | 603.70 | 1982.00 | 7711.0 | 26430.0 | 910.60 | 766.00 | 4291.00 | 2531.00 | 3310.00 | 9449.0 | 1484.0 | 740.40 | 39470.00 | 40320 |
| 2018 | 4369.0 | 2942.0 | 2534.0 | 19340.00 | 6915.0 | 782.20 | 786700.00 | 406.10 | 1160.0 | 21000.0 | 601.70 | 1775.00 | 7757.0 | 26180.0 | 930.00 | 730.00 | 4320.00 | 2251.00 | 3505.00 | 9667.0 | 1607.0 | 750.90 | 39410.00 | 38970 |
| 2019 | 4141.0 | 3040.0 | 2819.0 | 19860.00 | 7125.0 | 825.30 | 826900.00 | 451.00 | 1129.0 | 19670.0 | 583.50 | 1425.00 | 7912.0 | 25330.0 | 1011.00 | 722.00 | 4546.00 | 2225.00 | 3828.00 | 9064.0 | 1349.0 | 768.40 | 32350.00 | 40900 |
| 2020 | 3508.0 | 2820.0 | 2634.0 | 22050.00 | 6625.0 | 825.30 | 858200.00 | 451.00 | 1227.0 | 18130.0 | 569.70 | 1425.00 | 7059.0 | 24490.0 | 1011.00 | 725.00 | 4546.00 | 2310.00 | 3901.00 | 8192.0 | 1272.0 | 768.40 | 40810.00 | 40690 |
| 2021 | 4671.0 | 2820.0 | 2634.0 | 23790.00 | 6625.0 | 825.30 | 853000.00 | 451.00 | 1227.0 | 16160.0 | 569.70 | 1425.00 | 7059.0 | 23790.0 | 1011.00 | 701.30 | 4546.00 | 2310.00 | 3901.00 | 8609.0 | 1272.0 | 768.40 | 44390.00 | 41200 |

Table 1: Dataset

Here is some more information about our dataset.

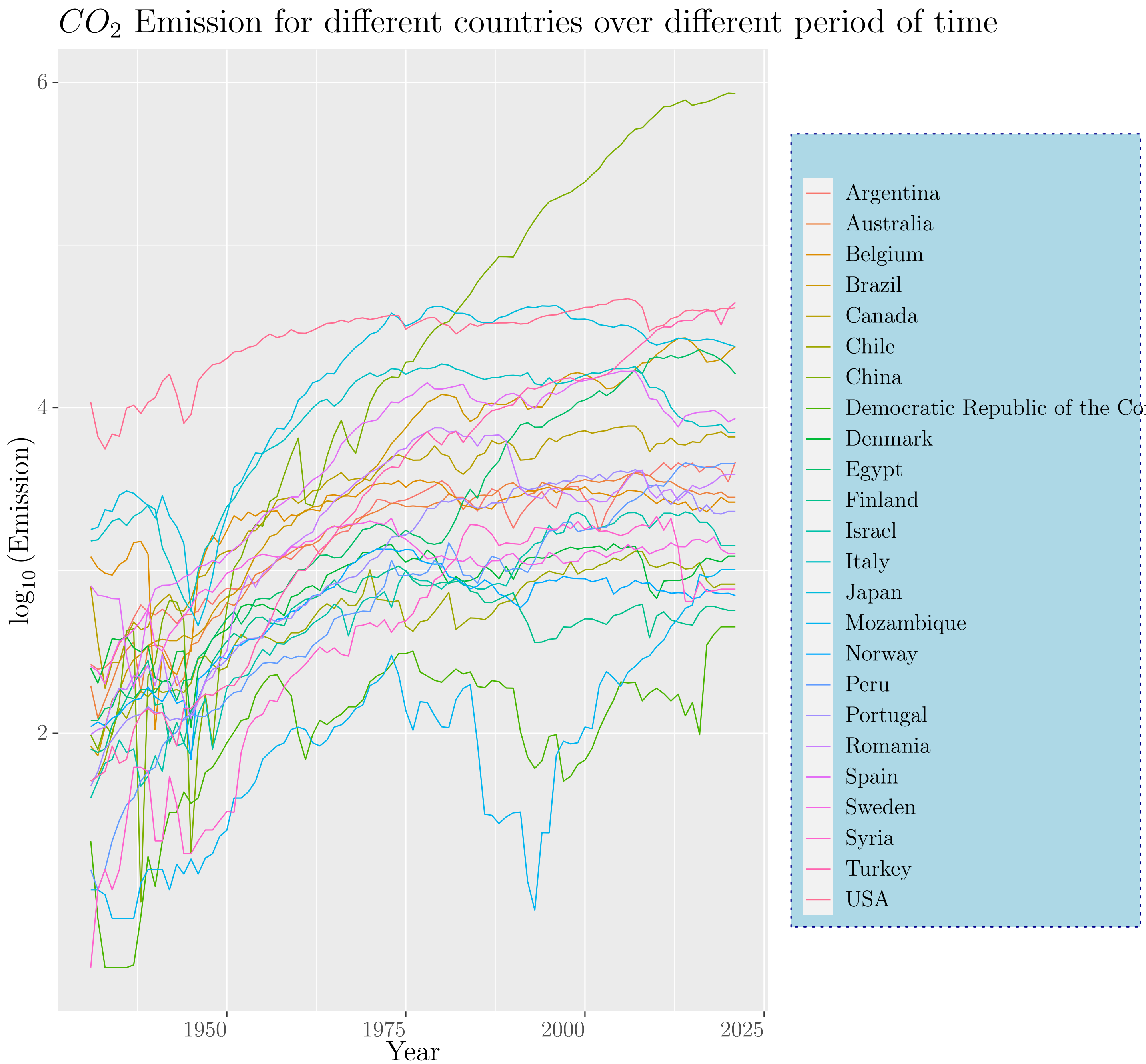
```
## 'data.frame': 91 obs. of  25 variables:
##  $ Year                            : num  1931 1932 1933 1934 1935 ...
##  $ Argentina                       : num  265 247 254 280 356 ...
##  $ Australia                       : num  196 124 160 207 276 ...
##  $ Belgium                         : num  1218 1039 963 938 1087 ...
##  $ Brazil                          : num  83.6 72.7 109 159.9 181.6 ...
##  $ Canada                          : num  800 363 189 273 273 ...
##  $ Chile                           : num  50.9 54.5 69 101.7 141.6 ...
##  $ China                           : num  97.9 79.6 113.2 97.9 156.1 ...
##  $ Democratic Republic of the Congo: num  21.81 7.27 3.63 3.63 3.63 ...
##  $ Denmark                         : num  251 204 273 382 374 ...
##  $ Egypt                           : num  120 120 142 145 189 ...
##  $ Finland                         : num  80 76.2 80 112.7 134.3 ...
##  $ Israel                          : num  40 50.9 65.4 69 90.9 ...
##  $ Italy                           : num  1519 1545 1756 2013 2086 ...
##  $ Japan                           : num  1788 1843 2366 2304 2904 ...
##  $ Mozambique                      : num  10.9 10.9 10.18 7.27 7.27 ...
##  $ Norway                          : num  110 117 111 124 131 ...
##  $ Peru                            : num  14.5 10.9 14.5 21.8 29.1 ...
##  $ Portugal                        : num  47.2 58.1 80 90.9 105.4 ...
##  $ Romania                         : num  98.1 105.4 109 156.3 189 ...
##  $ Spain                           : num  807 705 694 672 669 ...
##  $ Sweden                          : num  258 241 201 287 367 ...
##  $ Syria                           : num  3.63 10.9 14.54 10.9 14.54 ...
##  $ Turkey                          : num  50.8 54.5 58.1 83.6 65.4 ...
##  $ USA                             : num  10810 6656 5587 6900 6684 ...
```

As we can see we only have numeric data. Data are collected for 91 years and across 24 different countries.

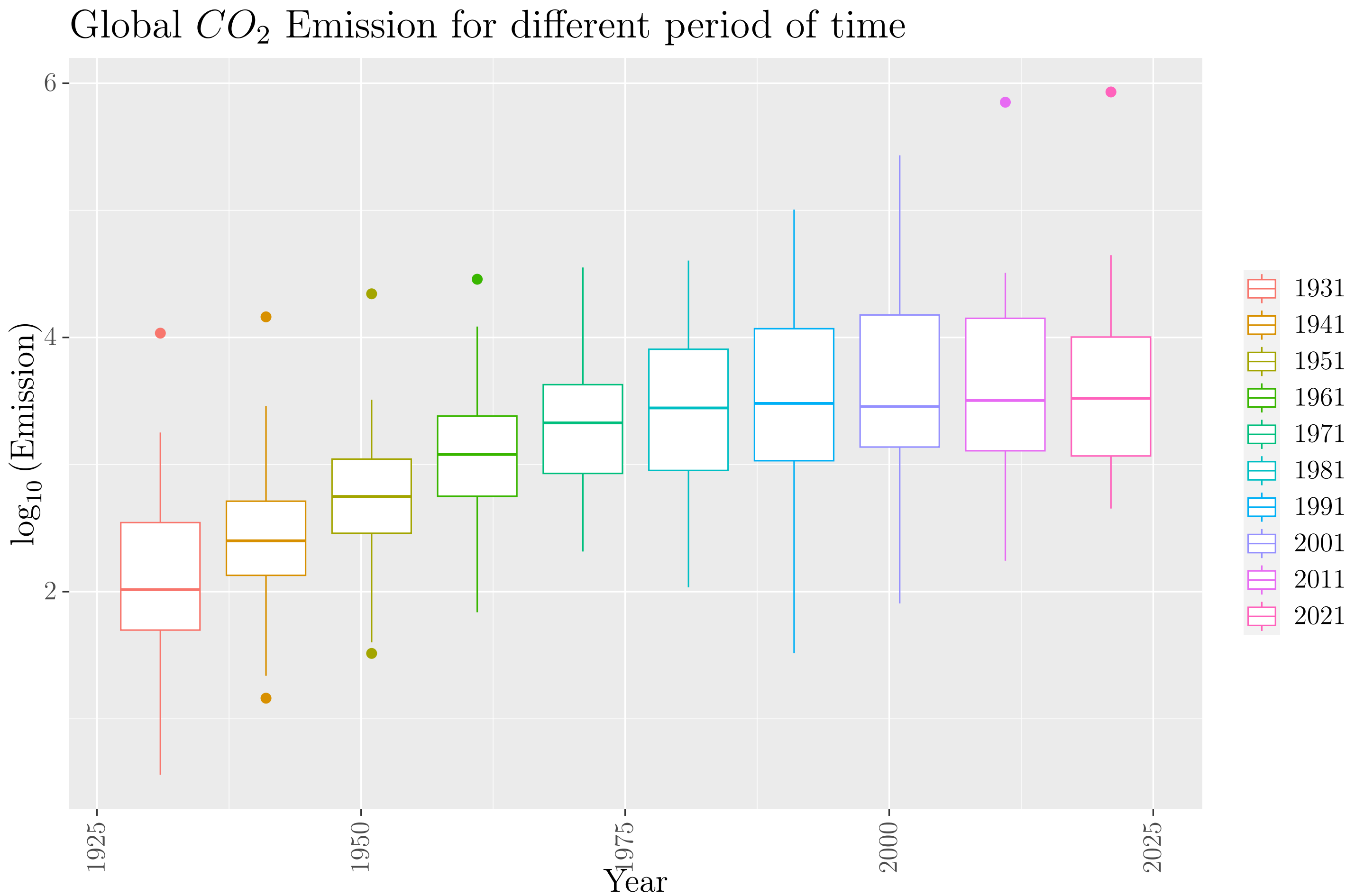
### 0.3.2 Data Visualization

Here are some visualizations of our data.

A glimpse of $CO_2$ emission for different countries over different time stamps.



An idea of distribution of decades-long global $CO_2$ emission.




## 0.4 Sign Test (Test for Location)

The sign test is a statistical method to test for consistent differences between pairs of observations, based on counting statistics. Given pairs of observations, the sign test determines if one member of the pair tends to be greater than (or less than) the other member of the pair. For example, we are testing for the differences of $CO_2$ emissions between two years with lag 10 years, for all the countries and between two countries.

### 0.4.1 Assumptions

The paired observations are denoted as $(X, Y)$. For comparisons of paired observations $(X, Y)$, define $Z_i = X_i - Y_i, \forall\ i = 1, .., n$. The basic assumptions are:

1. For a fixed $p, 0 < p < 1$, the distribution of $Z$ has a common (unknown) quantile, say $\theta$, such that $F_i(\theta) = P(Z_i < \theta) = p, \forall\ i$.

2. $F$ is continuous in the neighborhood of the unknown median of $Z$, say $M$, i.e. $P(Z < M) = P(Z > M) = \frac{1}{2}$, $P(Z = M) = 0$.

### 0.4.2 Test and Method

The null hypothesis is $H_0 : M = 0$ . The alternative hypothesis may be one sided or two sided. The null hypothesis states that given a random pair of measurements $(x_i, y_i)$, $x_i$ and $y_i$ are equally likely to be larger than the other.

- To test the null hypothesis, pairs of sample data $(x_i, y_i)$ are collected. Then calculate $z_i = x_i - y_i, \forall\, i$.
- Consider the ordered arrangement. Let $K$ be the number of observations for which $z_i > 0$, then under $H_0$, $K \sim Bin(n, \frac{1}{2})$.
- While performing exact test at level of significance $\alpha$, for the alternative hypothesis $H_1 : M > 0$, $\omega : K_{obs.} \geq k_\alpha$, where $k_\alpha$ is the minimum integer $\ni P_{H_0}(K \geq k_\alpha) \leq \alpha$.
- For the alternative hypothesis $H_2 : M < 0$, $\omega : K_{obs.} \leq k'_\alpha$, where $k'_\alpha$ is the maximum integer $\ni P_{H_0}(K \leq k'_\alpha) \leq \alpha$.
- For the alternative hypothesis $H_3 : M \neq 0$, $\omega : K_{obs.} \leq k_{\alpha/2}$ or $K_{obs.} \geq k'_{\alpha/2}$ , where $k_{\alpha/2}$ is the maximum integer $\ni P_{H_0}(K \leq k_{\alpha/2}) \leq \frac{\alpha}{2}$ and $k'_{\alpha/2}$ is the minimum integer $\ni P_{H_0}(K \geq k'_{\alpha/2}) \leq \frac{\alpha}{2}$.
- One can also use the randomized test to achieve the exact level of significance.
- Under $H_0$, $E\,(K) = \frac{n}{2}$ and $Var\,(K) = \frac{n}{4}$.
- The normal approximation to the binomial distribution can be used for asymptotic tests using the null moments and finite population correction, which is fine for $n \geq 12$.

### 0.4.3 Findings

**For decades-long global $CO_2$ emission:**

We first conduct both sided test for year 1931 and 1941.

```
##
##  One-sample Sign-Test
##
## data:  z
## S = 4, number of differences = 23, p-value = 0.002599
## alternative hypothesis: true median is not equal to 0
## 97.7 percent confidence interval:
##  -145.30   -3.64
## sample estimates:
```

```
## median of the differences
##                    -70.85
```

As we can see that *p-value* is less than 0.05, we reject the null hypothesis that $CO_2$ emission doesn't differ, at 5% level of significance.

Moreover, we are interested to know whether $CO_2$ emission for year 1931 is less than the $CO_2$ emission for year 1941.

```
##
##  One-sample Sign-Test
##
## data:  z
## S = 4, number of differences = 23, p-value = 0.0013
## alternative hypothesis: true median is less than 0
## 96.8 percent confidence interval:
##    -Inf -18.18
## sample estimates:
## median of the differences
##                    -70.85
```

Here, *p-value* is less than 0.05 and we reject the null hypothesis that $CO_2$ emission doesn't differ, at 5% level of significance. Thus we can conclude that there is a significant increase in $CO_2$ emission.

The table below shows the decisions for all possible pairs. Note that we have performed exact test here.

| | Decision made |
|---|---|
| 1931 vs 1941 | There is significant increase in $CO_2$ emission |
| 1941 vs 1951 | There is significant increase in $CO_2$ emission |
| 1951 vs 1961 | There is significant increase in $CO_2$ emission |
| 1961 vs 1971 | There is significant increase in $CO_2$ emission |
| 1971 vs 1981 | There is no significant change in $CO_2$ emission |
| 1981 vs 1991 | There is no significant change in $CO_2$ emission |
| 1991 vs 2001 | There is significant increase in $CO_2$ emission |
| 2001 vs 2011 | There is no significant change in $CO_2$ emission |
| 2011 vs 2021 | There is no significant change in $CO_2$ emission |

Table 2: Decision using Sign test

**For Countries:**

We first conduct both sided test for *Argentina* and *Australia*.

```
##
##  One-sample Sign-Test
##
## data:  z
## S = 55, number of differences = 91, p-value = 0.05857
## alternative hypothesis: true median is not equal to 0
## 96.5 percent confidence interval:
##   -4.0 148.9
## sample estimates:
## median of the differences
##                      87.2
```

As we can see that *p-value* is greater than 0.05 we fail to reject the null hypothesis that $CO_2$ emission doesn't differ, at 5% level of significance.

The table below shows the decisions for first ten possible pairs. Note that we have performed asymptotic test here.

| | Decision made |
|---|---|
| Argentina vs Australia | There is no significant change in $CO_2$ emission |
| Argentina vs Belgium | There is significant increase in $CO_2$ emission |
| Argentina vs Brazil | There is significant increase in $CO_2$ emission |
| Argentina vs Canada | There is significant increase in $CO_2$ emission |
| Argentina vs Chile | There is significant decrease in $CO_2$ emission |
| Argentina vs China | There is significant increase in $CO_2$ emission |
| Argentina vs Democratic Republic of the Congo | There is significant decrease in $CO_2$ emission |
| Argentina vs Denmark | There is significant decrease in $CO_2$ emission |
| Argentina vs Egypt | There is no significant change in $CO_2$ emission |
| Argentina vs Finland | There is significant decrease in $CO_2$ emission |

Table 3: Decision from sign test

## 0.5 Wilcoxon Rank-Sum (Mann-Whitney U) Test

The Wilcoxon Rank-Sum or Mann-Whitney U test is perhaps the most common non-parametric test (for location) for unrelated samples. We use it when the two groups are independent of each other, for example, in our dataset, testing for differences in $CO_2$ emissions between two different years (e.g. 1941 and 1951). It can be used even when the two groups are of different sizes.

### 0.5.1 Method

- First, consider the ordered arrangement of the combined sample of $m$ $X'$s and $n$ $Y'$s. Equal values are allocated the average of the ranks they would have if there was tiny differences between them.
- Next we sum the ranks for $X$ observations and denote it by $W$. Then the minimum value of $W$ is $\frac{m(m+1)}{2}$ and the maximum value of $W$ is $\frac{m(m+1)}{2}+mn$. The distribution of $W$ is symmetric under $H_0$.
- For exact test, at the level of significance $\alpha$, the null hypothesis is $H_0 : F_Y(x) = F_X(x)$. When the alternative hypothesis is $H_1 : F_Y(x) \leq F_X(x)$ , then $\omega : W_{obs.} \leq w_\alpha$, where $w_\alpha$ is the maximum integer $\ni P_{H_0}(W \leq w_\alpha) \leq \alpha$.
- For the alternative hypothesis is $H_2 : F_Y(x) \geq F_X(x)$ , then $\omega : W_{obs.} \geq w'_\alpha$, where $w'_\alpha$ is the minimum integer $\ni P_{H_0}(W \geq w'_\alpha) \leq \alpha$.
- For the alternative hypothesis is $H_3 : F_Y(x) \neq F_X(x)$, then $\omega : W_{obs.} \leq w_{\alpha/2}$ or $W_{obs.} \geq w'_{\alpha/2}$ , where $w_{\alpha/2}$ is the maximum integer $\ni P_{H_0}(W \leq w_{\alpha/2}) \leq \frac{\alpha}{2}$ and $w'_{\alpha/2}$ is the minimum integer $\ni P_{H_0}(W \geq w'_{\alpha/2}) \leq \frac{\alpha}{2}$.
- Under $H_0$, $E\,(W) = \frac{m(m+n+1)}{2}$ and $Var\,(W) = \frac{mn(m+n+1)}{12}$.
- Moreover, one can use the normality assumption of the distribution of $W$ for asymptotic test using null moments and f.p.c.
- We conclude based on the cut off point and observed test statistic.

### 0.5.2 Findings

In this example we did a two-tailed test to measure if there is a difference in emission between all the pairs of the countries and the decades. Our null hypothesis $H_0$ is that there is no difference in the location.

**For decades-long global $CO_2$ emission:**

Here we have shown the output for the years 1931 and 1941.

```
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  x and y
## V = 55, p-value = 0.0121
## alternative hypothesis: true location shift is not equal to 0
## 95 percent confidence interval:
##  -198.18501  -21.84994
## sample estimates:
## (pseudo)median
##      -81.73654
```

As the *p-value* is less than 0.05, we reject $H_0$ at 5% level of significance.

Then we performed one-sided test to check which is larger. The alternative hypothesis for the one sided test is $CO_2$ emission for the year 1931 is less than that of 1941.

```
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  x and y
## V = 55, p-value = 0.00605
## alternative hypothesis: true location shift is less than 0
## 95 percent confidence interval:
##       -Inf -36.34491
## sample estimates:
## (pseudo)median
##      -81.73654
```

The *p-value* is less than 0.05, so we reject $H_0$ at 5% level of significance and conclude that in 1941, $CO_2$ emission was higher than 1931.

The table below shows the results for all decades.

| | Decision made |
|---|---|
| 1931 vs 1941 | There is significant increase in $CO_2$ emission |
| 1941 vs 1951 | There is significant increase in $CO_2$ emission |
| 1951 vs 1961 | There is significant increase in $CO_2$ emission |
| 1961 vs 1971 | There is significant increase in $CO_2$ emission |
| 1971 vs 1981 | There is significant increase in $CO_2$ emission |
| 1981 vs 1991 | There is no significant change in $CO_2$ emission |
| 1991 vs 2001 | There is significant increase in $CO_2$ emission |
| 2001 vs 2011 | There is no significant change in $CO_2$ emission |
| 2011 vs 2021 | There is no significant change in $CO_2$ emission |

Table 4: Decision values using Wilcoxon Rank-Sum test

**For Countries:**

Here we give a snapshot for *Australia* and *Argentina*.

```
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  x and y
## V = 2583, p-value = 0.0527
```

```
## alternative hypothesis: true location shift is not equal to 0
## 95 percent confidence interval:
##   -0.4999682 171.0000579
## sample estimates:
## (pseudo)median
##       83.60025
```

The *p-value* for the both sided test is greater than 0.05. So we accept $H_0$ that there is indeed no significant difference between the $CO_2$ emission of Australia and Argentina, at 5% level of significance.

The below table shows the result for some pairs of countries.

| | Decision made |
|---|---|
| Argentina vs Australia | There is no significant change in $CO_2$ emission |
| Argentina vs Belgium | There is significant increase in $CO_2$ emission |
| Argentina vs Brazil | There is significant increase in $CO_2$ emission |
| Argentina vs Canada | There is significant increase in $CO_2$ emission |
| Argentina vs Chile | There is significant decrease in $CO_2$ emission |
| Argentina vs China | There is significant increase in $CO_2$ emission |
| Argentina vs Democratic Republic of the Congo | There is significant decrease in $CO_2$ emission |
| Argentina vs Denmark | There is significant decrease in $CO_2$ emission |
| Argentina vs Egypt | There is significant increase in $CO_2$ emission |
| Argentina vs Finland | There is significant decrease in $CO_2$ emission |

Table 5: Decision values using Wilcoxon Rank-Sum test

## 0.6 Mood's Test

The Mood's Test is a useful test to detect relationship between the dispersion of 2 different samples using a linear rank statistic. This considers the ranks of observations in the combined ordering of the 2 samples. Before applying this test, we assume the 2 samples are from the populations having same location. The sample observations can be adjusted to have equal location if they are not specified, to have equal location parameter.

### 0.6.1 Method

- First, consider the ordered arrangement of the combined sample of $m$ $X'$s and $n$ $Y'$s. Let, $m+n = N$.
- Next we calculate the test statistic $M = \sum_{i=1}^{N}\left(i - \frac{N+1}{2}\right)^2 Z_i$, where $Z_i = 1$, if the $i$-th observation of the ordered combined sample is an $X$ observation and $Z_i = 0$,otherwise.

- The test statistic will have a low value when $X$ has lower variability and have a high value when $X$ has higher variability than $Y$ and the rejection region is determined accordingly.

- For exact test, at the level of significance $\alpha$, the null hypothesis is $H_0 : F_Y(x)$ and $F_X(x)$ have equal dispersion. When the alternative hypothesis is $H_1 : X$ has lower variability than $Y$, then $\omega : M_{obs.} \leq m_\alpha$, where $m_\alpha$ is the maximum value $\ni P_{H_0}(M \leq m_\alpha) \leq \alpha$.

- For the alternative hypothesis is $H_2 : X$ has higher variability than $Y$ , then $\omega : M_{obs.} \geq m'_\alpha$, where $m'_\alpha$ is the minimum value $\ni P_{H_0}(M \geq m'_\alpha) \leq \alpha$.

- For the alternative hypothesis is $H_3 : X$ and $Y$ have unequal variability, then $\omega : M_{obs.} \leq m_{\alpha/2}$ or $M_{obs.} \geq m'_{\alpha/2}$ , where $m_{\alpha/2}$ is the maximum value $\ni P_{H_0}(M \leq m_{\alpha/2}) \leq \frac{\alpha}{2}$ and $m'_{\alpha/2}$ is the minimum value $\ni P_{H_0}(M \geq m'_{\alpha/2}) \leq \frac{\alpha}{2}$.

- Under $H_0$, $E(M) = \frac{m(N^2-1)}{12}$ and $Var(M) = \frac{mn(N+1)(N^2-4)}{180}$.

- Here also, we can use the normality assumption on the distribution of $M$ for asymptotic test.

### 0.6.2 Findings

Here we have tested if the data from 2 different countries or 2 different years, have equal dispersion or not. For that we have tested for those pairs of years (or countries) for which the null hypothesis of Wilcoxon's Rank Sum test is accepted, i.e. they do not have significant difference in location.

**For decades-long global $CO_2$ emission:**

Only for three pairs (1981, 1991), (2001, 2011) and (2011, 2021) we have performed Mood's Test. The result for the years 1981 and 1991 are shown below:

```
##
##  Mood two-sample test of scale
##
## data:  x and y
## Z = -0.36967, p-value = 0.7116
## alternative hypothesis: two.sided
```

The *p-value* is much larger than 0.05 and hence we accept the null hypothesis at 5% level of significance and conclude that there is no significant difference between the variability of the year 1981 and 1991.

The results for all 3 pairs are presented here.

| | Decision made |
|---|---|
| 1981 vs 1991 | There is no significant change in $CO_2$ emission |
| 2001 vs 2011 | There is no significant change in $CO_2$ emission |
| 2011 vs 2021 | There is no significant change in $CO_2$ emission |

Table 6: Decisions using Mood's test

**For Countries:**

From the results of Wilcoxon Sign Rank, 15 pairs of countries have no significant difference in $CO_2$ emission like, (Australia, Argentina), (Belgium, Romania) etc.

We have shown the result for Australia and Argentina.

```
##
##  Mood two-sample test of scale
##
## data:  x and y
## Z = -0.16643, p-value = 0.8678
## alternative hypothesis: two.sided
```

As the *p-value* is much larger than 0.05, so we conclude that there is no significant difference in variability of Australia and Argentina.

Also the results for 15 pairs are presented,

| | Decision made |
|---|---|
| Argentina vs Australia | There is no significant change in $CO_2$ emission |
| Belgium vs Romania | There is significant increase in $CO_2$ emission |
| Brazil vs Turkey | There is significant increase in $CO_2$ emission |
| Canada vs Egypt | There is significant increase in $CO_2$ emission |
| Chile vs Finland | There is significant decrease in $CO_2$ emission |
| Chile vs Norway | There is no significant change in $CO_2$ emission |
| Democratic Republic of the Congo vs Mozambique | There is significant increase in $CO_2$ emission |
| Denmark vs Israel | There is significant increase in $CO_2$ emission |
| Denmark vs Peru | There is significant increase in $CO_2$ emission |
| Denmark vs Syria | There is significant increase in $CO_2$ emission |
| Egypt vs Romania | There is significant decrease in $CO_2$ emission |
| Israel vs Norway | There is significant decrease in $CO_2$ emission |
| Norway vs Syria | There is significant increase in $CO_2$ emission |
| Peru vs Sweden | There is significant decrease in $CO_2$ emission |
| Spain vs Turkey | There is significant increase in $CO_2$ emission |

Table 7: Decisions using Mood's test

## 0.7 Kruskal-Wallis Test

Kruskal-Wallis test by rank (Kruskal–Wallis H test) is a non-parametric alternative to one-way ANOVA test, which extends the two-samples Wilcoxon test in the situation where there are more than two groups. It's recommended when the assumptions of one-way ANOVA test are not met.

A significant Kruskal–Wallis test indicates that at least one sample stochastically dominates one other sample. The test does not identify where this stochastic dominance occurs or for how many pairs of groups stochastic dominance obtains.

Since it is a nonparametric method, the Kruskal–Wallis test does not assume a normal distribution of the residuals, unlike the analogous one-way analysis of variance. If the researcher can make the assumptions of an identically shaped and scaled distribution for all groups, except for any difference in medians, then the null hypothesis is that the median of all groups are equal, and the alternative hypothesis is that the population median of at-least one group is different from the population median of at-least one other group. Otherwise, it is impossible to say, whether the rejection of the null hypothesis comes from the shift in locations or group dispersions.

### 0.7.1 Method

- Rank all data from all groups together; i.e., rank the data from 1 to $N$ ignoring group membership. Assign any tied values the average of the ranks they would have received had they not been tied.

- The test statistic is given by

$$H = \frac{12}{N(N-1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)$$

  where $N$ is the total sample size, $k$ is the number of groups we are comparing, $R_i$ is the sum of ranks for group $i$, and $n_i$ is the sample size of group $i$ .

- The decision to reject or not the null hypothesis at the level of significance $\alpha$ is made by comparing $H$ to a critical value $H_\alpha$ obtained from a table or a software. So, $\omega : H_{obs.} \geq H_\alpha$, where $H_\alpha$is the minimum value $\ni P_{H_0}(H \geq H_\alpha) \leq \alpha$.

- It can be shown that, $H \sim \chi^2_{k-1}$, asymptotically, which works fine for $n_i \geq 5,\ k \geq 3$. Then $\omega : H_{obs.} \geq \chi^2_{1-\alpha,k-1}$, where $P(\chi^2_{k-1} \geq \chi^2_{1-\alpha,k-1}) = \alpha$.

- If the statistic is not significant, then there is no evidence of stochastic dominance between the samples. However, if the test is significant then at least one sample stochastically dominates another sample.

### 0.7.2 Findings

First we found the sample standard deviation. Then we have performed Kruskal-Wallis test for those group (size $> 2$), for which the standard deviations are close enough. The countries Argentina, Peru, Portugal have close standard deviation .

| | Standard Deviation |
|---|---|
| Argentina | 1327.8789 |
| Australia | 1245.9507 |
| Belgium | 885.7870 |
| Brazil | 7882.2382 |
| Canada | 2393.8287 |
| Chile | 343.2244 |
| China | 266589.4357 |
| Democratic Republic of the Congo | 104.2538 |
| Denmark | 392.7684 |
| Egypt | 7270.9458 |
| Finland | 251.4007 |
| Israel | 762.6959 |
| Italy | 6102.5108 |
| Japan | 14658.8081 |
| Mozambique | 246.2908 |
| Norway | 352.0404 |
| Peru | 1365.9741 |
| Portugal | 1368.9408 |
| Romania | 2203.5197 |
| Spain | 5345.5497 |
| Sweden | 448.3529 |
| Syria | 736.1023 |
| Turkey | 12109.3962 |
| USA | 10951.2528 |

For decades we haven't found any set of interest as the standard deviations are not that much close.

| | Standard Deviation |
|---|---|
| 1931 | 2194.929 |
| 1941 | 2939.747 |
| 1951 | 4412.352 |
| 1961 | 6129.387 |
| 1971 | 9117.920 |
| 1981 | 11615.976 |
| 1991 | 21798.270 |
| 2001 | 54726.254 |
| 2011 | 143328.878 |
| 2021 | 172785.052 |

The *p-value* of this test for these 3 countries is much smaller than 0.05, so we reject $H_0$ at 5% level of significance and conclude that there is significant difference in $CO_2$ emission between Argentina, Peru, Portugal.

```
##
##  Kruskal-Wallis rank sum test
##
## data:  dat_gather_2$Emission and as.factor(dat_gather_2$Country)
## Kruskal-Wallis chi-squared = 23.74, df = 2, p-value = 6.997e-06
```

## 0.8 Test for Independence (using Kendall Rank Correlation Coefficient)

The Kendall rank correlation coefficient, commonly referred to as Kendall's $\tau$ coefficient, is a statistic used to measure the ordinal association between two measured quantities. The Kendall's $\tau$ test is a non-parametric hypothesis test for statistical dependence based on the $\tau$ coefficient. Intuitively, Kendall coefficient between two variable will be high, when observations between the two variables have a similar rank and low, when observations have different rank between the two variables.

### 0.8.1 Method

Let $(X_1, Y_1), (X_2, Y_2), ..., (X_n, Y_n)$ be a set of observations from the joint distribution function $F_{X,Y}$, where we test for the null hypothesis $H_0 : F_{X,Y}(x, y) = F_X(x)F_Y(y)$ against $H_a$ : Not $H_0$. A pair of observations $(X_i, Y_i)$ and $(X_j, Y_j)$, where $i < j$, are said to be **Concordant**, if the sort order of $(X_i, X_j)$ and $(Y_i, Y_j)$ agrees, that is if either both $X_i > X_j$ and $Y_i > Y_j$ or $X_i < X_j$ and $Y_i < Y_j$, otherwise they are said to be **Discordant**.

Let, $\pi_c$ be the probability of concordance and $\pi_d$ be the probability of discordance. Then **Kendall's** $\tau$ is defined as; $\tau = \pi_c - \pi_d$. For a sample of size $N$, the sample estimate of $\tau$ is defined as:

$$T = \frac{\sum\sum_{1\leq i<j\leq N} A_{ij}}{\binom{N}{2}},$$

where $A_{ij} = sign(X_j - X_i)\ sign(Y_j - Y_i)$ which to equivalent to saying

$$A_{ij} = \begin{cases} 1 & \text{, if pairs are concordant with probability } \pi_c \\ 0 & \text{, if atleast one component is tied with probability } \pi_t\text{=1-}\pi_c\text{-}\pi_d \\ -1 & \text{, if pairs are discordant with probability } \pi_d \end{cases}$$

- $A_{ij}$ is an unbiased estimator of $\tau$ and $T$ is in the range $[-1, 1]$.

- If the agreement between two rankings is perfect (two rankings are the same), the coefficient has value 1.
- If the disagreement between two rankings is perfect (one ranking is the reverse of the other), the coefficient has value $-1$.
- At the level of significance $\alpha$, for the alternative hypothesis $H_a : X$ and $Y$ have positive association, $\omega : T \geq T_\alpha$, where $T_\alpha$ is the minimum value $\ni P_{H_0}(T \geq T_\alpha) \leq \alpha$.
- For the alternative hypothesis $H_a : X$ and $Y$ have negative association, $\omega : T \leq T_\alpha$, where $T_\alpha$ is the maximum value $\ni P_{H_0}(T \leq T_\alpha) \leq \alpha$.
- For the alternative hypothesis $H_a : X$ and $Y$ have some association, $\omega : T \leq T_{\alpha/2}$ or $T \geq T'_{\alpha/2}$, where $T_{\alpha/2}$ is the maximum value $\ni P_{H_0}(T \leq T_{\alpha/2}) \leq \alpha/2$ and $T'_{\alpha/2}$ is minimum value $\ni P_{H_0}(T \geq T'_{\alpha/2}) \leq \alpha/2$.
- Under $H_0$, $E(T) = 0, Var(T) = \frac{2(2N+5)}{9N(N-1)}$. Using the null moments and normality assumption on the distribution of T, we can apply the asymptotic test for large $N$.

In case of tie in $X$ or $Y$ or across population, the null distribution of $T$ and exact test procedure does not change. If there are too many ties, then one can use randomization.

### 0.8.2 Findings

**For Countries:**

Here we give a snapshot for *Australia* and *Argentina*.

```
##
##  Kendall's rank correlation tau
##
## data:  x and y
## z = 9.661, p-value < 2.2e-16
## alternative hypothesis: true tau is not equal to 0
## sample estimates:
##       tau
## 0.6884085
##
##  Kendall's rank correlation tau
##
## data:  x and y
## z = 9.661, p-value < 2.2e-16
## alternative hypothesis: true tau is greater than 0
## sample estimates:
```

```
##       tau
## 0.6884085
```

The *p-value* for the both sided test is less than 0.05. So we can reject $H_0$ and also for greater than test *p-value* is less than 0.05 which implies $H_0$ is rejected also and thus we can conclude that there is indeed positive association between the $CO_2$ emission of Australia and Argentina, at 5% level of significance.

The below table shows the result for some pairs of countries.

| | Decision made |
|---|---|
| Argentina vs Australia | There is positive association |
| Argentina vs Belgium | There is positive association |
| Argentina vs Brazil | There is positive association |
| Argentina vs Canada | There is positive association |
| Argentina vs Chile | There is positive association |
| Argentina vs China | There is positive association |
| Argentina vs Democratic Republic of the Congo | There is positive association |
| Argentina vs Denmark | There is positive association |
| Argentina vs Egypt | There is positive association |
| Argentina vs Finland | There is positive association |

Table 8: Decisions from Kendall Rank Correlation Coefficient

Notice that there is only positive association present in all the pairs.

```
## Decision made
## There is positive association
##                           276
```

Thus, we can conclude that $CO_2$ emission is moving in same direction for all the countries.

## 0.9 Conclusion

We have done a non-parametric study on the $CO_2$ emissions with respect to the nine decades (1931 - 2021) as well as with respect to the 24 countries.

With respect to the nine decades,

The Sign test and Wilcoxon Rank-Sum test agrees for all pairs of decades except 1971 and 1981. As the latter test takes into consideration both the rank and sign of the observations, we conclude based on the results given by the Wilcoxon Rank-Sum test. We conclude that, over the nine decades, the global $CO_2$ emission from cement production has increased from 1931 to 2010, with not a significant change during 1981-2000, and then remained same 2011 onwards. We have applied the Mood's test on the consecutive decades which do not show a significant change in the emission, i.e., 1981 vs 1991, 2001 vs 2011 and 2011 vs 2021. In all these years, the variability of $CO_2$ emission is approximately same.

With respect to countries,

The Wilcoxon Rank-Sum test depicts that there are several pairs with significant increase, decrease or no change in $CO_2$ emission. As we have large number (276) of pair of countries, we do not have any definite interpretation of the result. From the Mood's test, we culminate that the emission from Argentina and Australia as well as Chile and Norway are same in terms of location as well as scale, while the other countries vary. The variance of emission from Argentina, Peru and Portugal are nearly same and hence the Kruskal-Wallis test shows that there is significant difference in the amount of $CO_2$ emissions from these 3 countries. Moreover, we deduce that there is positive association between the emission of all pairs of countries from Kendall rank correlation, which is quite obvious as the $CO_2$ emission of each country has increased over the years due to increase in cement production.