OPTIMIZATION:
(CONSTRAINT and INTEGER LINEAR PROGRAMMING)

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Executive Summary

Signal Communication done via telecommunication satellite consists of a set of reception channels and emitter channels. The satellite transmits signals using emitter channels and receives signals using reception channels. Amplifiers are introduced into this design to increase the intensity of the signals. This basic design of communication must take care of the constraints:

1. Each signal must be assigned to only one reception and emission channel
2. Each reception channel is connected to an amplifier, this path is known as RApath
3. Each emission channel is connected to an amplifier, this path is known as EApath
4. Any two paths namely, RApaths or AEpaths are incompatible if they have the same source or destination respectively or they have common equipment connected
5. The Goal is to assign each signal a RApathand AEpath such that they are connected to the same amplifier

Goal: Minimize the length of all the connections established (RApath + AEpath)

In order to solve such problems optimization methodologies are used, such as Constraint Programming and Integer Linear Programming.

	IP	CP
Variables	0-1	Finite domain
Constraints	Linear equations and inequalities	Arithmetic constraints Symbolic/global constraints

- Constraint Programming (CP) - methodology encourages formulating both 'mathematical model' and the 'desired solution' technique

- Linear Programming (LP) - is the class of problems that can be expressed as the optimization of a linear objective function subject to a set of linear constraints (i.e., linear equations and inequalities) over real numbers, i.e. continuous variables. IP is the name given to LP problems which have the additional constraint that some or all the variables have to be integer

A mathematical model is a simplified representation ofthe problem that captures the essential mathematical relationships between various components ofthe system. Finally, afeasible solution is a set ofvalues for the model variables that satisfies all the constraints.

For the above stated problem a feasible solution will present below for both Constraint and Integer Linear Programming.

1 Constraint Programming

Problem Description 1: Complete the file 'telecom_cp_squelette.mod' to get the modeling of the problem in constraint programming framework and solve instances sat0.dat –to– sat5.dat. Additionally improve the computation time for instances by adding constrains in how the search is being pursued.

The mathematical formulation for the stated problem using constraint programming is as given below:

```
/*** CRITERION ***/

minimizesum(iinSig) (RAlength[rapath[i]] +AElength[aepath[i]]);

/*** CONSTRAINTS ***/

constraints{

/*** for a signal i, the destination of the path used by signal i-rapath
should be the same as the source of the path used by signal i-aepath***/

forall(iinSig)RAdst[rapath[i]] ==AEsrc[aepath[i]];

/*** We have told that i cannot be = j and then said that
use all values in the matrix where the result is 1 ***/

forall(i,jinSig:i<j) {
       RAcomp[rapath[i],rapath[j]] ==1;
       AEcomp[aepath[i],aepath[j]] ==1;
  }

/*** for a signal i, (currently we can have paths calculated in different orders)
to minimize computation time, we impose an order on the source and dst paths ***/

forall(iinSig:i<Ns)rapath[i] <rapath[i+1];
}
```

1. The Goal of the problem is to minimize the length of all the connections established (RApath + AEpath). This is done by adding the paths assigned for each signal
   ```
   minimizesum(iinSig) (RAlength[rapath[i]] +AElength[aepath[i]]);
   ```
2. To assign each signal a RApath and AEpath such that they are connected to the same amplifier
   ```
   forall(iinSig)RAdst[rapath[i]] ==AEsrc[aepath[i]];
   ```
3. Any two RApaths or two AEpaths are incompatible if they have the same source or destination respectively or they have common equipment connected. (Note: the compatibility Matrix is given)
   ```
   forall(i,jinSig:i<j) {
          RAcomp[rapath[i],rapath[j]] ==1;
          AEcomp[aepath[i],aepath[j]] ==1;
   }
   ```
4. Further the computation time isimproved by adding constrains in how the search is being pursued – Symmetry Breaking strategy.
   ```
   forall(iinSig:i<Ns)rapath[i] <rapath[i+1];
   ```

The above model is run for different data files.

Note: SAT-3.dat and SAT-5.dat files have no solution, the code run for 12, 40 hours straight and no feasible solution was found by the model.

SAT-0.dat

Execution time without Symmetry Breaking:

```
// solution with objective 6
rapath = [3 5 1];
aepath = [4 6 1];


! ----------------------------------------------------------------------------
! Minimization problem - 6 variables, 9 constraints
 ! Initial process time : 0.00s (0.00s extraction + 0.00s propagation)
 !  . Log search space  : 16.2 (before), 16.2 (after)
 !  . Memory usage      : 598.2 kB (before), 598.2 kB (after)
 ! Using parallel search with 8 workers.
 ! ----------------------------------------------------------------------------
 !          Best Branches  Non-fixed    W       Branch decision
 *             6        6 0.01s         1             -
 ! ----------------------------------------------------------------------------
 ! Search terminated normally, 1 solution found.
 ! Best objective         : 6 (optimal - effective tol. is 0)
 ! Number of branches     : 122
 ! Number of fails        : 66
 ! Total memory usage     : 5.7 MB (5.3 MB CP Optimizer + 0.4 MB Concert)
! Time spent in solve    : 0.02s (0.02s engine + 0.00s extraction)
 ! Search speed (br. / s) : 6,100.0
 ! ----------------------------------------------------------------------------
```

Execution time with Symmetry Breaking:

```
// solution with objective 6
rapath = [1 3 5];
aepath = [1 4 6];


! ----------------------------------------------------------------------------
! Minimization problem - 6 variables, 11 constraints
 ! Initial process time : 0.00s (0.00s extraction + 0.00s propagation)
 !  . Log search space  : 12.8 (before), 12.8 (after)
 !  . Memory usage      : 598.3 kB (before), 598.3 kB (after)
 ! Using parallel search with 8 workers.
 ! ----------------------------------------------------------------------------
 !          Best Branches  Non-fixed    W       Branch decision
 *             6        5 0.01s         1             -
 ! ----------------------------------------------------------------------------
 ! Search terminated normally, 1 solution found.
 ! Best objective         : 6 (optimal - effective tol. is 0)
 ! Number of branches     : 118
 ! Number of fails        : 71
 ! Total memory usage     : 5.7 MB (5.2 MB CP Optimizer + 0.4 MB Concert)
 ! Time spent in solve    : 0.01s (0.01s engine + 0.00s extraction)
 ! Search speed (br. / s) : 11,800.0
 ! ----------------------------------------------------------------------------
```

SAT-1.dat

Execution time without Symmetry Breaking:

```
// solution with objective 36
rapath = [6 5 1];
aepath = [5 3 4];


! ----------------------------------------------------------------------------
! Minimization problem - 6 variables, 9 constraints
 ! Initial process time : 0.00s (0.00s extraction + 0.00s propagation)
 !  . Log search space  : 16.2 (before), 16.2 (after)
 !  . Memory usage      : 598.2 kB (before), 598.2 kB (after)
 ! Using parallel search with 8 workers.
 ! ----------------------------------------------------------------------------
 !          Best Branches  Non-fixed    W       Branch decision
                    1,000          6    3             -
 *            36         2 0.07s        1             -
 ! ----------------------------------------------------------------------------
 ! Search terminated normally, 1 solution found.
 ! Best objective         : 36 (optimal - effective tol. is 0)
 ! Number of branches     : 12,233
 ! Number of fails        : 6,549
 ! Total memory usage     : 6.9 MB (6.4 MB CP Optimizer + 0.4 MB Concert)
! Time spent in solve    : 0.09s (0.08s engine + 0.01s extraction)
 ! Search speed (br. / s) : 152,912.5
 ! ----------------------------------------------------------------------------
```

Execution time with Symmetry Breaking:

```
// solution with objective 36
rapath = [1 5 6];
aepath = [4 3 5];

! ----------------------------------------------------------------------------
! Minimization problem - 6 variables, 11 constraints
 ! Initial process time : 0.00s (0.00s extraction + 0.00s propagation)
 !  . Log search space  : 13.5 (before), 12.8 (after)
 !  . Memory usage      : 598.3 kB (before), 598.3 kB (after)
 ! Using parallel search with 8 workers.
 ! ----------------------------------------------------------------------------
 !          Best Branches  Non-fixed    W       Branch decision
 *            36         7 0.05s        1             -
 ! ----------------------------------------------------------------------------
 ! Search terminated normally, 1 solution found.
 ! Best objective         : 36 (optimal - effective tol. is 0)
 ! Number of branches     : 12,103
 ! Number of fails        : 6,742
 ! Total memory usage     : 6.7 MB (6.2 MB CP Optimizer + 0.4 MB Concert)
! Time spent in solve    : 0.08s (0.08s engine + 0.00s extraction)
 ! Search speed (br. / s) : 151,287.5
 ! ----------------------------------------------------------------------------
```

SAT-2.dat

```
// solution with objective 29
rapath = [7 9 23 24 54];
aepath = [12 25 2 4 20];
```

Execution time without Symmetry Breaking:

```
! ----------------------------------------------------------------------------
 ! Search terminated normally, 2 solutions found.
 ! Best objective         : 29 (optimal - effective tol. is 0)
 ! Number of branches     : 34,624,220
 ! Number of fails        : 17,184,367
 ! Total memory usage     : 180.8 MB (180.3 MB CP Optimizer + 0.5 MB Concert)
! Time spent in solve    : 18139.86s (18139.85s engine + 0.01s extraction)
 ! Search speed (br. / s) : 1,908.7
 ! ----------------------------------------------------------------------------
```

Execution time with Symmetry Breaking:

```
! ----------------------------------------------------------------------------
 ! Search terminated normally, 2 solutions found.
 ! Best objective         : 29 (optimal - effective tol. is 0)
 ! Number of branches     : 132,737
 ! Number of fails        : 66,719
 ! Total memory usage     : 11.2 MB (10.7 MB CP Optimizer + 0.5 MB Concert)
! Time spent in solve    : 53.83s (53.83s engine + 0.00s extraction)
 ! Search speed (br. / s) : 2,465.9
 ! ----------------------------------------------------------------------------
```

SAT-3.dat

No solution for SAT-3.dat

```
! ----------------------------------------------------------------------------
! Search terminated normally, model has no solution.
 ! Number of branches     : 357,027
 ! Number of fails        : 179,110
 ! Total memory usage     : 18.6 MB (18.0 MB CP Optimizer + 0.7 MB Concert)
 ! Time spent in solve    : 915.69s (915.68s engine + 0.01s extraction)
 ! Search speed (br. / s) : 389.9
 ! ----------------------------------------------------------------------------
 ! ----------------------------------------------------------------------------
! Conflict refining - 531 constraints
 ! ----------------------------------------------------------------------------
*         189                           29
 ! Conflict refining terminated
 ! ----------------------------------------------------------------------------
 ! Conflict status           : Terminated normally, conflict found
 ! Conflict size             : 29 constraints
 ! Number of iterations      : 189
 ! Total memory usage        : 1.8 MB
 ! Conflict computation time : 3506.10s
 ! ----------------------------------------------------------------------------
```

SAT-4.dat

Execution time with Symmetry Breaking:

```
// solution with objective 77
rapath = [13 16 21 28 31 52 58];
aepath = [67 31 22 72 68 28 53];

! ----------------------------------------------------------------------------
! Minimization problem - 14 variables, 55 constraints
 ! Initial process time : 0.00s (0.00s extraction + 0.00s propagation)
 !  . Log search space  : 84.5 (before), 84.5 (after)
 !  . Memory usage      : 3.1 MB (before), 3.2 MB (after)
 ! Using parallel search with 8 workers.
 ! ----------------------------------------------------------------------------
 !          Best Branches  Non-fixed    W       Branch decision
    ……
             77     227k          6    1        34 != rapath(6)
 ! ----------------------------------------------------------------------------
 ! Search terminated normally, 5 solutions found.
 ! Best objective          : 77 (optimal - effective tol. is 0)
 ! Number of branches      : 1,880,659
 ! Number of fails         : 935,234
 ! Total memory usage      : 23.2 MB (22.6 MB CP Optimizer + 0.6 MB Concert)
! Time spent in solve     : 2632.29s (2632.28s engine + 0.01s extraction)
 ! Search speed (br. / s) : 714.5
 ! ----------------------------------------------------------------------------
```

SAT-5.dat

No solution for SAT-5.dat.The tool ran the data file for over 40 hours and the program still continues to run.

```
! ----------------------------------------------------------------------------
! Search terminated normally, model has no solution.
 ! Number of branches      : 14,259,126
 ! Number of fails         : 7,158,287
 ! Total memory usage      : 115.4 MB (113.6 MB CP Optimizer + 1.8 MB Concert)
 ! Time spent in solve     : 642664.10s (642664.05s engine + 0.04s extraction)
 ! Search speed (br. / s) : 22.2
 ! ----------------------------------------------------------------------------
 ! ----------------------------------------------------------------------------
 ! Conflict refining - 881 constraints
 ! ----------------------------------------------------------------------------
 !   Iteration      Number of constraints
 *          1                         881
```

2 Integer Linear Programming

<u>Problem Description 2</u>: The above problem can be solved using Integer Linear Programming.

The mathematical formulation for the stated problem using constraint programming is as given below:

```
/*** CRITERION ***/

minimizesum(iinSig,linRApaths) (rapath[i,l] *RAlength[l]) +sum(iinSig,kinAEpaths)
(aepath[i,k]*AElength[k]);

/*** CONSTRAINTS ***/

constraints{

/*** for every Signal there exist only one connection (hence only one RAPath, AEPath) ***/

forall(iinSig)sum(linRApaths) (rapath[i,l]) ==1;

forall(iinSig)sum(kinAEpaths) (aepath[i,k]) ==1;

/*** for a signal i, the destination of the path used by signal i-rapath
should be the same as the source of the path used by signal i-aepath***/

forall(iinSig){
      sum(jinRApaths) (rapath[i,j] *RAdst[j]) ==sum(kinAEpaths)(aepath[i,k] *AEsrc[k]);
}

/*** Check for non-compatibility
We have told that i cannot be = j (for 2 different signals)
2 RApaths are non-compatible if either of the 2 or both paths are not enabled ***/

forall(i,jinSig,k,linRApaths:i<j&&RAcomp[k,l] ==0){
rapath[i,k] +rapath[j,l] <=1;
}

forall(i,jinSig,k,linAEpaths:i<j&&AEcomp[k,l] ==0){
aepath[i,k] +aepath[j,l] <=1;
}

/*** for a signal i, (currently we can have paths calculated in different orders)
to minimize computation time, we impose an order on the source and dst paths...***/

forall(iinSig:i<Ns) {
sum(kinRApaths) (rapath[i,k]*k-rapath[i+1,k] *k) <= -1;
}
```

The above model is run for different data files.

<u>Note</u>: SAT-3.dat and SAT-5.dat files have no solution, the code run for 12, 40 hours straight and no feasible solution was found by the model.

SAT-0.dat

```
// solution (optimal) with objective 6
// Quality Incumbent solution:
// MILP objective                                  6.0000000000e+00
// MILP solution norm |x| (Total, Max)             6.00000e+00  1.00000e+00
// MILP solution error (Ax=b) (Total, Max)         0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)                 0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)           0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)             0.00000e+00  0.00000e+00
//

rapath = [[1 0 0 0 0 0]
[0 0 1 0 0 0]
[0 0 0 0 0 1]];
aepath = [[1 0 0 0 0 0 0]
[0 0 0 1 0 0 0]
[0 0 0 0 0 0 1]];
```

Execution time without Symmetry Breaking:

```
Root node processing (before b&c):
  Real time             =    0.05 sec. (0.59 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    0.05 sec. (0.59 ticks)
```

Execution time with Symmetry Breaking:

```
Root node processing (before b&c):
  Real time             =    0.02 sec. (0.27 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    0.02 sec. (0.27 ticks)
```

SAT-1.dat

```
// solution (optimal) with objective 36
// Quality Incumbent solution:
// MILP objective                                  3.6000000000e+01
// MILP solution norm |x| (Total, Max)             6.00000e+00  1.00000e+00
// MILP solution error (Ax=b) (Total, Max)         0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)                 0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)           0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)             0.00000e+00  0.00000e+00
//

rapath = [[1 0 0 0 0 0]
[0 0 0 0 1 0]
[0 0 0 0 0 1]];
aepath = [[0 0 0 1 0 0 0]
[0 0 1 0 0 0 0]
[0 0 0 0 1 0 0]];
```

Execution time without Symmetry Breaking:

```
Root node processing (before b&c):
  Real time             =    0.00 sec. (0.76 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    0.00 sec. (0.76 ticks)
```

Execution time with Symmetry Breaking:

```
Root node processing (before b&c):
  Real time             =    0.00 sec. (0.66 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    0.00 sec. (0.66 ticks)
```

SAT-2.dat

```
// solution (optimal) with objective 29
// Quality Incumbent solution:
// MILP objective                                2.9000000000e+01
// MILP solution norm |x| (Total, Max)           1.00000e+01  1.00000e+00
// MILP solution error (Ax=b) (Total, Max)       0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)               0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)         0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)           0.00000e+00  0.00000e+00
//

rapath = [[0
             0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]];
aepath = [[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]];
```

Execution time without Symmetry Breaking:

```
Root node processing (before b&c):
  Real time             =    1.69 sec. (1516.00 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.44 sec. (584.77 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    2.13 sec. (2100.77 ticks)
```

Execution time with Symmetry Breaking:

```
Root node processing (before b&c):
  Real time             =    1.42 sec. (1294.27 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.41 sec. (480.38 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    1.83 sec. (1774.65 ticks)
```

SAT-3.dat

NO feasible solution obtained.

Execution time without Symmetry Breaking:

```
Root node processing (before b&c):
  Real time             =   28.92 sec. (26753.42 ticks)
Parallel b&c, 8 threads:
  Real time             =   26.80 sec. (29747.05 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =   55.72 sec. (56500.47 ticks)
```

```
Root node processing (before b&c):
  Real time             =    8.90 sec. (9723.88 ticks)
Parallel b&c, 8 threads:
  Real time             =   35.05 sec. (44827.52 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =   43.96 sec. (54551.40 ticks)
```

Execution time with Symmetry Breaking:

```
Root node processing (before b&c):
  Real time             =    8.07 sec. (10560.83 ticks)
Parallel b&c, 8 threads:
  Real time             =   44.86 sec. (71461.73 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =   52.93 sec. (82022.56 ticks)
```

SAT-4.dat

```
// solution (optimal) with objective 77
// Quality Incumbent solution:
// MILP objective                                7.7000000000e+01
// MILP solution norm |x| (Total, Max)           1.40000e+01  1.00000e+00
// MILP solution error (Ax=b) (Total, Max)       0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)               0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)         0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)           0.00000e+00  0.00000e+00
//

rapath = [[0
             0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]];
aepath = [[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
             0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
             0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]];
```

<u>Execution time without Symmetry Breaking</u>:

```
Root node processing (before b&c):
  Real time             =    4.59 sec. (4237.64 ticks)
Parallel b&c, 8 threads:
  Real time             =    1.78 sec. (2148.87 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    6.37 sec. (6386.51 ticks)
```

<u>Execution time with Symmetry Breaking</u>:

```
Root node processing (before b&c):
  Real time             =    5.07 sec. (4522.30 ticks)
Parallel b&c, 8 threads:
  Real time             =    3.29 sec. (3831.53 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    8.36 sec. (8353.83 ticks)
```

<u>SAT-5.dat</u>

NO feasible solution obtained.

<u>Execution time without Symmetry Breaking</u>:

```
Elapsed time = 70979.59 sec. (55793641.37 ticks, tree = 1.03 MB)
 2815364   396        0.0000   151                    0.0000 9.32e+08
 2817070   343        0.0000   140                    0.0000 9.33e+08
 2818456   317    infeasible                          0.0000 9.33e+08
 2819890   323        0.0000   132                    0.0000 9.34e+08
 2821515   431        0.0000   129                    0.0000 9.34e+08
```

The program was run for over 22 hours and was yet not completed

<u>Execution time with Symmetry Breaking</u>:

```
Root node processing (before b&c):
  Real time             =  632.00 sec. (681951.02 ticks)
Parallel b&c, 8 threads:
  Real time             = 33854.90 sec. (18748523.78 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) = 34486.90 sec. (19430474.80 ticks)
```

3 Conclusion

Configuration File	Constraint Programming		Integer Linear Programming	
	Without Symmetry Breaking	With Symmetry Breaking	Without Symmetry Breaking	With Symmetry Breaking
SAT - 0	0.02s	0.01s	0.05 sec (0.59 ticks)	0.02 sec (0.27 ticks)
SAT - 1	0.09s	0.08s	0.00 sec. (0.76 ticks)	0.00 sec. (0.66 ticks)
SAT - 2	18139.86s	53.83s	2.13 sec. (2100.77 ticks)	1.83 sec. (1774.65 ticks)
SAT - 3	No solution	No solution	No solution [99,68 sec]	No solution [52.93 sec]
SAT - 4	Logs got corrupted	2632.29s	6.37 sec. (6386.51 ticks)	8.36 sec. (8353.83 ticks)
SAT - 5	No solution	No solution	No solution	No solution

- As seen above Integer Linear Programming takes lesser time than Constraint Programming and symmetry breaking helps in much faster computation.
- There exist No solution for data files SAT – 3 and SAT – 5
- With ILP very huge data files file SAT – 3 are computed within sec. but the same computation nearly takes 20 hours with CP
- SAT – 5 data file is comparatively the largest and even after 40 hours there was no feasible solution obtained
- The solutions obtained when symmetry breaking is used are ordered i.e. the least weighted path is listed first
- The solution obtained with CP and ILP is more or less the same, with ILP we obtained the results in matrix form with 0..1 values; whereas with CP we obtain the values as Integers