Thresholding by quantization

Let p be the picture histogram, so that $p(x)$ is the number of pixels of value x, for $x = 0, \ldots, M$. We are looking for a threshold value t and two values q_1, q_2, such that all pixels in the range $0 \leq x < t$ are replaced with q_1, and all pixels in the range $t \leq x \leq M$ are replaced with q_2. Define the following expression as the total error:

$$E(t, q_1, q_2) = \sum_{x=0}^{t-1} (x - q_1)^2 p(x) + \sum_{x=t}^{M} (x - q_2)^2 p(x).$$

For each t we can compute the minimum of E by choosing the "best possible" values for q_1, q_2. These are computed by taking the derivatives of E with respect to q_1, q_2.
Taking the derivative of e with respect to q_1 we have:

$$\frac{\partial e}{\partial q_1} = 2 \sum_{x=0}^{t-1} x p(x) - 2 q_1 \sum_{x=0}^{t-1} p(x).$$

The requirement that $\frac{\partial e}{\partial q_1} = 0$ gives:

$$q_1 = \frac{\sum_{x=0}^{t-1} x p(x)}{\sum_{x=0}^{t-1} p(x)}$$

and similarly:

$$q_2 = \frac{\sum_{x=t}^{M} x p(x)}{\sum_{x=t}^{M} p(x)}$$

Therefore, we can compute the value of E for any given value of t by first computing q_1, q_2 and then substituting their values in the above expression for E. Since there are only 255 possible values for t the minimizer of t can be determined by examining all values of $E(t)$ for $t = 1..255$.