02/02/2019 Home Work-2 Varun Mehrotra

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Question-1.

Quantization Error,

$$E_t(q_1,q_2) = \sum_{x=0}^{t-1} h(x)\,|x-q_1| + \sum_{x=t}^{255} h(x)\,|x-q_2|$$

To find q_1 and q_2 :-

(i) Derivative w.r.t q_1

$$\frac{\partial E_t}{\partial q_1} = \frac{\partial}{\partial q_1}\left[\sum_{x=0}^{t-1} h(x)\,|x-q_1|\right] + 0$$

$$= -\sum_{x=0}^{t-1} \frac{h(x)\,(x-q_1)}{|x-q_1|}$$

$$\therefore \frac{\partial E}{\partial q_1} = 0 \Rightarrow q_1 = \frac{\sum_{x=0}^{t-1} x\,h(x)}{\sum_{x=0}^{t-1} h(x)}.$$ where $x \neq q_1$

(ii) Derivative w.r.t q_2

$$\frac{\partial E_t}{\partial q_2} = 0 - \sum_{x=t}^{M} \frac{h(x)\,(x-q_2)}{|x-q_2|}$$

$$\therefore q_2 = \frac{\sum_{x=t}^{M} x\,h(x)}{\sum_{x=t}^{m} h(x)}.$$

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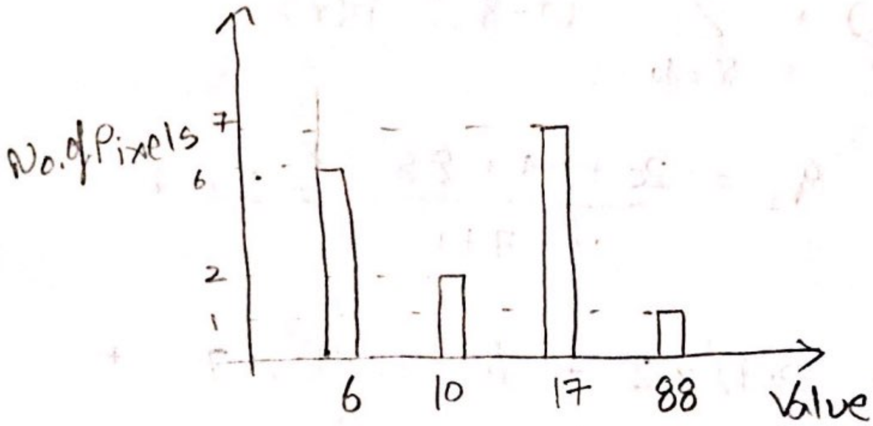
Question-2

1.



2. Optimal Thresholding Algorithm.

$$E(t,q_1,q_2) = \sum_{x=0}^{t-1} (x-q_1)^2 p(x) + \sum_{x=t}^{M} (x-q_2)^2 p(x)$$

(a) Let, $t=6$

$$E(6,q_1,q_2) = \sum_{x=0}^{5} (x-q_1)^2 p(x) + \sum_{x=6}^{M} (x-q_2)^2 p(x) = \sum_{x=6}^{M} (x-q_2)^2 p(x)$$

$$q_2 = \frac{\sum_{x=t}^{M} x\,p(x)}{\sum_{x=t}^{M} p(x)} = \frac{36+20+119+88}{6+2+7+1} = 16.4375$$

$$\therefore E(6,0,16.4375) = (6-16.4375)^2 \times 6 + (10-16.4375)^2 \times 2 + (17-16.4375)^2 \times 7 + (88-16.4375)^2$$

$$= 653.65 + 82.88 + 2.215 + 5121.19$$

$$= 5859.97$$

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(b) Let, $t = 10$

$$E(10, q_1, q_2) = \sum_{x=0}^{q} (x-q_1)^2 p(x) + \sum_{x=10}^{M} (x-q_2)^2 p(x)$$

$$q_1 = \frac{36}{6} = 6 \qquad q_2 = \frac{20+119+88}{2+7+1} = 22.7$$

$$\text{Error} = (6-6)^2 \times 6 + (10-22.7)^2 \times 2 + (17-22.7)^2 \times 7 + (88-22.7)^2$$

$$= 0 + 322.58 + 227.43 + 4264.09$$

$$= 4814.1$$

(c) Let, $t = 17$

$$E(17, q_1, q_2) = \sum_{x=0}^{16} (x-q_1)^2 p(x) + \sum_{x=17}^{M} (x-q_2)^2 p(x)$$

$$q_1 = \frac{36+20}{8} = 8 \qquad q_2 = \frac{119+88}{8} = 25.875$$

$$\therefore \text{Error} = (6-8)^2 \times 6 + (10-8)^2 \times 2 + (17-25.875)^2 \times 7 + (88-25.875)^2 \times 1$$

$$= 24 + 8 + 551.36 + 3859.52$$

$$= 4442.88$$

(d) Let, $t = 88$

$$E(88, q_1, q_2) = \sum_{x=0}^{87} (x-q_1)^2 p(x) + \sum_{x=88}^{m} (x-q_2)^2 p(x)$$

$$q_1 = \frac{36 + 20 + 119}{15} = 11.67 \quad , \quad q_2 = \frac{88}{1} = 88$$

$$\therefore \text{Error} = (6-11.67)^2 \times 6 + (10-11.67)^2 \times 2 + (17-11.67)^2 \times 7 + (88-88)^2 \times 1$$

$$= 192.89 + 5.58 + 198.86 + 0$$

$$= 397.33$$

$\therefore$ Error value is least when $t = 88$.

$\therefore$ Threshold = 88

Image :-

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	1

3. Linear Scaling.

$$x \to \frac{(x-a)(B-A)}{b-a} + A$$

$A = 0$
$B = 255$

As per image, $a = 6$ and $b = 88$

$$\therefore x \to \frac{(x-6) \times 255}{82}$$

$x = 6 \Rightarrow \frac{(6-6) \times 255}{82} = 0$

$x = 17 \Rightarrow \frac{11 \times 255}{82} = 34.207$
≈ 34

$x = 10 \Rightarrow \frac{4 \times 255}{82}$
$= 12.439$
≈ 12

$x = 88 \Rightarrow \frac{82 \times 255}{82} = 255$

0	0	0	12
0	0	0	12
34	34	34	34
34	34	34	255

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4. Histogram Equalization

i	h(i)	f(i)	$\frac{f(i-1)+f(i)}{2} \times \frac{256}{16}$	floor	
0	0	0	0	0	$0 \Rightarrow 0$
1	0	0	0	0	$1 \Rightarrow 0$
⋮					
6	6	6	48	48	$6 \Rightarrow 48$
7	0	6	96	96	$7 \Rightarrow 96$
⋮				⋮	
10	2	8	112	112	$10 \Rightarrow 112$
⋮				⋮	
17	7	15	168	184	$17 \Rightarrow 184$
⋮				⋮	
88	1	16	248	248	$88 \Rightarrow 248$

Image:-

48	48	48	112
48	48	48	112
184	184	184	184
184	184	184	248

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