Histogram flattening

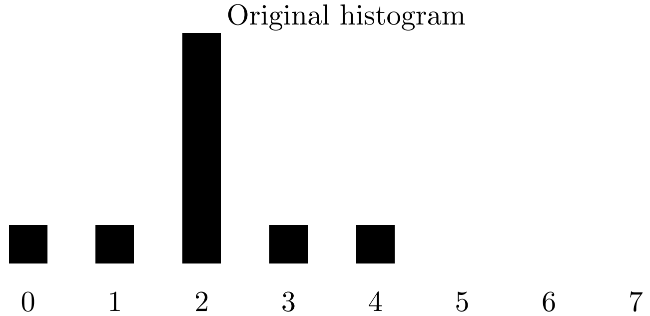
Here we are interested in changing the gray levels of an image so that the histogram of the new image is flat. If the image has n pixels (rows times columns) and the display range has k values (typically 256), then the result would be an image with approximately n/k pixels of each value.

Motivation: A picture with a flattened histogram takes advantage of all the display gray levels, and the result is invariant to any monotone change in the gray levels. The algorithm requires that pixels of the same gray level are transformed into pixels of different gray level.

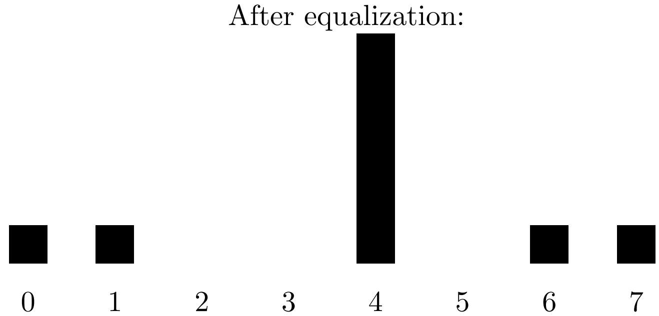
Histogram equalization

Histogram flattening is easy to understand but difficult and inefficient to implement. Histogram equalization is easy and efficient to implement but more difficult to explain.

Original histogram



After equalization:



i	$h(i)$	$f(i)$	$\frac{f(i-1)+f(i)}{2}\frac{8}{10}$	floor	
0	1	1	0.4	0	$0 \rightarrow 0$
1	1	2	1.2	1	$1 \rightarrow 1$
2	6	8	4	4	$2 \rightarrow 4$
3	1	9	6.8	6	$3 \rightarrow 6$
4	1	10	7.6	7	$4 \rightarrow 7$
5	0	10	8	7	$5 \rightarrow 7$
6	0	10	8	7	$6 \rightarrow 7$
7	0	10	8	7	$7 \rightarrow 7$

The derivation

A picture with n pixels. Pixel values are in the range $[0, k-1]$. (k is usually 256.)

$h(i)$**:** The histogram. Number of pixels of value i.

$f(i)$**:** Number of pixels in the range $[0, i]$. We have: $f(i) = \sum_{t=0}^{i} h(t)$.

$g(j)$**:** The equivalent of $f(i)$ in the flattened histogram. We have: $g(j) = \frac{(j+1)n}{k}$

There are two conditions that the requirement $i \to j$ should satisfy in the ideal case:

$$\begin{array}{lll} \text{condition 1:} & f(i) \approx g(j) & = \frac{(j+1)n}{k} \\ \text{condition 2:} & f(i-1) \approx g(j-1) & = \frac{jn}{k} \end{array}$$

This gives:

$$j \approx \frac{k(f(i-1) + f(i))}{2n} - 0.5$$

Therefore, the approximate value of j can be computed by rounding the expression on the right. But this is the same as taking floor of the following expression, with the "understanding" that the floor of k is $k-1$.

$$j = \lfloor \frac{k(f(i-1) + f(i))}{2n} \rfloor$$