Homework 4

Q1) Given Image —

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
(255, 0, 0)	(255, 0, 0)	(255, 0, 0)	(255, 0, 0)
(100, 100, 100)	(100, 100, 100)	(100, 100, 100)	(100, 100, 100)
(0, 100, 100)	(0, 100, 100)	(0, 100, 100)	(0, 100, 100)

Convert to non-linear RGB.

Divide each pixel by 255

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
(0.39, 0.39, 0.39)	(0.39, 0.39, 0.39)	(0.39, 0.39, 0.39)	(0.39, 0.39, 0.39)
(0, 0.39, 0.39)	(0, 0.39, 0.39)	(0, 0.39, 0.39)	(0, 0.39, 0.39)

Convert to Linear RGB ~~by~~

Perform inverse gamma correction

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
(0.126, 0.126, 0.126)	(0.126, 0.126, 0.126)	(0.126, 0.126, 0.126)	(0.126, 0.126, 0.126)
(0, 0.126, 0.126)	(0, 0.126, 0.126)	(0, 0.126, 0.126)	(0, 0.126, 0.126)

Convert to XYZ

Perform Linear Transform

for pixel with value (0, 0, 0) the XYZ values after transform = (0,0,0)

for Pixel with RGB value (1,0,0) the XYZ values after transform =

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{bmatrix} 0.413 & 0.358 & 0.180 \\ 0.213 & 0.715 & 0.072 \\ 0.019 & 0.119 & 0.950 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.413 \\ 0.213 \\ 0.019 \end{bmatrix}$$

for Pixel with RGB value (0.126, 0.126, 0.126)

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{bmatrix} 0.413 & 0.358 & 0.180 \\ 0.213 & 0.715 & 0.072 \\ 0.019 & 0.119 & 0.950 \end{bmatrix} \times \begin{bmatrix} 0.126 \\ 0.126 \\ 0.126 \end{bmatrix}$$

$$= \begin{bmatrix} 0.119 \\ 0.126 \\ 0.137 \end{bmatrix}$$

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For Pixel with value (0, 0.126, 0.126)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.413 & 0.358 & 0.180 \\ 0.213 & 0.715 & 0.721 \\ 0.019 & 0.119 & 0.950 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0.126 \\ 0.126 \end{bmatrix}$$

$$= \begin{bmatrix} 0.068 \\ 0.099 \\ 0.135 \end{bmatrix}$$

Hence, XYZ matrix is

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
(0.413, 0.213, 0.019)	(0.413, 0.213, 0.019)	(0.413, 0.213, 0.019)	(0.413, 0.213, 0.019)
(0.119, 0.126, 0.137) ~~(0.068, 0.099, 0.135)~~	(0.19, 0.126, 0.137)	(0.119, 0.126, 0.137)	(0.119, 0.126, 0.137)
(0.068, 0.099, 0.135)	(0.068, 0.099, 0.135)	(0.068, 0.099, 0.135)	(0.068, 0.099, 0.135)

Convert to xyY

Use formula

$$Y = Y \qquad x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z}$$

for pixel (0, 0, 0) $\quad xyY = (0, 0, 0)$

for pixel (0.413, 0.213, 0.019)

$$\underline{Y = 0.213} \qquad x = \frac{0.413}{0.645} = \underline{0.640} \qquad y = \frac{0.213}{0.645} = \underline{0.330}$$

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For pixel (0.119, 0.126, 0.137) we get

$Y = 0.126$ $\quad x = \frac{0.119}{0.382} = 0.312$ $\quad y = \frac{0.126}{0.382} = \cancel{0.330}\ 0.330$

For pixel (0.068, 0.099, 0.135)

$Y = 0.099$ $\quad x = \frac{0.068}{0.302} = 0.225$ $\quad y = \frac{0.099}{0.302} = 0.\cancel{447}\ 328$

Hence xyY image is

(0,0,0) (0,0,0)

(0.225, 0.3~~88~~2, 0.099) (0.312, 0.330, 0.126)

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
(0.640, 0.330, 0.213)	(0.640, 0.330, 0.213)	(0.640, 0.330, 0.213)	(0.640, 0.330, 0.213)
(0.312, 0.330, 0.126)	(0.312, 0.330, 0.126)	(0.312, 0.330, 0.126)	(0.312, 0.330, 0.126)
(0.225, 0.328, 0.099)	(0.225, 0.328, 0.099)	(0.225, 0.328, 0.099)	(0.225, 0.328, 0.099)

* Convert Image to ~~XYZ~~ Luv

$(X_w, Y_w, Z_w) = (0.95, 1.0, 1.09)$

$$u_w = \frac{4X_w}{X_w + 15Y_w + 3Z_w} = \frac{3.8}{19.22} = 0.198$$

$$v_w = \frac{9Y_w}{X_w + 15Y_w + 3Z_w} = \underline{0.468}$$

For pixel $(0,0,0)$

$t = Y/Y_w = 0$ $\quad L = 0$

$u = 0$ $\quad v = 0$

For pixel $(\cancel{0.640}, \cancel{0.330}, \cancel{0.213})$ $(0.413 \quad 0.213 \quad 0.019)$

$t = Y/Y_w = \cancel{0.330}\ 0.213$.

$L = 116 \cdot t^{1/3} - 16 = \cancel{64.16}\ \underline{53.28}$

$d = X + 15Y + 3Z = 4.247$

$u' = 4X/d = \underline{0.389}$ $\quad v' = 9Y/d = \underline{0.451}$

$\cancel{u = u' - u_w = 0.191}$

$u = 13L(u' - u_w) = \underline{132.29}$ $\quad v = 13L(v' - v_w) = \underline{-11.77}$

For pixel $(0.119, 0.126, 0.137)$

$t = Y/Y_w = 0.126$

$L = 116t^{1/3} - 16 = \underline{42.15}$

$d = X + 15Y + 3Z = 2.42$

$u' = 4X/d = 0.196$ $\quad v' = 9Y/d = 0.469$

$u = 13L(u' - u_w) = -1.095 = \underline{-1.10}$

$v = 13L(v' - v_w) = 0.547 = \underline{0.55}$

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For pixel $(0.068, 0.091, 0.135)$

$t = Y/Y_w = 0.099$

$L = 116 t^{1/3} - 16 = \underline{37.66}$

$d = X + 15Y + 3Z = 1.958$

$u' = 4X/d = 0.1389 \qquad v' = 9Y/d = 0.455$

$u = 13L(u' - u_w) = \underline{-28.89} \qquad v = 13L(v' - v_w) = \underline{-6.37}$

Hence the image in Luv format is.

(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
(53.28, 132.29, -11.77)	(53.28, 132.29, -11.77)	(53.28, 132.29, -11.77)	(53.28, 132.29, -11.77)
(42.15, -1.10, 0.55)	(42.15, -1.10, 0.55)	(42.15, -1.10, 0.55)	(42.15, -1.10, 0.55)
	~~(37.66, -28.89, -6.37)~~	~~(37.66, -28.89, -6.39)~~	~~(37.66, -28.89, -6.37)~~
(37.66, -28.89, -6.37)	(37.66, -28.89, -6.39)	(37.66, -28.99, -6.37)	(37.66, -28.99, -6.37)

To increase the luminosity we need to stretch ~~To perform~~ L values over 0 - 100. We will do this with histogram equalisation. For that we first perform quantisation of L values (Round it to nearest whole integer)

Hence L values for the image are

0	0	0	0
53	53	53	53
42	42	42	42
38	38	38	38

Image histogram

Pixel values	# Pixels.
0	4
38	4
42	4
53	4

# of grades (k) (0 – 100) = 101

pixels (n) = 16 $\quad k/n = \frac{101}{16} = 6.313$

i	$h(i)$	$f(i)$	$\frac{f(i)+f(i-1)}{2} \cdot \frac{k}{n}$	floor	
0	4	4	$2 \times 6.313 = 12.626$	12	$0 \to 12$
38	4	8	$6 \times 6.313 = 37.878$	37	$38 \to 37$
42	4	12	$10 \times 6.313 = 63.13$	63	$42 \to 63$
53	4	16	$14 \times 6.313 = 88.38$	88	$53 \to 88$

Hence L'uv image is →

(12, 0, 0)	(12, 0, 0)	(12, 0, 0)	(12, 0, 0)
(88, 132.29, -11.77)	(88, 132.29, -11.77)	(88, 132.29, -11.77)	(88, 132.29, -11.77)
(63, -1.10, 0.55)	(63, -1.10, 0.55)	(63, -1.10, 0.55)	(63, -1.10, 0.55)
(37, -28.89, -6.37)	(37, -28.89, -6.37)	(37, -28.89, -6.37)	(37, -28.89, -6.37)

L'uv to XYZ conversion

$$u' = \frac{13 u_w L + u}{13L} \qquad v' = \frac{v + 13 v_w L}{13L}$$

For pixel (12, 0, 0)

~~u' = 33.462~~ ~~v' =~~

u' = 0.198 v' = 0.468

Y = ~~0.016~~ 0.014

X = 0.013 Z = ~~1.092~~ 0.015

For pixel (88, 132.29, -11.77)

u' = 0.313 v' = 0.458

Y = 0.72

X = 1.107 ≈ 1.0 Z = 0.747

For pixel (63, -1.10, 0.55)

u' = 0.197 v' = 0.4689

Y = 0.316

X = 0.29 Z = 0.346

For pixel (37, -28.89, -6.37)

u' = 0.137 v' = 0.454

Y = 0.095

X = 0.067 Z = 0.131

Hence XYZ image is

(0.013, 0.014, 0.015)	(0.013, 0.014, 0.015)	(0.013, 0.014, 0.015)	(0.013, 0.014, 0.015)
(1.0, 0.72, 0.747)	(1.0, 0.72, 0.747)	(1.0, 0.72, 0.747)	(1.0, 0.72, 0.747)
(0.29, 0.316, 0.346)	(0.29, 0.316, 0.346)	(0.29, 0.316, 0.346)	(0.29, 0.316, 0.346)
(0.067, 0.095, 0.131)	(0.067, 0.095, 0.131)	(0.067, 0.095, 0.131)	(0.067, 0.095, 0.131)

Q2]

$g_1 = f_1 \otimes g$

$=$

1	0	2

$\otimes$

1	1	1	1	1
0	1	2	3	4
0	0	0	0	0
2	2	2	2	2

$g_1 =$

2	2	3	3	3	1	1
0	2	4	7	10	3	4
0	0	0	0	0	0	0
4	4	6	6	6	2	2

$h = f \times g$

$= f' \otimes g$

$=$

2	0	1

$\otimes$

1	1	1	1	1
0	1	2	3	4
0	0	0	0	0
2	2	2	2	2

$=$

1	1	3	3	3	2	2
0	1	2	5	8	6	8
0	0	0	0	0	0	0
2	2	6	6	6	4	4

$$g_2 = f_2 \otimes g_1$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \otimes \begin{bmatrix} 2 & 2 & 3 & 3 & 3 & 1 & 1 \\ 0 & 2 & 4 & 7 & 10 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 6 & 6 & 6 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & -3 & -3 & -3 & -1 & -1 & 0 \\ 2 & 2 & 1 & -1 & -4 & 1 & -2 & 1 \\ 0 & 2 & 6 & 11 & 17 & 13 & 7 & 4 \\ -4 & -4 & -6 & -6 & -6 & -2 & -2 & 0 \\ 4 & 8 & 10 & 12 & 12 & 8 & 4 & 2 \end{bmatrix}$$

Q2] Find f such that $g_2 = f \otimes g$

$$g_2 = f_2 \otimes g_1$$

$$= f_2 \otimes (f_1 \otimes g)$$

$$= f_2' * (f_1' * g)$$ ← Change from cross corelation to convolution.

$$= (f_2' * f') * g$$ ← rearrange paranthesis.

$$g_2 = (f_2' * f')' \otimes g$$

$$= f \otimes g$$

Thus $f = (f_2' * f_1')'$

$= ((f_2')' \otimes f_1')'$

$= (f_2 \otimes f_1')'$

$f' = f_2 \otimes f_1' =$

1	1
0	-1

2	0	1

$f' =$

-2	0	-1	0
2	2	1	1

$f =$

1	1	2	2
0	-1	0	-2

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