

CS221 Fall 2018 Homework [car]

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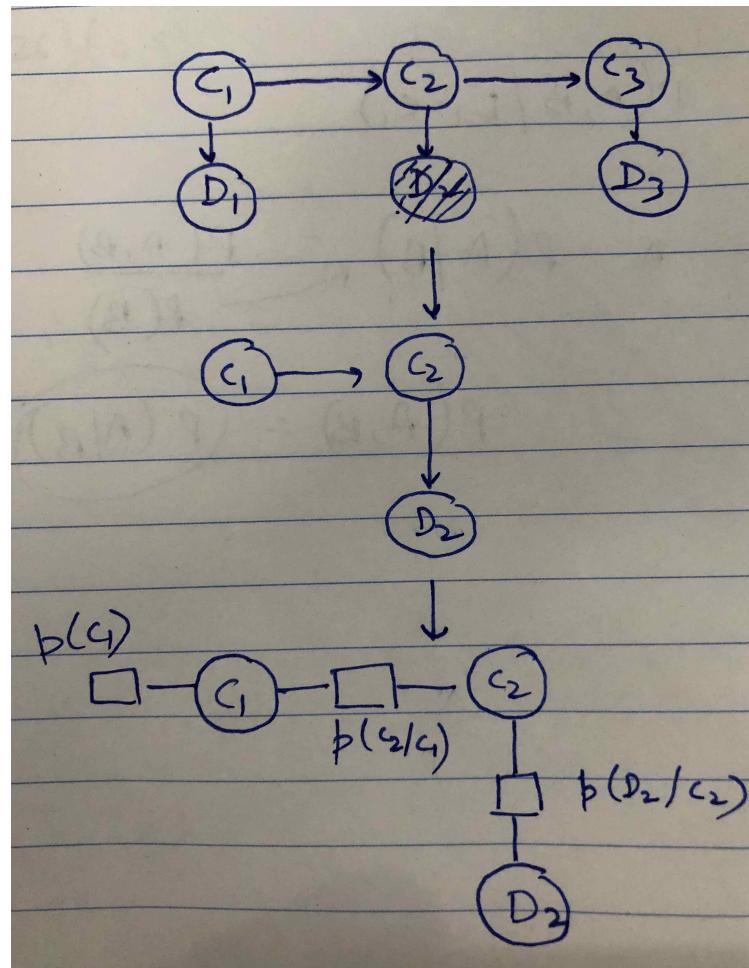
By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

Problem 1

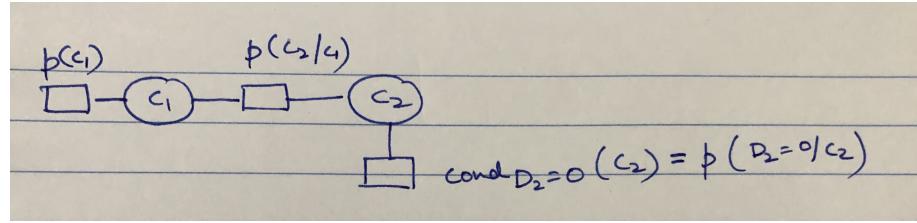
- (a) Step 1: Remove variables that are not ancestors

Step 2: Converting to factor graph

Step 1 and step 2 are shown in diagram below:



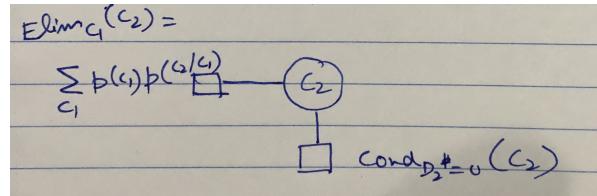
Step3: Conditioning on $D_2 = 0$



Condition variable D_2 on value $D_2 = 0$, replacing it with a factor $\text{cond}_{D_2=0}(C_2)$, we get

$$\begin{array}{ll} \text{cond}_{D_2=0}(C_2) & C_2 \\ 1 - \eta & 0 \\ \eta & 1 \end{array}$$

Step4: Eliminate C_1



$$\begin{aligned} \text{elim}_{C_1}(C_2) &= \sum_{C_1} p(C_1)p(C_2|C_1) \\ &= 0.5 \sum_{C_1} p(C_2|C_1) \end{aligned}$$

This is given from the below table:

$$\begin{array}{ll} \text{elim}_{C_1}(C_2) & C_2 \\ 0.5(1 - \epsilon + \epsilon) = 0.5 & 0 \\ 0.5(\epsilon + 1 - \epsilon) = 0.5 & 1 \end{array}$$

Therefore, now that we know $\text{elim}_{C_1}(C_2)$ and $\text{cond}_{D_2=0}(C_2)$,

$$p(C_2/D_2 = 0) = \text{elim}_{C_1}(C_2) * \text{cond}_{D_2=0}(C_2)$$

$$\begin{array}{ll} p(C_2/D_2 = 0) & C_2 \\ 0.5(1 - \eta) & 0 \\ 0.5\eta & 1 \end{array}$$

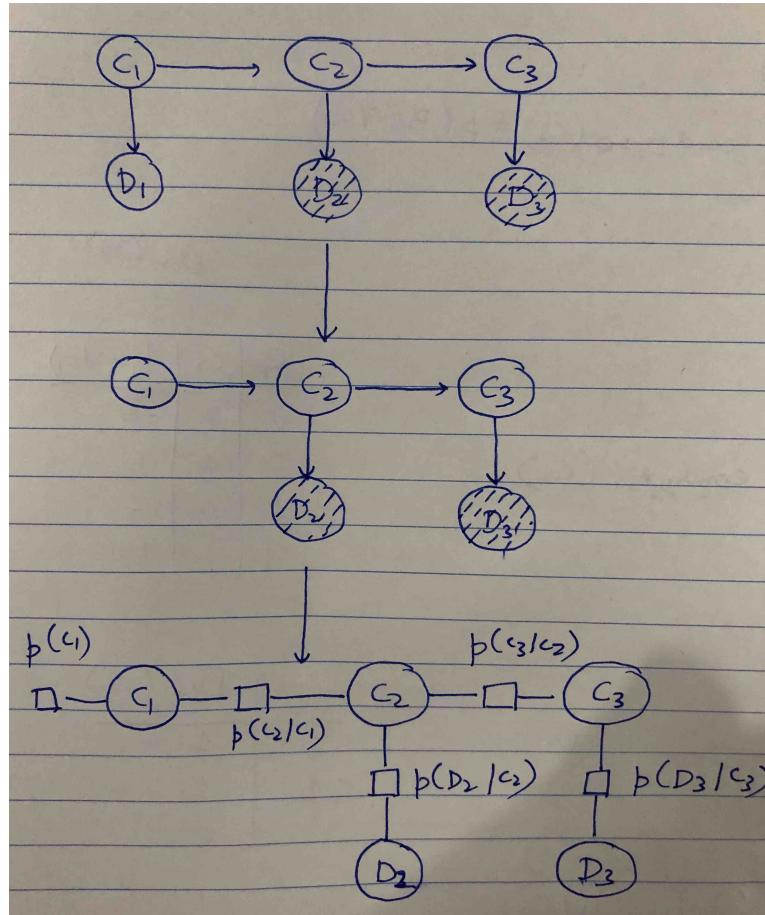
Hence, the given query,

$$\begin{aligned} p(C_2 = 1 / D_2 = 0) &= \frac{0.5\eta}{0.5\eta + 0.5(1 - \eta)} \\ &= \eta \end{aligned}$$

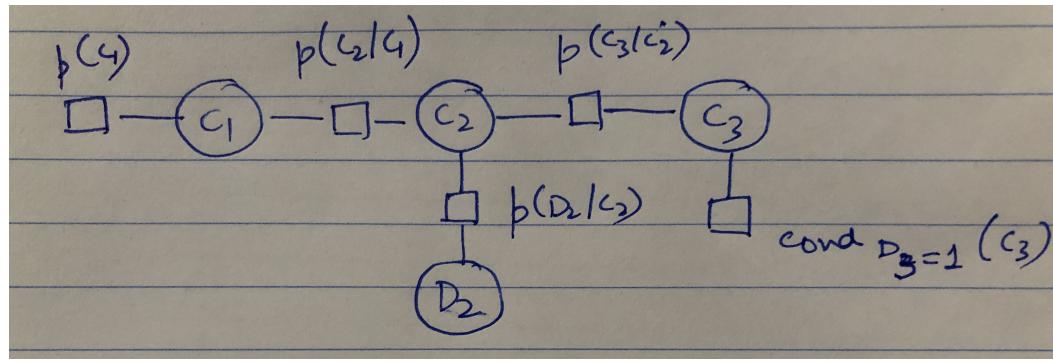
(b) **Step1:** Remove variables that are not ancestors

Step2: Converting to factor graph

Step 1 and step 2 are illustrated below:



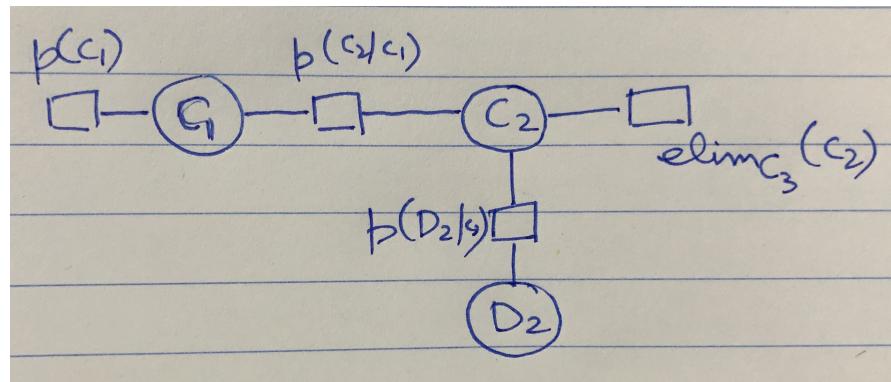
Step3: Conditioning on $D_3 = 1$



Conditioning on variable D_3 , and replacing it with a factor $\text{cond}_{D_3=1}(C_3)$, we get

$$\begin{array}{ll} \text{cond}_{D_3=1}(C_3) & C_3 \\ \eta & 0 \\ 1 - \eta & 1 \end{array}$$

Step4: Eliminating C_3



Defining function $\text{elim}_{C_3}(C_2)$ in order to eliminate node C_3 as

$$\text{elim}_{C_3}(C_2) = \sum_{C_3} \text{cond}_{D_3=1}(C_3)p(C_3|C_2)$$

The probability distribution $p(C_3|C_2)$ is given by:

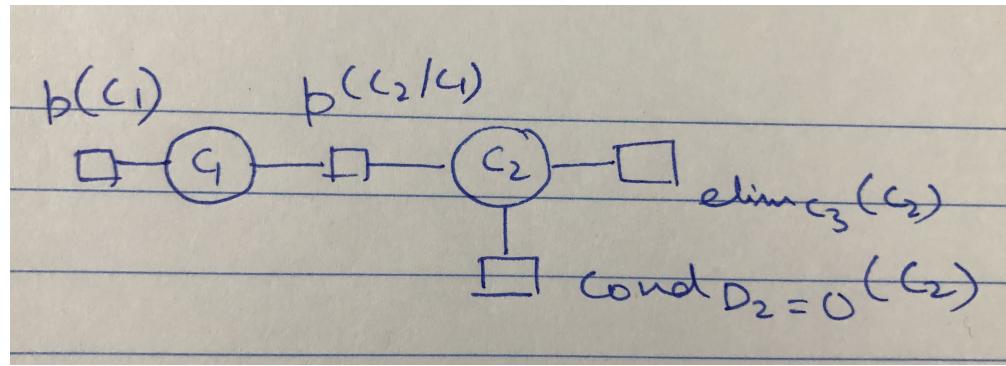
C2	C3	$p(C_3 C_2)$
0	0	$1 - \epsilon$
0	1	ϵ
1	0	ϵ
1	1	$1 - \epsilon$

The probability distribution $\text{cond}_{D_3=1}(C_3)$ is defined in Step 3.

Combining both and substituting in equation 1, and doing summation over values of C_3 , we will have probability distribution of $\text{elim}_{C_3}(C_2)$ is given by:

$$\begin{array}{ll} C_2 & \text{elim}_{C_3}(C_2) \\ 0 & (1 - \epsilon)\eta + \epsilon(1 - \eta) \\ 1 & \epsilon\eta + (1 - \eta)(1 - \epsilon) \end{array}$$

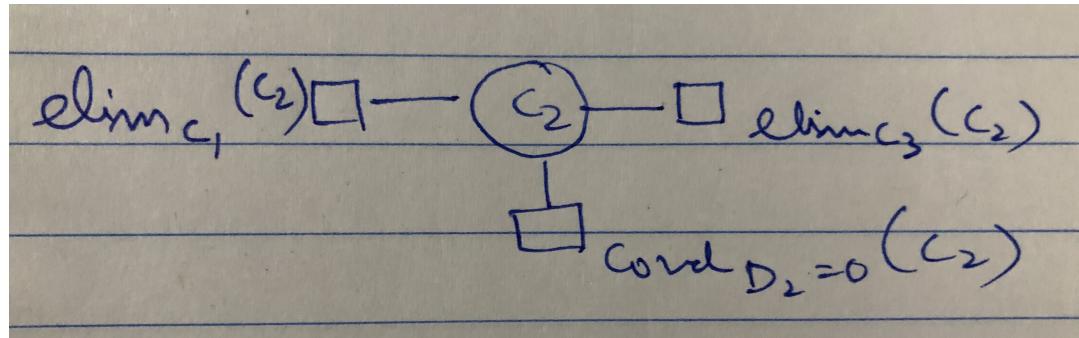
Step5: Conditioning on $D_2 = 0$



Condition variable D_2 on value $D_2 = 0$, replacing it with a factor $\text{cond}_{D_2=0}(C_2)$, we get

$$\begin{array}{ll} \text{cond}_{D_2=0}(C_2) & C_2 \\ 1 - \eta & 0 \\ \eta & 1 \end{array}$$

Step6: Eliminate C_1



$$\begin{aligned} \text{elim}_{C_1}(C_2) &= \sum_{C_1} p(C_1)p(C_2/C_1) \\ &= 0.5 \sum_{C_1} p(C_2/C_1) \end{aligned}$$

This is given from the below table:

$\text{elim}_{C_1}(C_2)$	C_2
$0.5(1 - \epsilon + \epsilon) = 0.5$	0
$0.5(\epsilon + 1 - \epsilon) = 0.5$	1

Step7: Combining all factors of C_2

Therefore, now that we know $\text{elim}_{C_1}(C_2)$, $\text{cond}_{D_2=0}(C_2)$ and $\text{elim}_{C_3}(C_1)$,

$$p(C_2/D_2 = 0, D_3 = 1) = \text{elim}_{C_1}(C_2) * \text{cond}_{D_2=0}(C_2) * \text{elim}_{C_3}(C_2)$$

$p(C_2/D_2 = 0, D_3 = 1)$	C_2
$0.5((1 - \epsilon)\eta + \eta(1 - \epsilon))(1 - \eta)$	0
$0.5(\epsilon\eta + (1 - \eta)(1 - \epsilon))\eta$	1

Therefore,

$$\begin{aligned} P(C_2 = 1/D_2 = 0, D_3 = 1) &= \frac{0.5(\epsilon\eta + (1 - \eta)(1 - \epsilon))\eta}{0.5(\epsilon\eta + (1 - \eta)(1 - \epsilon))\eta + 0.5((1 - \epsilon)\eta + \epsilon(1 - \eta))(1 - \eta)} \\ &= \frac{(\epsilon\eta + (1 - \eta)(1 - \epsilon))\eta}{(\epsilon\eta + (1 - \eta)(1 - \epsilon))\eta + ((1 - \epsilon)\eta + \epsilon(1 - \eta))(1 - \eta)} \end{aligned}$$

(c) i.

$$\begin{aligned} P(C_2 = 1/D_2 = 0) &= 0.2 \\ P(C_2 = 1/D_2 = 0, D_3 = 1) &= 0.4157 \end{aligned}$$

- ii. Adding second sensor reading increased the probability from 0.2 to 0.4157. Since D_3 is equal to 1, it means we observed the location to be 1 at location 3. This would increase the probability of $C_3 = 1$ since the emission probability $p(d_t/c_t)$ favours similar values with higher probability. $C_3 = 1$ increases the probability of $C_2 = 1$, since the transition probability $p(c_t/c_{t-1})$ favours same location with higher probability.
- iii. Both the probabilities would be same when the sensor reading at D_3 doesn't matter. This won't matter when the transition probabilities $p(c_t/c_{t-1})$ are equal meaning no matter what is the value of c_3 out of all the possible values, we will get constant transition probability. This would happen when $\epsilon = 1 - \epsilon$, therefore when $\epsilon = 0.5$.

Problem 2

(a) (your solution)

(b) (your solution)