CS221 Fall 2018 Homework [scheduling]

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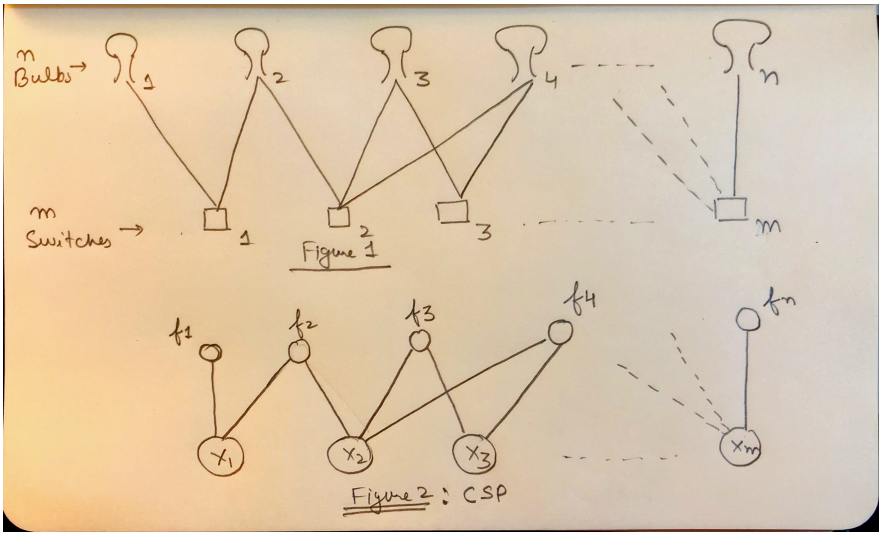
By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

Problem 0

(a) The problem statement can be visualised as below:



Variables: The variables for the CSP are the m switches, $X_1, X_2...X_m$. The domain of these variables are Domains $\epsilon\{0, 1\}$, 0 being the "off" state for a switch and 1 being the "on" state.

Constraints: The constraints or the factors are created among a bulb and it's controlling switch(es). For example, from above Figure 1, if switch 1 controls bulb 1 and bulb 2, and switch 2 controls bulb 2 and bulb 3, a factor corresponding to bulb 1 would have switch 1 value as it's parameter (be dependent on switch 1) and bulb 2 would have values of both switch 1 and switch 2 as it's parameters. Since T_j (for each button $j = 1, ...m$) defines every set of light bulbs a switch controls, factor f_k (for each light bulb $k = 1, ...n$) would depend on variable j if k in T_j. The value of this factor should return an odd number, so that even numbers render the state of the bulb to be "off" and last number makes the bulb "on".

$$f_k(X_1[k \text{ in } T_1], X_2[k \text{ in } T_2], ...X_m[k \text{ in } T_m]) \\ = sum(X_1[k \text{ in } T_1], X_2[k \text{ in } T_2], ...X_m[k \text{ in } T_m])\%2 == 1$$

Table 1: Table depicting unigram costs assigned to words

x1	x2	x3	t1(x)	t2(x)	**Consistency**
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	0	1	0
1	1	1	0	0	0

(b) (a) For finding consistent assignments, we draw the table for finding values of t_1 and t_2. From the Table 1, we can see that there are two consistent solutions for x_1, x_2, x_3, one being $\{0, 1, 0\}$ and other being $\{1, 0, 1\}$, respectively.

(c) For fixed variables X_1, X_2, X_3, backtrack will be called in following ways:

$$
\begin{aligned}
backtrack(\phi, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\}) \\
backtrack(\{x_1 : 0\}, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\}) \\
backtrack(\{x_1 : 0, x_3 : 0\}, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\}) \\
backtrack(\{x_1 : 0, x_2 : 1, x_3 : 0\}, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\}) \\
backtrack(\{x_1 : 0, x_3 : 1\}, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\}) \\
backtrack(\{x_1 : 1\}, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\}) \\
backtrack(\{x_1 : 1, x_3 : 0\}, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\}) \\
backtrack(\{x_1 : 1, x_3 : 1\}, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\}) \\
backtrack(\{x_1 : 1, x_2 : 0, x_3 : 1\}, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\})
\end{aligned}
$$

Therefore, backtrack algorithm is called 9 times.

(d) When lookahead is enabled (AC3):

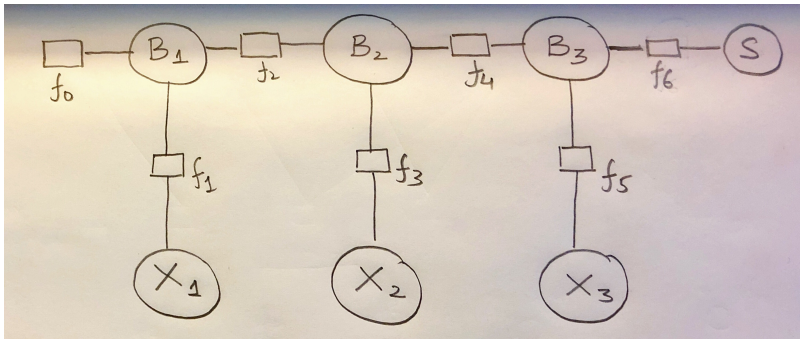
$$
\begin{aligned}
backtrack(\phi, 1, \{x_1 : [0,1], x_2 : [0,1], x_3 : [0,1]\}) \\
backtrack(\{x_1 : 0\}, 1, \{x_1 : [0], x_2 : [1], x_3 : [0]\}) \\
backtrack(\{x_1 : 0, x_3 : 0\}, 1, \{x_1 : [0], x_2 : [1], x_3 : [0]\}) \\
backtrack(\{x_1 : 0, x_2 : 1 x_3 : 0\}, 1, \{x_1 : [0], x_2 : [1], x_3 : [0]\}) \\
backtrack(\{x_1 : 1\}, 1, \{x_1 : [1], x_2 : [0], x_3 : [1]\}) \\
backtrack(\{x_1 : 1, x_3 : 1\}, 1, \{x_1 : [1], x_2 : [0], x_3 : [1]\}) \\
backtrack(\{x_1 : 1, x_2 : 0 x_3 : 1\}, 1, \{x_1 : [1], x_2 : [0], x_3 : [1]\})
\end{aligned}
$$

Therefore, backtrack algorithm with AC3 is called 7 times.

Problem 2

(a) **Reducing the CSP:** In order to reduce an N-ary CSP to unary or binary factors, we can think of introducing another set of variables (B_1, B_2, B_3 and S) called auxiliary variables to keep a track of total sum of variables X_1, X_2 and X_3 as we traverse in the graph.

Auxiliary variables introduction: Following is the graphical representation of auxiliary variables B_1, B_2, B_3 and S and given variables X_1, X_2 and X_3.



Domains: X_1, X_2 and X_3 are the given variables with domains $\{0, 1, 2\}$ each. As discussed earlier, auxiliary variables B_1, B_2 and B_3 keep a track of the total sum. Therefore, these are two dimensional, one element carrying the total sum of previous auxiliary variable and other keeping a record after adding value of attached variable X_i. Both $B_n[0]$ and $B_n[1]$ will have a range from $0, 1, ...K$, where K being the maximum sum constraint. S is the sum variable, having domain $0, 1, ...K$

Constraints: There are going to be unary and binary constraints as follows(referencing above image):

f_0 states that $B_1[0]$ should be 0, since we start with sum = 0. Therefore, $f_0(B1) = B_1[0] == 0$

f_1, f_3 and f_5 make sure that adding X_i to first coordinate of B_i gives us second coordinate of B_i. Hence, $f_1(B_1, X_1) = B_1[0] + X_1 == B_1[1]$, similarly for $f_3(B_2, X_2) = B_2[0] + X_2 == B_2[1]$, and for $f_5(B_3, X_3) = B_3[0] + X_3 == B_3[1]$.

f_2 and f_4 make sure that first index of B_{i+1} is equal to last index of B_i. Hence, $f_2(B_2, B_1) = B_2[0] == B_1[1]$ and $f_4(B_3, B_2) = B_3[0] == B_2[1]$.

f_6 maintains that last index of B_3 and the value of sum variable S are equal. Therefore, $f_6(B_3, S) = B_3[1] == S$.

This scheme works because it has added maximum value constraint over the sum of all three variables through auxiliary sum variable S, along with propagating sum of variables one by one from X_1 to X_3.