CS221 Fall 2018 Homework [car]

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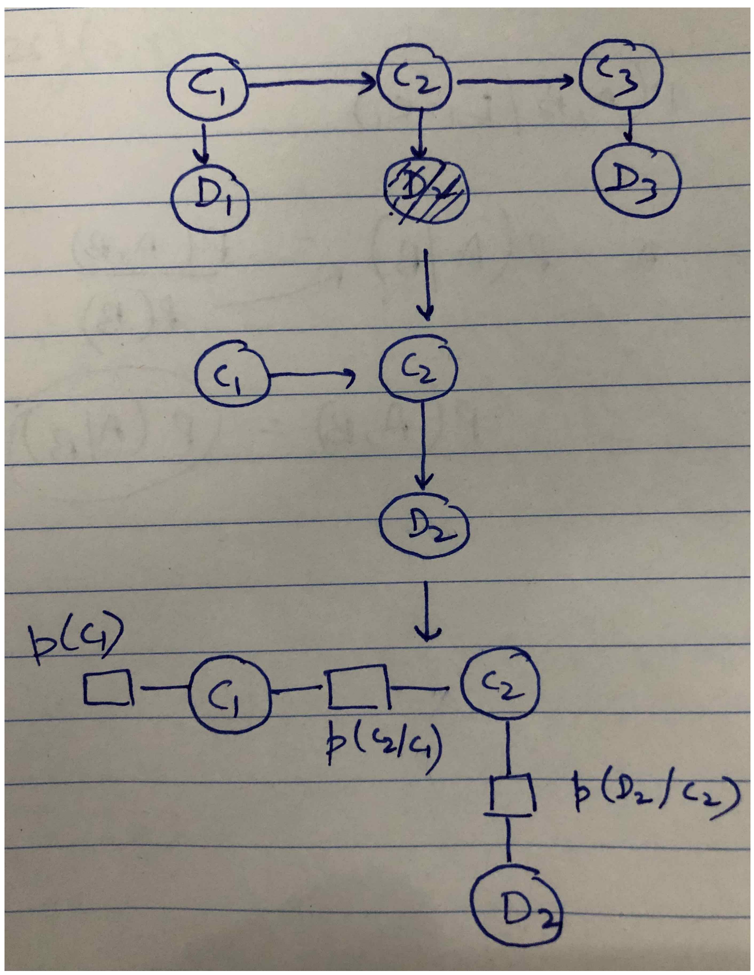
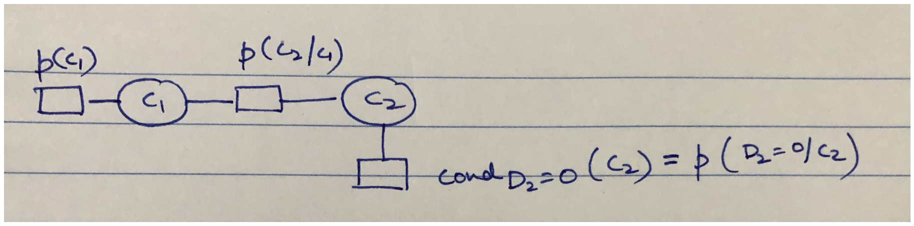
By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

Problem 1

(a) **Step 1: Remove variables that are not ancestors**
Step 2: Converting to factor graph
Step 1 and step 2 are shown in diagram below:



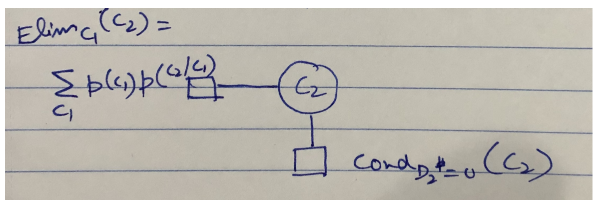
Step3: Conditioning on $D_2 = 0$



Condition variable D_2 on value $D_2 = 0$, replacing it with a factor $\text{cond}_{D_2=0}(C_2)$, we get

$\text{cond}_{D_2=0}(C_2)$	C_2
$1-\eta$	0
η	1

Step4: Eliminate C_1



$$\begin{aligned} elim_{C_1}(C_2) &= \sum_{C_1} p(C_1)p(C_2/C_1) \\ &= 0.5 \sum_{C_1} p(C_2/C_1) \end{aligned}$$

This is given from the below table:

$\text{elim}_{C_1}(C_2)$	C_2
$0.5(1-\epsilon+\epsilon) = 0.5$	0
$0.5(\epsilon+1-\epsilon) = 0.5$	1

Therefore, now that we know $\text{elim}_{C_1}(C_2)$ and $\text{cond}_{D_2=0}(C_2)$,

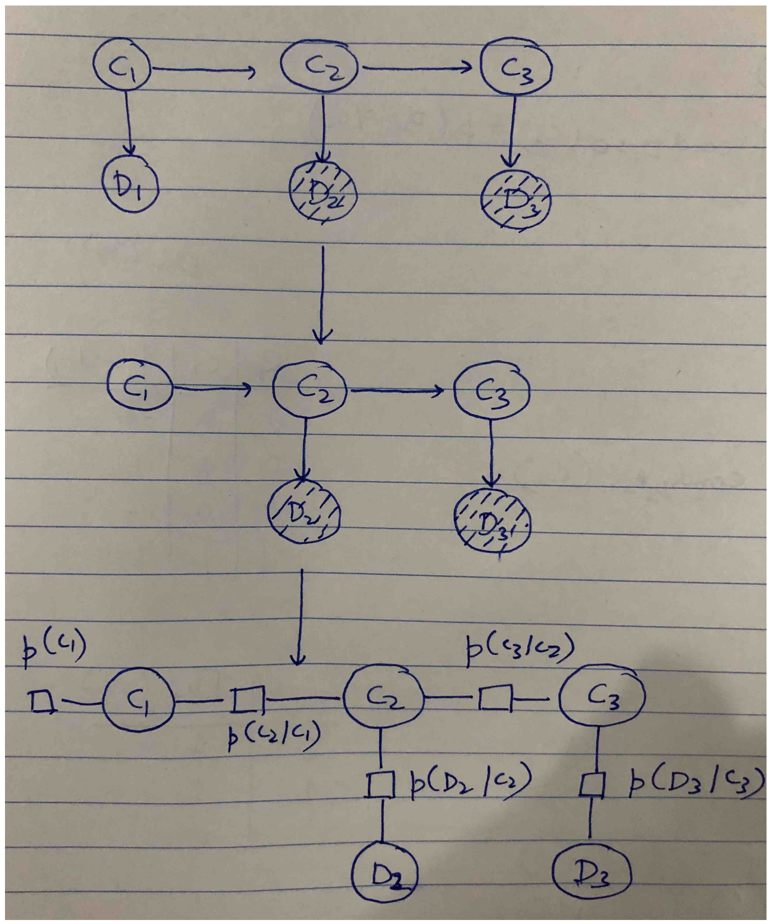
$$p(C_2/D_2=0) = \text{elim}_{C_1}(C_2) * \text{cond}_{D_2=0}(C_2)$$

$p(C_2/D_2=0)$	C_2
$0.5(1-\eta)$	0
0.5η	1

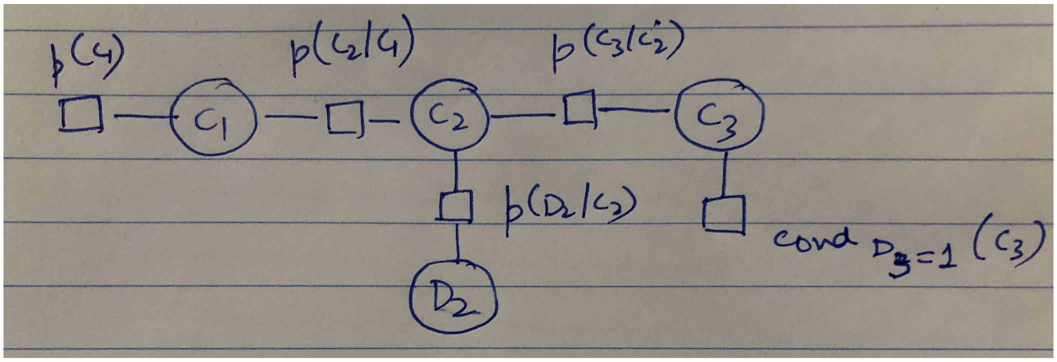
Hence, the given query,

$$p(C_2 = 1/D_2 = 0) = \frac{0.5\eta}{0.5\eta + 0.5(1-\eta)}$$
$$= \eta$$

(b) **Step1: Remove variables that are not ancestors**
Step2: Converting to factor graph
Step 1 and step 2 are illustrated below:



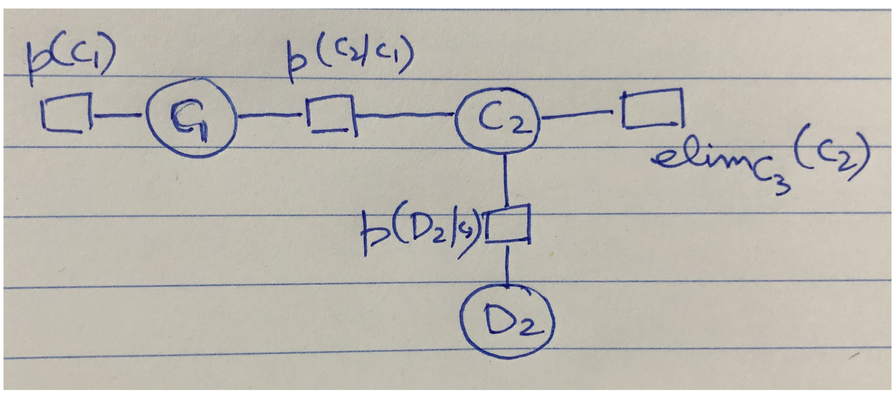
Step3: Conditioning on $D_3 = 1$



Conditioning on variable D_3, and replacing it with a factor $\text{cond}_{D_3=1}(C_3)$, we get

$\text{cond}_{D_3=1}(C_3)$	C_3
η	0
$1-\eta$	1

Step4: Eliminating C_3



Defining function $\text{elim}_{C_3}(C_2)$ in order to eliminate node C_3 as

$$\text{elim}_{C_3}(C_2) = \sum_{C_3} \text{cond}_{D_3=1}(C_3) p(C_3/C_2)$$

The probability distribution $p(C_3/C_2)$ is given by:

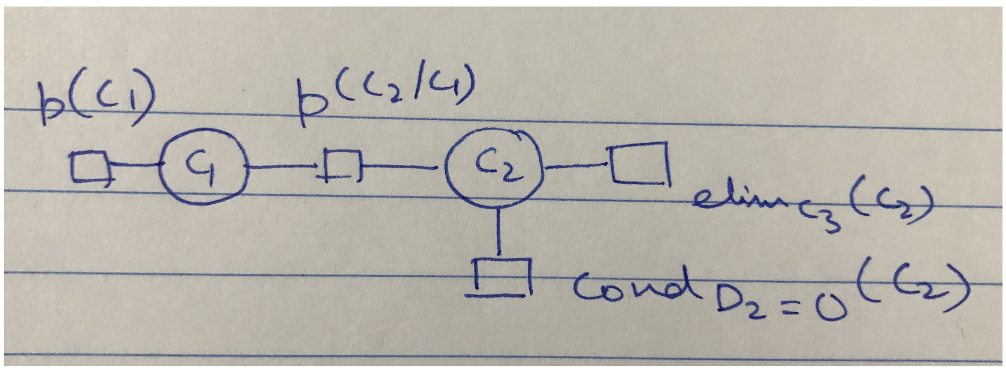
C2	C3	p(C3/C2)
0	0	$1-\epsilon$
0	1	ϵ
1	0	ϵ
1	1	$1-\epsilon$

The probability distribution $\text{cond}_{D_3=1}(C_3)$ is defined in Step 3.

Combining both and substituting in equation 1, and doing summation over values of C_3, we will have probability distribution of $\text{elim}_{C_3}(C_2)$ is given by:

C_2	$\text{elim}_{C_3}(C_2)$
0	$(1-\epsilon)\eta + \epsilon(1-\eta)$
1	$\epsilon\eta + (1-\eta)(1-\epsilon)$

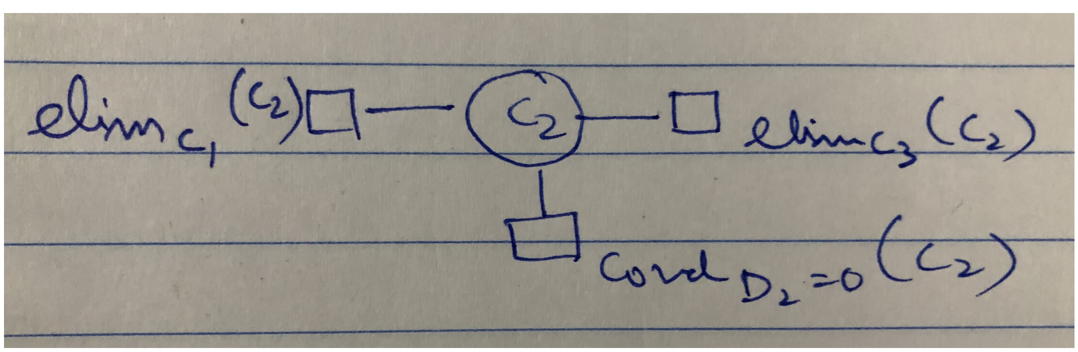
Step5: Conditioning on $D_2 = 0$



Condition variable D_2 on value $D_2 = 0$, replacing it with a factor $\text{cond}_{D_2=0}(C_2)$, we get

$\text{cond}_{D_2=0}(C_2)$	C_2
$1-\eta$	0
η	1

Step6: Eliminate C_1



$$
\begin{aligned}
elim_{C_1}(C_2) &= \sum_{C_1} p(C_1)p(C_2/C_1) \\
&= 0.5 \sum_{C_1} p(C_2/C_1)
\end{aligned}
$$

This is given from the below table:

$\text{elim}_{C_1}(C_2)$	C_2
$0.5(1-\epsilon+\epsilon)=0.5$	0
$0.5(\epsilon+1-\epsilon)=0.5$	1

Step7: Combining all factors of C_2

Therefore, now that we know $\text{elim}_{C_1}(C_2)$, $\text{cond}_{D_2=0}(C_2)$ and $\text{elim}_{C_3}(C_1)$,

$$p(C_2/D_2=0, D_3=1) = \text{elim}_{C_1}(C_2) * \text{cond}_{D_2=0}(C_2) * \text{elim}_{C_3}(C_2)$$

$p(C_2/D_2=0, D_3=1)$	C_2
$0.5((1-\epsilon)\eta+\eta(1-\epsilon))(1-\eta)$	0
$0.5(\epsilon\eta+(1-\eta)(1-\epsilon))\eta$	1

Therefore,

$$
\begin{aligned}
P(C_2=1/D_2=0, D_3=1) &= \frac{0.5(\epsilon\eta+(1-\eta)(1-\epsilon))\eta}{0.5(\epsilon\eta+(1-\eta)(1-\epsilon))\eta + 0.5((1-\epsilon)\eta+\epsilon(1-\eta))(1-\eta)} \\
&= \frac{(\epsilon\eta+(1-\eta)(1-\epsilon))\eta}{(\epsilon\eta+(1-\eta)(1-\epsilon))\eta + ((1-\epsilon)\eta+\epsilon(1-\eta))(1-\eta)}
\end{aligned}
$$

(c) i.

$$
\begin{aligned}
P(C_2=1/D_2=0) &= 0.2 \\
P(C_2=1/D_2=0, D_3=1) &= 0.4157
\end{aligned}
$$

ii. Adding second sensor reading increased the probability from 0.2 to 0.4157. Since D_3 is equal to 1, it means we observed the location to be 1 at location 3. This would increase the probability of $C_3=1$ since the emission probability $p(d_t/c_t)$ favours similar values with higher probability. $C_3=1$ increases the probability of $C_2=1$, since the transition probability $p(c_t/c_{t-1})$ favours same location with higher probability.

iii. Both the probabilities would be same when the sensor reading at D_3 doesn't matter. This won't matter when the transition probabilities $p(c_t/c_{t-1})$ are equal meaning no matter what is the value of c_3 out of all the possible values, we will get constant transition probability. This would happen when $\epsilon = 1-\epsilon$, therefore when $\epsilon = 0.5$.

Problem 5

(a) Simplifying the equation $P(C_{11}, C_{12}|E_1 = e_1)$ using the Bayes Theorem, we have

$$P(C_{11}, C_{12}|E_1 = e_1) = P(C_{11}/E_1 = e_1|C_{12}/E_1 = e_1) * P(C_{12}/E_1 = e_1)$$

Since the location of both the cars are independent of each other (C_{11} doesn't depend in any way on C_{12} and vice versa), therefore, $C_{11}/E_1 = e_1$ and $C_{12}/E_1 = e_1$ are independent events.

$$\begin{aligned} P(C_{11}, C_{12}|E_1 = e_1) &= P(C_{11}/E_1 = e_1|C_{12}/E_1 = e_1) * P(C_{12}/E_1 = e_1) \\ &= P(C_{11}/E_1 = e_1) * P(C_{12}/E_1 = e_1) \\ &\propto p(c_{11})p_N(e_{11}, ||a_1 - c_{11}||, \sigma^2)p(c_{12})p_N(e_{12}, ||a_1 - c_{12}||, \sigma^2) \end{aligned}$$

(b) (your solution)