CS221 Fall 2018 Homework [scheduling]

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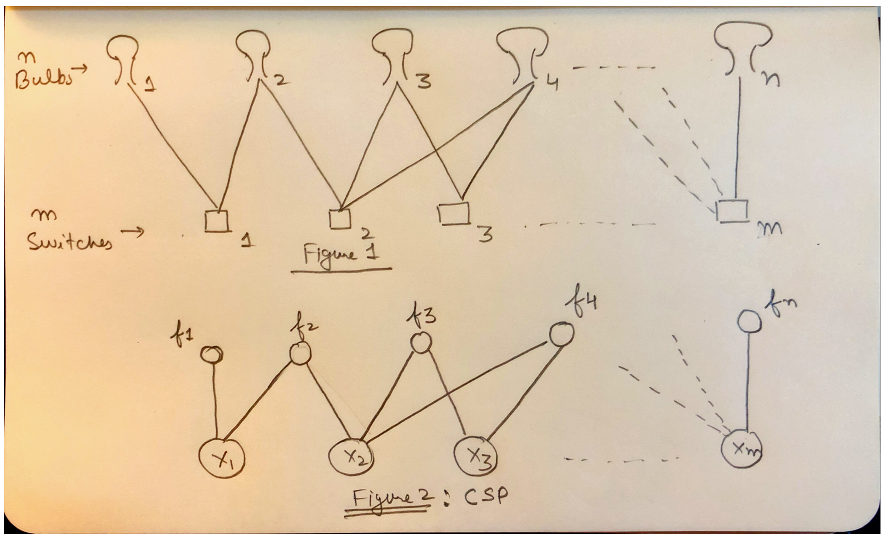
By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

Problem 0

(a) The problem statement can be visualised as below:



Variables: The variables for the CSP are the m switches, $X_1, X_2 ... X_m$. The domain of these variables are Domains $\epsilon \{0, 1\}$, 0 being the "off" state for a switch and 1 being the "on" state.
Constraints: The constraints or the factors are created among a bulb and it's controlling switch(es). For example, from above Figure 1, if switch 1 controls bulb 1 and bulb 2, and switch 2 controls bulb 2 and bulb 3, a factor corresponding to bulb 1 would have switch 1 value as it's parameter (be dependent on switch 1) and bulb 2 would have values of both switch 1 and switch 2 as it's parameters. Since T_j (for each button $j = 1, ...m$) defines every set of light bulbs a switch controls, factor f_k (for each light bulb $k = 1, ...n$) would depend on variable j if k in T_j. The value of this factor should return an odd number, so that even numbers render the state of the bulb to be "off" and last number makes the bulb "on".

$$f_k(X_1[k \text{ in } T_1], X_2[k \text{ in } T_2], ...X_m[k \text{ in } T_m])$$
$$= sum(X_1[k \text{ in } T_1], X_2[k \text{ in } T_2], ...X_m[k \text{ in } T_m])\%2 == 1$$

Table 1: Table depicting consistency calculation for XOR operation

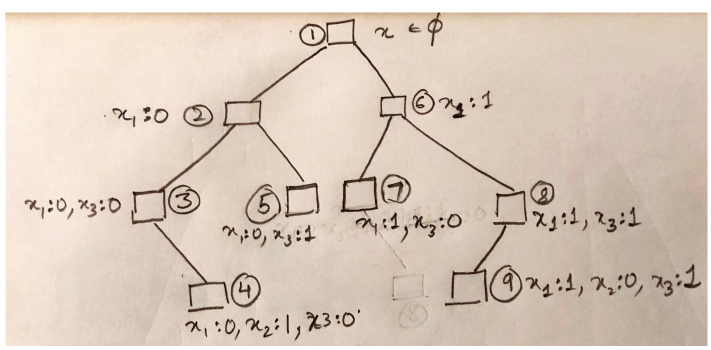
x1	x2	x3	t1(x)	t2(x)	**Consistency**
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	0	1	0
1	1	1	0	0	0

(b) i. For finding consistent assignments, we draw the table for finding values of t_1 and t_2. From the Table 1, we can see that there are two consistent solutions for x_1, x_2, x_3, one being $\{0, 1, 0\}$ and other being $\{1, 0, 1\}$, respectively.

ii. For fixed variables X_1, X_2, X_3, backtrack will be called in following ways:

$$backtrack(\phi, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{1}$$
$$backtrack(\{x_1 : 0\}, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{2}$$
$$backtrack(\{x_1 : 0, x_3 : 0\}, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{3}$$
$$backtrack(\{x_1 : 0, x_2 : 1, x_3 : 0\}, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{4}$$
$$backtrack(\{x_1 : 0, x_3 : 1\}, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{5}$$
$$backtrack(\{x_1 : 1\}, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{6}$$
$$backtrack(\{x_1 : 1, x_3 : 0\}, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{7}$$
$$backtrack(\{x_1 : 1, x_3 : 1\}, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{8}$$
$$backtrack(\{x_1 : 1, x_2 : 0, x_3 : 1\}, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{9}$$

Therefore, backtrack algorithm is called 9 times. Here's the graph visually:



Have numbered each call stack for backtrack but due to space constraints haven't wrote down domains in the drawn graph. Domains will remain same at every step as defined in callstack $(\{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\})$.

iii. When lookahead is enabled (AC3), domains are changed on each step. Here's the callstack:

$$backtrack(\phi, 1, \{x_1 : [0, 1], x_2 : [0, 1], x_3 : [0, 1]\}) \tag{1}$$

$$backtrack(\{x_1 : 0\}, 1, \{x_1 : [0], x_2 : [1], x_3 : [0]\}) \tag{2}$$

$$backtrack(\{x_1 : 0, x_3 : 0\}, 1, \{x_1 : [0], x_2 : [1], x_3 : [0]\}) \tag{3}$$

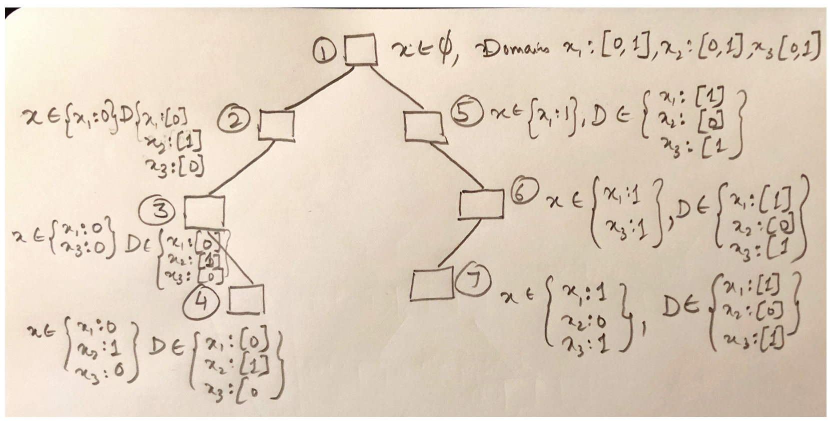
$$backtrack(\{x_1 : 0, x_2 : 1x_3 : 0\}, 1, \{x_1 : [0], x_2 : [1], x_3 : [0]\}) \tag{4}$$

$$backtrack(\{x_1 : 1\}, 1, \{x_1 : [1], x_2 : [0], x_3 : [1]\}) \tag{5}$$

$$backtrack(\{x_1 : 1, x_3 : 1\}, 1, \{x_1 : [1], x_2 : [0], x_3 : [1]\}) \tag{6}$$

$$backtrack(\{x_1 : 1, x_2 : 0x_3 : 1\}, 1, \{x_1 : [1], x_2 : [0], x_3 : [1]\}) \tag{7}$$

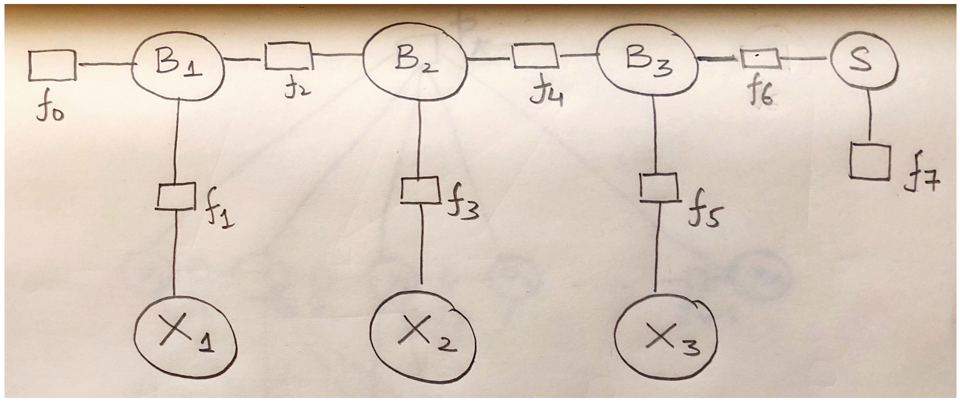
Therefore, backtrack algorithm with AC3 is called 7 times. Here's the drawn graph with domains and assignments (numbered 1 to 7):



Problem 2

(a) **Reducing the CSP:** In order to reduce an N-ary CSP to unary or binary factors, we can think of introducing another set of variables (B_1, B_2, B_3 and S) called auxiliary variables to keep a track of total sum of variables X_1, X_2 and X_3 as we traverse in the graph.

Auxiliary variables introduction: Following is the graphical representation of auxiliary variables B_1, B_2, B_3 and S and given variables X_1, X_2 and X_3.



Domains: X_1, X_2 and X_3 are the given variables with domains $\{0, 1, 2\}$ each. As discussed earlier, auxiliary variables B_1, B_2 and B_3 keep a track of the total sum. Therefore, these are two dimensional, one element carrying the total sum of previous auxiliary variable and other keeping a record after adding value of attached variable X_i. Both $B_n[0]$ and $B_n[1]$ will have a range from $0, 1, ...M$, where M being the maximum sum (for our case where K is the maximum sum, $M \geq K$). S is the sum variable, having domain $0, 1, ...M$ ($M \geq K$) since it tries to bind the variables based on total sum.

Constraints: There are going to be unary and binary constraints as follows(referencing above image):

f_0 states that $B_1[0]$ should be 0, since we start with sum = 0. Therefore, $f_0(B1) = B_1[0] == 0$

f_1, f_3 and f_5 make sure that adding X_i to first coordinate of B_i gives us second coordinate of B_i. Hence, $f_1(B_1, X_1) = B_1[0] + X_1 == B_1[1]$, similarly for $f_3(B_2, X_2) = B_2[0] + X_2 == B_2[1]$, and for $f_5(B_3, X_3) = B_3[0] + X_3 == B_3[1]$.

f_2 and f_4 make sure that first index of B_{i+1} is equal to last index of B_i. Hence, $f_2(B_2, B_1) = B_2[0] == B_1[1]$ and $f_4(B_3, B_2) = B_3[0] == B_2[1]$.

f_6 maintains that last index of B_3 and the value of sum variable S are equal. Therefore, $f_6(B_3, S) = B_3[1] == S$.

Finally, we can have a constraint on the sum variable S, such that it is less than or equal to K. Therefore, this is represented by f_7 by $f_7(S) = S <= K$.

This scheme works because it has added maximum value constraint over the sum of all three variables through auxiliary sum variable S, along with propagating sum of variables one by one from X_1 to X_3 using B_1, B_2 and B_3.

Problem 3

(c) Since I am planning to pursue graduate certificate by taking up courses every quarter(except summer break), with propensity towards CS221 and CS224N, I listed out all the courses offered and let the system suggest what courses should I pick up when. Here's the profile I created:

```
# Unit limit per quarter. You can ignore this for the first
# few questions in problem 2.
minUnits 3
maxUnits 3

# These are the quarters that I need to fill. It is assumed that
# the quarters are sorted in chronological order.
register Aut2018
register Win2019
register Spr2019
register Aut2019

# Courses I've already taken
taken CS107
taken CS103
taken CS106X
taken CS106B
taken CS103
taken CS124

# Courses that I'm requesting
request CS221 in Aut2018 weight 2
request CS157
request CS223A
request CS224N in Aut2019 weight 2
request CS228
request CS229
request CS231A
request CS227B
```

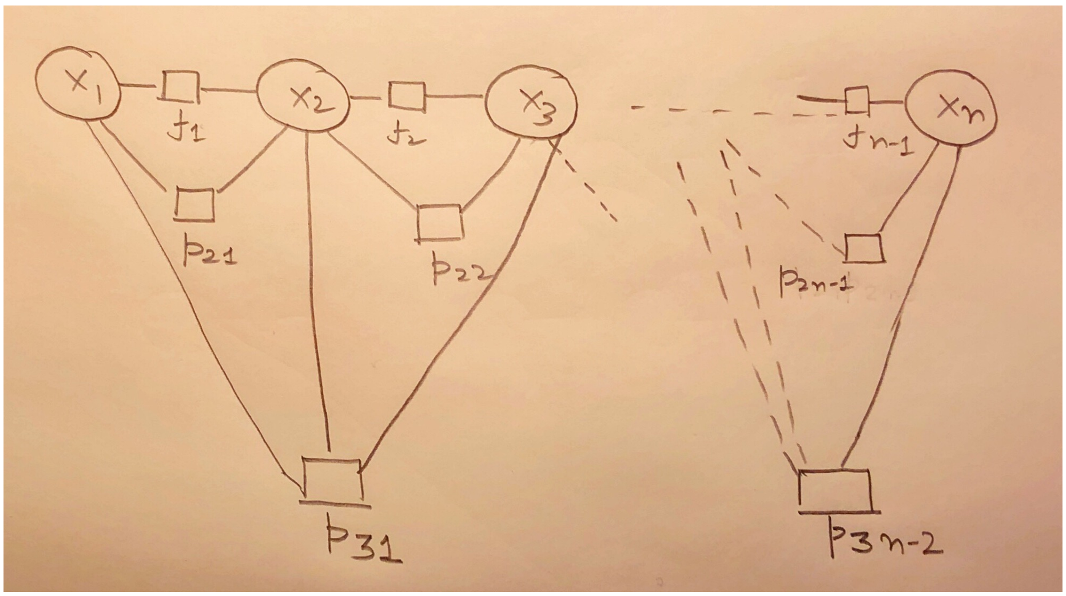
The best schedule that the system suggested is depicted in Table 2. It makes sense because I had given higher weights to CS221 and CS224N, since I personally wanted to take these courses up with a vision to finish the graduate certificate as soon as possible. As I am working, I wanted to restricted the courses to 1 per quarter, therefore added maxUnits and minUnits for this profile as 3.

Table 2: Table Depicting best schedule suggested by system

Quarter	Units	Course
Aut2018	3	CS221
Win2019	3	CS223A
Spr2019	3	CS227B
Aut2019	3	CS224N

Problem 4

(a) If we were to include notable patterns as factors to CSP, the graph would look like the following:



where X_1, X_2 ... X_n are variables, consecutive pairs connected by factors f_1, f_2 ... f_{n-1}. The notable patterns, when introduced as constraints, have arity based on the length of the pattern. For example, in the above image, one type of constraints are binary constraints(p_{21}, p_{22} ... p_{2n-1}) occurring over each consecutive pair, checking for existence of a sequence [x, y] being satisfied. Other type are ternary constraints occurring on all possible 3 consecutive variables(p_{31}, p_{32} ... p_{3n-2}).
In worst case scenario would be a notable pattern constraint expecting all variables to take a particular value (meaning having length equal to number of variables). Tree width in this case would be n, since variable elimination cannot be done with any of the variables as the pairs are connected via multiple constraints.

(b) Let p_t be the factors introduced by notable patterns. These are N-ary constraints, added to all sequences of variables. They have arity based on the length of the pattern(described in 4a). The algorithm would be as follows:

best_weight = 0
best_assignment = {}
max_weight_assignment(x, f, n, Domains)

If x is complete assignment and $f\gamma^n$ is greater than best_weight :

best_weight = $f\gamma^n$ and best_assignment = x and return

Choose unassigned VARIABLE X_i which has least consistent values

Order VALUES $Domain_i$ of chosen X_i through least constrained value

For each value v in that order:

$\delta \leftarrow \pi_{f_j \epsilon D(x,X_i)} f_j(x \bigcup \{X_i : v\})$

If $\delta = 0$: continue

$N \leftarrow \sum_{p_t \epsilon D(x,X_i)} p_j(x \bigcup \{X_i : v\})$

Domains' ← Domains via AC-2

max_weight_assignment$(x \bigcup \{X_i : v\}, w\delta, n + N, Domains')$

max_weight_assignment({}, 1, 0, { $X_1 : [1, 2...K]$, ..., $X_n : [1, 2...K]$})

The algorithm starts by checking if x(current assignment) is a complete assignment. If yes and the weight (which is $f\gamma^n$) is greater than best_weight until that point, then it updates the current best_weight and best_assignment with $f\gamma^n$ and x respectively. Else, it continues and picks up an unassigned variable which has fewest consistent values(most constrained variable), and then picks up values based on least constrained value(order by least constrained value). Then for each value v it calculates the product of new factors f_j which are dependent on the new assignment after $X_i = v$ is added to the existing assignment x. If $\delta = 0$, then we skip the current assignment else we continue and find out the value of N which is sum of the new notable pattern factors p_j which are dependent on newly assigned variable. Furthermore, we update the domains via AC-3. And recursively call the max_weight_assignment function with new assignment, multiplying existing f_i factors weight f by δ, add N to observed sequences previously and with the updated domains via AC-3. The algorithm is initialised with empty assignment, 1 as factor weight of f, 0 as sum of notable patterns and original domain for every variable $X_i : [1, 2...K]$.

Time complexity would be $O(K^n)$, since it can be visualised as a graph with depth n and branching factor K. Space complexity would be $O(n)$ since we store the assignments as we perform them.