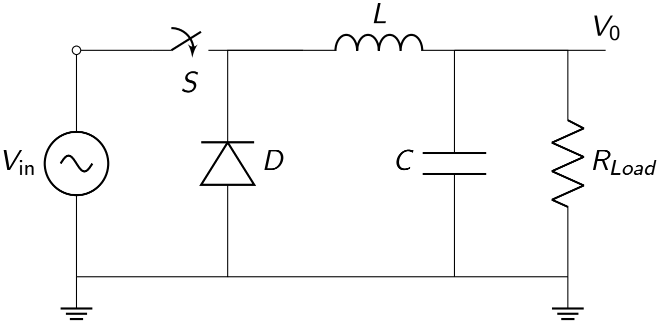
BUCK FINAL

EE23BTECH11210, EE23BTECH11215, EE23BTECH11207

May 2025

BUCK CONVERTER



Buck Converter Operation

Consider the case when S1 is closed, i.e switch is ON:

$$V_L(on) = V_{in} - V_o$$

$$I_C(on) = I_L - I_o$$

where,
V_{in} : Input Voltage to the buck converter
V_o : Output Voltage
I_o : Output load current
V_L : Voltage across the inductor
I_L : Inductor current
I_c : capacitor Current
Now we will consider the case when the switch is open i.e switch is OFF:

$$V_L(off) = -V_o$$

$$I_c(off) = I_L - I_o$$

Finding the Output Voltage

We know that

$$V = L\frac{di}{dt}$$

Now consider over a time period T:

$$L\int_0^T di = \int_0^{DT} V_L(on).dt + \int_{DT}^T V_L(off).dt$$

$$L(i(T) - i(0)) = (V_{in} - V_o)(DT) + (-V_o)(T - DT)$$

$$i(T) = i(0)$$

$$V_o = D.V_{in}$$

Finding the Output Current

In steady state, the net charge stored on the capacitor over one switching period:

$$\int_0^T i_c(t)dt = 0$$

$$I_c(t) = I_L(t) - I_o$$

$$\int_0^T (I_L(t) - I_o)dt = 0$$

$$\int_0^T i_L(t)\,dt = I_0 \cdot T$$

$$I_o = \frac{1}{T}\int_0^T i_L(t)\,dt$$

So the output current is the average value of the current through the inductor over a time T:

$$I_o = I_L$$

Ripple in Inductor Current

Ripple in On state:

$$v_L = V_{\text{in}} - V_0 = V_{\text{in}}(1 - D)$$

$$\frac{di_L}{dt} = \frac{v_L}{L}$$

$$\Delta I_{L,\text{on}} = \frac{V_{\text{in}}(1 - D)}{L} \cdot DT$$

$$= \frac{D(1 - D)V_{\text{in}}}{L \times f}$$

Ripple in Off state:

$$v_L = -V_0 = -DV_{\text{in}}$$

$$\Delta I_{L,\text{off}} = \frac{-DV_{\text{in}}}{L} \cdot (1 - D)T$$

$$= \frac{-D(1 - D)V_{\text{in}}}{L \times f}$$

Inductor Current Extremes

$$I_{L,\max} = I_0 + \frac{\Delta I_L}{2}$$
$$I_{L,\min} = I_0 - \frac{\Delta I_L}{2}$$

- I_L: Current through the inductor
- ΔI_L: Peak-to-peak inductor current ripple

Ripple in Capacitor Current:

$$I_c(t) = I_L(t) - I_0$$

$$\Delta I_C = \Delta I_L$$

Voltage Ripple

OFF TIME

$$\Delta Q = \frac{1}{2} \cdot \Delta I_L \cdot (1-D)T$$

$$\Delta V_0 = \frac{\Delta Q}{C} = \frac{1}{2} \cdot \frac{\Delta I_L \cdot (1-D)T}{C}$$

Substitute:

$$\Delta I_L = \frac{(1-D)DV_{\text{in}}}{fL}$$

Then:

$$\Delta V_0 = \frac{1}{2} \cdot \frac{(1-D)DV_{\text{in}}}{fL} \cdot \frac{(1-D)T}{C}$$

$$= \frac{D(1-D)^2 V_{\text{in}}}{2f^2 LC}$$

Solve for critical capacitance:

$$C_c = \frac{D(1-D)^2 V_{\text{in}}}{2f^2 L \Delta V_0}$$

ON TIME

$$\Delta Q = \frac{1}{2} \cdot \Delta I_L \cdot DT$$

$$\Delta V_0 = \frac{\Delta Q}{C} = \frac{1}{2} \cdot \frac{\Delta I_L \cdot DT}{C}$$

Substitute:

$$\Delta I_L = \frac{(1 - D)DV_{\text{in}}}{fL}$$

Then:

$$\begin{aligned} \Delta V_0 &= \frac{1}{2} \cdot \frac{(1 - D)DV_{\text{in}}}{fL} \cdot \frac{DT}{C} \\ &= \frac{D^2(1 - D)V_{\text{in}}}{2f^2LC} \end{aligned}$$

Total Ripple:

$$\Delta V_0 = \frac{D^2(1 - D)V_{\text{in}}}{2f^2LC}$$

Critical Inductance and Capacitance

Why Critical Inductance L_c?

To ensure Continuous Conduction Mode (CCM) operation:

$$I_0 = \frac{\Delta I_L}{2}$$

From output current:

$$I_0 = \frac{V_0}{R_{Load}} = \frac{DV_{\text{in}}}{R_{Load}}$$

Ripple current:

$$\Delta I_L = \frac{(1-D)DV_{\text{in}}}{fL}$$

Equating:

$$\frac{DV_{\text{in}}}{R_{Load}} = \frac{1}{2} \cdot \frac{(1-D)DV_{\text{in}}}{fL_c}$$

$$\Rightarrow \boxed{L_c = \frac{(1-D)DV_{in}}{2I_o \times f}}$$

Critical Capacitance Derivation

Why Critical Capacitance C_c?

To limit voltage ripple:

$$\Delta Q = \frac{1}{2} \cdot \Delta I_L \cdot (1 - D)T$$

$$\Delta V_0 = \frac{\Delta Q}{C}$$

Substitute:

$$\Delta I_L = \frac{(1 - D)DV_{\text{in}}}{fL_c}, \quad T = \frac{1}{f}$$

Then:

$$\Delta V_0 = \frac{1}{2} \cdot \frac{(1 - D)DV_{\text{in}}}{fL_c} \cdot \frac{(1 - D)}{fC} = \frac{D(1 - D)^2 V_{\text{in}}}{2f^2 LC}$$

Solving for C:

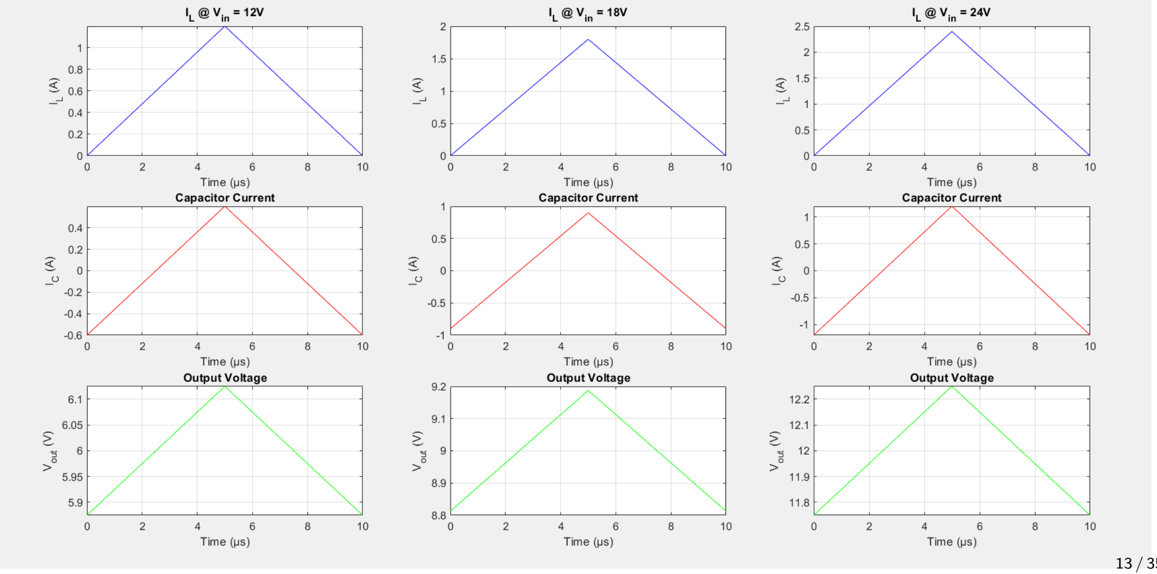
$$\boxed{C_c = \frac{D(1 - D)^2 V_{\text{in}}}{2f^2 L \Delta V_0}}$$

Calculated Values

V_{in} (V)	V_{out} (V)	I_{out} (A)	L (µH)	ΔI_L (A)	C (µF)	ΔV_{out} (V)	C_c (µF)	L_c (µH)
12	6	0.6	25	1.2	6.0	0.25	6	25
18	9	0.9	25	1.8	13.5	0.375	9	25
24	12	1.2	25	2.4	24.0	0.50	12	25

Table: Calculated Buck Converter Parameters for Various Input Voltages

Matlab simulations



LT spice values

V_{in} (V)	V_{out} (V)	I_{out} (A)	L (μH)	ΔI_L (A)	C (μF)	ΔV_{out} (V)	C_c (μF)	L_c (μH)
12	6	0.6	25	1.11	6.0	0.228	6	25
18	9	0.9	25	1.71	13.5	0.273	9	25
24	12	1.2	25	2.24	24.0	0.50	12	25

Table: Calculated Buck Converter Parameters for Various Input Voltages

Conclusions

- When the capacitance decreases the voltage ripple increases.
- When the Inductor decreases, the current ripple increases.
- If the inductance is less than the critical value the inductor enters the Discontinuous conduction mode

Advancements of Buck converter

Buck Converter Basics and Inductor Ripple Current

- Ideal output voltage:

$$V_o = D \cdot V_{\text{in}} \Rightarrow D = \frac{V_o}{V_{\text{in}}}$$

- D: Duty cycle, V_o: output voltage

$$\Delta I_L = \frac{(V_{\text{in}} - V_o) \cdot D \cdot T_s}{L} \Rightarrow \frac{V_o(V_{\text{in}} - V_o) \cdot T_s}{LV_{\text{in}}}$$

- ΔI_L increases with V_{in}
- Higher ripple increases the likelihood of DCM

CCM/DCM Boundary Condition and Non-Ideal MOSFET Effects

$$\frac{V_o}{R} = \frac{\Delta I_L}{2} \Rightarrow \frac{1}{R} = \frac{(V_{\text{in}} - V_o) \cdot T_s}{2LV_{\text{in}}}$$

Solving:

$$V_{\text{in}} = \frac{V_o}{1 - \frac{2L}{RT_s}}$$

- Non-ideal switch has finite $R_{\text{DS(on)}}$, related to W/L:

$$R_{\text{DS(on)}} \approx \frac{1}{\mu_n C_{\text{ox}} \frac{W}{L} (V_{GS} - V_{th})}$$

- Causes additional voltage drop when ON:

$$V_{\text{sw}} = I_L \cdot R_{\text{DS(on)}} \Rightarrow V_o = D \cdot (V_{\text{in}} - V_{\text{sw}})$$

Effect on Ripple and Mode with Design Implications

- Modified ripple:

$$\Delta I_L = \frac{(V_{\text{in}} - V_o - V_{\text{sw}}) \cdot D \cdot T_s}{L}$$

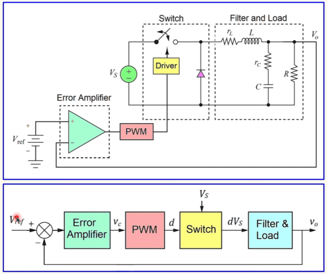
- $V_{\text{sw}} = I_L \cdot R_{\text{DS(on)}} \Rightarrow$ Depends on W/L
- Lower $W/L \Rightarrow$ Higher $R_{\text{DS(on)}} \Rightarrow$ Larger losses and ripple
- Increases tendency to enter DCM

- To minimize $R_{\text{DS(on)}}$, increase W/L
- Trade-off: Larger $W \rightarrow$ more gate capacitance $\rightarrow$ slower switching
- Select W/L based on:
  - Required efficiency
  - Expected load and switching frequency

Summary

- Higher V_{in} increases ripple ΔI_L
- Mode shifts from CCM to DCM if $\frac{V_o}{R} < \frac{\Delta I_L}{2}$
- MOSFET's $R_{\text{DS(on)}}$ worsens losses and ripple
- $R_{\text{DS(on)}} \propto \left(\frac{W}{L}\right)^{-1}$
- Choose optimal W/L to balance conduction loss and switching speed.

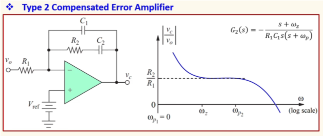
Controller Circuits



- Error Amplifier
- PWM
- Driver
- Switch
- Filter and Load

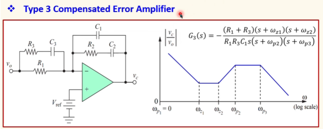
$$V_{\text{ref}}, \quad V_s, \quad V_o$$

Transfer functions: 1/3



Type 2 Compensator Amplifier

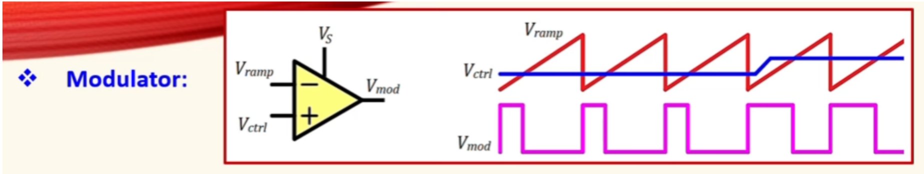
$$G_2(s) = -\frac{s+\omega_z}{R_1 C_1 s(s+\omega_p)}$$



Type 3 Compensator Amplifier

$$G_3(s) = -\frac{(R_1+R_3)(s+\omega_{z1})(s+\omega_{z2})}{R_1 R_3 C_1 s(s+\omega_{p2})(s+\omega_{p3})}$$

Transfer functions: 2/3



Modulator

$$V_{\text{mod}} = \frac{1}{T}\int_0^T v_{\text{mod}}(t)\,dt = \frac{1}{T}\int_0^{t_{\text{on}}} V_s\,dt$$

$$M(s) = \frac{dV_{\text{mod}}}{dV_{\text{ctrl}}} = \frac{V_s}{V_{\text{ramp}}}$$

Transfer functions: 3/3

Filter & Load

$$F(s) = \frac{V_o}{V_D} = \frac{1 + r_c Cs}{1 + \frac{r_L}{R} + \left(\frac{L}{R} + (r_L + r_c)C + \frac{r_L r_c C}{R}\right) s + \left(1 + \frac{r_c}{R}\right) LCs^2}$$

Compensator

$$G(s) = \frac{V_c}{V_o} = \frac{A_{OL}(s)}{1 + A_{OL}(s)\beta(s)} \approx \frac{1}{\beta(s)}$$

$$A_{OL}(s) = \frac{A_{DC}}{1 + \frac{s}{\omega_{pe}}}, \quad A_{OL}(s)\beta(s) \gg 1$$

Design 1: Specifications

Design Objective

- Type 3 Compensated Error Amplifier for a stable control system.
- Design for a crossover frequency of 10 kHz and a phase margin of 55.

Given:
- DC input voltage $V_S = 60\ \text{V}$
- Average output voltage $V_o = 15\ \text{V}$
- Average output current $I_o = 2\ \text{A}$
- Maximum percentage peak-peak output ripple voltage $\Delta V_o / V_o = 0.01\ (1\%)$

Buck Converter Component Values & Selections:

$L = 300\ \mu\text{H}$ $\quad r_L = 25\ \text{m}\Omega$ $\quad R = V_o / I_o = 7.5\ \Omega$

$C = 20\ \mu\text{F}$ $\quad r_C = 400\ \text{m}\Omega$ $\quad f_s = 100\ \text{kHz}$ (selected)

Design specifications

Given:

- $V_s = 60$ V, $V_o = 15$ V, $I_o = 2$ A
- $\frac{\Delta V_o}{V_o} = 0.01$

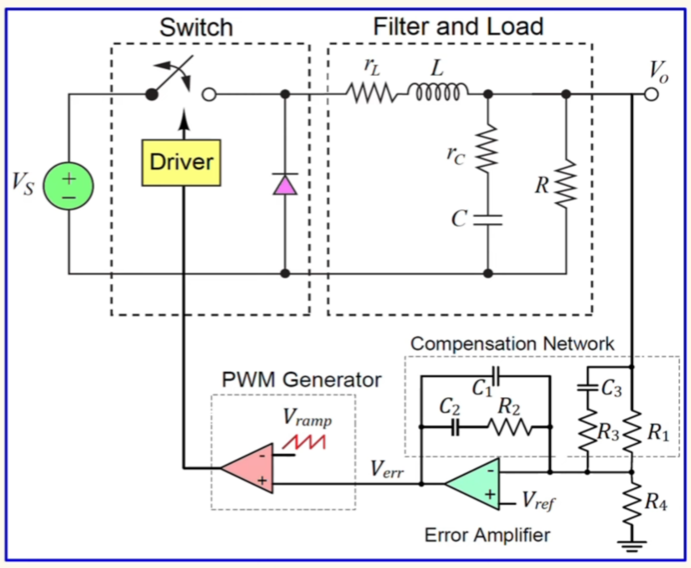
Component Values:

$$L = 300\ \mu H,\ r_L = 25\ m\Omega,\ R = \frac{V_o}{I_o} = 7.5\ \Omega,\ C = 20\ \mu F$$

and

$$r_C = 400\ m\Omega,\ f_s = 100\ \text{kHz}$$

Feedback loop system



Design analysis

$$L(s) = M(s)F(s)G_3(s)$$

$$G_3(s) \approx \frac{1}{R_3 C_1} \cdot \frac{(s + \frac{1}{R_1 R_3 C_3})(s + \frac{1}{R_2 C_2})}{s(s + \frac{1}{R_3 C_3})(s + \frac{1}{R_2 C_1})}$$

Phase Analysis

$$\arg(L(j\omega)) = \arg(M(j\omega)) + \arg(F(j\omega)) + \arg(G_3(j\omega))$$

$$\arg(M(j\omega)) = 0^\circ, \arg(F(j\omega_t)) = -146^\circ, PM = 180^\circ + \arg(L(j\omega_t)) = 55^\circ$$

$$\Rightarrow \arg(G_3(j\omega_t)) = 21^\circ, \phi_{\text{comp}} = 201^\circ$$

Gain Calculation

$$|L(j\omega)| = |M(j\omega)| \cdot |F(j\omega)| \cdot |G_3(j\omega)|$$

$$M = \frac{V_s}{V_{\text{ramp}}} = 15, \quad |F(j\omega)| = 0.04636$$

$$|G_3(j\omega)| = \frac{1}{15 \cdot 0.04636} = 1.438 \quad (3.155 \text{ dB})$$

Transfer function

K-Factor Method Step 1: Calculate K

$$K = \tan^2\left(\frac{\phi_{\text{comp}} + 90^\circ}{4}\right) = \tan^2\left(\frac{201^\circ + 90^\circ}{4}\right) = 10.4$$

Step 2: Calculations

$$R_2 = \frac{|G_3(j\omega_t)| \cdot R_1}{\sqrt{K}} = \frac{1.438 \cdot 200\,\text{k}\Omega}{\sqrt{10.4}} = 89.18\,\text{k}\Omega$$

Step 3: $\mathrm{C}_1 = \frac{1}{\omega_t R_2 \sqrt{K}} = 55.34\,\text{pF}$

Step 4: $\mathrm{C}_2 = \frac{\sqrt{K}}{2\pi \cdot 10^4 \cdot 89.18 \cdot 10^3} = 575.5\,\text{pF}$

Step 5: $\mathrm{C}_3 = \frac{\sqrt{K}}{2\pi \cdot 10^4 \cdot 200 \cdot 10^3} = 256.6\,\text{pF}$

Step 6: $\mathrm{R}_3 = \frac{1}{\omega_t C_3 \sqrt{K}} = 19.23\,\text{k}\Omega$

Step 7: $\mathrm{R}_4 = \frac{V_{\text{ref}}}{V_o - V_{\text{ref}}} \cdot R_1 = \frac{0.8}{15 - 0.8} \cdot 200\,\text{k}\Omega = 11.27\,\text{k}\Omega$

Transfer function

$$H(s) = \frac{Z_f}{Z_i} = \frac{\left(\frac{1}{sC_1} + R_2\right)\frac{1}{sC_3}}{\left(\frac{1}{sC_2} + R_1\right)R_1 \Big/ \left(\frac{1}{sC_2} + R_3 + R_1\right)}$$

We obtain a more familiar expression:

$$H(s) = \frac{(sC_2(R_1 + R_3) + 1)(sC_1 R_2 + 1)}{(sR_1(C_1 + C_3))(sC_2 R_3 + 1)\left(\frac{sC_1 C_3 R_2}{C_1 + C_3} + 1\right)}$$

$$\approx \frac{(sC_2(R_1 + R_3) + 1)(sC_1 R_2 + 1)}{(sR_1 C_1)(sC_2 R_3 + 1)(sR_2 C_3 + 1)}, \quad \text{if } (\ C_1 \gg C_3)$$

Expected Circuit Behavior

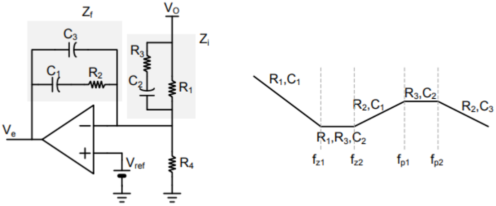
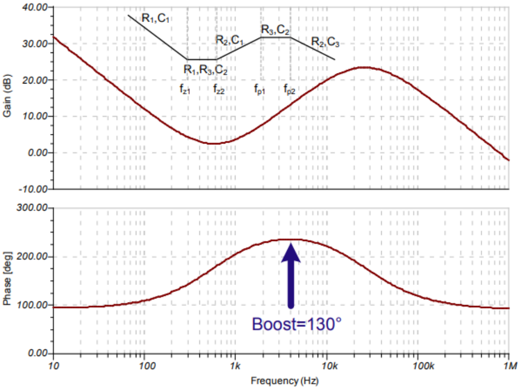
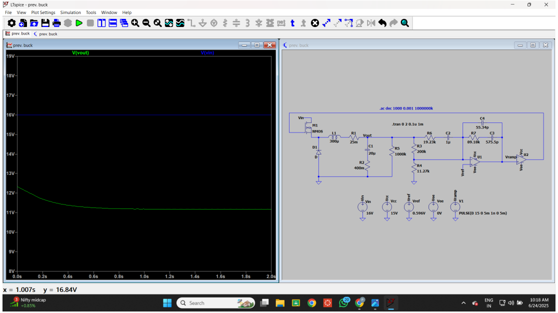
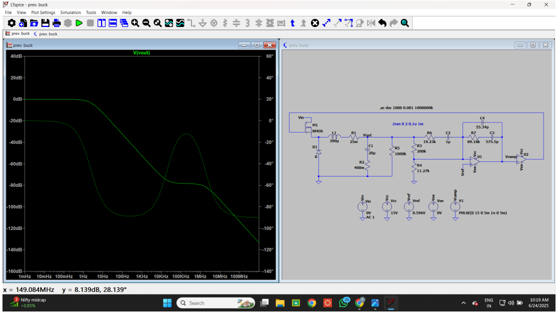


Figure 7. Type III Compensator with Gain Curve



Experimental circuit behavior





Design 2: Texas Instruments based datasheet

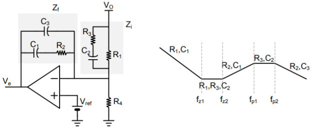


Figure 7. Type III Compensator with Gain Curve

$$f_{p0} = \frac{1}{2\pi R_1 (C_1 + C_3)} \approx \frac{1}{2\pi R_1 C_1} = 10\ Hz \tag{31}$$

$$f_{p1} = \frac{1}{2\pi R_3 C_2} = 10\ MHz \tag{32}$$

$$f_{p2} = \frac{1}{2\pi R_2 \left(\frac{C_1 C_3}{C_1 + C_3}\right)} = \frac{1}{2\pi R_2 \left(\frac{1}{\frac{1}{C_1} + \frac{1}{C_3}}\right)} \approx \frac{1}{2\pi R_2 C_3} = 100\ MHz \tag{33}$$

$$f_{z1} = \frac{1}{2\pi (R_1 + R_3) C_2} = 100\ Hz \tag{34}$$

$$f_{z2} = \frac{1}{2\pi R_2 C_1} = 1000\ Hz \tag{35}$$

Components selection

We can find the required C_1, C_2, C_3, R_2, and R_3 once we select R_1 with the desired $f_{p0}, f_{p1}, f_{p2}, f_{z1}, f_{z2}$ as follows:

$$C_1 = \frac{f_{p2} - f_{z2}}{2\pi R_1 f_{p0} f_{p2}} = 79.577nF \tag{36}$$

$$C_2 = \frac{f_{p1} - f_{z1}}{2\pi R_1 f_{p1} f_{z1}} = 7.9577nF \tag{37}$$

$$C_3 = \frac{f_{z2}}{2\pi R_1 f_{p0} f_{p2}} = 0.79577nF \tag{38}$$

$$R_2 = \frac{R_1 f_{p0}}{(f_{p2} - f_{z2}) f_{z2}} = 2k\Omega \tag{39}$$

$$R_3 = \frac{R_1 f_{z1}}{f_{p1} - f_{z1}} = 2\Omega \tag{40}$$

$$R_4 = 35.294k\Omega \ (for \ \ Vref = 0.6v) \tag{40}$$

Experimental circuit behavior