GATE - EC 27

EE23BTECH11215 - Penmetsa Srikar Varma

Question

Q27) Let m(t) be a strictly band-limited signal with bandwidth B and energy E. Assuming $\omega_0 = 10B$, the energy in the signal $m(t)\cos(\omega_0 t)$

(A) $\frac{E}{4}$

(B) $\frac{E}{2}$

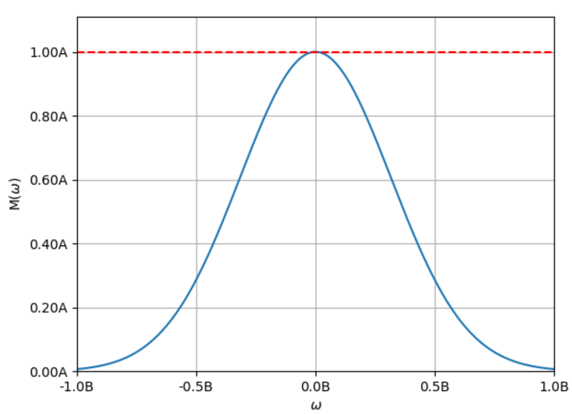
(C) E

(D) 2E (GATE EC 2023)

Solution

Variables	Conditions
A	amplitude of signal $M(\omega)$
ω_0	angular frequency
y(t)	$y(t)=m(t)\cos(\omega_0 t)$
$M(\omega)$	fourier transform of m(t)
$Y(\omega)$	fourier transform of y(t)
E_1	energy of y(t)

Table of Parameters



Energy (E) of the signal m(t) is given by,

$$E = \int_{-\infty}^{\infty} |m(t)|^2 \, dt \tag{1}$$

According to Parvesal's theorem,

$$E = \int_{-\infty}^{\infty} |m(t)|^2 \, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |M(\omega)|^2 d\omega \;\propto\; A^2 \tag{2}$$

Fourier transform of y(t) is given by,

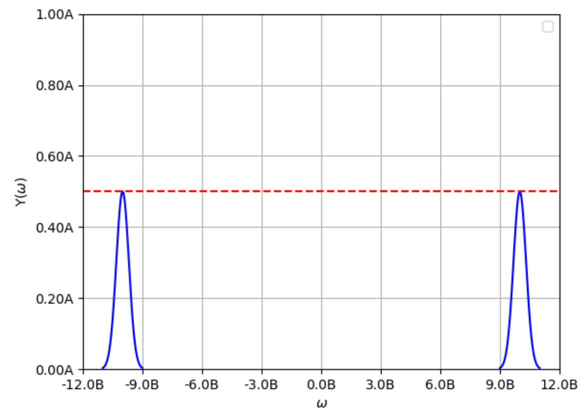
$$Y(\omega) = \int_{-\infty}^{\infty} m(t)\cos(\omega_0 t)\, e^{j\omega t} dt \tag{3}$$

$$Y(\omega) = \frac{1}{2}\int_{-\infty}^{\infty} m(t)\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) e^{j\omega t} dt \tag{4}$$

$$Y(\omega) = \frac{1}{2}\int_{-\infty}^{\infty} m(t)\left(e^{j(\omega+\omega_0)t} + e^{j(\omega-\omega_0)t}\right) dt \tag{5}$$

We have $\omega_0 = 10B$,

$$Y(\omega) = \frac{1}{2}\left(M(\omega+\omega_0) + M(\omega-\omega_0)\right) \tag{6}$$



Energy (E_1) of the signal y(t) is given by sum of Energies of individual bandwidth signals,

$$E_1 \propto \left(\frac{A}{2}\right)^2 + \left(\frac{A}{2}\right)^2 = \frac{A^2}{2}$$

from (2),

$$E_1 = \frac{E}{2}$$

So, option B is correct