GATE - EC 27

EE23BTECH11215 - Penmetsa Srikar Varma

Question

Q27) Let m(t) be a strictly band-limited signal with bandwidth B and energy E. Assuming $\omega_0 = 10B$, the energy in the signal $m(t)\cos(\omega_0 t)$

(A) $\frac{E}{4}$

(B) $\frac{E}{2}$

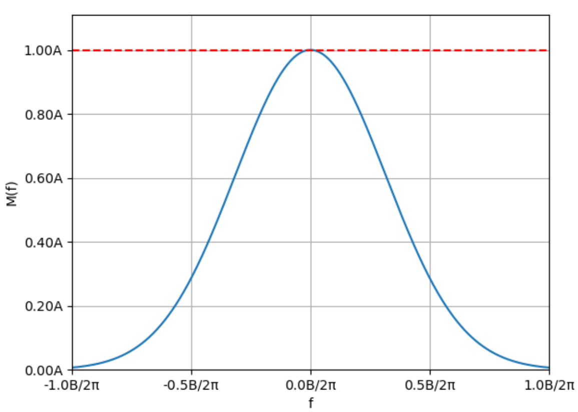
(C) E

(D) 2E (GATE EC 2023)

Solution

Variables	Conditions
A	amplitude of M(f)
$f_0=\frac{10}{2\pi}B$	band-width frequency
y(t)	$y(t)=m(t)\cos(2\pi f_0 t)$
M(f)	fourier transform of m(t)
Y(f)	fourier transform of y(t)
E_1	energy of y(t)

Table of Parameters



Energy (E) of the signal m(t) is given by,

$$E = \int_{-\infty}^{\infty} |m(t)|^2 \, dt \tag{1}$$

According to Parseval's theorem,

$$E = \int_{-\infty}^{\infty} |m(t)|^2 \, dt = \int_{-\infty}^{\infty} |M(f)|^2 df \; \propto \; A^2 \tag{2}$$

Fourier transform of y(t) is given by,

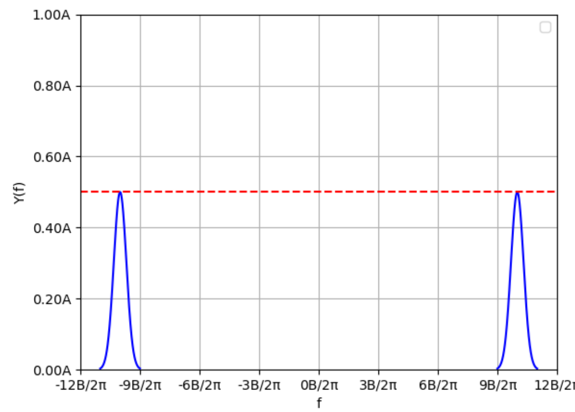
$$Y(f) = \int_{-\infty}^{\infty} m(t)\cos(2\pi f_0 t)\, e^{j2\pi ft} dt \tag{3}$$

$$Y(f) = \frac{1}{2}\int_{-\infty}^{\infty} m(t)\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right) e^{j2\pi ft} dt \tag{4}$$

$$Y(f) = \frac{1}{2}\int_{-\infty}^{\infty} m(t)\left(e^{j(2\pi f + 2\pi f_0)t} + e^{j(2\pi f - 2\pi f_0)t}\right) dt \tag{5}$$

We have $f_0=\frac{10}{2\pi}B$,

$$Y(f) = \frac{1}{2}\left(M(2\pi f + 2\pi f_0) + M(2\pi f - 2\pi f_0)\right) \tag{6}$$



Energy (E_1) of the signal y(t) is given by sum of Energies of individual bandwidth signals,

$$E_1 \propto \left(\frac{A}{2}\right)^2 + \left(\frac{A}{2}\right)^2 = \frac{A^2}{2}$$

from (2),

$$E_1 = \frac{E}{2}$$

So, option B is correct