

GATE - EC 27

EE23BTECH11215 - Penmetsa Srikar Varma

QUESTION

Q27) Let $m(t)$ be a strictly band-limited signal with bandwidth B and energy E . Assuming $\omega_0 = 10B$, the energy in the signal $m(t) \cos(\omega_0 t)$

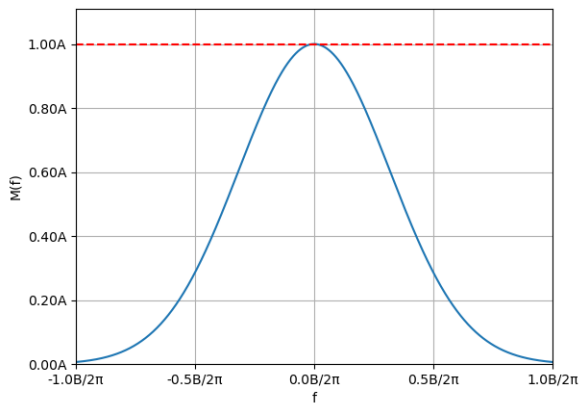
- (A) $\frac{E}{4}$
 (B) $\frac{E}{2}$
 (C) E
 (D) $2E$

(GATE EC 2023)

SOLUTION

Variables	Conditions
A	amplitude of $M(f)$
$f_0 = \frac{10}{2\pi} B$	band-width frequency
$y(t)$	$y(t) = m(t) \cos(2\pi f_0 t)$
$M(f)$	fourier transform of $m(t)$
$Y(f)$	fourier transform of $y(t)$
E_1	energy of $y(t)$

Table of Parameters



Energy (E) of the signal $m(t)$ is given by,

$$E = \int_{-\infty}^{\infty} |m(t)|^2 dt \quad (1)$$

According to Parseval's theorem,

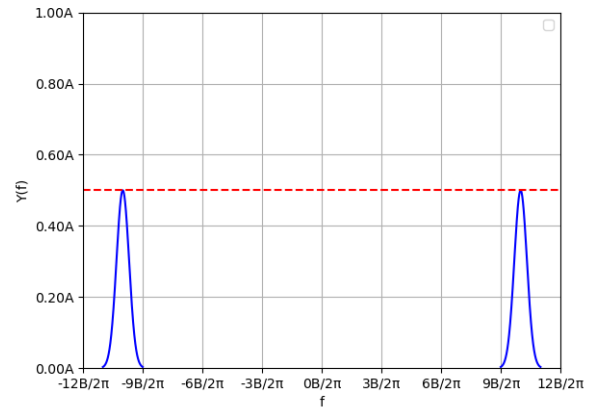
$$E = \int_{-\infty}^{\infty} |M(f)|^2 df \propto A^2 \quad (2)$$

Fourier transform of $y(t)$ is given by,

$$Y(f) = M(f) * \frac{1}{2} (\delta(f_0 + f) + \delta(f - f_0)) \quad (3)$$

We have $f_0 = \frac{10}{2\pi} B$,

$$Y(f) = \begin{cases} \frac{M(f)}{2} & \frac{9B}{2\pi} \leq |f| \leq \frac{12B}{2\pi} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$



Energy (E_1) of the signal $y(t)$ is given by sum of energies of individual bandwidth signals,

$$E_1 \propto \left(\frac{A}{2}\right)^2 + \left(\frac{A}{2}\right)^2 = \frac{A^2}{2} \quad (5)$$

from (2),

$$E_1 = \frac{E}{2}$$

So, option B is correct