Assignment 10.5.4-2

ee23btech11215 - Penmetsa Srikar Varma

Question:

Q2) The sum of the third and the seventh terms of AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP

Solution:

Table of Parameters

Input Variables	Input Condition
x(2)+x(6)	6
x(2).x(6)	8

Then general term x(n) of arithmetic progression is given by:

$$x(n) = x(0) + n.d \tag{1}$$

Then from (1):

$$x(2) = x(0) + 2.d \tag{2}$$

$$x(6) = x(0) + 6.d \tag{3}$$

Then from table of parameters,

$$x(2) = 6 - x(6) \tag{4}$$

From (4)

$$x(2).x(6) = 8$$

$$x(6).(6 - x(6)) = 8$$

$$x(6) = 2 \text{ or } 4 \tag{5}$$

Then from (4) and (5)

$$x(2) = 4 \text{ or } 2 \tag{6}$$

from (2),(3) and (5),(6)
for x(2) = 2 and x(6) = 4

$$x(0) = 1,\ d = \frac{1}{2} \tag{7}$$

for x(2) = 4 and x(6) = 2

$$x(0) = 5,\ d = -\frac{1}{2} \tag{8}$$

We know that the sum of first n terms of arithmetic progression is given by:

$$S(n) = \frac{n}{2}(2.x(0) + (n-1).d) \tag{9}$$

Then from (9) let sum of first 16 terms of arithmetic progression be S_{16}:

$$S(16) = \frac{16}{2}(2.x(0) + 15d) \tag{10}$$

Hence from (10), for $x(0)$=1,d=$\frac{1}{2}$

$$S_1(16) = 76 \tag{11}$$

or from (10), for $x(0)$=5,d=-$\frac{1}{2}$

$$S_2(16) = 20 \tag{12}$$

The general term of AP x(n) and sum of first n terms of AP S(n) are given by:

$$x_1(n) = \frac{n+2}{2} \quad and \quad S_1(n) = \frac{n.(n+3)}{4} \tag{13}$$

$$x_2(n) = \frac{10-n}{2} \quad and \quad S_2(n) = \frac{n.(21-n)}{4} \tag{14}$$

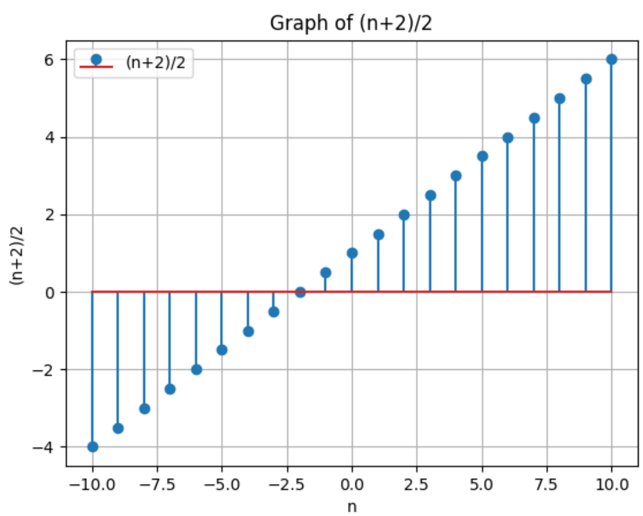

Fig. 0. *

Graph of $\frac{n+2}{2}$

We know that Z-Transform of x(n) is given by:

$$X(z) = \sum_{k=-\infty}^{\infty} x(k).z^{-k} \tag{15}$$

where, we assume that x(k)=0 for ($k < 0$)
Then, (15) modifiy as follows:

$$X_1(z) = \sum_{k=0}^{\infty} x_1(k).z^{-k} \tag{16}$$

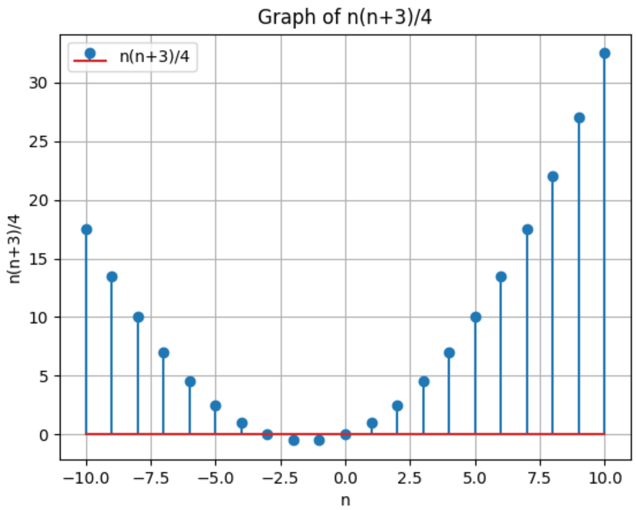

Fig. 0. *

Graph of $\frac{n(n+3)}{4}$

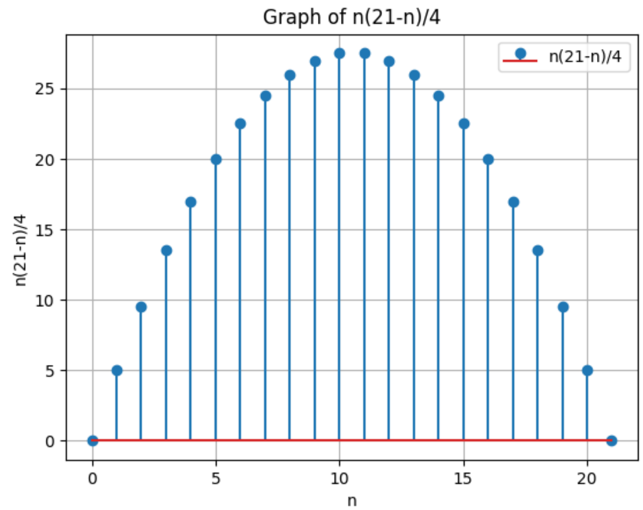

Fig. 0. *

Graph of $\frac{n(21-n)}{4}$

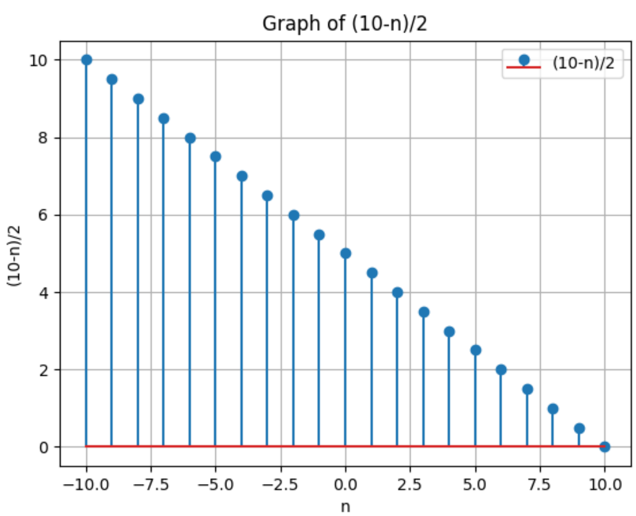

Fig. 0. *

Graph of $\frac{10-n}{2}$

$$X_1(z) = \sum_{k=0}^{\infty} \left(\frac{k+2}{2}\right).z^{-k} \tag{17}$$

$$X_1(z) = \frac{1}{2}\left(\sum_{k=0}^{\infty} k.z^{-k}\right) + \sum_{k=0}^{\infty} z^{-k} \tag{18}$$

$$X_1(z) = \frac{1}{2}\left(\frac{z^{-1}}{(1-z^{-1})^2}\right) + \frac{1}{1-z^{-1}} \tag{19}$$

So,

$$X_1(Z) = \frac{2-z^{-1}}{2.(1-z^{-1})^2} \quad for\ |z^{-1}| < 1 \tag{20}$$

or,

$$X_1(Z)\ \ is\ not\ defined \qquad for\ |z^{-1}| > 1 \tag{21}$$

From (16),

$$X_2(z) = \sum_{k=0}^{\infty} x_2(k).z^{-k} \tag{22}$$

$$X_2(Z) = \sum_{k=0}^{\infty} \left(\frac{10-k}{2}\right).z^{-k} \tag{23}$$

$$X_2(Z) = 5.\left(\sum_{k=0}^{\infty} z^{-k}\right) - \frac{1}{2}\left(\sum_{k=0}^{\infty} k.z^{-k}\right) \tag{24}$$

$$X_2(Z) = 5.\left(\frac{1}{1-z^{-1}}\right) - \frac{1}{2}\left(\frac{z^{-1}}{(1-z^{-1})^2}\right) \tag{25}$$

So,

$$X_2(Z) = \frac{11.\left(1-z^{-1}\right)-1}{2.(1-z^{-1})^2} \quad for\ |z^{-1}| < 1 \tag{26}$$

or,

$$X_2(Z)\ \ is\ not\ defined \quad for \qquad |z^{-1}| > 1 \tag{27}$$

Let, $\mathcal{Z}$ tranform of s(n) be S(Z):
from (15):

$$S(Z) = \sum_{k=-\infty}^{\infty} s(k)\ z^{-k} = \sum_{k=0}^{\infty} s(k)\ z^{-k} \quad (s(k) = 0\ for\ k < 0)$$

from (9)

$$S(Z) = \sum_{k=-\infty}^{\infty} \left(\left(x(0) - \frac{d}{2}\right)k + \frac{d}{2}\ k^2\right).z^{-k} \tag{28}$$

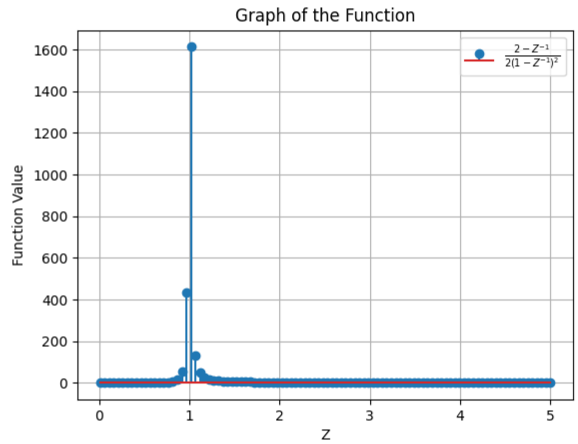

Fig. 0. *

Graph of $\frac{2-z^{-1}}{2.(1-z^{-1})^2}$

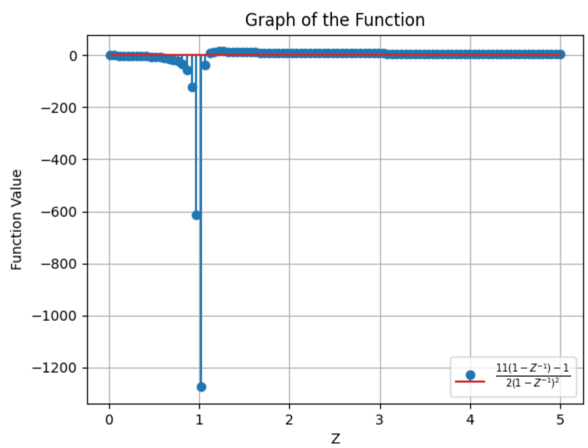

Fig. 0. *

Graph of $\frac{11.(1-z^{-1})-1}{2.(1-z^{-1})^2}$

$$S\,(Z) = \left(x\,(0) - \frac{d}{2}\right)\left(\sum_{k=0}^{\infty} k.Z^{-k}\right) + \frac{d}{2}.\left(\sum_{k=0}^{\infty} k^2.Z^{-k}\right) \tag{29}$$

$$S\,(Z) = \left(x\,(0) - \frac{d}{2}\right)\left(\frac{z^{-1}}{(1-z^{-1})^2}\right) + \frac{d}{2}.\left(\frac{z^{-1}\left(1+z^{-1}\right)}{(1-z^{-1})^3}\right) \tag{30}$$

$$s\,(Z) = x\,(0)\left(\frac{z^{-1}}{(1-z^{-1})^2}\right) + d\left(\frac{z^{-2}}{(1-z^{-1})^3}\right) \tag{31}$$

$$s\,(Z) = x\,(0)\left(\frac{z}{(z-1)^2}\right) + d\left(\frac{z}{(z-1)^3}\right) \tag{32}$$

for x(0)=1 and $d = \frac{1}{2}$

$$S_1\,(Z) = \frac{z^{-1}\left(1 - \frac{1}{2}z^{-1}\right)}{(1-z^{-1})^3} \tag{33}$$

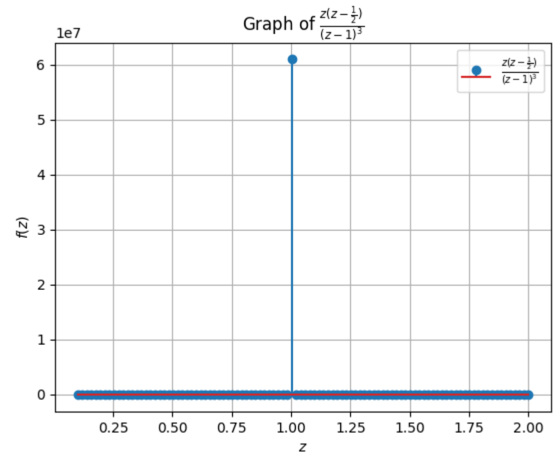

Fig. 0. *

Graph of $\frac{z^{-1}(1-\frac{1}{2}z^{-1})}{(1-z^{-1})^3}$

for x(0)=5 and $d = -\frac{1}{2}$

$$S_2\,(Z) = \frac{z^{-1}\left(5 - \frac{11}{2}z^{-1}\right)}{(1-z^{-1})^3} \tag{34}$$

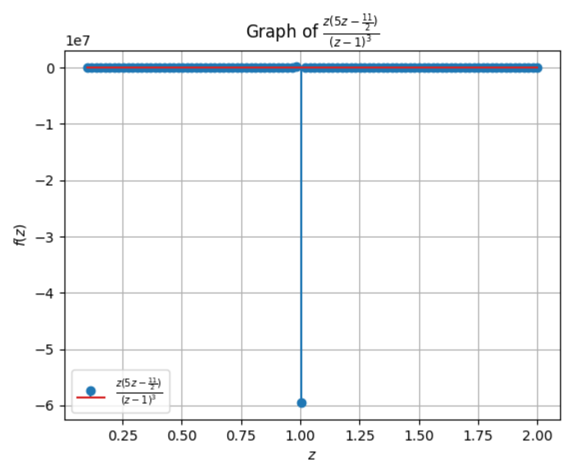

Fig. 0. *

Graph of $\frac{z^{-1}(5-\frac{11}{2}z^{-1})}{(1-z^{-1})^3}$

We know that Inverse $\mathcal{Z}$ - transform of $S\,(z)$ say $s\,(n)$, by counter integral method is given by:

$$s\,(n) = \oint_C S\,(z)\; z^{n-1}\, dz \tag{35}$$

from (32),

$$s\,(n) = x\,(0)\oint_C\left(\frac{z}{(z-1)^2}dz\right) + d\left(\oint_C \frac{z}{(z-1)^3}\,dz\right) \tag{36}$$

From residue theorem for similar residues:

$$s(n) = x(0)\left|\frac{1}{1!}\frac{d}{dz}(z^n)\right|_{z=1} + d\left|\frac{1}{2!}\frac{d^2}{dz^2}(z^n)\right|_{z=1} \tag{37}$$

$$s(n) = n.x(0) + n(n-1)\frac{d}{2} \tag{38}$$

$$s_1(n) = \frac{n(n+3)}{4}, s_1(16) = 76 \tag{39}$$

and,

$$s_2(n) = \frac{n(21-n)}{4}, s_2(16) = 20 \tag{40}$$

we can observe that (11) and (39),(12) and (40) are the same results