Assignment

10.5.4-2

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Question:

Q2) The sum of the third and the seventh terms of AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP

Solution:

Table of Parameters

Input Variables	Input Condition
x(2)+x(6)	6
x(2).x(6)	8
$x_i(n)$	general term of i$^{\text{th}}$ AP sequence
$y_i(n)$	sum of first n terms of i$^{\text{th}}$ AP sequence
$x_i(0)$	first term of i$^{\text{th}}$ AP sequence
d_i	common difference of i$^{\text{th}}$ AP sequence
$x_i(n) \overset{\mathcal{Z}}{\longleftrightarrow} X_i(z)$	z-transform of $x_i(n)$
$y_i(n) \overset{\mathcal{Z}}{\longleftrightarrow} Y_i(z)$	z-transform of $y_i(n)$

Then general term x(n) of arithmetic progression is given by:

$$x_i(n) = x_i(0) + n\ d_i \tag{1}$$

Then from table of parameters,

$$x_i(6)(6 - x_i(6)) = 8 \tag{2}$$

$$x_1(6) = 4 \text{ and } x_2(6) = 2 \tag{3}$$

Then from table and (3)

$$x_1(2) = 2 \text{ and } x_2(2) = 4 \tag{4}$$

for $x_1(2) = 2$ and $x_1(6) = 4$

$$x_1(0) = 1,\ d_1 = \frac{1}{2} \tag{5}$$

for $x_2(2) = 4$ and $x_2(6) = 2$

$$x_2(0) = 1,\ d_2 = -\frac{1}{2} \tag{6}$$

We know that the sum of first n terms of arithmetic progression is given by:

$$y_i(n) = \frac{n}{2}(2x_i(0) + (n-1)d_i)u(n) \tag{7}$$

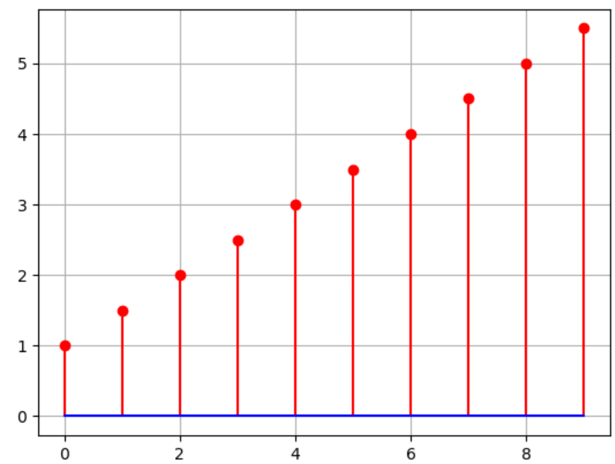
$$y_1(16) = 76 \tag{8}$$

$$y_2(16) = 20 \tag{9}$$

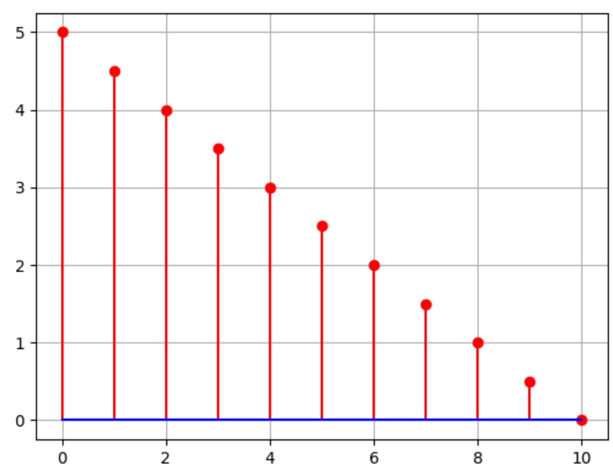
The general term of AP $x_i(n)$ and sum of first n terms of AP $y_i(n)$ are given by:

$$x_1(n) = \left(\frac{n+2}{2}\right)u(n) \text{ and } y_1(n) = \left(\frac{n(n+3)}{4}\right)u(n) \tag{10}$$

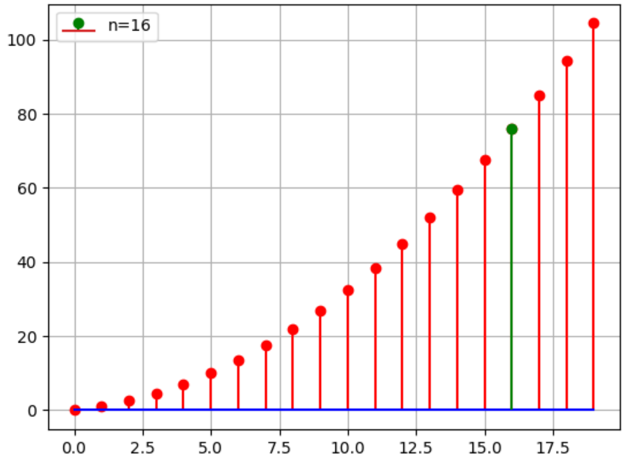
$$x_2(n) = \left(\frac{10-n}{2}\right)u(n) \text{ and } y_2(n) = \left(\frac{n(21-n)}{4}\right)u(n) \tag{11}$$



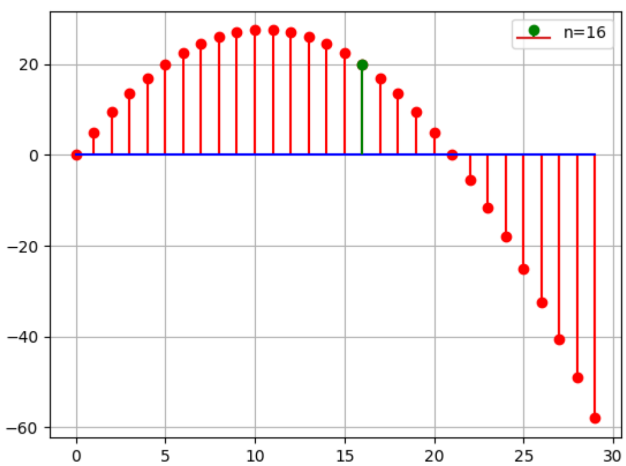
Graph of $x_1(n)$



Graph of $x_2(n)$



Graph of $y_1(n)$



Graph of $y_2(n)$

z-Transform of x_1 (n) is given by:

$$X_1(z) = \sum_{k=-\infty}^{\infty} x_1(k) z^{-k} \tag{12}$$

$$\tag{13}$$

$$X_1(z) = \frac{2 - z^{-1}}{2.(1 - z^{-1})^2}, \quad |z^{-1}| < 1 \tag{14}$$

Similarly,

$$X_1(z) = \sum_{k=-\infty}^{\infty} x_2(k) z^{-k} \tag{15}$$

$$\tag{16}$$

$$X_2(z) = \frac{10 - 11z^{-1}}{2.(1 - z^{-1})^2}, \quad |z^{-1}| < 1 \tag{17}$$

Similarly for sum of first n terms of AP,

$$Y_i(z) = \sum_{k=-\infty}^{\infty} y_i(k) z^{-k} \tag{18}$$

$$Y_i(z) = x_i(0)\left(\frac{z}{(z-1)^2}\right) + d_i\left(\frac{z}{(z-1)^3}\right) \tag{19}$$

$$Y_1(z) = \frac{z\left(z - \frac{1}{2}\right)}{(z-1)^3}, \quad |z| > 1 \tag{20}$$

$$Y_2(z) = \frac{z\left(5z - \frac{11}{2}\right)}{(z-1)^3}, \quad |z| > 1 \tag{21}$$

Inverse z-transform of $y_i(z)$ by counter integral method is given by:

$$y_i(n) = \oint_C Y_i(z)\, z^{n-1}\, dz \tag{22}$$

$$y_i(n) = x_i(0) \oint_C \left(\frac{z}{(z-1)^2} dz\right) + d_i\left(\oint_C \frac{z}{(z-1)^3}\, dz\right) \tag{23}$$

We can observe that pole is repeated 2,3 times so, m=2 and 3 respectively:

$$y_i(n) = x_i(0) \left| \frac{1}{1!} \frac{d}{dz} (z^n) \right|_{z=1} + d_i \left| \frac{1}{2!} \frac{d^2}{dz^2} (z^n) \right|_{z=1} \tag{24}$$

$$y_i(n) = \left(n\ x_i(0) + n(n-1)\frac{d_i}{2}\right) u(n) \tag{25}$$

$$y_1(n) = \frac{n(n+3)}{4} u(n), \quad y_1(16) = 76 \tag{26}$$

$$y_2(n) = \frac{n(21-n)}{4} u(n), \quad y_2(16) = 20 \tag{27}$$