Assignment

10.5.4-2

ee23btech11215 - Penmetsa Srikar Varma

Question:

Q2) The sum of the third and the seventh terms of AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP

Solution:

Table of Parameters

Input Variables	Input Condition
x(2)+x(6)	6
x(2).x(6)	8
$x_i(n)$	general term of i^{th} AP sequence
$y_i(n)$	sum of first n terms of i^{th} AP sequence
$x_i(0)$	first term of i^{th} AP sequence
d_i	common difference of i^{th} AP sequence

Then from table of parameters,

$$x_i^2(6) - 6\,x_i(6) + 8 = 0 \tag{1}$$

$$x_i(6) = 2 \; or \; 4 \tag{2}$$

1)

$$\left(\, x_i(2), x_i(6) \,\right) = \begin{cases} (2,4) & \text{if } i = 1 \\ (4,2) & \text{if } i = 2 \end{cases} \tag{3}$$

2)

$$\left(\, x_i(0), d_i \,\right) = \begin{cases} \left(1, \frac{1}{2}\right) & \text{if } i = 1 \\ \left(5, -\frac{1}{2}\right) & \text{if } i = 2 \end{cases} \tag{4}$$

3)

$$x_i(n) = \begin{cases} \left(\frac{n+2}{2}\right) u(n) & \text{if } i = 1 \\ \left(\frac{10-n}{2}\right) u(n) & \text{if } i = 2 \end{cases} \tag{5}$$

z-Transform of x_1 (n) is given by:

$$X_1(z) = \frac{1 - \frac{z^{-1}}{2}}{(1 - z^{-1})^2}, \quad |z^{-1}| < 1 \tag{6}$$

Similarly,

$$X_2(z) = \frac{5 - \frac{11z^{-1}}{2}}{(1 - z^{-1})^2}, \quad |z^{-1}| < 1 \tag{7}$$

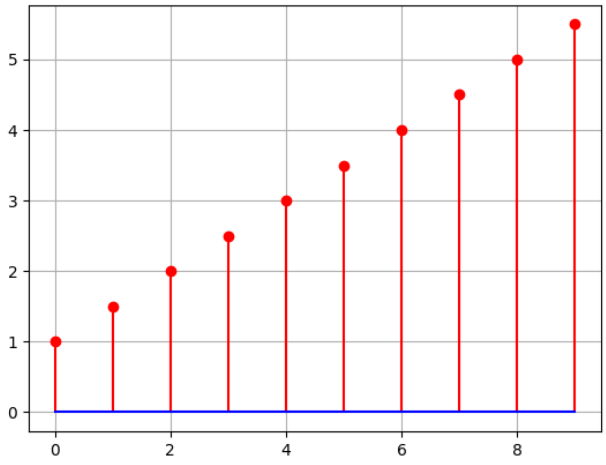
Similarly for sum of first n terms of AP,

$$y_i(n) = x_1(n) * u(n) \tag{8}$$

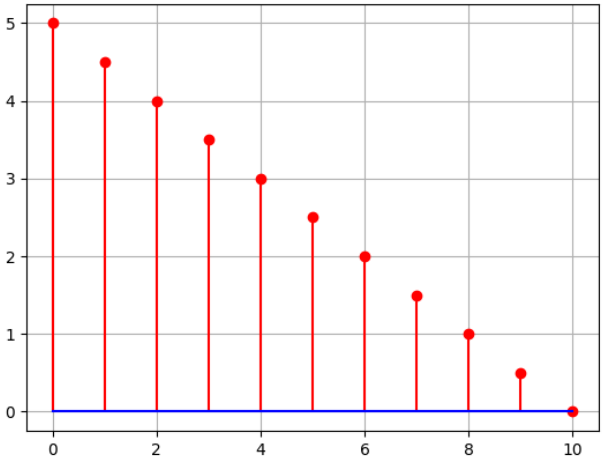
$$Y_i(z) = \frac{X_i(z)}{(1 - z^{-1})} \tag{9}$$

$$Y_1(z) = \frac{1 - \frac{z^{-1}}{2}}{(1 - z^{-1})^3}, \quad |z| > 1 \tag{10}$$

$$Y_2(z) = \frac{5 - \frac{11z^{-1}}{2}}{(1 - z^{-1})^3}, \quad |z| > 1 \tag{11}$$



Graph of $x_1(n)$



Graph of $x_2(n)$

Inverse z-transform by counter integral method for $y_1(z)$,

Since n starts from 0 to n-1 for $x_1(n)$ so, n $\rightarrow$ n-1 so that $y_1(n)$ starts from 1 to n for given n,

$$y_1(16) = \oint_C \frac{z^3\left(1-\frac{z^{-1}}{2}\right)}{(z-1)^3} z^{14}\,dz \tag{12}$$

$$y_1(16) = \frac{1}{2!}\left(\frac{d^2}{dz^2}z^{17} - \frac{1}{2}\frac{d^2}{dz^2}z^{16}\right)_{z=1} \tag{13}$$

$$y_1(16) = 76 \tag{14}$$

Similarly for $y_2(z)$,

$$y_2(16) = \oint_C \frac{z^3\left(5-\frac{11z^{-1}}{2}\right)}{(z-1)^3} z^{14}\,dz \tag{15}$$

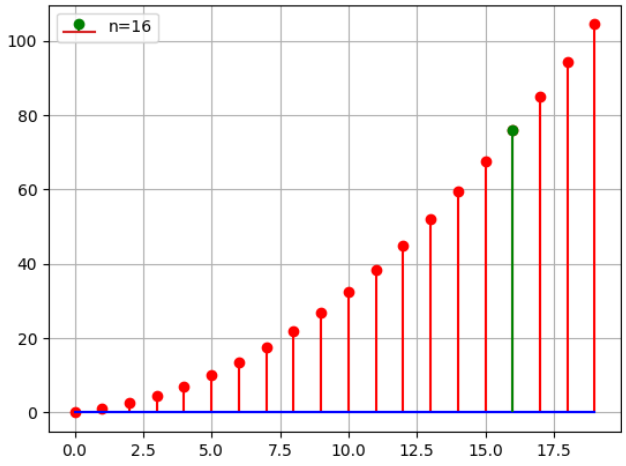
$$y_2(16) = \frac{1}{2!}\left(5\,\frac{d^2}{dz^2}z^{17} - \frac{11}{2}\frac{d^2}{dz^2}z^{16}\right)_{z=1} \tag{16}$$

$$y_2(16) = 20 \tag{17}$$

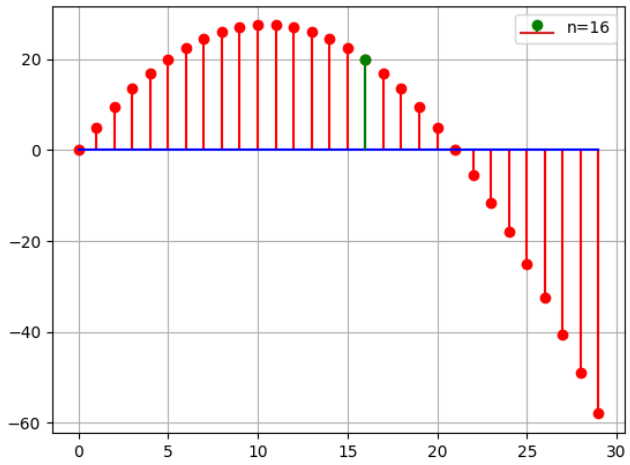
In fact,

$$y_1(n) = \frac{n(n+3)}{4} \tag{18}$$

$$y_2(n) = \frac{n(21-n)}{4} \tag{19}$$



Graph of $y_1(n)$



Graph of $y_2(n)$