Assignment
10.5.4-2

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Question:

Q10) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Solution:

Table of Parameters

Input Variable	Condition
$x(0),\ x(n)$	first term and general term of a GP
r	common ratio of a GP
$x(0), x(1), x(2)$	three terms in a GP
$x(0), x(1), x(2)$	$x(0) + x(1) + x(2) = 56$
$x(0) - 1, x(1) - 7, x(2) - 21$	form an AP
$x_i(n)$	general term of i$^{\text{th}}$ GP sequence
$x_i(0)$	first term of i$^{\text{th}}$ GP sequence
r_i	common ratio of i$^{\text{th}}$ GP sequence
$u(n)$	Unit step function
$x_i(n) \overset{Z}{\longleftrightarrow} X_i(z)$	$X_i(z) = x_i(0) \sum_{k=0}^{k=\infty} \left(\frac{z}{r}\right)^{-k}$

(n+1)$^{\text{th}}$ term of GP x(n) is given by:

$$x(n) = x(0)\ \ r^n \tag{1}$$

Then from given table of parameters,

$$x(0) + x(1) + x(2) = 56 \tag{2}$$

$$x(0) \implies \frac{56}{(1 + r + r^2)} \tag{3}$$

and from given another case following are in AP,

$$x(0) - 1, x(1) - 7, x(2) - 21$$

$$2(x(1) - 7) = x(0) - 1 + x(2) - 21 \tag{4}$$

$$x(0)\left(r^2 - 2r + 1\right) = 8 \tag{5}$$

and from (3)

$$\frac{56.\left(r^2 - 2r + 1\right)}{(1 + r + r^2)} = 8 \tag{6}$$

$$r_1 = 2\ ,\ r_2 = \frac{1}{2} \tag{7}$$

so from (3),

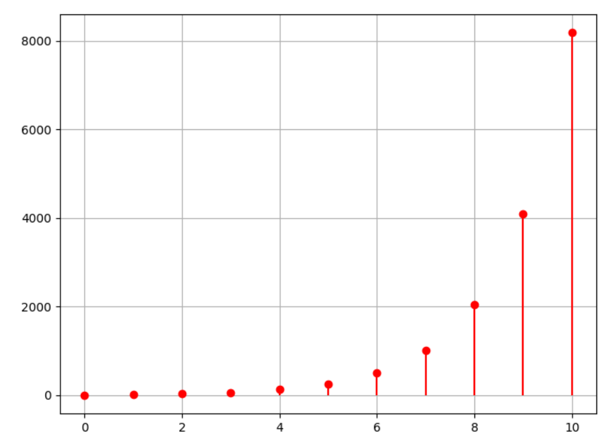
$$x_1(0) = 8\ ,\ x_2(0) = 32 \tag{8}$$

Then from (1)

$$x_1(n) = 8.2^n = 2^{n+3}\ \ u(n) \tag{9}$$

$$x_2(n) = 32.\left(\frac{1}{2}\right)^n u(n) = 2^{5-n}\ \ u(n) \tag{10}$$

$x_1(0)$, $x_1(1)$ and $x_1(2)$ are 8, 16, 32 (or) $x_2(0)$, $x_2(1)$ and $x_2(2)$ are 32, 16, 8 respectively



Graph of $x_1(n)$

z-transform of $x_1(n)$ is given by:

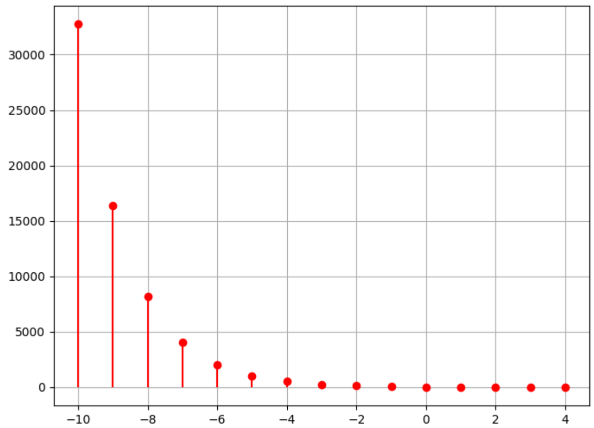
$$X_1(z) = \sum_{k=-\infty}^{\infty} x_1(k)\,.z^{-k} \tag{11}$$

from (9),

$$X_1(z) = \sum_{k=0}^{\infty} 2^{k+3}\ \ z^{-k} \tag{12}$$

Hence,

$$X_1(z) = \frac{8}{1 - 2z^{-1}},\quad |2z^{-1}| < 1 \tag{13}$$



Graph of $x_2(n)$

and also from (10),

$$X_2(z) = \sum_{k=-\infty}^{\infty} x_2(k) . z^{-k} \tag{14}$$

$$X_2(z) = \sum_{k=0}^{\infty} 2^{5-k} \ z^{-k} \tag{15}$$

Hence,

$$X_2(z) = \frac{32}{1-(2z)^{-1}}, \quad |(2z)^{-1}| < 1 \tag{16}$$