Discrete Assignment

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Question 2 Exercise 5.4 chapter 5: Arithmetic Progressions of class 10

Q2) The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP

Answer:
Let us assume the first term of given arithmetic progression be $a(0)$ and common difference be 'd'

Let the terms of AP be:

$$a(0), a(1), a(2)...a(k-1)$$

Input Table:

Input Variables	Input Condition
$a(2)$	third term of AP
$a(6)$	seventh term of AP
$a(2)$+$a(6)$	6
$a(2)$.$a(6)$	8
$S(16)$	sum of first 16 terms of AP

from above we can observe that n^{th} term of AP and sum of first n terms of AP are :

$$a(n-1) \quad and \quad S(n) \tag{1}$$

then general term $a(n)$ of arithmetic progression is given by:

$$a(n) = a(0) + n.d \tag{2}$$

So from the given information the third term and seventh term of arithmetic progression be $a(2)$ and $a(6)$ respectively, Then from (**??**):

$$a(2) = a(0) + 2d \tag{3}$$

$$a(6) = a(0) + 6d \tag{4}$$

Then from (**??**) and (**??**)

$$a(2) + a(6) = 6 \tag{5}$$

$$a(2).a(6) = 8 \tag{6}$$

or we can say from (**??**),

$$2a(0) + 8d = 6$$

$$a(0) + 4d = 3$$
$$a(0) = 3 - 4d \tag{7}$$

or we can say from (**??**),

$$(a(0) + 2d)(a(0) + 6d) = 8$$

and from (**??**),

$$(3 - 2d).(3 + 2d) = 8$$
$$9 - 4d^2 = 8$$
$$d^2 = \frac{1}{4}$$
$$d = \frac{1}{2}, -\frac{1}{2} \tag{8}$$

Then from (**??**),

$$a(0) = 1, 5 \tag{9}$$

We know that the sum of first n terms of arithmetic progression is given by:

$$S(n) = \frac{n}{2}(2.a(0) + (n-1)d) \tag{10}$$

Then from (**??**) let sum of first 16 terms of arithmetic progression be S_{16}:

$$S(16) = \frac{16}{2}(2a(0) + 15d) \tag{11}$$

Hence from (**??**), for $a(0)$=1,d=$\frac{1}{2}$

$$S(16) = 76$$

or from (**??**), for $a(0)$=5,d=-$\frac{1}{2}$

$$S(16) = 20$$

The general term of AP (a_n) and sum of first n terms of AP (S_n) are given by:

$$a(n) = a(0) + n.d \quad and \quad S(n) = \frac{n}{2}(2.a(0) + (n-1).d)$$

$$a(n) = \frac{n+2}{2} \quad and \quad S(n) = \frac{n.(n+3)}{4} \quad for\ (a(0) = 1, d = \frac{1}{2})$$
$$a(n) = \frac{10-n}{2} \quad and \quad S(n) = \frac{n.(21-n)}{4} \quad for\ (a(0) = 5, d = -\frac{1}{2})$$

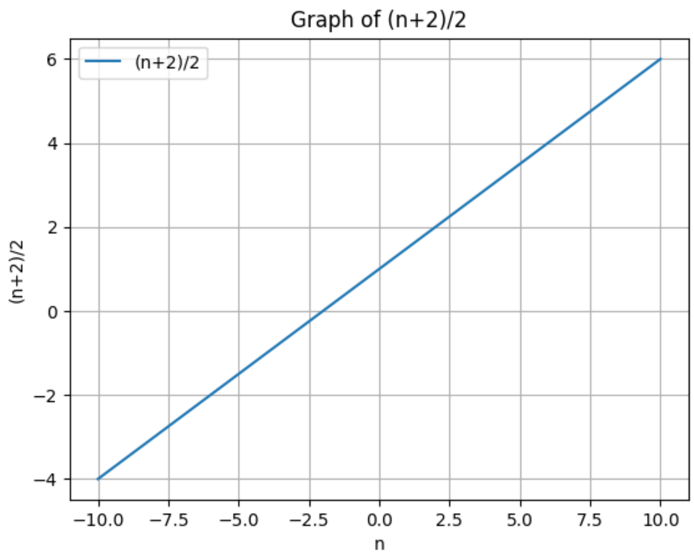

Figure 1: Graph of $\left(\frac{n+2}{2}\right)$

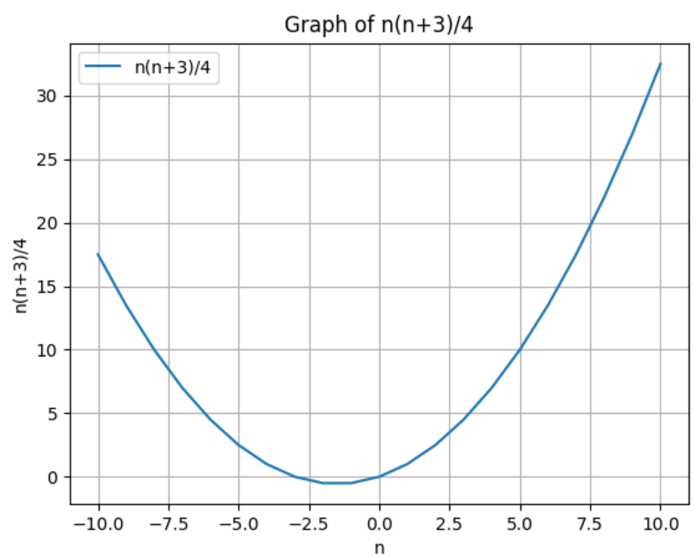

Figure 2: Graph of $\left(\frac{n.(n+3)}{4}\right)$

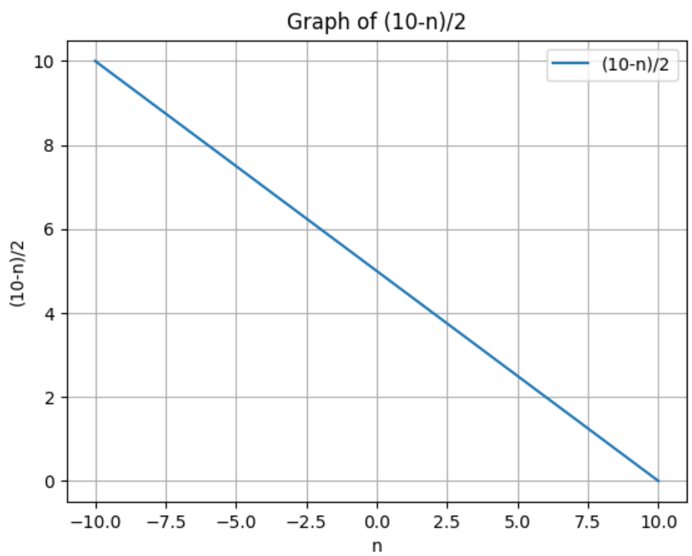

Figure 3: Graph of $\left(\frac{10-n}{2}\right)$

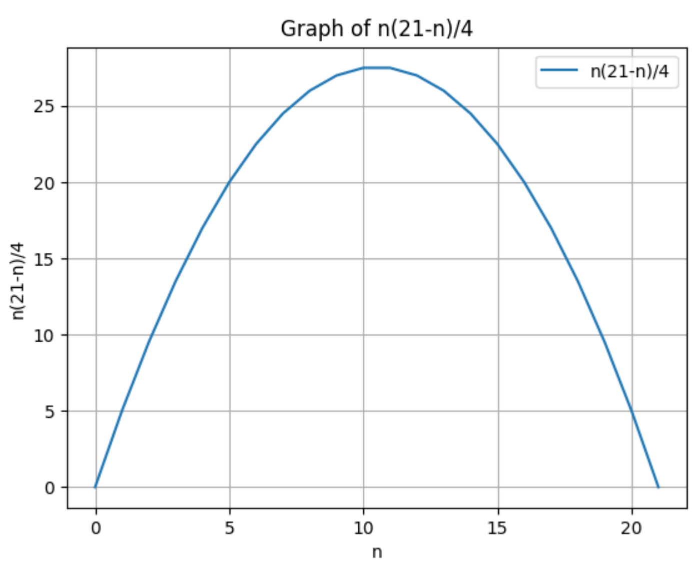

Figure 4: Graph of $\left(\frac{n.(21-n)}{4}\right)$