Assignment

10.5.4-2

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Question:

Q10) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Solution:

Table of Parameters

Input Variable	Condition
$x(0)$	first term of AP
r	common ratio of GP
$x(0), x(1), x(2)$	three terms in GP
$x(0), x(1), x(2)$	$x(0) + x(1) + x(2) = 56$
$x(0) - 1, x(1) - 7, x(2) - 21$	form an AP
$x(n)$	$(n+1)^{th}$ term of GP
$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z)$	$X(z) = x(0)\sum_{k=0}^{k=\infty}\left(\frac{z}{r}\right)^{-k}$

We know that, if three numbers p,q and r are in arithmetic progression then,

$$2q = p + r \tag{1}$$

Then $(n+1)^{th}$ term of GP x(n) is given by:

$$x(n) = x(0).r^n \tag{2}$$

Then from given table of parameters,

$$x(0) + x(1) + x(2) = 56 \tag{3}$$

$$x(0) \implies \frac{56}{(1 + r + r^2)} \tag{4}$$

and from given another case following are in AP,

$$x(0) - 1, x(1) - 7, x(2) - 21$$

Then from (1),

$$2(x(1) - 7) = x(0) - 1 + x(2) - 21 \tag{5}$$

$$x(0)\left(r^2 - 2r + 1\right) = 8 \tag{6}$$

and from (4)

$$\frac{56.\left(r^2 - 2r + 1\right)}{(1 + r + r^2)} = 8 \tag{7}$$

$$r = 2, \frac{1}{2} \tag{8}$$

so from (4),

$$x(0) = 8, 32 \tag{9}$$

Then from (2)

$$x_1(n) = 8.2^n = 2^{n+3}\ u(n) \tag{10}$$

$$x_2(n) = 32.\left(\frac{1}{2}\right)^n\ u(n) = 2^{5-n}\ u(n) \tag{11}$$

$x(0)$, $x(1)$, $x(2)$ are 8, 16, 32 (or) 32, 16, 8 respectively

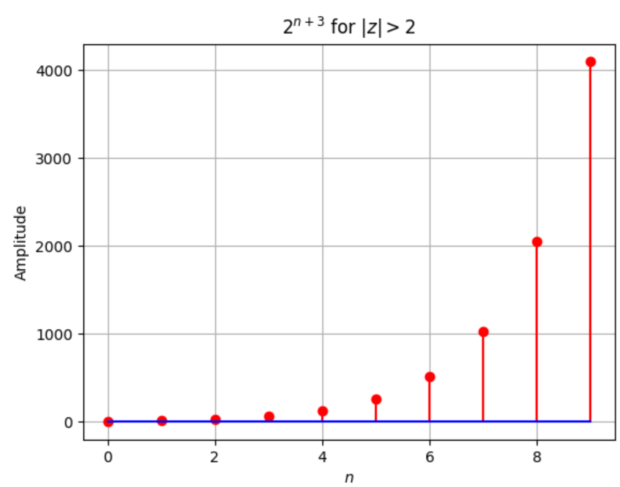

Fig. 0. *

Graph of 2^{n+3}

$$X(z) = \sum_{k=-\infty}^{\infty} x(k).z^{-k} \tag{12}$$

$$X_1(z) = \sum_{k=-\infty}^{\infty} x_1(k).z^{-k} \tag{13}$$

from (10),

$$X_1(z) = \sum_{k=0}^{\infty} 2^{k+3}\ u(k)\ z^{-k} \tag{14}$$

Hence,

$$X_1(z) = \frac{8}{1 - 2z^{-1}},\ \text{ROC} \implies \{|2z^{-1}| < 1\} \tag{15}$$

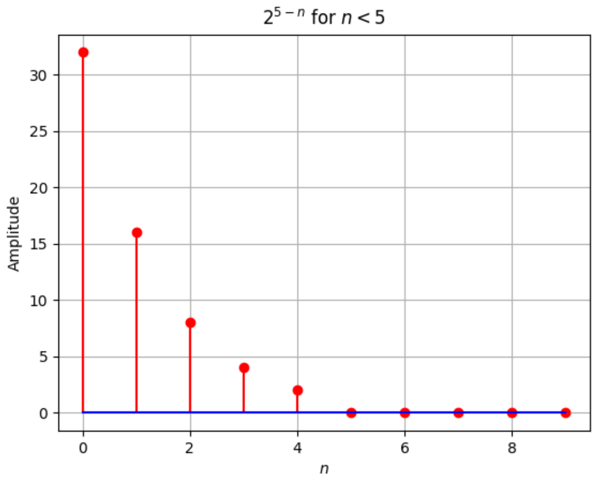

Fig. 0. *

Graph of 2^{5-n}

and also from (11),

$$X_2(z) = \sum_{k=-\infty}^{\infty} x_2(k) .z^{-k} \tag{16}$$

$$X_2(z) = \sum_{k=0}^{\infty} 2^{5-k}\ u(k)\, z^{-k} \tag{17}$$

Hence,

$$X_2(z) = \frac{32}{1-(2z)^{-1}},\ \text{ROC} \implies \{|(2z)^{-1}| < 1\} \tag{18}$$

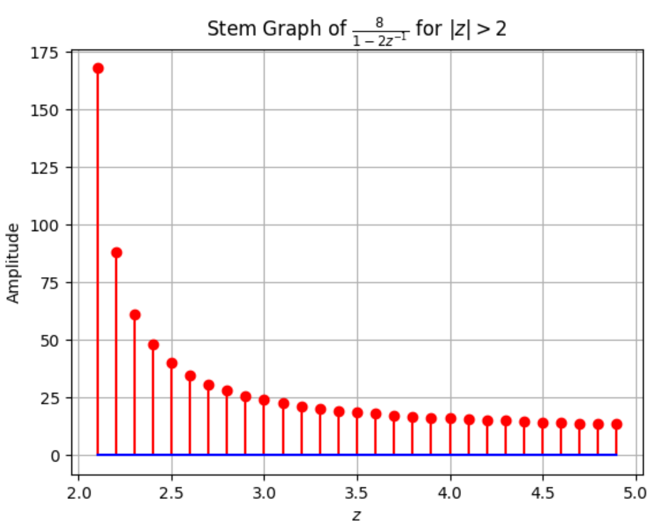

Fig. 0. *

Graph of $X_1(Z)$

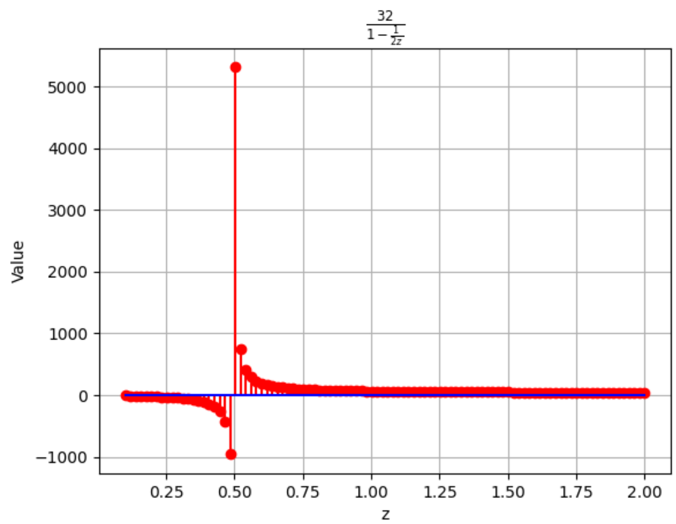

Fig. 0. *

Graph of $X_2(Z)$