Assignment
10.5.4-2

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Question:

Q10) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Solution:

Table of Parameters

Input Variable	Condition
x(0)	first term of GP
r	common ratio of GP
$x(0), x(1), x(2)$	three terms in GP
$x(0), x(1), x(2)$	$x(0) + x(1) + x(2) = 56$
$x(0) - 1, x(1) - 7, x(2) - 21$	form an AP
x(n)	$(n+1)^{th}$ term of GP
$x(n) \overset{Z}{\longleftrightarrow} X(z)$	z-transform of x(n)

We know that, if three numbers p,q and r are in arithmetic progression then,

$$2q = p + r \tag{1}$$

Then $(n+1)^{th}$ term of GP x(n) is given by:

$$x(n) = x(0).r^n \tag{2}$$

Then from given,

$$x(0) + x(1) + x(2) = 56$$

$$x(0).\left(1 + r + r^2\right) = 56$$

$$x(0) = \frac{56}{(1 + r + r^2)} \tag{3}$$

and from given another case following are in AP,

$$x(0) - 1, x(1) - 7, x(2) - 21$$

Then from (1),

$$2(x(1) - 7) = x(0) - 1 + x(2) - 21$$

$$2rx(0) - 14 = x(0) + x(0)r^2 - 22$$

$$x(0) + x(0)r^2 - 2rx(0) = 8$$

$$x(0)\left(r^2 - 2r + 1\right) = 8$$

and from (3)

$$\frac{56.\left(r^2 - 2r + 1\right)}{(1 + r + r^2)} = 8$$

$$7\left(r^2 - 2r + 1\right) = \left(1 + r + r^2\right)$$

$$6r^2 - 15r + 6 = 0$$

$$2r^2 - 5r + 2 = 0$$

$$(2r - 1)(r - 2) = 0$$

$$r = \frac{1}{2}, 2 \tag{4}$$

so from (3),

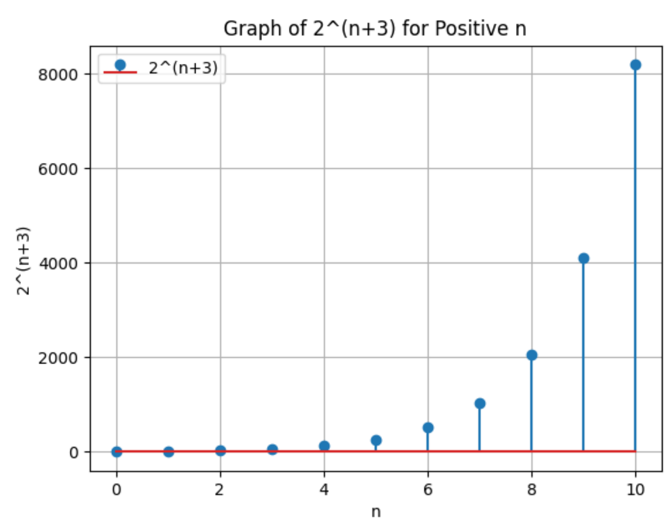
$$x(0) = 8$$

Then from (2)

$$x(n) = 8.2^n = 2^{n+3} \qquad for\,(r = 2) \tag{5}$$

$$x(n) = 32.\left(\frac{1}{2}\right)^n = 2^{5-n} \quad for\left(r = \frac{1}{2}\right) \tag{6}$$

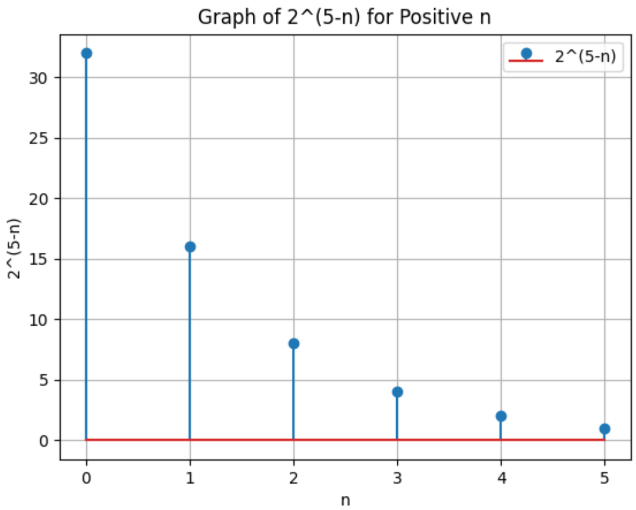
$x(0)$,$x(1)$ and $x(2)$ are 8,16 and 32 (or) 32,16 and 8 respectively We know that Z-Transform of x(n) is given by:



$$X(z) = \sum_{k=-\infty}^{\infty} x(k).z^{-k} \tag{7}$$

where, we assume that x(k)=0 for $(k < 0)$
(7) Then, modify as follows:

$$X(z) = \sum_{k=0}^{\infty} x(k).z^{-k} \tag{8}$$



from (5),

$$X(z) = \sum_{k=0}^{\infty} 8.2^k.z^{-k}$$

$$X(z) = 8.\sum_{k=0}^{\infty} 2^k.z^{-k}$$

$$X(z) = 8.\sum_{k=0}^{\infty}\left(\frac{2}{z}\right)^k$$

or,

$$X(z) = \lim_{n\to\infty} 8.\sum_{k=0}^{n}\left(\frac{2}{z}\right)^k$$

$$X(z) = 8.\lim_{n\to\infty}\sum_{k=0}^{n}\left(\frac{2}{z}\right)^k$$

$$X(z) = 8.\lim_{n\to\infty}\frac{\left(2.z^{-1}\right)^{n+1} - 1}{2.z^{-1} - 1}$$

Hence,

$$X_1(z) = \frac{8}{1 - 2z^{-1}} \quad if \ \left(|2z^{-1}| < 1\right)$$

or,

$$X_1(z) \ \ is \ undefined \quad if \ \left(|2z^{-1}| > 1\right)$$

and also from (6),

$$X(z) = \sum_{k=0}^{\infty} 32.\left(\frac{1}{2}\right)^k.z^{-k}$$

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$$X(z) = 32.\lim_{n\to\infty}\sum_{k=0}^{n}\left(\frac{1}{2z}\right)^k$$

$$X(z) = 32.\lim_{n\to\infty}\frac{\left((2z)^{-1}\right)^{n+1} - 1}{(2z)^{-1} - 1}$$

Hence,

$$X_2(z) = \frac{32}{1 - (2z)^{-1}} \quad if \ \left(|(2z)^{-1}| < 1\right)$$

or,

$$X_2(z) \ \ is \ undefined \quad if \ \left(|(2z)^{-1}| > 1\right)$$