Assignment
10.5.4-2

ee23btech11215 - Penmetsa Srikar Varma

Question:

Q10) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Solution:

Table of Parameters

Input Variable	Condition
$x(0)$	first term of AP
r	common ratio of GP
$x(0), x(1), x(2)$	three terms in GP
$x(0), x(1), x(2)$	$x(0) + x(1) + x(2) = 56$
$x(0) - 1, x(1) - 7, x(2) - 21$	form an AP
$x(n)$	$(n+1)^{th}$ term of GP
$x(n) \overset{Z}{\longleftrightarrow} X(z)$	$X(z) = x(0)\sum_{k=0}^{k=\infty}\left(\frac{z}{r}\right)^{-k}$

We know that, if three numbers p,q and r are in arithmetic progression then,

$$2q = p + r \tag{1}$$

Then $(n+1)^{th}$ term of GP x(n) is given by:

$$x(n) = x(0).r^n \tag{2}$$

Then from given,

$$x(0) + x(1) + x(2) = 56 \tag{3}$$

$$x(0).\left(1 + r + r^2\right) = 56 \tag{4}$$

$$x(0) \implies \frac{56}{(1 + r + r^2)} \tag{5}$$

and from given another case following are in AP,

$$x(0) - 1, x(1) - 7, x(2) - 21$$

Then from (1),

$$2(x(1) - 7) = x(0) - 1 + x(2) - 21 \tag{6}$$

$$x(0)\left(r^2 - 2r + 1\right) = 8 \tag{7}$$

and from (5)

$$\frac{56.\left(r^2 - 2r + 1\right)}{(1 + r + r^2)} = 8 \tag{8}$$

$$7\left(r^2 - 2r + 1\right) = \left(1 + r + r^2\right) \tag{9}$$

$$6r^2 - 15r + 6 = 0 \tag{10}$$

$$2r^2 - 5r + 2 = 0 \tag{11}$$

$$(2r - 1)(r - 2) = 0 \tag{12}$$

$$r = 2, \frac{1}{2} \tag{13}$$

so from (5),

$$x(0) = 8, 32 \tag{14}$$

Then from (2)

$$x_1(n) = 8.2^n = 2^{n+3} \qquad for\,(r = 2) \tag{15}$$

$$x_2(n) = 32.\left(\frac{1}{2}\right)^n = 2^{5-n} \quad for\left(r = \frac{1}{2}\right) \tag{16}$$

$x(0)$,$x(1)$ and $x(2)$ are 8,16 and 32 (or) 32,16 and 8 respectively We know that Z-Transform of x(n) is given by:

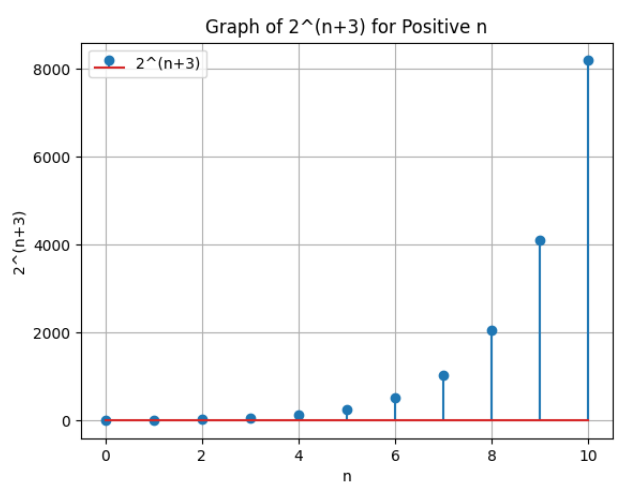

Fig. 0. *

Graph of 2^{n+3}

$$X(z) = \sum_{k=-\infty}^{\infty} x(k).z^{-k} \tag{17}$$

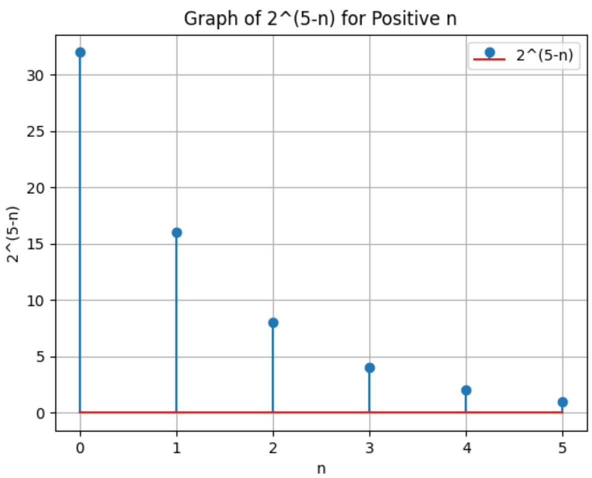

Fig. 0. *

Graph of 2^{5-n}

where, we assume that x(k)=0 for ($k < 0$)
(17) Then, modify as follows:

$$X_1(z) = \sum_{k=0}^{\infty} x_1(k).z^{-k} \quad (18)$$

from (15),

$$X_1(z) = \sum_{k=0}^{\infty} 8.2^k.z^{-k} \quad (19)$$

$$X_1(z) = 8.\sum_{k=0}^{\infty} 2^k.z^{-k} \quad (20)$$

$$X_1(z) = 8.\sum_{k=0}^{\infty} \left(\frac{2}{z}\right)^k \quad (21)$$

or,

$$X_1(z) = \lim_{n\to\infty} 8.\sum_{k=0}^{n} \left(\frac{2}{z}\right)^k \quad (22)$$

$$X_1(z) = 8.\lim_{n\to\infty} \sum_{k=0}^{n} \left(\frac{2}{z}\right)^k \quad (23)$$

$$X_1(z) = 8.\lim_{n\to\infty} \frac{\left(2.z^{-1}\right)^{n+1} - 1}{2.z^{-1} - 1} \quad (24)$$

Hence,

$$X_1(z) = \frac{8}{1 - 2z^{-1}} \quad if \;\; (|)\, 2z^{-1}| < 1 \quad (25)$$

or,

$$X_1(z) \;\; is\ undefined \quad if \;\; \left(|2z^{-1}| > 1\right)$$

and also from (16),

$$X_2(z) = \sum_{k=0}^{\infty} 32.\left(\frac{1}{2}\right)^k.z^{-k} \quad (26)$$

$$X_2(z) = 32.\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k.z^{-k} \quad (27)$$

$$X_2(z) = 32.\sum_{k=0}^{\infty} \left(\frac{1}{2z}\right)^k \quad (28)$$

or,

$$X_2(z) = \lim_{n\to\infty} 32.\sum_{k=0}^{n} \left(\frac{1}{2z}\right)^k \quad (29)$$

$$X_2(z) = 32.\lim_{n\to\infty} \sum_{k=0}^{n} \left(\frac{1}{2z}\right)^k \quad (30)$$

$$X_2(z) = 32.\lim_{n\to\infty} \frac{\left((2z)^{-1}\right)^{n+1} - 1}{(2z)^{-1} - 1} \quad (31)$$

Hence,

$$X_2(z) = \frac{32}{1 - (2z)^{-1}} \quad if \;\; \left(|(2z)^{-1}| < 1\right) \quad (32)$$

or,

$$X_2(z) \;\; is\ undefined \quad if \;\; \left(|(2z)^{-1}| > 1\right)$$

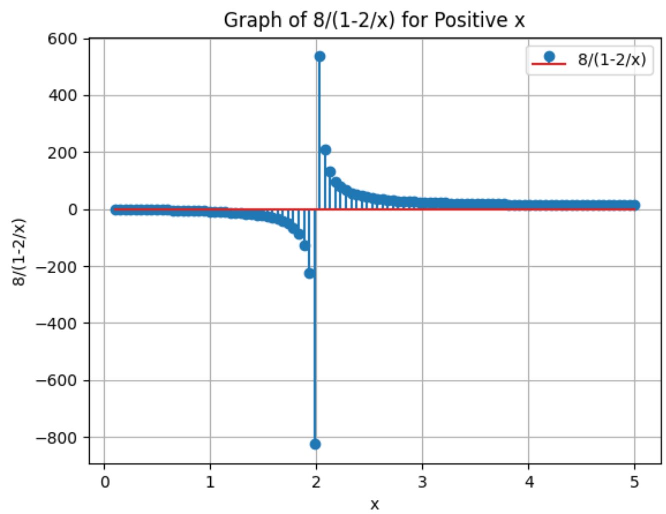

Fig. 0. *

Graph of $\frac{8}{1-2Z^{-1}}$

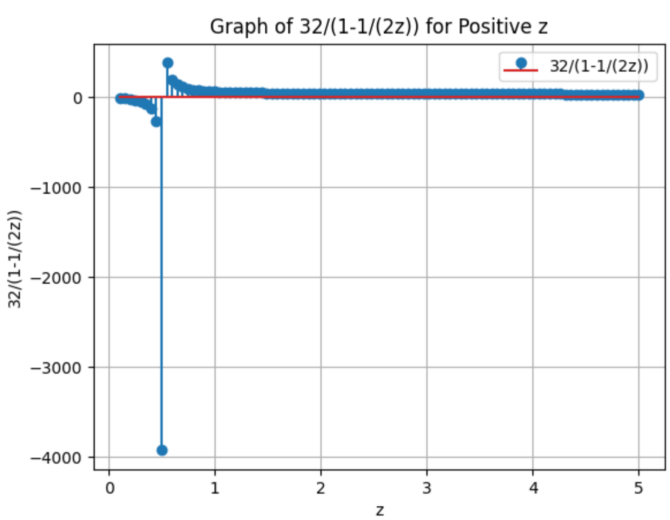

Fig. 0. *

Graph of $\frac{32}{1-(2Z)^{-1}}$