Assignment

10.5.4-2

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Question:

Q2) The sum of the third and the seventh terms of AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP

Solution:

Table of Parameters

Input Variables	Input Condition
y(2)+y(6)	6
y(2).y(6)	8
$y_1(n)$	general term of first AP
$y_2(n)$	general term of second AP
$S_1(n)$	sum of first n terms first AP
$S_2(n)$	sum of first n terms of second AP
$y_1(0)$	first term of first AP
$y_2(0)$	first term of second AP
d_1	common difference of first AP
d_1	common differenece of second AP
$y_{1,2}(n) \overset{\mathcal{Z}}{\longleftrightarrow} Y_{1,2}(z)$	z-transform of $y_{1,2}(n)$
$s_{1,2}(n) \overset{\mathcal{Z}}{\longleftrightarrow} S_{1,2}(z)$	z-transform of $s_{1,2}(n)$

Then general term y(n) of arithmetic progression is given by:

$$y_{1,2}(n) = y_{1,2}(0) + n\ d_{1,2} \tag{1}$$

Then from table of parameters,

$$y_{1,2}(6)(6 - y_{1,2}(6)) = 8 \tag{2}$$

$$y_{1,2}(6) = 2 \text{ or } 4 \tag{3}$$

Then from table and (3)

$$y_{1,2}(2) = 4 \text{ or } 2 \tag{4}$$

for $y_1(2) = 2$ and $y_1(6) = 4$

$$y_1(0) = 1,\ d_1 = \frac{1}{2} \tag{5}$$

for $y_2(2) = 4$ and $y_2(6) = 2$

$$y_2(0) = 1,\ d_2 = -\frac{1}{2} \tag{6}$$

We know that the sum of first n terms of arithmetic progression is given by:

$$S_1(n) = \frac{n}{2}(2y_1(0) + (n-1)d_1)u(n) \tag{7}$$

Then from (7)

$$S_1(16) = \frac{16}{2}(2y_1(0) + 15d_1) \tag{8}$$

Hence from (8), for $y_1(0) = 1$, $d_1 = \frac{1}{2}$

$$S_1(16) = 76 \tag{9}$$

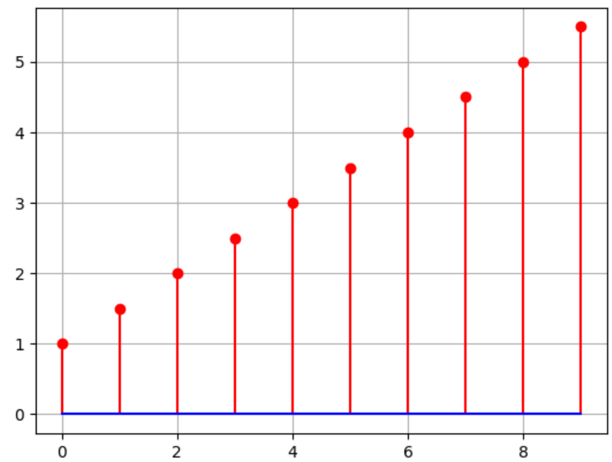
or from (8), for $y_1(0)$=5, $d_2 = -\frac{1}{2}$

$$S_2(16) = 20 \tag{10}$$

The general term of AP $y_{1,2}(n)$ and sum of first n terms of AP $S_{1,2}(n)$ are given by:

$$y_1(n) = \left(\frac{n+2}{2}\right)u(n) \text{ and } S_1(n) = \left(\frac{n(n+3)}{4}\right)u(n) \tag{11}$$

$$y_2(n) = \left(\frac{10-n}{2}\right)u(n) \text{ and } S_2(n) = \left(\frac{n(21-n)}{4}\right)u(n) \tag{12}$$



Graph of $y_1(n)$

z-Transform of y(n) is given from table :

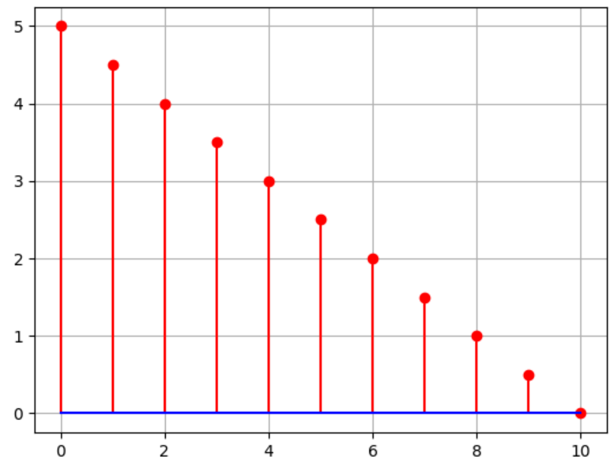
$$Y_1(z) = \sum_{k=0}^{\infty}\left(\frac{k+2}{2}\right) z^{-k} \tag{13}$$

$$Y_1(z) = \frac{1}{2}\left(\sum_{k=0}^{\infty} k\ z^{-k}\right) + \sum_{k=0}^{\infty} z^{-k} \tag{14}$$

$$Y_1(z) = \frac{1}{2}\left(\frac{z^{-1}}{(1-z^{-1})^2}\right) + \frac{1}{1-z^{-1}} \tag{15}$$

So,

$$Y_1(z) = \frac{2-z^{-1}}{2.(1-z^{-1})^2}, \quad |z^{-1}| < 1 \tag{16}$$



Graph of $y_2(n)$

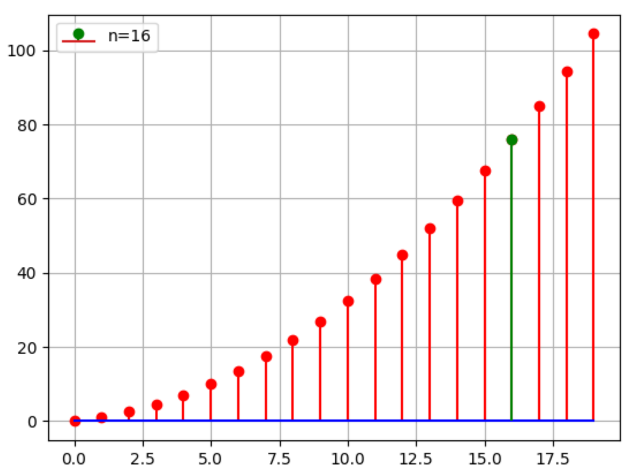
Similarly,

$$Y_2(z) = \sum_{k=0}^{\infty} \left(\frac{10-k}{2} \right) z^{-k} \tag{17}$$

$$Y_2(z) = 5\left(\sum_{k=0}^{\infty} z^{-k} \right) - \frac{1}{2}\left(\sum_{k=0}^{\infty} k\ z^{-k} \right) \tag{18}$$

So,

$$Y_2(z) = \frac{10 - 11z^{-1}}{2.(1-z^{-1})^2}, \quad |z^{-1}| < 1 \tag{19}$$



Graph of $S_1(n)$

Similarly for sum of first n terms of AP,

$$S(z) = \sum_{k=-\infty}^{\infty} \left(\left(y(0) - \frac{d}{2} \right) k + \frac{d}{2}\ k^2 \right) u(k)\, z^{-k} \tag{20}$$

$$S(z) = \left(y(0) - \frac{d}{2} \right) \left(\sum_{k=0}^{\infty} k\ z^{-k} \right) + \frac{d}{2} \left(\sum_{k=0}^{\infty} k^2\ z^{-k} \right) \tag{21}$$

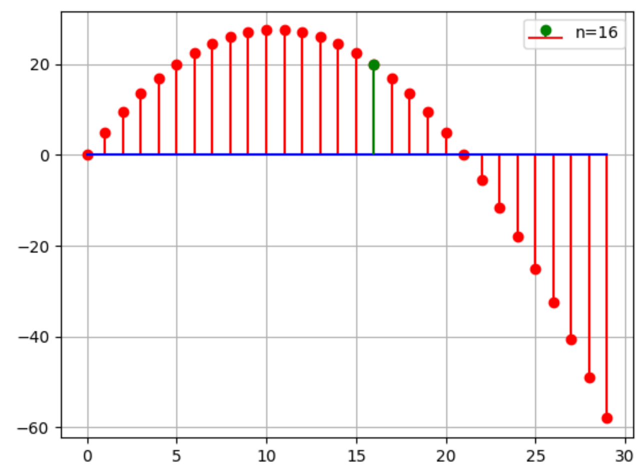
$$S(z) = y(0) \left(\frac{z}{(z-1)^2} \right) + d \left(\frac{z}{(z-1)^3} \right) \tag{22}$$

for $y_1(0) = 1$ and $d_1 = \frac{1}{2}$

$$S_1(z) = \frac{z\left(z - \frac{1}{2}\right)}{(z-1)^3}, \quad |z| > 1 \tag{23}$$

for $y_2(0) = 5$ and $d_2 = -\frac{1}{2}$

$$S_2(z) = \frac{z\left(5z - \frac{11}{2}\right)}{(z-1)^3}, \quad |z| > 1 \tag{24}$$



Graph of $S_2(n)$

Inverse z-transform of $S_{1,2}(z)$ by counter integral method is given by:

$$s_{1,2}(n) = \oint_C S_{1,2}(z)\ z^{n-1}\, dz \tag{25}$$

from (22),

$$s_{1,2}(n) = y_{1,2}(0) \oint_C \left(\frac{z}{(z-1)^2} dz \right) + d_{1,2} \left(\oint_C \frac{z}{(z-1)^3}\, dz \right) \tag{26}$$

We can observe that pole is repeated 2,3 times so, m=2 and 3 respectively:

$$s_{1,2}(n) = y_{1,2}(0) \left| \frac{1}{1!} \frac{d}{dz} (z^n) \right|_{z=1} + d_{1,2} \left| \frac{1}{2!} \frac{d^2}{dz^2} (z^n) \right|_{z=1} \tag{27}$$

$$s_{1,2}(n) = \left(n\ y_{1,2}(0) + n(n-1) \frac{d_{1,2}}{2} \right) u(n) \tag{28}$$

$$s_1(n) = \frac{n(n+3)}{4} u(n), \quad s_1(16) = 76 \tag{29}$$

and,

$$s_2(n) = \frac{n(21-n)}{4} u(n), \quad s_2(16) = 20 \tag{30}$$

we can observe that (9) and (29),(10) and (30) are the same results