Assignment 10.5.4-2

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Question:

Q2) The sum of the third and the seventh terms of AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP

Solution:

Table of Parameters

Input Variables	Input Condition
x(2)	third term of AP
x(6)	seventh term of AP
x(2)+x(6)	6
x(2).x(6)	8
S(16)	sum of first 16 terms of AP
x(0)	first term of AP
x($n-1$)	n^{th} term of AP
S(n)	Sum of n terms of AP

Then general term x($n-1$) of arithmetic progression is given by:

$$x(n-1) = x(0) + (n-1).d \quad (or) \quad x(n) = x(0) + (n).d \tag{1}$$

So, from the table, third term and seventh term of arithmetic progression be $a(2)$ and $a(6)$ respectively,
Then from (1):

$$x(n-1) = x(0) + 2.d \tag{2}$$

$$x(6) = x(0) + 6.d \tag{3}$$

Then from (2) and (3)

$$x(2) + x(6) = 6 \tag{4}$$

$$x(2).x(6) = 8 \tag{5}$$

or we can say from (4),

$$2x(0) + 8d = 6$$

$$x(0) + 4d = 3$$

$$x(0) = 3 - 4d \tag{6}$$

or we can say from (5),

$$(x(0) + 2d).(x(0) + 6d) = 8$$

and from (6),

$$(3 - 2d).(3 + 2d) = 8$$

$$9 - 4d^2 = 8$$

$$d^2 = \frac{1}{4}$$

$$d = \frac{1}{2}, -\frac{1}{2} \tag{7}$$

Then from (6),

$$x(0) = 1, 5 \tag{8}$$

We know that the sum of first n terms of arithmetic progression is given by:

$$S(n) = \frac{n}{2}(2.x(0) + (n-1).d) \tag{9}$$

Then from (9) let sum of first 16 terms of arithmetic progression be S_{16}:

$$S(16) = \frac{16}{2}(2.x(0) + 15d) \tag{10}$$

Hence from (10), for $x(0)$=1,d=$\frac{1}{2}$

$$S(16) = 76$$

or from (10), for $x(0)$=5,d=-$\frac{1}{2}$

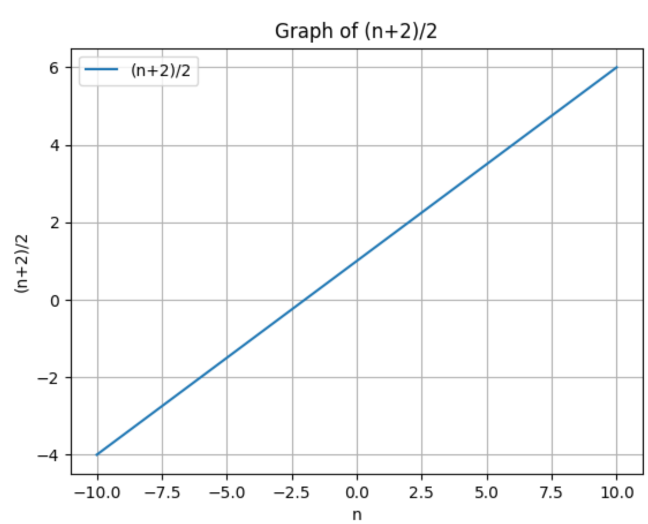
$$S(16) = 20$$

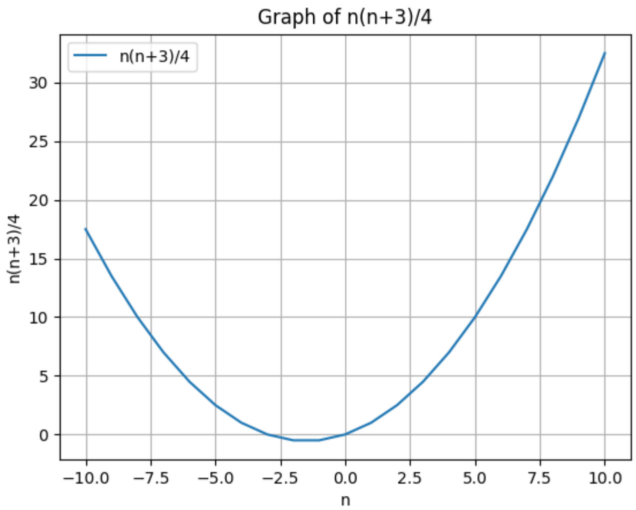
The general term of AP (a_n) and sum of first n terms of AP (a_n) are given by:

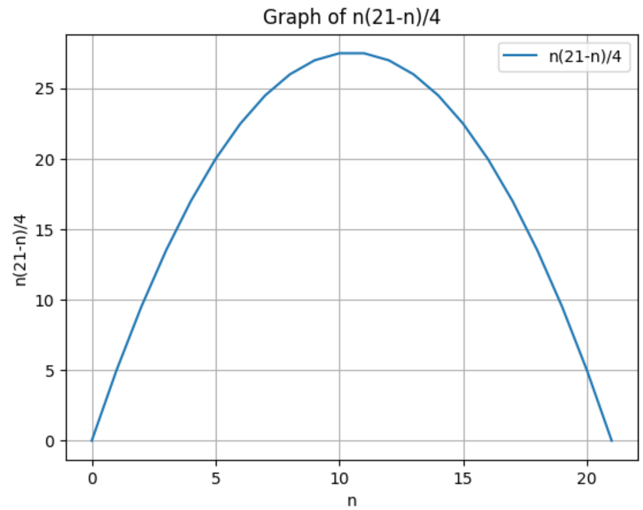
$$x(n) = x(0) + n.d \quad and \quad S(n) = \frac{n}{2}(2.x(0) + (n-1).d)$$

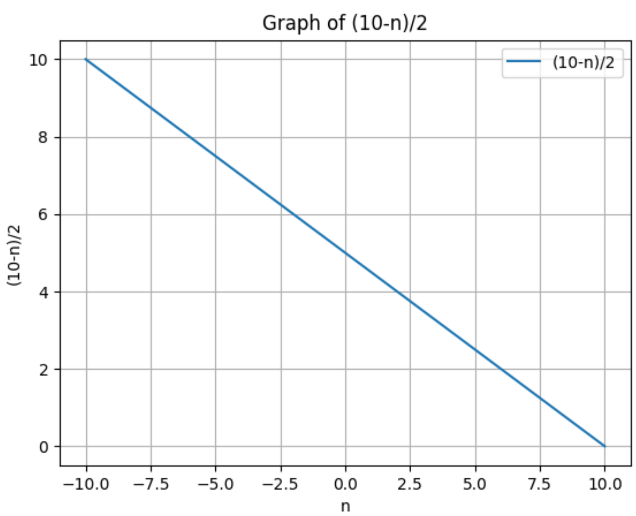
$$x(n) = \frac{n+2}{2} \quad and \quad S(n) = \frac{n.(n+3)}{4} \quad for\,(x(0) = 1, d = \frac{1}{2})$$

$$x(n) = \frac{10-n}{2} \quad and \quad S(n) = \frac{n.(21-n)}{4} \quad for\,(x(0) = 5, d = -\frac{1}{2})$$









We know that Z-Transform of x(n) is given by:

$$X(z) = \sum_{k=-\infty}^{\infty} x(k).z^{-k} \tag{11}$$

where, we assume that x(k)=0 for ($k < 0$)
Then, (11) modifiy as follows:

$$X(z) = \sum_{k=0}^{\infty} x(k).z^{-k} \tag{12}$$

$$X(z) = \sum_{k=0}^{\infty} \left(\frac{k+2}{2}\right).z^{-k}$$

$$X(z) = \frac{1}{2}\left(\sum_{k=0}^{\infty} k.z^{-k}\right) + \sum_{k=0}^{\infty} z^{-k} \tag{13}$$

Then consider sum, say S:

$$S = \left(\sum_{k=0}^{\infty} k.z^{-k}\right)$$

$$S = 0 + \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3}....\infty \tag{14}$$

$$\frac{S}{z} = 0 + \frac{1}{z^2} + \frac{2}{z^3} + \frac{3}{z^4}....\infty \tag{15}$$

Then from (15)-(14)

$$S\left(1 - z^{-1}\right) = \sum_{k=0}^{\infty} z^{-k}$$

$$S = \frac{\sum_{k=0}^{\infty} z^{-k}}{1 - z^{-1}} \tag{16}$$

So,(16) in (13) Then,

$$X(n) = \sum_{k=0}^{\infty} z^{-k}\left(\frac{1}{2.(1 - z^{-1})} + 1\right)$$

$$X(n) = \lim_{n\to\infty} \sum_{k=0}^{n} z^{-k}\left(\frac{1}{2.(1 - z^{-1})} + 1\right)$$

$$X(n) = \left(\frac{1}{2.(1 - z^{-1})} + 1\right). \lim_{n\to\infty} \sum_{k=0}^{n} z^{-k}$$

$$X(n) = \left(\frac{1}{2.(1 - z^{-1})} + 1\right). \lim_{n\to\infty} \left(\frac{1 - \left(z^{-1}\right)^{n+1}}{1 - z^{-1}}\right)$$

So,

$$X_1(n) = \frac{1 + 2.\left(1 - z^{-1}\right)}{2.(1 - z^{-1})^2} \quad for\ |z^{-1}| < 1$$

or,

$$X_1(n)\ \ is\ not\ defined \quad for\ |z^{-1}| > 1$$

From (12),

$$X(z) = \sum_{k=0}^{\infty} \left(\frac{10-k}{2} \right) . z^{-k}$$

$$X(n) = 5. \left(\sum_{k=0}^{\infty} z^{-k} \right) - \frac{1}{2} \left(\sum_{k=0}^{\infty} k. z^{-k} \right)$$

Then from (16),

$$X(n) = \left(5 - \frac{1}{2.(1-z^{-1})} \right) \sum_{k=0}^{\infty} z^{-k}$$

$$X(n) = \left(5 - \frac{1}{2.(1-z^{-1})} \right) \lim_{n \to \infty} \sum_{k=0}^{n} z^{-k}$$

$$X_2(n) = \frac{10.\left(1-z^{-1}\right) - 1}{2.(1-z^{-1})^2} \quad for \quad |z^{-1}| < 1$$

$$X_2(n) \;\; is \; not \; defined \quad for \qquad |z^{-1}| > 1$$