

Assignment

10.5.4-2

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QUESTION:

Q10) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

SOLUTION:

Table of Parameters

Input Variable	Condition
$x(0)$	first term of GP
r	common ratio of GP
$\frac{x(0)}{r}, x(0), x(0) \cdot r$	three terms in GP
$\frac{x(0)}{r}, x(0), x(0) \cdot r$	$\frac{x(0)}{r} + x(0) + x(0) \cdot r = 56$
$\frac{x(0)}{r} - 1, x(0) - 7, x(0) \cdot r - 21$	form an AP
$x(n-1)$	n^{th} term of GP
$x(n) \xleftrightarrow{Z} X(z)$	z-transform of $x(n)$

We know that, if three numbers p,q and r are in arithmetic progression then,

$$2q = p + r \quad (1)$$

Then n^{th} term of GP $x(n-1)$ is given by:

$$x(n-1) = x(0) \cdot r^{n-1} \quad \text{or} \quad x(n) = x(0) \cdot r^n \quad (2)$$

Then from given,

$$\frac{x(0)}{r} + x(0) + x(0) \cdot r = 56$$

$$x(0) \cdot \left(\frac{1}{r} + 1 + r \right) = 56$$

$$x(0) = \frac{56}{\left(\frac{1}{r} + 1 + r \right)} \quad (3)$$

and from given another case following are in AP,

$$\frac{x(0)}{r} - 1, x(0) - 7, x(0) \cdot r - 21$$

Then from (1),

$$2 \cdot (x(0) - 7) = \frac{x(0)}{r} - 1 + x(0) \cdot r - 21$$

$$2 \cdot x(0) - 14 = \frac{x(0)}{r} + x(0) \cdot r - 22$$

$$\frac{x(0)}{r} + x(0) \cdot r - 2 \cdot x(0) = 8$$

$$x(0) \left(\frac{1}{r} + r - 2 \right) = 8$$

and from (3)

$$\frac{56 \cdot \left(\frac{1}{r} + r - 2 \right)}{\left(\frac{1}{r} + 1 + r \right)} = 8$$

$$7 \left(\frac{1}{r} + r - 2 \right) = \left(\frac{1}{r} + 1 + r \right)$$

$$\frac{6}{r} + 6 \cdot r - 15 = 0$$

$$6 \cdot r^2 - 15 \cdot r + 6 = 0 = 2 \cdot r^2 - 5 \cdot r + 2$$

$$(2 \cdot r - 1) \cdot (r - 2) = 0$$

$$r = \frac{1}{2}, 2 \quad (4)$$

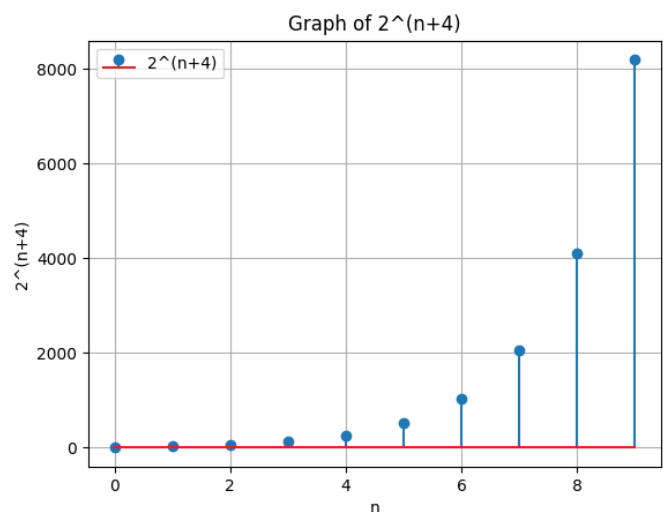
so from (3),

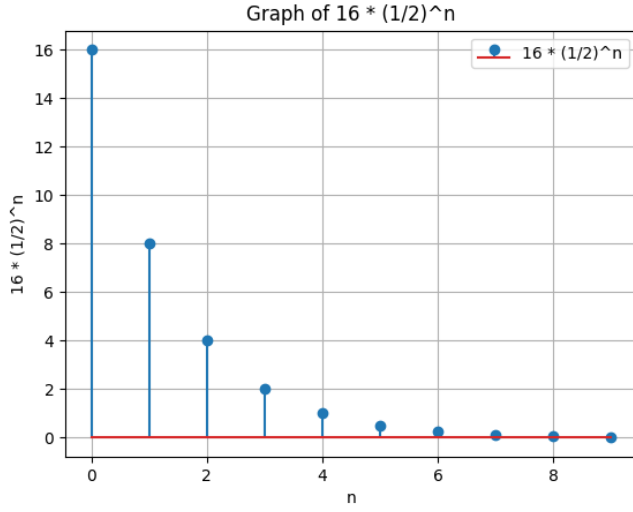
$$x(0) = 16$$

Then from (2),

$$x(n) = 16 \cdot 2^n = 2^{n+4} \quad \text{for } (r = 2) \quad (5)$$

$$x(n) = 16 \cdot \left(\frac{1}{2} \right)^n = 2^{4-n} \quad \text{for } \left(r = \frac{1}{2} \right) \quad (6)$$





or,

$$X(z) = 16. \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k .z^{-k}$$

$$X(z) = 16. \sum_{k=0}^{\infty} \left(\frac{1}{2z}\right)^k$$

$$X(z) = \lim_{n \rightarrow \infty} 16. \sum_{k=0}^n \left(\frac{1}{2z}\right)^k$$

$$X(z) = 16. \lim_{n \rightarrow \infty} \sum_{k=0}^n \left(\frac{1}{2z}\right)^k$$

$$X(z) = 16. \lim_{n \rightarrow \infty} \frac{((2z)^{-1})^{n+1} - 1}{(2z)^{-1} - 1}$$

Hence,

$$X_2(z) = \frac{16}{1 - (2z)^{-1}} \quad \text{if } |(2z)^{-1}| < 1$$

or,

$$X_2(z) \text{ is undefined if } |(2z)^{-1}| > 1$$

We know that Z-Transform of $x(n)$ is given by:

$$X(z) = \sum_{k=-\infty}^{\infty} x(k) .z^{-k} \quad (7)$$

where, we assume that $x(k)=0$ for $(k < 0)$
(7) modify as follows:

$$X(z) = \sum_{k=0}^{\infty} x(k) .z^{-k} \quad (8)$$

from (5),

$$X(z) = \sum_{k=0}^{\infty} 16.2^k .z^{-k}$$

$$X(z) = 16. \sum_{k=0}^{\infty} 2^k .z^{-k}$$

$$X(z) = 16. \sum_{k=0}^{\infty} \left(\frac{2}{z}\right)^k$$

or,

$$X(z) = \lim_{n \rightarrow \infty} 16. \sum_{k=0}^n \left(\frac{2}{z}\right)^k$$

$$X(z) = 16. \lim_{n \rightarrow \infty} \sum_{k=0}^n \left(\frac{2}{z}\right)^k$$

$$X(z) = 16. \lim_{n \rightarrow \infty} \frac{(2.z^{-1})^{n+1} - 1}{2.z^{-1} - 1}$$

Hence,

$$X_1(z) = \frac{16}{1 - 2z^{-1}} \quad \text{if } |2z^{-1}| < 1$$

or,

$$X_1(z) \text{ is undefined if } |2z^{-1}| > 1$$

and also from (6),

$$X(z) = \sum_{k=0}^{\infty} 16. \left(\frac{1}{2}\right)^k .z^{-k}$$

