

Assignment

10.5.4-2

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QUESTION:

Q10) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

SOLUTION:

Table of Parameters

Input Variable	Condition
$x(0), x(n)$	first term and general term of a GP
r	common ratio of a GP
$x(0), x(1), x(2)$	three terms in GP
$x_i(n)$	general term of i^{th} GP sequence
$x_i(0)$	first term of i^{th} GP sequence
r_i	common ratio of i^{th} GP sequence

$(n+1)^{\text{th}}$ term of GP $x(n)$ is given by:

$$x(n) = x(0) r^n u(n) \quad (1)$$

Then from given table of parameters,

$$x(0) + x(1) + x(2) = 56 \quad (2)$$

$$x(0) \Rightarrow \frac{56}{(1 + r + r^2)} \quad (3)$$

and from given another case following are in AP,

$$x(0) - 1, x(1) - 7, x(2) - 21$$

$$2(x(1) - 7) = x(0) - 1 + x(2) - 21 \quad (4)$$

$$x(0)(r^2 - 2r + 1) = 8 \quad (5) \quad \text{from (9),}$$

and from (3)

$$\frac{56 \cdot (r^2 - 2r + 1)}{(1 + r + r^2)} = 8 \quad (6)$$

Hence,

$$r_1 = 2, r_2 = \frac{1}{2} \quad (7)$$

so from (3),

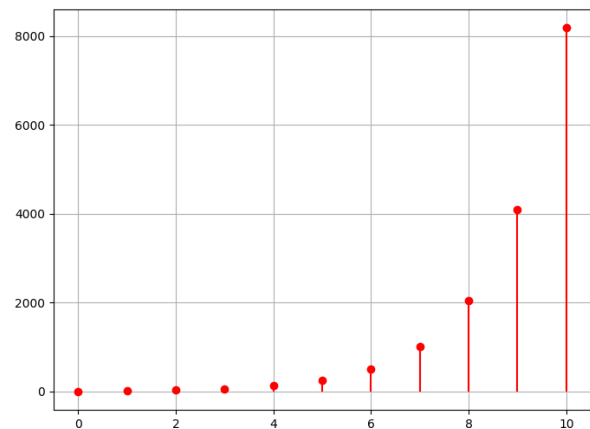
$$x_1(0) = 8, x_2(0) = 32 \quad (8)$$

Then from (1)

$$x_1(n) = 8 \cdot 2^n = 2^{n+3} u(n) \quad (9)$$

$$x_2(n) = 32 \cdot \left(\frac{1}{2}\right)^n u(n) = 2^{5-n} u(n) \quad (10)$$

$x_1(0), x_1(1)$ and $x_1(2)$ are 8, 16, 32 (or) $x_2(0), x_2(1)$ and $x_2(2)$ are 32, 16, 8 respectively



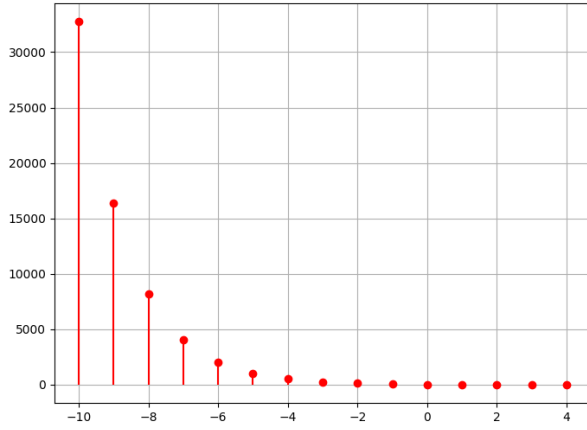
Graph of $x_1(n)$

z -transform of $x_1(n)$ is given by:

$$X_1(z) = \sum_{k=-\infty}^{\infty} x_1(k) \cdot z^{-k} \quad (11)$$

$$X_1(z) = \sum_{k=0}^{\infty} 2^{k+3} z^{-k} \quad (12)$$

$$X_1(z) = \frac{8}{1 - 2z^{-1}}, \quad |2z^{-1}| < 1 \quad (13)$$



Graph of $x_2(n)$

and also from (10),

$$X_2(z) = \sum_{k=-\infty}^{\infty} x_2(k) \cdot z^{-k} \quad (14)$$

$$X_2(z) = \sum_{k=0}^{\infty} 2^{5-k} z^{-k} \quad (15)$$

Hence,

$$X_2(z) = \frac{32}{1 - (2z)^{-1}}, \quad |(2z)^{-1}| < 1 \quad (16)$$