Discrete Assignment

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Question 10 of Exercise 5 of Chapter 9: Progressions and Series of class 11

Q10) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Answer:

Let us assume the first term of given geometric progression be $a(0)$ and common ratio be 'r'

Let the terms of GP be:

$$a(0), a(1), a(2)...a(k)$$

From above we can say that n^{th} term of GP a(n) is given by:

$$a(n) = a(0).r^n \tag{1}$$

Input table:

Input Variable	Input Condition
$a(0)$	first term of GP
r	common ratio of GP
$\frac{a(0)}{r}, a(0), a(0).r$	$\frac{a(0)}{r} + a(0) + a(0).r = 56$
$\frac{a(0)}{r} - 1, a(0) - 7, a(0).r - 21$	form an AP

We know that, if three numbers p,q and r are in arithmetic progression then,

$$2q = p + r \tag{2}$$

Let us assume three numbers which are in GP,

$$\frac{a(0)}{r}, a(0), a(0).r$$

Then from given,

$$\frac{a(0)}{r} + a(0) + a(0).r = 56$$

$$a(0).\left(\frac{1}{r} + 1 + r\right) = 56$$

$$a(0) = \frac{56}{\left(\frac{1}{r} + 1 + r\right)} \tag{3}$$

and from given another case following are in AP,

$$\frac{a(0)}{r} - 1, a(0) - 7, a(0).r - 21$$

Then from (**??**),

$$2.(a(0) - 7) = \frac{a(0)}{r} - 1 + a(0).r - 21$$

$$2.a(0) - 14 = \frac{a(0)}{r} + a(0).r - 22$$

$$\frac{a(0)}{r} + a(0).r - 2.a(0) = 8$$

$$a(0)\left(\frac{1}{r} + r - 2\right) = 8$$

and from (**??**)

$$\frac{56.\left(\frac{1}{r} + r - 2\right)}{\left(\frac{1}{r} + 1 + r\right)} = 8$$

$$7\left(\frac{1}{r} + r - 2\right) = \left(\frac{1}{r} + 1 + r\right)$$

$$\frac{6}{r} + 6.r - 15 = 0$$

$$6.r^2 - 15.r + 6 = 0 = 2.r^2 - 5.r + 2$$

$$(2.r - 1).(r - 2) = 0$$

$$r = \frac{1}{2}, 2 \tag{4}$$

so from (**??**),

$$a(0) = 16$$

Then from (**??**),

$$a(n) = 16.2^n = 2^{n+4} \quad for\ (r = 2) \qquad and \qquad a(n) = 16.\left(\frac{1}{2}\right)^n = 2^{4-n} \quad for\ (r = \frac{1}{2})$$

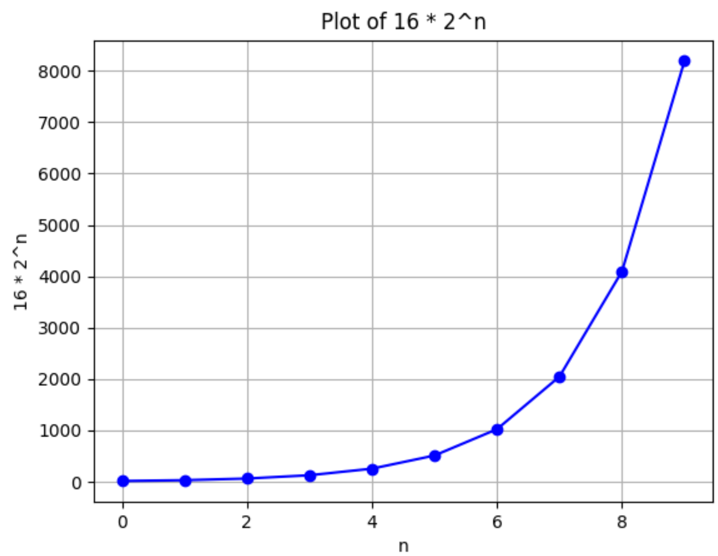

Figure 1: Graph of 2^{n+4}

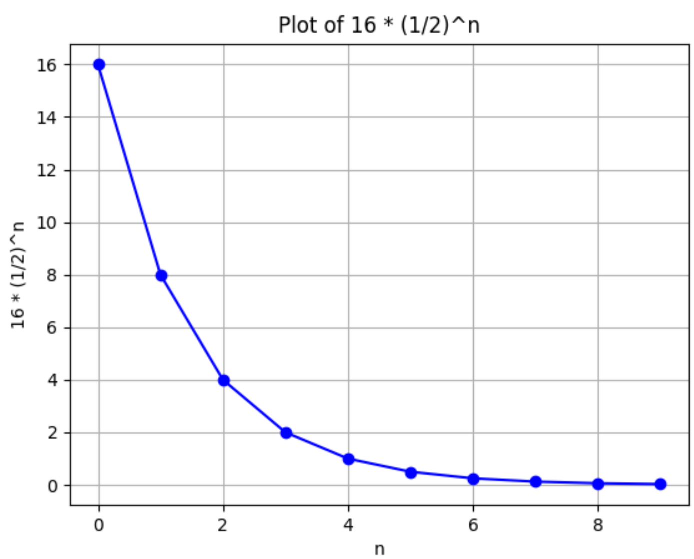

Figure 2: Graph of 2^{4-n}