Assignment

10.5.4-2

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QUESTION:

Q2) The sum of the third and the seventh terms of AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP

SOLUTION:

Table of Parameters

Input Variables	Input Condition
x(2)+x(6)	6
x(2).x(6)	8

Then general term x(n) of arithmetic progression is given by:

$$x(n) = x(0) + n.d \tag{1}$$

Then from table of parameters,

$$x(2) = 6 - x(6) \tag{2}$$

From (2)

$$x(2).x(6) = 8 \tag{3}$$

$$x(6) = 2 \text{ or } 4 \tag{4}$$

Then from (2) and (4)

$$x(2) = 4 \text{ or } 2 \tag{5}$$

for x(2) = 2 and x(6) = 4

$$x(0) = 1,\ d = \frac{1}{2} \tag{6}$$

for x(2) = 4 and x(6) = 2

$$x(0) = 5,\ d = -\frac{1}{2} \tag{7}$$

We know that the sum of first n terms of arithmetic progression is given by:

$$S(n) = \frac{n}{2}(2x(0) + (n-1)d)u(n) \tag{8}$$

Then from (8)

$$S(16) = \frac{16}{2}(2.x(0) + 15d) \tag{9}$$

Hence from (9), for $x(0)$=1,d=$\frac{1}{2}$

$$S_1(16) = 76 \tag{10}$$

or from (9), for $x(0)$=5,d=-$\frac{1}{2}$

$$S_2(16) = 20 \tag{11}$$

The general term of AP x(n) and sum of first n terms of AP S(n) are given by:

$$x_1(n) = \left(\frac{n+2}{2}\right)u(n) \text{ and } S_1(n) = \left(\frac{n(n+3)}{4}\right)u(n) \tag{12}$$

$$x_2(n) = \left(\frac{10-n}{2}\right)u(n) \text{ and } S_2(n) = \left(\frac{n(21-n)}{4}\right)u(n) \tag{13}$$

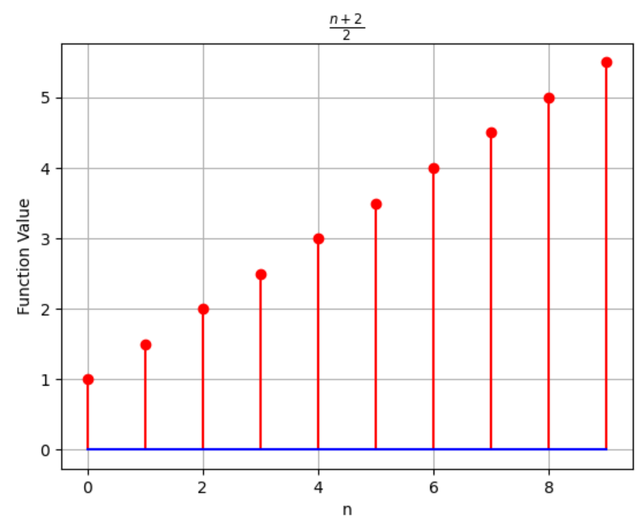

Fig. 0. *

Graph of $x_1(n)$

We know that Z-Transform of x(n) is given by:

$$X(z) = \sum_{k=-\infty}^{\infty} x(k)z^{-k} \tag{14}$$

$$X_1(z) = \sum_{k=0}^{\infty} x_1(k)z^{-k} \tag{15}$$

$$X_1(z) = \sum_{k=0}^{\infty}\left(\frac{k+2}{2}\right)u(k)z^{-k} \tag{16}$$

$$X_1(z) = \frac{1}{2}\left(\sum_{k=0}^{\infty} k\ u(k)z^{-k}\right) + \sum_{k=0}^{\infty} u(k)z^{-k} \tag{17}$$

$$X_1(z) = \frac{1}{2}\left(\frac{z^{-1}}{(1-z^{-1})^2}\right) + \frac{1}{1-z^{-1}} \tag{18}$$

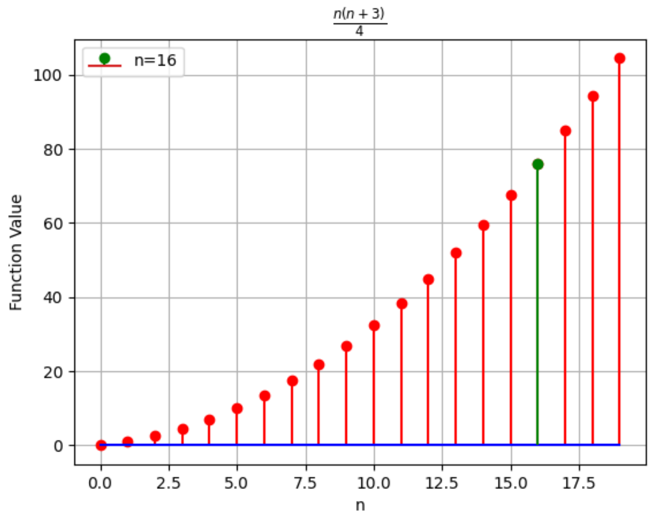

Fig. 0. *

Graph of $S_1(n)$

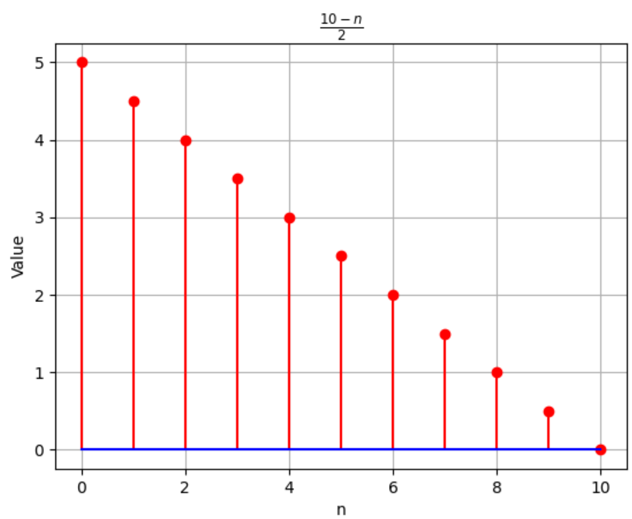

Fig. 0. *

Graph of $x_2(n)$

So,

$$X_1(Z) = \frac{2 - z^{-1}}{2.\,(1 - z^{-1})^2}, \text{ROC} \implies \{|z^{-1}| < 1\} \tag{19}$$

From (15),

$$X_2(z) = \sum_{k=0}^{\infty} x_2(k)\, z^{-k} \tag{20}$$

$$X_2(Z) = \sum_{k=0}^{\infty} \left(\frac{10-k}{2}\right) u(k)\, z^{-k} \tag{21}$$

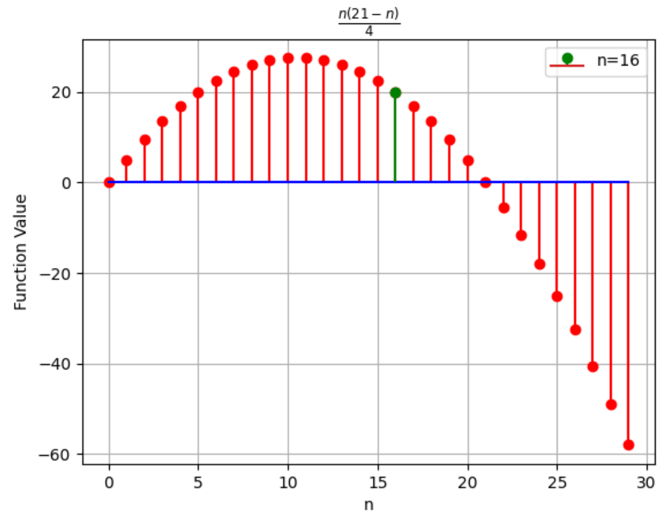

Fig. 0. *

Graph of $S_2(n)$

$$X_2(Z) = 5\left(\sum_{k=0}^{\infty} u(k)\, z^{-k}\right) - \frac{1}{2}\left(\sum_{k=0}^{\infty} k\ u(k)\, z^{-k}\right) \tag{22}$$

So,

$$X_2(Z) = \frac{10 - 11z^{-1}}{2.\,(1 - z^{-1})^2}, \text{ROC} \implies \{|z^{-1}| < 1\} \tag{23}$$

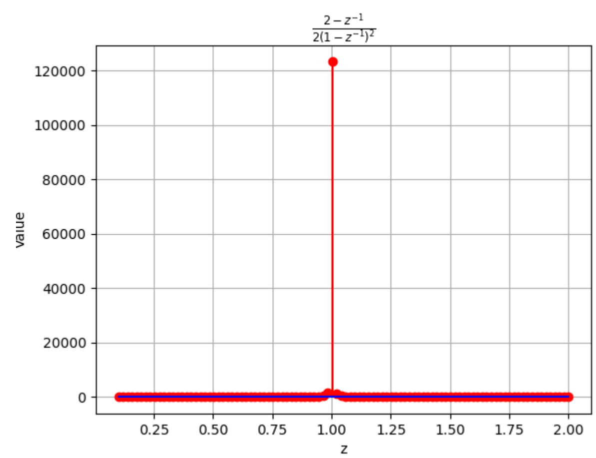

Fig. 0. *

Graph of $X_1(Z)$

Let, $\mathcal{Z}$ transform of s(n) be S(Z):
from (14):

$$S(Z) = \sum_{k=-\infty}^{\infty} s(k)\ z^{-k}$$

from (8)

$$S(Z) = \sum_{k=-\infty}^{\infty} \left(\left(x(0) - \frac{d}{2}\right)k + \frac{d}{2}\ k^2\right) u(k)\, z^{-k} \tag{24}$$

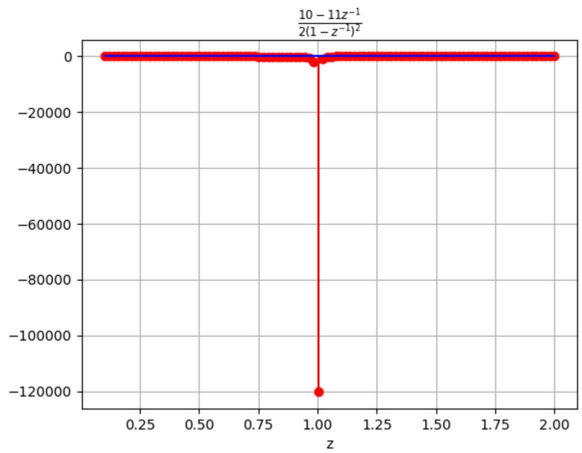

Fig. 0. *

Graph of $X_2(Z)$

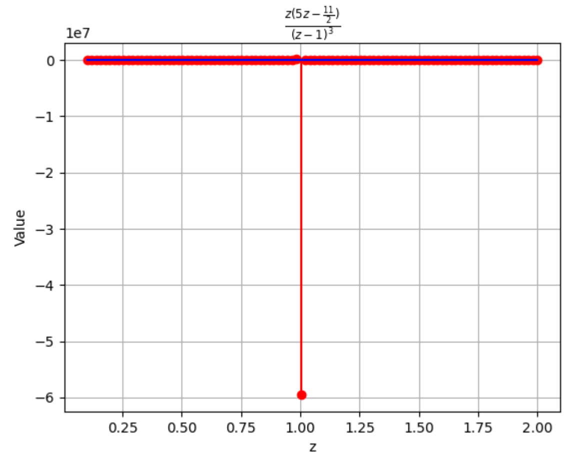

Fig. 0. *

Graph of $S_2(Z)$

$$S(Z) = \left(x(0) - \frac{d}{2}\right)\left(\sum_{k=0}^{\infty} k z^{-k}\right) + \frac{d}{2}\left(\sum_{k=0}^{\infty} k^2 z^{-k}\right) \tag{25}$$

$$s(Z) = x(0)\left(\frac{z}{(z-1)^2}\right) + d\left(\frac{z}{(z-1)^3}\right) \tag{26}$$

for x(0)=1 and $d = \frac{1}{2}$

$$S_1(Z) = \frac{z\left(z - \frac{1}{2}\right)}{(z-1)^3}, \text{ROC} \implies \{|z| > 1\} \tag{27}$$

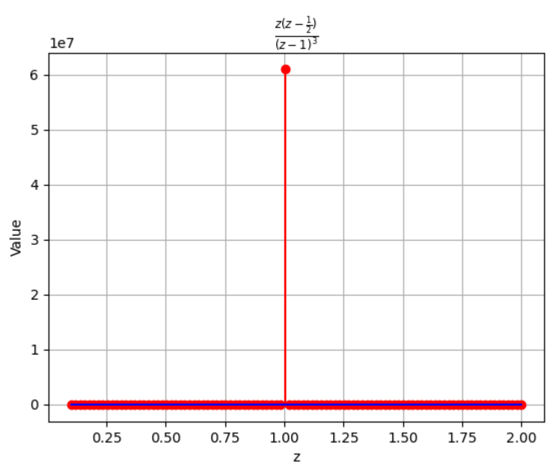

Fig. 0. *

Graph of $S_1(Z)$

for x(0)=5 and $d = -\frac{1}{2}$

$$S_2(Z) = \frac{z\left(5z - \frac{11}{2}\right)}{(z-1)^3}, \text{ROC} \implies \{|z| > 1\} \tag{28}$$

We know that Inverse $\mathcal{Z}$ - transform of $S(z)$ say $s(n)$, by counter integral method is given by:

$$s(n) = \oint_C S(z)\, z^{n-1}\, dz \tag{29}$$

from (26),

$$s(n) = x(0)\oint_C \left(\frac{z}{(z-1)^2} dz\right) + d\left(\oint_C \frac{z}{(z-1)^3}\, dz\right) \tag{30}$$

We can observe that pole is repeated 2,3 times
so, m=2 and 3 respectively:

$$s(n) = x(0)\left|\frac{1}{1!}\frac{d}{dz}(z^n)\right|_{z=1} + d\left|\frac{1}{2!}\frac{d^2}{dz^2}(z^n)\right|_{z=1} \tag{31}$$

$$s(n) = \left(n\ x(0) + n(n-1)\frac{d}{2}\right)u(n) \tag{32}$$

$$s_1(n) = \frac{n(n+3)}{4}u(n), s_1(16) = 76 \tag{33}$$

and,

$$s_2(n) = \frac{n(21-n)}{4}u(n), s_2(16) = 20 \tag{34}$$

we can observe that (10) and (33),(11) and (34) are the same results