Assignment 10.5.4-2

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Question:

Q10) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Solution:

Table of Parameters

Input Variable	Condition
$x(0)$	first term of AP
r	common ratio of GP
$x(0), x(1), x(2)$	three terms in GP
$x(0), x(1), x(2)$	$x(0)+x(1)+x(2)=56$
$x(0)-1, x(1)-7, x(2)-21$	form an AP
$x(n)$	$(n+1)^{th}$ term of GP
$x(n) \overset{Z}{\longleftrightarrow} X(z)$	$X(z) = x(0)\sum_{k=0}^{k=\infty}\left(\frac{z}{r}\right)^{-k}$

We know that, if three numbers p,q and r are in arithmetic progression then,

$$2q = p + r \tag{1}$$

Then $(n+1)^{th}$ term of GP x(n) is given by:

$$x(n) = x(0).r^n \tag{2}$$

Then from given,

$$x(0) + x(1) + x(2) = 56 \tag{3}$$

$$x(0).\left(1 + r + r^2\right) = 56 \tag{4}$$

$$x(0) \implies \frac{56}{(1 + r + r^2)} \tag{5}$$

and from given another case following are in AP,

$$x(0) - 1, x(1) - 7, x(2) - 21$$

Then from (1),

$$2(x(1) - 7) = x(0) - 1 + x(2) - 21 \tag{6}$$

$$x(0)\left(r^2 - 2r + 1\right) = 8 \tag{7}$$

and from (5)

$$\frac{56.\left(r^2 - 2r + 1\right)}{(1 + r + r^2)} = 8 \tag{8}$$

$$7\left(r^2 - 2r + 1\right) = \left(1 + r + r^2\right) \tag{9}$$

$$6r^2 - 15r + 6 = 0 \tag{10}$$

$$r = 2, \frac{1}{2} \tag{11}$$

so from (5),

$$x(0) = 8, 32 \tag{12}$$

Then from (2)

$$x_1(n) = 8.2^n = 2^{n+3} \tag{13}$$

$$x_2(n) = 32.\left(\frac{1}{2}\right)^n = 2^{5-n} \tag{14}$$

$x(0)$,$x(1)$ and $x(2)$ are 8,16 and 32 (or) 32,16 and 8 respectively We know that Z-Transform of x(n) is given by:

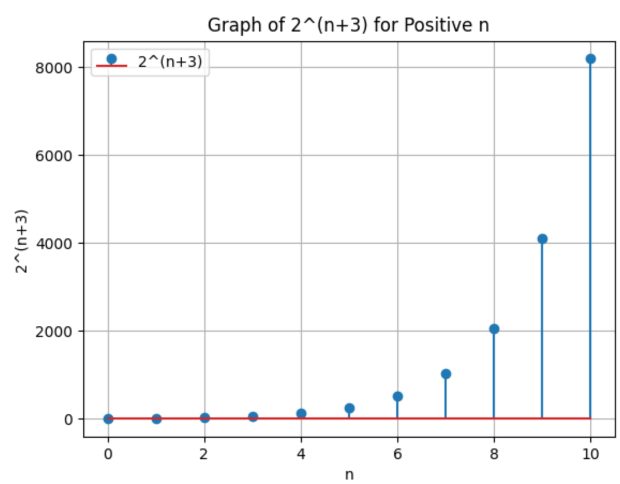

Fig. 0. *

Graph of 2^{n+3}

$$X(z) = \sum_{k=-\infty}^{\infty} x(k).z^{-k} \tag{15}$$

where, we assume that x(k)=0 for ($k < 0$)
(15) Then, modify as follows:

$$X_1(z) = \sum_{k=0}^{\infty} x_1(k).z^{-k} \tag{16}$$

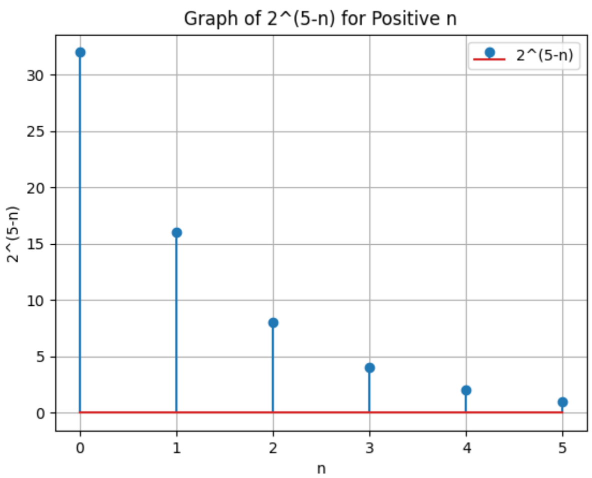

Fig. 0. *

Graph of 2^{5-n}

from (13),

$$X_1(z) = \sum_{k=0}^{\infty} 8.2^k.z^{-k} \tag{17}$$

$$X_1(z) = 8.\sum_{k=0}^{\infty}\left(\frac{2}{z}\right)^k \tag{18}$$

$$X_1(z) = 8.\lim_{n\to\infty}\left(\frac{1-\left(2.z^{-1}\right)^{n+1}}{1-2.z^{-1}}\right) \tag{19}$$

Hence,

$$X_1(z) = \frac{8}{1-2z^{-1}} \quad if \ \left(|2z^{-1}| < 1\right) \tag{20}$$

or,

$$X_1(z) \ \ is \ undefined \quad if \ \left(|2z^{-1}| > 1\right)$$

and also from (14),

$$X_2(z) = \sum_{k=0}^{\infty} x_2(k).z^{-k} \tag{21}$$

$$X_2(z) = \sum_{k=0}^{\infty} 32.\left(\frac{1}{2}\right)^k.z^{-k} \tag{22}$$

$$X_2(z) = 32.\sum_{k=0}^{\infty}\left(\frac{1}{2z}\right)^k \tag{23}$$

or,

$$X_2(z) = 32.\lim_{n\to\infty}\left(\frac{1-\left((2z)^{-1}\right)^{n+1}}{1-(2z)^{-1}}\right) \tag{24}$$

Hence,

$$X_2(z) = \frac{32}{1-(2z)^{-1}} \quad if \ \left(|(2z)^{-1}| < 1\right) \tag{25}$$

or,

$$X_2(z) \ \ is \ undefined \quad if \ \left(|(2z)^{-1}| > 1\right)$$

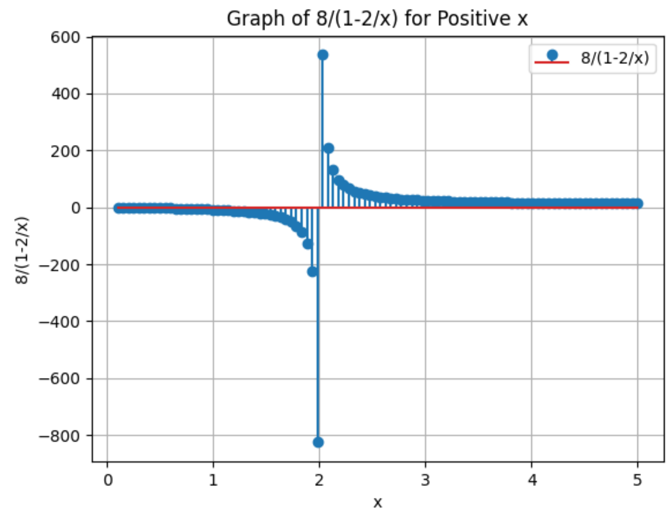

Fig. 0. *

Graph of $\frac{8}{1-2Z^{-1}}$

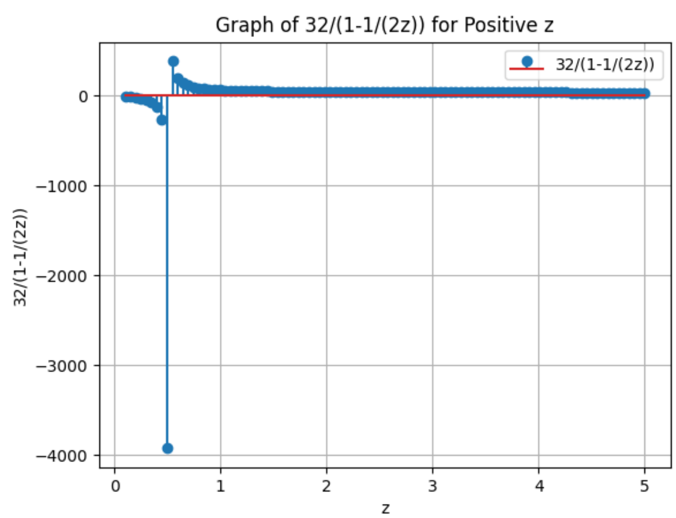

Fig. 0. *

Graph of $\frac{32}{1-(2Z)^{-1}}$