Assignment
10.5.4-2

ee23btech11215 - Penmetsa Srikar Varma

Question:

Q10) The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Solution:

Table of Parameters

Input Variable	Condition
x(0)	first term of AP
r	common ratio of GP
$\frac{x(0)}{r}, x(0), x(0).r$	three terms in GP
$\frac{x(0)}{r}, x(0), x(0).r$	$\frac{x(0)}{r} + x(0) + x(0).r = 56$
$\frac{x(0)}{r} - 1, x(0) - 7, x(0).r - 21$	form an AP
x$(n-1)$	n^{th} term of GP
$x(n) \overset{Z}{\longleftrightarrow} X(z)$	z-transform of x(n)

We know that, if three numbers p,q and r are in arithmetic progression then,

$$2q = p + r \tag{1}$$

Then n^{th} term of GP x$(n-1)$ is given by:

$$x(n-1) = x(0).r^{n-1} \quad or \quad x(n) = x(0).r^{n} \tag{2}$$

Then from given,

$$\frac{x(0)}{r} + x(0) + x(0).r = 56$$

$$x(0).\left(\frac{1}{r} + 1 + r\right) = 56$$

$$x(0) = \frac{56}{\left(\frac{1}{r} + 1 + r\right)} \tag{3}$$

and from given another case following are in AP,

$$\frac{x(0)}{r} - 1, x(0) - 7, x(0).r - 21$$

Then from (1),

$$2.(x(0) - 7) = \frac{x(0)}{r} - 1 + x(0).r - 21$$

$$2.x(0) - 14 = \frac{x(0)}{r} + x(0).r - 22$$

$$\frac{x(0)}{r} + x(0).r - 2.x(0) = 8$$

$$x(0)\left(\frac{1}{r} + r - 2\right) = 8$$

and from (3)

$$\frac{56.\left(\frac{1}{r} + r - 2\right)}{\left(\frac{1}{r} + 1 + r\right)} = 8$$

$$7\left(\frac{1}{r} + r - 2\right) = \left(\frac{1}{r} + 1 + r\right)$$

$$\frac{6}{r} + 6.r - 15 = 0$$

$$6.r^2 - 15.r + 6 = 0 = 2.r^2 - 5.r + 2$$

$$(2.r - 1).(r - 2) = 0$$

$$r = \frac{1}{2}, 2 \tag{4}$$

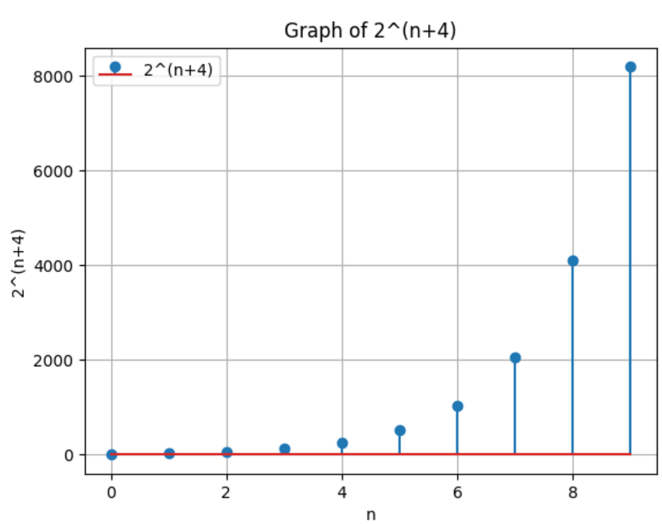
so from (3),

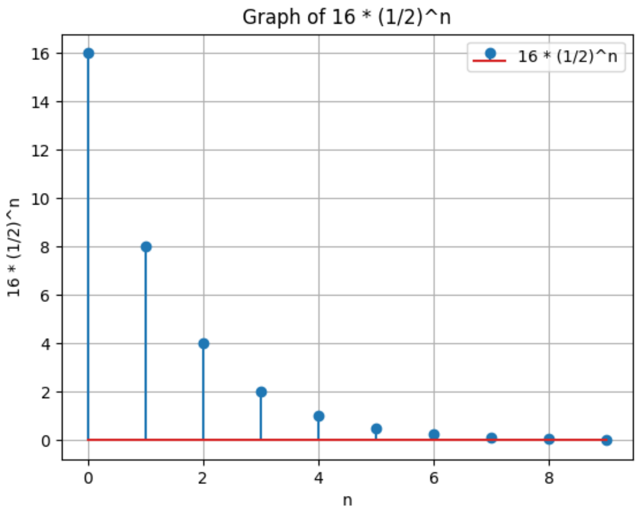
$$x(0) = 16$$

Then from (2),

$$x(n) = 16.2^n = 2^{n+4} \qquad for\,(r = 2) \tag{5}$$

$$x(n) = 16.\left(\frac{1}{2}\right)^n = 2^{4-n} \quad for\left(r = \frac{1}{2}\right) \tag{6}$$





We know that Z-Transform of x(n) is given by:

$$X(z) = \sum_{k=-\infty}^{\infty} x(k).z^{-k} \tag{7}$$

where, we assume that x(k)=0 for ($k < 0$)
(7) modify as follows:

$$X(z) = \sum_{k=0}^{\infty} x(k).z^{-k} \tag{8}$$

from (5),

$$X(z) = \sum_{k=0}^{\infty} 16.2^{k}.z^{-k}$$

$$X(z) = 16.\sum_{k=0}^{\infty} 2^{k}.z^{-k}$$

$$X(z) = 16.\sum_{k=0}^{\infty} \left(\frac{2}{z}\right)^{k}$$

or,

$$X(z) = \lim_{n\to\infty} 16.\sum_{k=0}^{n} \left(\frac{2}{z}\right)^{k}$$

$$X(z) = 16.\lim_{n\to\infty} \sum_{k=0}^{n} \left(\frac{2}{z}\right)^{k}$$

$$X(z) = 16.\lim_{n\to\infty} \frac{\left(2.z^{-1}\right)^{n+1} - 1}{2.z^{-1} - 1}$$

Hence,

$$X_1(z) = \frac{16}{1 - 2z^{-1}} \quad if \ \left(|2z^{-1}| < 1\right)$$

or,

$$X_1(z) \ is \ undefined \quad if \ \left(|2z^{-1}| > 1\right)$$

and also from (6),

$$X(z) = \sum_{k=0}^{\infty} 16.\left(\frac{1}{2}\right)^{k}.z^{-k}$$

$$X(z) = 16.\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{k}.z^{-k}$$

$$X(z) = 16.\sum_{k=0}^{\infty} \left(\frac{1}{2z}\right)^{k}$$

or,

$$X(z) = \lim_{n\to\infty} 16.\sum_{k=0}^{n} \left(\frac{1}{2z}\right)^{k}$$

$$X(z) = 16.\lim_{n\to\infty} \sum_{k=0}^{n} \left(\frac{1}{2z}\right)^{k}$$

$$X(z) = 16.\lim_{n\to\infty} \frac{\left((2z)^{-1}\right)^{n+1} - 1}{(2z)^{-1} - 1}$$

Hence,

$$X_2(z) = \frac{16}{1 - (2z)^{-1}} \quad if \ \left(|(2z)^{-1}| < 1\right)$$

or,

$$X_2(z) \ is \ undefined \quad if \ \left(|(2z)^{-1}| > 1\right)$$