MI2_07_TheChantastic4

June 20, 2017

1 Fast ICA & the independent components of image patches

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```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from IPython.display import Audio
        from sklearn.decomposition import FastICA
        from time import time
        from sklearn.feature_extraction.image import extract_patches_2d

        %matplotlib inline
```

1.2 7.1 Negentropy is scale-invariant

```
In [2]: a = plt.imread("analytical.jpg")
```

```
In [23]: plt.figure(figsize=(18,21))
         plt.imshow(a)
         plt.axis("off")
         plt.show()
```

7.1. Negentropy is scale invariant

$J(Ax) = H(Ax_{Gauss}) - H(Ax)$ by the definition

- Let's first compute $H(Ax)$:

For a linear transformation of x, e.g $y = A.x$, the relation between the probability densities $p_x(x)$ and $p_y(y)$ can be written as follows

$$p_y(y) = \frac{1}{|\det A|} \cdot p_x(x)$$

Now, expressing entropy as an expectation

$H(A.x) = H(y) = -E\{\log p_y(y)\}$, we get

$$E\{\log p_y(y)\} = E\left\{\log\left(\frac{p_x(x)}{|\det A|}\right)\right\} = E\{\log p_x(x)\} - \log|\det A|$$

Thus we obtain the relation between the entropies as

$$H(A.x) = H(x) + \log|\det A|$$

- Now let's compute $H(Ax_{Gauss})$ using the form in the exercise sheet

$$H(A.x_{Gauss}) = \frac{1}{2}\log(\det(A\Sigma A^T)) + \frac{n}{2}(1+\log 2\pi)$$

$$= \frac{1}{2}\log(\det\Sigma) + 2\cdot\frac{1}{2}\log|\det A| + \frac{n}{2}(1+\log 2\pi)$$

Thus ; $H(A.x_{Gauss}) = H(x_{Gauss}) + \log|\det A|$

- Combining the two terms we see that

$$J(A.x) = H(x_{Gauss}) + \cancel{\log|\det A|} - H(x) - \cancel{\log|\det A|}$$

$$= H(x_{Gauss}) - H(x) = J(x) \checkmark$$

The negentropy is scale invariant !

1.3 7.2 fastICA vs. Infomax

```
In [4]: # Helper function for the transformed data
        def logistic(y):
            return 1/(1+np.exp(-y))

        # Helper fucntion for invertible matrix
        def invert(N=3):
            #matrix = np.random.normal(loc=mu, scale=sigma, size=(N, N)) #scipy.stats.ortho_grou
            matrix = np.random.uniform(0, 1, (N, N))
            exception = True
            while exception:
                try:
                    np.linalg.inv(matrix)
                except:
                    matrix = np.random.uniform(0, 1, (N, N))
                else:
                    exception = False
            return matrix

        # natural gradient ascent
        def nat_meng(data, w0, eps=0.01, lamb=0.9999):
            """
            Computes natural gradient ascent on an array of mixed sounds in order to obtain the
            data: an array containing mixed sources.
            w0: initial weight matrix, randomly generated.
            returns
            ws: an array of all the evolution of weights.
            """
        #     ws = np.zeros((data.shape[1]+1, data.shape[0], data.shape[0]))
        #      ws[0] = w0
            dim = data.shape[0]
            length = data.shape[1]
            W = w0
            for t in np.arange(length):
                x = data[:, t]
        #         W = ws[t].copy()
                eps = eps * lamb
                #eps = eps0/(t+1)
                delta = np.identity(dim)
                u = np.dot(W, x)
                update = np.dot(delta, W) + np.dot((1- 2*logistic(u)).reshape(dim,1), np.dot(u.r
                delta_W = eps * update
                W += delta_W
        #         ws[t+1] = W
```

```
        return W
```

In [5]:
```
# Load sounds
source1 = np.loadtxt('sound1.dat')
source2 = np.loadtxt('sound2.dat')
original = np.array([source1, source2])
```

In [6]:
```
# Create an invertible mixing matrix
A = invert(2)

# Mix the sources
mixed = np.dot(A, original)

# Center the data
mixed_c = np.subtract(mixed.T, mixed.mean(axis=1)).T

# Initialize weights
W0 = invert(2)
```

In [7]:
```
start1 = time()

# Run natural gradient
W = nat_meng(mixed_c, W0)

# Unmixed
# W = Ws[-1]
unmixed = np.dot(W, mixed_c)

end1 = time()
```

In [8]:
```
start2 = time()

# Run FastICA
ica = FastICA(2)
S_ = ica.fit_transform(mixed.T).T
A_ = ica.mixing_

end2 = time()
```

In [9]:
```
# Compare computational cost
nat_time = end1-start1
fast_time = end2-start2
print("Computational time")
print("Natural gradient: %.4f" %nat_time)
print("Fast ICA: %.4f" %fast_time)
```

```
Computational time
Natural gradient: 0.4923
Fast ICA: 0.0236
```

Try audios

```
In [10]: Audio(unmixed[1], rate=8192)
```

```
Out[10]: <IPython.lib.display.Audio object>
```

```
In [11]: Audio(S_[0], rate=8192)
```

```
Out[11]: <IPython.lib.display.Audio object>
```

Fast ICA algorithm is always better in speed and source identification.

1.4 7.3 ICA on Image Patches

```
In [12]: # Read images
         nature = []    # 13 nature images
         for i in range(13):
             nature.append(plt.imread("imgpca/n"+str(i+1)+".jpg", format='jpg'))

         buildings = []    # 10 building images, without considering the zooms of the 9th img.
         for i in range(10):
             buildings.append(plt.imread("imgpca/b"+str(i+1)+".jpg", format='jpg'))

         text = []    # 14 text images
         for i in range(14):
             text.append(plt.imread("imgpca/t"+str(i+1)+".jpg", format='jpg'))
```

1.4.1 a) Sample P patches of $\sqrt{N}$ x $\sqrt{N}$

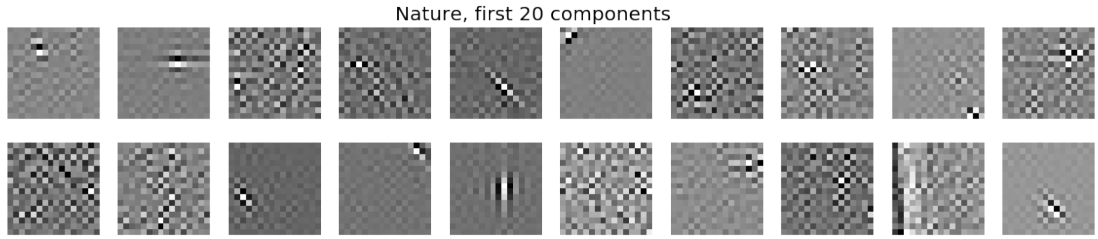
```
In [13]: # Patches
         P = 5000
         pix = 16
         N = pix*pix

         n_patches = np.zeros((len(nature), P, N))
         b_patches = np.zeros((len(buildings), P, N))
         t_patches = np.zeros((len(text), P, N))

         images_ = [nature, buildings, text]
         patches_ = [n_patches, b_patches, t_patches]

         for i in range(3):
             images = images_[i]
             patches = patches_[i]
             for j in range(len(images)):
                 patches[j,:,:] = extract_patches_2d(images[j], (pix, pix), max_patches=P).resha
```

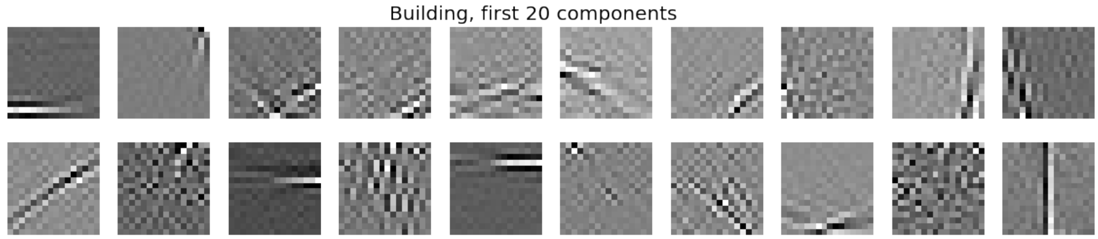
1.4.2 b) Independent features of the patches

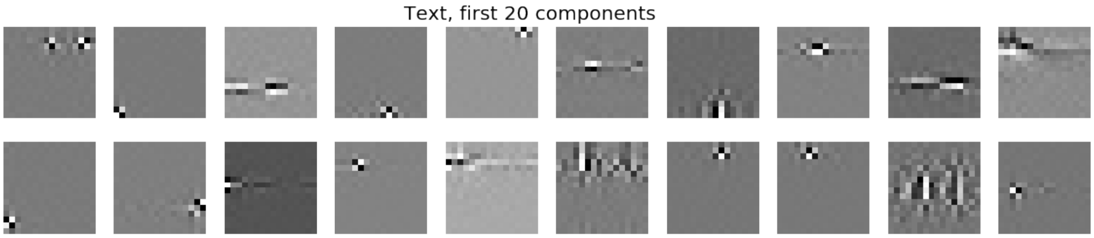
In [14]:
```
# Calculating independent features of the images using FastICA
# Standard setting of the FastICA function is whitening and contrast function logcosh a
rs = []
As = []
for i in range(3):
    patches = patches_[i].reshape(-1, N)
    patches = np.subtract(patches, np.mean(patches, axis=0))

    ica = FastICA(n_components=N, tol=0.01, max_iter=1000)
    #r_ = ica.fit_transform(patches.T)
    A_ = ica.fit(patches).components_
    #rs.append(r_)
    As.append(A_)
```

1.4.3 c) First 20 independent features

In [15]:
```
titles = ('Nature', 'Building', 'Text')
for i in range(3):
    f, axes = plt.subplots(2, 10, figsize=(20,4))
    axes = axes.ravel()
    for j in range(20):
        axes[j].imshow(As[i][j,:].reshape((pix,pix)),'gray')#, vmin=np.min(As[i]), vmax
        axes[j].axis("off")
    plt.suptitle(titles[i]+", first 20 components", fontsize=20, y=.95)
    plt.show()
```



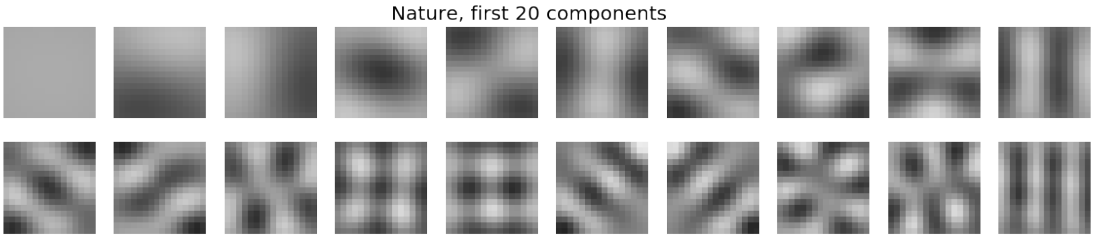




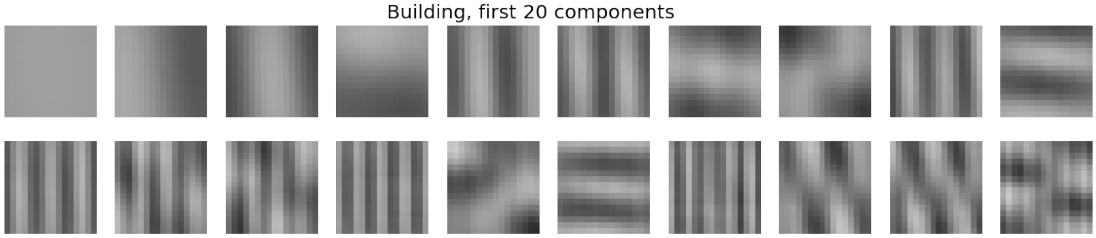
1.4.4 d) PCA on the same set of patches

```
In [16]: ls = []
         ws = []

         for i in range(3):
             #image = images_[i][0]
             #size = image.shape[0] * image.shape[1]
             patches = patches_[i].reshape(-1, N)
             patches = np.subtract(patches, np.mean(patches, axis=0))

             # PCA
             cov = np.cov(patches.T)
             l, w = np.linalg.eig(cov)

             # Sorted eigenvectors
             w = w[:,np.flipud(np.argsort(l))]

             ls.append(l)
             ws.append(w)
```

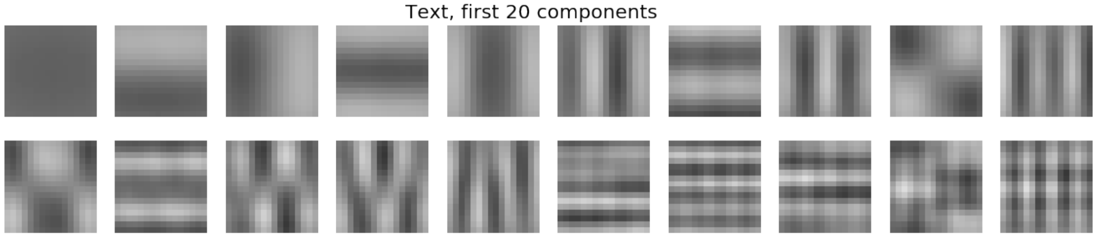
```
In [17]: titles = ('Nature', 'Building', 'Text')
         for i in range(3):
             f, axes = plt.subplots(2, 10, figsize=(20,4))
             axes = axes.ravel()
             for j in range(20):
                 axes[j].imshow(ws[i][:,j].reshape((pix,pix)),'gray', vmin=np.min(ws[i]), vmax=n
                 axes[j].axis("off")
             plt.suptitle(titles[i]+", first 20 components", fontsize=20, y=.95)
             plt.show()
```



Building, first 20 components



Text, first 20 components



1.4.5 Discussion

The first 20 independent components are different from the corresponding principal components; for fast ICA the resolution is higher and the components appear to concentrate in smaller details. Although the overall structure coincides, i.e. we observe circle for natural images, mainly vertical lines for buildings and mainly horizontal ones for text. The detail can be observe, for example, in the case of buildings lines can be recognise depending on the position and not just on the direction.

We expected a similar output of fast ICA and PCA, actually we discussed throughly the application of the methods. We have confidence in the results, but fail on explaining why are the components so different.

```
In [ ]:
```