MI2_ES06_TheChantastic4

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1 MI2 - ES06: ICA2, Noise and Kurtosis

1.1 The chantastic 4: Elisabeth Kress, Paola Suárez, Jianmeng Wu and Esra Zihni

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import Audio
from scipy.io import loadmat
from scipy.stats import kurtosis
```

1.2 1. Natural Gradient

1.2.1 a) ICA-learning scheme based on natural gradient

In [2]:
```
# Helper function for the transformed data
def logistic(y):
    return 1/(1+np.exp(-y))

# Helper fucntion for invertible matrix
def invert(N=3):
    #matrix = np.random.normal(loc=mu, scale=sigma, size=(N, N)) #scipy.sta
    matrix = np.random.uniform(0, 1, (N, N))
    exception = True
    while exception:
        try:
            np.linalg.inv(matrix)
        except:
            matrix = np.random.uniform(0, 1, (N, N))
        else:
            exception = False
    return matrix
```

In [3]:
```
def nat_meng(data, w0, eps=0.5, lamb=0.9):
    """
    Computes natural gradient ascent on an array of mixed sounds in order t
    data: an array containing mixed sources.
    w0: initial weight matrix, randomly generated.
    returns
```

```
        ws: an array of all the evolution of weights.
        """
        ws = np.zeros((source1.shape[0]+1, 3, 3))
        ws[0] = w0
        for t in np.arange(source1.shape[0]):
            x = data[:, t]
            W = ws[t].copy()
            eps = eps * lamb
            #eps = eps0/(t+1)
            delta = np.identity(3)
            u = np.dot(W, x)
            update = np.dot(delta, W) + np.dot((1- 2*logistic(u)).reshape(3,1),
            delta_W = eps * update
            W += delta_W
            ws[t+1] = W
        return ws
```

1.2.2 b) ICA on sounds and noise

```
In [4]: # Load sounds
        source1 = np.loadtxt('sounds/sound1.dat')
        source2 = np.loadtxt('sounds/sound2.dat')
        original = np.array([source1, source2])
```

```
In [5]: # Create Gaussian noise using mu and sigma from the first two sources
        mu = np.mean(original.mean(axis=1))
        sigma = np.mean(original.std(axis=1))
        noise_g = np.random.normal(loc=mu, scale=sigma, size=source1.shape)
```

```
In [8]: # Add all sources
        original = np.array([source1, source2, noise_g])
```

```
In [124]: # Create an invertible mixing matrix
          A = invert()
```

```
In [125]: # Mix the sources
          mixed = np.dot(A, original)
```

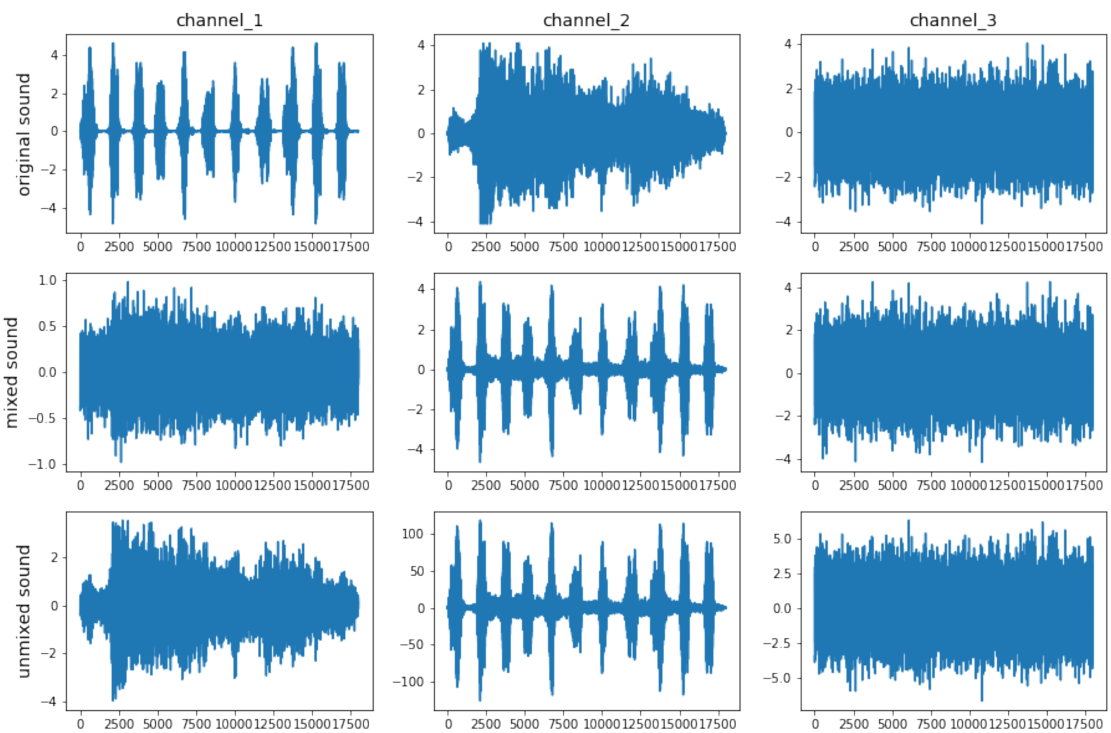
```
In [126]: # Center the data
          mixed_c = np.subtract(mixed.T, mixed.mean(axis=1)).T
```

```
In [127]: # Initialize weights
          W0 = invert()
```

```
In [128]: # Run natural gradient
          Ws = nat_meng(mixed_c, W0)
```

```
In [129]: # Unmixed
          W = Ws[-1]
          unmixed = np.dot(W, mixed_c)
```

```
In [130]: # Plot
          fig, axes = plt.subplots(3, 3, figsize=(15, 10))
          sounds = [original, mixed, unmixed]
          labels1 = ['channel_1', 'channel_2', 'channel_3']
          labels2 = ["original", "mixed", "unmixed"]
          for i in range(3):
              for j in [0, 1, 2]:
                  axes[0,j].set_title("%s" %labels1[j], size=14)
                  axes[i,j].plot(sounds[i][j])
                  axes[i,0].set_ylabel("%s sound" %labels2[i], size=14)
          plt.show()
```




```
In [131]: Audio(unmixed[2], rate=8192)

Out[131]: <IPython.lib.display.Audio object>
```

1.2.3 c) Repeat with a different noise source

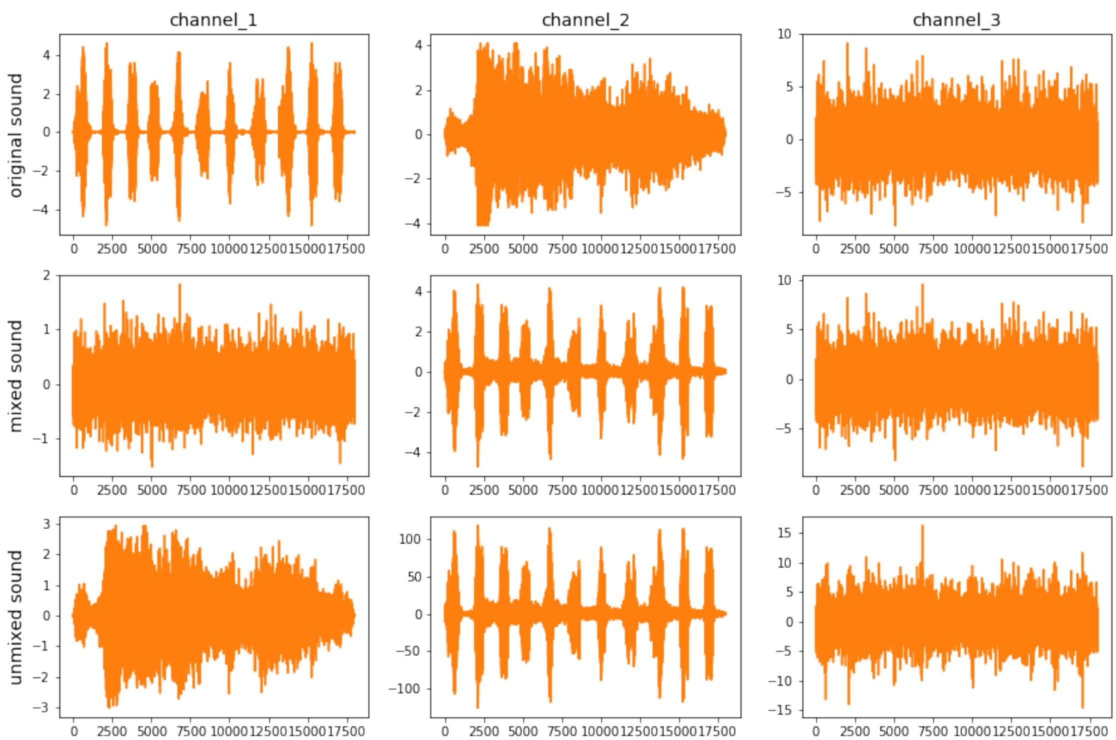
```
In [168]: noise_l = np.random.laplace(loc=mu, scale=sigma, size=source1.shape)

In [169]: # Add all sources
          original_l = np.array([source1, source2, noise_l])

In [170]: # Mix the sources
          mixed_l = np.dot(A, original_l)
```

In [171]:
```
# Center the data
mixed_cl = np.subtract(mixed_l.T, mixed_l.mean(axis=1)).T
```

In [172]:
```
# Run natural gradient
Ws_l = nat_meng(mixed_cl, W0)
```

In [173]:
```
# Unmixed
W_l = Ws_l[-1]
unmixed_l = np.dot(W_l, mixed_cl)
```

In [174]:
```
# Plot
fig, axes = plt.subplots(3, 3, figsize=(15, 10))
sounds = [original_l, mixed_l, unmixed_l]
labels1 = ['channel_1', 'channel_2', 'channel_3']
labels2 = ["original", "mixed", "unmixed"]
for i in range(3):
    for j in [0, 1, 2]:
        axes[0,j].set_title("%s" %labels1[j], size=14)
        axes[i,j].plot(sounds[i][j], c="C1")
        axes[i,0].set_ylabel("%s sound" %labels2[i], size=14)
plt.show()
```

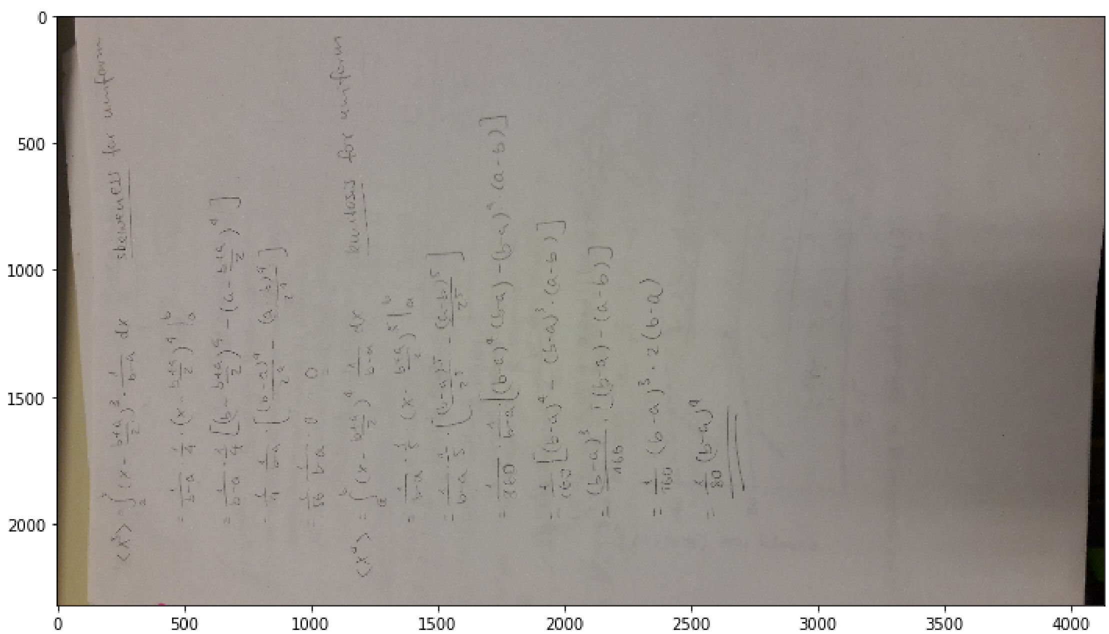


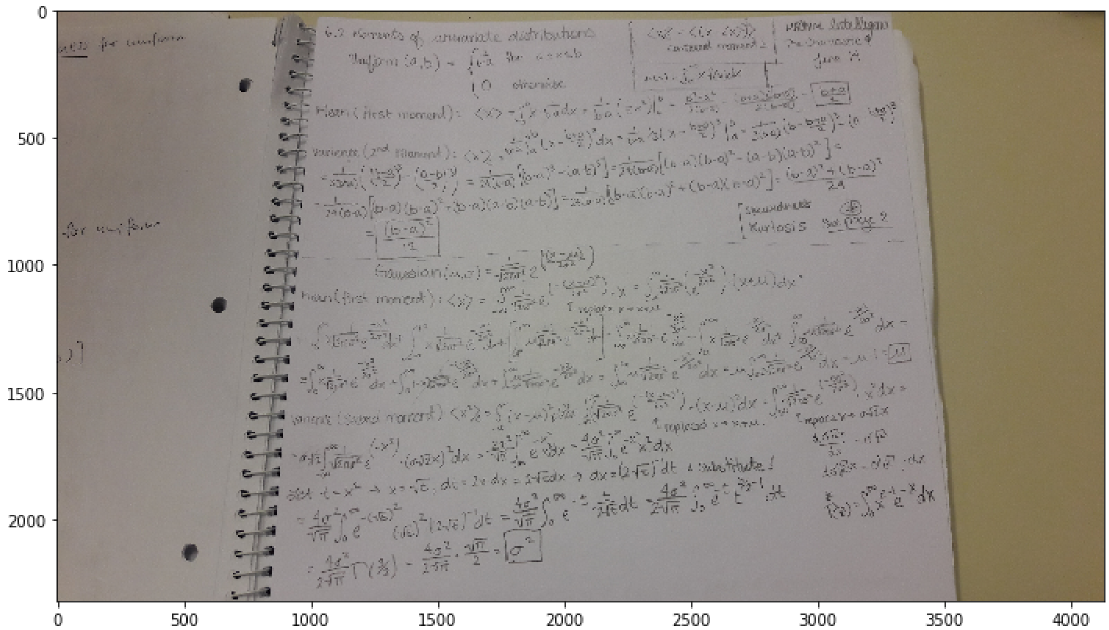
In [175]:
```
Audio(unmixed_l[2], rate=8192)
```

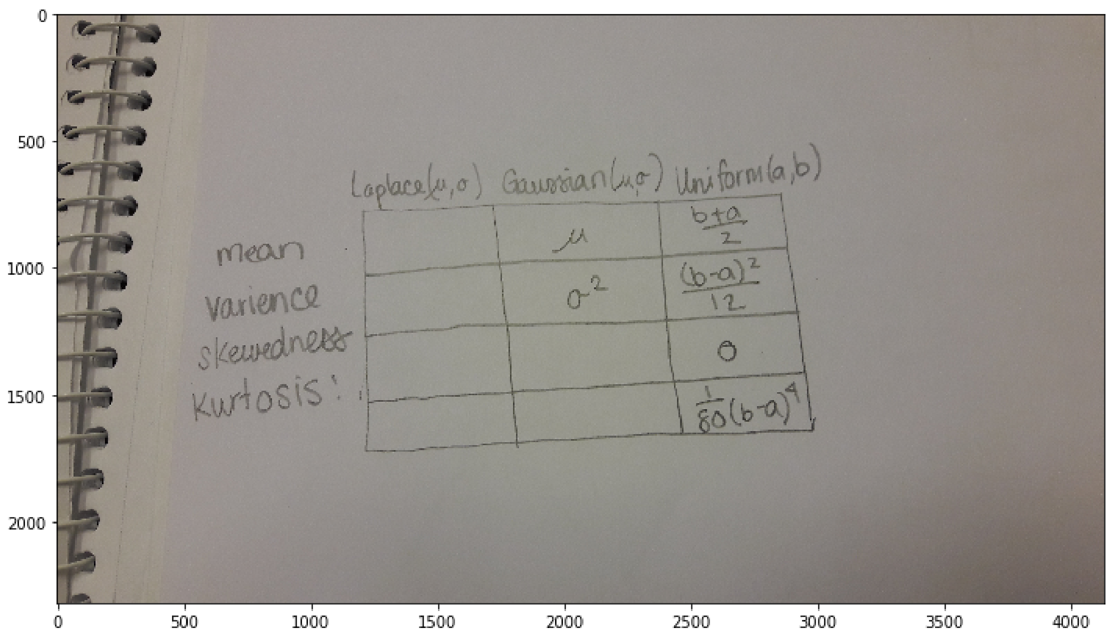
Out[175]: <IPython.lib.display.Audio object>

1.3 2. Moments of univariate distributions

In [176]:
```
plt.figure(figsize=(12,20))
f = plt.imread('1.jpg' , format='jpg')
plt.imshow(f)
plt.show()

plt.figure(figsize=(12,20))
f = plt.imread('2.jpg' , format='jpg')
plt.imshow(f)
plt.show()

plt.figure(figsize=(12,20))
f = plt.imread('3.jpg' , format='jpg')
plt.imshow(f)
plt.show()
```







1.4 3. Kurtosis of toy data

```
In [177]: # Load data
          data = loadmat("distrib.mat")
          uni = data["uniform"]
```

```
nor = data["normal"]
lap = data["laplacian"]
```

```
In [178]: s = np.array([uni, nor, lap])
```

1.4.1 a) Apply mixing matrix

```
In [179]: A = np.array([[4, 3], [2, 1]])

          mix_s = np.array([np.dot(A, s[i]) for i in range(3)])
```

1.4.2 b) Center the data

```
In [180]: mix_c = np.array([(np.subtract(mix_s[i].T, mix_s.mean(axis=2)[i])).T for
```

1.4.3 c) Decorrelate the data

```
In [181]: # PCA
          covs = np.array([np.cov(mix_c[i]) for i in range(3)])
          ls = np.array([np.linalg.eig(covs[i])[0] for i in range(3)])
          es = np.array([np.linalg.eig(covs[i])[1] for i in range(3)])
```

```
In [182]: # Projection
          sol = np.array([np.dot(es[i].T, mix_c[i]) for i in range(3)])
```

1.4.4 d) Scale the data

```
In [183]: # Whiten
          mix_w = np.array([np.dot(np.diag(ls[i]**(-1/2)), sol[i]) for i in range(3
```

1.4.5 e) Rotate and calculate kurtosis

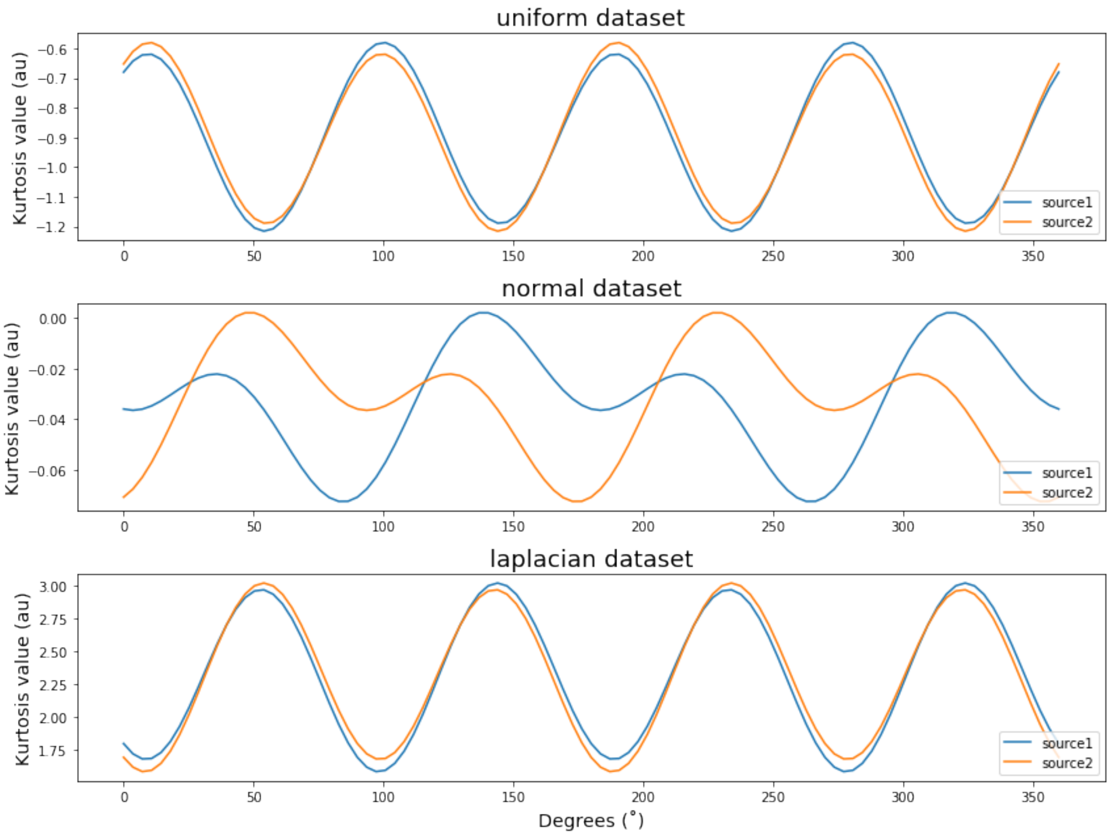
```
In [184]: angles = np.linspace(0, 2*np.pi, 101) #np.pi/50)
          #angles = np.pi

          R = np.array([[np.cos(angles), np.sin(angles)], [-np.sin(angles), np.cos(
```

```
In [185]: # Rotation
          sol_R = np.array([np.array([np.dot(R[j], mix_w[i]) for j in range(101)])
```

```
In [186]: # Kurtosis, k_uni < 0; k_nor ~ 0; k_lap >0
          kurts = kurtosis(sol_R, axis=3)
```

```
In [187]: f, axes = plt.subplots(3, 1, figsize=(12,9))
          labels = ["uniform", "normal", "laplacian"]
          for i in range(3):
              axes[i].plot(np.rad2deg(angles), kurts[i,:,:])
              axes[i].set_title("%s dataset" %labels[i], size=18)
              axes[2].set_xlabel("Degrees (°)", size=14)
              axes[i].set_ylabel("Kurtosis value (au)", size=14)
```

```
    axes[i].legend(["source1", "source2"], loc='lower right')
plt.tight_layout()
plt.show()
```



Discuss what the extrama of kurtosis mean

1.4.6 f) Min and max kurtosis

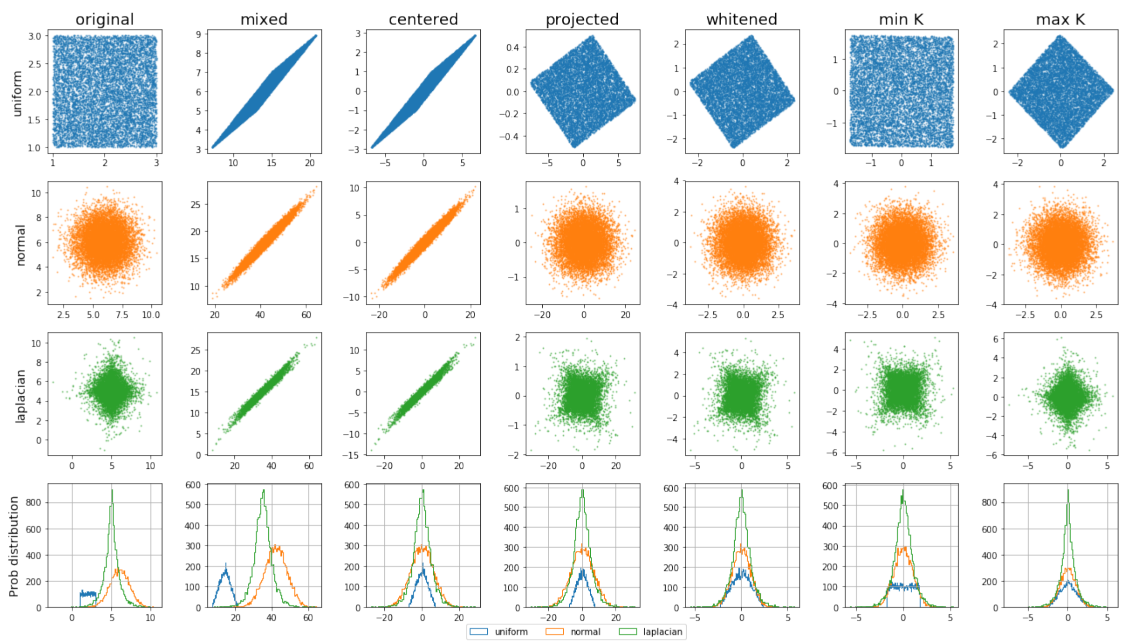
```
In [188]: # Extrema of the kurtosis correspond to independent sources if model assu
          min_k = kurts[:,:,0].argmin(axis=1)
          max_k = kurts[:,:,0].argmax(axis=1)
```

```
In [189]: # Reduced the dimensions of sol_R to be coherent and the region of intere
          sol_min = np.array([sol_R[i,min_k[i],:,:] for i in range(3)])
          sol_max = np.array([sol_R[i,max_k[i],:,:] for i in range(3)])
```

1.4.7 Plots

```
In [190]: # A lot of plots
          f, axes = plt.subplots(4, 7, figsize=(18, 10))
          to_plot = [s, mix_s, mix_c, sol, mix_w, sol_min, sol_max]
```

```
titles = ["original", "mixed", "centered", "projected", "whitened", "min
labels = ["uniform", "normal", "laplacian"]
colors = ["C0", "C1", "C2"]
for i in range(3):
    for j, d in enumerate(to_plot):
        axes[i,j].scatter(d[i,0,:], d[i,1,:], s=1, c=colors[i], alpha=0.5
        axes[0,j].set_title("%s" %titles[j], size=18)
        axes[i,0].set_ylabel("%s" %labels[i], size=14)

        axes[3,j].hist(d[i,0,:], bins=100, histtype='step', label=labels
        axes[3,j].grid()
axes[3,0].set_ylabel("Prob distribution", size=14)
plt.suptitle('')
plt.legend(ncol=3, bbox_to_anchor=(-2.75,-0.1))
plt.tight_layout()
plt.show()
```



Compare the histograms after rotation by θ_{min} and θ_{max} for the different distributions ** uniform **: rotation by θ_{min} results in good marginal histogram as compared to the original data, whereas rotation by θ_{max} leads to rather poor recovery of the original data. This is due to the nature of the uniform distribution that has low kurtosis value, i.e. it has only few outliers (or is flat).

** normal **: both θ_{min} and θ_{max} are suitable for reproducing the marginal distribution, although the latter retains the amplitude of the orginal data, better.

** laplacian **: rotation by θ_{max} results in better marginal distribution, as the laplacian distribution has high kurtosis, i.e. many outliers (or is peaked).

```
In [ ]:
```