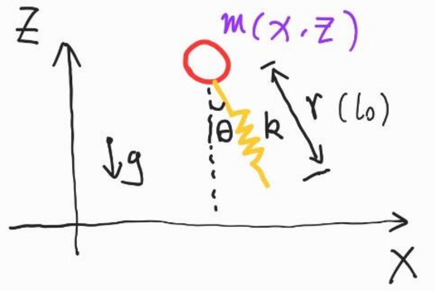


① Flight phase

$$X_f = \begin{pmatrix} x \\ z \\ \dot{x} \\ \dot{z} \end{pmatrix}$$

$$\dot{X}_f = \begin{pmatrix} \dot{x} \\ \dot{z} \\ 0 \\ -g \end{pmatrix}$$

② Stance phase

$$X_s = \begin{pmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{pmatrix}$$

$$\begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} -r \cdot \sin\theta \\ r \cdot \cos\theta \end{pmatrix}$$

Kinetic energy

$$T = \frac{m}{2} \cdot (\dot{r}^2 + r^2 \cdot \dot{\theta}^2)$$

Potential energy

$$U = m \cdot g \cdot r \cdot c\theta + \frac{k}{2} (l_0 - r)^2$$

$\Rightarrow$ Lagrange formulation

$$\ddot{r} = r \cdot \dot{\theta}^2 - g \cdot c\theta + \frac{k}{m} \cdot (l_0 - r)$$

$$\ddot{\theta} = \frac{-2}{r} \cdot \dot{r} \cdot \dot{\theta} + \frac{g}{r} \cdot s\theta$$

① → ② Flight → stance

Assuming the spring is ideal, so no energy loss

$$r = l_0$$

$$\dot{r} = -s\theta \cdot \dot{x} + c\theta \cdot \dot{z}$$

$$\dot{\theta} = -\frac{1}{r} \cdot (\dot{x} \cdot c\theta + \dot{z} \cdot s\theta)$$

θ = control input

② → ① Stance → Flight

$$x = -l_0 \cdot s\theta$$

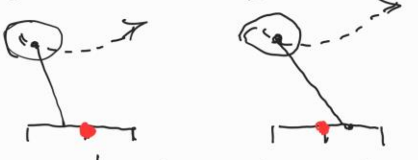
$$z = l_0 \cdot c\theta$$

$$\dot{x} = -\dot{r} \cdot s\theta - r \cdot c\theta$$

$$\dot{z} = \dot{r} \cdot c\theta - r \cdot s\theta$$

Raibert style controller

a. b.



accelerate deaccelerate

• leg's neutral point

$$x_f = \frac{\dot{x} \cdot T_s}{2} + k \cdot err + i \cdot e_{sum}$$

$$\theta = \operatorname{atan}\left(\frac{x_f}{l_0}\right)$$

$$err = \dot{x} - \dot{x}_t$$

$$e_{sum} = \sum err$$

$\dot{x}$: x velocity

$\dot{x}_t$: x target velocity

T_s: time of stance phase

Ref

M. Raibert, M. Chepponis and H. Brown, "Running on four legs as though they were one," in *IEEE Journal on Robotics and Automation*, vol. 2, no. 2, pp. 70-82, June 1986, doi: 10.1109/JRA.1986.1087044.

http://www.cim.mcgill.ca/~aki/research/SLIP_Hopper.htm

http://underactuated.mit.edu/simple_legs.html#section3