Diverse weighting of shared input noise prevents information saturation in a population code

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Correlated Cortical Variability

Neurons in cortex have been observed to exhibit pairwise correlated variability[1]. These correlations are called **noise correlations**. They are of theoretical interest because their structure influences the fidelity of a population code[2].

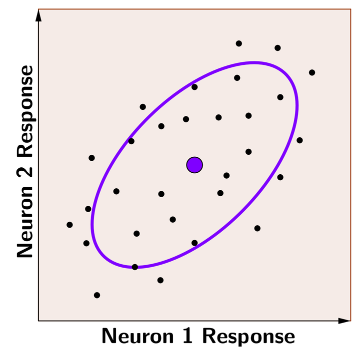


Figure 1. *Noise Correlation*

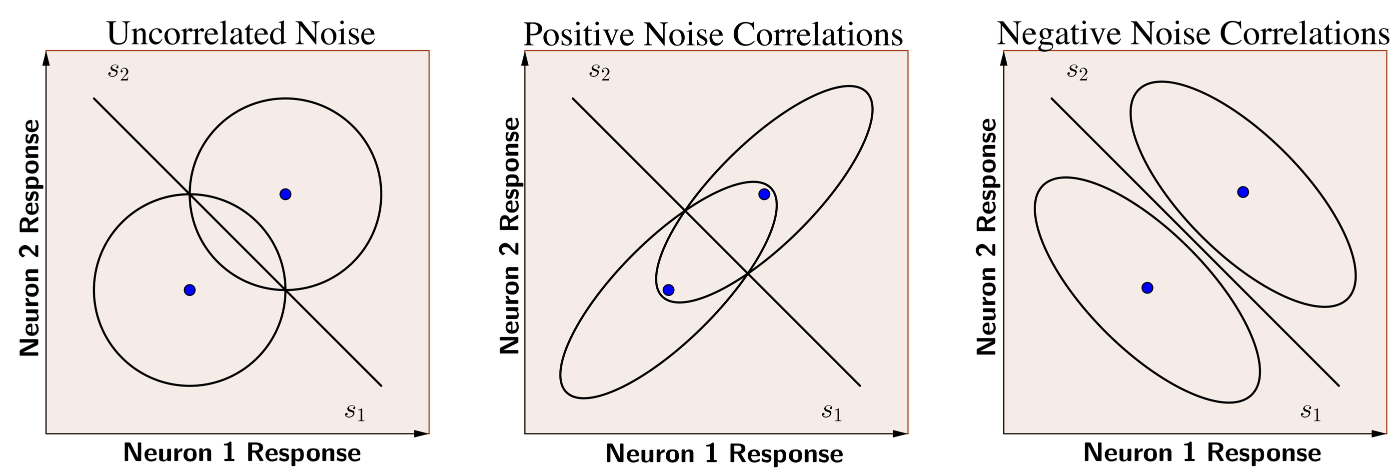


Figure 2. *Noise correlations and their structure.*

Differential Correlations

For a tuning curve $\mathbf{f}(s)$, **differential correlations** take on the form $\mathbf{f}'(s)\mathbf{f}'(s)^T$ in the covariance matrix[3]:

$$\boldsymbol{\Sigma}(s) = \boldsymbol{\Sigma}_0(s) + \mathbf{f}'(s)\mathbf{f}'(s)^T.$$

Differential correlations lie parallel to the tuning curve and are detrimental to a population code. They can be induced by **shared input noise**.

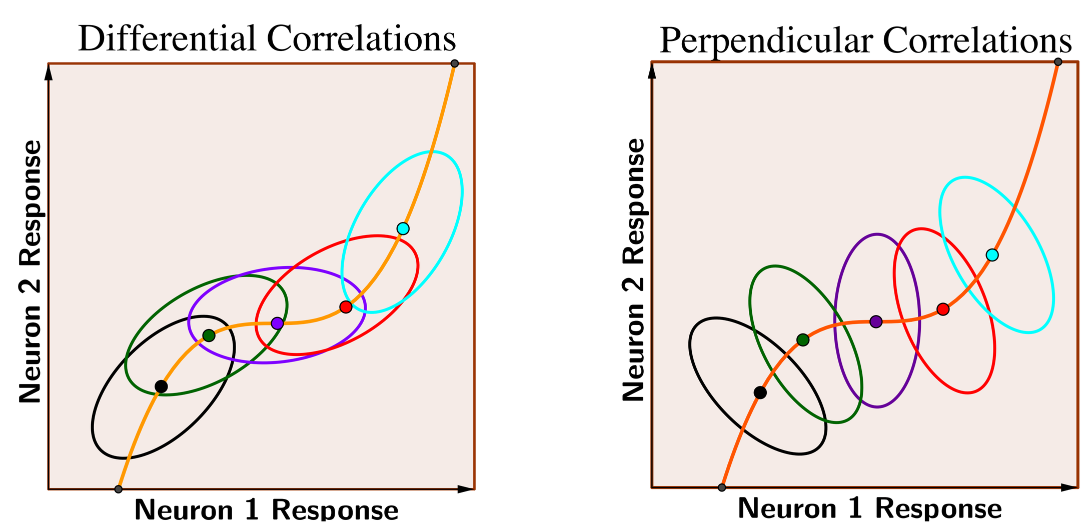


Figure 3. *Noise correlation structure is important for neural coding.*

Fisher Information

To quantify a population code, we use **Fisher information** (FI), $I_F(s)$, or the lower bound for the variance of an unbiased estimator $\hat{s}$:

$$\operatorname{Var}(\hat{s}) \geq \frac{1}{I_F(s)}.$$

A common approximation to the FI called the **linear Fisher information**:

$$I_F(s) = \mathbf{f}'(s)^T \boldsymbol{\Sigma}^{-1}(s)\mathbf{f}'(s).$$

The presence of differential correlations causes $I_F(s)$ to saturate[3] as a function of the number of neurons N.

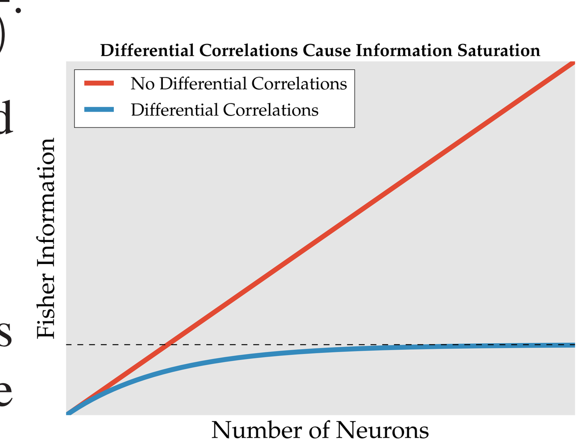


Figure 4. *Information Saturation*

References

[1] Cohen M. & Kohn A., Nat. Rev. Neurosci. 7, 811 (2011).
[2] Averbeck B., et al., Nat. Rev. Neurosci. 7, 358 (2006).
[3] Moreno-Bote R. et al., Nat. Neurosci. 17, 1410 (2014).

Goals

Major Question: How can a neural population avoid the harmful effects of differential correlations due to shared input noise?

Approach: We design a simple linear-nonlinear network which accepts shared noise as an input. We explore how varying the linear filter affects the strength of its population code.

Network Architecture

Our network consists of N neurons each accepting a stimulus s and shared noise ξ_S (mean 0, variance σ_S). Neuron i performs the computation:

$$s, \xi_S \xrightarrow[\text{linear filter}]{} \ell_i = v_i s + w_i \xi_S \xrightarrow[\text{nonlinearity}]{} r_i = g_i(\ell_i) \xrightarrow[\text{draw from distribution}]{} \begin{cases} \mathcal{N}(r_i, \sigma_G) \\ \text{Poisson}(r_i) \end{cases}.$$

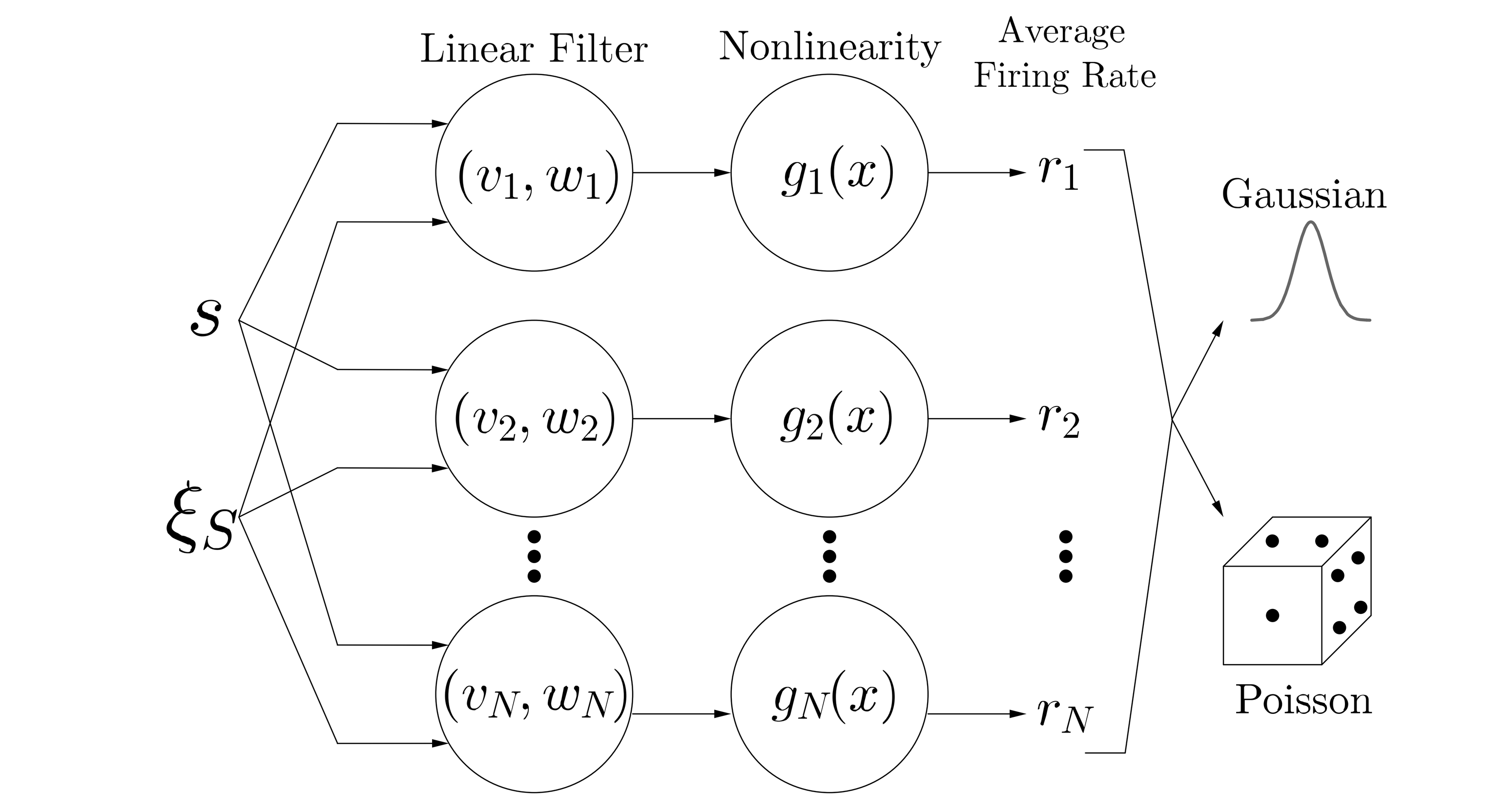


Figure 5. *Linear-Nonlinear Architecture.*

Our goal is to study how a choice of weights $(\mathbf{v}, \mathbf{w})$ affects the FI of this network.

Model Parameters

Structured Weights

To facilitate analytic calculations of the FI (for comparison with numerical calculations), we choose weights which take on the structure indicated to the right. The weight vector is parameterized by the splitting k.

$$\begin{array}{r l} N/k \text{ times} & \left\{ \begin{matrix} 1 \\ \vdots \\ 1 \end{matrix} \right. \\ N/k \text{ times} & \left\{ \begin{matrix} 2 \\ \vdots \\ 2 \end{matrix} \right. \\ & \quad \vdots \\ N/k \text{ times} & \left\{ \begin{matrix} k \\ \vdots \\ k \end{matrix} \right. \end{array}$$

$$k = 1, \mathbf{w} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \quad k = N, \mathbf{w} = \begin{pmatrix} 1 \\ 2 \\ \vdots \\ N-1 \\ N \end{pmatrix}$$

$$k = N, \mathbf{w} = \begin{pmatrix} 1 \\ \vdots \\ 1 \\ 2 \\ \vdots \\ 2 \end{pmatrix} \quad k = N, \mathbf{w} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ \vdots \\ N/2 \\ N/2 \end{pmatrix}$$

Figure 6. *Structured Weights.*

Unstructured Weights

Next, we consider weights drawn from a Gaussian with mean μ and standard deviation σ_w truncated to the range $[1, \infty)$. We study the impact μ and σ_w have on the FI.

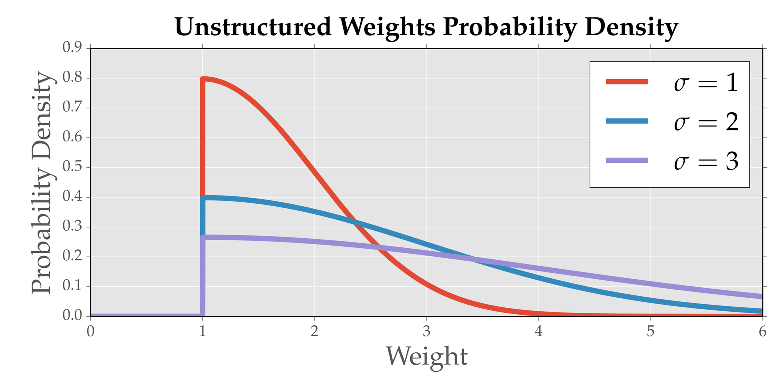


Figure 7. *Unstructured Weights.*

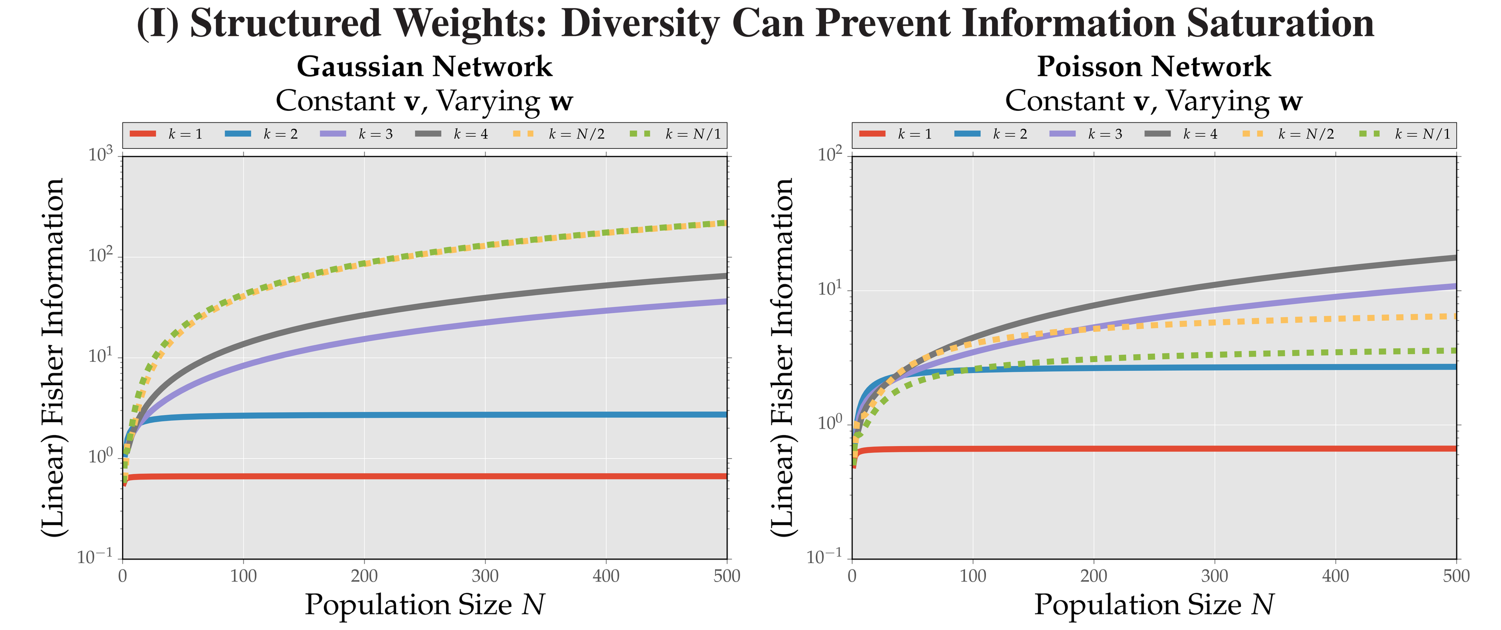
Other Parameters

Nonlinearity: All neurons have their nonlinearity set to $g_i(x) = x^2$.

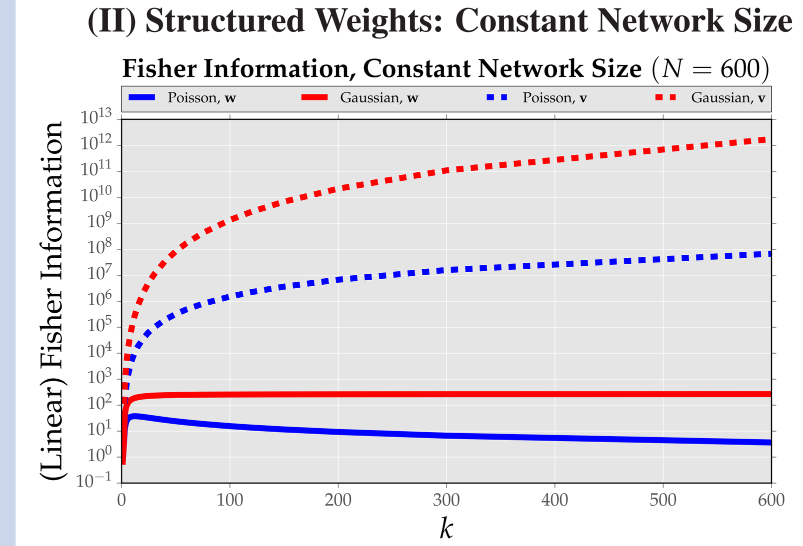
Noise Terms: For the results shown, we set $\sigma_S = \sigma_G = 1$.

Results

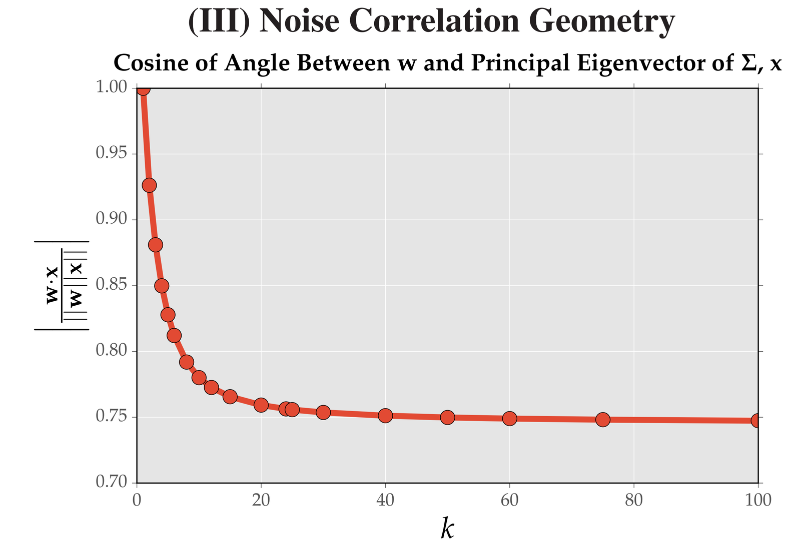
(I) Structured Weights: Diversity Can Prevent Information Saturation



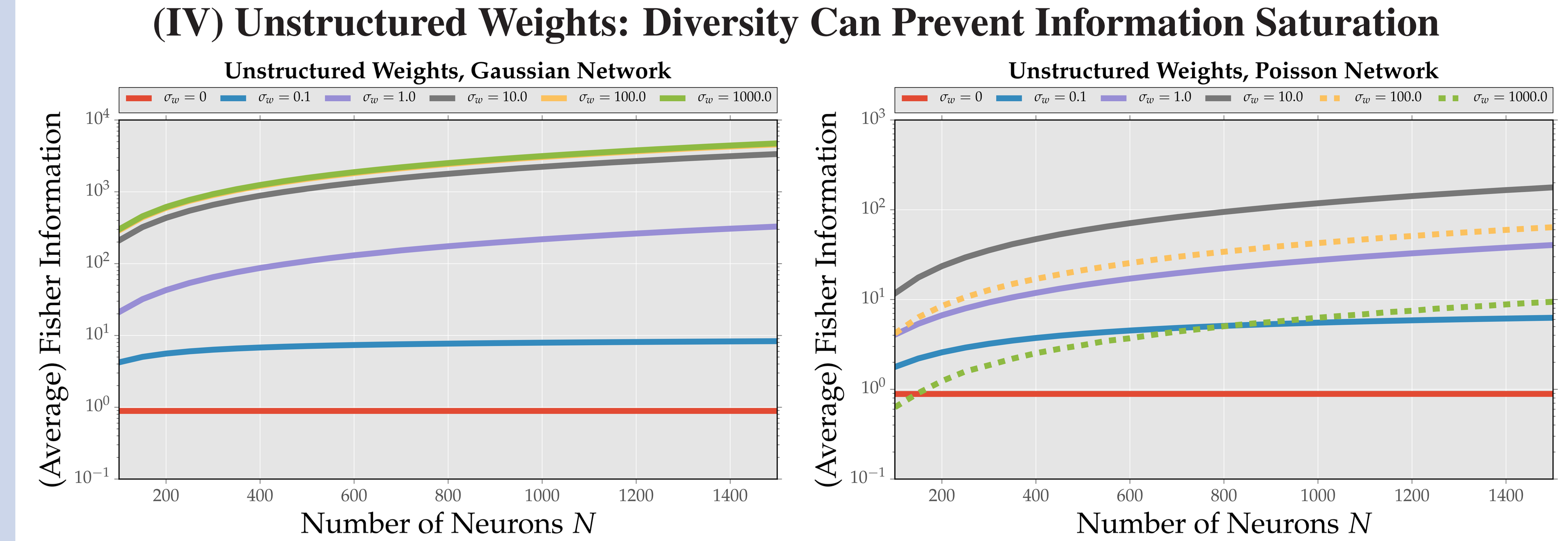
(II) Structured Weights: Constant Network Size



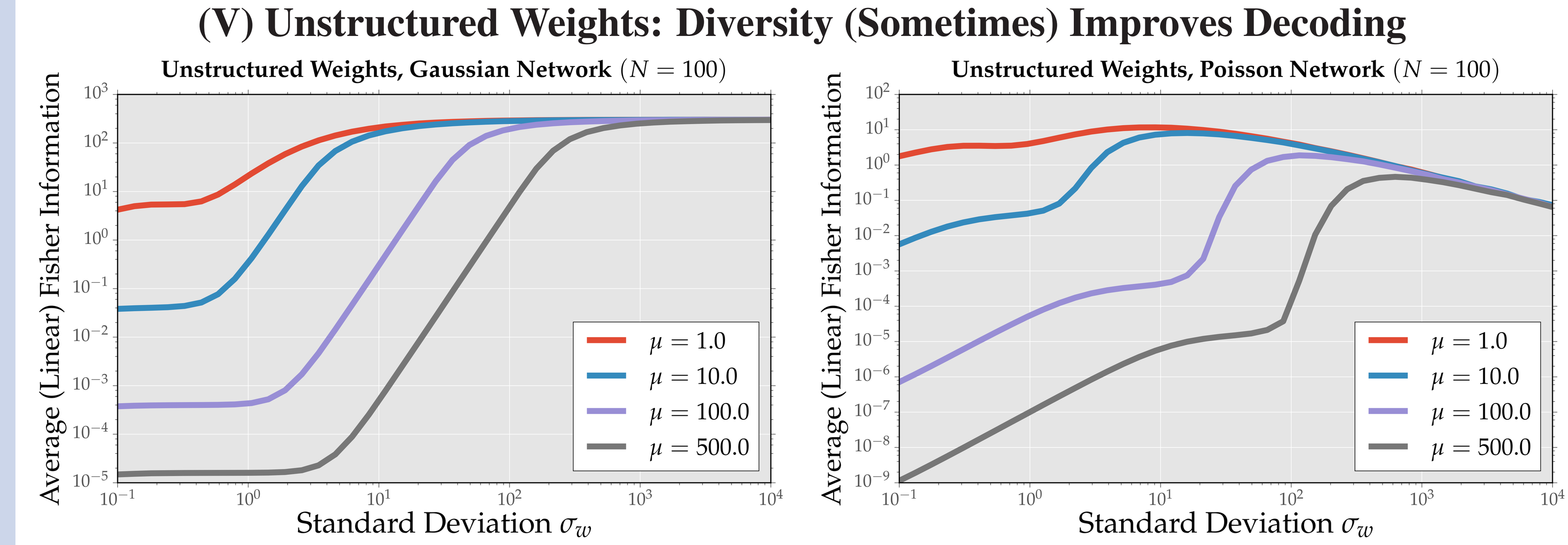
(III) Noise Correlation Geometry



(IV) Unstructured Weights: Diversity Can Prevent Information Saturation



(V) Unstructured Weights: Diversity (Sometimes) Improves Decoding



Discussion

(1) By diversifying synaptic weights, a population of neurons can remove harmful effects imposed by afferents uninformative about a stimulus.

(2) Diversity of synaptic weights may be harmful to a population code, depending on the character of the noise.

(3) In future work we will analyze how weight diversity plays a role in other nonlinearities, such as a Gaussian or a quadratic threshold. Our current progress suggests that the behavior may be more subtle.

(4) Furthermore, in future work we will compare the behavior of Fisher information to the mutual information between the stimulus and response.