

$$Q1. A = \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 0 \end{bmatrix} \xrightarrow{\substack{-R_1+R_2 \\ -2R_1+R_3}} \begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\begin{matrix} & \underbrace{\quad}_{\substack{\{ \\ \downarrow \\ A}} & \\ L_1 & & U \end{matrix}$$

$$\begin{bmatrix} 1 & & \\ -1 & 1 & \\ -2 & & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow{-\frac{2}{3}R_2+R_3} \begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{14}{3} \end{bmatrix}$$

$$\begin{matrix} & \underbrace{\quad}_{\substack{\downarrow \\ L_1}} & & A & & U \\ L_2 & & & & & \end{matrix}$$

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & -\frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ -1 & 1 & \\ -2 & & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{14}{3} \end{bmatrix}$$

$$L = L_1^{-1} L_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{2}{3} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & \frac{2}{3} & 1 \end{bmatrix}$$

$$Q2. Ax = \begin{bmatrix} 0 \\ 3 \\ 16 \end{bmatrix} = b = LUx$$

Let $Ux = y$ and $Ly = b$

Solve $Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 16 \end{bmatrix}$$

$$\begin{aligned} y_1 &= 0 \\ y_1 + y_2 &= 3 \Rightarrow y_2 = 3 \\ 2y_1 + \frac{2}{3}y_2 + y_3 &= 16 \Rightarrow y_3 = 14 \end{aligned}$$

Solve $Ux = y$

$$\begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{14}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 14 \end{bmatrix}$$

$$\begin{aligned} \frac{14}{3}x_3 &= 14 \Rightarrow x_3 = 3 \\ 3x_2 - x_3 &= 3 \Rightarrow x_2 = 2 \\ 4x_1 + x_2 - 2x_3 &= 0 \Rightarrow x_1 = 1 \end{aligned}$$

→ The solution is $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$