CS3200 – Extra Credit Written Report
Skylar Shyu

Question 1:

To be able to solve for x in the given systems of equations, it was necessary to be able to generate matricies A and b for any given n-size. I implemented generateA_b.m to do this; the eponymous function takes in an n-size and returns A and b matricies.

The A and b matricies generated are then used as arguments for the pentsolve function, which returns x, the goal of question 1. x is stored in the first column of the results matrix, in preparation for question 2.

Here is an example of the generated matricies:

Table 1: Matricies A and b (n = 7)

Matrix A							**Matrix b**
9	-4	1	0	0	0	0	0.00041649312786339O
-4	6	-4	1	0	0	0	0.00041649312786339O
1	-4	6	-4	1	0	0	0.00041649312786339O
0	1	1	1	1	1	0	0.00041649312786339O
0	0	1	-4	6	-4	1	0.00041649312786339O
0	0	0	1	-4	5	-2	0.00041649312786339O
0	0	0	0	1	-2	1	0.00041649312786339O

Question 2:

Using the A and b matricies generated for question 1, the systems of equations was solved using Matlab's built-in solver; thus, the result of A\b was stored in column 2 of the results matrix. Here are some comparisons between the solutions generate from the Matlab solver and the full solver:

Table 2: solutions for system of equations (n = 7)

pentsolver solution	MatLab solution
-0.00021968868282904l	-0.00021968868282904l
-0.00109844341414520	-0.00109844341414521
-0.0020000823832560б	-0.0020000823832560б
-0.00187193065160579	-0.00187193065160579
0.00075517984722482 8	0.00075517984722482 8
0.00463176972964561	0.00463176972964562
0.00892485273992978	0.00892485273992980

Table 3: solutions for system of equations (n = 100)

pentsolver solution	MatLab solution
-5.31914893617020e-09	-5.31914893617021e-09
-2.48226950354610e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971632e-08	-2.80141843971631e-08
5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453901e-08
-2.48226950354610e-08	-2.48226950354610e-08

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-4.14184397163120e-08	-4.14184397163121e-08
-2.80141843971632e-08	-2.80141843971631e-08
5.24822695035459e-08	5.24822695035460e-08
5.17730496453902e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971632e-08	-2.80141843971631e-08
5.24822695035460e-08	5.24822695035460e-08
5.17730496453901e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971632e-08	-2.80141843971631e-08
5.24822695035460e-08	5.24822695035460e-08
5.17730496453901e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163121e-08
-2.80141843971632e-08	-2.80141843971631e-08
5.24822695035459e-08	5.24822695035460e-08
5.17730496453902e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035459e-08	5.24822695035460e-08
5.17730496453902e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035459e-08	5.24822695035460e-08
5.17730496453902e-08	5.17730496453901e-08
-2.48226950354610e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035459e-08	5.24822695035460e-08
5.17730496453902e-08	5.17730496453901e-08
-2.48226950354610e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035459e-08	5.24822695035460e-08
5.17730496453902e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035459e-08	5.24822695035460e-08
5.17730496453902e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971632e-08	-2.80141843971631e-08

5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971632e-08	-2.80141843971631e-08
5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453901e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163121e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453902e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163121e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453902e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163121e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453902e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163121e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453902e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163121e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453902e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163120e-08
-2.80141843971631e-08	-2.80141843971631e-08
5.24822695035458e-08	5.24822695035460e-08
5.17730496453903e-08	5.17730496453902e-08
-2.48226950354609e-08	-2.48226950354610e-08
-4.14184397163120e-08	-4.14184397163121e-08
-2.80141843971631e-08	-2.80141843971631e-08
-4.60992907801422e-09	-4.60992907801419e-09

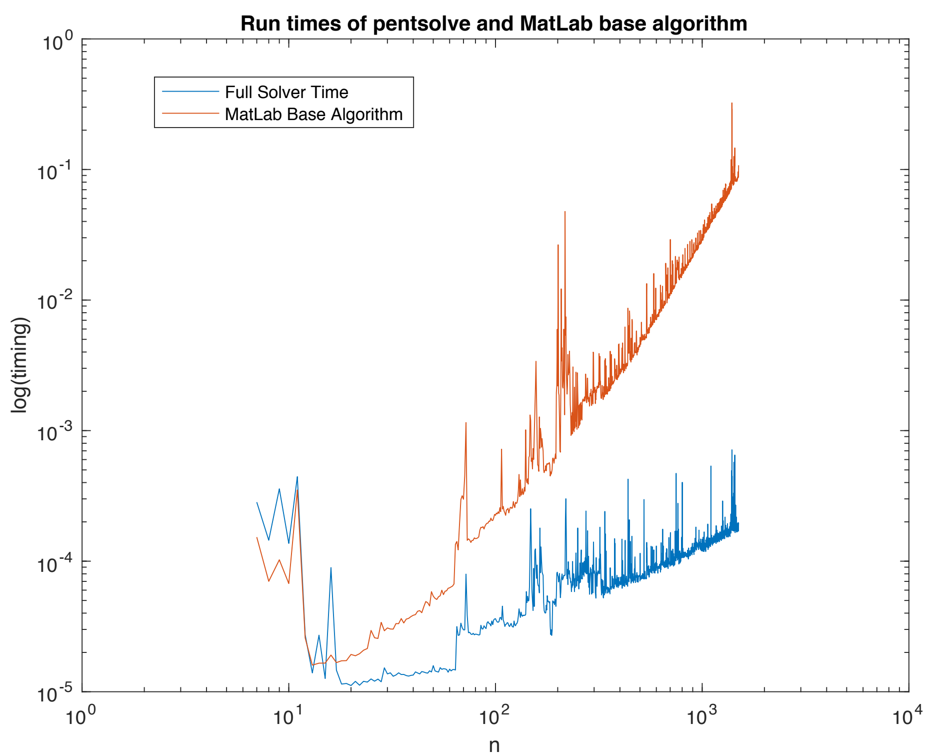
As seen above, the highlighted portions in the tables note the discrepancies in the solutions obtained using the two methods; I didn't highlight all of them, as it would be too time consuming. These discrepancies, while small, indicate that the solutions generated are not equal.

Question 3:

To obtain the run times of both the full solver and the MatLab method, each respective method was wrapped between a tic (timer start) and toc (timer end) call as the solutions were being processed. My laptop, unfortunately, was unable to handle running large matricies. Below is a plot of their timings in a loglog plot:

From the plot, we can see that the MatLab timings are different than the ones from the full solver timings. This can be attributed to the implementation of both methods, which has resulted in a higher runtime for the full solver compared to the MatLab solver.

Graph 1: Running time of pentsolve and MatLab Base Algorithm



Question 4:

On examining the pentsolve method, it appears that its computational complexity is roughly $O(N^2)$. The observed timings of pentsolve is not the same as the base MatLab solver. The former is costlier due to checks on matrix A to see whether it's symmetrical or not, and then executing the appropriate algorithm.

Question 5:

Given R, then R^T is transposed. Let the following numbers be represented in the following way, with blank cells representing 0:

- a = 2
- b = 1

R can now be represented as the following:

a	-a	b					
	b	-a	b				
		b	⋱	⋱			
			⋱	⋱	⋱		
				⋱	⋱	b	
					b	-a	b
						b	-a
							b

Thus R^T is the following matrix:

a							
-a	b						
b	-a	b					
	b	-a	⋱				
		b	⋱	⋱			
			⋱	⋱	b		
				⋱	-a	b	
					b	-a	b

The multiplication of RR^T yields the following:

$a^2+2a+2b$	$-2ab$	b^2					
$-2ab$	$2b^2+a^2$	$-2ab$	b^2				
b^2	$-2ab$	$2b^2+a^2$	⋱	⋱			
	b^2	⋱	⋱	⋱	⋱		
		⋱	⋱	⋱	⋱	b^2	
			⋱		$2b^2+a^2$	$-2ab$	b^2
				b^2	$-2ab$	a^2+b	$-ab^2$
					b^2	$-ab^2$	b

Thus, A may be broken down into the format of RR^T.

Question 6:

Because R and R^T are the upper and lower triangles, respectively, due to the decomposition of A, Gaussian elimination may be used to eliminate the diagonals starting at -2ab above. Backward substitution can then be used on the resulting matrix to obtain the solution to the systems of equation.

Sources of Information:

Sources of information include the slides posted on Canvas, as well as MatLab's documentation on routines such as zero() and fsolve(). Also, I did browse the Discussion forum on Canvas for tips and help, as well as going to TA hours to obtain assistance

I did not collaborate with any classmates on this assignment.

As far as source codes went, I implemented all the functions that didn't already come with MatLab or were not handed out. In this case, I implemented the function *generateA_b.m* while *pentsolve.m* was doled out for our use.