

Given

- A water distribution network $W := (V, E)$ where V is a set of nodes (junctions, tanks, reservoirs) and $E \subseteq V \times V$ is a set of edges (pipes, pumps, valves) connecting the nodes.
- A number of sensors $N \leq |V| \in \mathbb{N}$
- A number of measured properties per sensor $C \in \mathbb{N}$
- A set of sensors $\Sigma := \{\sigma_1, \dots, \sigma_N\}$ with $\sigma_n \in \mathbb{R}^C \forall n \in 1, \dots, N$
- A set of predictions from a leakage detection model $\Pi := \{\pi_1, \dots, \pi_N\}$ with $\pi_n \in \mathbb{R}^C \forall n \in \{1, \dots, N\}$
- A set of thresholds $\{\tau_1, \dots, \tau_N\}$ with $\tau_n \in \mathbb{R}^+ \forall n \in \{1, \dots, N\}$
- A metric function to measure divergence between sensor measurements and model predictions

$$d_{div} : \mathbb{R}^{C \times T} \times \mathbb{R}^{C \times T} \mapsto \mathbb{R}^+$$

The model is classified as under attack **iff**

$$|\{n \in \{1, \dots, N\} | d_{div}(\sigma_n, \pi_n) > \tau_n\}| > n_{alarm}$$

for some hyperparameter n_{alarm}

Let $v_s \in V$ be a node at which one wants the network to be more sensitive to potential attacks. Then one could...

1. introduce a virtual sensor at that node. The simulated measurements of that sensor are constructed using some function depending on the other sensors:

$$\sigma_{n+1} = f_\sigma(\sigma_1, \dots, \sigma_n)$$

2. Make a prediction for that node, that is based on the other predictions of the model

$$\pi_{n+1} = f_\pi(\pi_1, \dots, \pi_n)$$

3. define a reasonable threshold τ_{n+1} for that sensor.