Given

- A water distribution network $W := (V, E)$ where V is a set of nodes (junctions, tanks, reservoirs) and $E \subseteq V \times V$ is a set of edges (pipes, pumps, valves) connecting the nodes.
- A leak $L := (s, l)$ with a size $s \in \mathbb{R}^+$ and a location $l \in V$
- A number of timesteps $T \in \mathbb{N}$
- A number of sensors $N \leq |V| \in \mathbb{N}$
- A number of measured properties per sensor $C \in \mathbb{N}$
- A set of sensors $\Sigma := \{\sigma_1(L), \ldots, \sigma_N(L)\}$ where each sensor is a function depending on the leak size and location and yielding real-valued measurements:
$$\sigma_n : \mathbb{R} \times V \mapsto \mathbb{R}^{C \times T} \qquad \forall n \in 1, \ldots, N$$
- A hacked sensor $\sigma_h \in \Sigma$
- A set of predictions from a leakage detection model $\Pi := \{\pi_1, \ldots, \pi_N\}$ with $\pi_n \in \mathbb{R}^{C \times T}$ $\forall n \in \{1, \ldots, N\}$
- A set of thresholds $\{\tau_1, \ldots, \tau_N\}$ with $\tau_n \in \mathbb{R}^+$ $\forall n \in \{1, \ldots, N\}$
- A metric function to measure divergence between sensor measurements and model predictions
$$d_{div} : \mathbb{R}^{C \times T} \times \mathbb{R}^{C \times T} \mapsto \mathbb{R}^+$$
- A sensor placement function
$$p_s : \Sigma \mapsto V$$
- A metric function on the network
$$d_{net} := V \times V \mapsto \mathbb{R}^+$$

Then the leakage radius of the hacked sensor can be defined as

$$\max_{l} \; d_{net}(l, p_s(\sigma_h)) \quad \text{s.t.} \quad d_{div}(\sigma_n(l, s), \pi_n) \leq \tau_n \; \forall n \in \{1, \ldots, N\} \tag{1}$$

for a fixed leak size s