Supplementary Materials: Superfluid vortex reconnections at non-zero temperatures - TBD

P. Z. Stasiak,[1] Y. Xing,[2,3] Y. Alihosseini,[2,3] C.F. Barenghi,[1]
A. Baggaley,[1] W. Guo,[2,3] L. Galantucci,[4] and G. Krstulovic[5]

[1]*School of Mathematics, Statistics and Physics, Newcastle University,*
Newcastle upon Tyne, NE1 7RU, United Kingdom
[2]*National High Magnetic Field Laboratory, 1800 East Paul Dirac Drive, Tallahassee, Florida 32310, USA*
[3]*Mechanical Engineering Department, FAMU-FSU College of Engineering, Tallahassee, Florida 32310, USA*
[4]*Istituto per le Applicazioni del Calcolo "M. Picone" IAC CNR, Via dei Taurini 19, 00185 Roma, Italy*
[5]*Université Côte d'Azur, Observatoire de la Côte d'Azur, CNRS,Laboratoire Lagrangre,*
Boulevard de l'Observatoire CS 34229 - F 06304 NICE Cedex 4, France

EXPERIMENTAL METHOD

To produce solidified deuterium (D_2) tracer particles, we slowly inject a gas mixture of 5% D_2 and 95% ^{4}He into a superfluid helium bath. Our gas injection system is similar to that described by Fonda et al. [1]. A solenoid valve is installed to control the duration of the gas injection, and a needle valve is used to regulate the gas flow rate. The injected D_2 gas solidifies into small ice particles with a mean radius of 1.1 μm, derived from the particle settling velocities in quiescent superfluid helium [2]. A 473 nm continuous-wave laser sheet (thickness: 0.8 mm) illuminates the particles, and their motion in the laser sheet plane is recorded by a camera at 200 frames per second with a maximum resolution of 2560×1440 pixels. We then identify vortex reconnection events from the recorded videos, and manually track the coordinates of the trapped particles for the pre- (if captured) and post-reconnection. Knowing the particle coordinates, the minimum distance between the reconnecting vortices $\delta^{\pm}(t)$ can be measured. We calculate the prefactors $A^{\pm}$ using the slopes of linear fits to the $\delta^2(t)$ data.

NUMERICAL METHOD

Using Schwarz mesoscopic model [3], vortex lines can be described as space curves $\mathbf{s}(\xi, t)$ of infinitesimal thickness, with a single quantum of circulation $\kappa = h/m_4 = 9.97 \times 10^{-8}\text{m}^2/\text{s}$, where h is Planck's constant, $m_4 = 6.65 \times 10^{-27}\text{kg}$ is the mass of one helium atom, ξ is the natural parameterisation, arclength, and t is time. These conditions are a good approximation, since the vortex core radius of superfluid ^{4}He$(a_0 = 10^{-10}\text{m})$ is much smaller than any of the
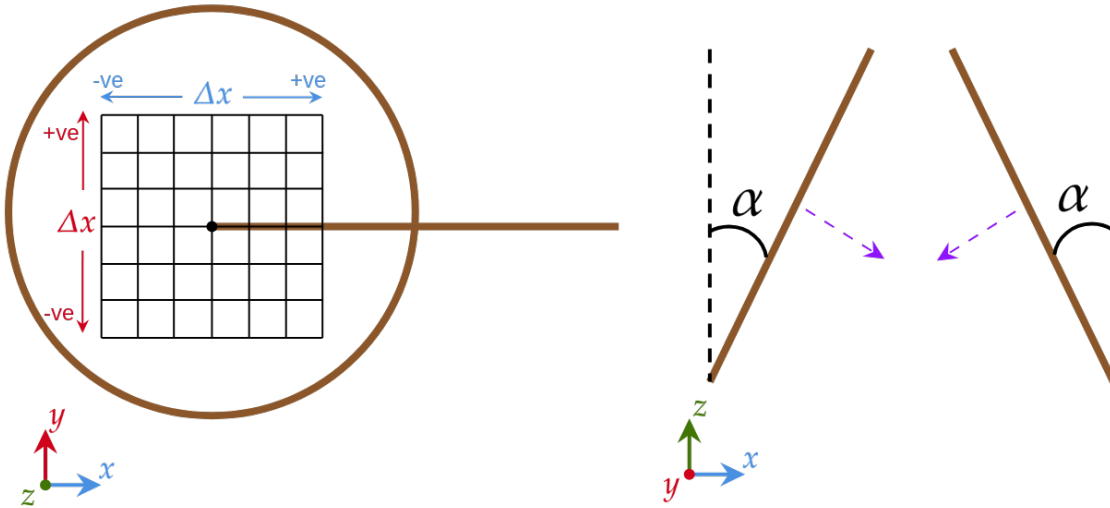

FIG. 1: Schematic diagram for numeric initial condition. *Left:* Hopf-link. *Right:* Oblique collision.

length scale of interest in turbulent flows. The equation of motion is

$$\dot{\mathbf{s}}(\xi, t) = \mathbf{v}_s + \frac{\beta}{1+\beta}\left[\mathbf{v}_{ns} \cdot \mathbf{s}'\right]\mathbf{s}' + \beta\mathbf{s}' \times \mathbf{v}_{ns} + \beta'\mathbf{s}' \times \left[\mathbf{s}' \times \mathbf{v}_{ns}\right],\tag{1}$$

where $\dot{\mathbf{s}} = \partial\mathbf{s}/\partial t$, $\mathbf{s}' = \partial\mathbf{s}/\partial\xi$ is the unit tangent vector, $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$, $\mathbf{v}_n$ and $\mathbf{v}_s$ are the normal fluid and superfluid velocities at $\mathbf{s}$ and β, β' are temperature and Reynolds number dependent mutual friction coefficients [4]. The superfluid velocity $\mathbf{v}_s$ at a point $\mathbf{x}$ is determined by the Biot-Savart law

$$\mathbf{v}_s(\mathbf{x}, t) = \frac{\kappa}{4\pi}\oint_{\mathcal{T}}\frac{\mathbf{s}'(\xi, t) \times [\mathbf{x} - \mathbf{s}(\xi, t)]}{|\mathbf{x} - \mathbf{s}(\xi, t)|}d\xi,\tag{2}$$

where $\mathcal{T}$ represents the entire vortex configuration. There is currently a lack of a well-defined theory of vortex reconnections in superfluid helium, like for the Gross-Pitaevskii equation [5–7]. An *ad hoc* vortex reconnection algorithm is employed to resolve the collisions of vortex lines [8].

A *two-way model* is crucial to understand the accurately interept the back-reaction effect of the normal fluid on the vortex line and vice-versa [9]. We self-consistently evolve the normal fluid $\mathbf{v}_n$ with a modified Navier-Stokes equation

$$\frac{\partial\mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\mathbf{v}_n = -\nabla\frac{p}{\rho} + \nu_n\nabla^2\mathbf{v}_n + \frac{\mathbf{F}_{ns}}{\rho_n},\tag{3}$$

$$\mathbf{F}_{ns} = \oint_{\mathcal{T}}\mathbf{f}_{ns}\delta(\mathbf{x} - \mathbf{x})d\xi, \quad \nabla \cdot \mathbf{v}_n = 0,\tag{4}$$

where $\rho = \rho_n + \rho_s$ is the total density, ρ_n and ρ_s are the normal fluid and superfluid densities, p is the pressure, ν_n is the kinematic viscosity of the normal fluid and $\mathbf{f}_{ns}$ is the local friction per unit length [10]

$$\mathbf{f}_{ns} = -D\mathbf{s}' \times [\mathbf{s}' \times (\dot{\mathbf{s}} - \mathbf{v}_n)] - \rho_n\kappa\mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}),\tag{5}$$

where D is a coefficient dependent on the vortex Reynolds number and intrinsic properties of the normal fluid.

The results in this Letter are reported in dimensionless units, where the characteristic length scale is $\tilde{\lambda} = D/D_0$, where $D^3 = (1 \times 10^{-3}\text{m})^3$ is the dimensional cube size, $D_0^3 = (2\pi)^3$ is the non-dimensional cubic computational domain. The time scale is given by $\tilde{\tau} = \tilde{\lambda}^2\nu_n^0/\nu_n$, where the non-dimensional viscosity ν_n^0 resolves the small scales of the normal fluid. In these simulations, these quantities are $\tilde{\lambda} = 1.59 \times 10^{-4}$cm, $\nu_n^0 = 0.16$ and $\tilde{\tau} = 0.183$s at $T = 0K$ and at $T = 1.9K$ and $\tilde{\tau} = 0.242$s at $T = 2.1K$. We consider two distinct initial vortex geometries at $T = 0K, 1.9K$ and $2.1K$. The first is a Hopf link, two linked rings of radius $R \approx 1$ with an offset in the xy-plane defined by parameters Δl_x and Δl_y. The offsets are chosen so that $(\Delta l_x, \Delta l_y) \in \{(0.125i, 0.125j)|i, j = -3, \cdots, 3\}$, a total of 49 reconnections for each temperature. The second geometry is a collision of vortex rings of radius $R \approx 1$ in a tent-like configuration (see Fig. **??**), making an angle α with the vertical. We take 12 realisations of α, such that $\alpha \in \{i\pi/13|i = 1, \cdots, 12\}$.

In both cases, normal fluid rings are initially superimposed to match the vortex lines, eliminating the transient phase of generating normal fluid structures. The Lagrangian discretisation of vortex lines is $\Delta\xi = 0.025$ (a total of 668 discretiation points) with a timestep of $\Delta t_{VF} = 1.25 \times 10^{-5}$. A total of $N = 256^3$ Eulerian mesh points were used for the normal fluid, with a timestep of $\Delta t_{NS} = 40\Delta t_{VF}$.

---

[1] E. Fonda, K. R. Sreenivasan, and D. P. Lathrop, Sub-micron solid air tracers for quantum vortices and liquid helium flows, Rev. Sci. Instrum. **87**, 025106 (2016).
[2] Y. Tang, W. Guo, H. Kobayashi, S. Yui, M. Tsubota, and T. Kanai, Imaging quantized vortex rings in superfluid helium to evaluate quantum dissipation, Nat. Commun. **14**, 2941 (2023).
[3] KW. Schwarz, Three-dimensional vortex dynamics in superfluid ^{4}He, Phys. Rev. B **38**, 2398 (1988).
[4] L. Galantucci, A. W. Baggaley, C. F. Barenghi, and G. Krstulovic, A new self-consistent approach of quantum turbulence in superfluid helium, Eur. Phys. J. Plus **135**, 547 (2020).
[5] A. Villois, D. Proment, and G. Krstulovic, Irreversible Dynamics of Vortex Reconnections in Quantum Fluids, Phys. Rev. Lett. **125**, 164501 (2020).
[6] A. Villois, D. Proment, and G. Krstulovic, Universal and nonuniversal aspects of vortex reconnections in superfluids, Phys. Rev. Fluids **2**, 044701 (2017).

[7] D. Proment and G. Krstulovic, Matching theory to characterize sound emission during vortex reconnection in quantum fluids, Phys. Rev. Fluids **5**, 104701 (2020).

[8] A. W. Baggaley, The sensitivity of the vortex filament method to different reconnection models, J. Low Temp. Phys. **168**, 18 (2012).

[9] P. Z. Stasiak, A. W. Baggaley, G. Krstulovic, C. F. Barenghi, and L. Galantucci, Cross-Component Energy Transfer in Superfluid Helium-4, J Low Temp Phys 10.1007/s10909-023-03042-5 (2024).

[10] L. Galantucci, M. Sciacca, and CF. Barenghi, Coupled normal fluid and superfluid profiles of turbulent helium II in channels, Phys Rev B **92**, 174530 (2015).