Punctuated energy injection in superfluid helium-4 vortex reconnections

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Introduction.— Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Main results.— The vast seperation of length scales between the vortex core a_0 and average distance between vortices in a tangle ℓ allows for vortices to be described as space curves $\mathbf{s}(\xi, t)$ with parameter ξ. The equation of motion of vortex lines follows from Schwarz [1]

$$\dot{\mathbf{s}}(\xi, t) = \mathbf{v}_s + \frac{\beta}{1+\beta}\left[\mathbf{v}_{ns} \cdot \mathbf{s}'\right]\mathbf{s}' + \beta\mathbf{s}' \times \mathbf{v}_{ns} + \beta'\mathbf{s}' \times \left[\mathbf{s}' \times \mathbf{v}_{ns}\right], \tag{1}$$

here $\dot{\mathbf{s}} = \partial\mathbf{s}/\partial t$, $\mathbf{s}' = \partial\mathbf{s}/\partial\xi$ is the unit tangent vector, $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$, $\mathbf{v}_n$ and $\mathbf{v}_s$ are the normal fluid and

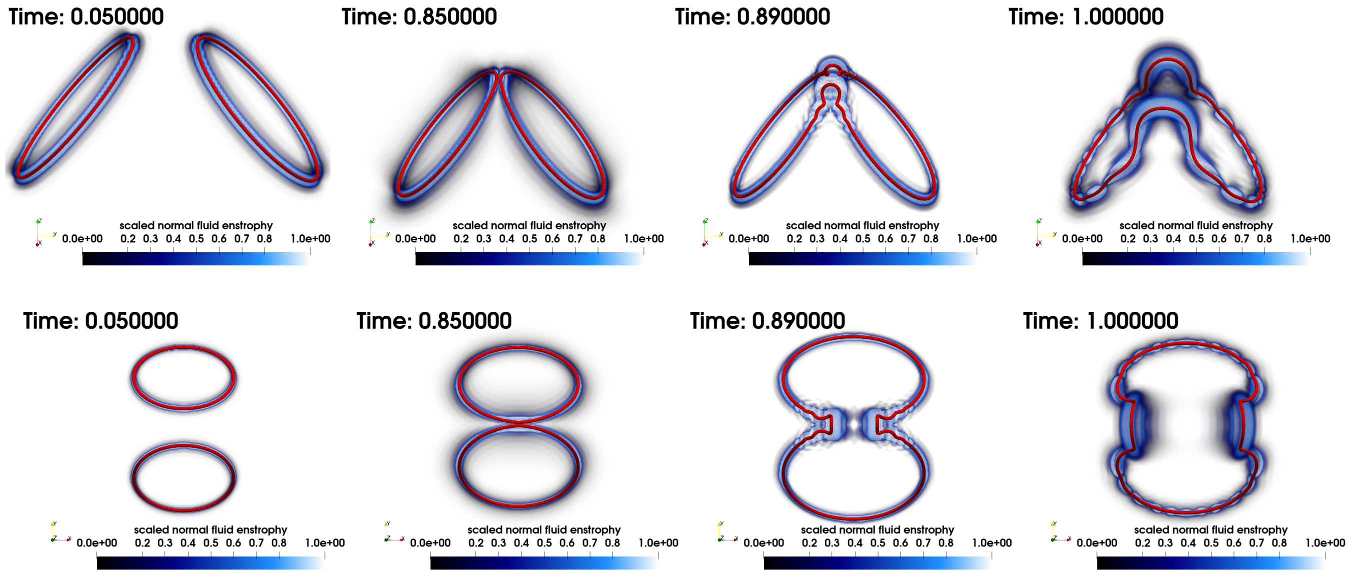

FIG. 1: 3D rendering of vortex ring collisions, from a tent-like initial condition. The red tube represents a superfluid vortex, where the radius has been greatly exaggerated for visual purposes, and the blue volume rendering represents the scaled normal fluid enstrophy $\boldsymbol{\omega}^2/\boldsymbol{\omega}^2_{max}$. *Top row:* Isometric view. *Bottom row:* View of the xy-plane.

superfluid velocities at $\mathbf{s}$ and β,β' are temperature and Reynolds number dependent mutual fricition coefficients [2].Superfluid vortices are coupled to a classical description of the incompressible ($\nabla \cdot \mathbf{v}_n = 0$) normal fluid via the mutual friciton force $\mathbf{F}_{ns}$

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\mathbf{v}_n = -\nabla \frac{p}{\rho} + \nu_n \nabla^2 \mathbf{v}_n + \frac{\mathbf{F}_{ns}}{\rho_n}, \qquad (2)$$

where $\rho = \rho_n + \rho_s$ is the total density, ρ_n and ρ_s are the normal fluid and superfluid densities, p is the pressure, ν_n is the kinematic viscosity of the normal fluid. We consider two distinct initial vortex geometries at $T = 0K, 1.9K$ and $2.1K$. The first is a Hopf link, two linked rings of radius $R \approx 1$ with an offset in the xy-plane defined by parameters Δl_x and Δl_y. The offsets are chosen so that $(\Delta l_x, \Delta l_y) \in \{(0.125i, 0.125j)|i, j = -3, \cdots, 3\}$, a total of 49 reconnections for each temperature. The second geometry is a collision of vortex rings of radius $R \approx 1$ in a tent-like configuration (see Fig. 1), making an angle α with the vertical. We take 12 realisations of α, such that $\alpha \in \{i\pi/13|i = 1, \cdots, 12\}$. See the supplementary material for details on the model and dimensionality [**?**]. In both cases, normal fluid rings are initially superimposed to match the vortex lines, eliminating the transient phase of generating normal fluid structures.

The linked structure of the Hopf link naturally decays due to self-induced propogation of vortex rings, leading to vortex reconnections. The smallest distance between two filaments around the reconnection event δ, referred to in this Letter simply as 'the minimum distance', has been shown to exhibit a power law behaviour

$$\delta^{\pm}(t) = A^{\pm}(\kappa|t - t_0|)^{1/2}, \qquad (3)$$

where κ is the quantum of circulation,t_0 is the reconnection time, $A^{\pm}$ are the dimensionless prefactors where $+$ represents the *seperation* of vortex filaments and $-$ the *approach*. A total of 147 simulations (49 across 3 temperatures) for the Hopf links are shown in Fig. 2a, where the prefactors which appear in the inset, are systematically computed using the shaded region for gradient estimation. The effect of viscous dissipation due to finite temperature effects is immediately clear in the pre-reconnection regime, a clear segregation of δ^- due to temperature. In stark contrast, any temperature-dependent segregation is lost after the reconnection event, bearing little to no resemblance to the order in the pre-reconnection regime. Interestingly, the A^+ distribution does not drastically change when including finite temperature effects, suggesting a minor role of the normal fluid in the reconnection dynamics. Our results confirm the irreversibility of vortex reconnections in our finite temperature model, for which $A^+ \geq A^-$.

As shown in Fig. 2b, the $T = 0K$ calculation for helium is in good agreement with the Gross-Pitaevskii (GP) model where $A^- \sim 0.4$-0.6, for both initial conditions. Recent investigation in the classical Navier-Stokes [3] have displayed a clear 1/2 power law scaling and prefactor ratio $A^- \sim 0.3$-0.4, which again shows good agreement with the results that we have presented here for finite temperature.

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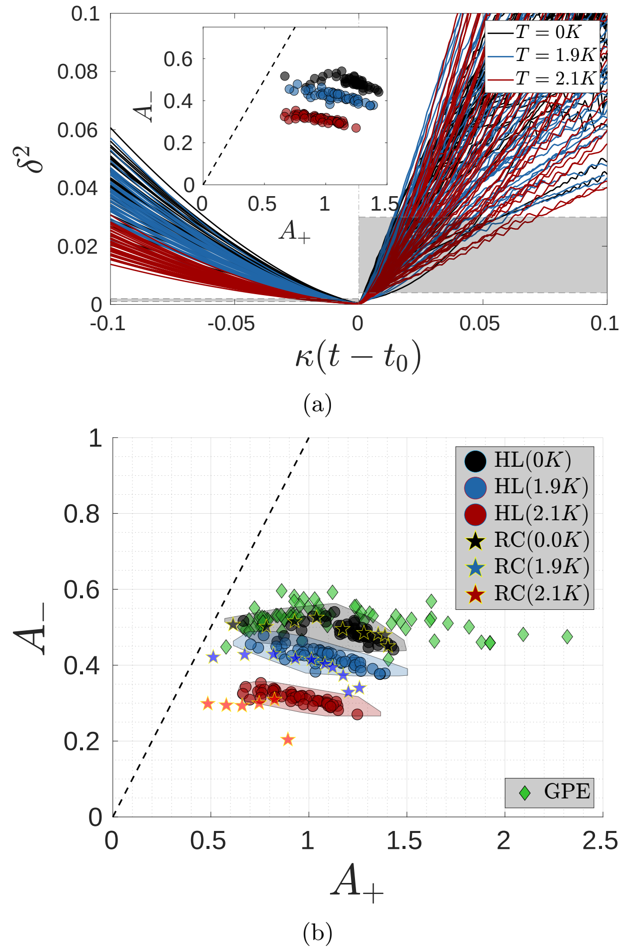

FIG. 2: *(a)*: Time evolution of the minimum distance squared δ^2 for the Hopf link initial conditions at $T = 0K, 1.9K$ and $2.1K$. The grey shaded area represens the vertical region used to estimate the prefactors $A^{\pm}$. *Inset:* Values of the seperation prefactor A^+ and approach prefactors A^-. *(b)*: Comparison of all prefactor values, *HL*-Hopf link (circles), *RC*-ring collision (stars with yellow outline), GPE-data from Gross-Pitaevskii simulations from Villois *et al.* [4]. The shaded areas associated with each colour represent the convex hull of errors for each temperature.

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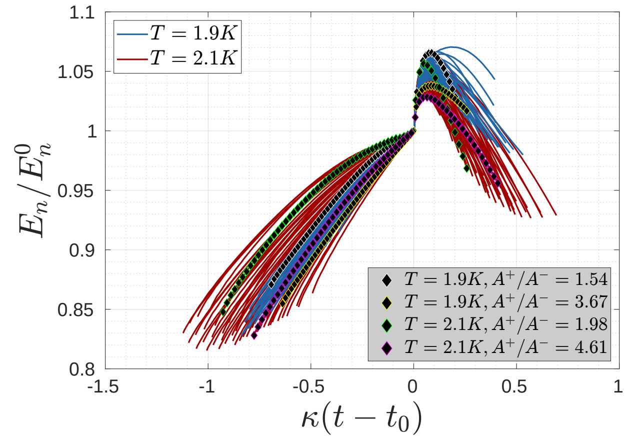

FIG. 3: Total normal fluid kinetic energy E_n scaled by the kinetic energy at reconnection time E_n^0. Black diamonds represent the simulations with minimum and maximum prefactor ratios A^+/A^- at $T = 1.9K$ and $T = 2.1K$ respectively.

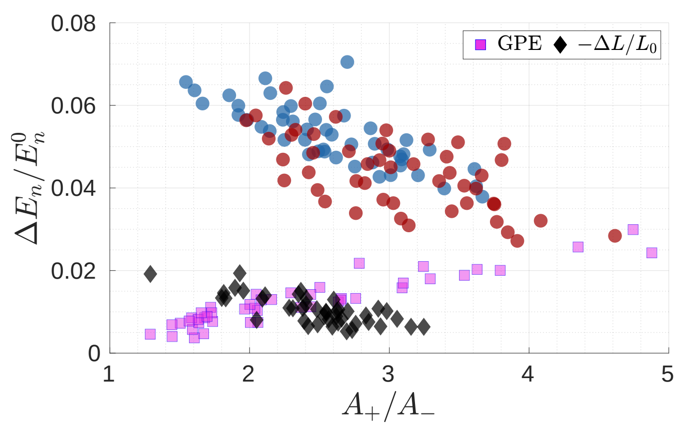

FIG. 4: The total energy jump $\Delta E_n = E_n^0 - E_n(t_{max})$, where t_{max} is such that $E_n(t_{max}) = \max(E_n(t > t_0))$. The solid black diamond represents the change in line length ΔL in the $T = 0K$ case, and the purple squares is from GP simulations from Villois *et al.* [4]

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Closing remarks.— Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem.

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[1] KW. Schwarz, Three-dimensional vortex dynamics in superfluid ^{4}He, Phys. Rev. B **38**, 2398 (1988).

[2] L. Galantucci, A. W. Baggaley, C. F. Barenghi, and G. Krstulovic, A new self-consistent approach of quantum turbulence in superfluid helium, Eur. Phys. J. Plus **135**, 547 (2020).

[3] J. Yao and F. Hussain, Separation scaling for viscous vortex reconnection, J. Fluid Mech. **900**, R4 (2020).

[4] A. Villois, D. Proment, and G. Krstulovic, Irreversible Dynamics of Vortex Reconnections in Quantum Fluids, Phys. Rev. Lett. **125**, 164501 (2020).