

# Supplementary Materials: Experimental and theoretical evidence of universality in superfluid vortex reconnections

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## EXPERIMENTAL METHOD

To produce solidified deuterium ( $D_2$ ) tracer particles, we slowly inject a gas mixture of 5%  $D_2$  and 95%  $^4He$  into a superfluid helium bath. Our gas injection system is similar to that described by Fonda et al. [1]. A solenoid valve is installed to control the duration of the gas injection, and a needle valve is used to regulate the gas flow rate. The injected  $D_2$  gas solidifies into small ice particles with a mean radius of  $1.1 \mu m$ , derived from the particle settling velocities in quiescent superfluid helium [2]. A 473 nm continuous-wave laser sheet (thickness: 0.8 mm) illuminates the particles, and their motion in the laser sheet plane is recorded by a camera at 200 frames per second with a maximum resolution of  $2560 \times 1440$  pixels. We then identify vortex reconnection events from the recorded videos, and manually track the coordinates of the trapped particles for the pre- (if captured) and post-reconnection. Knowing the particle coordinates, the minimum distance between the reconnecting vortices  $\delta^\pm(t)$  can be measured. We calculate the prefactors  $A^\pm$  using the slopes of linear fits to the  $\delta^2(t)$  data.

## NUMERICAL METHOD

Using Schwarz mesoscopic model [3], vortex lines can be described as space curves  $\mathbf{s}(\xi, t)$  of infinitesimal thickness, with a single quantum of circulation  $\kappa = h/m_4 = 9.97 \times 10^{-8} m^2/s$ , where  $h$  is Planck’s constant,  $m_4 = 6.65 \times 10^{-27} kg$  is the mass of one helium atom,  $\xi$  is the natural parameterisation, arclength, and  $t$  is time. These conditions are a good approximation, since the vortex core radius of superfluid  $^4He(a_0 = 10^{-10} m)$  is much smaller than any of the

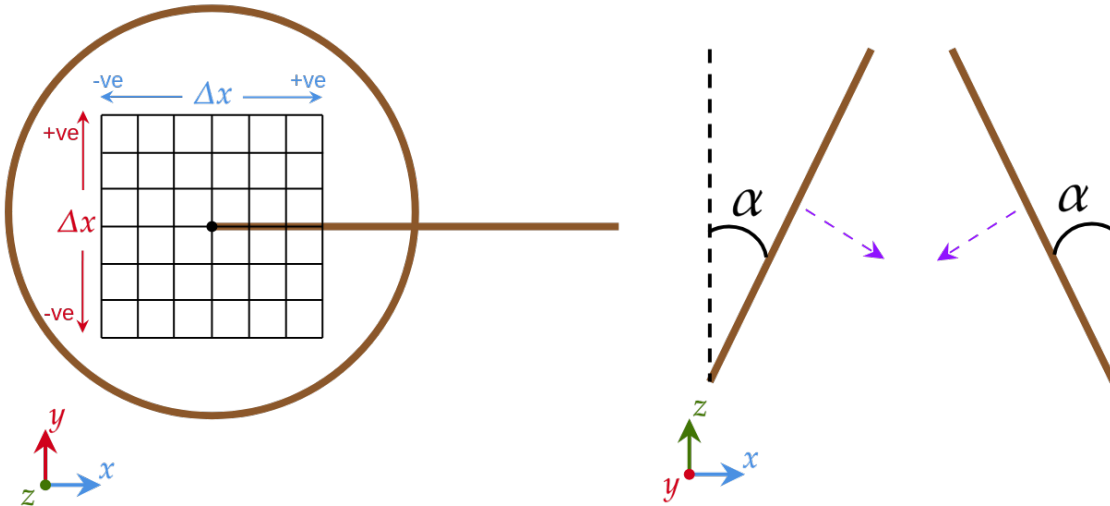


FIG. 1: Schematic diagram for numeric initial condition. *Left*: Hopf-link. *Right*: Oblique collision.

length scale of interest in turbulent flows. The equation of motion is

$$\dot{\mathbf{s}}(\xi, t) = \mathbf{v}_s + \frac{\beta}{1 + \beta} [\mathbf{v}_{ns} \cdot \mathbf{s}'] \mathbf{s}' + \beta \mathbf{s}' \times \mathbf{v}_{ns} + \beta' \mathbf{s}' \times [\mathbf{s}' \times \mathbf{v}_{ns}], \quad (1)$$

where  $\dot{\mathbf{s}} = \partial \mathbf{s} / \partial t$ ,  $\mathbf{s}' = \partial \mathbf{s} / \partial \xi$  is the unit tangent vector,  $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$ ,  $\mathbf{v}_n$  and  $\mathbf{v}_s$  are the normal fluid and superfluid velocities at  $\mathbf{s}$  and  $\beta, \beta'$  are temperature and Reynolds number dependent mutual friction coefficients [4]. The superfluid velocity  $\mathbf{v}_s$  at a point  $\mathbf{x}$  is determined by the Biot-Savart law

$$\mathbf{v}_s(\mathbf{x}, t) = \frac{\kappa}{4\pi} \oint_{\mathcal{T}} \frac{\mathbf{s}'(\xi, t) \times [\mathbf{x} - \mathbf{s}(\xi, t)]}{|\mathbf{x} - \mathbf{s}(\xi, t)|} d\xi, \quad (2)$$

where  $\mathcal{T}$  represents the entire vortex configuration. There is currently a lack of a well-defined theory of vortex reconnections in superfluid helium, like for the Gross-Pitaevskii equation [5–7]. An *ad hoc* vortex reconnection algorithm is employed to resolve the collisions of vortex lines [8].

A *two-way model* is crucial to understand the accurately interpret the back-reaction effect of the normal fluid on the vortex line and vice-versa [9]. We self-consistently evolve the normal fluid  $\mathbf{v}_n$  with a modified Navier-Stokes equation

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n = -\nabla \frac{p}{\rho} + \nu_n \nabla^2 \mathbf{v}_n + \frac{\mathbf{F}_{ns}}{\rho_n}, \quad (3)$$

$$\mathbf{F}_{ns} = \oint_{\mathcal{T}} \mathbf{f}_{ns} \delta(\mathbf{x} - \mathbf{x}) d\xi, \quad \nabla \cdot \mathbf{v}_n = 0, \quad (4)$$

where  $\rho = \rho_n + \rho_s$  is the total density,  $\rho_n$  and  $\rho_s$  are the normal fluid and superfluid densities,  $p$  is the pressure,  $\nu_n$  is the kinematic viscosity of the normal fluid and  $\mathbf{f}_{ns}$  is the local friction per unit length [10]

$$\mathbf{f}_{ns} = -D \mathbf{s}' \times [\mathbf{s}' \times (\dot{\mathbf{s}} - \mathbf{v}_n)] - \rho_n \kappa \mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}), \quad (5)$$

where  $D$  is a coefficient dependent on the vortex Reynolds number and intrinsic properties of the normal fluid.

The results in this Letter are reported in dimensionless units, where the characteristic length scale is  $\tilde{\lambda} = D/D_0$ , where  $D^3 = (1 \times 10^{-3} \text{m})^3$  is the dimensional cube size,  $D_0^3 = (2\pi)^3$  is the non-dimensional cubic computational domain. The time scale is given by  $\tilde{\tau} = \tilde{\lambda}^2 \nu_n^0 / \nu_n$ , where the non-dimensional viscosity  $\nu_n^0$  resolves the small scales of the normal fluid. In these simulations, these quantities are  $\tilde{\lambda} = 1.59 \times 10^{-4} \text{cm}$ ,  $\nu_n^0 = 0.16$  and  $\tilde{\tau} = 0.183 \text{s}$  at  $T = 0 \text{K}$  and at  $T = 1.9 \text{K}$  and  $\tilde{\tau} = 0.242 \text{s}$  at  $T = 2.1 \text{K}$ . We consider two distinct initial vortex geometries at  $T = 0 \text{K}$ ,  $1.9 \text{K}$  and  $2.1 \text{K}$ . The first is a Hopf link, two linked rings of radius  $R \approx 1$  with an offset in the  $xy$ -plane defined by parameters  $\Delta l_x$  and  $\Delta l_y$ . The offsets are chosen so that  $(\Delta l_x, \Delta l_y) \in \{(0.125i, 0.125j) | i, j = -3, \dots, 3\}$ , a total of 49 reconnections for each temperature. The second geometry is a collision of vortex rings of radius  $R \approx 1$  in a tent-like configuration (see Fig. 1), making an angle  $\alpha$  with the vertical. We take 12 realisations of  $\alpha$ , such that  $\alpha \in \{i\pi/13 | i = 1, \dots, 12\}$ .

In both cases, normal fluid rings are initially superimposed to match the vortex lines, eliminating the transient phase of generating normal fluid structures. The Lagrangian discretisation of vortex lines is  $\Delta \xi = 0.025$  (a total of 668 discretisation points) with a timestep of  $\Delta t_{VF} = 1.25 \times 10^{-5}$ . A total of  $N = 256^3$  Eulerian mesh points were used for the normal fluid, with a timestep of  $\Delta t_{NS} = 40 \Delta t_{VF}$ .

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