Supplementary Materials: Superfluid vortex reconnections at non-zero temperatures - TBD

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NUMERICAL METHOD

Using Schwarz mesoscopic model [1], vortex lines can be described as space curves $\mathbf{s}(\xi, t)$ of infinitesimal thickness, with a single quantum of circulation $\kappa = h/m_4 = 9.97 \times 10^{-8}\mathrm{m}^2/\mathrm{s}$, where h is Planck's constant, $m_4 = 6.65 \times 10^{-27}\mathrm{kg}$ is the mass of one helium atom, ξ is the natural parameterisation, arclength, and t is time. These conditions are a good approximation, since the vortex core radius of superfluid $^4\mathrm{He}(a_0 = 10^{-10}\mathrm{m})$ is much smaller than any of the length scale of interest in turbulent flows. The equation of motion is

$$\dot{\mathbf{s}}(\xi, t) = \mathbf{v}_s + \frac{\beta}{1+\beta}\left[\mathbf{v}_{ns} \cdot \mathbf{s}'\right]\mathbf{s}' + \beta \mathbf{s}' \times \mathbf{v}_{ns} + \beta' \mathbf{s}' \times \left[\mathbf{s}' \times \mathbf{v}_{ns}\right], \tag{1}$$

where $\dot{\mathbf{s}} = \partial \mathbf{s}/\partial t$, $\mathbf{s}' = \partial \mathbf{s}/\partial \xi$ is the unit tangent vector, $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$, $\mathbf{v}_n$ and $\mathbf{v}_s$ are the normal fluid and superfluid velocities at $\mathbf{s}$ and β,β' are temperature and Reynolds number dependent mutual fricition coefficients [2]. The superfluid velocity $\mathbf{v}_s$ at a point $\mathbf{x}$ is determined by the Biot-Savart law

$$\mathbf{v}_s(\mathbf{x}, t) = \frac{\kappa}{4\pi} \oint_{\mathcal{T}} \frac{\mathbf{s}'(\xi, t) \times [\mathbf{x} - \mathbf{s}(\xi, t)]}{|\mathbf{x} - \mathbf{s}(\xi, t)|} d\xi, \tag{2}$$

where $\mathcal{T}$ represents the entire vortex configuration. There is currently a lack of a well-defined theory of vortex reconnections in superfluid helium, like for the Gross-Pitaevskii equation [3–5]. An *ad hoc* vortex reconnection algorithm is employed to resolve the collisions of vortex lines [6].

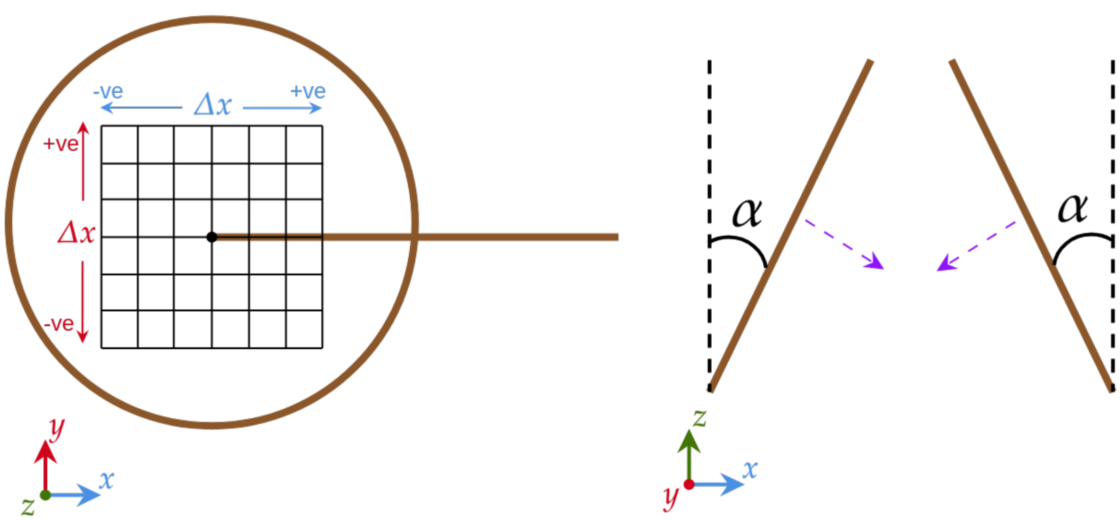

FIG. 1: Schematic diagram for numeric initial condition. *Left:* Hopf-link. *Right:* Oblique collision.

A *two-way model* is crucial to understand the accurately interept the back-reaction effect of the normal fluid on the vortex line and vice-versa [7]. We self-consistently evolve the normal fluid $\mathbf{v}_n$ with a modified Navier-Stokes equation

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\mathbf{v}_n = -\nabla \frac{p}{\rho} + \nu_n \nabla^2 \mathbf{v}_n + \frac{\mathbf{F}_{ns}}{\rho_n}, \tag{3}$$

$$\mathbf{F}_{ns} = \oint_{\mathcal{T}} \mathbf{f}_{ns} \delta(\mathbf{x} - \mathbf{x}) d\xi, \quad \nabla \cdot \mathbf{v}_n = 0, \tag{4}$$

where $\rho = \rho_n + \rho_s$ is the total density, ρ_n and ρ_s are the normal fluid and superfluid densities, p is the pressure, ν_n is the kinematic viscosity of the normal fluid and $\mathbf{f}_{ns}$ is the local friction per unit length [8]

$$\mathbf{f}_{ns} = -D\mathbf{s}' \times [\mathbf{s}' \times (\dot{\mathbf{s}} - \mathbf{v}_n)] - \rho_n \kappa \mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}), \tag{5}$$

where D is a coefficient dependent on the vortex Reynolds number and intrinsic properties of the normal fluid.

The results in this Letter are reported in dimensionless units, where the characteristic length scale is $\tilde{\lambda} = D/D_0$, where $D^3 = (1 \times 10^{-3}\mathrm{m})^3$ is the dimensional cube size, $D_0^3 = (2\pi)^3$ is the non-dimensional cubic computational domain. The time scale is given by $\tilde{\tau} = \tilde{\lambda}^2 \nu_n^0 / \nu_n$, where the non-dimensional viscosity ν_n^0 resolves the small scales of the normal fluid. In these simulations, these quantities are $\tilde{\lambda} = 1.59 \times 10^{-4}$cm, $\nu_n^0 = 0.16$ and $\tilde{\tau} = 0.183$s at $T = 0K$ and at $T = 1.9K$ and $\tilde{\tau} = 0.242$s at $T = 2.1K$. We consider two distinct initial vortex geometries at $T = 0K, 1.9K$ and $2.1K$. The first is a Hopf link, two linked rings of radius $R \approx 1$ with an offset in the xy-plane defined by parameters Δl_x and Δl_y. The offsets are chosen so that $(\Delta l_x, \Delta l_y) \in \{(0.125i, 0.125j) | i, j = -3, \cdots, 3\}$, a total of 49 reconnections for each temperature. The second geometry is a collision of vortex rings of radius $R \approx 1$ in a tent-like configuration (see Fig. **??**), making an angle α with the vertical. We take 12 realisations of α, such that $\alpha \in \{i\pi/13 | i = 1, \cdots, 12\}$.

In both cases, normal fluid rings are initially superimposed to match the vortex lines, eliminating the transient phase of generating normal fluid structures. The Lagrangian discretisation of vortex lines is $\Delta\xi = 0.025$ (a total of 668 discretiation points) with a timestep of $\Delta t_{VF} = 1.25 \times 10^{-5}$. A total of $N = 256^3$ Eulerian mesh points were used for the normal fluid, with a timestep of $\Delta t_{NS} = 40\Delta t_{VF}$.

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