Supplementary Materials: Experimental and theoretical evidence of universality in superfluid vortex reconnections

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EXPERIMENTAL METHOD

To produce solidified deuterium (D_2) tracer particles, we slowly inject a gas mixture of 5% D_2 and 95% ^{4}He into a superfluid helium bath. Our gas injection system is similar to that described by Fonda et al. [1]. A solenoid valve is installed to control the duration of the gas injection, and a needle valve is used to regulate the gas flow rate. The injected D_2 gas solidifies into small ice particles with a mean radius of 1.1 μm, derived from the particle settling velocities in quiescent superfluid helium [2]. A 473 nm continuous-wave laser sheet (thickness: 0.8 mm) illuminates the particles, and their motion in the laser sheet plane is recorded by a camera at 200 frames per second with a maximum resolution of 2560×1440 pixels. We then identify vortex reconnection events from the recorded videos, and manually track the coordinates of the trapped particles for the pre- (if captured) and post-reconnection. Knowing the particle coordinates, the minimum distance between the reconnecting vortices $\delta^{\pm}(t)$ can be measured. We calculate the prefactors $A^{\pm}$ using the slopes of linear fits to the $\delta^2(t)$ data.

NUMERICAL METHOD

Using Schwarz mesoscopic model [3], vortex lines can be described as space curves $\mathbf{s}(\xi, t)$ of infinitesimal thickness, with a single quantum of circulation $\kappa = h/m_4 = 9.97 \times 10^{-8}\mathrm{m}^2/\mathrm{s}$, where h is Planck's constant, $m_4 = 6.65 \times 10^{-27}$kg is the mass of one helium atom, ξ is the natural parameterisation, arclength, and t is time. These conditions are a good approximation, since the vortex core radius of superfluid ^{4}He($a_0 = 10^{-10}$m) is much smaller than any of the

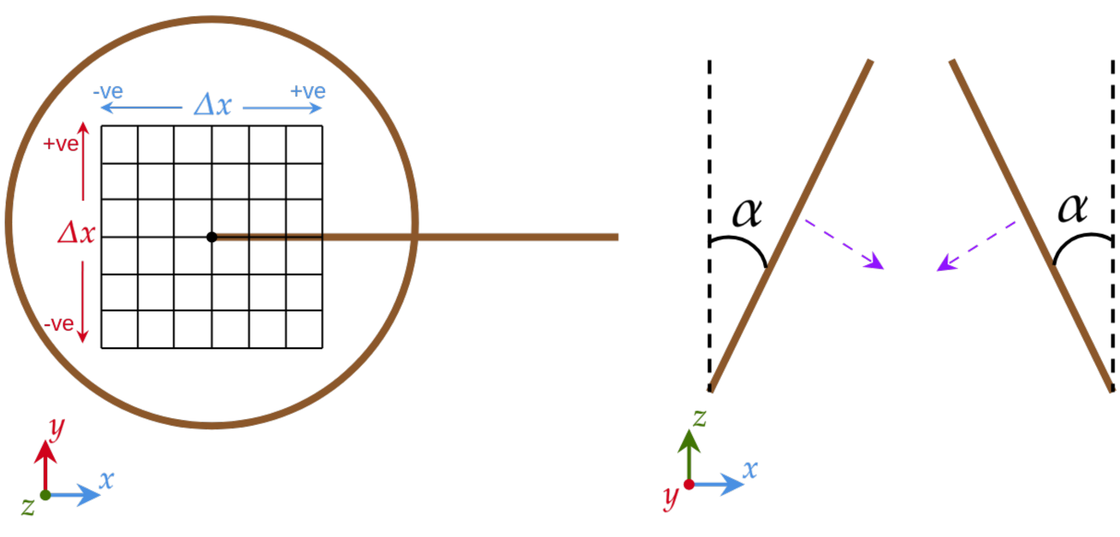

FIG. 1: Schematic diagram for numeric initial condition. *Left:* Hopf-link. *Right:* Oblique collision.

length scale of interest in turbulent flows. The equation of motion is

$$\dot{\mathbf{s}}(\xi,t) = \mathbf{v}_s + \frac{\beta}{1+\beta}\left[\mathbf{v}_{ns}\cdot\mathbf{s}'\right]\mathbf{s}' + \beta\mathbf{s}'\times\mathbf{v}_{ns} + \beta'\mathbf{s}'\times\left[\mathbf{s}'\times\mathbf{v}_{ns}\right], \tag{1}$$

where $\dot{\mathbf{s}} = \partial\mathbf{s}/\partial t$, $\mathbf{s}' = \partial\mathbf{s}/\partial\xi$ is the unit tangent vector, $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$, $\mathbf{v}_n$ and $\mathbf{v}_s$ are the normal fluid and superfluid velocities at $\mathbf{s}$ and β,β' are temperature and Reynolds number dependent mutual fricition coefficients [4]. The superfluid velocity $\mathbf{v}_s$ at a point $\mathbf{x}$ is determined by the Biot-Savart law

$$\mathbf{v}_s(\mathbf{x},t) = \frac{\kappa}{4\pi}\oint_{\mathcal{T}}\frac{\mathbf{s}'(\xi,t)\times[\mathbf{x}-\mathbf{s}(\xi,t)]}{|\mathbf{x}-\mathbf{s}(\xi,t)|}d\xi, \tag{2}$$

where $\mathcal{T}$ represents the entire vortex configuration. There is currently a lack of a well-defined theory of vortex reconnections in superfluid helium, like for the Gross-Pitaevskii equation [5–7]. An *ad hoc* vortex reconnection algorithm is employed to resolve the collisions of vortex lines [8].

A *two-way model* is crucial to understand the accurately interept the back-reaction effect of the normal fluid on the vortex line and vice-versa [9]. We self-consistently evolve the normal fluid $\mathbf{v}_n$ with a modified Navier-Stokes equation

$$\frac{\partial\mathbf{v}_n}{\partial t} + (\mathbf{v}_n\cdot\nabla)\mathbf{v}_n = -\nabla\frac{p}{\rho} + \nu_n\nabla^2\mathbf{v}_n + \frac{\mathbf{F}_{ns}}{\rho_n}, \tag{3}$$

$$\mathbf{F}_{ns} = \oint_{\mathcal{T}}\mathbf{f}_{ns}\delta(\mathbf{x}-\mathbf{x})d\xi, \quad \nabla\cdot\mathbf{v}_n = 0, \tag{4}$$

where $\rho = \rho_n + \rho_s$ is the total density, ρ_n and ρ_s are the normal fluid and superfluid densities, p is the pressure, ν_n is the kinematic viscosity of the normal fluid and $\mathbf{f}_{ns}$ is the local friction per unit length [10]

$$\mathbf{f}_{ns} = -D\mathbf{s}'\times[\mathbf{s}'\times(\dot{\mathbf{s}}-\mathbf{v}_n)] - \rho_n\kappa\mathbf{s}'\times(\mathbf{v}_n-\dot{\mathbf{s}}), \tag{5}$$

where D is a coefficient dependent on the vortex Reynolds number and intrinsic properties of the normal fluid.

The results in this Letter are reported in dimensionless units, where the characteristic length scale is $\tilde{\lambda} = D/D_0$, where $D^3 = (1\times10^{-3}\mathrm{m})^3$ is the dimensional cube size, $D_0^3 = (2\pi)^3$ is the non-dimensional cubic computational domain. The time scale is given by $\tilde{\tau} = \tilde{\lambda}^2\nu_n^0/\nu_n$, where the non-dimensional viscosity ν_n^0 resolves the small scales of the normal fluid. In these simulations, these quantities are $\tilde{\lambda} = 1.59\times10^{-4}$cm, $\nu_n^0 = 0.16$ and $\tilde{\tau} = 0.183$s at $T = 0K$ and at $T = 1.9K$ and $\tilde{\tau} = 0.242$s at $T = 2.1K$. We consider two distinct initial vortex geometries at $T = 0K, 1.9K$ and $2.1K$. The first is a Hopf link, two linked rings of radius $R \approx 1$ with an offset in the xy-plane defined by parameters Δl_x and Δl_y. The offsets are chosen so that $(\Delta l_x, \Delta l_y) \in \{(0.125i, 0.125j)|i,j = -3,\cdots,3\}$, a total of 49 reconnections for each temperature. The second geometry is a collision of vortex rings of radius $R \approx 1$ in a tent-like configuration (see Fig. 1), making an angle α with the vertical. We take 12 realisations of α, such that $\alpha \in \{i\pi/13|i = 1,\cdots,12\}$.

In both cases, normal fluid rings are initially superimposed to match the vortex lines, eliminating the transient phase of generating normal fluid structures. The Lagrangian discretisation of vortex lines is $\Delta\xi = 0.025$ (a total of 668 discretiation points) with a timestep of $\Delta t_{VF} = 1.25\times10^{-5}$. A total of $N = 256^3$ Eulerian mesh points were used for the normal fluid, with a timestep of $\Delta t_{NS} = 40\Delta t_{VF}$.

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