

Inverse energy transfer in three-dimensional quantum vortex flows

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Vortex reconnections play a fundamental role in fluids. They increase the complexity of flow and develop small-scale motions. In this work, we report that in superfluids, they can also excite large scales. We numerically illustrate that during a superfluid vortex reconnection energy is injected into the thermal (normal) component of helium II at small length scales, but is transferred nonlinearly to larger length scales, increasing the integral length scale of the normal fluid. We show that this inverse energy transfer is triggered by the helical imbalance generated in the normal fluid flow by the mutual friction force coupling the superfluid vortices and the normal component. We finally discuss the relevance of our findings to the problem of superfluid turbulence.

Turbulence is ubiquitous in the universe. It occurs in systems as large as nebulae of interstellar gas, and as small as clouds of few thousands atoms confined by lasers in the laboratory. Turbulence shapes patterns and properties of fluids of all kinds, from ordinary viscous fluids (Navier-Stokes turbulence[1]) to electrically conducting fluids (magneto-hydrodynamics turbulence [2]) to quantum fluids (quantum turbulence [3, 4]). All turbulent systems are characterised by the existence of a wide range of length scales across which inviscid conserved quantities are transferred without loss in the spirit of the cascade depicted by Richardson [5].

In three-dimensional (3D) classical fluids, turbulence is characterised by a direct (forward) cascade: the non-linear dissipationless transfer of kinetic energy from the scale of the large eddies (at which energy is injected) to the smallest length scales at which energy is dissipated into heat [5, 6]. The resulting distribution of energy across length scales is the celebrated Kolmogorov energy spectrum [1, 6].

Confining Navier-Stokes turbulence to two-dimensions (2D) entails fundamentally distinct physics: a dual cascade emerges of energy and enstrophy (mean squared vorticity) [7, 8], the two conserved quantities in ideal two-dimensional flows. While the enstrophy cascade is direct (from large to small scales), the energy cascade is inverse (from small to large scales) [9]. This inverse cascade may favour the generation and persistence of large coherent structures [10].

Remarkably, the same cascade phenomenology is ob-

served in turbulent flows of quantum fluids, *i.e.* fluids at very low temperatures whose physics is dominated by quantum effects. Examples of such fluids are superfluid helium and atomic Bose-Einstein Condensates (BECs). The dynamics of these systems can be successfully depicted in terms of a two-fluid model [11–13] describing the quantum fluid as the mixture of two components, the superfluid component and the thermal (or normal) component, which interact by means of a mutual friction force [14–16]. The superfluid component flows without viscosity and vanishing entropy; its vorticity is confined to effectively one-dimensional vortex filaments of atomic core thickness (called quantum vortices or vortex lines), around which the circulation of the velocity is quantised. In BECs the thermal component forms a ballistic gas, whereas in superfluid ^4He it can be described as a classical viscous fluid. Despite these significant differences with respect to ordinary fluids, the direct kinetic energy cascade has indeed been observed in three-dimensional superfluid turbulence [17–22]. Evidence of this direct cascade has been found also in three-dimensional turbulent BECs [23]. In this forward energy cascade a fundamental role is played by the dynamics and interactions of quantum vortices, in particular by their reconnections, phenomena where two vortex lines collide and recombine, exchanging heads and tails, altering the overall topology of the flow [24–30]. Reconnections occur frequently in 3D quantum turbulence and are essential to the energy transfer towards small scales by engendering the breakdown of coherent vortex structures and triggering

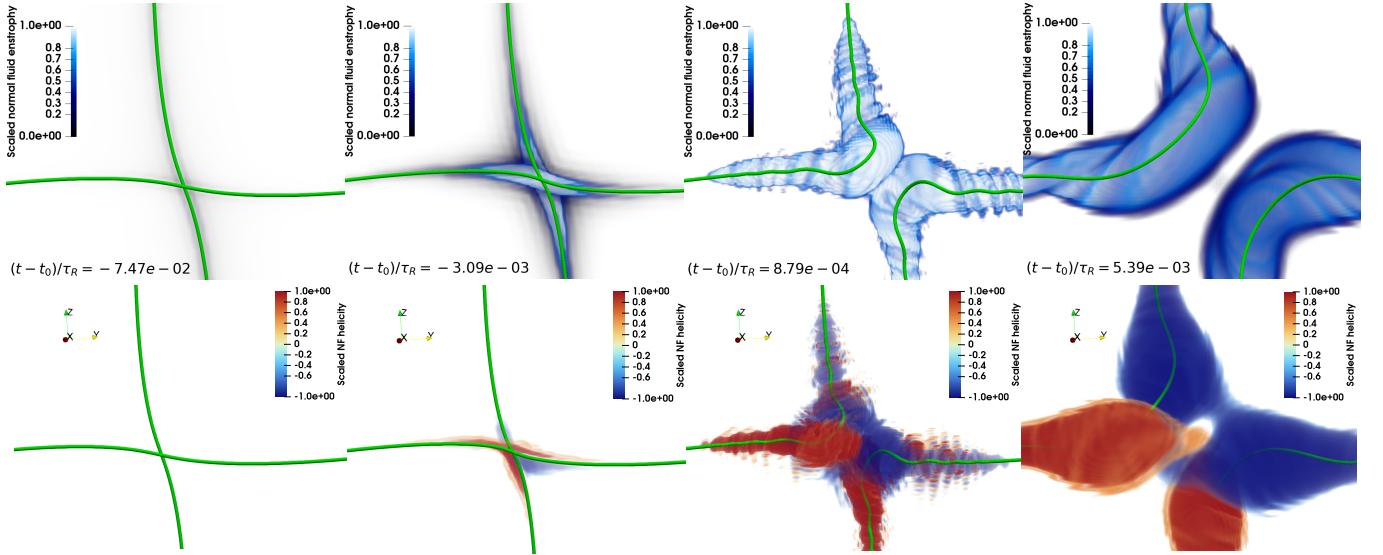


FIG. 1: Three-dimensional rendering of the time evolution of an initially orthogonal vortex configuration undergoing a vortex reconnection at $T = 1.9\text{K}$ at dimensionless times (from left to right) $(t - t_R)/\tau_R = -7.47 \times 10^{-2}$, -3.09×10^{-3} , 8.79×10^{-4} and 5.39×10^{-3} , where $\tau_R = L_R/E_R^{1/2}$. The quantities E_R and L_R are the energy E and integral length scales L of the normal fluid at reconnection time t_R . The green tubes represent the superfluid vortex lines (the tubes' radii have been greatly exaggerated for visual purpose). In the top sequence, the blue volume rendering represents the scaled normal fluid enstrophy ω^2/ω_{max}^2 . Note the Kelvin wave on the superfluid vortex at $(t - t_R)/\tau_R = 8.79 \times 10^{-4}$. In the bottom sequence, the red/blue volume rendering at the same times represent scaled positive/negative normal fluid helicity.

the Kelvin-wave cascade [31]. In 2D, similarly to classical turbulence, an inverse energy cascade characterises turbulence in two-dimensional BECs, as shown in theoretical [32–35] and experimental [36, 37] studies.

In turbulent systems, the type and the number of sign-defined ideal invariants determine the direction of cascades. Indeed, the famous Fjørtoft argument [38] predicts the existence of an inverse energy cascade in 2D classical turbulence. It also predicts an inverse particle and a direct energy cascade for 3D wave turbulent BECs, as recently addressed theoretically [39]. In 3D classical fluids, helicity, which is also an inviscid invariant, is not sign-defined and thus only a direct energy cascade is possible. However, recent studies have demonstrated that the direction of the energy cascade may be inverted by artificially controlling the chirality of the flow, *i.e.* the balance between positive and negative helical modes [40]. Indeed, by restricting the non-linear energy transfer to homochiral interactions via a suitable decimation of the Navier-Stokes equation [41, 42], by controlling the weight of homochiral interactions [43], or by the external injection of positive helical modes at all length scales [44], inverse energy cascades have been observed in three-dimensional turbulence of classical fluids. In brief, when the flow is synthetically designed to have an enhanced chirality, an inverse energy cascade can be observed.

In this work, we unveil a similar dynamics occurring

in superfluid helium (${}^4\text{He}$) as a result of vortex reconnections. We show that the mutual friction force arising from the vortex reconnection is chiral, injecting in the normal fluid prevalently helicity of a given sign. Thus, as a consequence of vortex reconnections, we observe an increase of the chiral imbalance of the quantum fluid, producing a transfer of kinetic energy from small to large scales, similarly to the phenomenology observed in 3D helical-decimated classical flows. Unlike classical fluids, such a chiral imbalance arises naturally as physical process in the normal fluid.

To model superfluid helium dynamics, we employ the recently developed FOUCAULT model [45]. In this approach, superfluid vortex lines are parametrized as one-dimensional space curves $\mathbf{s}(\xi, t)$, ξ and t being arclength and time respectively, exploiting the large separation of length scales between the vortex core radius, the Lagrangian discretisation along the vortex lines $\Delta\xi$, and the average radius of curvature R_c of the vortex lines. The vortex lines evolve according to the following equation of motion:

$$\dot{\mathbf{s}}(\xi, t) = \mathbf{v}_s + \frac{\beta}{1 + \beta} [\mathbf{v}_{ns} \cdot \mathbf{s}'] \mathbf{s}' + \beta \mathbf{s}' \times \mathbf{v}_{ns} + \beta' \mathbf{s}' \times [\mathbf{s}' \times \mathbf{v}_{ns}], \quad (1)$$

where $\dot{\mathbf{s}} = \partial\mathbf{s}/\partial t$, $\mathbf{s}' = \partial\mathbf{s}/\partial\xi$ is the unit tangent vector, \mathbf{v}_n and \mathbf{v}_s are the normal fluid and superfluid velocities at \mathbf{s} , $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$, and β, β' are temperature

and Reynolds number dependent mutual friction coefficients [45]. The calculation of the superfluid velocity \mathbf{v}_s is performed via the computation of the Biot-Savart integral de-singularised with standard techniques (see Supplementary Material [46]). The normal fluid is described classically using the incompressible ($\nabla \cdot \mathbf{v}_n = 0$) Navier-Stokes equation

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n = -\frac{1}{\rho} \nabla p + \nu_n \nabla^2 \mathbf{v}_n + \frac{\mathbf{F}_{ns}}{\rho_n} , \quad (2)$$

where ρ_n and ρ_s are the normal fluid and superfluid densities, $\rho = \rho_n + \rho_s$, p is the pressure, ν_n is the kinematic viscosity of the normal fluid, and the mutual friction force per unit volume, \mathbf{F}_{ns} , is the line integral of the mutual friction force per unit length, \mathbf{f}_{ns} [46]:

$$\mathbf{F}_{ns}(\mathbf{x}) = \oint_C \delta(\mathbf{x} - \mathbf{s}) \mathbf{f}_{ns}(\mathbf{s}) d\xi , \quad (3)$$

C representing the entire vortex configuration. The regularisation of mutual friction is performed using a physically self-consistent scheme [45]. We consider a periodical box of size 2π (so that wavevectors are integers).

To study the reconnection dynamics, we consider two pairs of initially orthogonal vortices (where the corresponding vortices of each pair have opposite circulation in order to preserve periodicity along the boundaries) at two distinct temperatures, $T = 1.9K$ and $T = 2.1K$. The vortex pairs are separated by the distance D_ℓ ; each vortex within each pair is initially at distance d_ℓ to the other vortex, such that $d_\ell \ll D_\ell$ to ensure that the dynamics in the vicinity of the reconnection is dominated by local interactions, and that the far-field contribution from the other vortex pair is negligible. All results reported hereafter refer only to the reconnection of one pair of vortices.

The evolution of the vortex reconnection of a single pair is reported in Fig. 1. The first row shows the reconnecting superfluid vortices (in green) accompanied by normal fluid structures generated by mutual friction, here displayed as enstrophy rendering $\omega(\mathbf{x})^2 = |\nabla \times \mathbf{v}_n|^2$. Such structures are the signature of the violent irreversible energy transfers in vortex reconnections [47]. The second row shows the rendering of the local helicity $H(\mathbf{x}) = \mathbf{v}_n \cdot \boldsymbol{\omega}$, where we observe a clear local helicity production, with an abrupt change of sign due to the rearrangement of the vortex topology. Remarkably, during reconnection there is a net sudden normal fluid helicity production, as shown in Fig. 2. We will come back to this finding later.

We now focus on the time evolution of the normal fluid energy spectrum $E(k)$, defined by

$$E = \frac{1}{(2\pi)^3} \int_V \frac{1}{2} |\mathbf{v}_n|^2 dV = \int_0^\infty E(k) dk \quad (4)$$

where E is the total normal fluid energy and k is the magnitude of the three-dimensional wavenumber. The

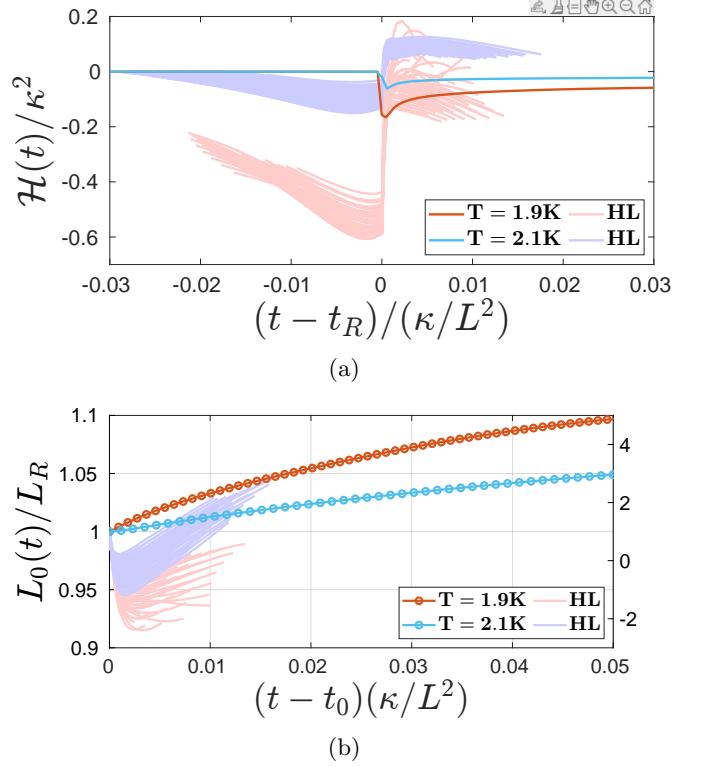


FIG. 2: Temporal evolution of the normal fluid helicity $\mathcal{H} = \int_V H(\mathbf{x}) dV$ computed over the entire volume V . For superfluid helium, it is proper to make the helicity dimensionless in terms of κ^2 .

energy at reconnection E_R is given by $E(t_R)$, where t_R is the time at reconnection. The energy spectrum $E(k)$ is displayed in Fig. 3a.

It clearly emerges that, during the reconnection, energy is predominantly injected into the normal fluid at intermediate and small length scales. For $k > 5$ in correspondence of the reconnection time t_R , we observe a significant increase of the normal fluid energy spectral density: $E(k, t \approx t_R)/E(k, t \ll t_R) \approx 10^2$. In the post-reconnection regime, we simultaneously observe a small decrease of the spectrum at intermediate and small scales ($k > 5$) and an increase at large scales, suggesting the existence of a mechanism by which energy generated at small length scales is transferred to larger scales. To shed light on this mechanism, as customary for turbulent flows, we analyse the spectral energy flux

$$\Pi(k) = \int_{|\mathbf{p}| < k} \hat{\mathbf{v}}_n^* \cdot \left[\widehat{(\mathbf{v}_n \cdot \nabla) \mathbf{v}_n} \right] d\mathbf{p} + c.c. , \quad (5)$$

where $\widehat{\cdot}$ indicates the Fourier transform.

We observe that $\Pi(k) < 0$ for all k during and after reconnection; we also observe that, near the time of reconnection, the peak value of $|\Pi(k)|$ is in the range $15 < k < 25$. The negative sign of $\Pi(k)$ is evidence of a flux of kinetic energy from small to large scales, exciting

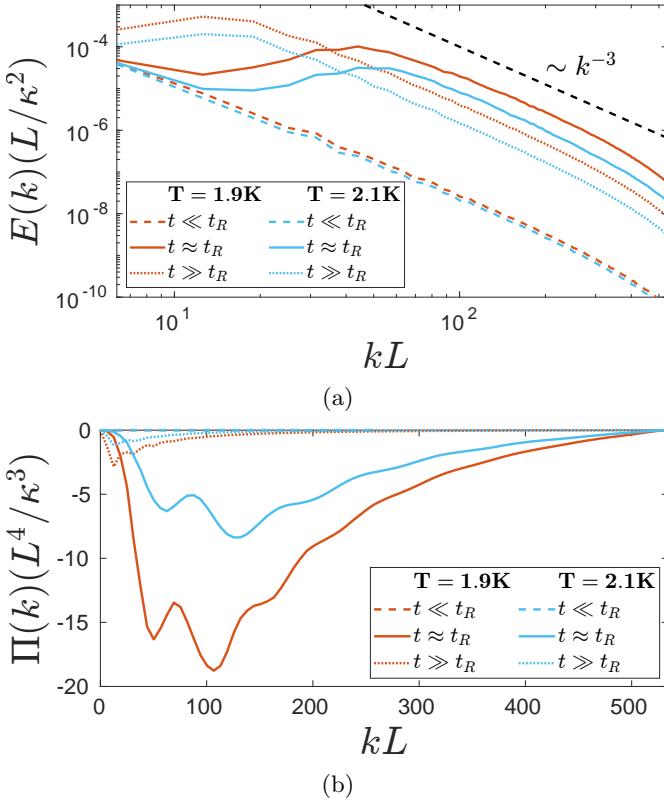


FIG. 3: (a): Normal fluid kinetic energy spectrum $E(k)$ before reconnection (dashed lines), at reconnection (solid lines) and after reconnection (dotted lines) for $T = 1.9\text{K}$ (red) and $T = 2.1\text{K}$ (blue). (b): Spectral normal fluid kinetic energy flux, $\Pi(k)$. It is normalised using the integral scale and the normal fluid energy at reconnection. *Inset:* Post reconnection evolution of the integral length scale, L . Times and temperatures are labelled as in Fig. 3a.

larger and larger scales. This behaviour is quantified by the evolution of the integral length scale L , defined as

$$L = \frac{\pi}{2E} \int_0^\infty \frac{E(k)}{k} dk, \quad (6)$$

The inset of Fig. 3b shows that L indeed increases steadily in the post-reconnection regime. Note that times have been normalised by the largest eddy-turnover-time at the reconnection event, evidencing its fast evolution.

To explain the inverse energy transfer shown in Fig. 3b, we look whether the reconnection triggers a chirality imbalance. We decompose the incompressible Fourier modes of the normal fluid velocity into helical modes [48]:

$$\hat{\mathbf{v}}_n(\mathbf{k}) = \hat{\mathbf{v}}_n^+(\mathbf{k}) + \hat{\mathbf{v}}_n^-(\mathbf{k}) = v_n^+(\mathbf{k})\mathbf{h}^+(\mathbf{k}) + v_n^-(\mathbf{k})\mathbf{h}^-(\mathbf{k}), \quad (7)$$

where $\mathbf{h}^\pm(\mathbf{k})$ are the two eigenvectors of the curl operator, *i.e.* $i\mathbf{k} \times \mathbf{h}^\pm(\mathbf{k}) = \pm k\mathbf{h}^\pm(\mathbf{k})$. Similarly, we decompose the transverse modes of the mutual friction

force: $\hat{\mathbf{F}}_{ns}^\perp(\mathbf{k}) = f^+(\mathbf{k})\mathbf{h}^+ + f^-(\mathbf{k})\mathbf{h}^-$ (the Fourier modes of \mathbf{F}_{ns} parallel to the wavenumber \mathbf{k} do not play any role in the time evolution of \mathbf{v}_n due to the incompressible constraint). Finally, the helical decomposition naturally allow us to decompose the total helicity as $\mathcal{H} = \mathcal{H}^+ - \mathcal{H}^-$ [44].

A chiral imbalance occurs if the mutual friction force is helical, *i.e.* if the ratio $|f^+|^2/|f^-|^2 \neq 1$, with $|f^\pm|^2$ the total squared norm of the mutual friction components. In Fig. 4, we show the temporal evolution of $|f^+|^2/|f^-|^2$, for both temperatures. It is apparent that

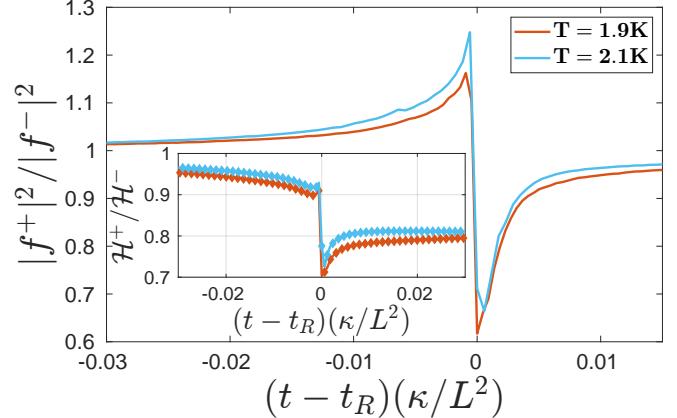


FIG. 4: Temporal evolution of projected mutual friction force components $f^\pm/|f^\pm|^2$. Inset: temporal evolution of total helical components.

during and after the reconnection, the mutual friction force is strongly chiral, injecting more negative helicity than positive helicity. As a result, the ratio $\mathcal{H}^+/\mathcal{H}^-$ (reported in the inset of Fig. 4) decreases significantly at reconnection and remains smaller than unity even at later times, indicating that the flow is persistently chiral. We conclude that the reconnection triggers indeed a chiral imbalance. For the sake of completeness, it is worth noting that the total helicity of the flow (obtained by adding the superfluid helicity to that of the normal fluid \mathcal{H}) is not conserved. From Fig. 4, we consider a time t^* after reconnection where the ratio $|f^+|^2/|f^-|^2$ has sufficiently decayed to a roughly 5% difference and we determine the non-dimensional timescale $\tau = (t^* - t_R)/\tau_R$ during which the mutual friction force is chiral as a result of reconnections: $\tau \approx 0.01$ and $\tau \approx 0.005$ for $T = 1.9\text{K}$ and $T = 2.1\text{K}$, respectively, corresponding dimensionally to $\tau \approx 0.1\text{s}$ for both temperatures. In superfluid turbulence, the timescale between two consecutive reconnections can be smaller than τ provided that the vortex line density \mathcal{L} (length of vortices per unit volume) is larger than 10^8m^{-2} [47, 49], a condition which is easily met in superfluid helium experiments [50, 51].

In conclusion, the reconnection of quantum vortices in the two-fluid regime ($T \gtrsim 1.5\text{K}$) not only injects punctu-

ated energy in the normal fluid [47], but also triggers in the normal fluid a transfer of kinetic energy towards the large scales. This inverse energy transfer is triggered by the helical character of the mutual friction, directly arising from the Kelvin waves released by the reconnecting cusp, which produces a chiral imbalance in the normal fluid, as previously observed in turbulent Navier-Stokes flows [41, 44].

Our findings have profound implications for the nature of turbulence in finite temperature superfluids, where vortex reconnections play a key role. In circumstances where the vortex density \mathcal{L} is large and where the isotropy of the vortex tangle is broken by external forcing (for instance thermal counterflows [52]), the chirality of the flow generated by the frequent non-symmetrical reconnections may be strong enough to induce an inverse energy cascade [53]. The microscopic mechanism that we have described may be relevant in triggering the inverse energy cascade which is observed numerically in large-scale simulations of counterflow turbulence at large heat fluxes [54], which is indeed inherently not isotropic. However, formalising the connection between high counterflow, unbalanced helicity injection and inverse energy cascade is challenging, as the model used in [54] and the one used in our work describe superfluid helium at very different scales. Our work hence motivates further detailed studies of the role played by helicity in superfluid dynamics [55, 56], moving the emphasis from few vortex systems [57] to fully coupled superfluid turbulence.

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