Emerging inverse energy transfer in superfluid helium-4 vortex reconnections

P. Z. Stasiak, A. Baggaley, and C.F. Barenghi
School of Mathematics, Statistics and Physics, Newcastle University,
Newcastle upon Tyne, NE1 7RU, United Kingdom

G. Krstulovic
Université Côte d'Azur, Observatoire de la Côte d'Azur, CNRS,Laboratoire Lagrangre,
Boulevard de l'Observatoire CS 34229 - F 06304 NICE Cedex 4, France

L. Galantucci
Istituto per le Applicazioni del Calcolo "M. Picone" IAC CNR, Via dei Taurini 19, 00185 Roma, Italy
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Introduction.— Homogeneous isotropic turbulence (HIT) in three-dimensional classical fluid dynamics is often characterised by a forward energy cascade with a Kolmogorov-like scaling $\sim k^{-5/3}$ in an inertial range, where typically energy is injected at the largest length scales [1]. Similar scaling laws have been found in mechanically [2, 3] and thermally [4] driven superfluid turbulence, as well as a unique ultra-cold regime with $\sim k^{-1}$ energy scaling, typically arising in the absence of a strong energy injection. In such cases, energy is transferred via non-local interactions from the large to small length scales.

In certain cases, like in two-dimensional turbulence [5], geophysical flows [6] and shallow fluid layers [7], a reversal of energy flux can be identified, for which energy is injected at small length scales, and is transferred by non-local interactions to large length scales. An inverse energy cascade in two-dimensional turbulence is well known in both classical and quantum fluid dynamics [8], and in recent years the role of helicity has been highlighted in the generation of inverse energy transfers for three-dimensional turbulence, without the constraint of quasi-two-dimensionality [9, 10].

Main results.— In this Letter, we use the Schwarz model [11] to evolve vortex filaments $\mathbf{s}(\xi,t)$, where ξ is the natural parametrisation of vortex lines, also known as the arclength. The normal fluid is coupled via the mutual friction force $\mathbf{f}_{ns}$ in a self-consistent manner using a recently developed technique in Ref. [12]. Further details of the method are outlined in Ref. [13] and the corresponding Supplementary Materials. We set two pairs of orthogonal vortices, where each vortex in the pair has the same circulation $\pm\kappa$, to preserve superfluid periodicity along the boundaries, and we consider two finite

temperatures, $T = 1.9K$ and $T = 2.1K$. Vortex pairs are seperated by distance D_ℓ, and each vortex within each pair is seperated by distance d_ℓ, such that $D_\ell \ll d_\ell$, see Fig. 1. The seperation of scales ensures that the dynamics in the vicinity of the reconnection are dominated by local interactions, and that far-reaching contributions from the other vortex pair are negligent. The normal fluid is initally quiescent, and evolves according to a classical Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\left(\frac{p}{\rho_n}\right) + \nu_n \nabla^2 \mathbf{v} + \frac{\mathbf{F}_{ns}}{\rho_n} \quad (1)$$

where $\mathbf{v}$ denotes the normal fluid velocity, p is the pressure, ρ_n is the normal fluid density, ν_n is the kinematic viscosity and $\mathbf{F}_{ns}$ is the mutual friction coupling, arising from the relative motion between the normal fluid and superfluid vortices $\mathbf{F}_{ns} \propto |\mathbf{v}_l - \mathbf{v}|$ where $\mathbf{v}_l$ is the velocity of vortex lines. The generation of the force is highly localised in space, at microscopic length scales on the order of the vortex core radius $\sim a_0$, and $\Delta x \gg a_0$, where Δx is a typical grid discretisation. Abrupt changes in the topology of vortex lines, such as during vortex reconnections, form highly curved cusps and strongly inject vorticity (see Fig. 1), since $|\mathbf{v}_l - \mathbf{v}| \approx |\mathbf{v}_l| \propto \zeta$ in the first approximation, where ζ is the curvature of vortex lines.

In this way, energy is injected into the normal fluid at the small length scales in a natural way in the absence of external property, an intrinsic and unique feature of quantum fluids. The injection of energy is clear in the kinetic energy spectrum E_k which is shown in in Fig. 2, at wavenumbers $k > 10$ a vast increase can be observed $E_k(t \approx t_0)/E_k(t \ll t_0) \sim 10^2$, evidence of very strong energy injection. Remarkably, in the post reconnection regime E_k appears to increase when k is small, and decay
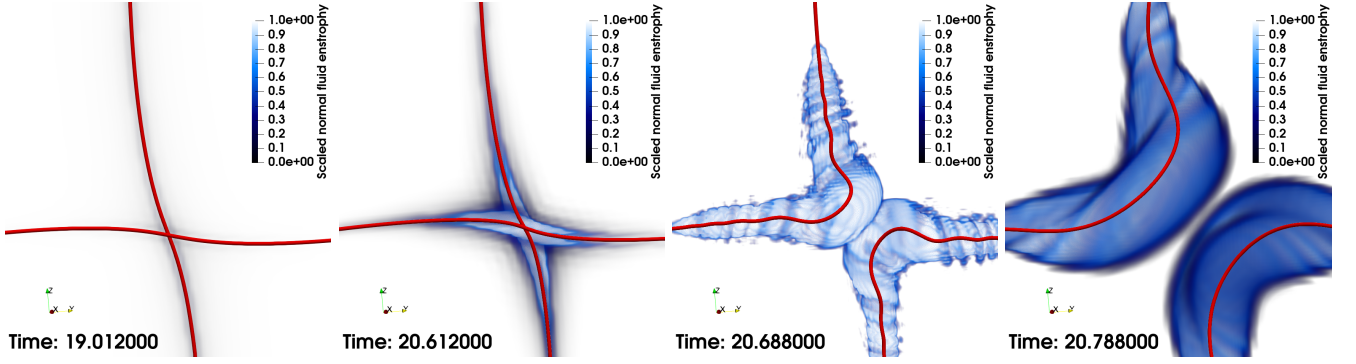
FIG. 1: 3D rendering of an orthogonal vortex configuration, undergoing a vortex reconnection. The red tube represents a superfluid vortex, where the radius has been greatly exaggerated for visual purposes, and the blue volume rendering represents the scaled normal fluid enstrophy ω^2/ω_{max}^2.

at large values of k, suggesting a possible mechanism by which energy generated at small length scales is transferred to larger scales. The energy spectrum E_k evolves according to the balance sources/sinks and internal transfers,

$$\frac{\partial E_k}{\partial t} = T_k - D_k + I_k \qquad (2)$$

where T_k is the spectral kinetic energy transfer function, $D_k = 2\nu_n k^2 E_k$ is the dissipation spectrum and $I_k = \mathrm{Re}(\hat{\mathbf{F}}_{ns}(\mathbf{k}) \cdot \hat{\mathbf{v}}^*(\mathbf{k}))$ is the injection spectrum due

FIG. 2: Normal fluid kinetic energy spectrum E_k before reconnection (dashed lines), at reconnection (solid lines) and after reconnection (dotted lines) for $T = 1.9K$ and $T = 2.1K$. *Inset:* Dissipation spectrum $D_k/2\nu_n = k^2 E_k$ at the same snapshots in time.

to the mutual friction $\mathbf{F}_{ns}$. The injection spectrum I_k as shown in Fig. 3a again shows that the injection of energy covers primarily the high k range, but is not limited to a thin band of injection. In fact, the energy flux $\Pi_E(k) = \int_k^\infty T_k dk$ as shown in Fig. 3b asserts a hallmark feature of inverse energy cascades, typical in 2D isotropic turbulence. A flux of $\Pi_E(k) < 0$ implies a negative flux of energy from small to large k, i.e. an inverse transfer of energy. The effect of kinetic energy transferral to large length scales results in the creation of large scale structures, evident in the evolution of the integral length scale $\mathcal{L}_E$, where

$$\mathcal{L}_E = \frac{\pi}{2\langle \mathbf{v}'^2 \rangle} \int_0^\infty \frac{E_k}{k} dk \qquad (3)$$

and $\langle \mathbf{v}'^2 \rangle$ represents the turbulent kinetic energy. In the inset of Fig. 3b, $\mathcal{L}_E$ steadily increases in the post-reconnection region, implying a generation of large scale structure.

In order to explain the inverse energy transfer mechanism, we follow recent work in classical fluids outlined in Refs. [10, 14], where it is even possible to sustain an inverse energy cascade under a helical forcing applied at all length scales. Typically, velocity coefficients $\hat{\mathbf{v}}(\mathbf{k})$ can be decomposed into their helical modes, where $\hat{\mathbf{v}}(\mathbf{k}) = v^+(\mathbf{k})\mathbf{h}^+(\mathbf{k}) + v^-(\mathbf{k})\mathbf{h}^-(\mathbf{k})$ and satisfies $\mathbf{k} \cdot \hat{\mathbf{v}}(\mathbf{k}) = 0$, where $\mathbf{v}^\pm$ are complex scalars and $\mathbf{h}^\pm(\mathbf{k})$ are the two eigenvectors of the curl operator, such that $i\mathbf{k} \times \mathbf{h}^\pm(\mathbf{k}) = \pm k\mathbf{h}^\pm(\mathbf{k})$. To explain the inverse energy transfer in terms of helical modes, we show that in fact the driving force, which in our case is a punctuated burst due to superfluid vortex reconnections, is of a helical nature. The mutual friction modes $\hat{\mathbf{F}}_{ns}(\mathbf{k})$ are not incompressible, and so we take the projection of the modes orthogonal to the wavenumber $\mathbf{k}$,

$$\hat{\mathbf{f}}(\mathbf{k}) = \hat{\mathbf{F}}_{ns}(\mathbf{k}) - \frac{(\hat{\mathbf{F}}_{ns}(\mathbf{k}) \cdot \mathbf{k})}{k^2}\mathbf{k}. \qquad (4)$$

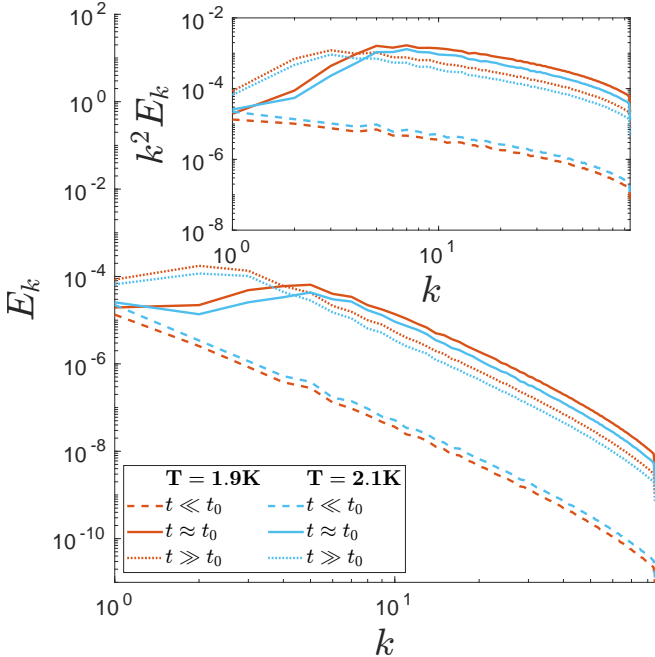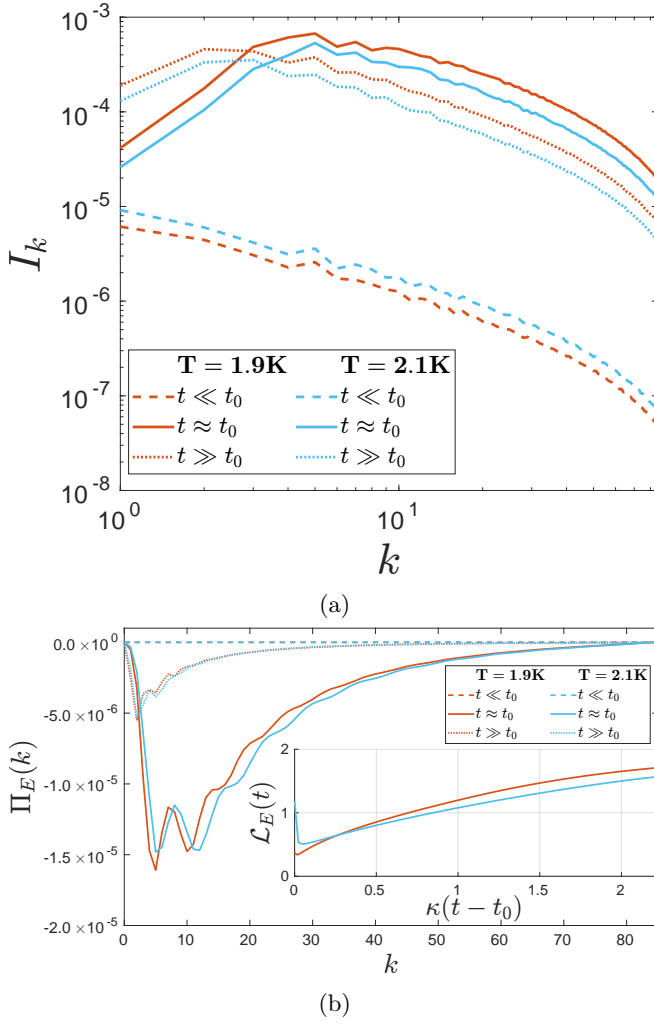The projected modes $\hat{\mathbf{f}}(\mathbf{k})$ are then decomposed into he-

(a)

(b)

FIG. 3: *Top:* Mutual friction injection spectrum I_k.
Bottom: Spectral normal fluid kinetic energy flux Π_E.
Inset: Post reconnection evolution of the integral length
scale $\mathcal{L}_E$.

k, which from Fig. 3a, it is evident that this indeed the
case.

Closing remarks.—   ...

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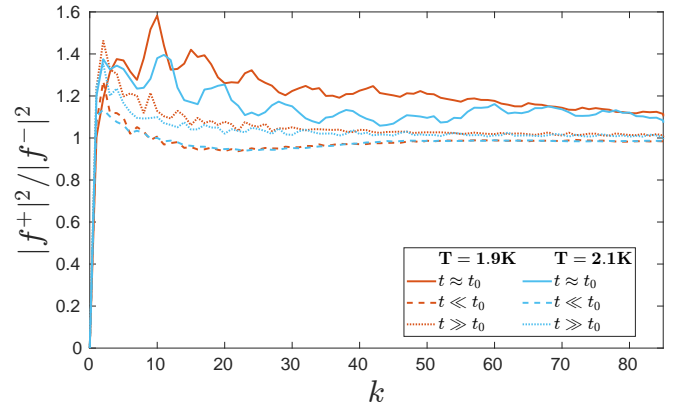
lical modes $\hat{\mathbf{f}}(\mathbf{k}) = f^+\mathbf{h}^+(\mathbf{k}) + f^-\mathbf{h}^-(\mathbf{k})$. The ratio of the
two helical modes $|f^+|^2/|f^-|^2$ are shown in Fig. 4. At
reconnection time t_0, the ratio is much larger, indicating
a clear inbalance that favours the injection of positive
helical modes. In the same way, helicity modes $\hat{\mathcal{H}}(\mathbf{k})$ can
be decomposed,

$$\hat{\mathcal{H}}(\mathbf{k}) = k(E^+(\mathbf{k}) - E^-(\mathbf{k})) = \mathcal{H}^+ + \mathcal{H}^- \qquad (5)$$

where $E^\pm = \frac{1}{2}|\mathbf{v}^\pm(\mathbf{k})|^2$ are the helical energy modes.
The evolution of the ratio $\mathcal{H}^+/\mathcal{H}^-$ is shown in Fig. 5.
The sharp increase at reconnection time t_0 is evidence
of a large influx of positive helical modes as a result of
the vortex reconnection, which is in agreement with the
conditions to facilitate an inverse energy transfer by a
helical injection. Finally, as observed in Ref. [14], it is
necessary for the forcing to cover the entire spectrum of

FIG. 4: The ratio of the projected helical mutual
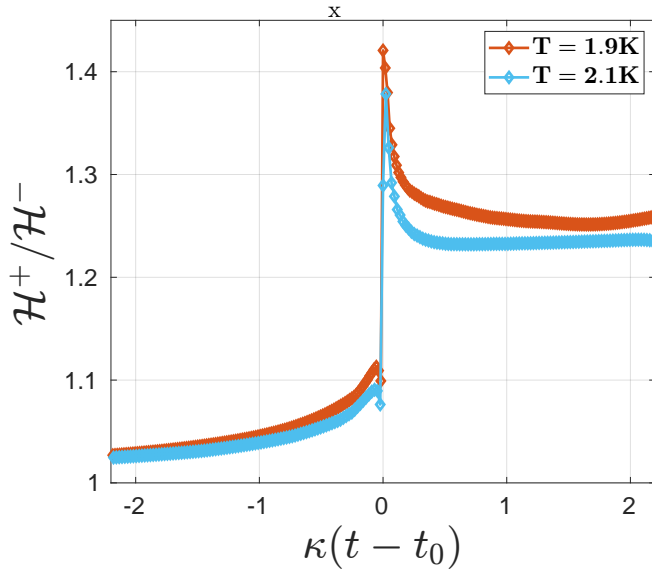friction modes $f^+(k)$ and $f^-(k)$

FIG. 5: Balance of helical helicity modes $\mathcal{H}^+$ and $\mathcal{H}^-$.

[13] Punctuated energy injection in superfluid helium-4 vortex reconnections.

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