Point-by-point response to referee comments

We thank the reviewers for taking the time to read over and give constructive feedback on our paper titled "*Inverse energy transfer in three-dimensional quantum vortex flows*", submitted to PRL. Please see below the point-by-point response to the comments made by the reviewers.

Referee A

For issue 1), authors commented "We omitted the plot of the superfluid helicity from the original manuscript," but I did not understand where or which part was omitted.

Originally we wanted to include the plot of the superfluid helicity in the plot of the normal fluid helicity. We decided against this as in this paper we decide to only focus on the normal fluid aspect. Nevertheless, we have included a sentence in the manuscript emphasizing that the total helicity of the flow, normal fluid plus superfluid, is nor conserved in the reconnection process.

For issue 2), the total sign of helicity for two reconnections is neutral; one is negative and the other is positive. Therefore, total symmetry breaking does not occur. This means that a lot of reconnections do not cause total symmetry breaking or chiral imbalance in quantum turbulence. As a result, the inverse cascade will not happen.

To generalise our findings, we perform the study of the chiral imbalance generated by reconnections occurring in Hopf-links, constituted by two, linked, orthogonal vortex rings, whose initial relative position is varied (see new figures in Supplemental material). This allows to observe reconnections taking place at different angles. Also in this geometrical setting, we observe a strong helicity injection, an increase of the integral scale of the flow, negative spectral fluxes and mutual friction force chirality. We have included the data referring to Hopf-link reconnections in the manuscript and in the Supplemental material. Hence, we can claim, with supporting evidence, that reconnections trigger inverse energy transfer. However, this transfer of energy towards large scales may lead to an inverse cascade only if reconnections are not exactly symmetrycal, *i.e.* (simplifying) if the number of reconnections of a given type is not exactly balanced by the same number of reconnections between vortices of opposite circulation. We suggest that this might be the case when symmetry is broken by external forcing, as in thermal counterflow.

Regarding issue 3), a new normalization does not improve the order of the kinetic energy spectrum and kinetic energy flux, although Fig. 1 with dimensions in the response letter indeed shows a reasonable order.

In the revised manuscript, we have adapted the figures in the paper in order to use a normalization that is temperature independent scaling. The figures now show the same order as one would find when using dimensional units.

The finding of inverse energy transfer by single reconnection is interesting. However, other events cause forward energy transfer of normal fluid, e.g., nonlinear interaction, wake propagation, energy dissipations by mutual friction and viscosity, and so on. Therefore, it is unclear whether the inverse energy cascade of normal fluid is generated by reconnections in quantum turbulence. This stage is premature to recommend to Physical Review Letters.

We think that by including the data regarding Hopf-link reconnections, we generalised our findings beyond a particular type of reconnection. This inverse energy transfer, triggered by classical-like phenomena (chiral imbalance, which as in classical turbulence produces energy flowing towards large scales), is strikingly dissimilar from the direct energy transfer observed in vortex reconnections occurring in classical viscous fluids. Our work shows that vortex reconnections in quantum fluids may possess different characteristics from the ones taking place in classical fluids, depending on the fluid component we focus on: if we analyse superfluid vortices, the approach and separation is akin to classical dynamics (see our recent work, Stasiak et al., PNAS Vol. 122 No. 21 e2426064122 (2025)); if we observe the normal fluid we observe a non-classical energy transfer. Furthermore, this work adds additional elements to the lively discussion in the community regarding similarities and differences between classical and quantum turbulence. We think that together with the suggestion that reconnections may trigger an inverse energy cascade, these are the main findings of our investigation, which are of relevance for a broad audience such as PRL's readers. We have slightly changed the conclusions to emphasise these aspects.

Referee B

We thank the Referee for reading our revised version. We would like to insist, as it seems that is has not yet realized by the Referee, that our work is not about explaining the findings of Polanco and Krstulovic about the inverse cascade observed in that work for high counterflow. We have only mentioned that there might be a connection, but no claim is made. As a matter of fact, the comment is mild and we suggest that further studies are needed, so we open an new possible avenue of research. We have not mentioned such a claim in the abstract and not even in the introduction.

The referee make a claim that on head-on collision of quantum rings. We believe that the Referee is missing an important point. One could perhaps find a specific configuration such that the helicity imbalance is weaken or even suppressed. However, our point is that such is not the generic case, and it will not be relevant for experiments in quantum turbulence. The Referee should notice that in the orthogonal reconnection, discussed in the first version of our paper, the helicity for each pair is zero. In this revised version, we have included around 50 new reconnections of a Hopf-Link, which are topologically very different as the vortex filaments do have a non-zero initial helicity. The simulations of the Hopf-Link reproduce what we have observed with the orthogonal reconnection. We thus conclude, that our fining are robust and generic and physically relevant.