Inverse energy transfer in finite-temperature superfluid vortex reconnections

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Vortex reconnections play a fundamental role in fluids. They increase the complexity of flow and develop small-scale motions. In this work, we show that in superfluids, they can also excite large scales. We numerically show that during a superfluid vortex reconnection energy is injected into the thermal (normal) component of helium II at small length scales, but is transferred nonlinearly to larger length scales, increasing the integral length scale of the normal fluid. We provide an explanation of this inverse energy transfer by decomposing the velocity and the mutual friction (which couples superfluid and normal fluid) into helical modes, showing that the imbalance of homochiral modes results from the punctuated energy and helicity injection during the reconnection. Finally, we discuss the relevance of our findings to the problem of superfluid turbulence.

Turbulence is ubiquitous in the universe. It occurs in systems as large as nebulae of interstellar gas, and as small as clouds of few thousands atoms confined by lasers in the laboratory. Turbulence shapes patterns and properties of fluids of all kinds, from ordinary viscous fluids (Navier-Stokes turbulence[1]) to electrically conducting fluids (magneto-hydrodynamics turbulence [2]) to quantum fluid (quantum turbulence [3, 4]). All turbulent systems are characterised by the existence of a wide range of length scales across which inviscid conserved quantities are transferred without loss in the spirit of the cascade depicted by Richardson [5].

In three-dimensional (3D) classical fluids, turbulence is characterised by a direct cascade: the non-linear dissipationless transfer of kinetic energy from the scale of the large eddies (at which energy is injected) to the smallest length scales at which energy is dissipated into heat [5, 6]. The resulting distribution of energy across length scales is the celebrated Kolmogorov energy spectrum [1, 6].

Confining Navier-Stokes turbulence to two-dimensions (2D) entails fundamentally distinct physics: a dual cascade emerges of energy and enstrophy (mean squared vorticity) [7, 8], the two conserved quantities in ideal two-dimensional flows. While the enstrophy cascade is direct (from large to small scales), the energy cascade is inverse (from small to large scales) [9]. This inverse cascade may favour the generation and persistence of large coherent structures [10].

Remarkably, the same cascade phenomenology is observed in turbulent flows of quantum fluids, *i.e.* fluids at very low temperatures whose physics is dominated by quantum effects. Examples of such fluids are superfluid helium and atomic Bose-Einstein Condensates (BECs). The dynamics of these systems can be successfully depicted in terms of a two-fluid model [11–13] describing the quantum fluid as the mixture of two components, the superfluid component and the thermal (or normal) component, which interact by means of a mutual friction force [14–16]. The superfluid component flows without viscosity and vanishing entropy; its vorticity is confined to effectively one-dimensional vortex filaments of atomic core thickness (called quantum vortices or vortex lines), around which the circulation of the velocity is quantised. In BECs the thermal component forms a ballistic gas, whereas in superfluid ^{4}He it can be described as a classical viscous fluid. Despite these significant differences with respect to ordinary fluids, the direct kinetic energy cascade has indeed been observed in three-dimensional superfluid turbulence [17–22]. Evidence of this direct cascade has been found also in three-dimensional turbulent BECs [23]. Similarly to 2D classical turbulence, an inverse energy cascade characterises two-dimensional BECs, as shown in theoretical [24–27] and experimental [28, 29] studies.

In turbulent systems, the type and the number of sign-defined ideal invariants determine the direction of cascades. Indeed, the famous Fjørtoft argument [30] predicts the existence of an inverse energy cascade in 2D classical turbulence. It also predicts an inverse particle and a direct energy cascade for 3D wave turbulent BECs, as recently addressed theoretically [31]. In 3D classical fluids, helicity, which is also an inviscid invariant, is not sign-defined and thus only a direct energy cascade is possible. However, recent studies have demonstrated that the direction of the energy cascade may be inverted

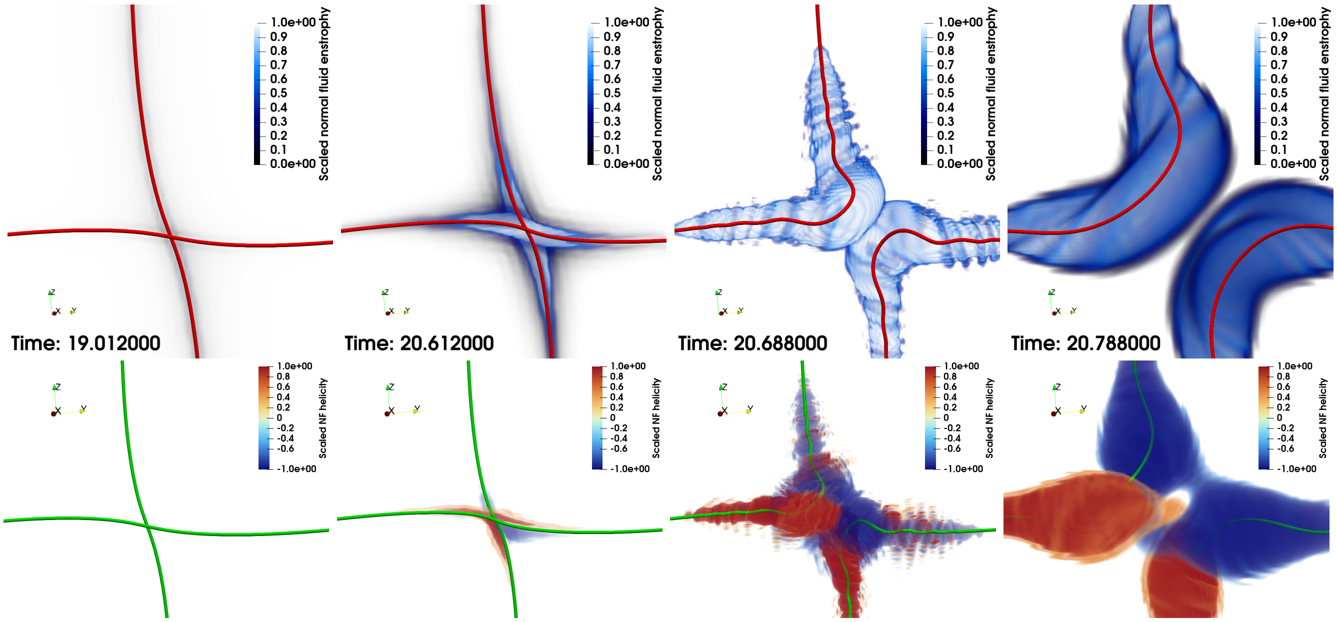

FIG. 1: Three-dimensional rendering of an orthogonal vortex configuration undergoing a vortex reconnection at $T = 1.9K$. The red tubes represent the superfluid vortex lines (the tubes' radii have been greatly exaggerated for visual purposes), and the blue volume rendering represents the scaled normal fluid enstrophy $\boldsymbol{\omega}^2/\boldsymbol{\omega}_{max}^2$. In the third panel, note the Kelvin waves on the superfluid vortices.

by artificially controlling the chirality of the flow, *i.e.* the balance between positive or negative helical modes [32]. Indeed, by restricting the non-linear energy transfer to homochiral interactions via a suitable decimation of the Navier-Stokes equation [33, 34], by controlling the weight of homochiral interactions [35], or by the external injection of positive helical modes at all length scales [36], inverse energy cascades have been observed in three-dimensional turbulence of classical fluids. In brief, when the flow is synthetically designed to have an enhanced chirality, an inverse energy cascade can observed.

In this work, we unveil a similar dynamics occurring in superfluid helium (^{4}He) as a result of vortex reconnections. Reconnections occur continuously in turbulence: they take place when two vortex lines collide and recombine, exchanging heads and tails, altering the overall topology of the flow [37–43]. We show that the mutual friction force arising from the vortex reconnection is chiral, injecting in the normal fluid prevalently helicity of a given sign. Thus, as a consequence of vortex reconnections, we observe an increase of the chiral imbalance of the quantum fluid, producing a transfer of kinetic energy from small to large scales, similarly to the phenomenology observed in 3D helical-decimated classical flows. Unlike classical fluids, such a chiral imbalance arises naturally as physical process in the normal fluid.

To model superfluid helium dynamics, we employ the recently developed FOUCAULT model [44]. In this approach, superfluid vortex lines are parametrized as one-dimensional space curves $\mathbf{s}(\xi,t)$, ξ and t being arclength and time respectively, exploiting the large separation of length scales between the vortex core radius, the Lagrangian discretisation along the vortex lines $\Delta\xi$, and the average radius of curvature R_c of the vortex lines. The vortex lines evolve according to the following equation of motion:

$$\dot{\mathbf{s}}(\xi,t) = \mathbf{v}_s + \frac{\beta}{1+\beta}\left[\mathbf{v}_{ns}\cdot\mathbf{s}'\right]\mathbf{s}' + \beta\mathbf{s}'\times\mathbf{v}_{ns} + \beta'\mathbf{s}'\times\left[\mathbf{s}'\times\mathbf{v}_{ns}\right], \tag{1}$$

where $\dot{\mathbf{s}} = \partial\mathbf{s}/\partial t$, $\mathbf{s}' = \partial\mathbf{s}/\partial\xi$ is the unit tangent vector, $\mathbf{v}_n$ and $\mathbf{v}_s$ are the normal fluid and superfluid velocities at $\mathbf{s}$, $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$, and β, β' are temperature and Reynolds number dependent mutual friction coefficients [44]. The calculation of the superfluid velocity $\mathbf{v}_s$ is performed via the computation of the Biot-Savart integral de-singularised with standard techniques (see Supplementary Material [45]). The normal fluid is described classically using the incompressible ($\nabla\cdot\mathbf{v}_n = 0$) Navier-Stokes equation

$$\frac{\partial\mathbf{v}_n}{\partial t} + (\mathbf{v}_n\cdot\nabla)\mathbf{v}_n = -\frac{1}{\rho}\nabla p + \nu_n\nabla^2\mathbf{v}_n + \frac{\mathbf{F}_{ns}}{\rho_n}\ , \tag{2}$$

where ρ_n and ρ_s are the normal fluid and superfluid densities, $\rho = \rho_n + \rho_s$, p is the pressure, ν_n is the kinematic viscosity of the normal fluid, and the mutual friction force per unit volume, $\mathbf{F}_{ns}$, is the line integral of the mutual friction force per unit length, $\mathbf{f}_{ns}$ [45]:

$$\mathbf{F}_{ns}(\mathbf{x}) = \oint_{\mathcal{C}} \delta(\mathbf{x}-\mathbf{s})\mathbf{f}_{ns}(\mathbf{s})d\xi, \tag{3}$$

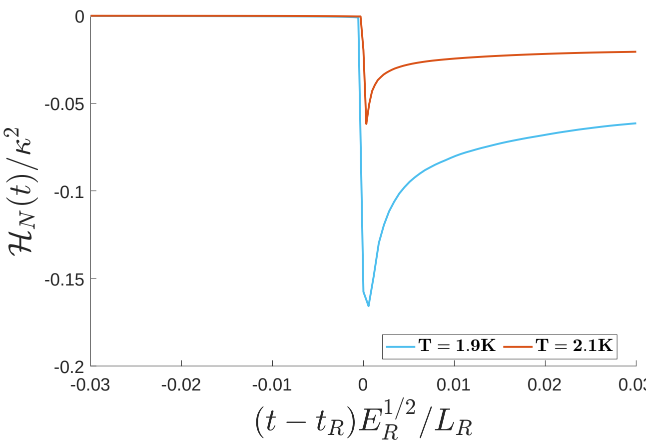

FIG. 2: Temporal evolution of the normal fluid helicity computed over half of the box to consider only one pair of vortices (gray zone). **[GK: in the text we call it $\mathcal{H}$ and in the figure $\mathcal{H}_N$] [GK: are you sure is correct to divide by κ? . In the code, the total helicity is computed per unit of volume (as usual in fluid mechanics) ie $[\mathcal{H}] = [\omega v] = L/T^2$]**

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$\mathcal{C}$ representing the entire vortex configuration. The regularisation of mutual friction is performed using a physically self-consistent scheme [44]. We consider a periodical box of size 2π (so that wavevectors are integers).

To study the reconnection dynamics, we consider two pairs of initially orthogonal vortices (where the corresponding vortices of each pair have opposite circulation in order to preserve periodicity along the boundaries) at two temperatures, $T = 1.9K$ and $T = 2.1K$. The vortex pairs are separated by the distance D_ℓ; each vortex within each pair is initially at distance d_ℓ to the other vortex, such that $d_\ell \ll D_\ell$ to ensures that the dynamics in the vicinity of the reconnection is dominated by local interactions, and that the far-field contribution from the other vortex pair is negligible.

The evolution of the vortex reconnection of a single pair is reported in Fig. 1. The first row shows the reconnecting superfluid vortices (in red) accompanied by normal fluid structures generated by mutual friction, here displayed as enstrophy rendering $\omega(\mathbf{x})^2 = |\nabla\times\mathbf{v}_n|^2$. Such structures are the signature of the violent irreversible energy transfers in vortex reconnections reported in [46]. The second row shows the rendering of the local helicity $H(\mathbf{x}) = \mathbf{v}_n \cdot (\nabla \times \mathbf{v}_n)$, where we observe a clear local helicity production, with an abrupt change of sign due to the rearrangement of the vortex topology. Remarkably, during reconnection there is a net sudden normal fluid helicity production, as shown in Fig. 2. We will come back to this finding later.

[Notes of CFB about today's meeting: Some notation needs fixing: The total helicity $\mathcal{H} = \int H dV$ should be defined in the text in terms of the helicity density $H(\mathbf{x})$ defined in the first column of page 3. In the caption of Fig.2 we can say that for superfluid helium it is proper to make the helicity dimensioness in terms of κ^2. The total energy $E = (1/V)\int v^2/2dV = \int_0^\infty E(k)dk$ should be defined together with the energy spectrum $E(k)$, as this allows to explain what is E_R (the energy at reconnection time) as well clarifying the energy spectrum. The energy spectrum is sometimes given by $E(k)$ and sometimes by E_k: we should be more consistent. The reconnection time t_R at some places is still called t_0, again we need consistency. The green text can be removed for clarity. We do not need to define the transfer function any longer. Finally we need a stronger conclusion]

We now focus on the time evolution of the normal fluid energy spectrum $E(k)$ (where k is the magnitude of the three-dimensional wavenumber), displayed in Fig. 3. It clearly emerges that, during the reconnection,

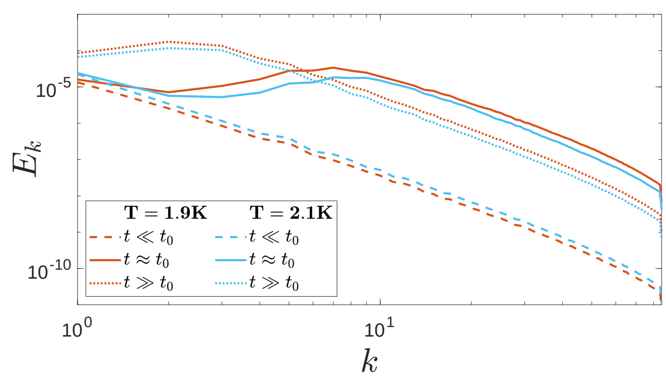

FIG. 3: a) Normal fluid kinetic energy spectrum $E(k)$ before reconnection (dashed lines), at reconnection (solid lines) and after reconnection (dotted lines) for $T = 1.9K$ (red) and $T = 2.1K$ (blue). **[GK: now, as we are not showing the velocity, we could put figure 4 together with figure 3.]**

energy is predominantly injected into the normal fluid at intermediate and small length scales. For $k > 5$ in correspondence of the reconnection time t_0, we observe a significant increase of the normal fluid energy spectral density: $E(k, t \approx t_0)/E(k, t \ll t_0) \approx 10^2$. In the post-reconnection regime, we simultaneously observe a small decrease of the spectrum at intermediate and small scales ($k > 5$) and an increase at large scales, suggesting the existence of a mechanism by which energy generated at small length scales is transferred to larger scales. To shed light on this mechanism, as customary for turbulent flows, we analyse the energy flux

$$\Pi(k) = \int_{|\mathbf{p}|<k} \hat{\mathbf{v}}_n^* \cdot \widehat{(\mathbf{v}_n \cdot \nabla)\mathbf{v}_n} d\mathbf{p} + c.c. \qquad (4)$$

We observe that $\Pi(k) < 0$ for all k during and after reconnection; we also observe that, near the time of

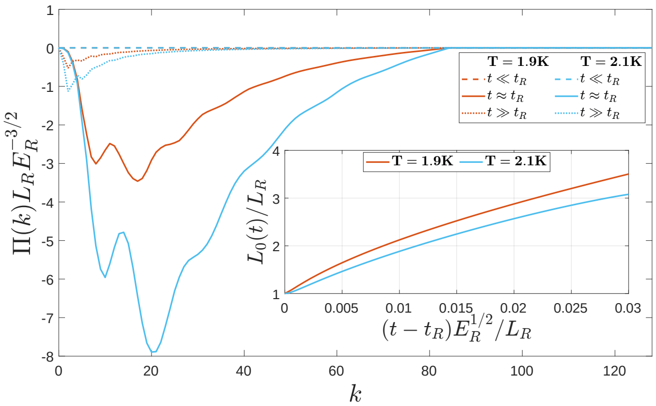

FIG. 4: *Top:* Mutual friction injection spectrum, I_k. *Bottom:* Spectral normal fluid kinetic energy flux, Π_E. It is normalised by using the integral scale and the normal fluid energy at reconnection. *Inset:* Post reconnection evolution of the integral length scale, L_0. Times and temperatures are labelled as in Fig. 3

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reconnection, the peak value of $|\Pi(k)|$ is in the range $5 < k < 15$. The negative sign of $\Pi(k)$ is evidence of a flux of kinetic energy from small to large scales, exciting larger and larger scales.This behaviour is quantified by the evolution of the integral length scale L_0, defined as

$$L_0 = \frac{\pi}{2} \int_0^\infty \frac{E(k)}{k} dk \Big/ \int_0^\infty E(k) dk. \tag{5}$$

The inset of Fig. 4 shows that L_0 indeed increases steadily in the post-reconnection regime. Note that times have been normalised by the largest eddy-turnover-time at the reconnection event, evidencing it quick evolution.

To explain the inverse energy transfer shown in Fig. 3, we look whether the reconnection triggers a chirality imbalance. We decompose the incompressible Fourier modes of the normal fluid velocity into helical modes [47]:

$$\hat{\mathbf{v}}_n(\mathbf{k}) = \hat{\mathbf{v}}_n^+(\mathbf{k}) + \hat{\mathbf{v}}_n^-(\mathbf{k}) = v_n^+(\mathbf{k})\mathbf{h}^+(\mathbf{k}) + v_n^-(\mathbf{k})\mathbf{h}^-(\mathbf{k}), \tag{6}$$

where $\mathbf{h}^\pm(\mathbf{k})$ are the two eigenvectors of the curl operator, *i.e.* $i\mathbf{k} \times \mathbf{h}^\pm(\mathbf{k}) = \pm k \mathbf{h}^\pm(\mathbf{k})$. Similarly, we decompose the Fourier of the transverse mutual friction force: $\hat{\mathbf{F}}_{ns}^\perp(\mathbf{k}) = f^+(\mathbf{k})\mathbf{h}^+ + f^-(\mathbf{k})\mathbf{h}^-$ (the Fourier modes of $\mathbf{F}_{ns}$ parallel to the wavemumber $\mathbf{k}$ do not play any role in the time evolution of $\mathbf{v}_n$ due to the incompressible constraint). Finally, the helical decomposition naturally allow us decompose the total helicity as $\mathcal{H} = \mathcal{H}^+ - \mathcal{H}^-$.

A chiral imbalance occurs if the mutual friction force is helical, *i.e.* if the ratio $|f^+|^2/|f^-|^2 \neq 1$, with $|f^\pm|^2$ the total squared norm the component. In Fig. 5, we show the temporal evolution of $|f^+|^2/|f^-|^2$, for both temperatures. It is apparent that during and after the reconnection, the mutual friction force is chiral, injecting more negative helicity than positive helicity.

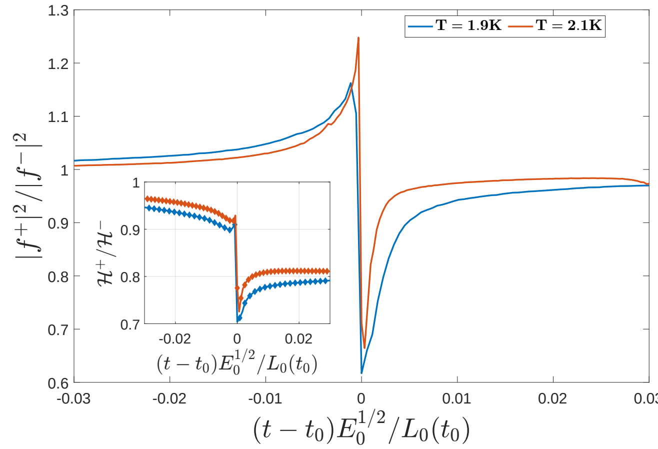

FIG. 5: Temporal evolution of projected mutual friction force components $f^\pm$/ Inset: temporal evolution of total helical components.

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As a result, the ratio $\mathcal{H}^+/\mathcal{H}^-$ (reported in the inset of Fig. 5) decreases significantly at reconnection and remains smaller than unity even at later times, indicating that the flow is chiral. We conclude that the reconnection triggers indeed a chiral imbalance.

In conclusion, the reconnection of quantum vortices in the two-fluid regime ($T \gtrsim 1.5$K) not only injects punctuated energy in the normal fluid [46], but also triggers in the normal fluid a transfer of kinetic energy towards the large scales. This inverse energy transfer arises from the fact that the mutual friction force (injecting energy and helicity in the normal fluid) is helical, as Kelvin waves develop on the vortices. The helical character of the mutual friction produces a chiral imbalance in the normal fluid, driving this inverse cascade as previously observed in turbulent Navier-Stokes flows [33, 36]. Our findings have profound implications **[GK: which ones?]** for the nature of finite temperature superfluid turbulence and motivate a detailed studied of fully coupled quantum turbulence to understand how energy transfer and dissipation is augmented by vortex reconnections.

[Can a flow sustain a constant injection with multiple reconnections? Maybe by ring injection? From last peter's plots, the injection last for about κ.1 (and much less the strong imbalance). How is this time compared with typical reconnection time in experiments? What would be the ultimate turbulent state/spectrum?]

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