Emerging inverse energy transfer in finite-temperature superfluid vortex reconnections

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Introduction.— Turbulence is a fundamental phenomenon that governs fluid motion across a vast range of scales, influencing everything from atmospheric dynamics to industrial processes. Understanding turbulence and the mechanisms of energy cascades is crucial for predicting flow behavior, optimizing performance, and advancing technologies in fields such as aerospace engineering, meteorology, and oceanography. In three-dimensional classical fluid dynamics, turbulence is often characterised by a forward cascade - a transfer of an inviscid, conserved quantity (typically energy) from large eddies to increasingly smaller eddies by nonlinear interactions of fluid elements, until dissipation occurs at the smallest length scales [1]. The distribution of energy across length scales in fluid dynamics is often referred to as the Kolmogorov spectrum, where the energy spectrum $E(k)$ scales according to $\sim k^{-5/3}$ at the intermediate length scales.

I would enlarge quite a lot the introductory part, in order to make more emphasis on the direct-inverse cascades and in general the fluid dynamics importance of our work. I would first of all say what a cascade is, and then start with 3D classical turbulence and say that we have 1 inviscid conserved quantity (energy) and hence a cascade of this quantity, in this case from large to small scales, and cite Richardson's cascade and then explain that this leads to the Kolmogorov spectrum. Then I would describe 2D classical turbulence, hence why we have a double cascade and its characteristics. At this point I would switch to quantum turbulence and say that surprisingly, despite the significant differences in the type of fluids, we recover the same behaviour (quickly explain the peculiarities of quantum fluids). Describe when and how we find 3d turbulence with direct energy cascade and cite also more recent experimental evidence (eg the work by people in Grenoble, Salort, Roche etc). The same for two-dimensional quantum turbulence. In BEC you can cite Bradley PRX 2012 and Simula PRL 2014 for the theory (together with Reeves PRL 2013), and then Johnstone Science 2019 and Gauthier Science 2019 for the experimental counterpart.

At this point you can describe, as you already do, that actually recently inverse cascades have been observed in 3d classical turbulence because of carefully numerical procedures employed (decimation, forcing like Plunian etc) based on the generation of a chiral imbalance in the flow. You then say that we show that this phenomena, ie inverse cascade by chiral imbalance is actually naturally oberved in superfluids, in reconnections (you have to explain what a reconnection is). And then you continue with the last part of the Introduction which you already have (a sentence appears to be truncated).

Homogeneous isotropic turbulence (HIT) in three-dimensional classical fluid dynamics is often characterised by a forward energy cascade with a Kolmogorov-like scaling $\sim k^{-5/3}$ in an inertial range, where typically energy is injected at the largest length scales [2]. Similar scaling laws have been found in mechanically [3, 4] and thermally [5] driven superfluid turbulence, as well as a unique ultra-cold regime with $\sim k^{-1}$ energy scaling, typically arising in the absence of a strong energy injection. In such cases, energy is transferred via non-local interactions from the large to small length scales.

In certain cases, like in two-dimensional turbulence [6], geophysical flows [7] and shallow fluid layers [8], a reversal of energy flux can be identified, for which energy is injected at small length scales, and is transferred by non-local interactions to large length scales. An inverse energy cascade in two-dimensional turbulence is well known in both classical and quantum fluid dynamics [9], and in recent years the role of helicity has been highlighted in the generation of inverse energy transfers for three-dimensional turbulence, without the constraint of quasi-two-dimensionality [10, 11]. Statistically stationary inverse energy transfer has been sustained in a three-dimensional system by an imbalance of non-local homochiral modes through decimating certain triadic interactions [11], and more recently by a strong, external injection of positive helical modes at all length scales [12].

In principle, to facilitate an inverse transfer of energy in a classical fluid requires a precise construction of conditions. In this Letter, we aim to show that an inverse energy transfer arises naturally in the normal fluid phase of superfluid helium-4 as a result of superfluid vortex reconnections, an intrinsic property of all superfluids. We will show that energy is injected at the small length scales immediately in the post reconnection regimes and moves towards larger length scales, increasing the integral length scale $\mathcal{L}_E$ of the normal fluid component. We provide an explanation of the inverse energy transfer by decomposing the velocity and mutual friction fields (the governing interaction force between the two-fluid components) into helical modes, showing that the imbalance of homochiral modes resulting from the punctuated energy and helicity injection during the reconnection process. Finally, we discuss the relevance of our findings to the broader field of transitions to superfluid turbulence.

Main results.— Now that we have decided to write a more fluid dynamics paper, you need to explain better the method as if the reader does not have an idea of what a superlfuid is. If not explicitly the Biot-Savart law, which is well known in fluid dynamics and which can be put in the supplementary, the vortex equation of motion is needed.

In this Letter, we use the Schwarz model [13] to evolve vortex filaments $\mathbf{s}(\xi, t)$, where ξ is the natural parametrisation of vortex lines, also known as the arclength. The normal fluid is coupled via the mutual friction force $\mathbf{f}_{ns}$ in a self-consistent manner using a recently developed technique in Ref. [14]. here you need to specify both the vortex equation of motion and the Navier-Stokes modified equation. As it is a fluid dynamics paper, I would specify that the mutual friction is the line integral of the mutual friction per unit length and write that the regularisation is performed using a physically self-consistent scheme (Casciola's group in Rome). Normally we don't specify it because in the superfluid community this is not really appreciated. Once this is done, then we start with the initial condistions. We set two pairs of orthogonal vortices, where corresponding vortices in each pair have oppposite circulation in order to preserve superfluid periodicity along the boundaries, and we consider two distinct temperatures, $T = 1.9K$ and $T = 2.1K$. Vortex pairs are seperated by distance D_ℓ, and each vortex within each pair is seperated by distance d_ℓ, such that $d_\ell \ll D_\ell$. The seperation of scales ensures that the dynamics in the vicinity of the reconnection are dominated by local interactions, and that far-reaching contributions from the other vortex pair are neglible.

The evolution of the vortex reconnection of a single pair is reported in Fig. 1 .

I would move this part on the normal fluid solver before the initial condition description.

The normal fluid is initally quiescent, and evolves according to a classical Navier-Stokes equation

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\mathbf{v}_n = -\nabla\left(\frac{p}{\rho_n}\right) + \nu_n \nabla^2 \mathbf{v}_n + \frac{\mathbf{F}_{ns}}{\rho_n} \quad (1)$$

where $\mathbf{v}_n$ denotes the normal fluid velocity, p is the pressure, ρ_n is the normal fluid density, ν_n is the kinematic viscosity and $\mathbf{F}_{ns}$ is the mutual friction coupling, arising from the relative motion between the normal fluid and superfluid vortices here I would express $\mathbf{F}_{ns}$ as a linear integral, report its expression and details in the Supplementary, say a few words on its regularisation as pointed out earlier and then continue with saying that $\mathbf{F}_{ns} \propto |\mathbf{v}_l - \mathbf{v}_n|$ $\mathbf{F}_{ns} \propto |\mathbf{v}_l - \mathbf{v}_n|$ where $\mathbf{v}_l$ is the velocity of vortex lines. I would not include the next sentence, might raise unnecessary questions from the referees. Maybe some details on why the mutual friction exists (scattering of rotons) can be reported in the supplementary..The generation of the force is highly localised in space, at microscopic length scales on the order of the vortex core radius $\sim a_0$, and $\Delta x \gg a_0$, where Δx is a typical grid discretisation.

I would remove the part on vorticity as we don't include its spectrum anymore and I would explain more detail the injection mechanism: reconnection implies cusp which then relaxes in structures with small radii of curvature. As $F_{ns} \propto |\mathbf{v}_l - \mathbf{v}| \approx |\mathbf{v}_l| \propto 1/R_c$, the scale of energy injection is small as R_c is small. This can be seen in Figure 2. As time goes by, the friction damps the smallest perturbations, and this is why the injection spectrum moves towards large scales. We also see an inverse transfer of energy, which does not arise entirely because the injection moves towards large scales. In fact if we compute the energy flux ...

Abrupt changes in the topology of vortex lines, such as during vortex reconnections, form highly curved cusps and strongly inject vorticity (see Fig. 1), since $|\mathbf{v}_l - \mathbf{v}| \approx |\mathbf{v}_l| \propto \zeta$ in the first approximation, where ζ is the curvature of vortex lines.

In this way, energy is injected into the normal fluid at the small length scales in a natural way in the absence of external property, an intrinsic and unique feature of quantum fluids. The injection of energy is clear in the kinetic energy spectrum E_k which is shown in in

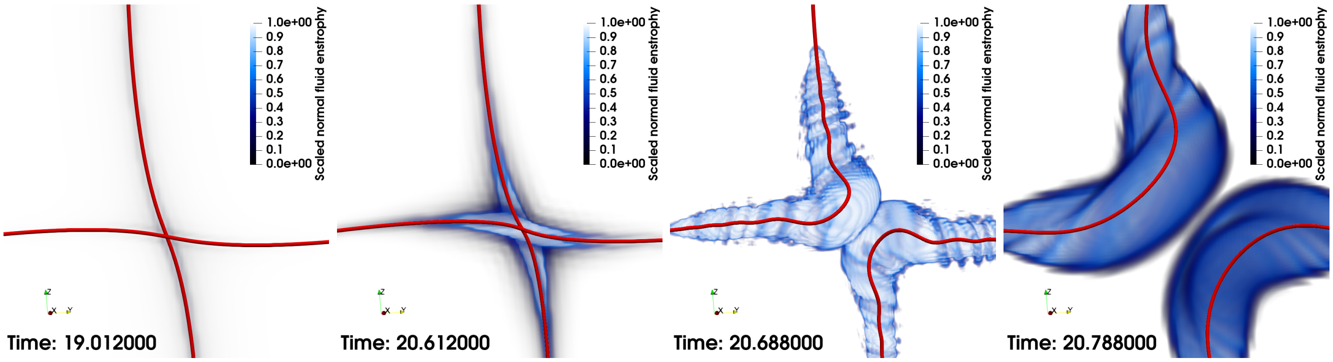

FIG. 1: 3D rendering of an orthogonal vortex configuration, undergoing a vortex reconnection. The red tube represents a superfluid vortex, where the radius has been greatly exaggerated for visual purposes, and the blue volume rendering represents the scaled normal fluid enstrophy $\boldsymbol{\omega}^2/\boldsymbol{\omega}^2_{max}$.

Fig. 2, at wavenumbers $k > 10$ a vast increase can be observed $E_k(t \approx t_0)/E_k(t \ll t_0) \sim 10^2$, evidence of very strong energy injection. Remarkably, in the post reconnection regime E_k appears to increase when k is small, and decay at large values of k, suggesting a possible mechanism by which energy generated at small length scales is transferred to larger scales. The energy spectrum E_k evolves according to the balance

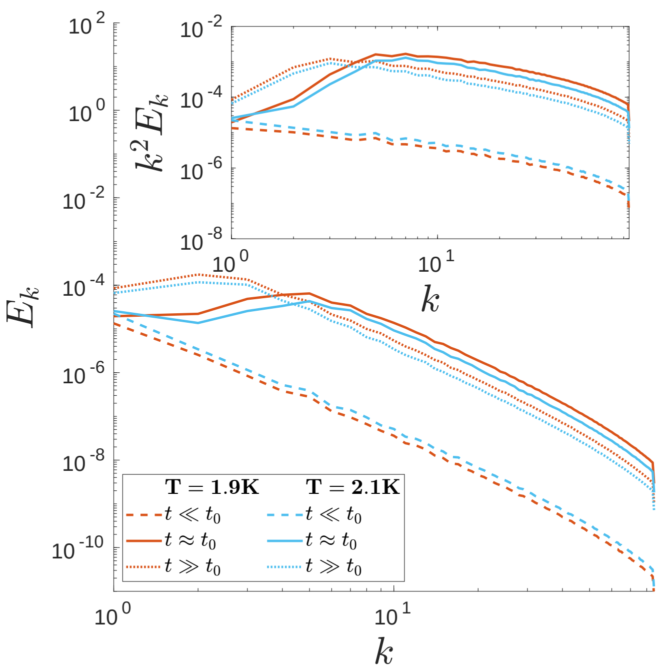

FIG. 2: Normal fluid kinetic energy spectrum E_k before reconnection (dashed lines), at reconnection (solid lines) and after reconnection (dotted lines) for $T = 1.9K$ and $T = 2.1K$. *Inset:* Dissipation spectrum $D_k/2\nu_n = k^2 E_k$ at the same snapshots in time.

sources/sinks and internal transfers,

$$\frac{\partial E_k}{\partial t} = T_k - D_k + I_k \tag{2}$$

where T_k is the spectral kinetic energy transfer function, $D_k = 2\nu_n k^2 E_k$ is the dissipation spectrum and $I_k = \mathrm{Re}(\hat{\mathbf{F}}_{ns}(\mathbf{k}) \cdot \hat{\mathbf{v}}^*(\mathbf{k}))$ is the injection spectrum due to the mutual friction $\mathbf{F}_{ns}$. The injection spectrum I_k as shown in Fig. 3a again shows that the injection of energy covers primarily the high k range, but is not limited to a thin band of injection. In fact, the energy flux $\Pi_E(k) = \int_k^\infty T_k dk$ as shown in Fig. 3b asserts a hallmark feature of inverse energy cascades, typical in 2D isotropic turbulence. A flux of $\Pi_E(k) < 0$ implies a negative flux of energy from small to large k, i.e. an inverse transfer of energy. The effect of kinetic energy transferral to large length scales results in the creation of large scale structures, evident in the evolution of the integral length scale $\mathcal{L}_E$, where

$$\mathcal{L}_E = \frac{\pi}{2\langle \mathbf{v}'^2 \rangle} \int_0^\infty \frac{E_k}{k} dk \tag{3}$$

and $\langle \mathbf{v}'^2 \rangle$ represents the turbulent kinetic energy. In the inset of Fig. 3b, $\mathcal{L}_E$ steadily increases in the post-reconnection region, implying a generation of large scale structure.

In order to explain the inverse energy transfer mechanism, we look for a chirality unbalance as a result of the reconnection process. we decompose the force (explain how, as you do in the current version) and we indeed observe that the force is chiral (maybe the force ratio could be poltted in log/log plot?), which results in changing the chirality of the flow as seen in the inset of Figure ... we follow recent work in classical fluids outlined in Refs. [11, 12], where it is even possible to sustain an inverse energy cascade under a helical forcing applied at all length scales. Typically, velocity coefficients $\hat{\mathbf{v}}(\mathbf{k})$ can be decomposed into their helical modes, where $\hat{\mathbf{v}}(\mathbf{k}) = v^+(\mathbf{k})\mathbf{h}^+(\mathbf{k}) + v^-(\mathbf{k})\mathbf{h}^-(\mathbf{k})$ and

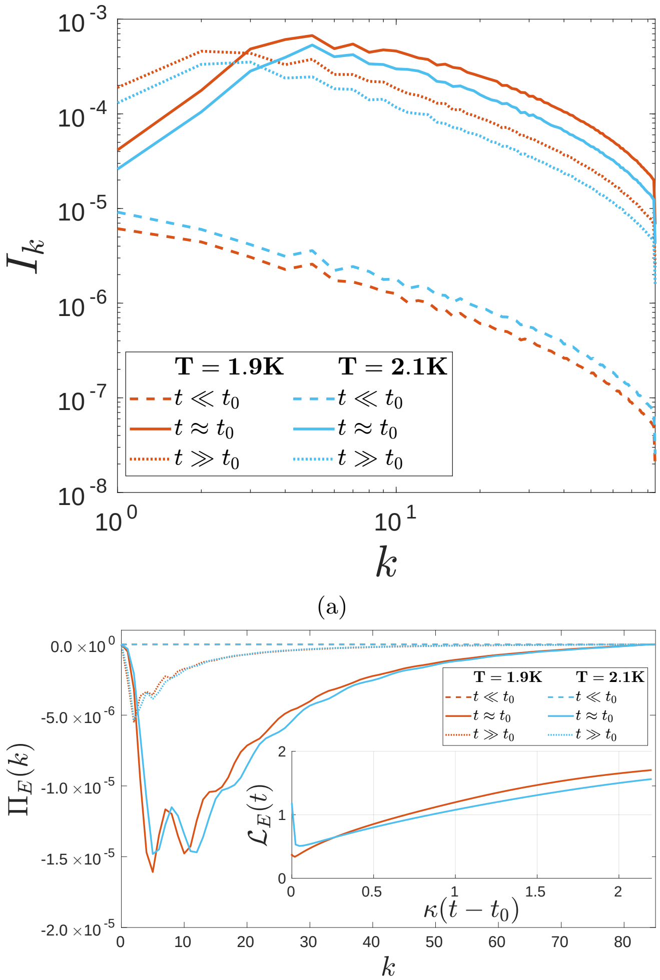

FIG. 3: *Top:* Mutual friction injection spectrum I_k. *Bottom:* Spectral normal fluid kinetic energy flux Π_E. *Inset:* Post reconnection evolution of the integral length scale $\mathcal{L}_E$.

satisfies $\mathbf{k} \cdot \hat{\mathbf{v}}(\mathbf{k}) = 0$, where $\mathbf{v}^{\pm}$ are complex scalars and $\mathbf{h}^{\pm}(\mathbf{k})$ are the two eigenvectors of the curl operator, such that $i\mathbf{k} \times \mathbf{h}^{\pm}(\mathbf{k}) = \pm k\mathbf{h}^{\pm}(\mathbf{k})$. To explain the inverse energy transfer in terms of helical modes, we show that in fact the driving force, which in our case is a punctuated burst due to superfluid vortex reconnections, is of a helical nature. The mutual friction modes $\hat{\mathbf{F}}_{ns}(\mathbf{k})$ are not incompressible, and so we take the projection of the modes orthogonal to the wavenumber $\mathbf{k}$,

$$\hat{\mathbf{f}}(\mathbf{k}) = \hat{\mathbf{F}}_{ns}(\mathbf{k}) - \frac{(\hat{\mathbf{F}}_{ns}(\mathbf{k}) \cdot \mathbf{k})}{k^2}\mathbf{k}. \tag{4}$$

The projected modes $\hat{\mathbf{f}}(\mathbf{k})$ are then decomposed into helical modes $\hat{\mathbf{f}}(\mathbf{k}) = f^+\mathbf{h}^+(\mathbf{k}) + f^-\mathbf{h}^-(\mathbf{k})$. The ratio of the two helical modes $|f^+|^2/|f^-|^2$ are shown in Fig. 4. At reconnection time t_0, the ratio is much larger, indicating a clear inbalance that favours the injection of positive helical modes. In the same way, helicity modes $\hat{\mathcal{H}}(\mathbf{k})$ can be decomposed,

$$\hat{\mathcal{H}}(\mathbf{k}) = k(E^+(\mathbf{k}) - E^-(\mathbf{k})) = \mathcal{H}^+ + \mathcal{H}^- \tag{5}$$

where $E^{\pm} = \frac{1}{2}|\mathbf{v}^{\pm}(\mathbf{k})|^2$ are the helical energy modes. The evolution of the ratio $\mathcal{H}^+/\mathcal{H}^-$ is shown in Fig. 5. The sharp increase at reconnection time t_0 is evidence of a large influx of positive helical modes as a result of the vortex reconnection, which is in agreement with the conditions to facilitate an inverse energy transfer by a helical injection. Finally, as observed in Ref. [12], it is necessary for the forcing to cover the entire spectrum of k, which from Fig. 3a, it is evident that this indeed the case.

Closing remarks.— ...

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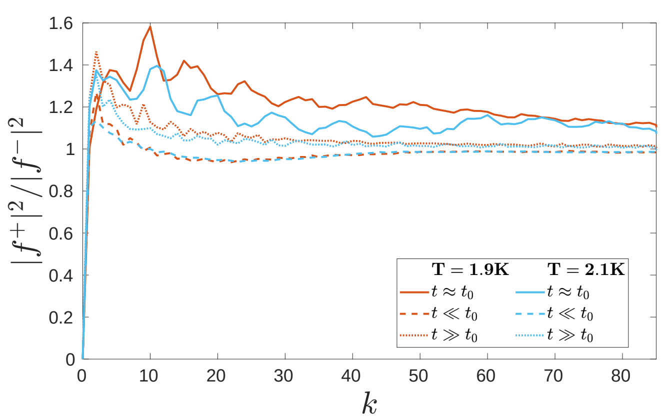

FIG. 4: The ratio of the projected helical mutual friction modes $f^+(k)$ and $f^-(k)$

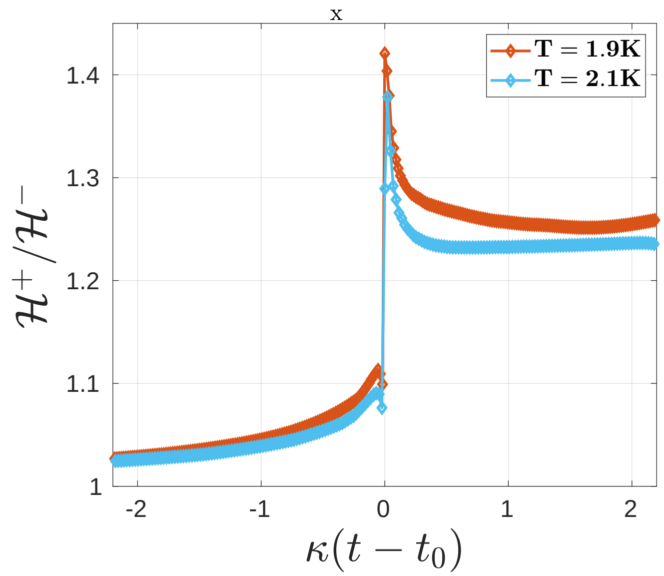

FIG. 5: Balance of helical helicity modes $\mathcal{H}^+$ and $\mathcal{H}^-$.

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