

PSTAT 120A, Summer 2022: Practice Problems 5*Week 2**Conceptual Review*

- (a) Why is a function of a random variable also a random variable?
- (b) If $Y := g(X)$ where the distribution of X is known, must we first find $f_Y(y)$ before computing $\mathbb{E}[Y]$?
- (c) How do transformations of discrete random variables work?
- (d) Will transformations of discrete random variables always be discrete? Will transformations of continuous random variables always be continuous?

Problem 1: Two Interesting Results

- (a) If $X \sim \text{Exp}(\lambda)$ and $Y := cX$ for some fixed constant $c > 0$, show that $Y \sim \text{Exp}(\lambda/c)$. For practice, derive the result in two ways: using the c.d.f. method, and using the Change of Variable formula.
- (b) If $X \sim \text{Gamma}(r, \lambda)$ and $Y := cX$ for some fixed constant $c > 0$, identify the distribution of Y **by name**, taking care to include any/all relevant parameter(s).

Problem 2: Raise The Roof- er, Ceiling!

Let $X \sim \text{Exp}(\lambda)$, and define $Y := \lceil X \rceil$. Identify the distribution of Y **by name**, taking care to include any/all relevant parameter(s). Recall that

$\lceil x \rceil :=$ smallest integer larger than or equal to x

so, for instance, $\lceil \pi \rceil = 4$.

Hint: Identify appropriate values for a and b such that

$$\{\lceil X \rceil = y\} = \{a < X \leq b\}$$

Extra Problems

Problem 3: Rounding

The true concentration of radiation in a particular room (measured in counts per second) is uniformly distributed on the interval $[0, 10]$. A Geiger counter is used to measure the radiation in this room, however it is very crude and only displays measurements rounded to the nearest integer value. Let X denote the true amount of radiation in the room, and Y denote the amount of radiation displayed on the Geiger counter.

- (a) Is X discrete or continuous? What about Y ?
- (b) Is it correct to say that Y is uniformly distributed on S_Y , the state space of Y ?
- (c) Now, find the p.m.f. of Y .

Problem 4: Transformations

(CB, 2.1)

In each of the following find the p.d.f. of Y . Show that the p.d.f. integrates to 1.

- (a) $Y = X^3$ and $f_X(x) = 42x^5(1-x)$, $0 < x < 1$
- (b) $Y = 4X + 3$ and $f_X(x) = 7e^{-7x}$, $0 < x < \infty$
- (c) $Y = X^2$ and $f_X(x) = 30x^2(1-x)^2$, $0 < x < 1$

Problem 5: Square-y Situation

Suppose $X \sim \text{Unif}[-1, 2]$ and $Y := X^2$.

- (a) Compute $\mathbb{E}[Y]$. **Hint:** If you remember certain properties about the uniform distribution, you can do this without computing any integrals.
- (b) Find $f_Y(y)$, the probability density function (p.d.f.) of Y .