HOMEWORK 1

PSTAT 120A: Summer 2022

Due: 11:59pm on Friday, June 28

Instructor: Ethan P. Marzban

Instructions:

- Please submit your work to Gradescope by no later than the due date posted above.
- Be sure to show your work; correct answers with no supporting work will not be awarded full points.
- 2 randomly selected questions will be graded, but you must still turn in your work for all problems in order to be eligible to earn full credit.

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1. **Calculus Crash-Course.** Compute the following:

   a) $\sum_{k=10}^{\infty} \frac{2}{3^k}$ **b** $\sum_{k=1}^{\infty} (-1)^k \frac{4^k}{k!}$ **c)** $\sum_{\substack{k=4 \\ \text{even}}}^{\infty} \frac{2}{3^k}$

   d) $\int_a^b xe^{-x^2}\,dx$ **e)** $\int_a^b xe^{-x}\,dx$ **f)** $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

2. **Refresher on Maclaurin Series Expansions**

   a) Consider the function $f(x) = \ln(1+x)$. Find an expression for $f^{(n)}(0)$, where $n \geq 0$ is an arbitrary integer. **Hint:** Your final answer should be piecewise-defined, with two cases.

   b) Using your answer from part (a), derive the Maclaurin Series Expansion of $f(x) = \ln(1+x)$.

   c) Use your answer from part (b) to evaluate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

3. **A Simple Experiment.** Suppose I toss a fair coin, roll a fair 4-sided die, and pick a number at random between 1 and 3 (inclusive), all at the same time.

   a) Write down a possible outcome space Ω. Be sure to clearly define your notation!

   b) Does it make sense to utilize the classical definition of probability for this problem? Explain why or why not.

   c) Compute the probabilities of the following events, using the classical definition of probability:

   (i) A is the event that the coin landed heads.

   (ii) B is the event that the die shows a number strictly smaller than the number selected from $\{1,2,3\}$.

(iii) C is the event that the coin landed heads, and the die shows a number strictly smaller than the number selected from $\{1, 2, 3\}$.

(iv) D is the event that the coin landed heads, or the die shows a number strictly smaller than the number selected from $\{1, 2, 3\}$.

4. Counting Students. In a particular section of PSTAT 120A, there are 100 students: 30 Freshman, 40 Sophomores , 20 Juniors, and 10 Seniors. Of the freshman, 20 are PSTAT majors; of the Sophomores, 10 are PSTAT majors; of the Juniors, 5 are PSTAT Majors; and of the Seniors, 2 are PSTAT Majors. A random subset of 10 of these students is to be selected.

(a) What is the probability that this sample contains only Freshman?

(b) What is the probability that this sample contains at least one student from each cohort (Freshman, Sophomore, Junior, Senior)?

(c) What is the probability that this sample contains only PSTAT Majors?

(d) Let A denote the event "the sample contains only Freshman" and B denote the event "the sample contains 5 PSTAT Majors." Compute $\mathbb{P}(A \cap B)$.