PSTAT 120A, Summer 2022: Practice Problems 5

Week 2

Conceptual Review

(a) Why is a function of a random variable also a random variable?

(b) If $Y := g(X)$ where the distribution of X is known, must we first find $f_Y(y)$ before computing $\mathbb{E}[Y]$?

(c) How to transformations of discrete random variables work?

(d) Will transformations of discrete random variables always be discrete? Will transformations of continuous random variables always be continuous?

Problem 1: Two Interesting Results

(a) If $X \sim \text{Exp}(\lambda)$ and $Y := cX$ for some fixed constant $c > 0$, show that $Y \sim \text{Exp}(\lambda/c)$. For practice, derive the result in two ways: using the c.d.f. method, and using the Change of Variable formula.

(b) If $X \sim \text{Gamma}(r, \lambda)$ and $Y := cX$ for some fixed constant $c > 0$, identify the distribution of Y **by name**, taking care to include any/all relevant parameter(s).

Problem 2: Raise The Roof- er, Ceiling!

Let $X \sim \text{Exp}(\lambda)$, and define $Y := \lceil X \rceil$. Identify the distribution of Y **by name**, taking care to include any/all relevant parameter(s). Recall that

$$\lceil x \rceil := \text{smallest integer larger than } x$$

so, for instance, $\lceil \pi \rceil = 4$.

Hint: *Identify appropriate values for a and b such that*

$$\{\lceil X \rceil = y\} = \{a < X \leq b\}$$

Extra Problems

Problem 3: Rounding

The true concentration of radiation in a particular room (measured in counts per second) is uniformly distributed on the interval $[0, 10]$. A Geiger counter is used to measure the radition in this room, however it is very crude and only displays measurements rounded to the nearest integer value. Let X denote the true amount of radiation in the room, and Y denote the amount of radiation displayed on the Geiger counter.

(a) Is X discrete or continuous? What about Y?

(b) Is it correct to say that Y is uniformly distributed on S_Y, the state space of Y?

(c) Now, find the p.m.f. of Y.

Problem 4: Transformations *(CB, 2.1)*

In each of the following find the p.d.f. of Y. Show that the p.d.f. integrates to 1.

(a) $Y = X^3$ and $f_X(x) = 42x^5(1-x),\ 0 < x < 1$

(b) $Y = 4X + 3$ and $f_X(x) = 7e^{-7x},\ 0 < x < \infty$

(c) $Y = X^2$ and $f_X(x) = 30x^2(1-x)^2,\ 0 < x < 1$

Problem 5: Square-y Situation

Suppose $X \sim \text{Unif}[-1, 2]$ and $Y := X^2$.

(a) Compute $\mathbb{E}[Y]$. **Hint:** If you remember certain properties about the uniform distribution, you can do this without computing any integrals.

(b) Find $f_Y(y)$, the probability density function (p.d.f.) of Y.