

PSTAT 120A, Summer 2022: Practice Problems 9*Week 6**Conceptual Review*

- (a) What is the definition of convergence in distribution? What about convergence in probability?
- (b) What is the Law of Large Numbers?
- (c) What is the Central Limit Theorem? How does it relate to the DeMoivre-Laplace Theorem?

Problem 1: Poisson Predicament

Provide a probabilistic proof for the identity

$$\sum_{k=0}^{\infty} e^{-n} \cdot \frac{n^k}{k!} \approx \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

Hint: Let $X_i \stackrel{\text{i.i.d.}}{\sim} \text{Pois}(1)$,
and $X := \sum_{i=1}^n X_i$

assuming n is quite large.

Problem 2: Money, Money, Money

Every day, Jack goes to the casino. Suppose that on any given day, the amount Jack earns is a random variable with mean \$10 and standard deviation \$2, independent of all other days. Approximate the probability that Jack will earn more than \$310 in a 30-day period.

Problem 3: Probabilistic Convergence

Prove the following theorem: if X_n is a random variable with mean μ that satisfies the property $\text{Var}(X_n) \rightarrow 0$ as $n \rightarrow \infty$, then $X_n \xrightarrow{P} \mu$.

Hint: Chebyshev's Inequality

Extra Problems

*Problem 4: Random Numbers**(modified from ASV, 4.16)*

We choose 500 numbers uniformly at random from the interval $[1.5, 4.8]$

- (a) Approximate the probability of the event that less than 65 of the numbers start with the digit 1, and justify your choice of approximation.
- (b) Approximate the probability of the event that more than 160 of the numbers start with the digit 3, and justify your choice of approximation.