PSTAT 120A, Summer 2022: Practice Problems 1

Week 1

WELCOME TO PSTAT 120A! Discussion worksheets are designed to give you additional practice with material covered in lecture the week prior. Please note that this worksheet contains material from Wednesday's lecture of Week 1.

A note on the "Extra Problems:" though these may not get covered in section, it is in your best interest to complete them as they are still fair game for quizzes/exams!

Conceptual Review

(a) What type of mathematical object (i.e. function, set, constant, etc.) is the **outcome space**?

(b) What type of mathematical object (i.e. function, set, constant, etc.) is an **event**?

(c) What are the **Axioms of Probability**?

Problem 1: Component Failures *(modified from Ross 2.5)*

A system is comprised of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.

a) How many outcomes are in the sample space of this experiment?

b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W.

c) Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A?

d) Write out all the outcomes in the event AW.

e) Compute $\mathbb{P}(A \cup W)$, assuming equally likely outcomes.

Problem 2: Poker Face

Suppose a hand of 5 cards is drawn from a standard deck of 52 playing cards. Compute the probabilities of the following hands:

a) Full House (3 of a kind and a 2 of a kind)

b) Three-of-a-kind

c) Two pairs (i.e. two distinct two-of-a-kinds)

Extra Problems

Problem 3: Further Practice with Sums

From the Calculus Review video, we saw the result:

$$\sum_{k=0}^{\infty} kr^k = \frac{r}{(1-r)^2}; \qquad |r| < 1 \tag{1}$$

a) Using a similar method used to derive (1), derive an expression for $\sum_{k=a}^{\infty} kr^k$. You may once again assume that the infinite sum and the derivative operator interchange.

b) Compute $E := \sum_{\substack{k=0 \\ \text{even}}}^{\infty} kr^k$ and $O := \sum_{\substack{k=1 \\ \text{odd}}}^{\infty} kp^k$.

Hint: Reindex the sums.

c) Which is larger, O or E? (Or are they the same value?) In other words, when summing over only the even natural numbers do we obtain a result that is equal to, larger than, or less that the result we would have obtained had we summed over only the *odd* natural numbers?

Problem 4 (Challenge): Castle Conundrum (modified from Ross 2.17)

A chessboard is a 8×8 board of 64 squares. Among the pieces on a chessboard is the **rook** (shaped like a castle); rooks can only move horizontally or vertically. If another chesspiece lies in the potential path of a rook, we say that the rook can "capture" the other chesspiece (see picture below).

Hint: Once you place a rook, you effectively "reduce" the size of the chessboard.

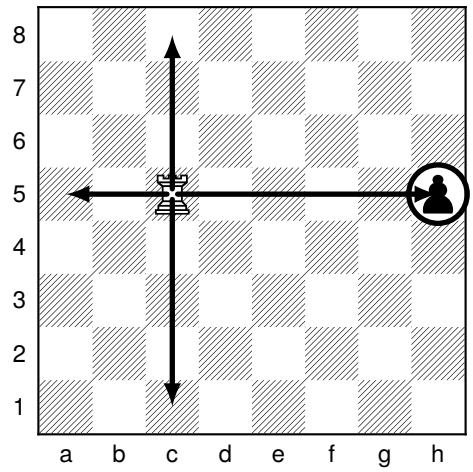

Fig. 1: *The pawn (circled) is in trouble...*

Suppose 8 rooks are to be placed on a chessboard. What is the chance that none of the 8 rooks can capture each other?