

HOMEWORK 1

PSTAT 120A: Summer 2022

Due: 11:59pm on Friday, June 28
Instructor: Ethan P. Marzban

Instructions:

- Please submit your work to Gradescope by no later than the due date posted above.
 - Be sure to show your work; correct answers with no supporting work will not be awarded full points.
 - 2 randomly selected questions will be graded, but you must still turn in your work for all problems in order to be eligible to earn full credit.
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1. Calculus Crash-Course. Compute the following:

a) $\sum_{k=10}^{\infty} \frac{2}{3^k}$

b) $\sum_{k=1}^{\infty} (-1)^k \frac{4^k}{k!}$

c) $\sum_{\substack{k=4 \\ \text{even}}}^{\infty} \frac{2}{3^k}$

d) $\int_a^b x e^{-x^2} dx$

e) $\int_a^b x e^{-x} dx$

f) $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$

2. Refresher on Maclaurin Series Expansions

- Consider the function $f(x) = \ln(1+x)$. Find an expression for $f^{(n)}(0)$, where $n \geq 0$ is an arbitrary integer. **Hint:** Your final answer should be piecewise-defined, with two cases.
- Using your answer from part (a), derive the Maclaurin Series Expansion of $f(x) = \ln(1+x)$.
- Use your answer from part (b) to evaluate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

3. A Simple Experiment. Suppose I toss a fair coin, roll a fair 4-sided die, and pick a number at random between 1 and 3 (inclusive), all at the same time.

- Write down a possible outcome space Ω . Be sure to clearly define your notation!
- Does it make sense to utilize the classical definition of probability for this problem? Explain why or why not.
- Compute the probabilities of the following events, using the classical definition of probability:
 - A is the event that the coin landed heads.
 - B is the event that the die shows a number strictly smaller than the number selected from $\{1, 2, 3\}$.

- (iii) C is the event that the coin landed heads, and the die shows a number strictly smaller than the number selected from $\{1, 2, 3\}$.
 - (iv) D is the event that the coin landed heads, or the die shows a number strictly smaller than the number selected from $\{1, 2, 3\}$.
4. **Counting Students.** In a particular section of PSTAT 120A, there are 100 students: 30 Freshman, 40 Sophomores, 20 Juniors, and 10 Seniors. Of the freshman, 20 are PSTAT majors; of the Sophomores, 10 are PSTAT majors; of the Juniors, 5 are PSTAT Majors; and of the Seniors, 2 are PSTAT Majors. A random subset of 10 of these students is to be selected.
- (a) What is the probability that this sample contains only Freshman?
 - (b) What is the probability that this sample contains at least one student from each cohort (Freshman, Sophomore, Junior, Senior)?
 - (c) What is the probability that this sample contains only PSTAT Majors?
 - (d) Let A denote the event “the sample contains only Freshman” and B denote the event “the sample contains 5 PSTAT Majors.” Compute $\mathbb{P}(A \cap B)$.