DISCUSSION WORKSHEET 01

PSTAT 120B: Mathematical Statistics, I
Summer Session A, 2024 with Instructor: Ethan P. Marzban

Welcome to our first PSTAT 120B Discussion Section! Discussion worksheets are designed to give you additional practice with material covered in lecture.

Conceptual Review

(a) What is a **conditional mass/density function**? For what values is it defined?

(b) What is a **conditional expectation**?

(c) What are the **Law of Iterated Expectations** and **Law of Total Variance**?

(d) What is the **Gamma distribution**? Specifically, what is its density function? What are its expectation and variance?

Problem 1: Conditional Distributions/Expectations

Let (X, Y) be a bivariate random vector with joint probability density function (p.d.f.) given by

$$f_{X,Y}(x, y) = \begin{cases} \lambda y e^{-y(x+\lambda)} & \text{if } x \geq 0,\ y \geq 0 \\ 0 & \textbf{otherwise} \end{cases}$$

where $\lambda > 0$ is a fixed constant.

a) Find $f_Y(y)$, the marginal density of Y and use this to identify Y as belonging to a known distribution. **Be sure to include any/all relevant parameter(s)!**

b) Find $f_{X|Y}(x \mid y)$, the density of $(X \mid Y = y)$, and use this to identify $(X \mid Y = y)$ as belonging to a known distribution. **Be sure to include any/all relevant parameter(s)!**

c) Set up **but do not evaluate** and integral corresponding to $\mathbb{E}[X]$, that only involves the marginal density function of Y.

Hint: Law of Iterated Expectations

Problem 2: The Gamma Distribution

Recall (from lecture) that if $X \sim$ Gamma(α, β), then X has density given by

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} \cdot \mathbb{1}_{\{x \geq 0\}}$$

where $\Gamma(\alpha)$ denotes the **gamma function**, defined as

$$\Gamma(r) := \int_0^\infty x^{r-1} e^{-x} \, \mathrm{d}x \text{if } r \geq 0$$

and $\Gamma(0) := 1$.

a) Show that X has MGF (moment generating function) given by

$$M_X(t) = \begin{cases} (1 - \beta t)^{-\alpha} & \text{if } t < 1/\beta \\ \infty & \text{otherwise} \end{cases}$$

b) Let $Y \sim \chi^2_\nu$. Use your answer to part (a) to derive the MGF $M_Y(t)$ of Y.

c) What is another name for the χ^2_2 distribution? Be sure to give the distribution's name and also list out any/all relevant parameter(s)!