PSTAT 120B ╱ **MIDTERM 2** ╱ Summer 2024   Instructor: **Ethan P. Marzban**

Name: _____   **NetID:** _____
                    (*First and Last*)   (*NOT Perm Number*)

Your Section: **2pm (Hyuk-Jean)**   **3pm (Hyuk-Jean)**   **4pm (Minwoo)**   **5pm (Minwoo)**
(*Circle One*)

Instructions:

- You will have **55 minutes** to complete this exam.

    – Nobody will be permitted to leave the exam room during the last 10 minutes of the exam.

- Please remember to write your name and NetID (not perm number) at the top of each sheet of this exam.

- You are allowed the use of a single **8.5 × 11-inch** sheet, front and back, of handwritten notes. You are also permitted the use of **calculators**; the use of any and all other electronic devices (laptops, cell phones, etc.) is prohibited.

    – You will be asked to turn in your note sheet with your exam.

- Unless otherwise specified, simplification is not needed; however, all integrals and infinite sums (unless otherwise specified) must be evaluated.

    – One exception is that, whenever applicable, answers may be left in terms of Φ, the standard normal c.d.f..

- **Good Luck!!!**

Honor Code: In signing my name below, I certify that all work appearing on this exam is entirely my own and not copied from any external source. I further certify that I have not received any unauthorized aid while taking this exam.

✕ _____

1. **(3 points)** Let $Y_1, \cdots Y_{10} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,9)$ [i.e. $\text{Var}(Y_i) = 9$ for all $i = 1, 2, \cdots, 10$]. Define U as

$$U := \sum_{i=1}^{10} Y_i^2$$

Identify the distribution of U by name, taking care to include any/all relevant parameter(s).

2. (4 points) The amount (in miles) Carl drives in a given day varies from day to day. Assume the amount he drives in one day is independent of the amount he drives in any other day. From prior experience, Carl knows that the true average amount he drives daily is 10 miles, with a standard deviation of 4 miles.

Suppose Carl selects a random 25-day period, over which he has enough gas to travel 260 miles. **What is the probability that Carl doesn't run out of gas during this period (i.e. that the <u>total</u> amount of miles he drives in this randomly-selected 25-day period does not exceed the 260 miles worth of gas he has)?**

Leave your answer in terms of $\Phi(\cdot)$, the standard normal CDF. If you use a theorem (e.g. Central Limit Theorem, Weak Law of Large Numbers, etc.), clearly state where in your work you use it.

3. Let $Y_1, Y_2 \overset{\text{i.i.d.}}{\sim} \text{Unif}[0, 1]$, and define $U := \ln(Y_2/Y_1)$.

    (a) (3 points) For $u < 0$, show that $F_U(U)$, the CDF (cumulative distribution function) of U at u, is given by

$$F_U(u) = \frac{1}{2}e^u$$

    (b) (3 points) For $u \geq 0$, show that $F_U(u)$, the CDF (cumulative distribution function) of U at u, is given by

$$F_U(u) = 1 - \frac{1}{2}e^{-u}$$

(c) (1 point) Is it true that the density of U can be expressed as:

$$f_U(u) = \frac{1}{2} e^{-|u|}$$

Justify your answer!

4. Let $Y_1, \cdots, Y_n \sim \text{Ray}(\theta)$. Recall that this is the notation used to denote the **Rayleigh distribution**, with density function given by

$$f_Y(y) = \left(\frac{2y}{\theta}\right) e^{-y^2/\theta} \cdot \mathbb{1}_{\{y \geq 0\}}$$

You may use, without proof, the fact that the expectation of the $\text{Ray}(\theta)$ distribution is given by

$$\mathbb{E}[Y_i] = \frac{\sqrt{\pi\theta}}{2}$$

(a) (6 points) Find $f_{Y_{(1)}}(y)$, the density of the first order statistic (i.e. the sample minimum). **Note:** This should show that $Y_{(1)} \sim \text{Ray}(\theta/n)$, a fact you may use in all future parts of this problem.

(b) (3 points) Is $Y_{(1)}$ an unbiased estimator for θ? Justify your answer.

(c) (2 points) Consider the following estimator for θ:

$$\widehat{\theta}_n = \frac{4}{\pi}(\overline{Y}_n)^2$$

Is this a consistent estimator for θ? Justify your answer carefully.

You may use the remainder of this page for scratch work; please note that nothing written on this page will be graded.

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