

Problem 1

(a) Find the smallest positive integer that has exactly 143 proper factors. An integer m is proper factor of another integer n if $1 < m < n$ and there is $n = m \times k$ for some positive integer k . So for example, 12 has proper factors 2, 3, 4 and 6.

(b) Show that for every n there are infinitely many integers with exactly n proper factors.

Problem 2

Find a sequence of real numbers from the interval $[0, 1]$ that has subsequences converging to infinitely many different values. For example, the sequence $1/2 + 1/3, 1/3 + 1/3, 1/2 + 1/4, 1/3 + 1/4, 1/2 + 1/5, 1/3 + 1/5, \dots$ has two convergent subsequences, the first consisting of the even numbered terms converges to $1/2$ and the one made up of the odd-numbered terms converges to $1/3$.