Problem

Find the smallest number that is divisible by all of the digits 2, ..., 9.

This problem is from The Morning Brew.

Solution

Our strategy will be to keep track of the required prime factors and their highest exponents. Suppose that x is divisible by each number in $D = \{2, ..., 9\}$. Then for each d in D, x's prime factorization must include all of the primes in d's prime factorization with equal or greater exponents. Starting with 2, that adds 2^1 to the required primes in x. Next is 3, where we add a similar term for 3. Continuing, 4 ups the required exponent on 2 to 2, 5 adds 5^1, 6 is covered (already have 2 and 3 required), 7 adds 7^1, 8 ups the exponent on 2 to 3, and 9 ups the exponent on 3 to 2. Putting this all together, to be divisible by each of the digits, x must be at least

$$2^3 3^2 5^1 7^1 = 2520$$