Problem

Find the smallest number that is divisible by all of the digits 2, ..., 9.

This problem was adapted from a problem from The Morning Brew.

Solution

Our strategy will be to keep track of the required prime factors and their highest exponents. Suppose that x is divisible by each number in $D = 2, ..., 9$. Then for each d in D, x's prime factorization must include all of the primes in d's prime factorization with equal or greater exponents.
Starting with 2, that adds 2^1 to the required primes in x. Next is 3, where we add a similar term for 3. Continuing, 4 ups the required exponent on 2 to 2, 5 adds 5^1, 6 is covered (already have 2 and 3 required), 7 adds 5^1, 8 ups the exponent on 2 to 3, and 9 ups the exponent on 3 to 2. Putting this all together, to be divisible by each of the digits, x must be at least

$$2^3 3^2 5^1 7^1 = 2520$$