

Problem

How many proper factors does one billion have? A *proper factor* of a positive integer n is a positive integer m such that $n = km$ where k is a positive integer greater than 1 and less than n .

Solution

$10^9 = 2^9 \cdot 5^9$. Every proper factor of 10^9 must have the form $2^m \cdot 5^n$ where m and n are between 0 and 9. The two combinations $m = n = 0$ and $m = n = 9$ must be excluded, but all others are valid and distinct factors. They are all distinct because 2 and 5 are primes (so two different products of them to different exponents cannot be the same, by uniqueness of prime factorization). Therefore, there are exactly $10 \cdot 10 - 2 = 98$ proper factors of one billion.