Problem

A set function f from the set of subsets of a finite set N to the positive real numbers is called *submodular* if for any $A \subseteq B \subseteq N$ and element u in N, we have

$$f(A \cup \{u\}) - f(A) \geq f(B \cup \{u\}) - f(B)$$

In many cases the greedy algorithm works well to optimize submodular objective functions. Find an example of a set N and submodular function f defined on N such that the greedy algorithm returns a subobptimal result when trying to maximize f.

Solution