

## Problem

Find the smallest number that is divisible by all of the digits 2, ..., 9.

This problem is from [The Morning Brew](#).

## Solution

Our strategy will be to keep track of the required prime factors and their highest exponents. Suppose that  $x$  is divisible by each number in  $D = \{2, \dots, 9\}$ . Then for each  $d$  in  $D$ ,  $x$ 's prime factorization must include all of the primes in  $d$ 's prime factorization with equal or greater exponents. Starting with 2, that adds  $2^1$  to the required primes in  $x$ . Next is 3, where we add a similar term for 3. Continuing, 4 ups the required exponent on 2 to 2, 5 adds  $5^1$ , 6 is covered (already have 2 and 3 required), 7 adds  $7^1$ , 8 ups the exponent on 2 to 3, and 9 ups the exponent on 3 to 2. Putting this all together, to be divisible by each of the digits,  $x$  must be at least

$$2^3 3^2 5^1 7^1 = 2520$$