Homework 2

Due 09/16/19

September 9, 2019

1. Use the formal definition of Big-Oh to prove that if f(n) and g(n) are nonnegative functions such that $f(n) = O(g(n))$, $f(n) + g(n) = O(g(n))$.

by definition, $f(n) \leq c(g(n))$ for all $n \geq n_0$

now, $f(n)+g(n) \leq c(g(n))+g(n)$ for all $n \geq n_0$

$f(n)+g(n) \leq (c+1)\, g(n)$ for all $n \geq n_0$

therefore, $\exists$ some $c_2 = c+1$ and $n_0 > 0$ such that $f(n)+g(n) \leq c_2(g(n))$. thus, $f(n)+g(n) = O(g(n))$

2. Use the formal definition of Big-Oh to prove that if f(n) and g(n) are nonnegative functions such that $f(n) = O(g(n))$, $f(n) + g(n) = \Omega(g(n))$.

by definition, $g(n) \geq c(g(n))$ for all $n \geq n_0$

now, $f(n)+g(n) \geq c(g(n))+f(n)$ for all $n \geq n_0$

because $f(n) \leq c_2\, g(n)$ for $n \geq n_0$

we can say that $c(g(n))$ dominates

therefore, $f(n)+g(n) \geq c(g(n))$ $\forall n \geq n_0$

therefore, $f(n)+g(n) = \Omega(g(n))$