Homework 10

Due December 4, 2019

In this assignment, you will prove that the Zero Sum problem is NP-Complete. The Zero Sum problem accepts an array of n integers and returns whether it is possible to negate some of those integers to make the array sum equal 0.

For example, the array data = [1, 3, 5, 7, 9, 11] has a zero sum if you negate the 7 and 11, while the array data = [1, 2, 5, 9, 13, 22] cannot be reduced to a sum of zero.

1. Prove that Zero Sum is NP.

```
|    sum = 0
|    for x in output:
|        if (-x not ∈ input || x not ∈ input):
|            return false
|        sum += x
|    if sum == 0:
|        return true
|    else:
|        return false
```

2. The Subset Sum problem is NPC. This takes in an array of n integers and a target t to determine if the array has a subset that sums to t.

   To prove that Zero Sum is NPC, would you reduce Zero Sum to Subset or Subset Sum to Zero Sum?

   You would need to reduce subset sum to Zero sum.

3. Give pseudocode for a reduction in the appropriate direction. *NOTE: This answer is based on the CORRECT answer for #2.*

   Subset Sum has two parameters (an array and a target value) while Zero Sum only has one (an array). If you are reducing ZS to SS, you will need to come up with a target value and possibly modify the array. If you are reducing SS to ZS, you will need to come up with an array that incorporates the target value.

```
Input: sub_arr, the subset array
Input: t, target sum
Output: result of ZeroSum, which tells weather or not sub_arr sums to t
SubSumReduction
Θ(1)          |    sum = 0
Θ(n)          |    for i in sub_arr
Θ(1)          |    sum += i
Θ(1)          |    full_arr = [sum, t]
O(ZeroSum)    |    return (ZeroSum(full_arr))
```

4. Show that your reduction is polynomial time

Reduction is Θ(n) + O (ZeroSum),

Therefore the reduction is polynomial.