1. Suppose that RecursionMystery is a function that accepts an array as a parameter. When called with an array of length 0, it executes 8 instructions. When called with an array of length $n > 0$, it executes of $8n + 64$ instructions, two of which are recursive function calls with an array of size $n - 1$. ($8n + 64$ includes the everything except the instructions executed during the recursive calls.)

   Prove that RecursionMystery executes $88(2^n) - 8n - 80$ instructions when its input has length n, for all $n \geq 0$.

Base Case: $n=0$

when array $=0$, ins $=8$

$$88(2^0)-8(0)-80=88-80=8$$

Assume $\text{RecursionMystery}(K)=88(2^K)-8K-80$

Prove $\text{RecursionMystery}(K+1)$

$$\text{RecursionMystery}(K+1)=8(K+1)+64+2(\text{RecursionMystery}(K))$$

$$=8(K+1)+64+2\{88(2^K)-8K-80\}$$

$$=8(K+1)+64+88(2^{K+1})-16K-160$$

$$=8K+\cancel{8}+\cancel{64}+88(2^{K+1})-16K-1\cancel{60}$$

$$=88(2^{K+1})-8K-88$$

$$=88(2^{K+1})-8K-8-80$$

$$=88(2^{K+1})-8(K+1)-80$$

thus, $\text{RecusionMystery}(n)=88(2^n)-8n-80$,

where $n=K+1$.

thus, RecursionMystery executes $88(2^n)-8n-80$ instructions for all $n\geq 0$

2. Suppose that DivisionMystery is a function that accepts an integer n as a parameter. When called with $n = 1$, it executes 2 instructions. When called with a larger value of n, it executes $10n + 30$ instructions, two of which are recursive calls to DivisionMystery(n/2).

   Prove that DivisionMystery(n) executes $10n \lg n + 32n - 30$ instructions for all $n \geq 1$. Note: we use $\lg n$ to represent the log base 2 of n (i.e., $\log_2(n)$).
   Hint: A useful property of logarithms is that $\lg(a/b) = \lg a - \lg b$ for any nonzero a and b.

Base Case: $n = 1$

When $n = 1$ ins $= 2$

$10(1)\lg_2(1) + 32(1) - 30 = 10(1)\cdot 0 + 32(1) - 30 = 2$

Assume DivisionMystery$(k) = 10k \cdot \lg k + 32k - 30$

Prove DivisionMystery$(k+1)$

DivisionMystery$(k+1) = 10(k+1) + 30 + 2\left(\text{DivMystery } \frac{k+1}{2}\right)$

$= 10(k+1) + 30 + 2\left\{10\left(\frac{k+1}{2}\right)\cdot \lg\left(\frac{k+1}{2}\right) + 32\left(\frac{k+1}{2}\right) - 30\right\}$

$= 10k + 40 + 10(k+1)\cdot \lg\left(\frac{k+1}{2}\right) + 32(k+1) - 60$

$= 10k + 40 + 10(k+1)(\lg(k+1) - \lg(2)) + 32(k+1) - 60$

$= 10k + 40 + 10(k+1)\lg(k+1) - 10(k+1)\lg(2) + 32(k+1) - 60$

$= \cancel{10k} + \cancel{40} + \cancel{10(k+1)\lg(k+1)} - \cancel{10k} - \cancel{10} + \cancel{32}k + \cancel{32} - \cancel{60}$

$= 10(k+1)\lg(k+1) + 2 + 32k$

$= 10(k+1)\lg(k+1) + 32k + 32 - 30$

$= 10(k+1)\lg(k+1) + 32(k+1) - 30$

thus, DivisionMystery$(n) = 10n\lg n + 32n - 30$, where $n = k+1$.

thus, DivisionMystery executes $10n\lg n + 32n - 30$ instructions for all $n \geq 1$.