Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Name and section: ______________________________

Instructor's name: ______________________________

1. Is it true that $x^n + y^n = z^n$ if x, y, z and n are positive integers? Explain.

2. Prove that the real part of all non-trivial zeros of the function $\zeta(z)$ is $\frac{1}{2}$.

3. Compute
$$\int_0^\infty \frac{\sin(x)}{x}$$

4. Prove the following identity.
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

5. Prove the following identity.
$$f(a) = \frac{1}{2\pi i}\oint \frac{f(z)}{z-a}dz$$

6. Prove the following identity.
$$\int_D (\nabla \cdot F)\,\mathrm{d}V = \int_{\partial D} F \cdot n\,\mathrm{d}S$$

7. Prove the following identity.
$$\vec{\nabla} \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$$

8. Prove the following identity.

$$(\nabla_X Y)^k = X^i(\nabla_i Y)^k = X^i\left(\frac{\partial Y^k}{\partial x^i} + \Gamma^k_{im} Y^m\right)$$

9. Prove the following identity.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

10. Prove the following identity.

$$\mathscr{L}\{f(t)\} = F(s)$$

11. Prove the following identity.

$$\vec{F}_{\text{ext}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t} = \frac{\mathrm{d}\vec{p}(t)}{\mathrm{d}t}$$

12. Prove the following identity.

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

13. Prove the following identity.

$$|x| = \begin{cases} x & x > 0 \\ -x & x \leq 0 \end{cases}$$

14. Prove the following identity.

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$