

## Lab 3 Part 3: PPG Analysis

In this part, we will examine the recovery time for human heart rate using our PPG sensor. We for a run for 20 minutes; after this workout, we immediately measured the heart rate over the next five minutes (sitting very still will gave us a good signal from the PPG, and also allowed our HR to recover as quickly as possible).

```
In [128... import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import find_peaks, get_window, welch, butter, filtfilt
from scipy.fft import fft, fftfreq
from scipy.optimize import curve_fit
import csv
```

We plot our HR as a function of time.

```
In [129... # Constants
f_s = 400 # Hz
min_hr = 30 # Minimum heart rate in beats per minute

# Imports
ppg_path = '/Users/egeturan/Documents/Sensing/SmartphoneSensors292S/ee292s/1

def parse_data(file_path):
    # Initialize lists to store the time and ecg_signal data
    time = []
    ecg_signal = []

    # Open and read the CSV file
    with open(file_path, 'r') as file:
        reader = csv.reader(file)
        next(reader) # Skip the header row

        for row in reader:
            # Append the parsed time and ecg_signal values
            time.append(float(row[0]))
            ecg_signal.append(float(row[1]))

    # Convert the lists to NumPy arrays
    time_array = np.array(time) - 64.84
    ecg_signal_array = np.array(ecg_signal)

    return time_array, ecg_signal_array
```

```
In [130... time, ppg = parse_data(ppg_path)

# Define minimum time difference between peaks (based on heart rate)
min_time_diff = min_hr / 60 # Minimum time difference in seconds
min_sample_diff = int(min_time_diff * f_s) # Convert to sample indices

# Find all peaks
```

```
peaks, properties = find_peaks(ppg, distance=min_sample_diff)

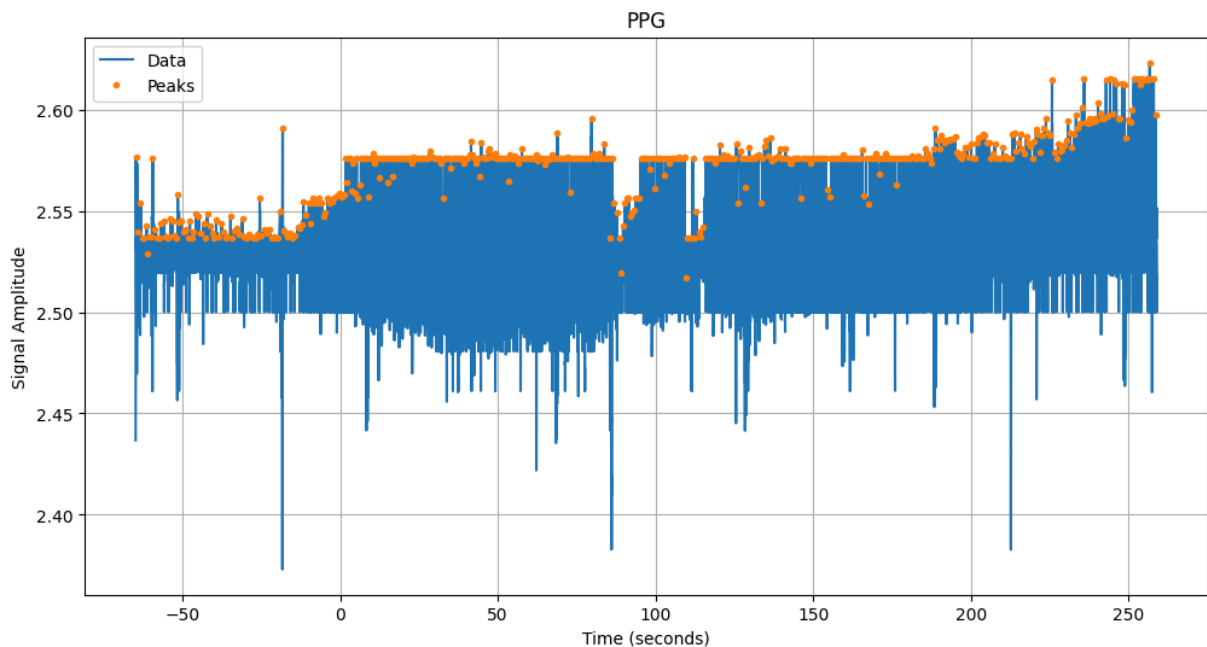
print("ppg:", ppg)
```

```
ppg: [2.43675499 2.45856255 2.46100395 ... 2.53913731 2.54259617 2.53676922]
```

```
In [131... # Plot the full data
plt.figure(figsize=(12, 6))
plt.plot(time, ppg, label="Data")
plt.plot(time[peaks], ppg[peaks], ".", label="Peaks") # Mark peaks with dot

# Add labels, legend, and title
plt.title("PPG")
plt.xlabel("Time (seconds)")
plt.ylabel("Signal Amplitude")
plt.legend()

# Show the plot
plt.grid(True)
plt.show()
```



```
In [132... # Calculate heart rate from peaks
if len(peaks) > 1:
    peak_intervals = np.diff(peaks) # Differences between consecutive peaks
    avg_peak_interval = np.mean(peak_intervals) # Average interval in samples
    avg_heart_rate = (60 * f_s) / avg_peak_interval # Convert to beats per minute
else:
    avg_heart_rate = 0 # Not enough peaks to calculate heart rate

print(f"Cumulative average Heart Rate: {avg_heart_rate:.2f} BPM")
```

Cumulative average Heart Rate: 93.81 BPM

Now, we plot our HR as a function of time. Smoothing the data using a moving average filter (deciding on the averaging length based on how noisy the data is).

```

In [133... # Calculate the time of each peak
peak_times = time[peaks]

# Calculate instantaneous heart rates (in BPM) from consecutive peak intervals
# Then, moving-average this
if len(peak_times) > 1:
    peak_intervals = np.diff(peak_times) # Differences between consecutive
    instantaneous_hr = 60 / peak_intervals # Convert to BPM

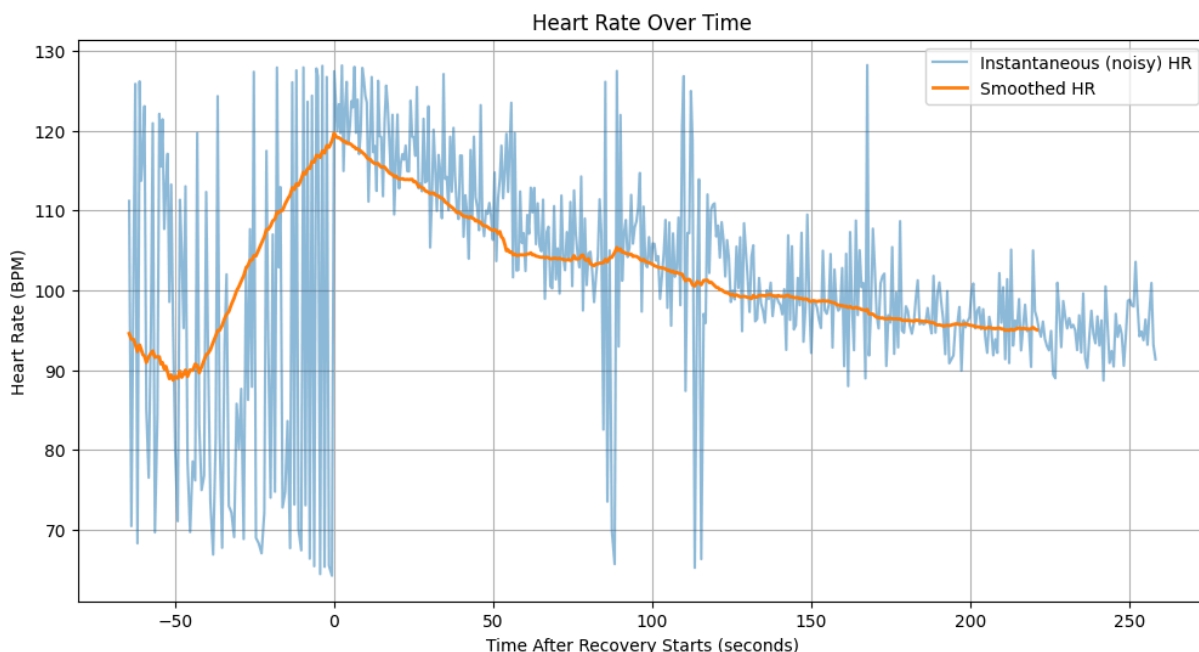
    # Associate each instantaneous HR with the midpoint of the peak interval
    hr_times = peak_times[:-1] + peak_intervals / 2

    # Apply a moving average filter to smooth the instantaneous heart rate
    avg_window_size = 60 # Adjust the window size based on noise level
    smoothed_hr = np.convolve(instantaneous_hr, np.ones(avg_window_size) / a

    # Adjust the time axis for the smoothed HR
    smoothed_hr_times = hr_times[:len(smoothed_hr)]
else:
    instantaneous_hr = []
    smoothed_hr = []
    smoothed_hr_times = []
    print("Not enough peaks detected to calculate instantaneous heart rate.")

# Plot the heart rate as a function of time
plt.figure(figsize=(12, 6))
if len(smoothed_hr) > 0: # Check if the smoothed_hr array has any values
    plt.plot(hr_times, instantaneous_hr, label="Instantaneous (noisy) HR", a
    plt.plot(smoothed_hr_times, smoothed_hr, label="Smoothed HR", linewidth=
    plt.title("Heart Rate Over Time")
    plt.xlabel("Time After Recovery Starts (seconds)")
    plt.ylabel("Heart Rate (BPM)")
    plt.legend()
else:
    plt.title("Heart Rate Over Time - No HR Calculated")
plt.grid(True)
plt.show()

```



```
In [134... # Find HRmax and its time
if len(smoothed_hr) > 0:
    hr_max_idx = np.argmax(smoothed_hr) # Index of maximum heart rate
    hr_max_time = smoothed_hr_times[hr_max_idx] # Time of HRmax
    hr_max_value = smoothed_hr[hr_max_idx] # HRmax value

    # Calculate HRR1 (during first recovery period)
    time_1_sec = hr_max_time + 52
    hr_1_idx = np.argmin(np.abs(smoothed_hr_times - time_1_sec)) # Closest
    hr_1_value = smoothed_hr[hr_1_idx]
    hrr1 = hr_max_value - hr_1_value

    # Calculate HRR2 (during second recovery period)
    time_2_sec = hr_max_time + 220
    hr_2_idx = np.argmin(np.abs(smoothed_hr_times - time_2_sec)) # Closest
    hr_2_value = smoothed_hr[hr_2_idx]
    hrr2 = hr_max_value - hr_2_value

    # Plot the results
    plt.figure(figsize=(12, 6))
    plt.plot(hr_times, instantaneous_hr, label="Instantaneous HR", alpha=0.5)
    plt.plot(smoothed_hr_times, smoothed_hr, label="Smoothed HR", linewidth=2)

    # Mark HRmax
    plt.scatter(hr_max_time, hr_max_value, color='red', label=f"HRmax: {hr_max_value:.2f} BPM")
    plt.annotate(f"HRmax\n({hr_max_time:.2f}s, {hr_max_value:.2f} BPM)",
                (hr_max_time, hr_max_value),
                textcoords="offset points", xytext=(0, 10), ha='center', color='red')

    # Mark HRR1
    plt.scatter(time_1_sec, hr_1_value, color='blue', label=f"HRR1: {hrr1:.2f} BPM")
    plt.annotate(f"HRR1\n({time_1_sec:.2f}s, {hr_1_value:.2f} BPM)",
                (time_1_sec, hr_1_value),
                textcoords="offset points", xytext=(0, 10), ha='center', color='blue')
```

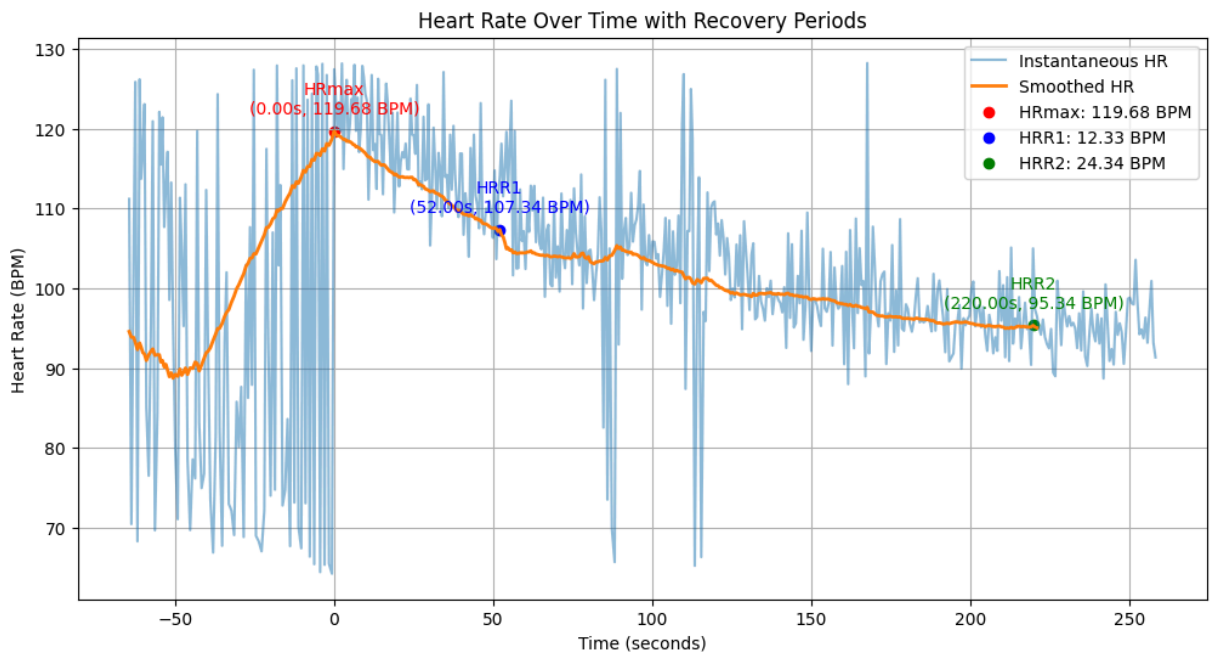
```

# Mark HRR2
plt.scatter(time_2_sec, hr_2_value, color='green', label=f"HRR2: {hrr2:.2f} BPM")
plt.annotate(f"HRR2\n({time_2_sec:.2f}s, {hr_2_value:.2f} BPM)",
            (time_2_sec, hr_2_value),
            textcoords="offset points", xytext=(0, 10), ha='center', cc

# Add labels, legend, and title
plt.title("Heart Rate Over Time with Recovery Periods")
plt.xlabel("Time (seconds)")
plt.ylabel("Heart Rate (BPM)")
plt.legend()
plt.grid(True)
plt.show()
else:
    print("Not enough data to calculate HRmax, HRR1, or HRR2.")

print(f"HRmax: {hr_max_value:.2f} BPM at {hr_max_time:.2f} seconds")
print(f"HRR1: {hrr1:.2f} BPM (52 seconds after HRmax)")
print(f"HRR2: {hrr2:.2f} BPM (220 seconds after HRmax)")

```



HRmax: 119.68 BPM at 0.00 seconds  
HRR1: 12.33 BPM (52 seconds after HRmax)  
HRR2: 24.34 BPM (220 seconds after HRmax)

Let's also include lines in each of the two recovery zones to show their average rates of recovery.

```

In [135... # Find HRmax and its time
if len(smoothed_hr) > 0:
    hr_max_idx = np.argmax(smoothed_hr) # Index of maximum heart rate
    hr_max_time = smoothed_hr_times[hr_max_idx] # Time of HRmax
    hr_max_value = smoothed_hr[hr_max_idx] # HRmax value

# Calculate HRR1 (during first recovery period)
time_1_sec = hr_max_time + 52
hr_1_idx = np.argmin(np.abs(smoothed_hr_times - time_1_sec)) # Closest

```

```

hr_1_value = smoothed_hr[hr_1_idx]
hrr1 = hr_max_value - hr_1_value

# Calculate HRR2 (during second recovery period)
time_2_sec = hr_max_time + 220
hr_2_idx = np.argmin(np.abs(smoothed_hr_times - time_2_sec)) # Closest
hr_2_value = smoothed_hr[hr_2_idx]
hrr2 = hr_max_value - hr_2_value

# Plot the results
plt.figure(figsize=(12, 6))
plt.plot(hr_times, instantaneous_hr, label="Instantaneous HR", alpha=0.5)
plt.plot(smoothed_hr_times, smoothed_hr, label="Smoothed HR", linewidth=2)

# Mark HRmax
plt.scatter(hr_max_time, hr_max_value, color='red', label=f"HRmax: {hr_max_value} BPM")
plt.annotate(f"HRmax\n({hr_max_time:.2f}s, {hr_max_value:.2f} BPM)",
            (hr_max_time, hr_max_value),
            textcoords="offset points", xytext=(0, 10), ha='center', ccolor='red')

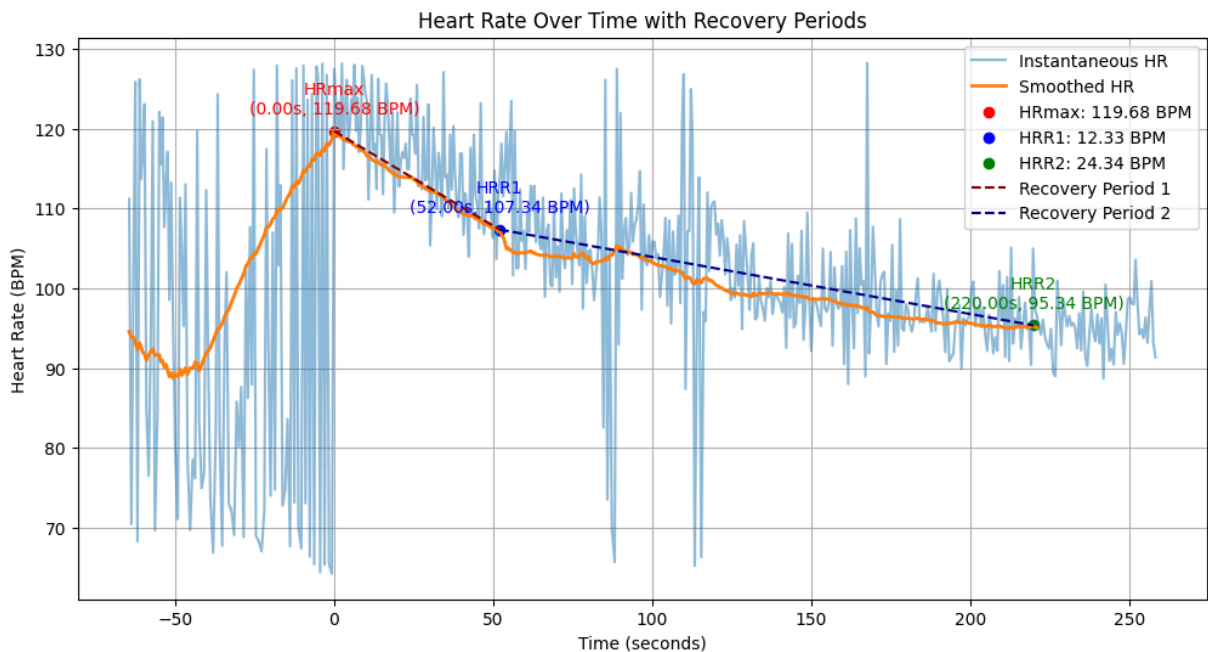
# Mark HRR1
plt.scatter(time_1_sec, hr_1_value, color='blue', label=f"HRR1: {hrr1:.2f} BPM")
plt.annotate(f"HRR1\n({time_1_sec:.2f}s, {hr_1_value:.2f} BPM)",
            (time_1_sec, hr_1_value),
            textcoords="offset points", xytext=(0, 10), ha='center', ccolor='blue')

# Mark HRR2
plt.scatter(time_2_sec, hr_2_value, color='green', label=f"HRR2: {hrr2:.2f} BPM")
plt.annotate(f"HRR2\n({time_2_sec:.2f}s, {hr_2_value:.2f} BPM)",
            (time_2_sec, hr_2_value),
            textcoords="offset points", xytext=(0, 10), ha='center', ccolor='green')

# Add lines
plt.plot([hr_max_time, time_1_sec], [hr_max_value, hr_1_value], color='red')
plt.plot([time_1_sec, time_2_sec], [hr_1_value, hr_2_value], color='darkblue')

# Add labels, legend, and title
plt.title("Heart Rate Over Time with Recovery Periods")
plt.xlabel("Time (seconds)")
plt.ylabel("Heart Rate (BPM)")
plt.legend()
plt.grid(True)
plt.show()
else:
    print("Not enough data to calculate HRmax, HRR1, or HRR2.")

```



Now, we can calculate the recovery time constant.

$$HR(t) = HR_{\infty} + (HR_{\max} - HR_{\infty}) \cdot e^{-(t/T)}$$

The recovery time constant ( $\tau$ ) represents how quickly the heart rate decreases after reaching its maximum value. It can be calculated by fitting the exponential decay model we have written above.

```
In [136... # Exponential decay model
def recovery_model(t, hr_inf, tau):
    return hr_inf + (hr_max_value - hr_inf) * np.exp(-t / tau)

# Define the recovery period data (after HRmax)
if len(smoothed_hr) > 0:
    recovery_mask = smoothed_hr_times > hr_max_time # Data after HRmax
    recovery_times = smoothed_hr_times[recovery_mask] - hr_max_time # Relative times
    recovery_hr = smoothed_hr[recovery_mask] # Heart rate during recovery

    if len(recovery_hr) > 2: # Ensure enough data for fitting
        # Initial guesses: HR_inf (minimum HR during recovery) and tau (arbitrary)
        hr_inf_guess = np.min(recovery_hr)
        tau_guess = 50 # Initial guess for time constant

        # Fit the model
        popt, pcov = curve_fit(recovery_model, recovery_times, recovery_hr,
                               hr_inf, tau = popt)

        # Plot the data and fit
        plt.figure(figsize=(12, 6))
        plt.plot(recovery_times, recovery_hr, 'o', label="Recovery Data", alpha=0.5)
        plt.plot(recovery_times, recovery_model(recovery_times, *popt), label="Fit", color='red')

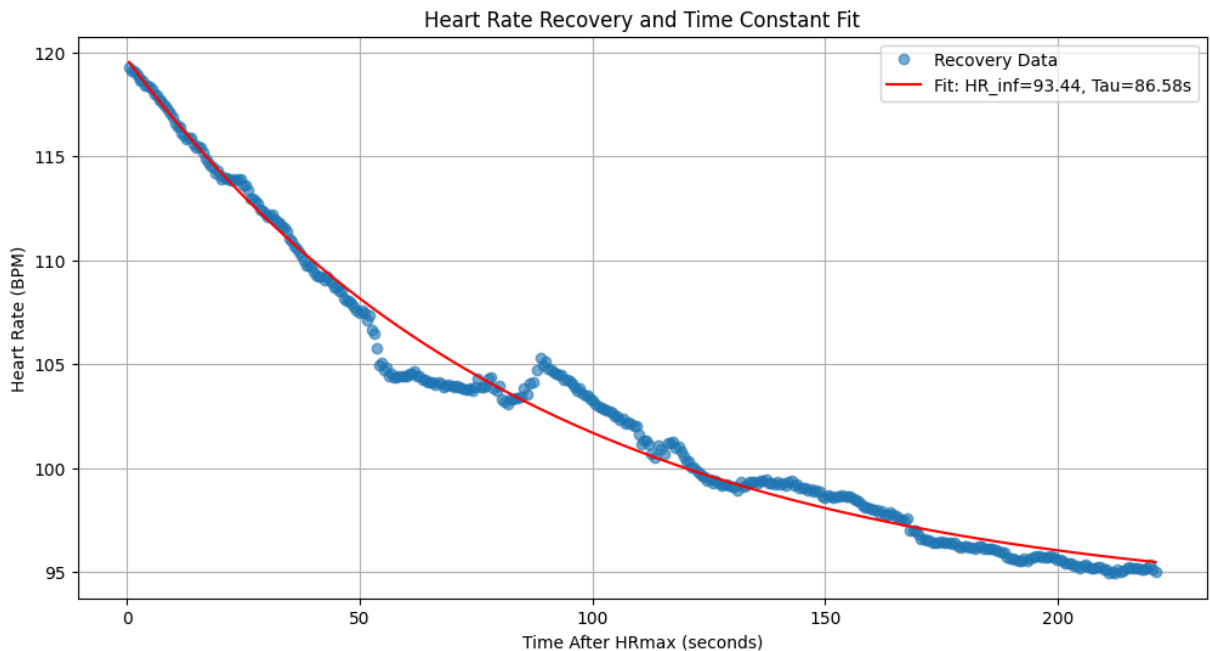
        # Add labels and legend
```

```

plt.title("Heart Rate Recovery and Time Constant Fit")
plt.xlabel("Time After HRmax (seconds)")
plt.ylabel("Heart Rate (BPM)")
plt.legend()
plt.grid(True)
plt.show()

print(f"Estimated Recovery Time Constant (Tau): {tau:.2f} seconds")
print(f"Asymptotic Heart Rate (HR_inf): {hr_inf:.2f} BPM")
else:
    print("Not enough data points in the recovery period to fit the model")
else:
    print("Not enough data to calculate the recovery time constant.")

```



Estimated Recovery Time Constant (Tau): 86.58 seconds  
 Asymptotic Heart Rate (HR\_inf): 93.44 BPM

Heart rate drop at some points of time:

```

In [137]: # Define times after HRmax for annotations (in seconds)
time_drops = [30, 60, 300] # 30 seconds, 1 minute, 5 minutes

# Calculate heart rate drops
if len(smoothed_hr) > 0:
    heart_rate_drops = {}
    for t_drop in time_drops:
        # Find the closest index for the specified time after HRmax
        target_time = hr_max_time + t_drop
        hr_idx = np.argmin(np.abs(smoothed_hr_times - target_time))
        hr_value = smoothed_hr[hr_idx]

        # Calculate the drop in heart rate
        hr_drop = hr_max_value - hr_value
        heart_rate_drops[t_drop] = (target_time, hr_value, hr_drop)

# Plot the data with annotations

```



```

plt.figure(figsize=(12, 6))
plt.plot(hr_times, instantaneous_hr, label="Instantaneous HR", alpha=0.5)
plt.plot(smoothed_hr_times, smoothed_hr, label="Smoothed HR", linewidth=2)

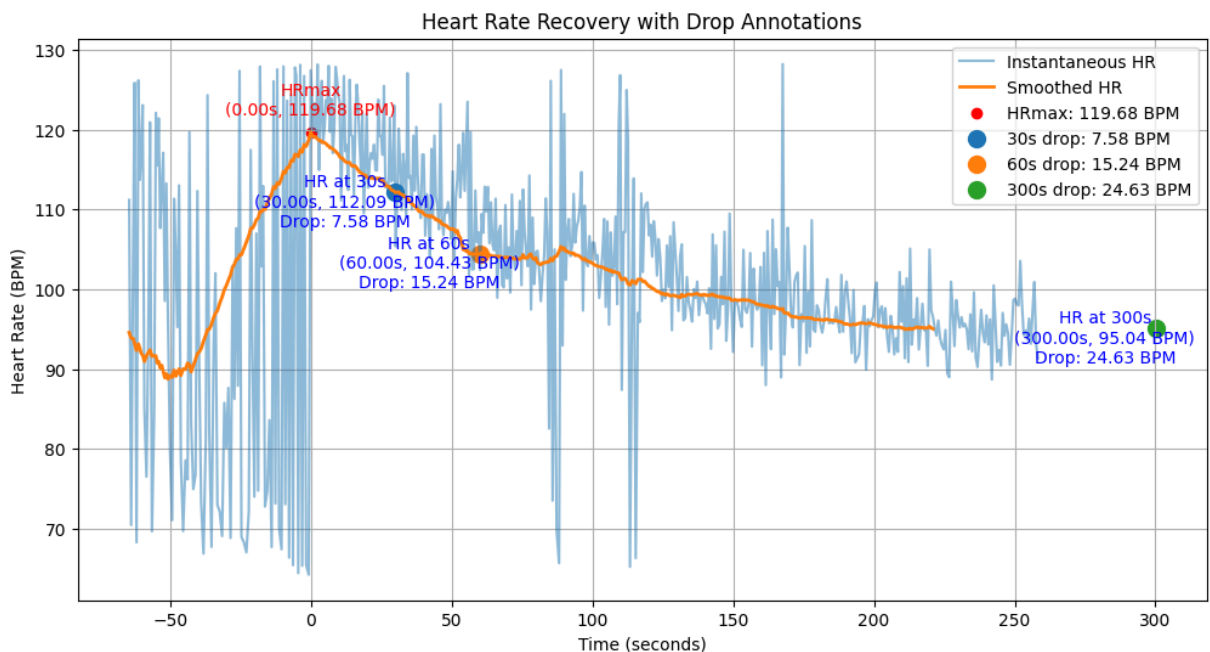
# Annotate HRmax
plt.scatter(hr_max_time, hr_max_value, color='red', label=f"HRmax: {hr_max_value:.2f} BPM")
plt.annotate(f"HRmax\n({hr_max_time:.2f}s, {hr_max_value:.2f} BPM)",
            (hr_max_time, hr_max_value),
            textcoords="offset points", xytext=(0, 10), ha='center', color='red')

# Annotate and mark heart rate drops
for t_drop, (t_time, t_hr_value, t_hr_drop) in heart_rate_drops.items():
    plt.scatter(t_time, t_hr_value, label=f"{t_drop}s drop: {t_hr_drop:.2f} BPM")
    plt.annotate(f"HR at {t_drop}s\n({t_time:.2f}s, {t_hr_value:.2f} BPM)",
                (t_time, t_hr_value),
                textcoords="offset points", xytext=(-30, -20), ha='center', color='blue')

# Add labels, legend, and title
plt.title("Heart Rate Recovery with Drop Annotations")
plt.xlabel("Time (seconds)")
plt.ylabel("Heart Rate (BPM)")
plt.legend()
plt.grid(True)
plt.show()

# Print the calculated drops
print(f"Heart Rate Drops After HRmax:")
for t_drop, (t_time, t_hr_value, t_hr_drop) in heart_rate_drops.items():
    print(f"- {t_drop} seconds: HR = {t_hr_value:.2f} BPM, Drop = {t_hr_drop:.2f} BPM")
else:
    print("Not enough data to calculate heart rate drops.")

```



Heart Rate Drops After HRmax:

- 30 seconds: HR = 112.09 BPM, Drop = 7.58 BPM
- 60 seconds: HR = 104.43 BPM, Drop = 15.24 BPM
- 300 seconds: HR = 95.04 BPM, Drop = 24.63 BPM