Central Limit Theorem
Practice Problems

7 — Sampling Distributions

7.1 Central Limit Theorem

The Central Limit Theorem is used to help us understand the following facts regardless of whether the population distribution is normal or not:

1. the mean of the sample means is the same as the population mean
2. the standard deviation of the sample means is always equal to the standard error (i.e. $SE = \frac{\sigma}{\sqrt{n}}$)
3. the distribution of sample means will become increasingly more normal as the sample size, n, increases.

Definition 7.1 — Sampling Distribution. The sampling distribution of a statistic is the distribution of that statistic. It may be considered as the distribution of the statistic for all possible samples from the same population of a given size.

■ **Example 7.1** We are interested in the average height of trees in a particular forest. To get results quickly we had 5 students go out and measure a sample of 20 trees. Each student returned with the average tree height from their samples.
Sample results : 35.23 , 36.71, 33.21, 38.2, 35.54
If it is known that the population average of tree heights in the forest is 36 feet with a standard deviation of 2 feet. How many Standard errors is the students average away from the population mean?

To solve this problem we first need to find the average of these students averages so

$$\bar{x} = \frac{35.23 + 36.71 + 33.21 + 38.2 + 35.54}{5} = 35.78$$

Now we find our Standard error of the sample:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{5} = 0.4$$

So now to get the number of standard errors away from the mean our observation is we can use the z-score formula:

$$\frac{35.78 - 36}{0.4} = -0.55$$

So our sample distribution is relatively close to the population distribution! ■

7.2 Practice Problems

Problem 7.1 The known average time it takes to deliver a pizza is 22.5 minutes with a standard deviation of 2 minutes. I ordered pizza every week for the last 10 weeks and got an average time of 18.5 minutes. What is the probability that get this average?

Problem 7.2 If I continue to order pizzas for eternity what could I expect this average to get close to?