2 — Hypothesis testing

2.1 What is a Hypothesis test?

A hypothesis test is used to test a claim that someone has about how an observation may be different from the known population parameter.

Definition 2.1 — Alpha level (α). The alpha level (α) of a hypothesis test helps us determine the critical region of a distribution.

Definition 2.2 — Null Hypothesis. The null hypothesis is always an equality. It is a the claim we are trying to provide evidence against. We commonly write the null hypothesis as one of the following:

$$H_0 : \mu_0 = \mu$$

$$H_0 : \mu_0 \geq \mu$$

$$H_0 : \mu_0 \leq \mu$$

Definition 2.3 — Alternative Hypothesis. The Alternative hypothesis is result we are checking against the claim. This is always some kind of inequality. We commonly write the alternative hypothesis as one of the following:

$$H_a : \mu_a \neq \mu$$

$$H_a : \mu_a > \mu$$

$$H_a : \mu_a < \mu$$

■ **Example 2.1** A towns census from 2001 reported that the average age of people living there was 32.3 years with a standard deviation of 2.1 years. The town takes a sample of 25 people and finds there average age to be 38.4 years. Test the claim that the average age of people in the town has increased. (Use an α level of 0.05)

First lets define our hypotheses:

$$H_0 : \mu_0 = 32.3 years$$

$$H_a : \mu_0 > 32.3 years$$

Now lets identify the important information:

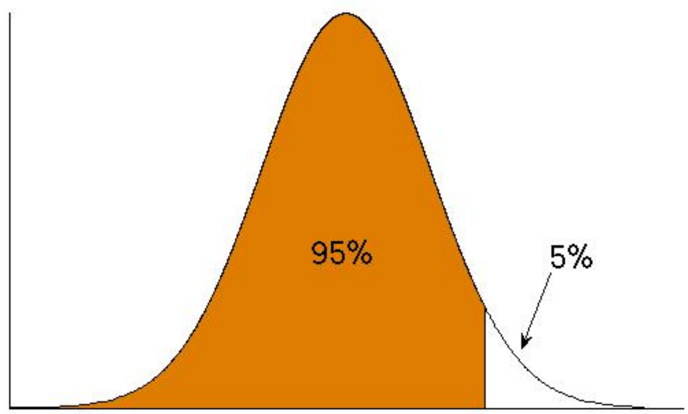
$$\bar{x} = 38.4$$

$$\sigma = 2.1$$

$$n = 25$$

$$SE = \frac{2.1}{\sqrt{25}} = 0.42$$

The last piece of important info we need is our critical value: Finding Z-critical value:



So we look up as close as we can to 95%
So that gives us a Z-crit of 1.64
Once we have all our important information we can now find our test statistic:

$$z - score = \frac{38.4 - 32.3}{0.42} = 14.5238$$

Since our z-score is much bigger than our z-crit we reject the claim (reject the null) that the average age of people living there was 32.3 years. ∎

2.1.1 Error Types

Definition 2.4 — Type I Error. A Type I Error is when you reject the null when the null hypothesis is actually true. The probability of committing a Type I error is α

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	[illegible]	[illegible]	[illegible]	[illegible]	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857

Definition 2.5 — Type II Error. A Type II Error is when you fail to reject the null when it is actually false. The probability of committing a Type II error is β

2.2 Practice Problems

Problem 2.1 An insurance company is reviewing its current policy rates. When originally setting the rates they believed that the average claim amount was $1,800. They are concerned that the true mean is actually higher than this, because they could potentially lose a lot of money. They randomly select 40 claims, and calculate a sample mean of $1,950. Assuming that the standard deviation of claims is $500, and set $\alpha = 0.05$, test to see if the insurance company should be concerned.

Problem 2.2 Explain a type I and type II error in context of the problem. Which is worse?