

950-565 Health Informatics

Biomedical Signal Processing

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Biomedical Signals

Outline

- EKG/ECG – Electrocardiogram
- IP – Impedance Pneumogram
- PPG – Photoplethysmogram
- EEG – Electroencephalogram
- Other signals

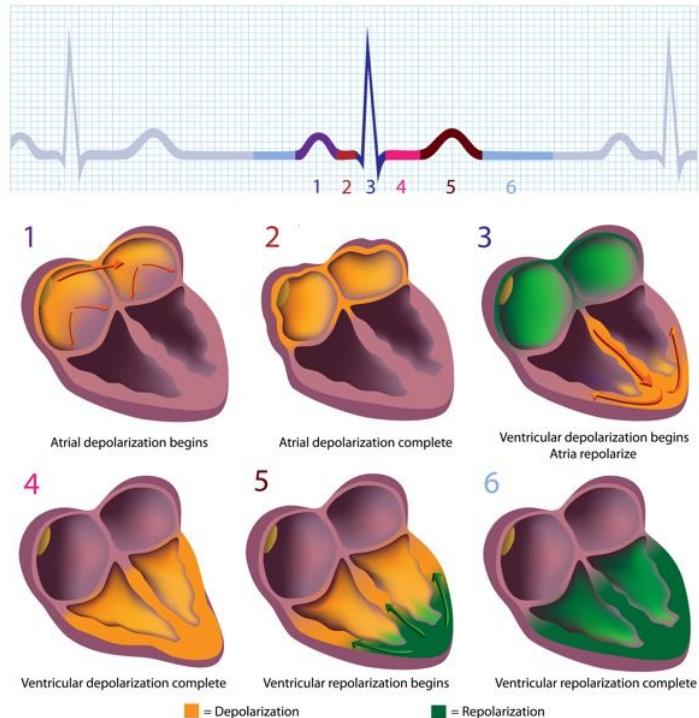
Electrocardiogram (EKG/ECG)

Electrocardiogram:

- *electro/elektrō* – electrical, electricity
- *cardio/kardia* – pertaining to the heart
- *gram* – written, drawn

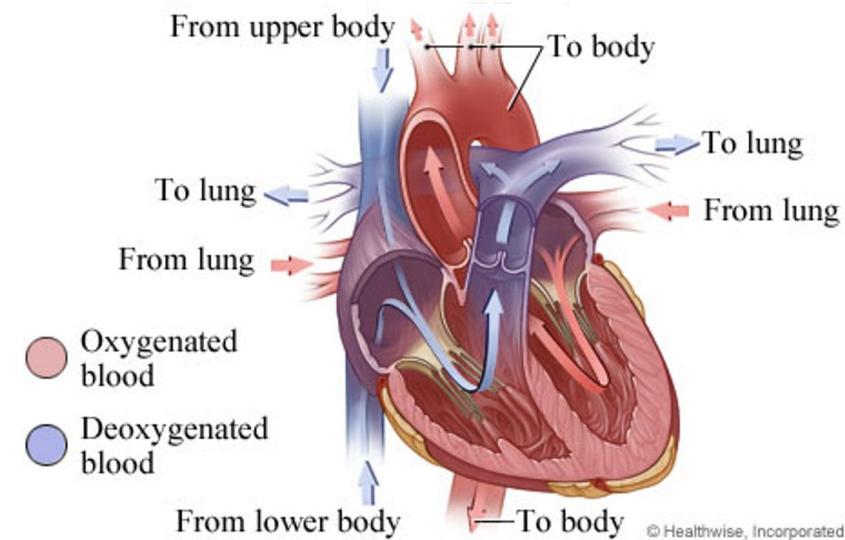
EKG/ECG a graph of voltage versus time of the electrical activity of the heart using electrodes placed on the skin.

ECG and electrical activity of the myocardium



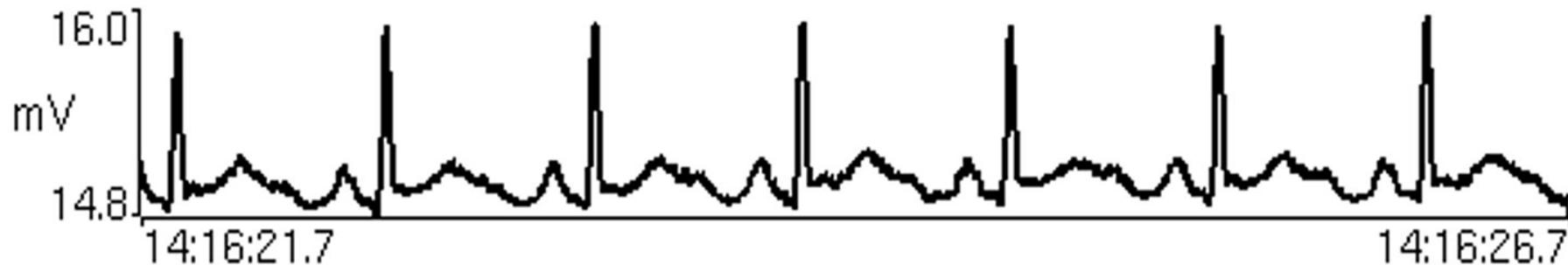
Electrocardiogram (EKG/ECG)

- The heart is an electrical organ, and its activity can be measured non-invasively
- Wealth of information related to:
 - The electrical patterns proper
 - The geometry and mechanical properties of the heart's tissue
 - The metabolic state of the heart
- The ECG is a standard tool used in a wide-range of medical evaluations

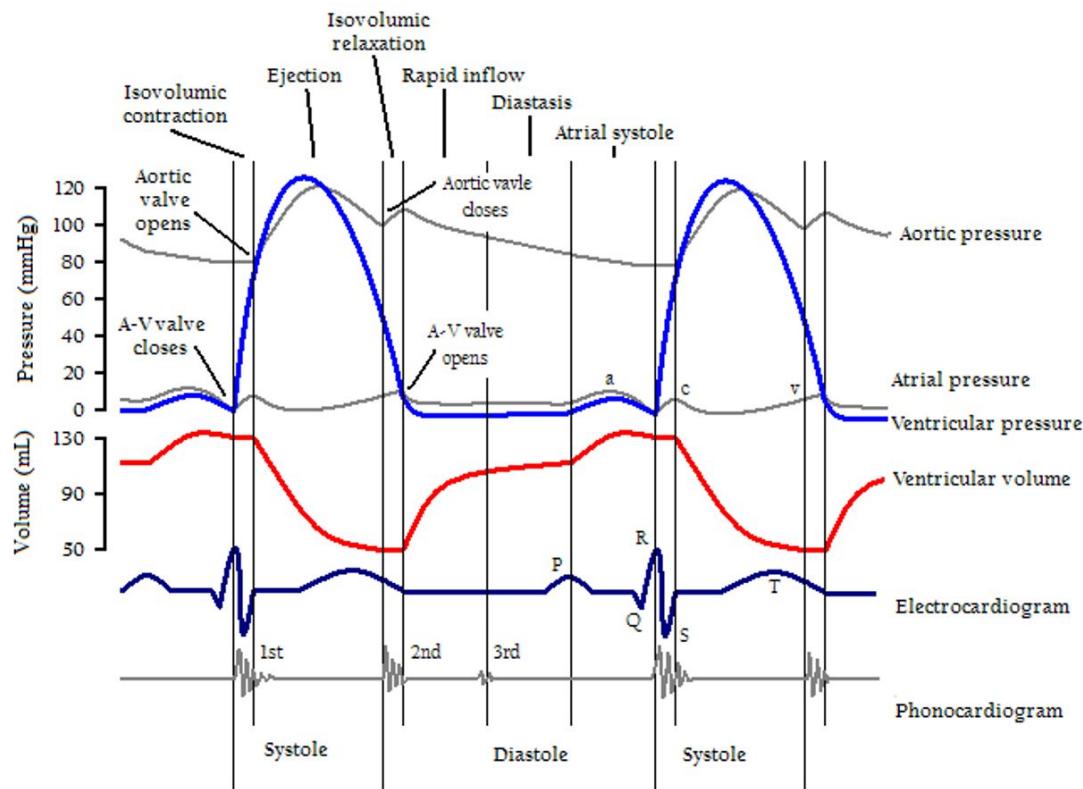


The Electrocardiogram

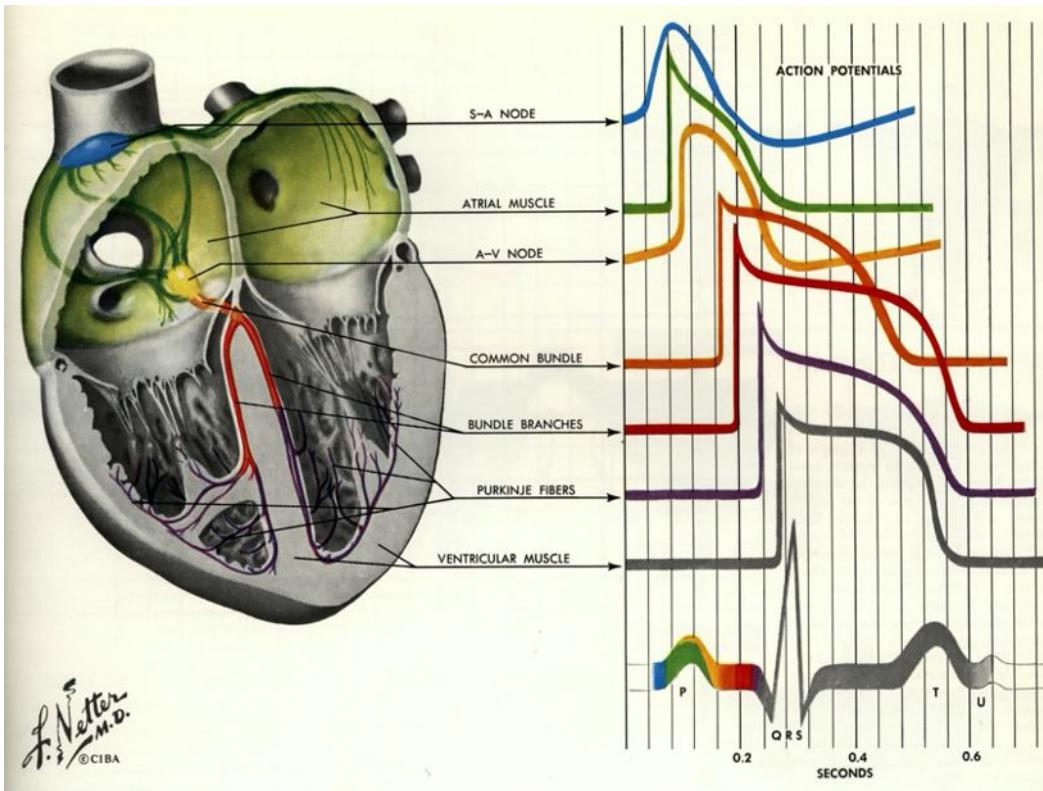
If two surface electrodes are attached to the upper body (thorax),
the following electrical signal is observed:



Pressure, ECG and sound of a beat

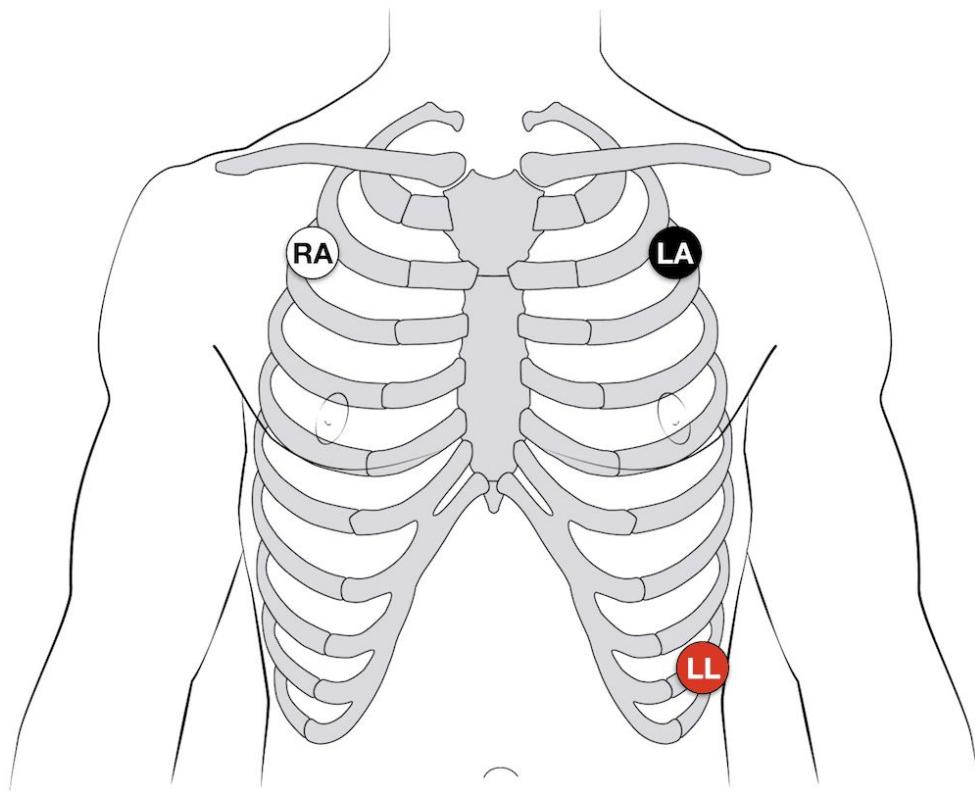


To understand the ECG

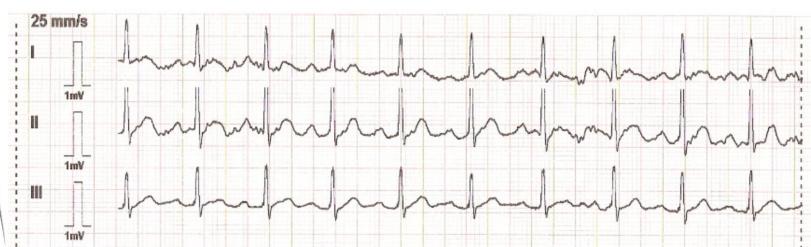


- Electrophysiology of a single cell
- How a wave of electrical current propagates through myocardium
- Specific structures of the heart through which the electrical wave travels
- How that leads to a measurable signal on the surface of the body

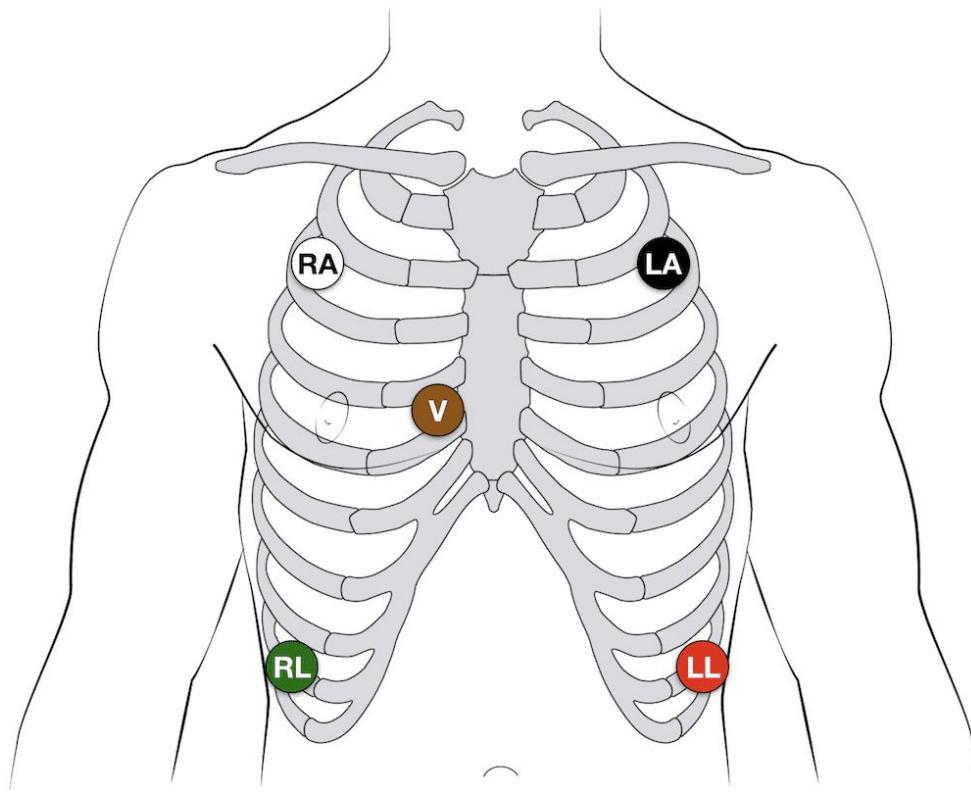
3-electrode system



- Uses 3 electrodes (RA, LA and LL)
- Used often for recording a 24-hour reading.
- Low diagnostic Capabilities: ischemia cannot be recognized and the informative value of arrhythmias and disorders is severely limited.

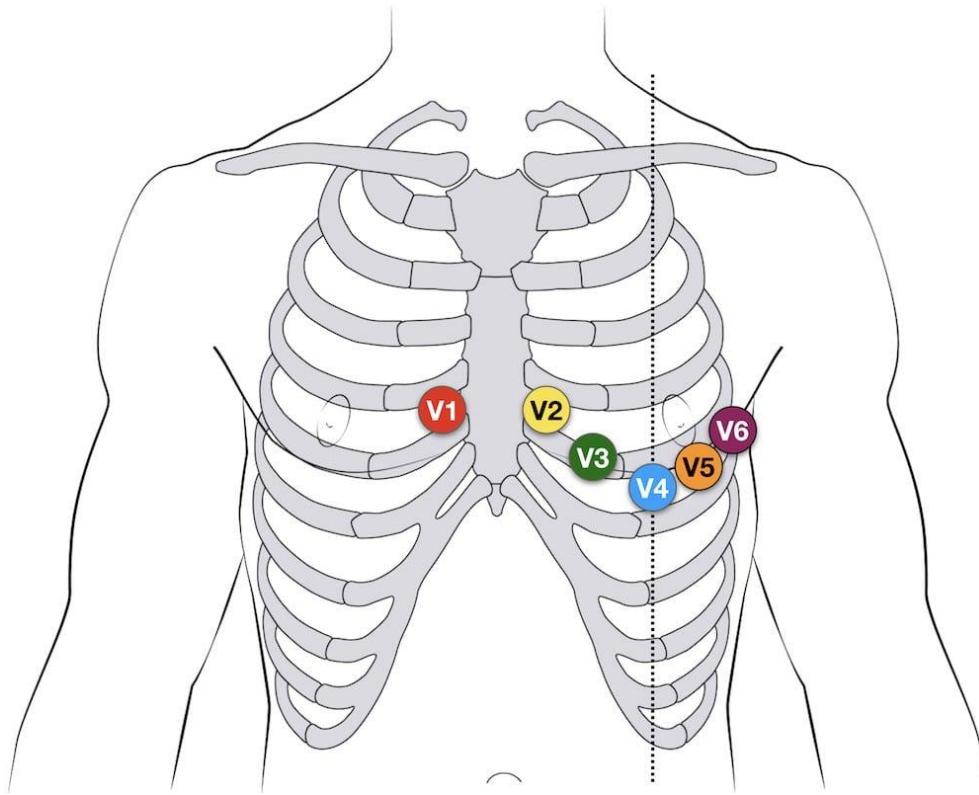


5-electrode system



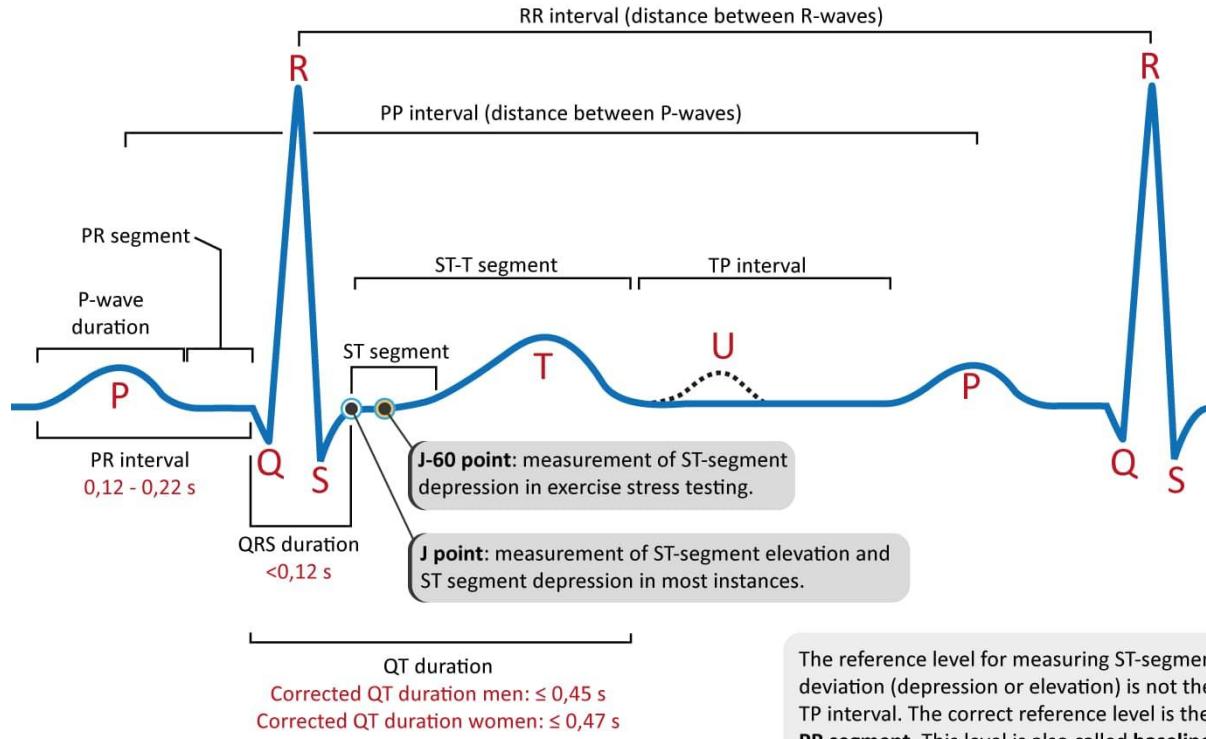
- Uses 5 electrodes (RA, RL, LA, LL and V)
- Enable the monitoring of extra leads and help improve ST elevation readings.
- Most current day intensive care monitors use a five electrode monitoring cable.
- More diagnosis capabilities compared to the 3-lead system.

12-lead ECG



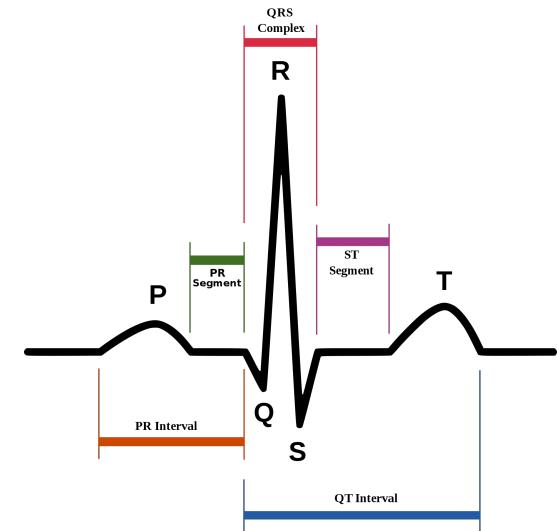
- 10 electrodes required to produce 12-lead ECG
- 4 Electrodes on all 4 limbs (RA, LL, LA, RL) and 6 Electrodes on precordium (V1–6)
- The 12-lead ECG is the gold standard for ECG diagnosis and is used for both resting and stress ECGs.
- All heart activities can be measured.

Typical ECG waveform

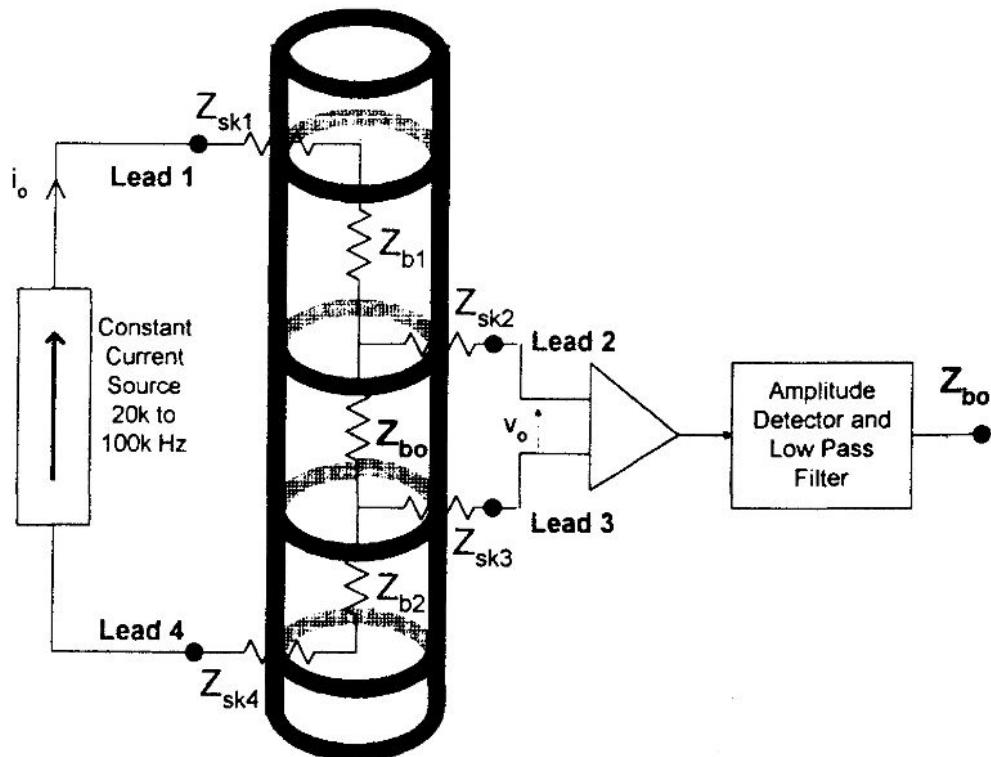


Components of ECG waveform

- **P-wave:** a small low-voltage deflection caused by the depolarisation of the atria prior to atrial contraction.
- **QRS complex:** the largest-amplitude portion of the ECG, caused by currents generated when the ventricles depolarise prior to their contraction.
- **T-wave:** ventricular repolarisation.
- **P-Q interval:** the time interval between the beginning of the P-wave and the beginning of the QRS complex.
- **Q-T interval:** characterises ventricular repolarisation.

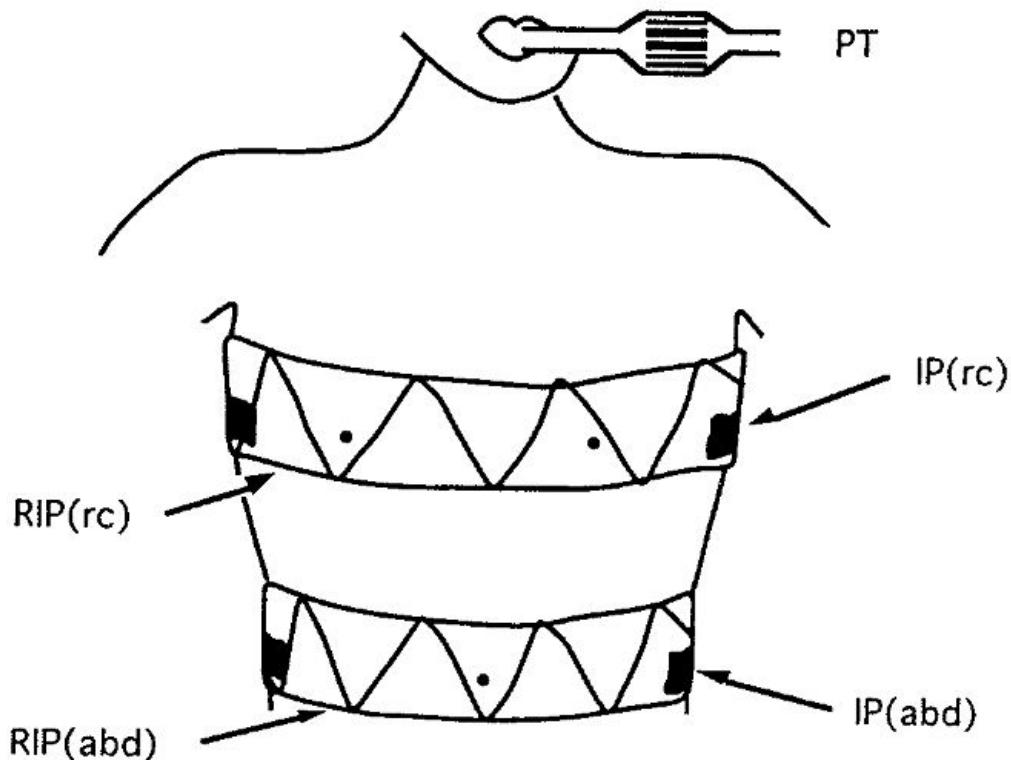


Impedance Pneumogram (IP)



- 4-electrode method to measure electrical impedance (Z) at 20 kHz
- Most of the changes in Z across the chest are due to breathing (although some are also caused by blood flow in and out of the heart)

Impedance Pneumogram (IP)



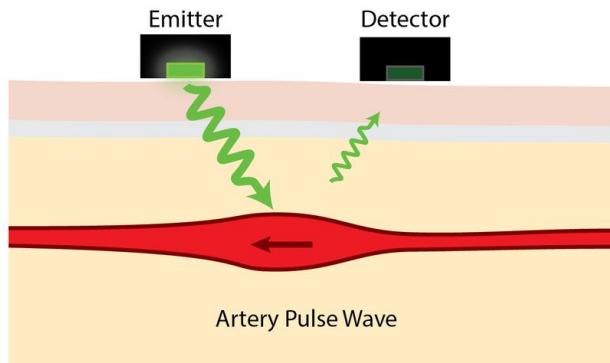
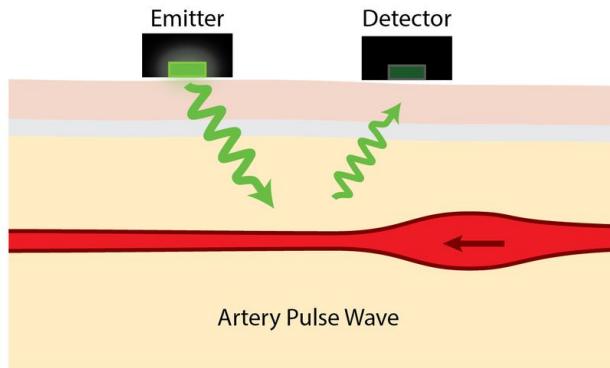
- There are no other clinically acceptable methods of monitoring respiration non-invasively.
- 2-electrode configuration (using ECG electrodes) is sufficient to obtain respiration rate.

Photoplethysmogram (PPG)

Photoplethysmogram:

- *photo* – light
- *plethysmo* – change in volume
- *gram* – written, drawn

The measurement of variations in blood volume
or blood flow in the body

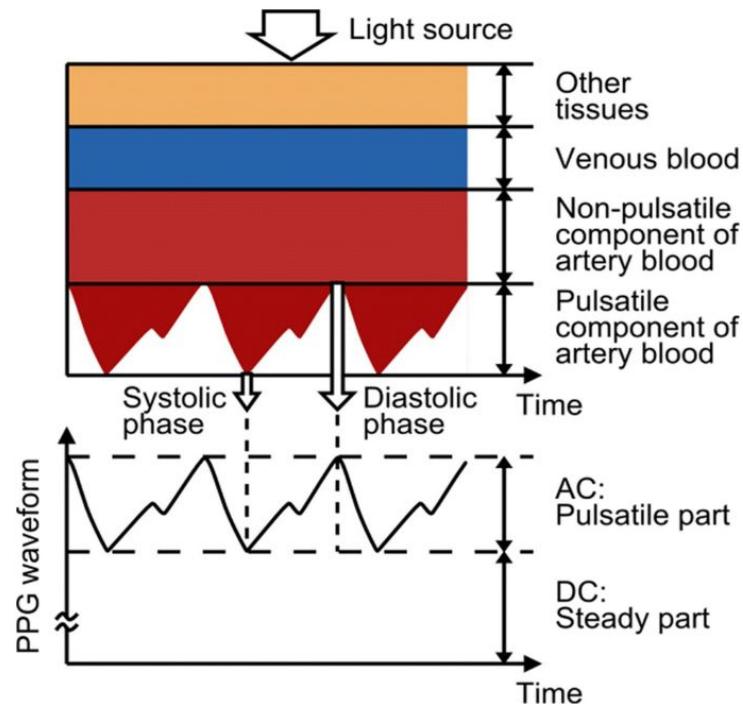


Tamura T, Maeda Y, Sekine M, Yoshida M. Wearable photoplethysmographic sensors-past and present.

Electronics. 2014;3(2):282–302.

Photoplethysmogram (PPG)

- The measurement of variations in blood volume or blood flow in the body
- PPG is a non-invasive method, using a light source and a high precision light sensor to detect volume of blood flow in order to understand the fluctuation in heart rate.
- Compared to other types, it is easy to set up, low in cost and does not require direct skin contact. PPG sensors are also less prone to motion artefact when body is in motion.

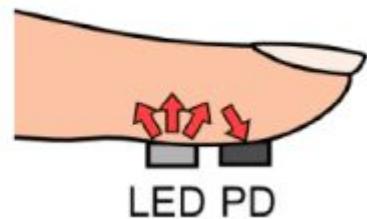
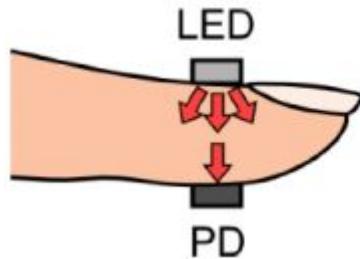


Optics

The wearable PPG has two modes

- **Transmission** – the light transmitted through the medium is detected by a PD opposite the LED source
- **Reflectance** – the PD detects light that is back-scattered or reflected from tissue, bone and/or blood vessels.

Transmission mode vs Reflectance mode



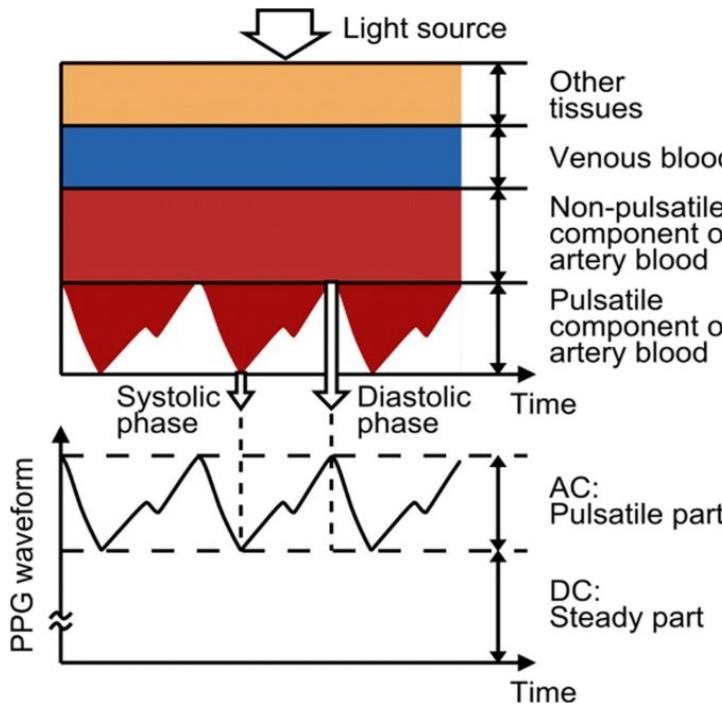
Transmission mode

- Good signal quality
- Limited measurement sites (only fingertip, nasal septum, cheek, tongue, or earlobe)

Reflectance mode

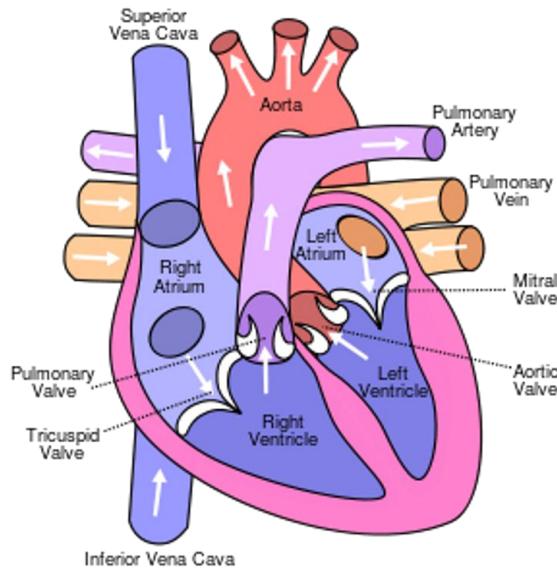
- Prone to artefacts

Light absorption

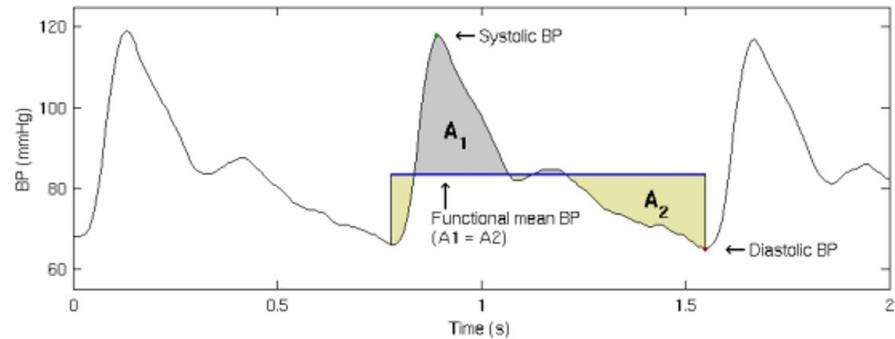


- The attenuation of light by the finger can be split into 3 independent components: arterial blood, venous blood and tissues
- Only that part of the signal directly related to the inflow of arterial blood into the body segment is used for the calculation of heart rate.

PPG Physiology

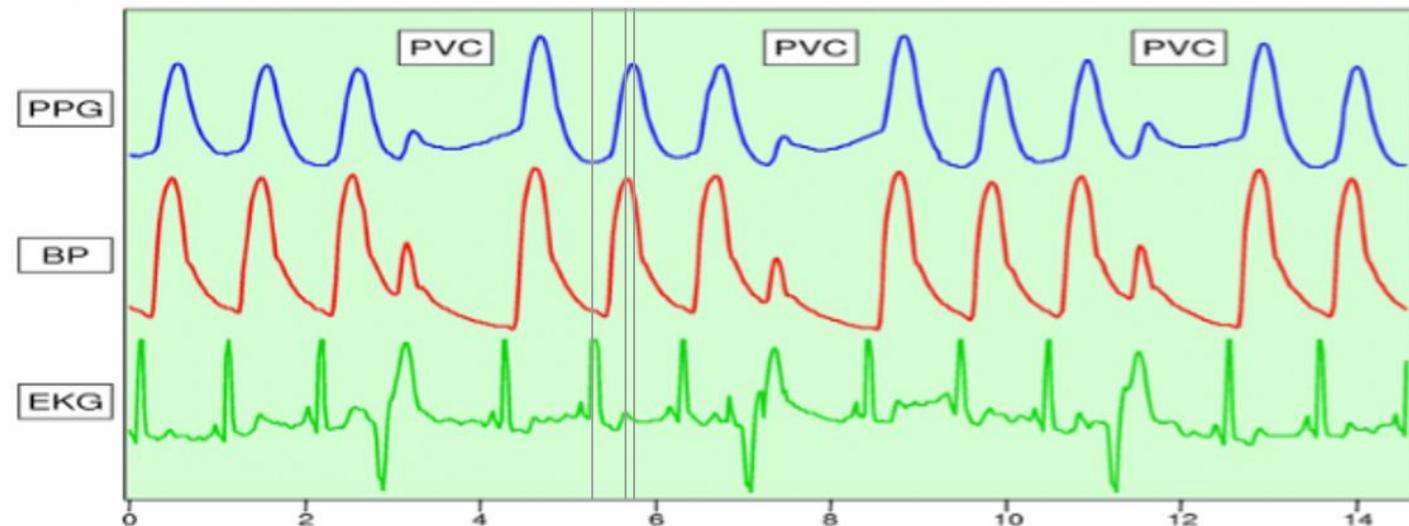


As blood is pumped through the cardiovascular system, the pressure pulse causes a change in the volume of blood at a given point.



PPG Physiology

As blood is pumped through the cardiovascular system, the pressure pulse causes a change in the volume of blood at a given point

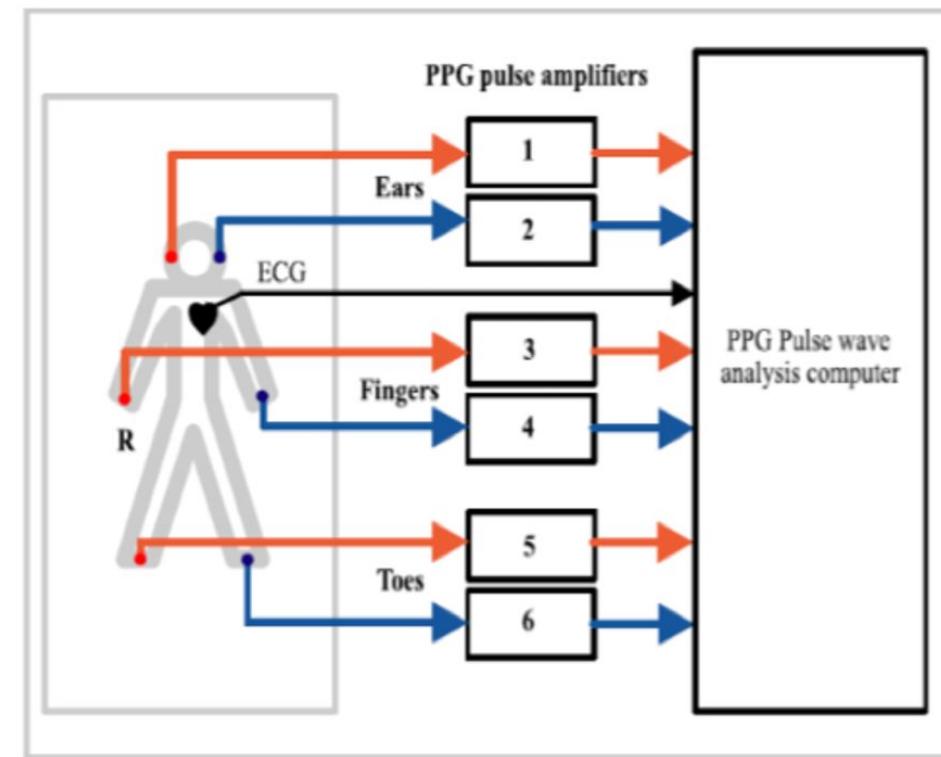


PPG – Why bother?

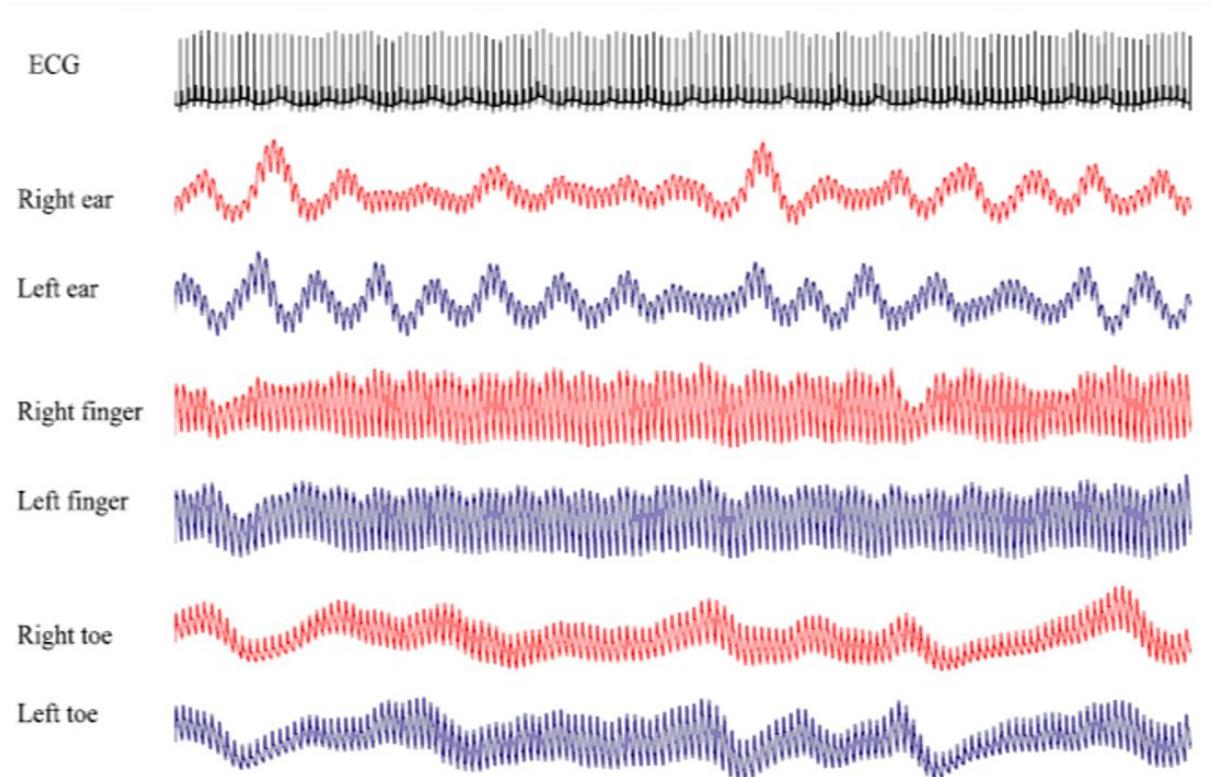
- Cheap and simple – widely used in smart watches
- Estimates of
 - Heart rate
 - Breathing rate (still difficult to estimate)
 - Blood pressure (requires at least two sensors placed at different locations)
- Is it clinically useful?

Practical considerations

- Placement of probe
- Interface pressure
- Movement artefact
- Subject posture



Location dependence of waveform morphology



Summary – PPG

- PPG is a measurement of the change in blood volume caused by cardiorespiratory events.
- Heart Rate, Heart Rate variability and Respiratory Rate can be derived from the signal
- Peripheral Oxygen Saturation (SpO_2) can be calculated by comparing the PPG at two- wavelengths.

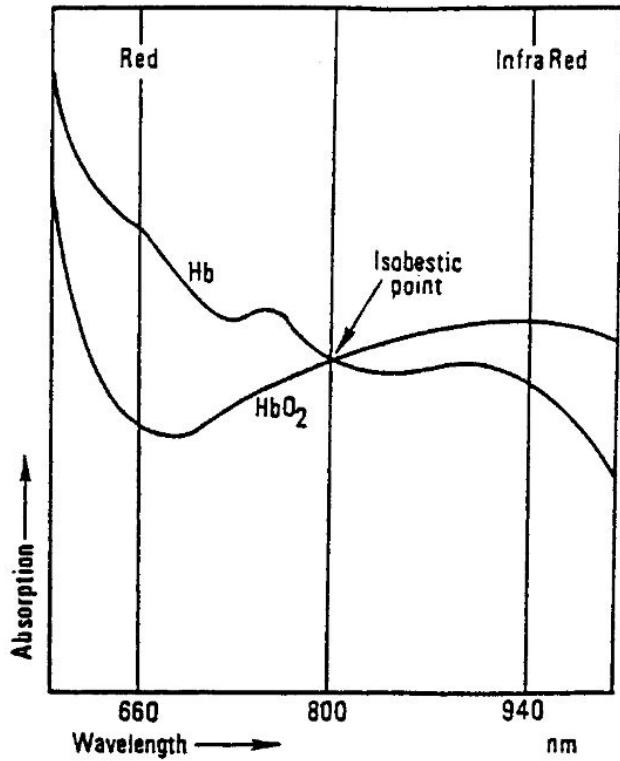
Blood oxygen saturation

The body closely monitors **blood oxygen** levels to keep them within a specific range, so that there is enough **oxygen** for the needs of every cell in the body. A person's **blood oxygen level** is an indicator of how well the body distributes **oxygen** from the lungs to the cells, and it can be **important** for people's health.

- Brain damage due to oxygen desaturation occurs within 5-minutes
- Chronic obstructive pulmonary disease (COPD) leads to low (<90%) oxygen saturation

Pulse oximetry (measurement of oxygen saturation)

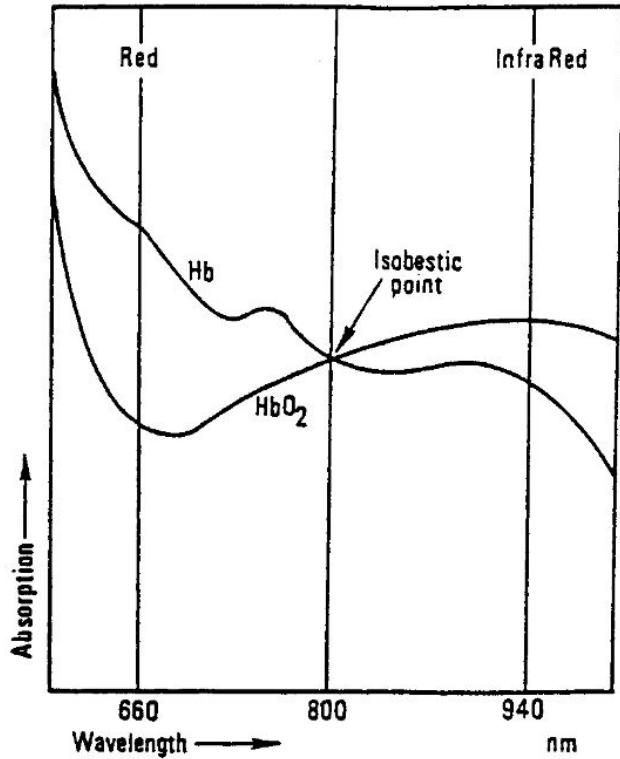
- Oximetry is a technique for measuring how much oxygen the blood is carrying, the oxygen saturation of the blood.
- The **haemoglobin** molecule (which is bound to the red blood cells) carries the **oxygen** in the blood.
- The two forms of the molecule (Hb and HbO_2) have different optical spectra.



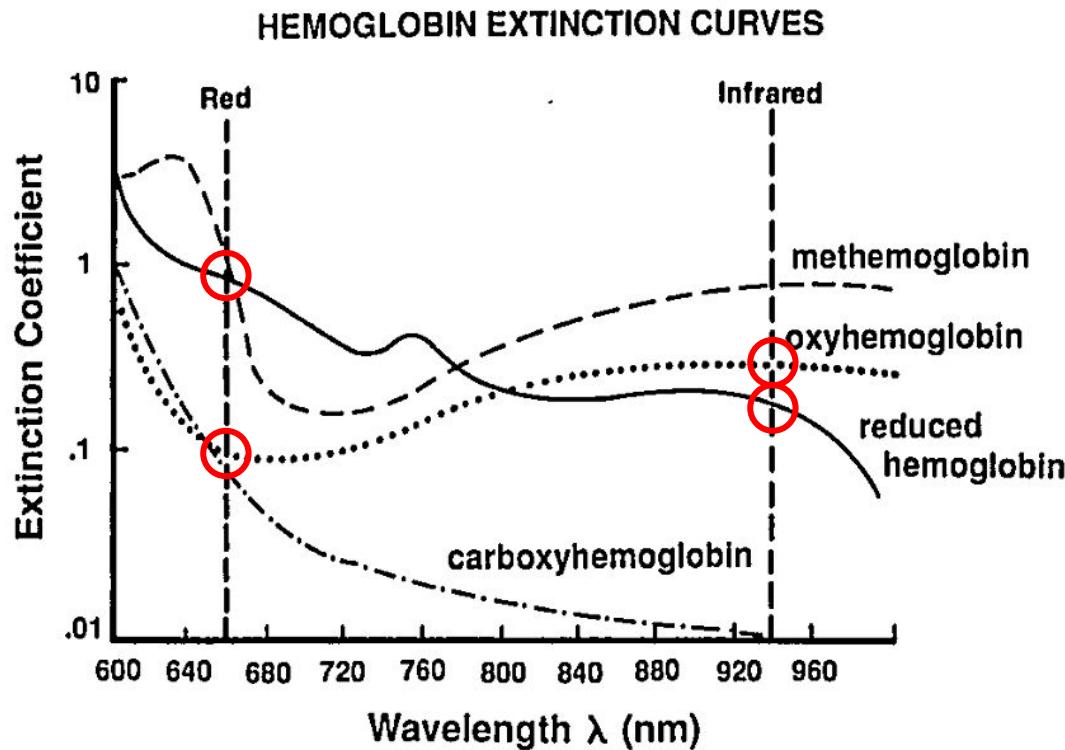
Pulse oximetry

- The wavelength range between 600 and 1,000 nm is also the range for which there is the least attenuation of light by body tissues.
- By measuring the light transmission through a body segment at two wavelengths within that range, the arterial SaO₂ can be determined.

Pulse oximetry is a non-invasive optical technique

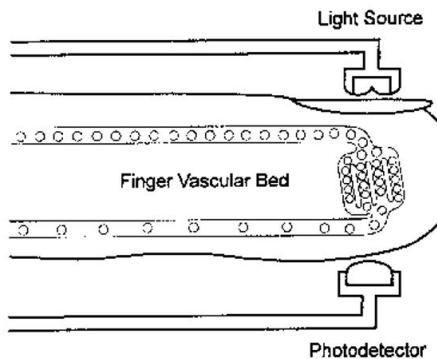
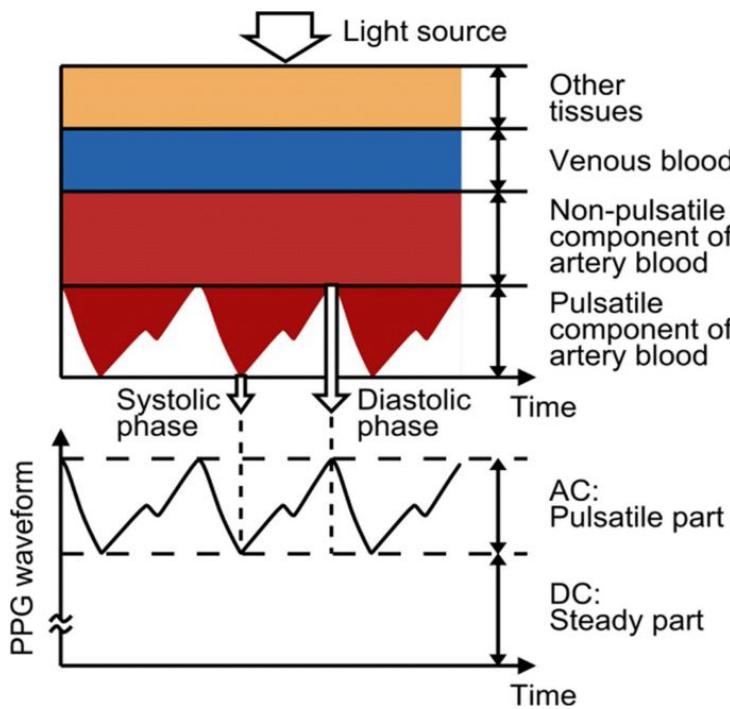


PPGs → Oxygen Saturation



Peripheral Oxygen Saturation (SpO_2) can be calculated by comparing the PPG at two wavelengths.

Measurement of oxygen saturation



$$S_pO_2 = \frac{HbO_2}{HbO_2 + Hb}$$

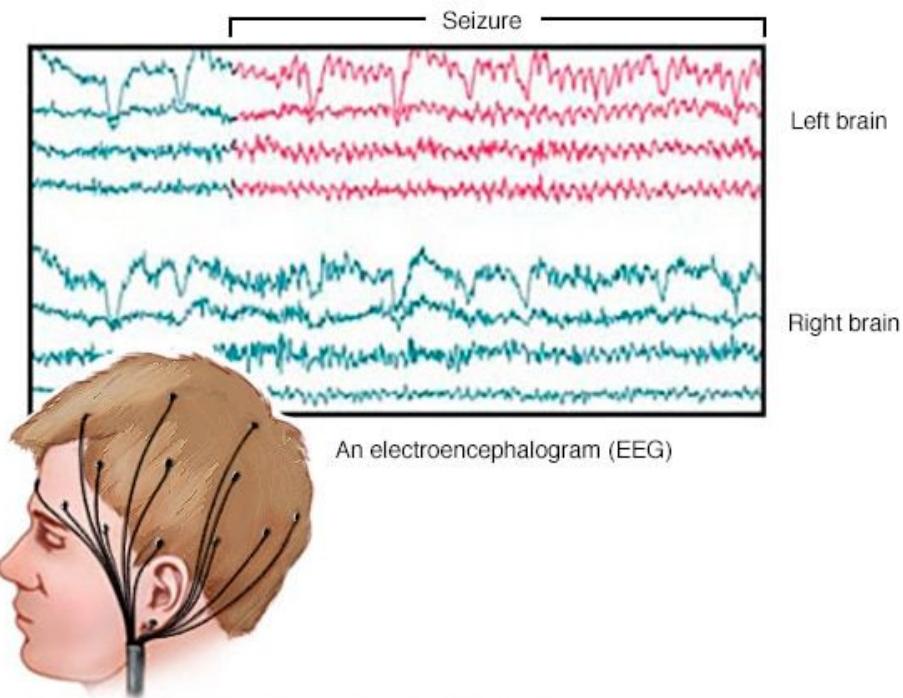
- Light absorption across the finger (or the earlobe) is measured using a single probe with two LEDs and a photodiode
- The amplitudes of the pulsatile component of light attenuation at the two wavelengths are used to derive arterial oxygen saturation

Electroencephalogram (EEG)

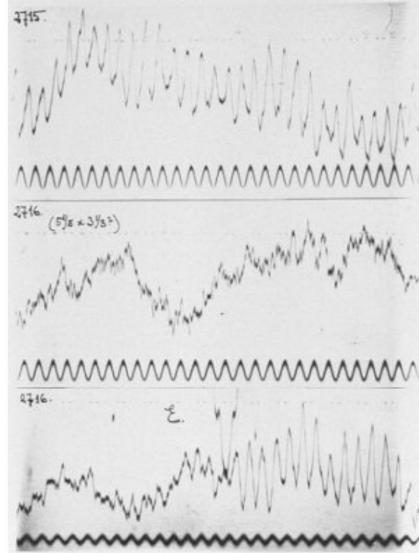
Electrocardiogram:

- *electro* – electrical, electricity
- *encephalo* – relating to the brain
- *gram* – written, drawn

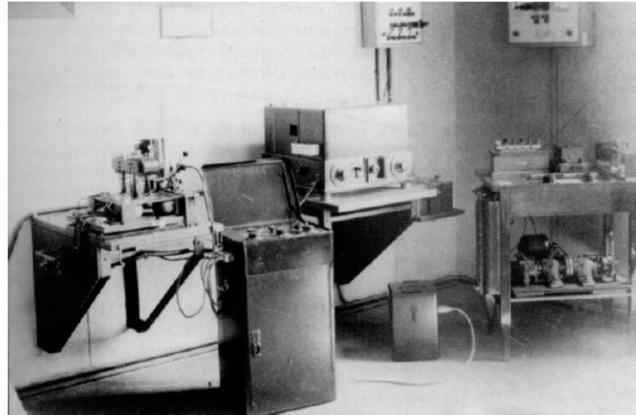
The EEG records the electrical activity of your brain via electrodes affixed to your scalp. EEG results show changes in brain activity that may be useful in diagnosing brain conditions, especially epilepsy and other seizure disorders.



History



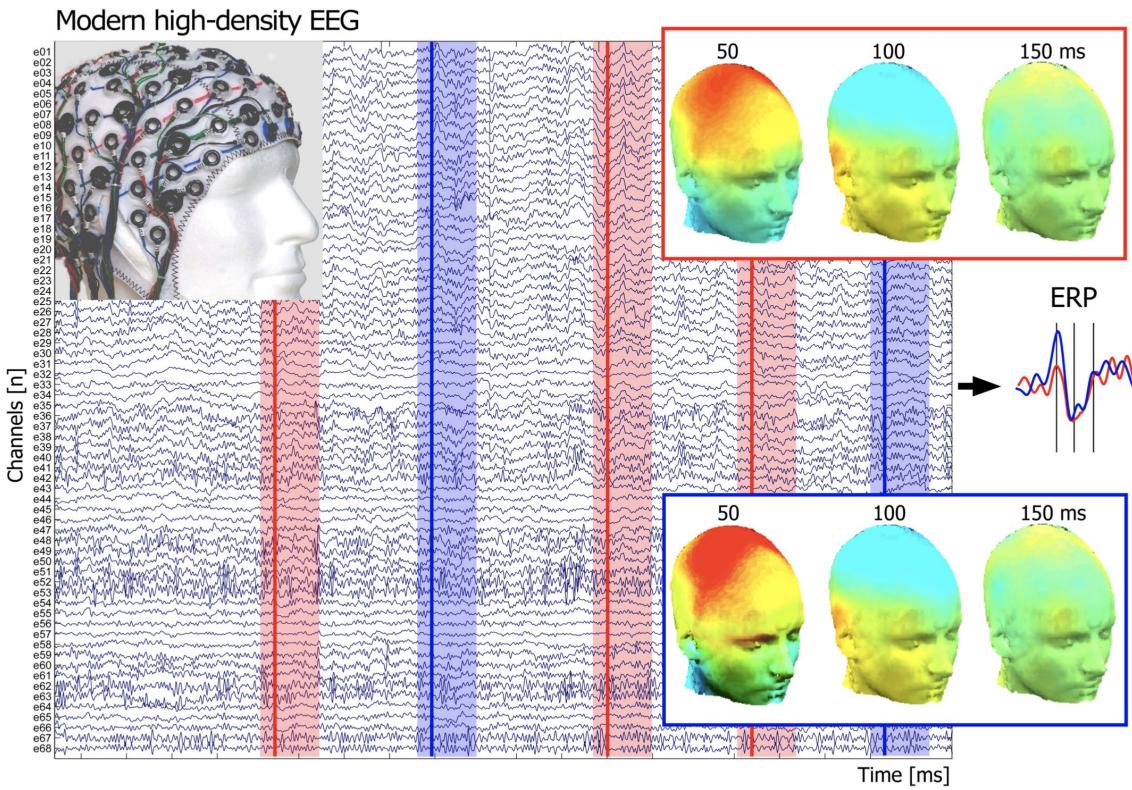
EEG recording from 1928



Hans Berger's lab, 1926



Present day high-density EEG



Electrical activity in the brain

What EEG does reflect?

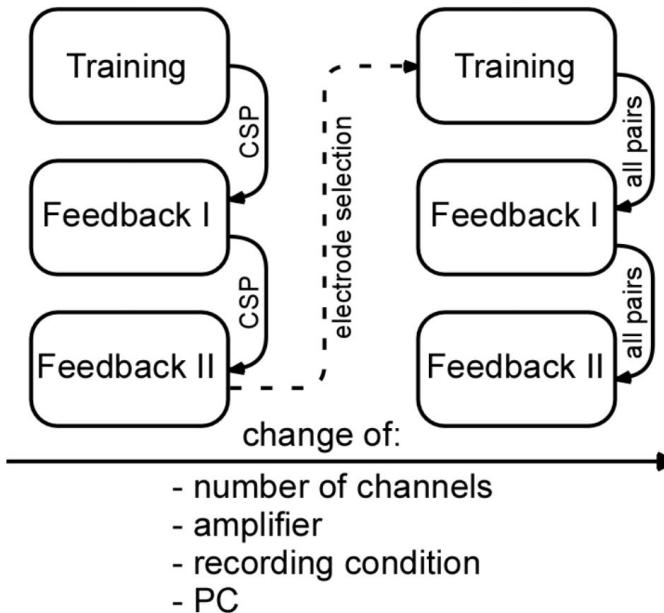
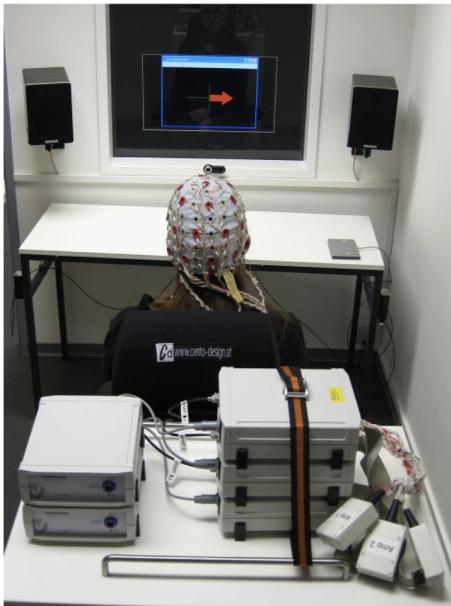
- Excitatory and inhibitory postsynaptic potentials at apikal dendrites of many synchronised pyramidal cells

What EEG doesn't reflect?

- Single neurons
- Asynchronous activities
- Action potentials
- Gila cells
- Subcortical structures

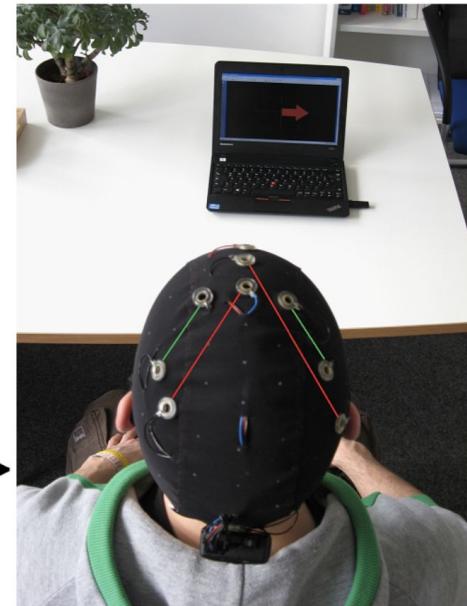
Motor imaginary study design

Day 1



EEG preparation time: 60 mins

Day 2-4



EEG preparation time: 5 mins

Neonatal brain monitoring



- EEG monitoring in neonatal intensive care unit (NICU)
- Treat seizures with medication
- Predict outcome
- No neurological experts present 24/7
- Artifacts obscure the EEG

Summary – EEG

- EEG can give a rough picture of what is going on in the brain.
- The EEG is used to evaluate several types of brain disorders. The test can also be used to diagnose other disorders that influence brain activity, such as Alzheimer's disease, certain psychoses, and a sleep disorder called narcolepsy.
- The EEG has been used for many years and is considered a safe procedure. The test causes no discomfort. The electrodes record activity. They do not produce any sensation. In addition, there is no risk of getting an electric shock.
- Signal processing becomes more sophisticated when you want to analyse single trial brain responses.

Other biomedical signals

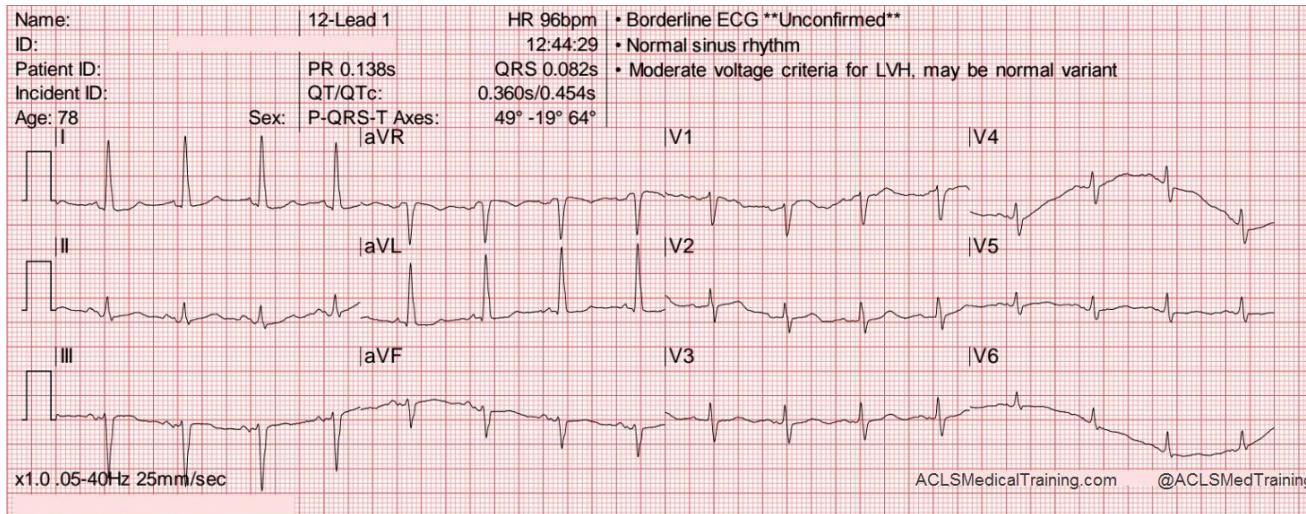
- Electromyogram (EMG) is used to identify muscle activity
- Electro-oculogram (EOG) is used to identify rapid eye movement (indicates dreaming)
- Electronystagmography (ENG) is a diagnostic test to record involuntary movements of the eye.
- Polysomnography – a multi-parametric sleep test
- Speech

Pre-processing

- Medical signals are often corrupted by
 - Movement artefact
 - Muscle tremor artifact
 - Mains frequencies noise

Movement artefact

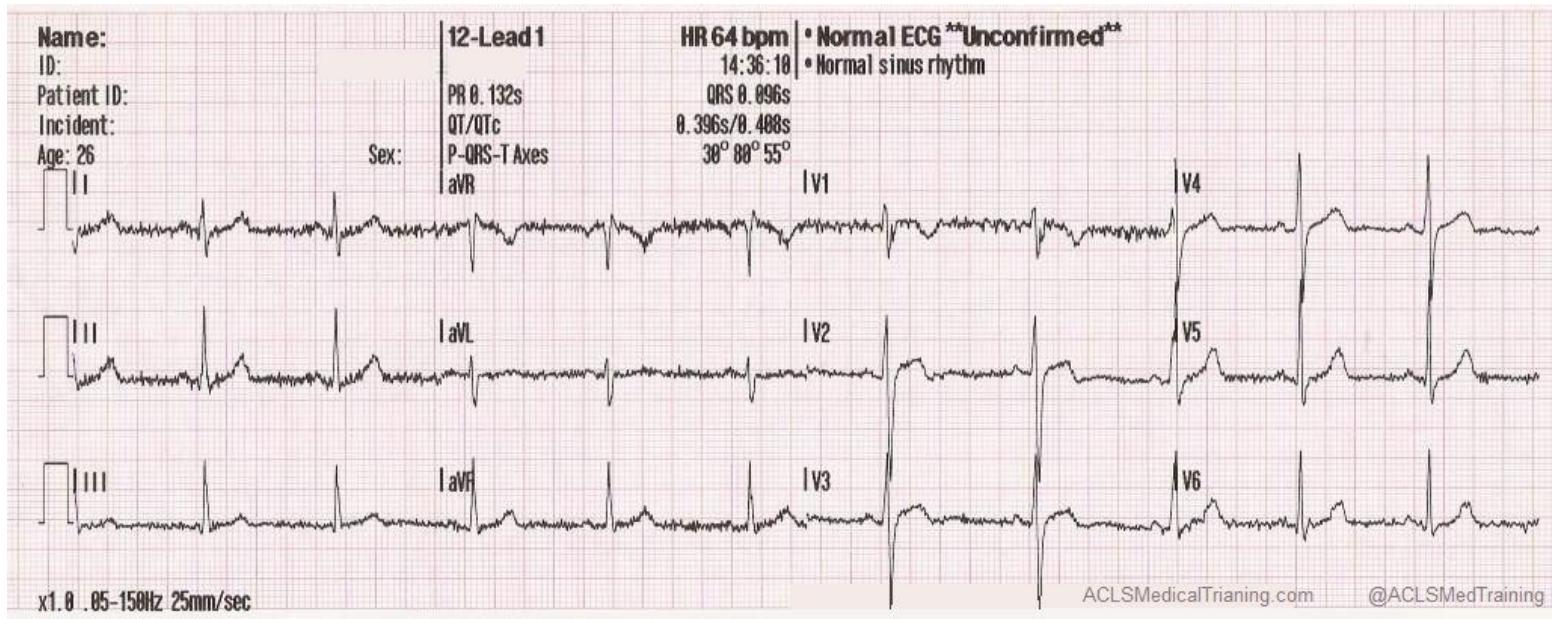
- Computation of heart rate is often corrupted by movement artefact



- Beat-to-beat heart rate is the reciprocal of the interval between two successive R-peaks in the ECG (x60 to give beats per minute)

Muscle tremor artifact

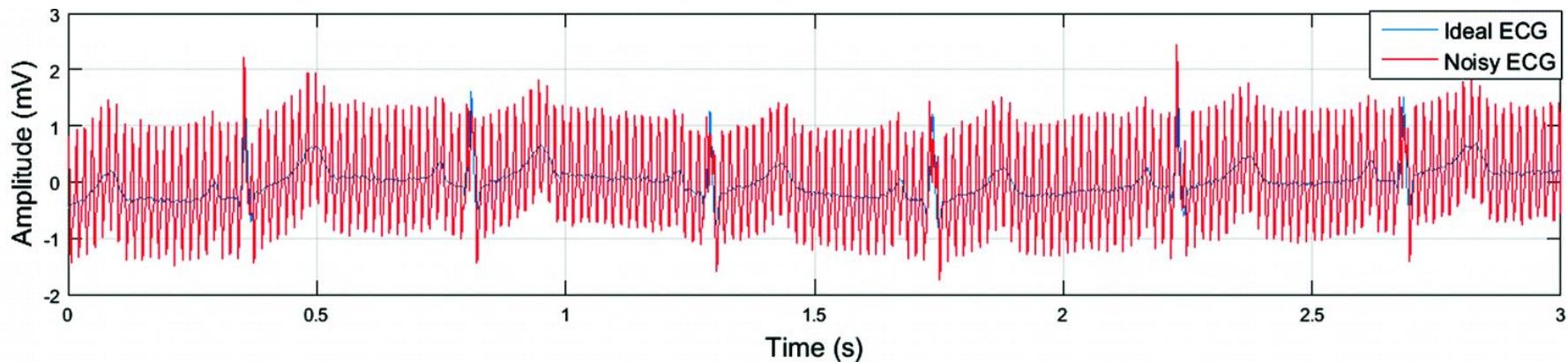
- Muscle tremor artifact is a type of motion artifact happening as a patient is cold and shivering.



Mains frequency noise

- Mains frequency noise usually results from electrical power lines, electrical equipment, and mobile telephones. In Thailand, this is referred to as 50 cycle interference. This can be removed by using signal filtering techniques.

Red – ECG suffered by 50Hz mains frequency noise
Blue – Filtered ECG signal



Pre-processing

- Medical signals are often corrupted by
 - Movement artefact
 - Muscle noise
 - Mains frequencies noise
- Noise can be removed using standard digital filtering techniques; an alternative is the use of Principal Components Analysis to smooth the signal.
- **Next: signal processing techniques**

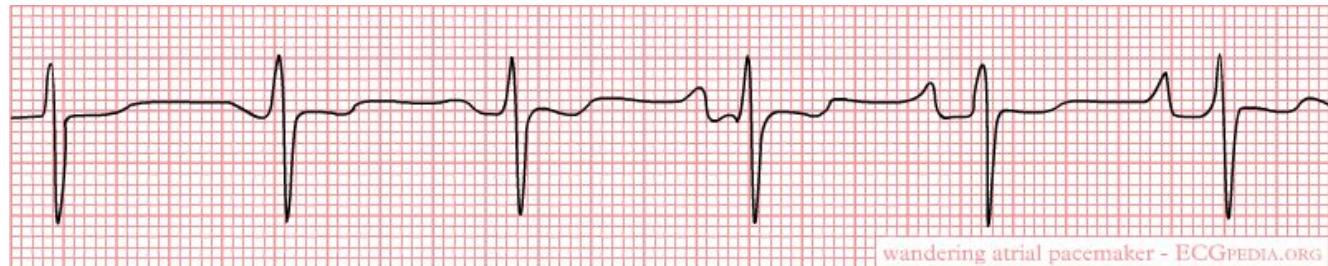
Signal Processing

Outline

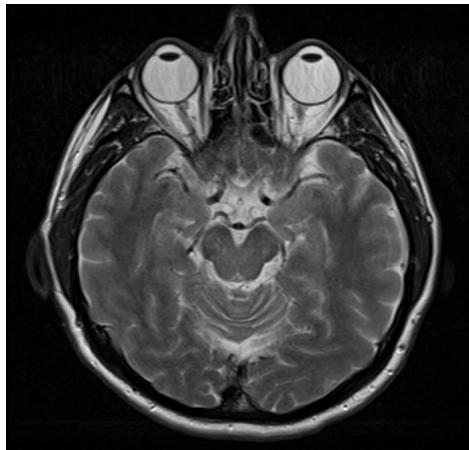
- Time and frequency domain
- Fourier's theory
- Fourier series
- Fourier transform
- Filtering
- Sampling
- Convolution
- Image reconstruction

Signals in time domain

Electrocardiogram (ECG) signal



Magnetic Resonance Image (MRI) data
as 2-dimensional signal



SET index in a day as signal (time series)

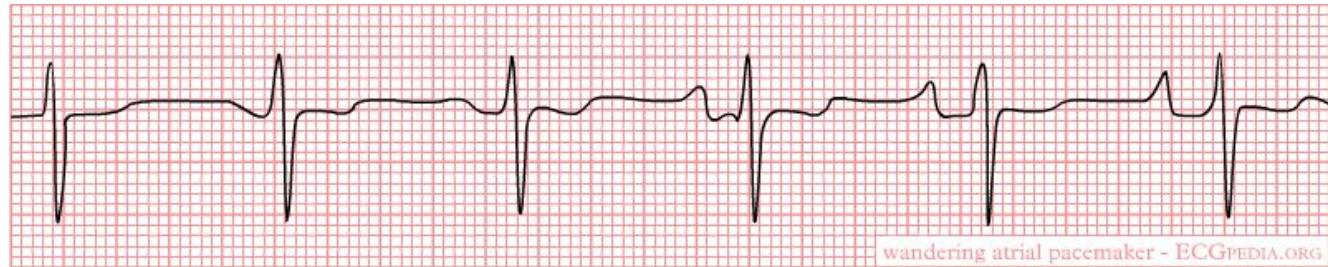


Signal classification

- Signal may be classified into:
 - Continuous-time and discrete-time signals
 - Analogue and digital signals
 - Periodic and aperiodic signals
 - Deterministic and probabilistic signals
 - Energy and power signals
 - Casual and non-causal
 - Even and odd signals

Continuous vs Discrete

Continuous-time signal
e.g. ECG signal

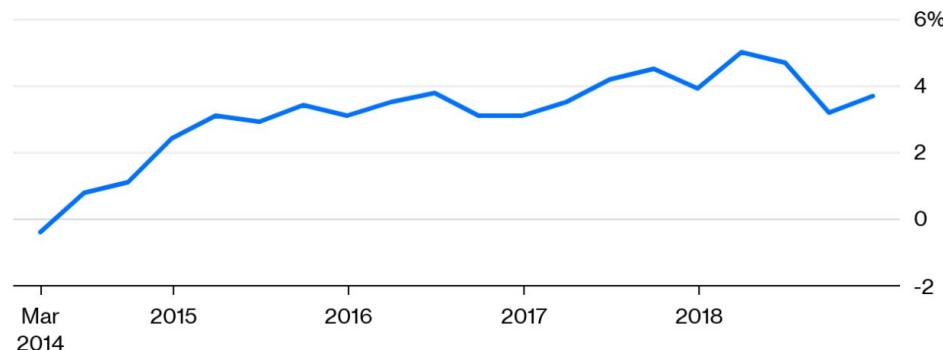


Discrete-time signal
e.g. GDP growth rate

Command Economy

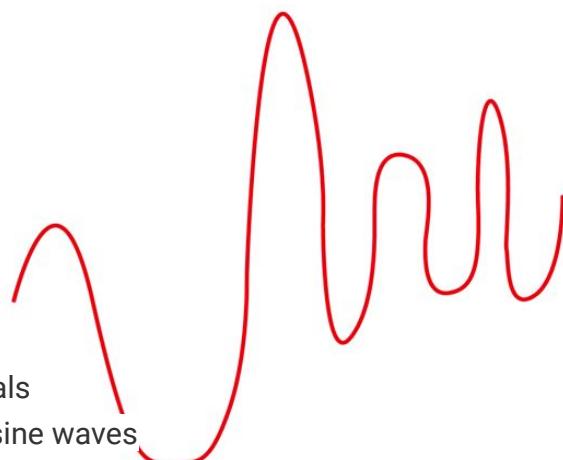
Thailand has grown steadily under its military government

GDP growth year-on-year



Analogue vs Digital

RED = ANALOG SIGNAL



Continuous signals

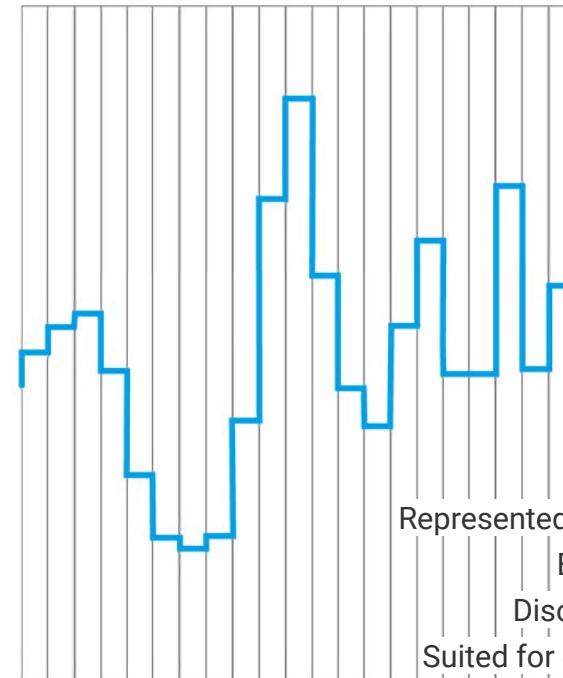
Represented by sine waves

Human voice, natural sound, analogue devices

Continuous range of values

Only be used in analog devices.

BLUE = DIGITAL QUANTIZED SIGNAL



Discrete signals

Represented by square waves

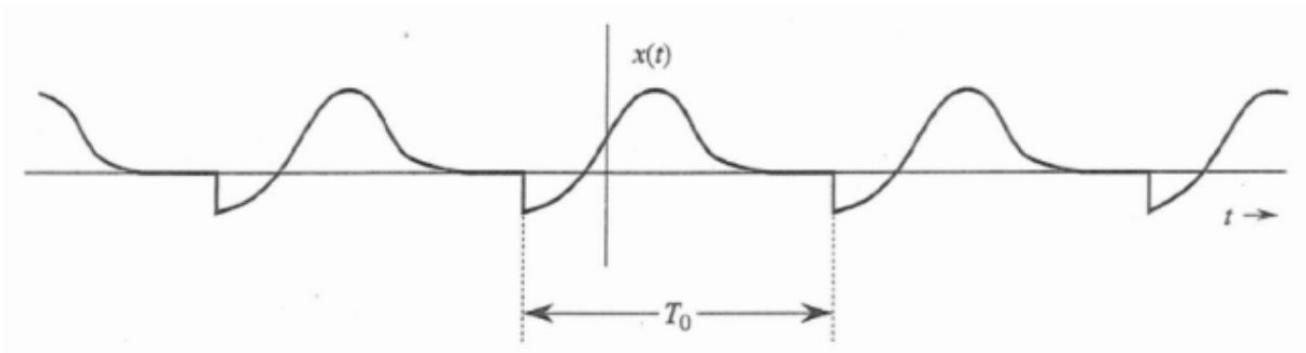
Electronic devices

Discontinuous values

Suited for digital electronics

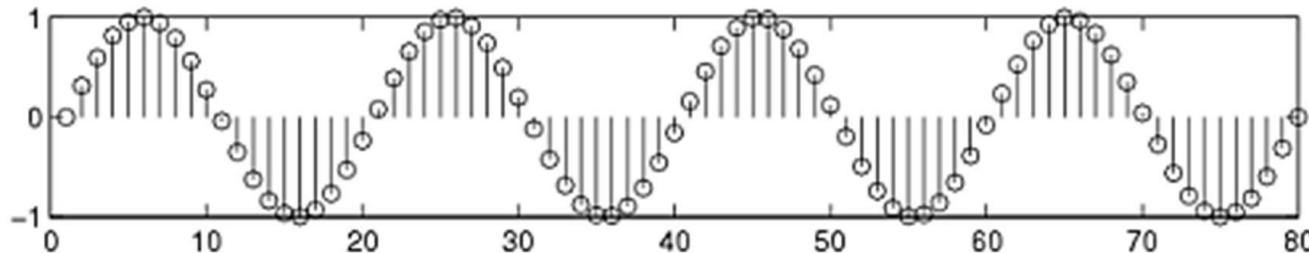
Periodic vs Aperiodic

- A signal $x(t)$ is said to be periodic if for some positive constant T_0
$$x(t) = x(t + T_0) \quad \text{for all } t$$
- The smallest value of T_0 that satisfies the periodicity condition of this equation is the **fundamental period** of $x(t)$.

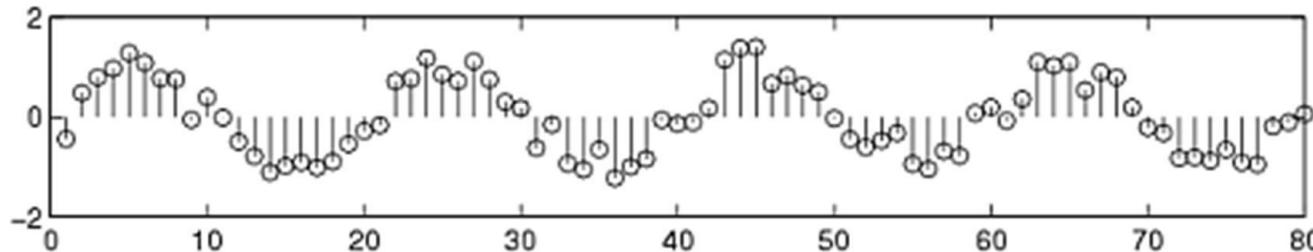


Deterministic vs Probabilistic

- A **deterministic signal** behaves in a fixed known way as a function of time.
 - There is no certainty about a deterministic signals value at any given time.



- A **random signal** is a signal with which there is uncertainty before it occurs.

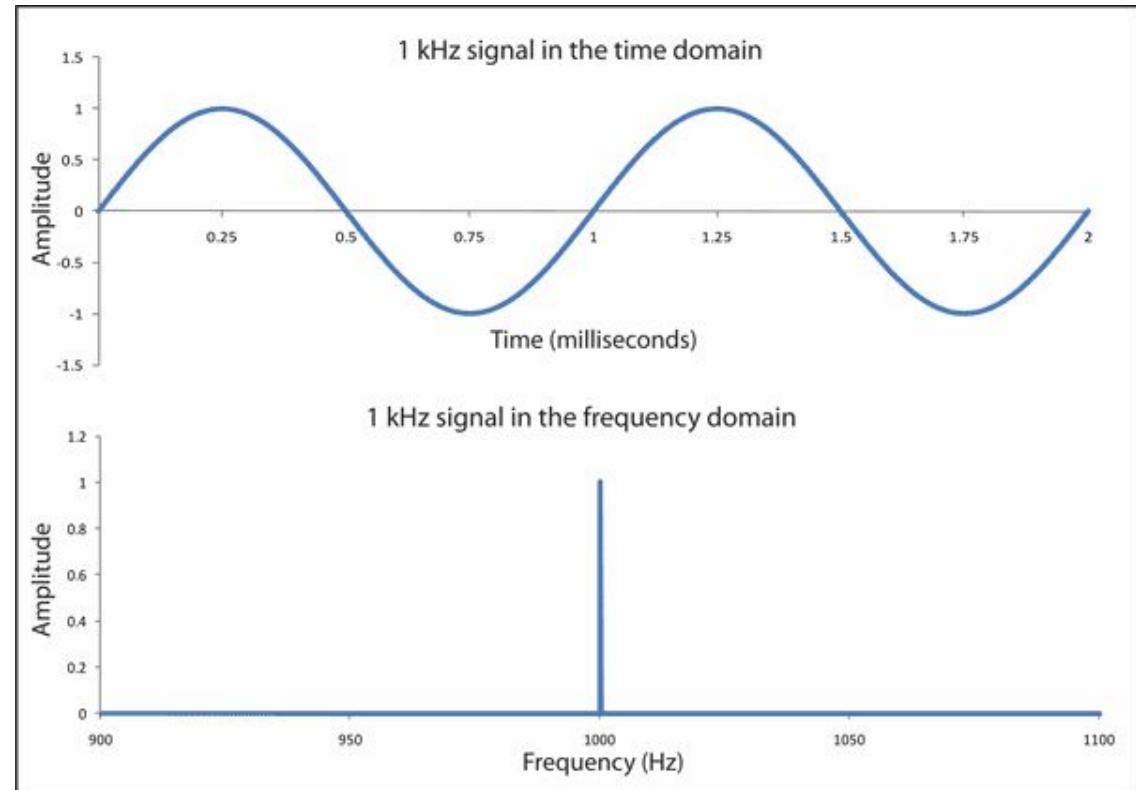


Sine wave – Time vs Frequency domain

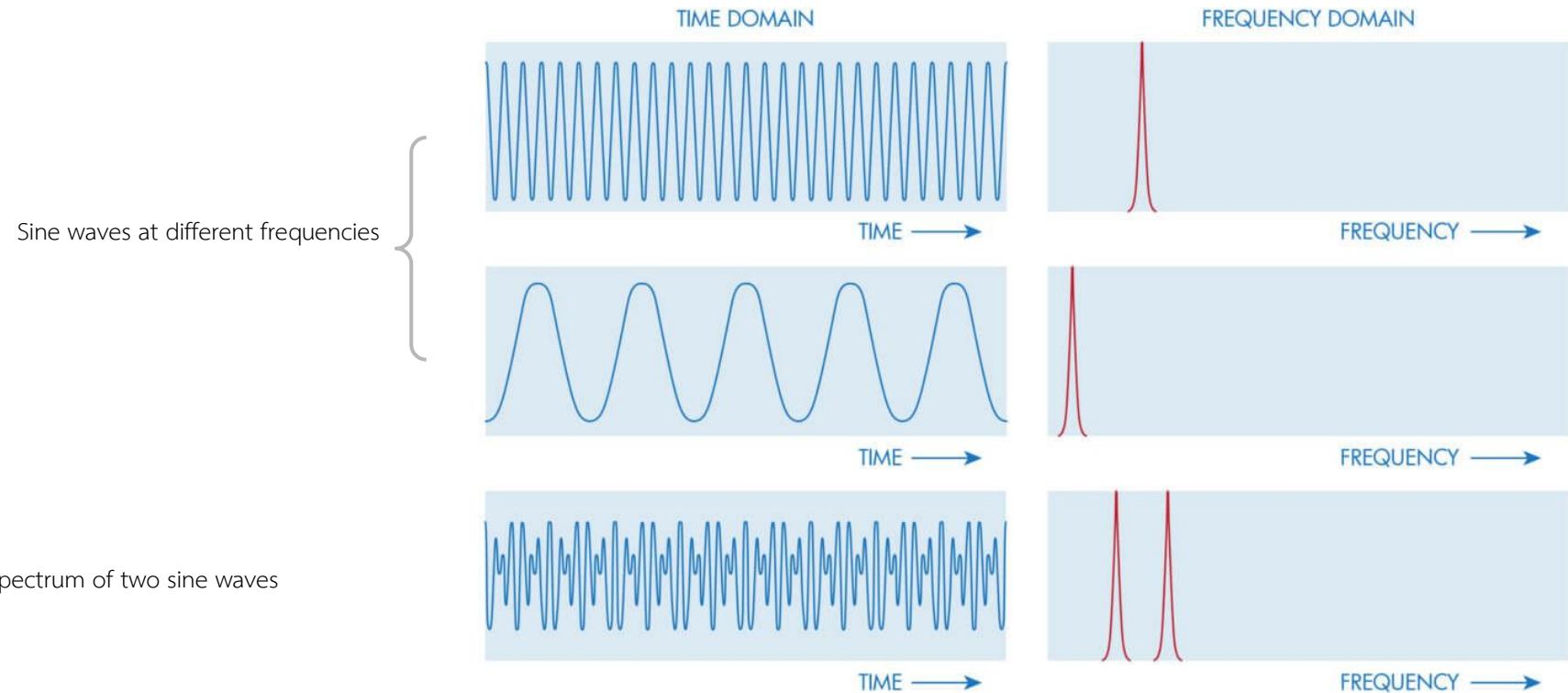
- Sine wave (sinusoidal signal) in time domain

$$y = \sin(2\pi t)$$

- Same sine wave in frequency domain



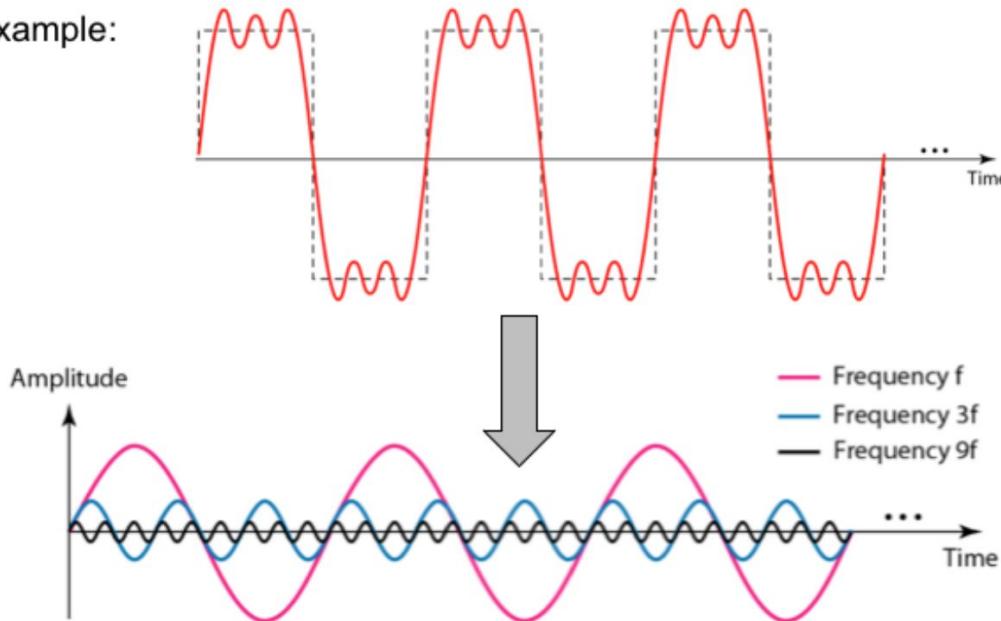
Two sine waves – Time vs Frequency domain



Fourier's theory

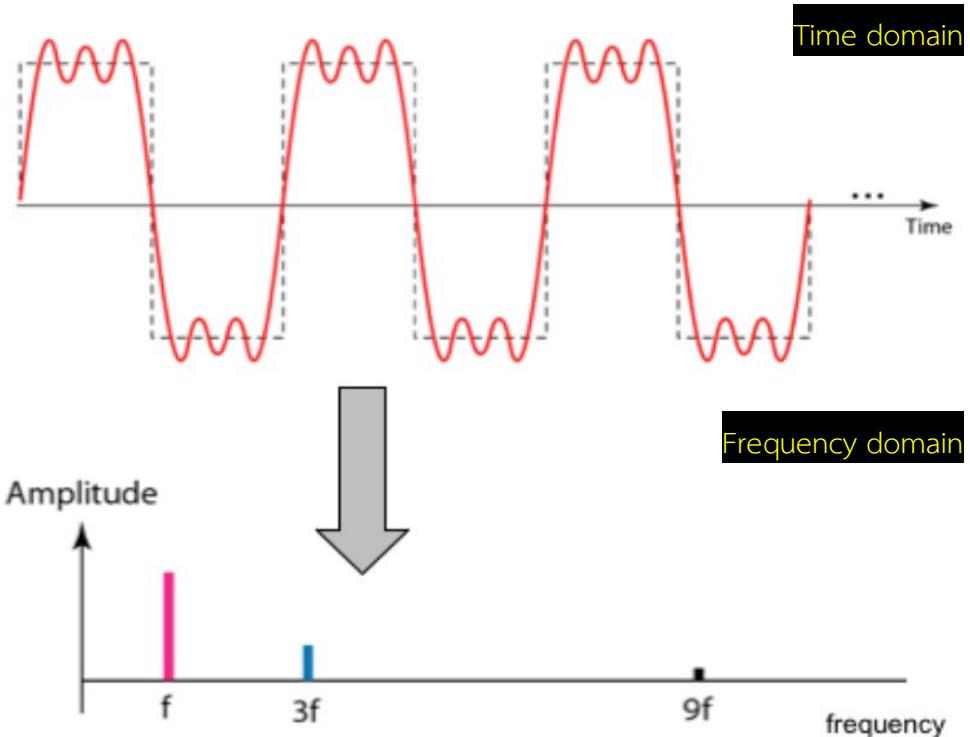
Any time domain signal can be constructed from weighted linear sum of sinusoidal signals (sine or cosine signals) at different frequencies.

For example:

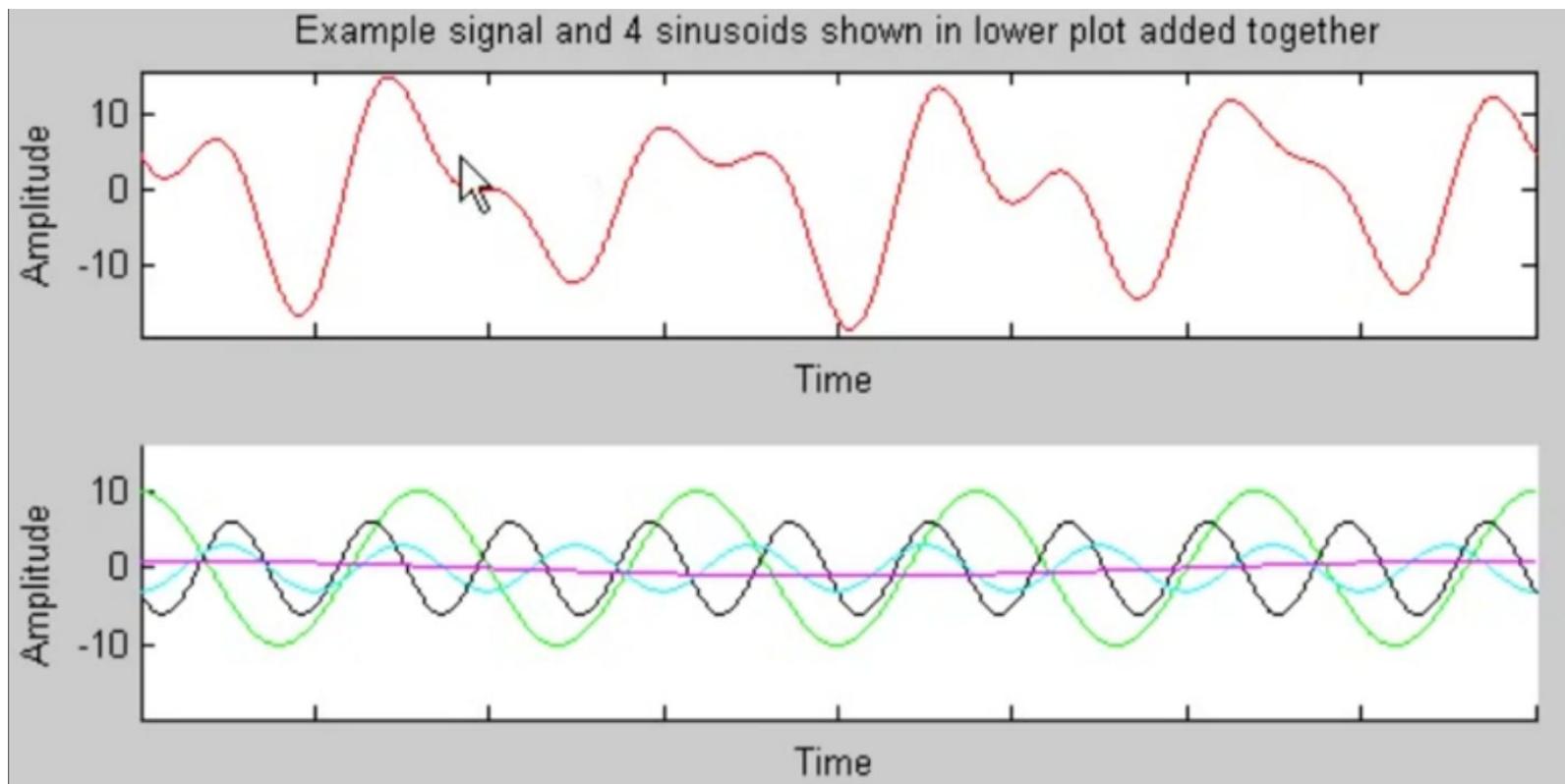


Frequency domain representation

- Instead of having to store individual time samples, we only need to store the amplitude, frequency and phase of each sinusoidal signal.



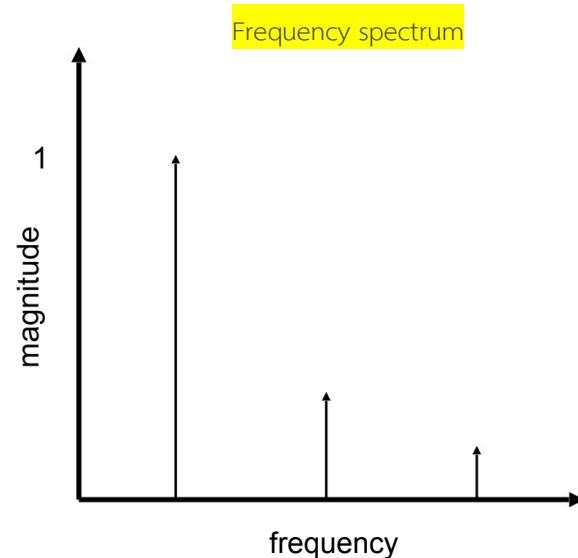
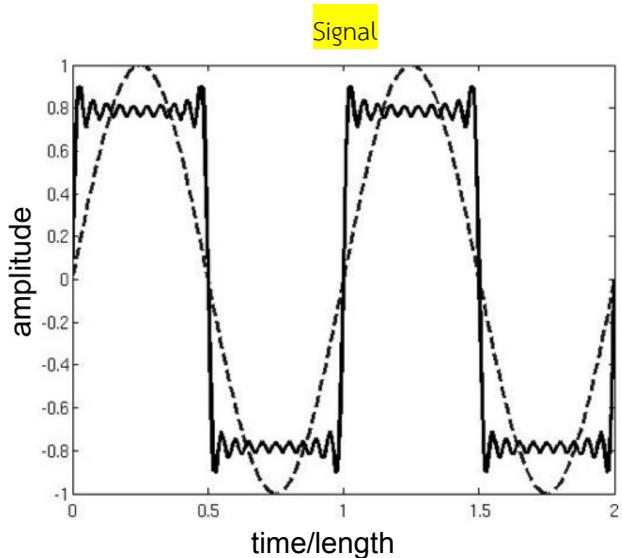
Another example – four sine waves



Fourier series

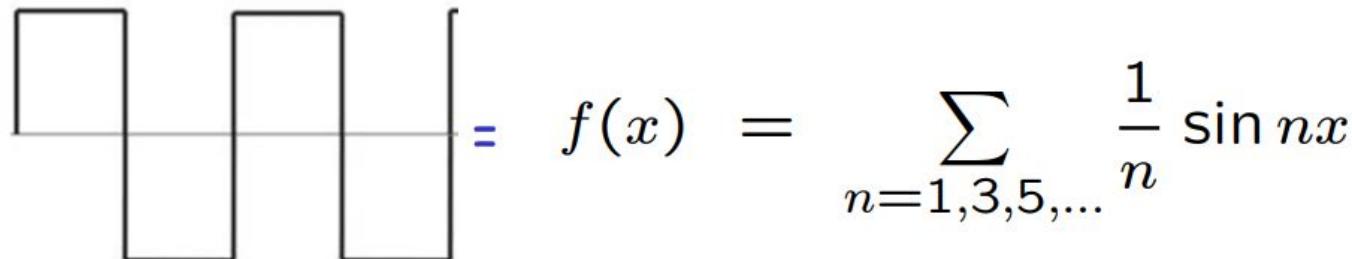
Approximate **periodic signals** with sines and cosines

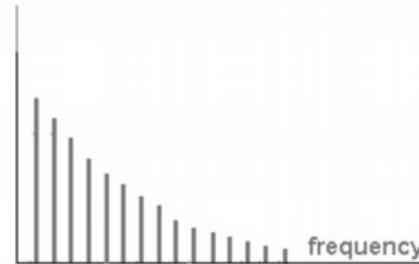
$$y(t) = \sum_n a_n \sin(nf \times 2\pi t) + \sum_n b_n \cos(nf \times 2\pi t)$$



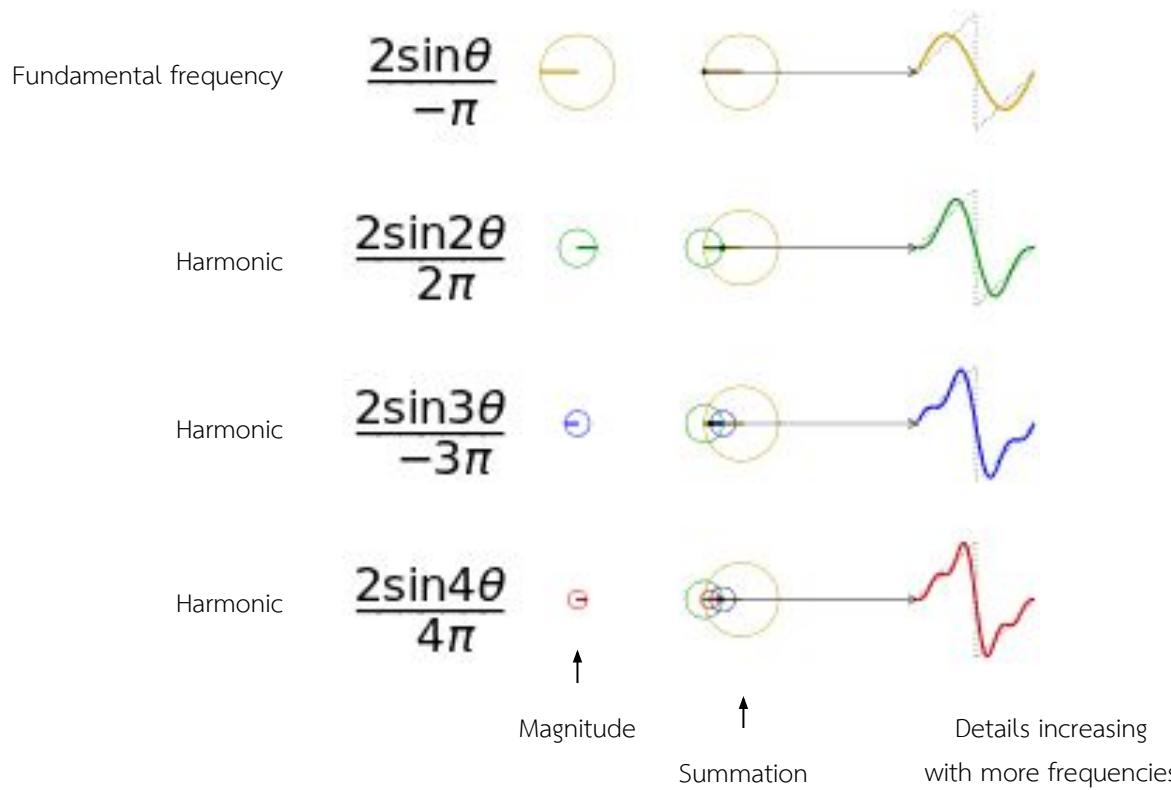
Fourier series

Fourier series for a square wave

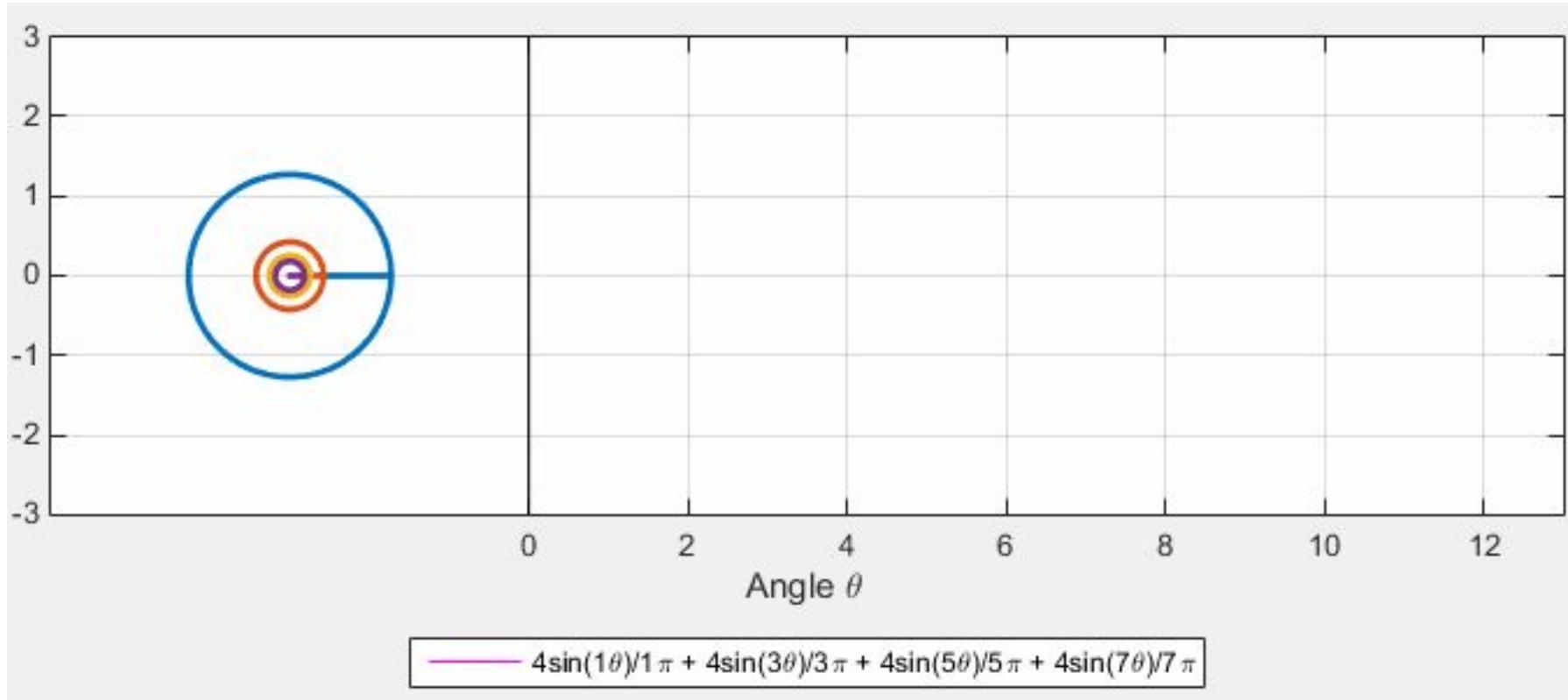

$$f(x) = \sum_{n=1,3,5,\dots} \frac{1}{n} \sin nx$$



Sines and cosines are building blocks



Sines and cosines are building blocks

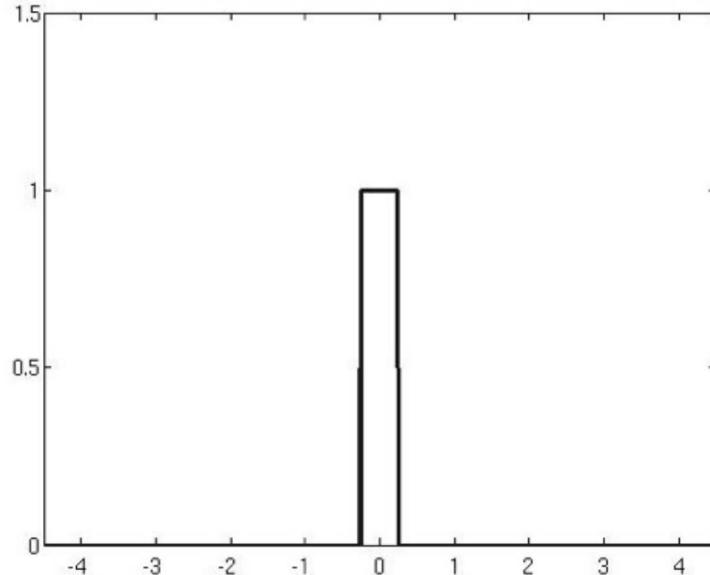


Fourier transform

Approximate **non-periodic signals** with sines and cosines

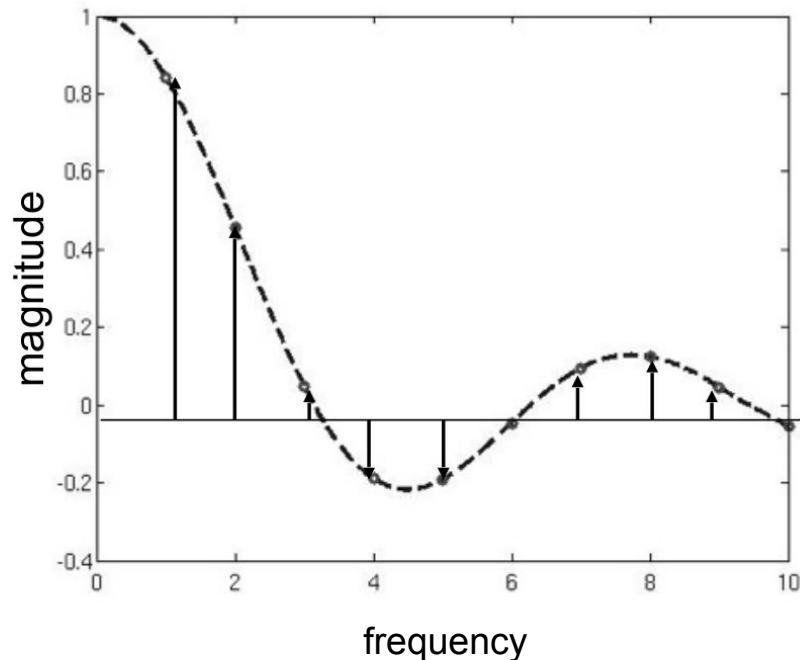
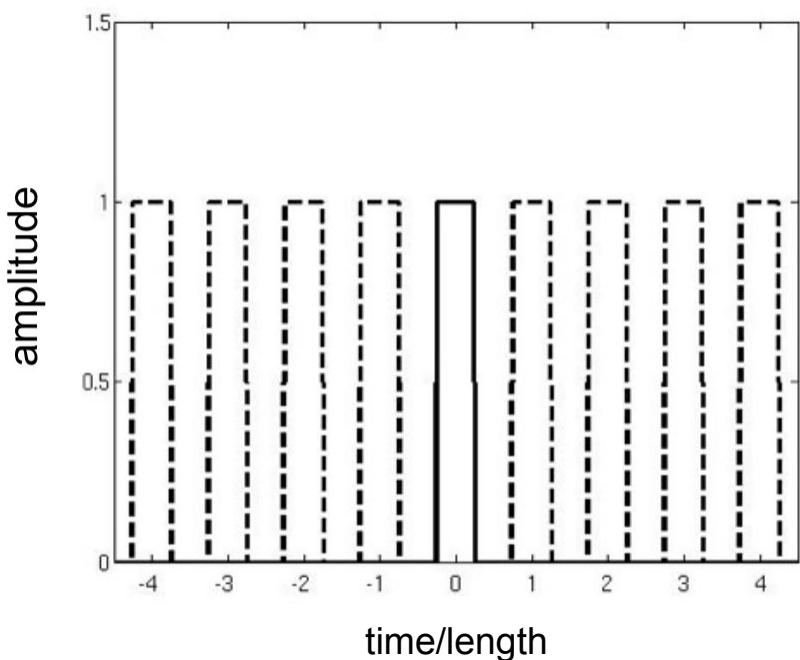
Fourier Series is good for periodic signals, what do we do if they are not?

Classic example:
the top hat function



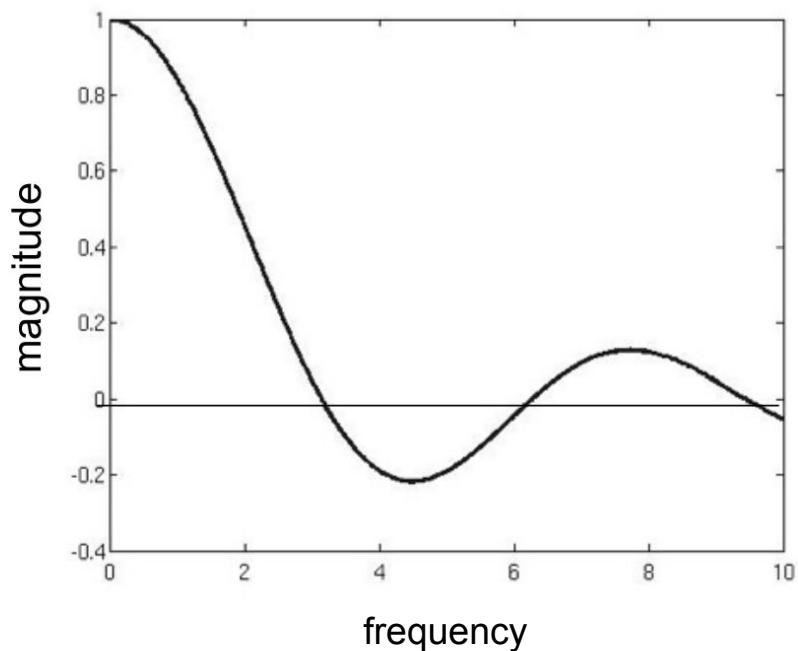
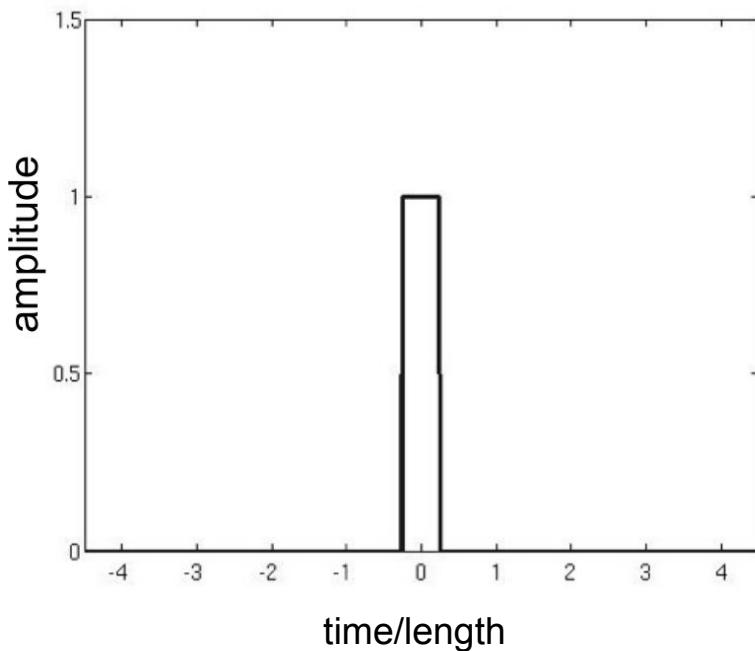
Fourier transform

Try making it periodic over a certain period



Fourier transform

In the limit ($T = \infty$) we get a smooth spectrum



Fourier transform

- Fourier series was a sum at specific frequencies

$$y(t) = \sum_n a_n \sin(nf \times 2\pi t) + \sum_n b_n \cos(nf \times 2\pi t)$$

- Fourier transform is a sum over all frequencies

$$y(t) = \int_{-\infty}^{\infty} A(f) e^{j2\pi f t} df$$

Annotations for the Fourier Transform formula:

- Negative frequencies: Points to the lower limit of the integral, $-\infty$.
- Sine/Cosine (compact notation): Points to the term $e^{j2\pi f t}$.
- Frequency: Points to the variable f in the term $e^{j2\pi f t}$.

- Note: this formula is usually called the inverse Fourier transform

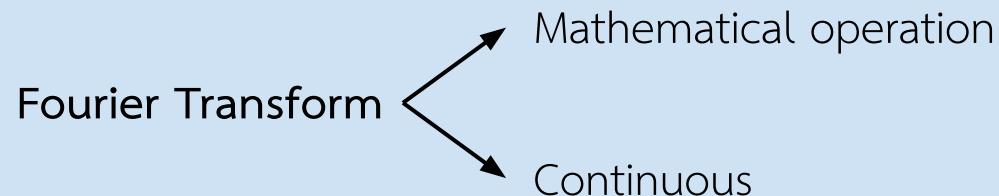
Fourier transform

- To find the frequency spectrum we need to do an integral

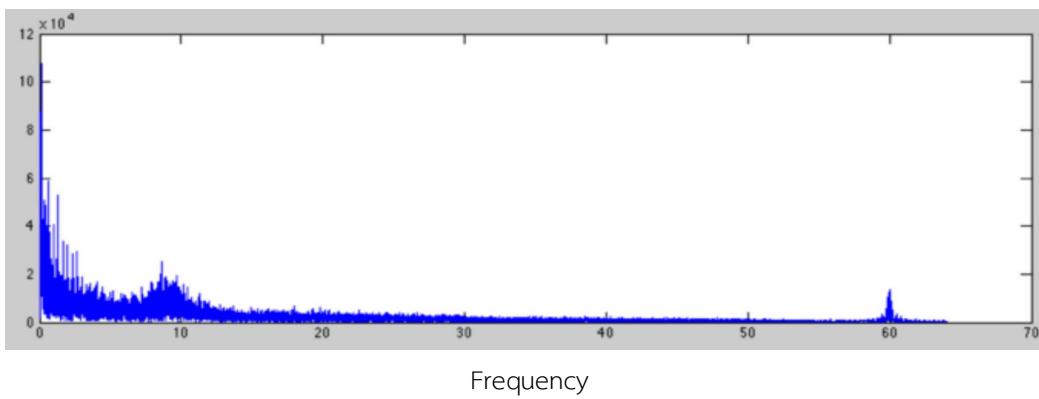
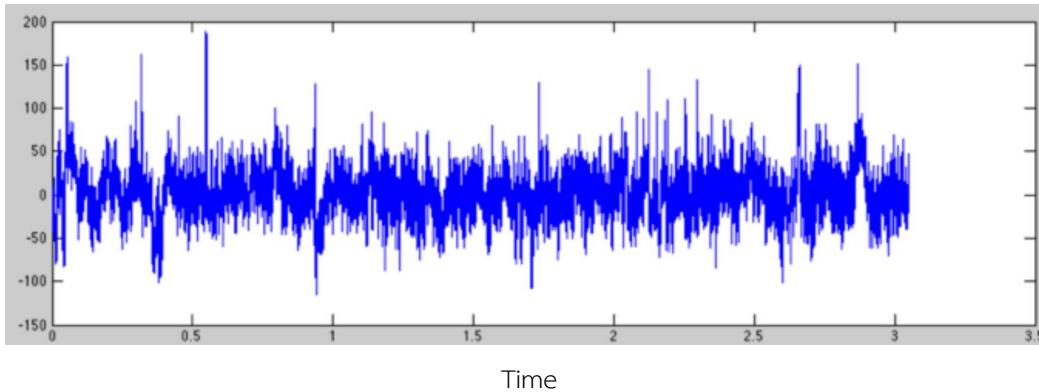
$$A(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi f t} dt$$

- Basically at frequency f , how much of $\sin(f \times 2\pi t)$ do I need?
- In practice, we get computers to deal with this using the **Fast Fourier Transform (FFT)** technique.

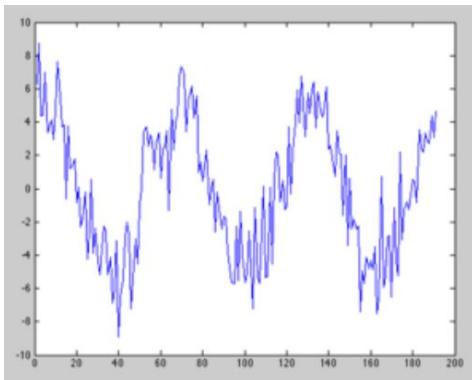
Fourier Transform vs Fast Fourier Transform



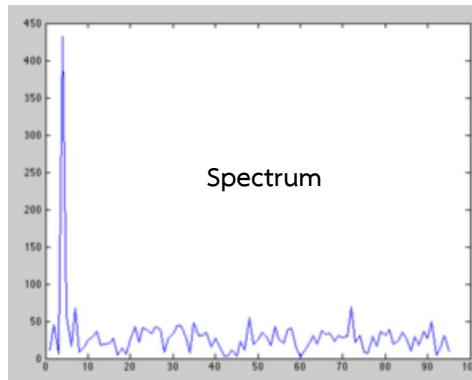
Fourier analysis – What's all about?



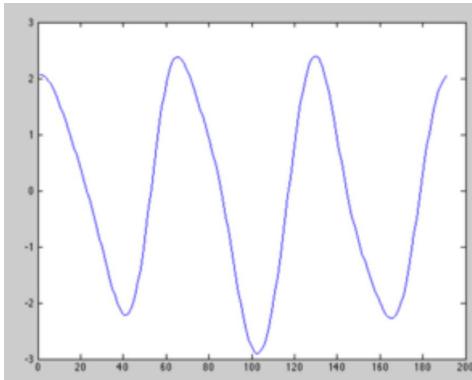
Fourier analysis – What's all about?



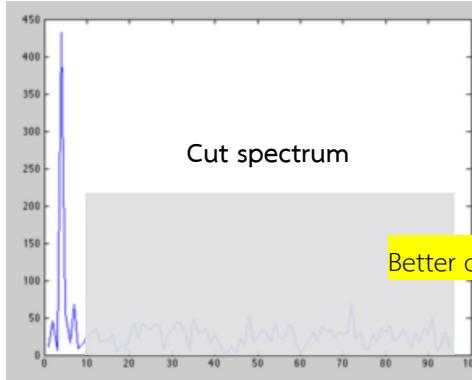
Fourier



Spectrum



Inverse

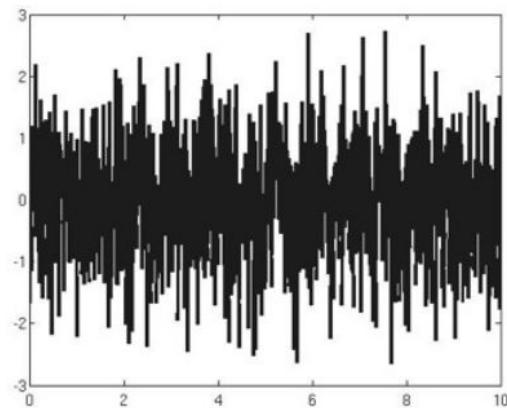


Cut spectrum

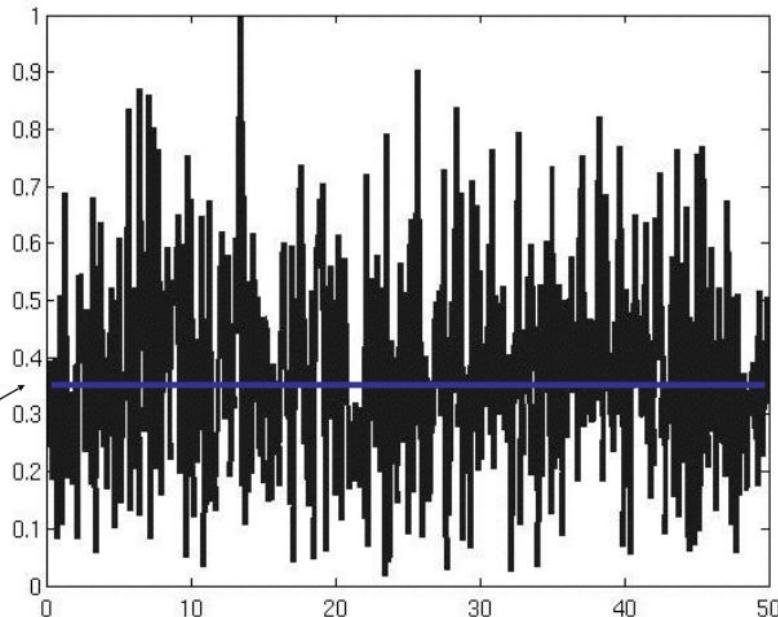
Better computational efficiency

Fourier transform

‘White’ noise



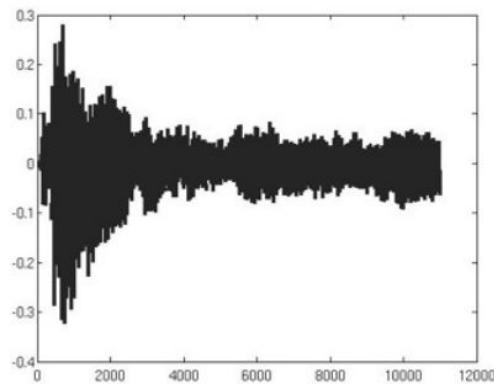
FFT Result



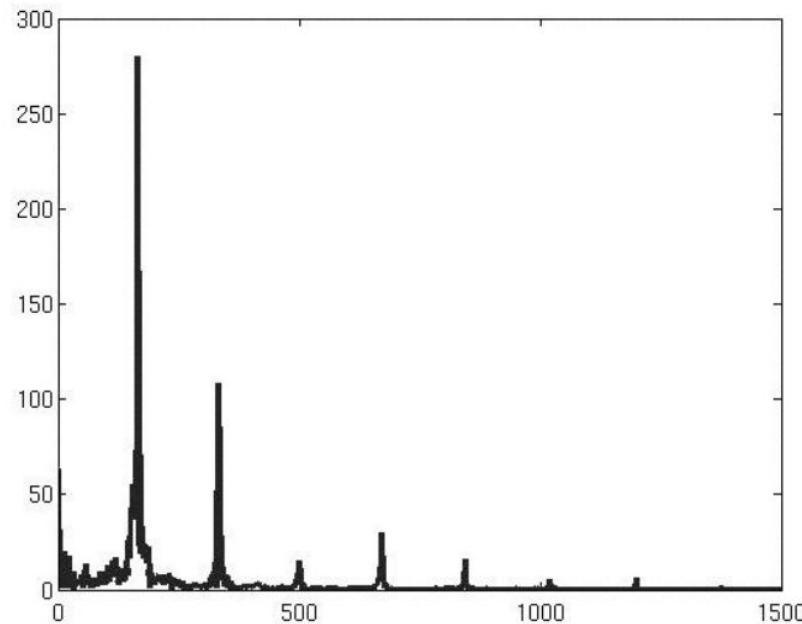
Ideally flat at all frequencies

Fourier transform – 1D

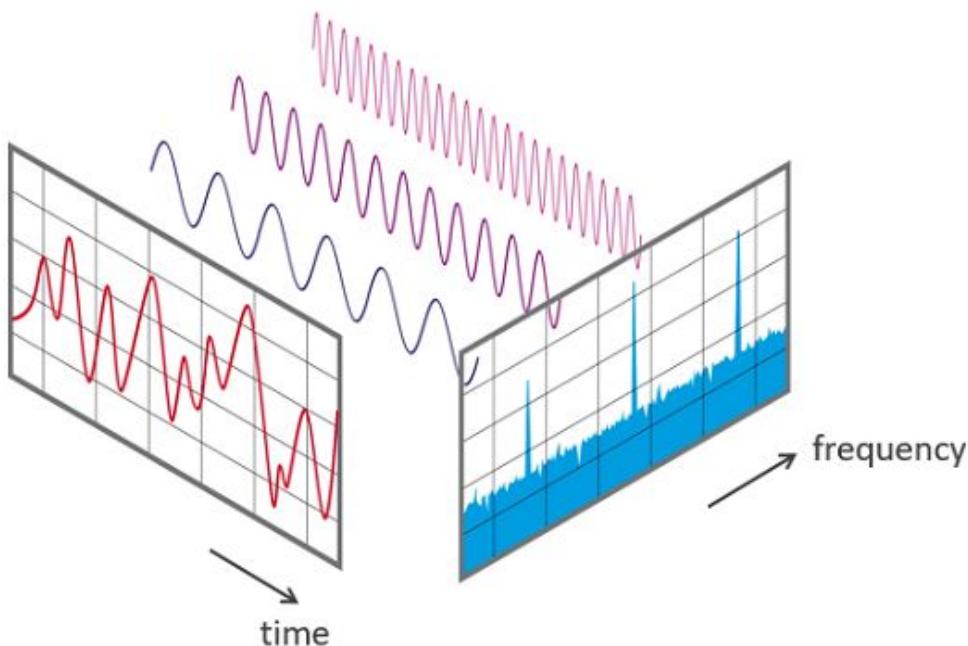
Piano note



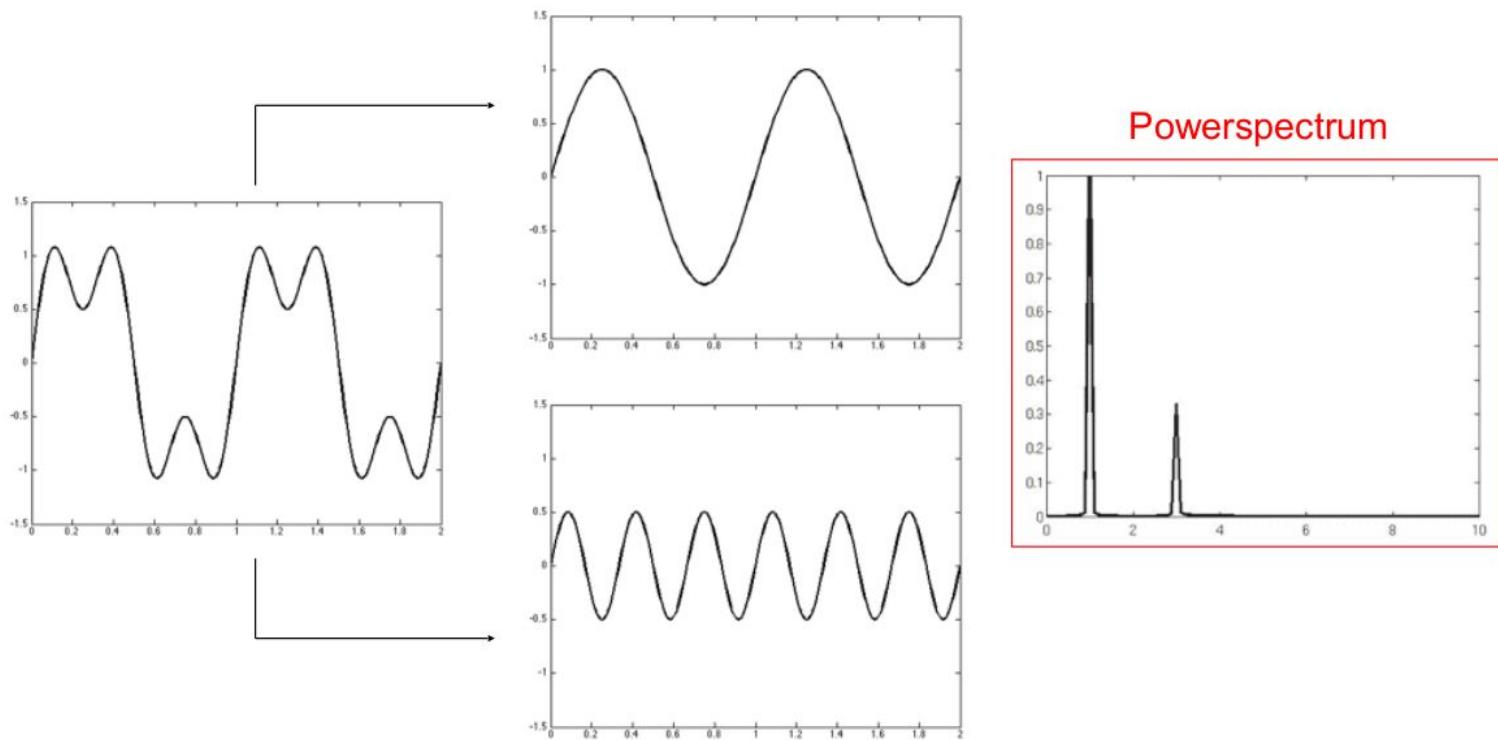
FFT Result



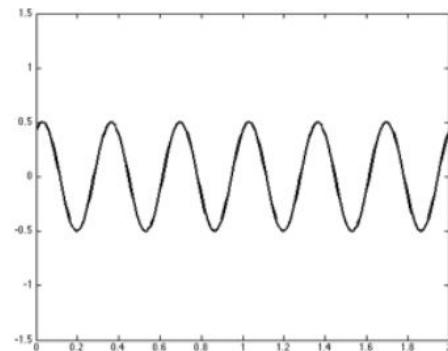
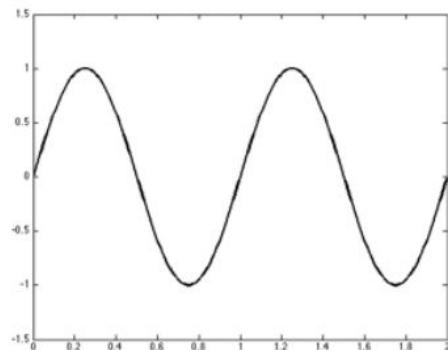
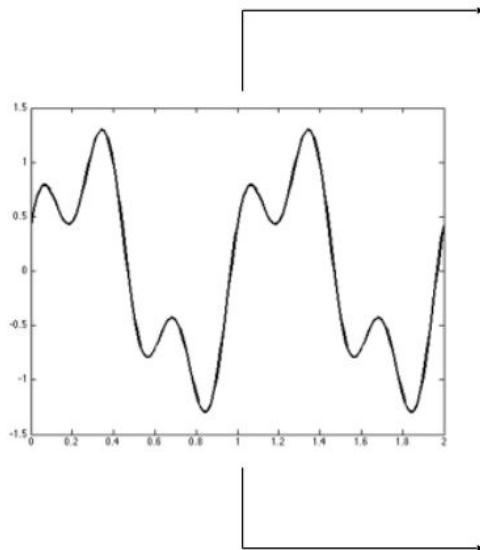
Fourier transform – Signal decomposition



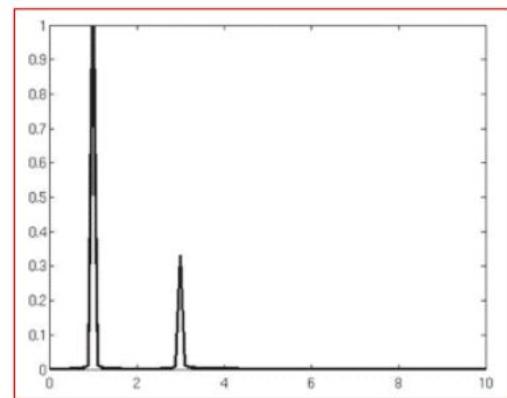
Fourier transform



Fourier transform



Powerspectrum



Power spectrum lacks
phase information

Power spectrum lacks phase information

Fourier
transform

$$A(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi f t} dt$$

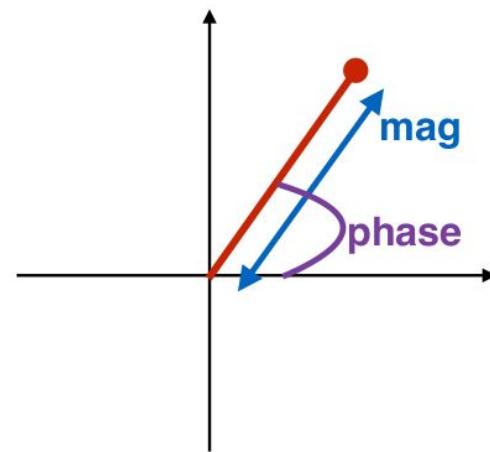
signal

complex
number

complex
number

Complex numbers easier to manipulate
than sine and cosine functions

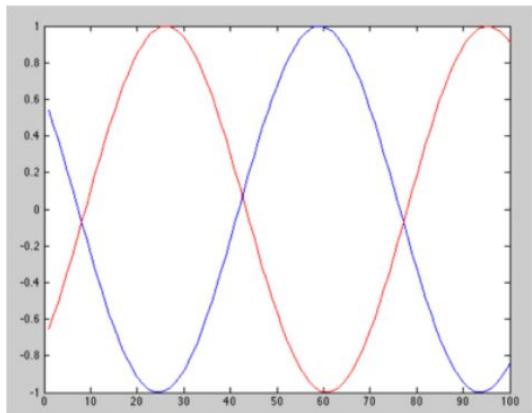
Information on phase and magnitude



Magnitude, Phase

What?

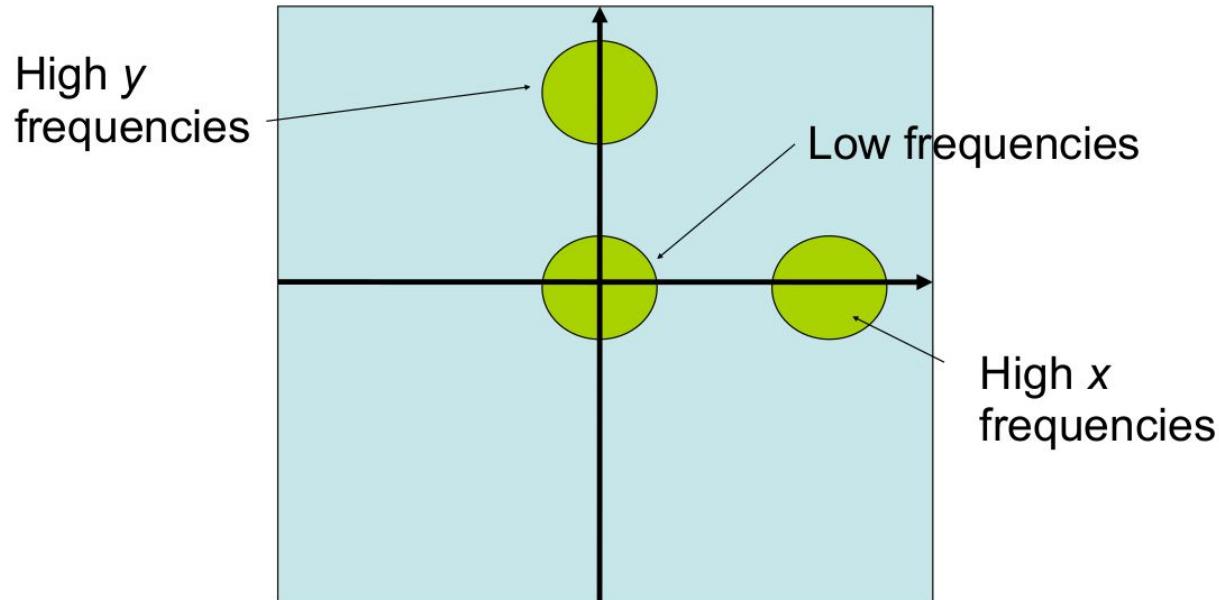
Where?



FFT → Same magnitude
Different phases

Fourier transform – 2D

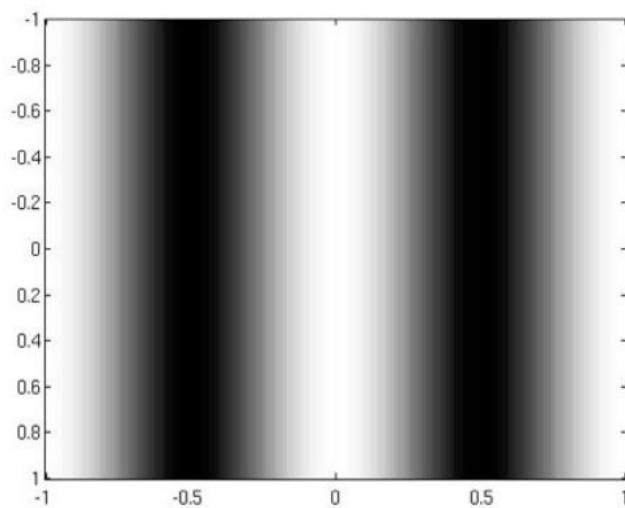
- 2D signals, i.e. images have 2D Fourier transforms
- We now have x and y frequencies:



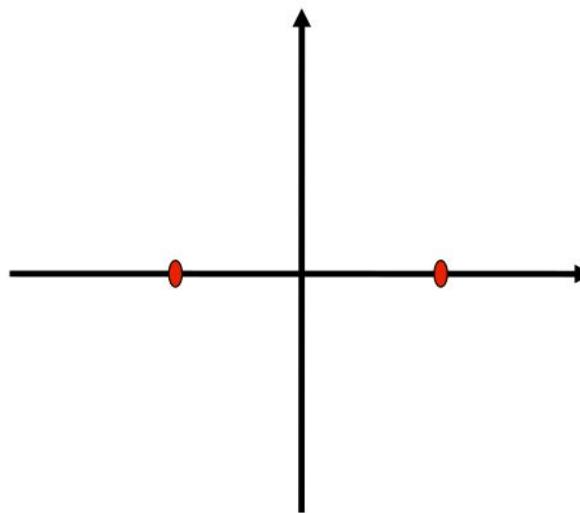
Fourier transform – 2D

Sinusoid in x direction

Image



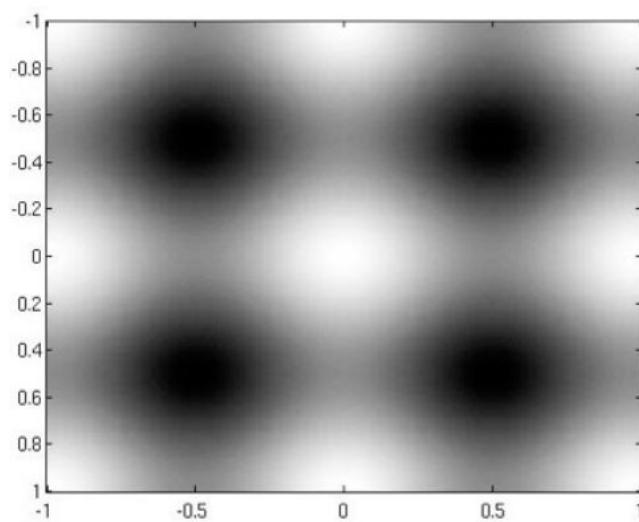
2D spectrum



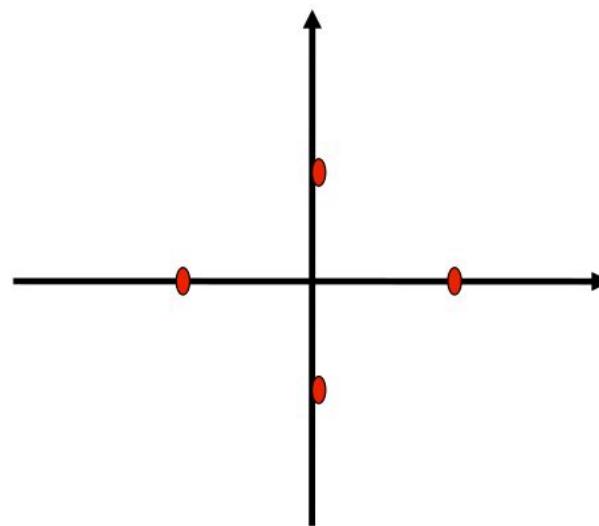
Fourier transform – 2D

Sinusoid in x direction + Sinusoid in y direction

Image

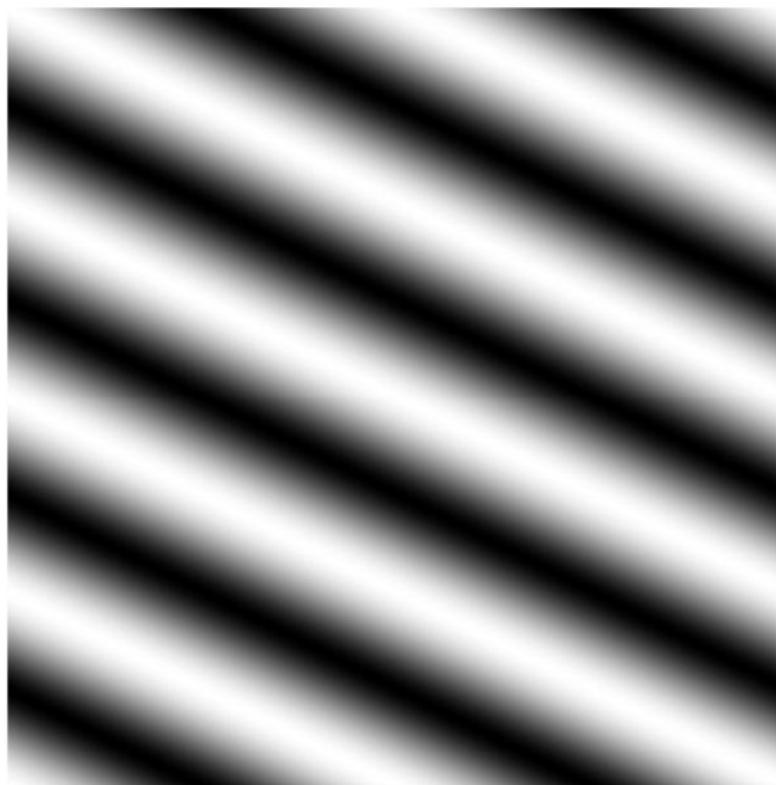


2D spectrum



Fourier transform – 2D

$$\begin{array}{|c|c|}\hline v & e^{-j\pi(ux+vy)} \\ \hline & u \\ \hline e^{j\pi(ux+vy)} & \\ \hline\end{array}$$



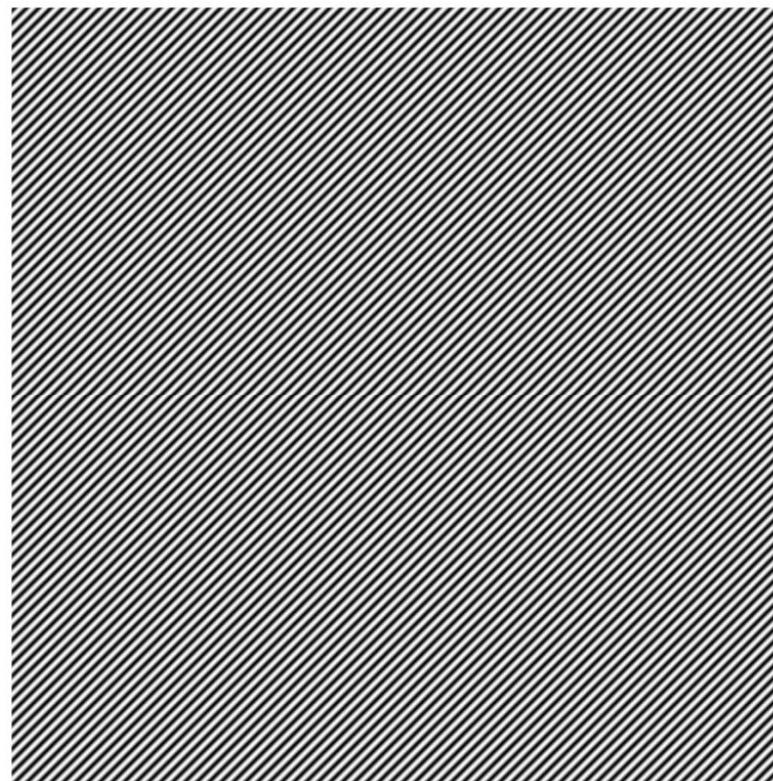
Fourier transform – 2D

	v
$e^{-j\pi(ux+vy)}$	
•	
	u
	•
	$e^{j\pi(ux+vy)}$



Fourier transform – 2D

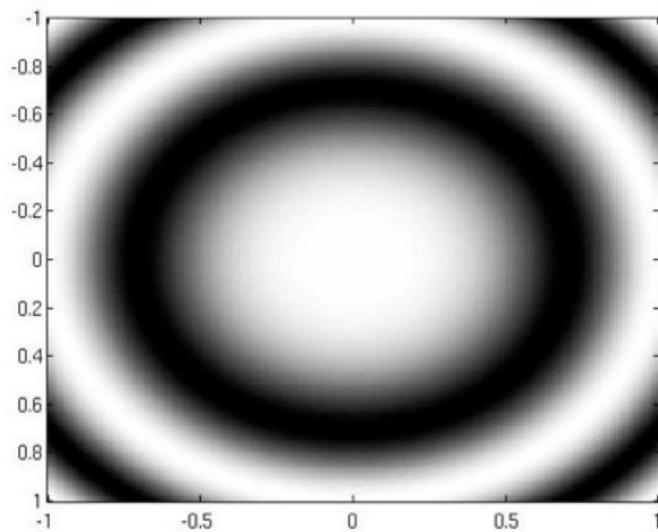
$$\begin{matrix} & e^{-j\pi(ux+vy)} \\ \bullet & v \\ \hline & u \\ \hline & e^{j\pi(ux+vy)} \\ \bullet & \end{matrix}$$



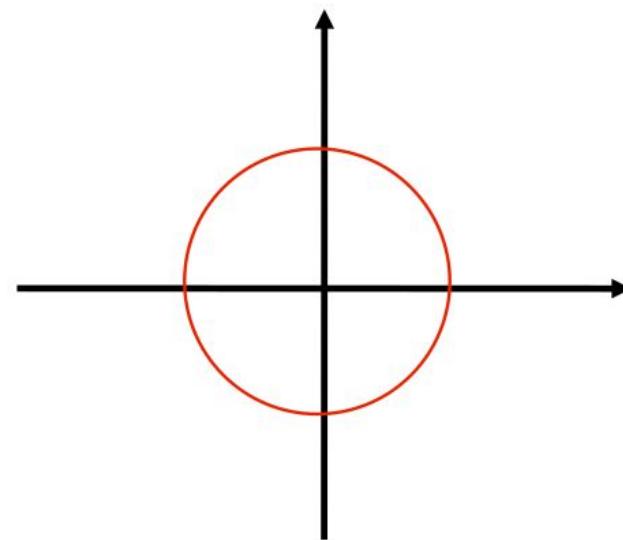
Fourier transform – 2D

Single frequency in all directions

Image



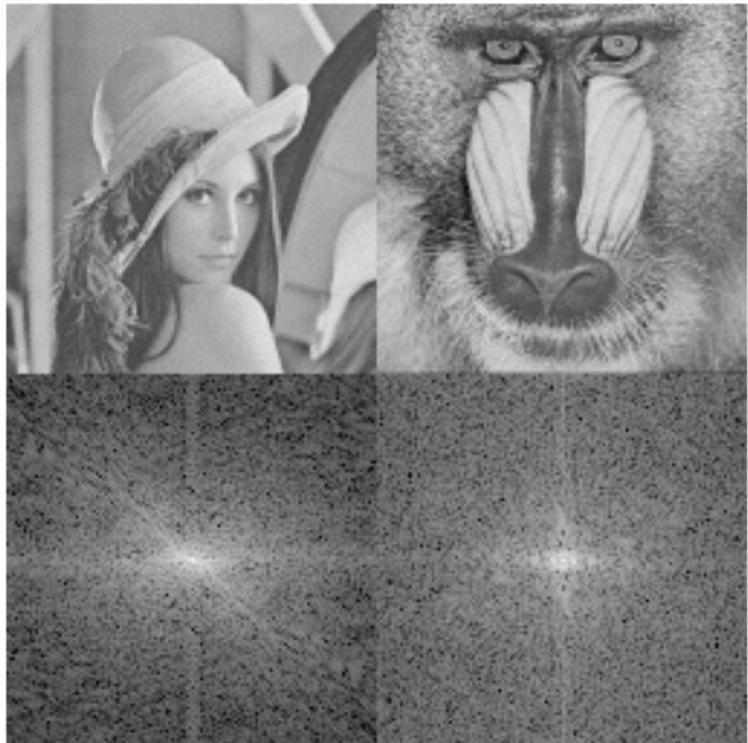
2D spectrum



Fourier transform – 2D

Examples:

- Spectrum is ‘bright’ in the centre.
- Detail involves high frequency.
- Spectrum is symmetric.



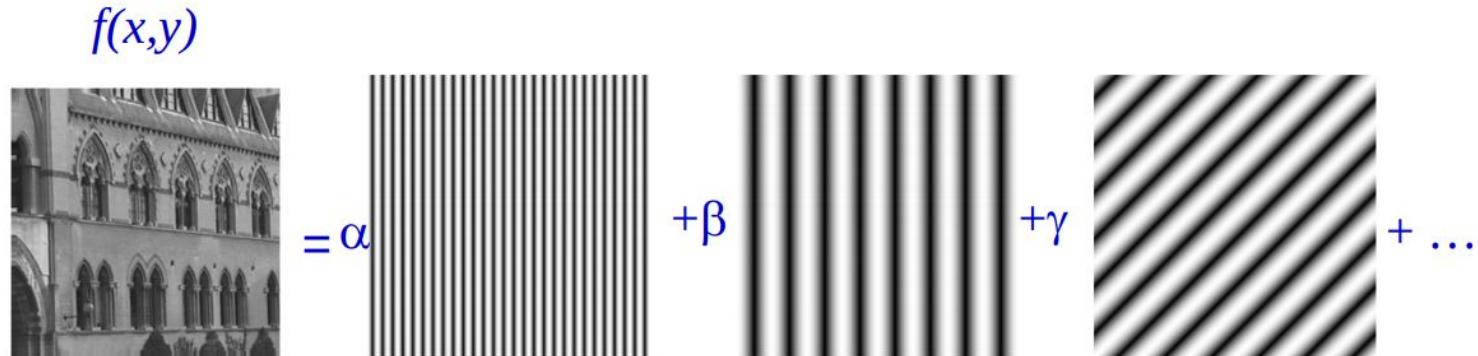
<http://www.cs.unm.edu/~brayer/vision/fourier.html>

Fourier transform – 2D

The spatial function $f(x, y)$

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$

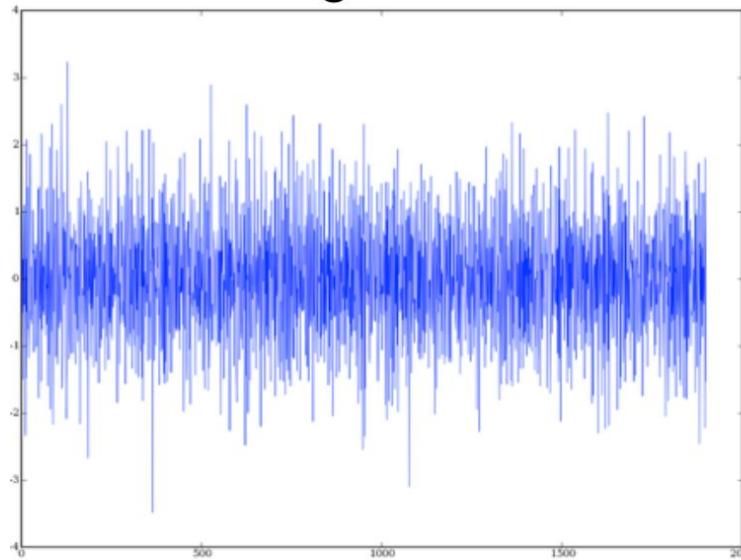
is decomposed into a weighted sum of 2D orthogonal basis functions in a similar manner to decomposing a vector onto a basis using scalar products.



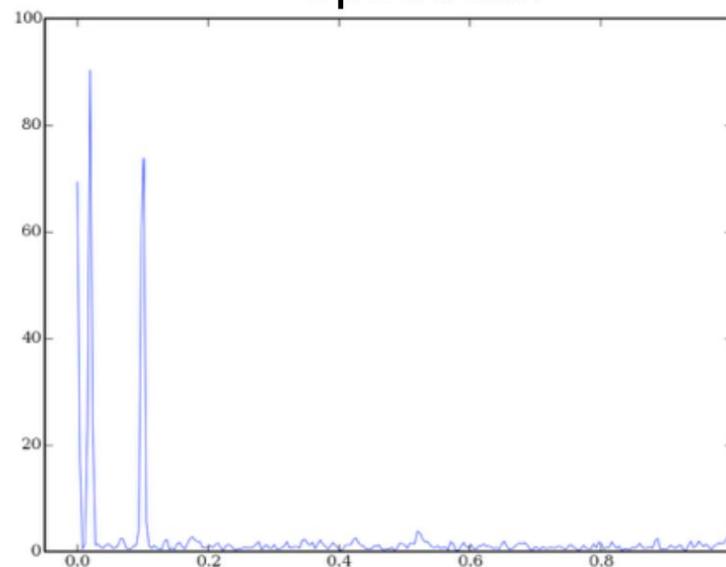
Filtering – 1D

- In Fourier, signals are mixture of different frequency components.
- Often, we want some components and not others

Signal



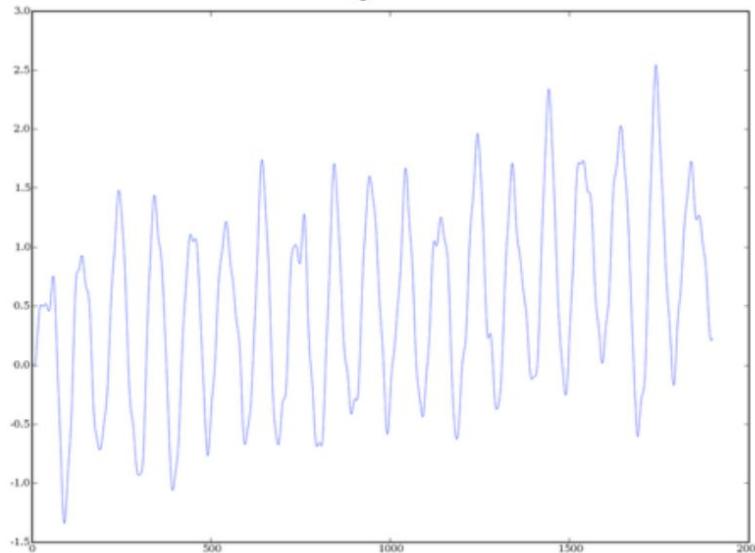
Spectrum



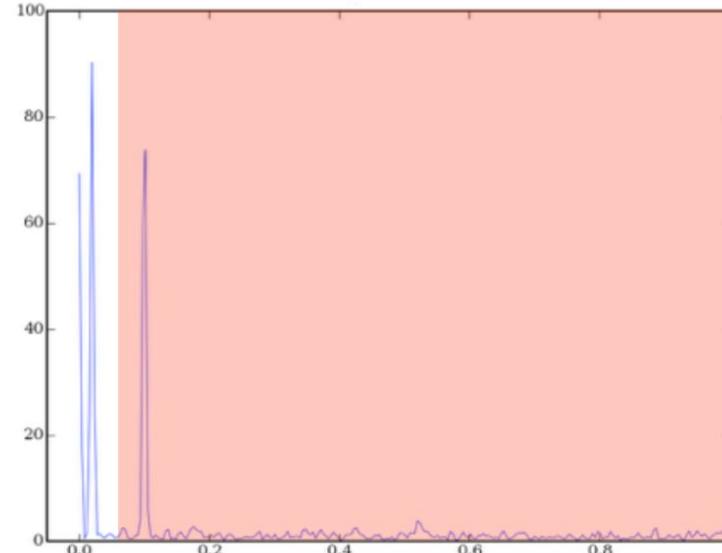
Filtering – 1D

- I want to get rid of high frequency noise components.
- **Low Pass** filter – throw away high frequencies

Signal



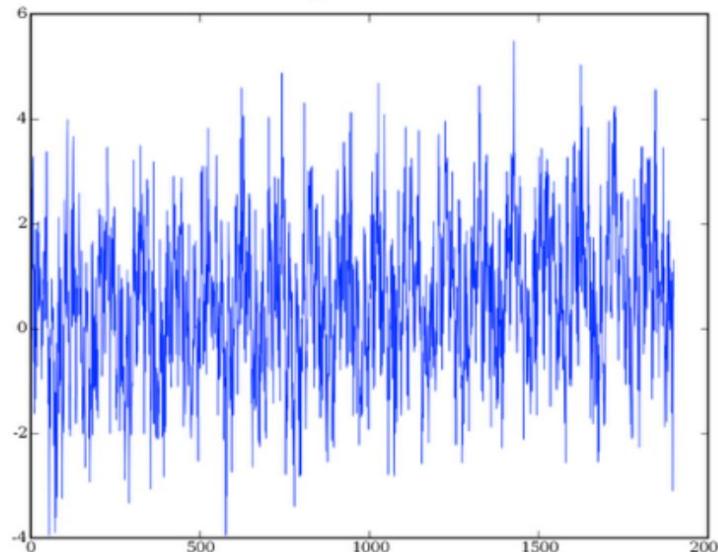
Spectrum



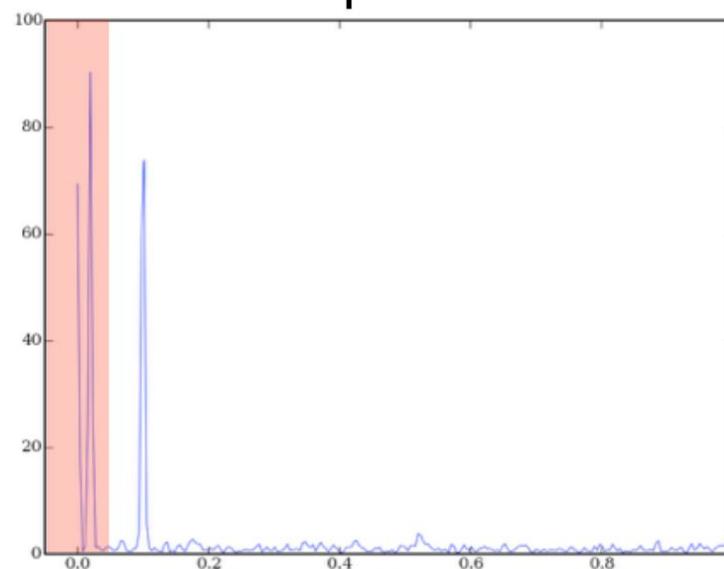
Filtering – 1D

- I want to get rid of low frequency noise/drift.
- **High Pass** filter – throw away high frequencies

Signal



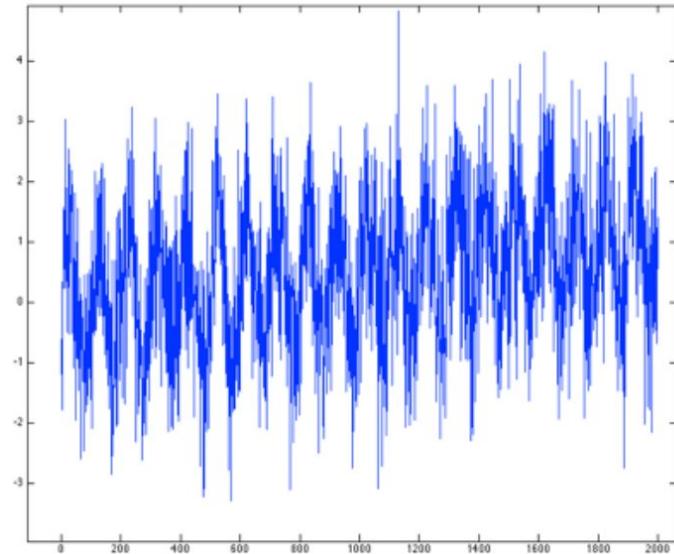
Spectrum



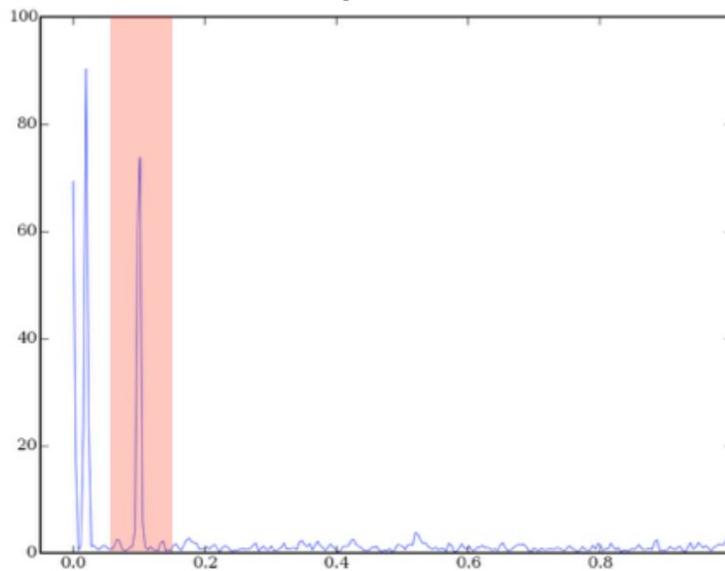
Filtering – 1D

- I want to get rid of that annoying component at ? Hz (e.g. 50hz mains noise).
- **Band Stop** filter – let everything through except ...

Signal



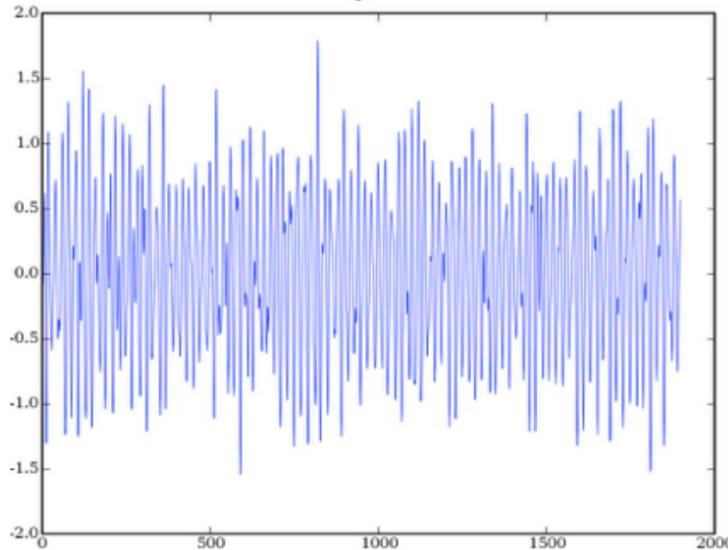
Spectrum



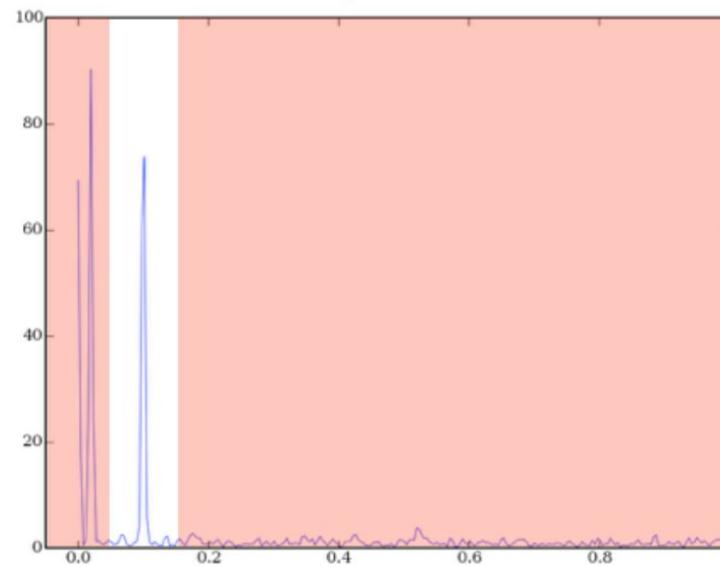
Filtering – 1D

- I want to get rid of all other stuff that is not my signal (not always this simple)
- **Band Pass** filter – get rid of everything but ...

Signal

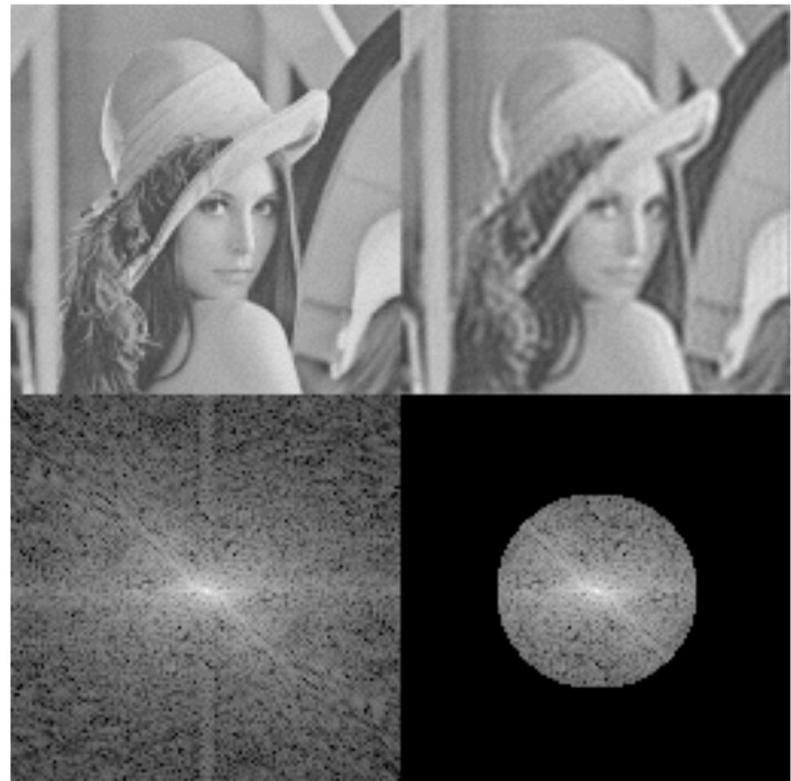


Spectrum



Filtering – 2D

- Same principles as in 1D.
- Low Pass
- Remove high frequencies.
- Loose detail.



Filtering – 2D

$f(x,y)$

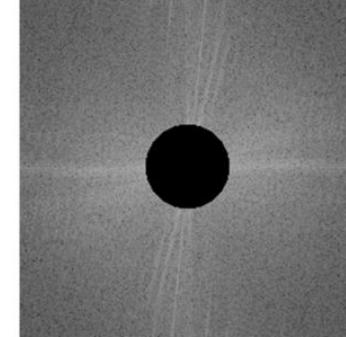
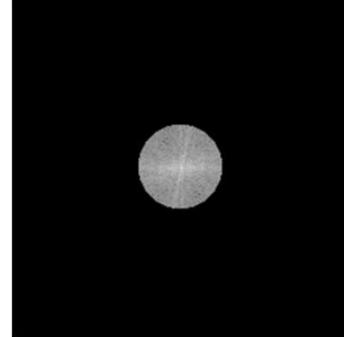
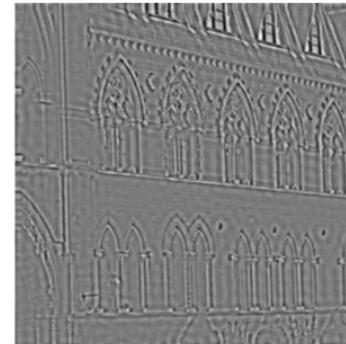
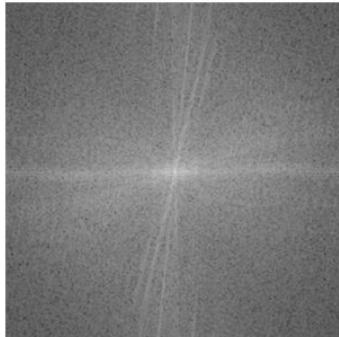


original

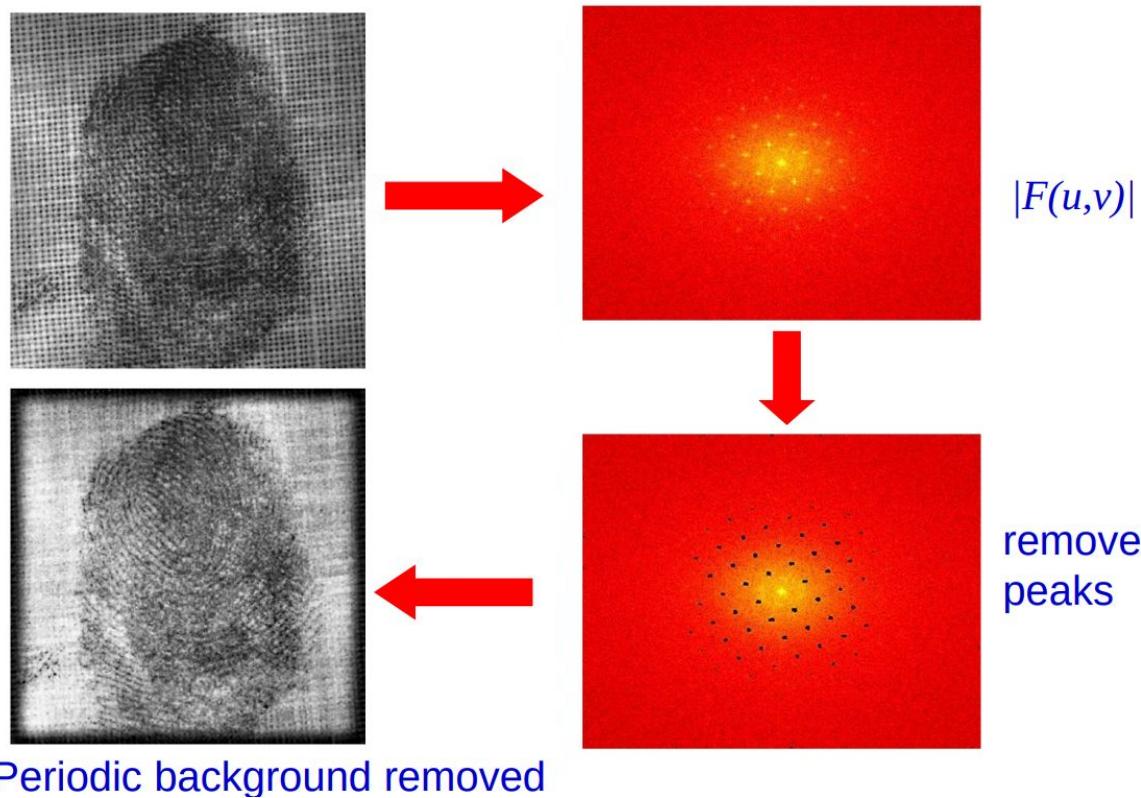
low pass

high pass

$|F(u,v)|$

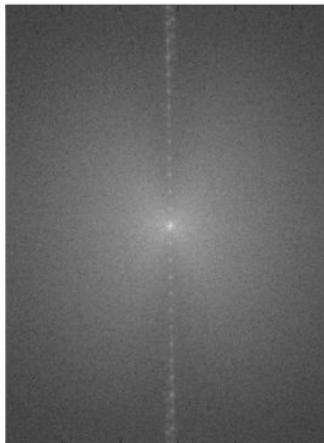
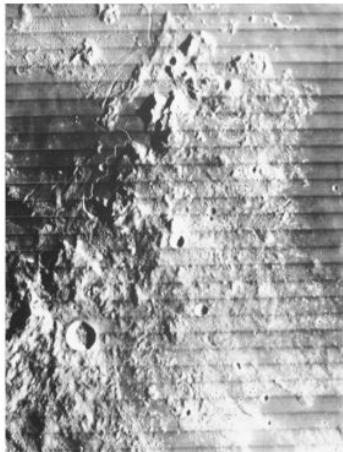


Example – Forensic application

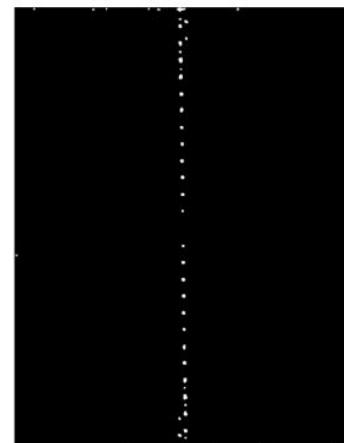


Example – Image processing

Lunar orbital image (1966)



$$|F(u,v)|$$



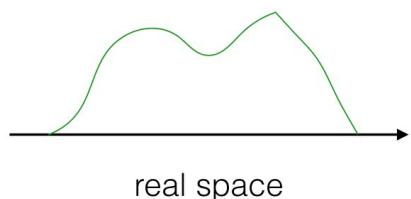
remove
peaks



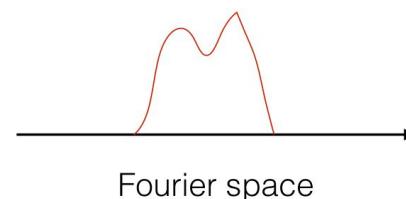
join lines
removed

Sampling

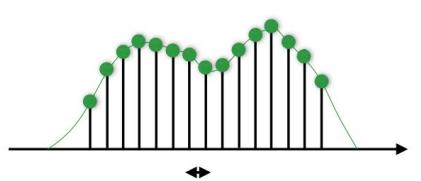
Sampling in real space is like repeating in Fourier space and vice versa



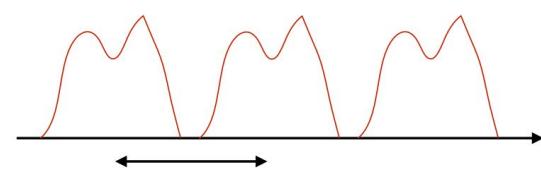
real space



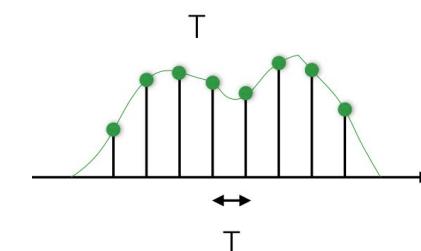
Fourier space



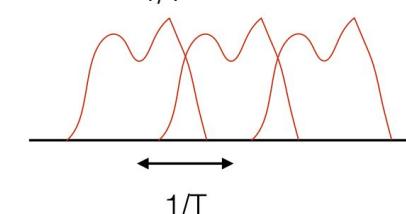
T



$1/T$



T



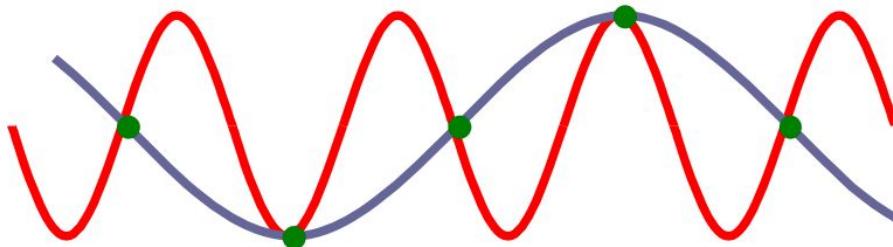
$1/T$

Frequencies above the Nyquist limit are ‘folded back’ corrupting the signal in the acceptable range.

The information in these frequencies is not correctly reconstructed.

Aliasing

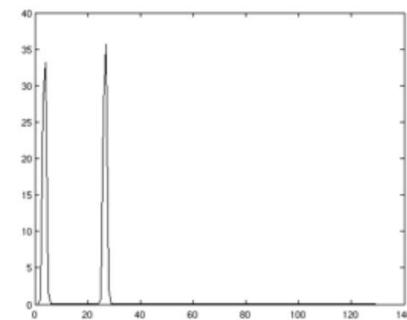
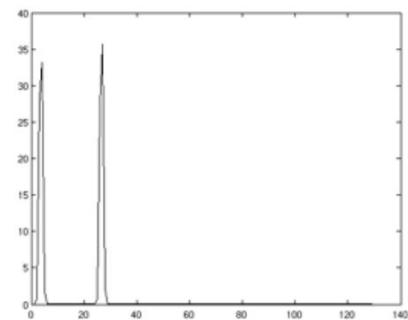
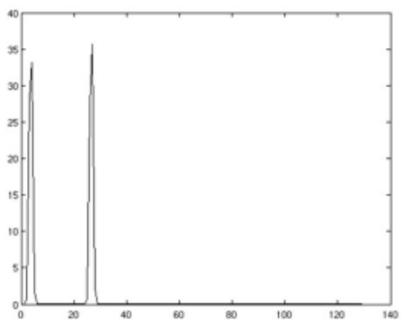
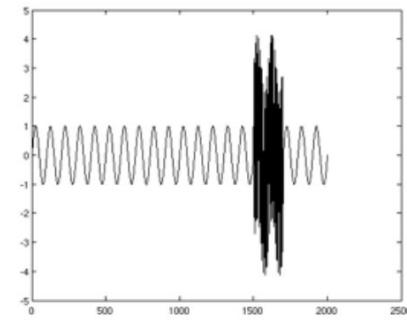
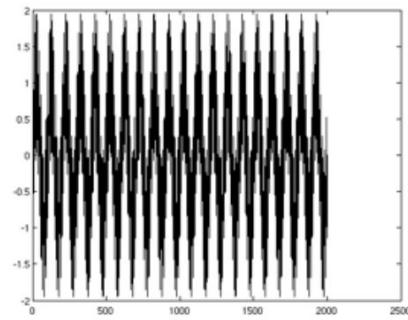
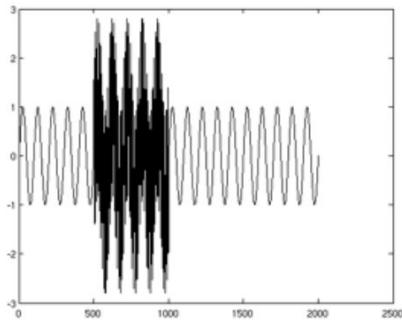
If the signal has frequencies above the Nyquist limit ...



Insufficient samples to distinguish the high and low frequency
aliasing: signals “travelling in disguise” as other frequencies

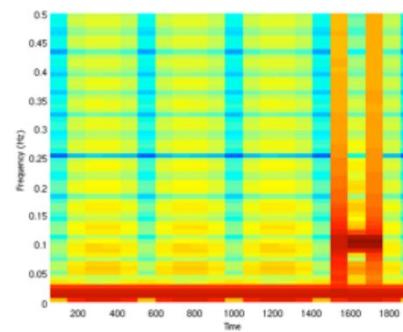
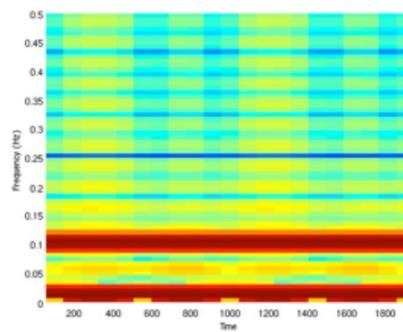
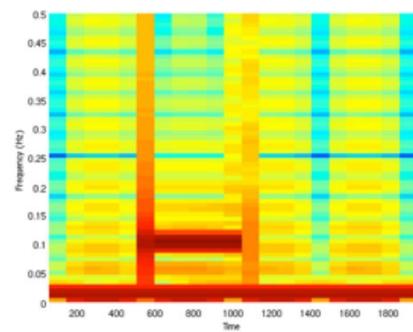
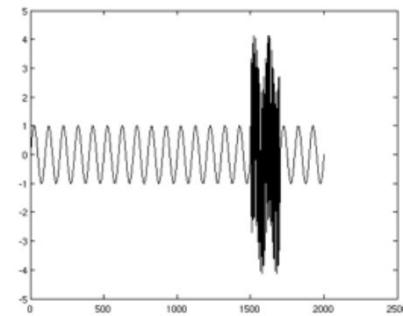
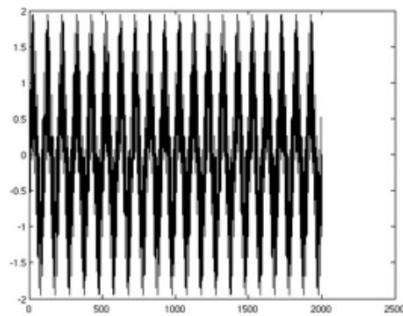
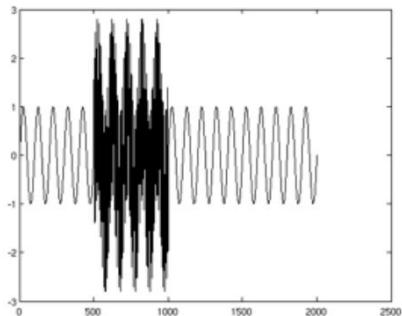
Time-frequency signals

Three different signals – same frequency spectrum.



Time-frequency signals

Need a dynamic time-frequency representation.



Convolution in 1D

$$g(x) = \sum_i f(x+i)h(i)$$

filtering $f(x)$ with $h(x)$

$f(x)$	100 200 100 200 90 80 80 100 100
$h(x)$	1/4 1/2 1/4
$g(x)$	150

molecule/template/kernel

$$g(x) = \int f(u)h(x-u) du \quad \text{convolution of } f(x) \text{ and } h(x)$$

$$\begin{aligned} &= \int f(x+u')h(-u') du' \quad \text{after change of} \\ &= \sum_i f(x+i)h(-i) \quad \text{variable } u' = u - x \end{aligned}$$

- note negative sign (which is a reflection in x) in convolution

- $h(x)$ is often symmetric (even/odd), and then (e.g. for even)

$$g(x) = \sum_i f(x+i)h(i)$$

Convolution in 2D

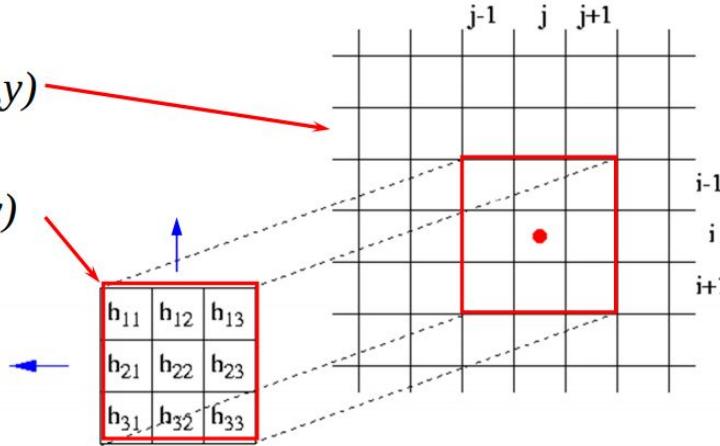
convolution

$$\begin{aligned}g(x, y) &= h(x, y) * f(x, y) = f(x, y) * h(x, y) \\&= \int \int f(u, v)h(x - u, y - v) du dv\end{aligned}$$

filtering

image $f(x, y)$

filter / kernel $h(x, y)$



$$\begin{aligned}g(x, y) = & h_{11} f(i - 1, j - 1) + h_{12} f(i - 1, j) + h_{13} f(i - 1, j + 1) + \\& h_{21} f(i, j - 1) + h_{22} f(i, j) + h_{23} f(i, j + 1) + \\& h_{31} f(i + 1, j - 1) + h_{32} f(i + 1, j) + h_{33} f(i + 1, j + 1)\end{aligned}$$

for convolution, reflect filter in x and y axes

Convolution in frequency domain

$$f(x, y) * h(x, y) \Leftrightarrow F(u, v)H(u, v)$$

Space convolution = frequency multiplication

In words: the Fourier transform of the convolution of two functions is the product of their individual Fourier transforms

Proof: exercise

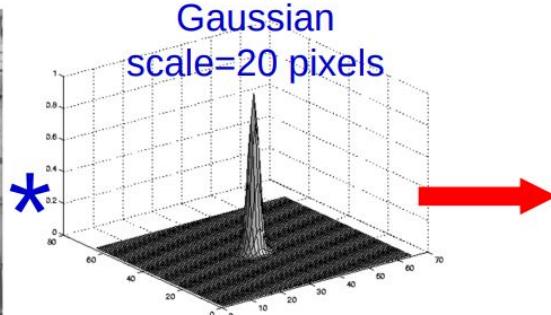
Why is this so important?

Because linear filtering operations can be carried out by simple multiplications in the Fourier domain

Importance of convolution

It establishes the link between operations in the frequency domain
and the action of linear spatial filters

Example smooth an image with a Gaussian spatial filter



1. Compute FT of image and FT of Gaussian
2. Multiply FT's
3. Compute inverse FT of the result.

$$f(x, y) * g(x, y) \Leftrightarrow F(u, v)G(u, v)$$

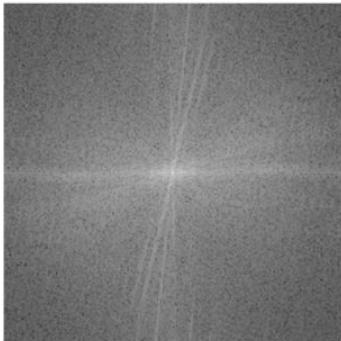
$f(x,y)$



$g(x,y)$

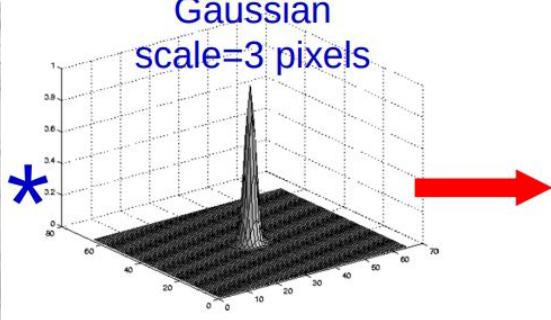


Fourier transform

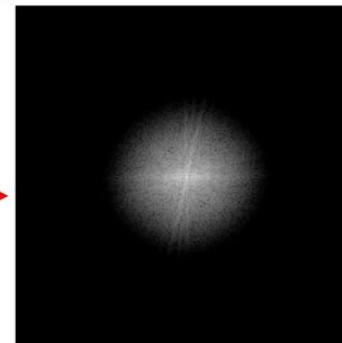


$|F(u,v)|$

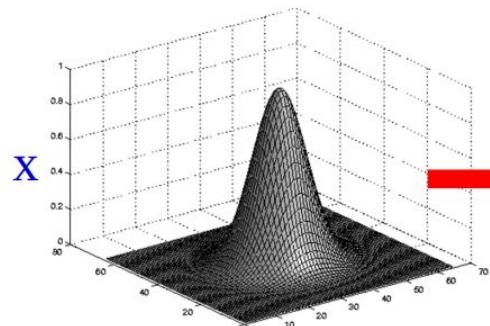
Gaussian
scale=3 pixels

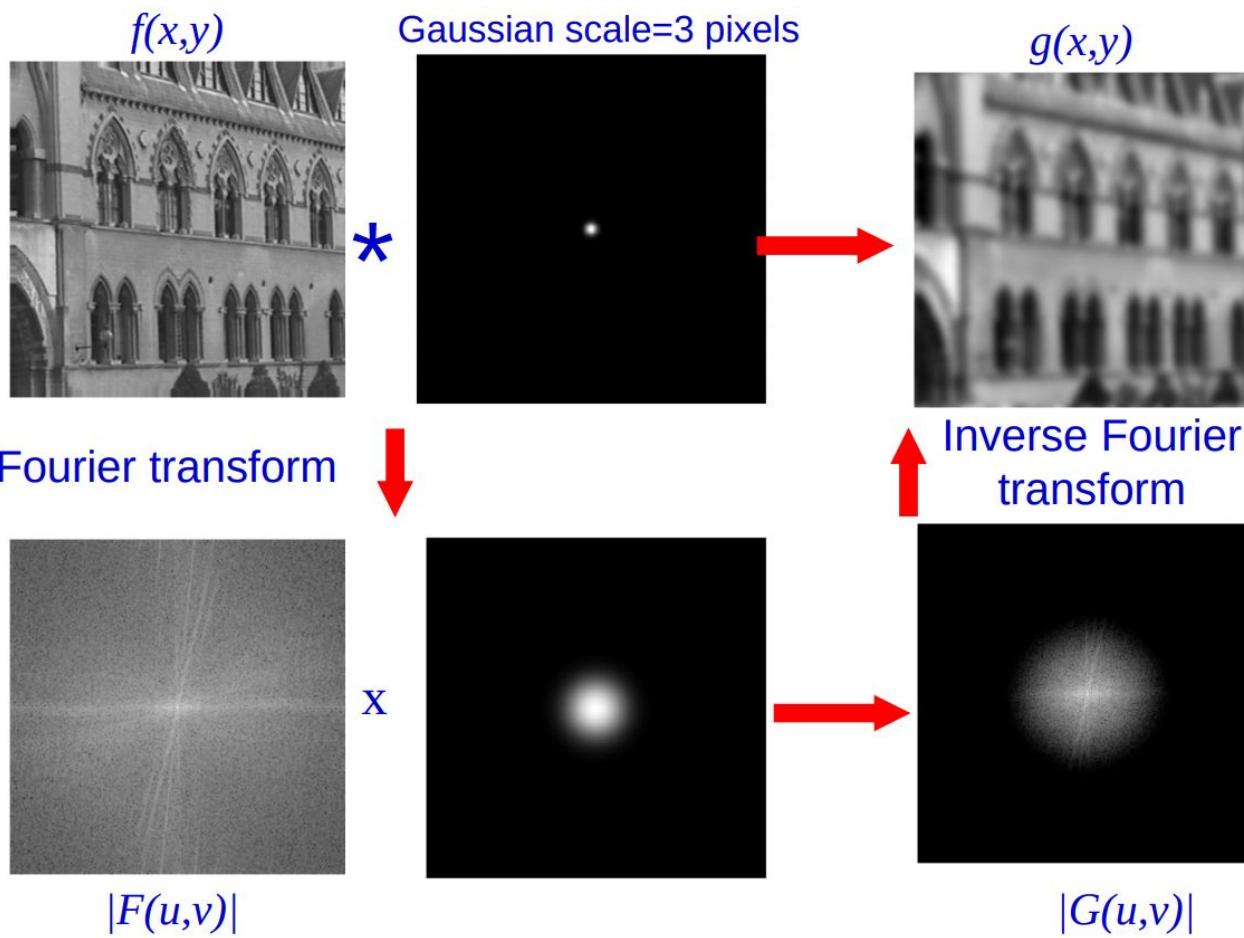


Inverse Fourier
transform



$|G(u,v)|$





Convolution

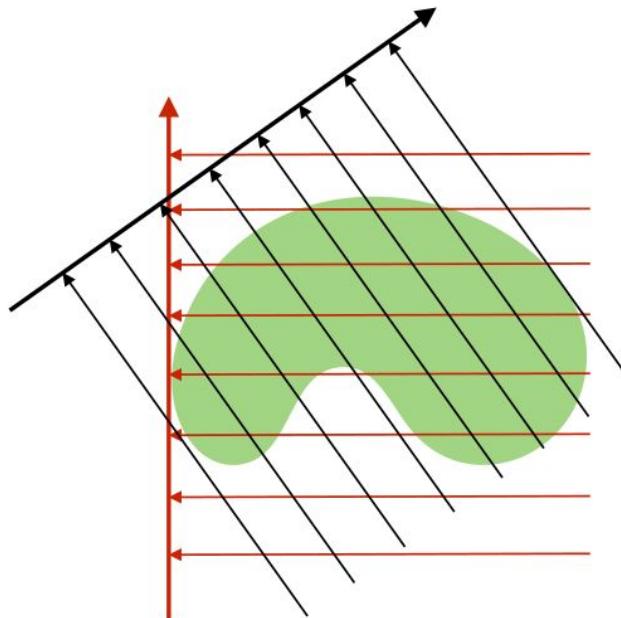
There are two equivalent ways of carrying out linear spatial filtering operations:

1. Spatial domain: convolution with a spatial operator
2. Frequency domain: multiply FT of signal and filter, and compute an inverse FT of product

Why choose one over the other?

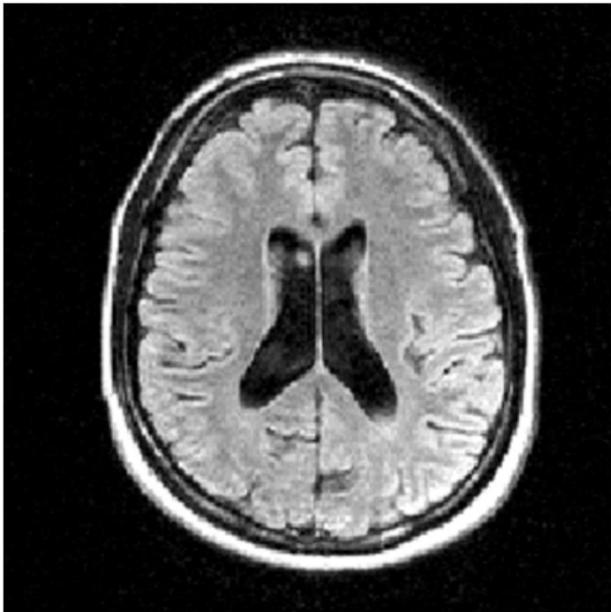
- The filter may be simpler to specify or compute in one of the domains
- Computational cost

Image reconstruction

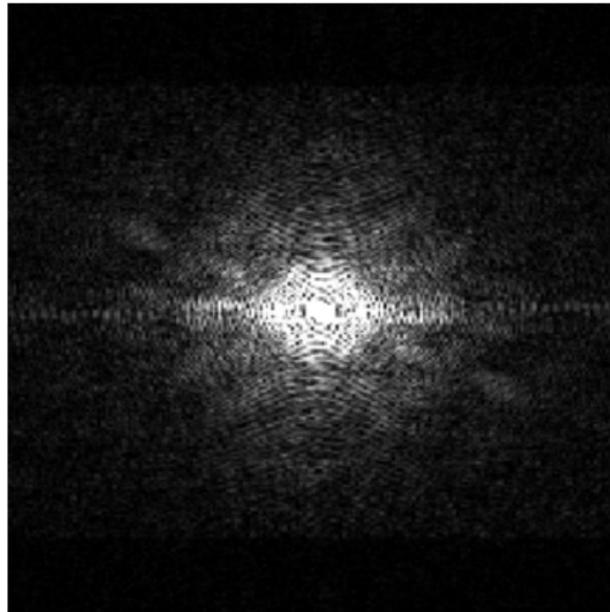


Radon transform (X-ray CT)

Magnetic Resonance Imaging (MRI)



Object of interest



Measurement
(Fourier space)

Want to learn more?

- http://www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/
 - Lectures and labs by Professor Peter Cheung at Imperial College London
- <http://www.robots.ox.ac.uk/~az/lectures/ia/>
 - Lectures by Professor Andrew Zisserman at the University of Oxford
- <https://users.fmrib.ox.ac.uk/~saad/>
 - Practicals by Professor Saad Jbabdi at the University of Oxford