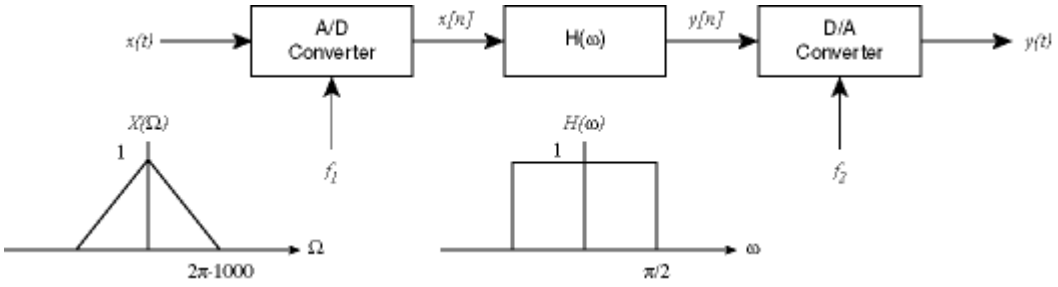
ENGR 451 – Problem Set #4

Problem 4-1

The discrete-time filtering system shown in Figure 6-68 comprises an A/D converter sampling at rate, f_1, a discrete time filter with frequency response, $H(\omega)$, and an ideal D/A converter reconstructing at rate, f_2. Ideal means that the converter contains an ideal lowpass reconstruction filter with a bandwidth of πf_2 and a gain of $1/f_2$. The spectrum of the input, $X(\Omega)$, is shown. Provide a fully labeled sketch of $X(\omega)$, $Y(\omega)$ and $Y(\Omega)$ for each of the following cases:



a) $f_1 = f_2 = 4000\,\text{Hz}$
b) $f_1 = f_2 = 2000\,\text{Hz}$
c) $f_1 = 4000\,\text{Hz}$, $f_2 = 2000\,\text{Hz}$
d) $f_1 = 2000\,\text{Hz}$, $f_2 = 4000\,\text{Hz}$

Problem 4-2

Given the discrete-time filtering system of Problem 1 with $x(t) = \cos 2\pi \cdot 1000t$, provide a fully labeled sketch of $X(\omega)$, $Y(\omega)$ and $Y(\Omega)$ and find $y(t)$ for each of the following cases:

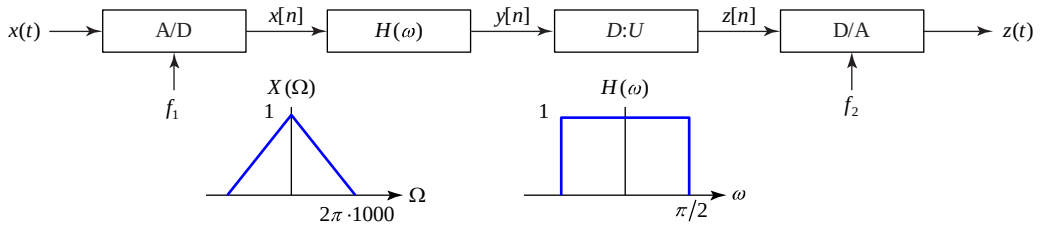
a) $f_1 = f_2 = 4000\text{Hz}$
b) $f_1 = f_2 = 2000\,\text{Hz}$
c) $f_1 = f_2 = 1333\,\text{Hz}$
d) $f_1 = f_2 = 1000\,\text{Hz}$

► Solution:

a) $y(t) = \cos 2\pi \cdot 1000t$
a) $y(t) = 0$
a) $y(t) = \cos 2\pi \cdot 500t$
a) $y(t) = 1$

Problem 4-3

A discrete-time filtering system shown in the figure below comprises an A/D converter sampling at rate, f_1, a discrete-time filter with frequency response, $H(\omega)$, a resampler that resamples at rate, D:U, and an ideal D/A converter at rate, f_2. "Ideal" means that the converter contains an ideal lowpass reconstruction filter with a bandwidth of πf_2 and a gain of $1/f_2$. Assume that the resampler is ideal (upsample by padding $y[n]$ with $U-1$ zeros, discrete-time filter with gain of U and bandwidth of $\pi/\max(U,D)$, downsample at D, tossing $D-1$ points). The spectrum of the input, $X(\Omega)$, is shown. For each of the following parts, plot the spectra $X(\omega)$, $Y(\omega)$, $Z(\omega)$ and $Z(\Omega)$.



a) $f_1 = 2000\,\text{Hz}$, $f_2 = 1000\,\text{Hz}$, $U = 1$, $D = 2$.
b) $f_1 = 2000\,\text{Hz}$, $f_2 = 4000\,\text{Hz}$, $U = 2$, $D = 1$.

Problem 4-4

A discrete-time filtering system shown in the figure below comprises an A/D converter sampling at rate, $f_1 = 8000Hz$, a 2:1 downsampler, a filter with frequency response, $H(\omega)$, and an ideal D/A converter reconstructing at rate, $f_2 = 4000Hz$. Ideal means that the converter contains an ideal lowpass reconstruction filter with a bandwidth of πf_2 and a gain of $1/f_2$. Provide a fully labeled sketch of $X(\omega), W(\omega), Y(\omega), Z(\omega)$ and $Z(\Omega)$ for each of the following cases:

a) $x(t) = \cos(2\pi \cdot 500t) + \cos(2\pi \cdot 1000t)$.
b) $x(t) = \cos(2\pi \cdot 1500t) + \cos(2\pi \cdot 3000t)$.