A Variational Analysis of Anisotropic, Inhomogeneous Dielectric Waveguides

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Abstract **—We derive a variational formulation for anisotropic, dielectric waveguides using only the (E_x, E_y) or only the (H_x, H_y) components of the electromagnetic field. We show that the (E_x, E_y) formulation is completely equivalent to the (H_x, H_y) formulation. In fact, they are the transpose problems of each other. Given the variational formulation, one can derive the finite element solution quite easily. We also show how to derive a variational expression where the natural boundary conditions are incorporated as an optimal solution of the variational expression. We illustrate our theory with a simple implementation of a finite element solution. Our solutions agree with previous results, and there is no occurrence of spurious modes. Furthermore, our formulation allows the easy inclusion of loss and frequency dispersion in the $\bar{\bar{\mu}}$ and $\bar{\bar{\epsilon}}$.**

I. INTRODUCTION

THERE HAVE been numerous papers published on the analysis of dielectric waveguides due to their growing importance in integrated optics and millimeter waves. Because of the complexity of the structure, numerical methods [1]–[15], such as the finite element method, are popular for solving this class of problems. In the finite element method, there is always an associated variational expression. Many different variational formulations have been introduced to derive the guided modes of a dielectric waveguide. Some of these formulations produce spurious modes [1]–[6], [8], which are not the physical modes of the dielectric waveguide. The spurious modes are usually observed with the (E_z, H_z) formulation where only the z components of the electromagnetic field are kept as unknowns [1], [2], [6], [8], or they are observed in a formulation where the $\nabla \cdot \boldsymbol{H} = 0$ condition is not imposed [4], [5].

Recently, several papers have been published in which the (H_x, H_y) formulation is advocated [11]–[13]. The authors showed that with the use of such a formulation, the spurious modes disappear. Even more recently, some workers have suggested using both (E_x, E_y) and (H_x, H_y) formulations to remove the spurious modes [14].

In all of the above formulations, the finite element method was used to solve the pertinent equations. Hence, one important consideration is the minimization of required computer memory. Therefore, it is most expedient to formulate the problem with a minimum number of the components of the electromagnetic fields. Some formulations use three components of the $\boldsymbol{H}$ field, and the unknowns for all three components of the $\boldsymbol{H}$ field have to be stored [4], [5], [12]. In other formulations, the unknowns for all four components of the fields, (H_x, H_y) and (E_x, E_y), have to be stored [14].

In this paper, we will use either the (E_x, E_y) only or the (H_x, H_y) only for the formulation of anisotropic, inhomogeneous waveguides. Contrary to popular belief, we show that the (E_x, E_y) formulation is as good as the (H_x, H_y) formulation. By properly defining the differential equations and the transpose problems [16], we show that the (E_x, E_y) and (H_x, H_y) formulations are completely equivalent to each other. Su [13] suggested that the (E_x, E_y) formulation was inherently less accurate than the (H_x, H_y) formulation. However, it can be shown that for the isotropic case, if the same basis set is used for the expansion functions and the testing functions, our (H_x, H_y) formulation is similar to Su's; hence, there is no reason to prefer the (H_x, H_y) formulation over the (E_x, E_y) formulation.

In all the formulations in which the spurious modes do not occur, either the $\nabla \cdot \boldsymbol{D} = 0$ or the $\nabla \cdot \boldsymbol{B} = 0$ condition is imposed. We believe that these conditions remove the possible accumulation of fictitious charges in the finite element approximation. These fictitious charges, which can store energy, can give rise to extraneous guided modes.

We will first derive the pertinent equations involving either the (E_x, E_y) or the (H_x, H_y) for an anisotropic waveguide with reflection symmetry [17]. Assuming that the field has $e^{ik_z z}$ dependence, our k_z^2 appears explicitly as the desired eigenvalue in the eigenequation. As opposed to formulations in which k^2 or the frequency appears explicitly as the desired eigenvalue [1], [4], [6], [8], [12], our formulation makes it possible to include loss and frequency dispersion.

After having derived the differential equations, we define their transpose problems [16] appropriately. Then, we derive the variational expression corresponding to the differential equations. We also discuss how the natural boundary conditions can be imposed in such a formulation. Finally, we show the numerical implementation of such a theory.

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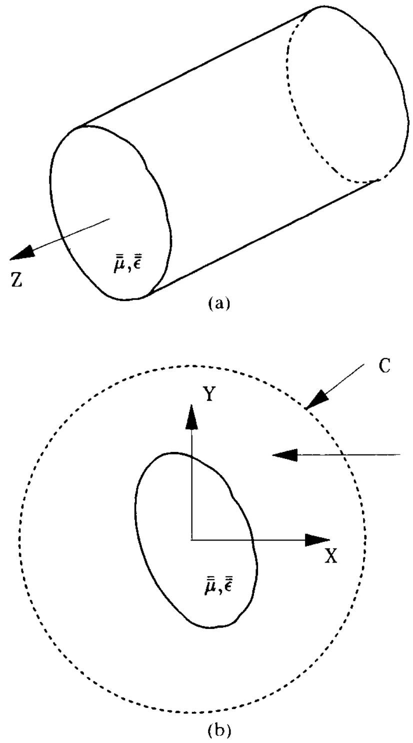

Fig. 1. Anisotropic, dielectric waveguide to be analyzed.

II. Derivation of the Differential Equations

In applying the finite element method, we have to first derive a variational expression. Before we do that, we need to derive the differential equation governing the field. Once the differential equation is derived, we can derive the variational expression quite easily once we identify the transpose problem (e.g., see [16] and [18]). In order to reduce the number of unknowns required, we reduce the differential equation to an equation involving (E_x, E_y) alone for the $\mathbf{E}$ field. The differential equation governing (H_x, H_y) can be derived via the duality principle.

The electric field in an inhomogeneous, anisotropic waveguide satisfies the vector wave equation

$$\nabla \times \bar{\bar{\mu}}^{-1}\cdot\nabla \times \mathbf{E}(\mathbf{r}) - \omega^2\bar{\bar{\epsilon}}\cdot\mathbf{E}(\mathbf{r}) = 0. \tag{1}$$

We will assume that

$$\bar{\bar{\mu}} = \begin{pmatrix} \bar{\bar{\mu}}_s & 0 \\ 0 & \mu_{zz} \end{pmatrix} \qquad \bar{\bar{\epsilon}} = \begin{pmatrix} \bar{\bar{\epsilon}}_s & 0 \\ 0 & \epsilon_{zz} \end{pmatrix} \tag{2}$$

are functions of position and that $\bar{\bar{\mu}}_s$ and $\bar{\bar{\epsilon}}_s$ are 2×2 tensors. In this manner, the geometry in Fig. 1 has reflection symmetry about the z axis; i.e., a mode propagating in the $+z$ direction is degenerate with a mode propagating in the $-z$ direction [17]. Equation (1) has three components of the electric field. To reduce the number of electric field components, we next proceed to separate the transverse and longitudinal components of (1). We assume that the field has $e^{ik_z z}$ dependence so that $\partial/\partial_z = ik_z$. Defining

$$\nabla = \nabla_s + \hat{z}ik_z \qquad \mathbf{E} = \mathbf{E}_s + \hat{z}E_z \tag{3}$$

where the subscript s denotes a vector transverse to z, the transverse components of (1) become

$$\nabla_s \times \nu_{zz}\nabla_s \times \mathbf{E}_s + ik_z\hat{z}\times\bar{\bar{\nu}}_s\cdot\nabla_s\times\hat{z}E_z - \omega^2\bar{\bar{\epsilon}}_s\cdot\mathbf{E}_s - k_z^2\hat{z}\times\bar{\bar{\nu}}_s\cdot\bar{\bar{z}}_s\cdot\hat{z}\times\mathbf{E}_s = 0 \tag{4}$$

where we have defined $\bar{\bar{\nu}}_s = \bar{\bar{\mu}}^{-1}$. We can remove E_z in (4) using the $\nabla\cdot\mathbf{D} = 0$ condition, from which we deduce that

$$E_z = -\frac{\nabla_s\cdot\bar{\bar{\epsilon}}_s\cdot\mathbf{E}_s}{ik_z\epsilon_{zz}} = \frac{i\kappa_{zz}}{k_z}\nabla_s\cdot\bar{\bar{\epsilon}}_s\cdot\mathbf{E}_s. \tag{5}$$

We have defined $\bar{\bar{\kappa}} = \bar{\bar{\epsilon}}^{-1}$ in the above. Consequently, (4) can be reduced to an equation involving only $\mathbf{E}_s$, viz.,

$$\nabla_s \times \nu_{zz}\nabla_s \times \mathbf{E}_s + \hat{z}\times\bar{\bar{\nu}}_s\cdot\hat{z}\times\nabla_s\left(\kappa_{zz}\nabla_s\cdot\bar{\bar{\epsilon}}_s\cdot\mathbf{E}_s\right) - \omega^2\bar{\bar{\epsilon}}_s\cdot\mathbf{E}_s - k_z^2\hat{z}\times\bar{\bar{\nu}}_s\cdot\hat{z}\times\mathbf{E}_s = 0. \tag{6}$$

The above is an eigenequation for k_z^2. The dependence on k_z^2 implies that $e^{\pm ik_z z}$ modes are degenerate. This would not have been possible without assuming the form of $\bar{\bar{\mu}}$ and $\bar{\bar{\epsilon}}$ in (2). In (6), k_z^2 multiplies a rather complicated expression. We can premultiply (6) by $\bar{\bar{\mu}}_s\cdot\hat{z}\times$ to arrive at

$$\bar{\bar{\mu}}_s\cdot\hat{z}\times\nabla_s\times\nu_{zz}\nabla_s\times\mathbf{E}_s - \hat{z}\times\nabla_s\kappa_{zz}\nabla_s\cdot\bar{\bar{\epsilon}}_s\cdot\mathbf{E}_s - \omega^2\bar{\bar{\mu}}_s\cdot\hat{z}\times\bar{\bar{\epsilon}}_s\cdot\mathbf{E}_s + k_z^2\hat{z}\times\mathbf{E}_s = 0. \tag{7}$$

Invoking the duality principle [18], we arrive at the equation for the transverse magnetic field:

$$\bar{\bar{\epsilon}}_s\cdot\hat{z}\times\nabla_s\times\kappa_{zz}\nabla_s\times\mathbf{H}_s - \hat{z}\times\nabla_s\nu_{zz}\nabla_s\cdot\bar{\bar{\mu}}_s\cdot\mathbf{H}_s - \omega^2\bar{\bar{\epsilon}}_s\cdot\hat{z}\times\bar{\bar{\mu}}_s\cdot\mathbf{H}_s + k_z^2\hat{z}\times\mathbf{H}_s = 0. \tag{8}$$

Equations (7) and (8) are very dissimilar in appearance. However, we shall show next that they are related via the definition of transpose operators [16].

III. The Transpose Problems

Equation (7) is a differential equation of the form

$$\mathscr{L}_e\cdot\mathbf{E}_s + k_z^2\mathscr{B}\cdot\mathbf{E}_s = 0 \tag{9}$$

where $\mathscr{L}_e$ and $\mathscr{B}$ are linear operators. Defining the inner product $\langle \mathbf{A}_s, \mathbf{B}_s\rangle = \int_S dS\mathbf{A}_s\cdot\mathbf{B}_s$, where S is the whole cross-sectional area of the waveguide (see Fig. 1(b)), the transpose operator[1] of $\mathscr{L}$ is an operator $\mathscr{L}^t$ such that $\langle \mathbf{A}_s, \mathscr{L}\cdot\mathbf{E}_s\rangle = \langle \mathbf{E}_s, \mathscr{L}^t\cdot\mathbf{A}_s\rangle$. An operator is symmetric if $\mathscr{L}^t = \mathscr{L}$. The operator $\mathscr{L}_e$ in this case is not symmetric because

$$\langle \mathbf{A}_s, \mathscr{L}_e\cdot\mathbf{E}_s\rangle \neq \langle \mathbf{E}_s, \mathscr{L}_e\cdot\mathbf{A}_s\rangle. \tag{10}$$

[1] We use the term *transpose* rather than *adjoint* because, to be strictly correct, the term *adjoint* is used only when we define the inner product as $\langle \mathbf{A}_s^*, \mathbf{B}_s\rangle$ [16], [20].

We can prove this readily because

$$\langle \boldsymbol{A}_s, \mathscr{L}_e \cdot \boldsymbol{E}_s \rangle = \int_S dS \boldsymbol{A}_s \cdot \bar{\bar{\mu}}_s \cdot \hat{z} \times \nabla_s \times \nu_{zz} \nabla_s \times \boldsymbol{E}_s - \int_S dS \boldsymbol{A}_s \cdot \hat{z} \times \nabla_s \kappa_{zz} \nabla_s \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s - \omega^2 \int_S dS \boldsymbol{A}_s \cdot \bar{\bar{\mu}}_s \cdot \hat{z} \times \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s. \tag{11}$$

Using integration by parts, or the appropriate vector identities, the above becomes

$$\begin{aligned}\langle \boldsymbol{A}_s, \mathscr{L}_e \cdot \boldsymbol{E}_s \rangle = &- \int_S dS \hat{z} \cdot \nabla_s \times \boldsymbol{E}_s \nu_{zz} \nabla_s \cdot \bar{\bar{\mu}}_s^t \cdot \boldsymbol{A}_s \\ &+ \int_S dS \hat{z} \cdot \nabla_s \times \boldsymbol{A} \kappa_{zz} \nabla_s \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s \\ &- \omega^2 \int_S dS \hat{z} \cdot (\bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s) \times (\bar{\bar{\mu}}_s^t \cdot \boldsymbol{A}_s) \\ &+ \int_C dl \hat{n} \cdot \bar{\bar{\mu}}_s^t \cdot \boldsymbol{A}_s \nu_{zz} \hat{z} \cdot \nabla_s \times \boldsymbol{E}_s \\ &- \int_C dl \hat{z} \cdot \hat{n} \times \boldsymbol{A}_s \kappa_{zz} \nabla_s \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s. \end{aligned} \tag{12}$$

If we think of $\boldsymbol{A}_s$ as a magnetic field, the last terms involving line integrals vanish if we have a PEC (perfect electric conductor) or PMC (perfect magnetic conductor) at C or if we assume C is at infinity. Then,

$$\begin{aligned}\langle \boldsymbol{A}_s, \mathscr{L}_e \cdot \boldsymbol{E}_s \rangle = &- \int_S dS \hat{z} \cdot \nabla_s \times \boldsymbol{E}_s \nu_{zz} \nabla_s \cdot \bar{\bar{\mu}}_s^t \cdot \boldsymbol{A}_s \\ &+ \int_S dS \hat{z} \cdot \nabla_s \times \boldsymbol{A} \kappa_{zz} \nabla_s \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s \\ &- \omega^2 \int_S dS \hat{z} \cdot (\bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s) \times (\bar{\bar{\mu}}_s^t \cdot \boldsymbol{A}_s). \end{aligned} \tag{12a}$$

Clearly, (10) holds true from the above expression.

Similarly, (8) is of the form

$$\mathscr{L}_h \cdot \boldsymbol{H}_s + k_z^2 \mathscr{B} \cdot \boldsymbol{H}_s = 0. \tag{13}$$

By the same token,

$$\langle \boldsymbol{C}_s, \mathscr{L}_h \cdot \boldsymbol{H}_s \rangle \neq \langle \boldsymbol{H}_s, \mathscr{L}_h \cdot \boldsymbol{C}_s \rangle. \tag{14}$$

In other words, $\mathscr{L}_h$ is not a symmetric operator. More specifically,

$$\begin{aligned}\langle \boldsymbol{C}_s, \mathscr{L}_h \cdot \boldsymbol{H}_s \rangle = &- \int_{S_0} dS \hat{z} \cdot \nabla_s \times \boldsymbol{H}_s \kappa_{zz} \nabla_s \cdot \bar{\bar{\epsilon}}_z^t \cdot \boldsymbol{C}_s \\ &+ \int_S dS \hat{z} \cdot \nabla_s \times \boldsymbol{C}_s \nu_{zz} \nabla_s \cdot \bar{\bar{\mu}}_s \cdot \boldsymbol{H}_s \\ &- \omega^2 \int_S dS \hat{z} \cdot (\bar{\bar{\mu}}_s \cdot \boldsymbol{H}_s) \times (\bar{\bar{\epsilon}}_s^t \cdot \boldsymbol{C}_s) \\ &+ \int_C dl \hat{n} \cdot \bar{\bar{\epsilon}}_s^t \cdot \boldsymbol{C}_s \kappa_{zz} \hat{z} \cdot \nabla_s \times \boldsymbol{H}_s \\ &- \int_C dl \hat{z} \cdot \hat{n} \times \boldsymbol{C}_s \nu_{zz} \nabla_s \cdot \bar{\bar{\mu}}_s \cdot \boldsymbol{H}_s. \end{aligned} \tag{15}$$

Again the line integrals vanish if we assume that C is a PEC or PMC and that $\boldsymbol{C}_s$ is an electric field, or that C is at infinity. Then,

$$\begin{aligned}\langle \boldsymbol{C}_s, \mathscr{L}_h \cdot \boldsymbol{H}_s \rangle = &- \int_S dS \hat{z} \cdot \nabla_s \times \boldsymbol{H}_s \kappa_{zz} \nabla_s \cdot \bar{\bar{\epsilon}}_z^t \cdot \boldsymbol{C}_s \\ &+ \int_S dS \hat{z} \cdot \nabla_s \times \boldsymbol{C}_s \nu_{zz} \nabla_s \cdot \bar{\bar{\mu}}_s \cdot \boldsymbol{H}_s \\ &- \omega^2 \int_S dS \hat{z} \cdot (\bar{\bar{\mu}}_s \cdot \boldsymbol{H}_s) \times (\bar{\bar{\epsilon}}_s^t \cdot \boldsymbol{C}_s). \end{aligned} \tag{15a}$$

Clearly, (14) holds true from the above expression.

In (12a) and (15a), $\mathscr{L}_e$ and $\mathscr{L}_h$ are functions of $\bar{\bar{\mu}}$ and $\bar{\bar{\epsilon}}$. By comparing (12a) and (15a) and letting $\boldsymbol{A} = \boldsymbol{H}_s$ in (12a) and $\boldsymbol{C}_s = \boldsymbol{E}_s$ in (15a), we notice that

$$\langle \boldsymbol{H}_s, \mathscr{L}_e(\bar{\bar{\mu}}, \bar{\bar{\epsilon}}) \cdot \boldsymbol{E}_s \rangle = \langle \boldsymbol{E}_s, -\mathscr{L}_h(\bar{\bar{\mu}}^t, \bar{\bar{\epsilon}}^t) \cdot \boldsymbol{H}_s \rangle. \tag{16}$$

Therefore, $-\mathscr{L}_h(\bar{\bar{\mu}}^t, \bar{\bar{\epsilon}}^t)$ is the transpose operator of $\mathscr{L}_e(\bar{\bar{\mu}}, \bar{\bar{\epsilon}})$. If the medium is reciprocal, then $\bar{\bar{\mu}}^t = \bar{\bar{\mu}}$ and $\bar{\bar{\epsilon}}^t = \bar{\bar{\epsilon}}$. In this case, $-\mathscr{L}_h$ is the transpose operator of $\mathscr{L}_e$ from the same medium. However, in general, we have

$$\mathscr{L}_e^t(\bar{\bar{\mu}}, \bar{\bar{\epsilon}}) = -\mathscr{L}_h(\bar{\bar{\mu}}^t, \bar{\bar{\epsilon}}^t). \tag{17}$$

In other words, the transpose of the operator $-\mathscr{L}_e$ in medium $(\bar{\bar{\mu}}, \bar{\bar{\epsilon}})$ is the operator $\mathscr{L}_h$ in medium $(\bar{\bar{\mu}}^t, \bar{\bar{\epsilon}}^t)$.

It is easy to show that the operator $\mathscr{B}$ in (9) and (13) satisfies

$$\langle \boldsymbol{A}, \mathscr{B} \cdot \boldsymbol{E}_s \rangle = \langle \boldsymbol{E}_s, -\mathscr{B} \cdot \boldsymbol{A} \rangle. \tag{18}$$

Therefore,

$$\mathscr{B}^t = -\mathscr{B}. \tag{19}$$

The above derivation establishes the transpose problems of (9) and (13), respectively. Given (9), its transpose problem exists, and can be associated with a physical problem. Hence, given

$$\mathscr{L}_e(\bar{\bar{\mu}}, \bar{\bar{\epsilon}}) \cdot \boldsymbol{E}_s + k_z^2 \mathscr{B} \cdot \boldsymbol{E}_s = 0 \tag{20}$$

the transpose problem is

$$\mathscr{L}_e^t(\bar{\bar{\mu}}, \bar{\bar{\epsilon}}) \cdot \boldsymbol{E}_s^t + k_z^2 \mathscr{B}^a \cdot \boldsymbol{E}_s^t = 0. \tag{21}$$

Using (17) and (19), the transpose problem is the same as

$$\mathscr{L}_h(\bar{\bar{\mu}}^t, \bar{\bar{\epsilon}}^t) \cdot \tilde{\boldsymbol{H}}_s + k_z^2 \mathscr{B} \cdot \tilde{\boldsymbol{H}}_s = 0. \tag{22}$$

Hence, the transpose solution, $\boldsymbol{E}_s^t$, is $\tilde{\boldsymbol{H}}_s$, which is the transverse magnetic field solution of a medium with the electromagnetic property of $(\bar{\bar{\mu}}^t, \bar{\bar{\epsilon}}^t)$. From this point onward, we use the tilde (~) to denote that $\tilde{\boldsymbol{H}}_s$ is the magnetic field solution with the medium $\bar{\bar{\mu}}^t, \bar{\bar{\epsilon}}^t$. If the medium is reciprocal, $\bar{\bar{\mu}}^t = \bar{\bar{\mu}}$ and $\bar{\bar{\epsilon}}^t = \bar{\bar{\epsilon}}$, the transpose solution is just the transverse magnetic field solution of the same medium, that is, $\tilde{\boldsymbol{H}}_s = \boldsymbol{H}_s$. Similarly, we can establish the transpose problem and the solution associated with (13). A similar result could be obtained if we use $\langle \boldsymbol{A}^*, \boldsymbol{B} \rangle$ as our inner products [16], [18], [20]. In this case, the term *transpose operators* will be replaced by *adjoint operators*. Equations (9) and (13) can be proved to be non-self-adjoint.

IV. Variational Principle

Given a nonsymmetric eigenequation (20) and its transpose equation (21), we can derive a variational expression for the eigenvalues k_z^2. The variational expression is

$$k_z^2 = -\frac{\langle \boldsymbol{E}_s^t, \mathscr{L}_e \cdot \boldsymbol{E}_s \rangle}{\langle \boldsymbol{E}_s^t, \mathscr{B} \cdot \boldsymbol{E}_s \rangle} = -\frac{\langle \tilde{\boldsymbol{H}}_s, \mathscr{L}_e \cdot \boldsymbol{E}_s \rangle}{\langle \tilde{\boldsymbol{H}}_s, \mathscr{B} \cdot \boldsymbol{E}_s \rangle}. \tag{23}$$

We can establish the variational property of the above expression about the exact solutions $\tilde{\boldsymbol{H}}_{se}$ and $\boldsymbol{E}_{se}$ by letting

$$\tilde{\boldsymbol{H}}_s = \tilde{\boldsymbol{H}}_{se} + \delta \boldsymbol{H} \qquad \boldsymbol{E}_s = \boldsymbol{E}_{se} + \delta \boldsymbol{E} \tag{24}$$

and showing that the first variation of k_z^2 is zero.[2]

Since (23) is a variational expression, we can apply the Rayleigh–Ritz procedure to find the optimal solution. To do this, we let

$$\boldsymbol{E}_s = \sum_{n=1}^{N} a_n \boldsymbol{E}_{ns} \qquad \tilde{\boldsymbol{H}}_s = \sum_{m=1}^{N} b_m \boldsymbol{H}_{ms}. \tag{25}$$

We next substitute (25) into (23) and require that the first variations of (23) with respect to the a_n's and b_m's vanish. We find that the optimal solution is given by the solution of the following matrix equations:

$$\left[\overline{\overline{L}}_e + k_z^2 \overline{\overline{B}}\right] \cdot \overline{a} = 0 \tag{26a}$$

$$\left[\overline{\overline{L}}{}_e^t + k_z^2 \overline{\overline{B}}{}^t\right] \cdot \overline{b} = 0 \tag{26b}$$

where

$$\left[\overline{\overline{L}}_e\right]_{mn} = \langle \boldsymbol{H}_{ms}, \mathscr{L}_e \cdot \boldsymbol{E}_{ns} \rangle \tag{27a}$$

$$\left[\overline{\overline{B}}\right]_{mn} = \langle \boldsymbol{H}_{ms}, \mathscr{B} \cdot \boldsymbol{E}_{ns} \rangle. \tag{27b}$$

In the above, $\overline{\overline{L}}_e$ and $\overline{\overline{B}}$ are also known as the matrix representations [21][3] of the operators $\mathscr{L}_e$ and $\mathscr{B}$. From the definition of transpose operators,

$$\begin{aligned}\langle \boldsymbol{H}_{ms}, \mathscr{L}_e \cdot \boldsymbol{E}_{ns} \rangle &= \langle \boldsymbol{E}_{ns}, \mathscr{L}_e^a \cdot \boldsymbol{H}_{ms} \rangle \\ &= \langle \boldsymbol{E}_{ns}, -\mathscr{L}_h(\overline{\overline{\mu}}{}^t, \overline{\overline{\epsilon}}{}^t) \cdot \boldsymbol{H}_{ms} \rangle \end{aligned} \tag{28}$$

or

$$\left[\overline{\overline{L}}_e\right]_{mn} = -\left[\overline{\overline{L}}_h\right]_{nm}$$

we see that

$$\overline{\overline{L}}{}_e^t = -\overline{\overline{L}}_h \tag{29}$$

where $\overline{\overline{L}}_h$ is the matrix representation of the operator $\mathscr{L}_h$. Since $\overline{\overline{B}}{}^t = -\overline{\overline{B}}$, (26b) can be rewritten as[4]

$$\left[\overline{\overline{L}}_h + k_z^2 \overline{\overline{B}}\right] \cdot \overline{b} = 0. \tag{30}$$

Equations (26a) and (30) can also be directly obtained from (9) and (13) by the application of the method of weighted residuals or the Petrov–Galerkin method [22] (also known as the method of moments [23]). In solving (9) using this method, for example, we will use $\boldsymbol{E}_{ns}$'s as expansion functions and $\boldsymbol{H}_{ms}$'s as weighting functions. Intuitively, for fast convergence, we would want the $\boldsymbol{E}_{ns}$'s to approximate $\boldsymbol{E}_s$ well, while we would want the $\boldsymbol{H}_{ms}$'s to approximate the transpose solution, $\tilde{\boldsymbol{H}}_s$, well. When we choose $\boldsymbol{H}_{ms}$ and $\boldsymbol{E}_{ns}$ to be the same basis set, the method is commonly known as Galerkin's method. When the basis functions are finite domain basis functions, the method is also known as the finite element method.

When the same basis set is used for $\boldsymbol{H}_{ms}$ and $\boldsymbol{E}_{ns}$, viz., using the Galerkin's method, our formulation here is equivalent to the $\boldsymbol{E}_s$ method where the testing field is also $\boldsymbol{E}_s$ or to the $\boldsymbol{H}_s$ method where the testing field is also $\boldsymbol{H}_s$.

Since the eigenequation (30) is the transpose of the eigenequation (26a), we need only solve one of them for the eigenvalues, since they share the same eigenvalues.[5] Furthermore, we can prove that for $\overline{a}_i$ and $\overline{b}_j$ corresponding to the ith and jth eigenvalues, respectively, $\overline{b}_j \cdot \overline{\overline{B}} \cdot \overline{a}_i = D_j \delta_{ij}$. Hence, once all the $\overline{a}_i$'s are found, we can find the $\overline{b}_j$'s easily. In addition, since $\overline{\overline{L}}_e$ is not a symmetric matrix and is non-Hermitian in general (a consequence of nonsymmetric and non-self-adjoint operator), k_z^2 can be complex. This corresponds to the complex modes of the structure which are not due to losses. The existence of these complex modes is a direct consequence of the non-self-adjoint natures of (9) and (13). A similar variational principle can be derived using $\langle \boldsymbol{A}^*, \boldsymbol{B} \rangle$ type inner products.

V. Natural Boundary Conditions

In the previous derivation, we have assumed C to be a PEC or a PMC. Under such an assumption, the line integral terms in (12) and (15) vanish. In choosing the basis set for $\boldsymbol{E}_s$ and $\tilde{\boldsymbol{H}}_s$, we have to ensure that the PEC or PMC boundary conditions are satisfied or that the fields vanish on C. Boundary conditions that are imposed by choosing the basis set are known as the essential boundary conditions (also known as forced or principal boundary conditions). However, it is sometimes desirable that the boundary conditions be a natural result of the optimal solution of a variational expression. These boundary conditions are known as the natural boundary conditions [18], [22]. We show next how such a variational expression can be derived, in which the boundary conditions are the natural result of the variational expression.

We can rewrite (12) with $\boldsymbol{A}_s = \tilde{\boldsymbol{H}}_s$ as

$$\begin{aligned}\langle \tilde{\boldsymbol{H}}_s, \mathscr{L}_e \cdot \boldsymbol{E}_s \rangle &= \langle \tilde{\boldsymbol{H}}_s, \mathscr{L}_e^o \cdot \boldsymbol{E}_s \rangle \\ &\quad + \int_C dl\, \hat{n} \cdot \overline{\overline{\mu}}{}_s^t \cdot \tilde{\boldsymbol{H}}_s \nu_{zz} \hat{z} \cdot \nabla_s \times \boldsymbol{E}_s \\ &\quad - \int_C dl\, \hat{z} \cdot \hat{n} \times \tilde{\boldsymbol{H}}_s \kappa_{zz} \nabla_s \cdot \overline{\overline{\epsilon}}_s \cdot \boldsymbol{E}_s \end{aligned} \tag{31}$$

where $\langle \tilde{\boldsymbol{H}}_s, \mathscr{L}_e^o \cdot \boldsymbol{E}_s \rangle$ is used to denote terms in (12) involving only integrals over S. Similarly, we can rewrite (15)

[2]To show that the first variation in k_z^2 vanishes, it is easiest to cross-multiply out (23) first.

[3]In quantum mechanics, a matrix representation is defined with the inner product $\langle \boldsymbol{A}^*, \boldsymbol{B} \rangle$.

[4]Note that the definitions of transpose operators and symmetric operators are very similar to the definitions of transpose matrices and symmetric matrices.

[5]This follows from $\det(\overline{\overline{L}}_e + k_z^2 \overline{\overline{B}}) = \det(\overline{\overline{L}}{}_e^t + k_z^2 \overline{\overline{B}}{}^t) = 0$.

with $\boldsymbol{C}_s = \boldsymbol{E}_s$ as

$$\langle \boldsymbol{E}_s, \mathscr{L}_h \cdot \tilde{\boldsymbol{H}}_s \rangle = \langle \boldsymbol{E}_s, \mathscr{L}_h^o \cdot \tilde{\boldsymbol{H}}_s \rangle + \int_C dl \hat{n} \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s \kappa_{zz} \hat{z} \cdot \nabla_s \times \tilde{\boldsymbol{H}}_s - \int_C dl \hat{z} \cdot \hat{n} \times \boldsymbol{E}_s \nu_{zz} \nabla_s \cdot \bar{\bar{\mu}}_s^t \cdot \tilde{\boldsymbol{H}}_s \quad (32)$$

where $\langle \boldsymbol{E}_s, \mathscr{L}_h^o \cdot \tilde{\boldsymbol{H}}_s \rangle$ is used to denote terms in (15) involving only integrals over S. If we have an impedance boundary condition on C, the line integral terms in (31) and (32) need not necessarily vanish. To impose the impedance boundary conditions, we can assume that

$$H_z = -Y_s \hat{z} \cdot \hat{n} \times \boldsymbol{E}_s \quad (33a)$$

$$\tilde{E}_z = \tilde{Z}_s \hat{z} \cdot \hat{n} \times \tilde{\boldsymbol{H}}_s \quad (33b)$$

for the original and transpose problems, respectively. After some manipulations, it is clear that the expression

$$k_z^2 = -\frac{\langle \tilde{\boldsymbol{H}}_s, \mathscr{L}_e^o \cdot \boldsymbol{E}_s \rangle - \langle \tilde{\boldsymbol{H}}_s, \partial \mathscr{L}_e \cdot \boldsymbol{E}_s \rangle}{\langle \tilde{\boldsymbol{H}}_s, \mathscr{B} \cdot \boldsymbol{E}_s \rangle} \quad (34)$$

is the required variational expression [18]. In the above, we define

$$\langle \tilde{\boldsymbol{H}}_s, \partial \mathscr{L}_e \cdot \boldsymbol{E}_s \rangle = i\omega \int_C dl \hat{n} \cdot \bar{\bar{\mu}}_s^t \cdot \tilde{\boldsymbol{H}}_s Y_s \hat{z} \cdot \hat{n} \times \boldsymbol{E}_s - i\omega \int_C dl \hat{n} \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s \tilde{Z}_s \hat{z} \cdot \hat{n} \times \tilde{\boldsymbol{H}}_s. \quad (35)$$

The functional is chosen judiciously so that symmetric boundary conditions will ensue for the actual and transpose problems, as we shall see later.

Taking the first variation of (34) with respect to the exact solution, we have

$$\langle \delta \boldsymbol{H}_s, \mathscr{L}_e^o \cdot \boldsymbol{E}_{se} \rangle + \langle \tilde{\boldsymbol{H}}_{se}, \mathscr{L}_e \cdot \delta \boldsymbol{E}_s \rangle - \langle \delta \boldsymbol{H}_s, \partial \mathscr{L}_e \cdot \boldsymbol{E}_{se} \rangle - \langle \tilde{\boldsymbol{H}}_{se}, \partial \mathscr{L}_e \cdot \delta \boldsymbol{E}_s \rangle + k_{ze}^2 \langle \delta \boldsymbol{H}_s, \mathscr{B} \cdot \boldsymbol{E}_{se} \rangle + k_{ze}^2 \langle \tilde{\boldsymbol{H}}_{se}, \mathscr{B} \cdot \delta \boldsymbol{E}_s \rangle + \delta k_z^2 \langle \tilde{\boldsymbol{H}}_{se}, \mathscr{B} \cdot \boldsymbol{E}_{se} \rangle = 0. \quad (36)$$

For arbitrary variations of $\delta \boldsymbol{E}_s$ and $\delta \boldsymbol{H}_s$, we find that the condition for δk_z^2 to vanish is that

$$\langle \delta \boldsymbol{H}_s, \mathscr{L}_e^o \cdot \boldsymbol{E}_{se} \rangle - \langle \delta \boldsymbol{H}_s, \partial \mathscr{L}_e \cdot \boldsymbol{E}_{se} \rangle + k_{ze}^2 \langle \delta \boldsymbol{H}_s, \mathscr{B} \cdot \boldsymbol{E}_{se} \rangle = 0 \quad (37a)$$

$$\langle \tilde{\boldsymbol{H}}_{se}, \mathscr{L}_e^o \cdot \delta \boldsymbol{E}_s \rangle - \langle \tilde{\boldsymbol{H}}_{se}, \partial \mathscr{L}_e \cdot \delta \boldsymbol{E}_s \rangle + k_{ze}^2 \langle \tilde{\boldsymbol{H}}_{se}, \mathscr{B} \cdot \delta \boldsymbol{E}_s \rangle = 0. \quad (37b)$$

Using (31) for the definition of $\langle \delta \boldsymbol{H}_s, \mathscr{L}_e^o \cdot \boldsymbol{E}_{se} \rangle$, and (35) for the definition of $\langle \delta \boldsymbol{H}_s, \partial \mathscr{L}_e \cdot \boldsymbol{E}_{se} \rangle$, we see that (37a) is satisfied only if

$$\langle \delta \boldsymbol{H}_s, \mathscr{L}_e \cdot \boldsymbol{E}_{se} \rangle + k_{ze}^2 \langle \delta \boldsymbol{H}_s, \mathscr{B} \cdot \boldsymbol{E}_{se} \rangle = 0 \quad (38)$$

and

$$\int_C dl \hat{n} \cdot \bar{\bar{\mu}}_s^t \cdot \delta \boldsymbol{H}_s \nu_{zz} \hat{z} \cdot \nabla_s \times \boldsymbol{E}_s - \int_C dl \hat{z} \cdot \hat{n} \times \delta \boldsymbol{H}_s \kappa_{zz} \nabla_s \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s = i\omega \int_C dl \hat{n} \cdot \bar{\bar{\mu}}_s^t \cdot \delta \boldsymbol{H}_s Y_s \hat{z} \cdot \hat{n} \times \boldsymbol{E}_s - i\omega \int_C dl \hat{n} \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s \tilde{Z}_s \hat{z} \cdot \hat{n} \times \delta \boldsymbol{H}_s. \quad (39)$$

For arbitrary $\delta \boldsymbol{H}_s$, (38) and (39) imply that

$$\mathscr{L}_e \cdot \boldsymbol{E}_{se} + k_{ze}^2 \mathscr{B} \cdot \boldsymbol{E}_{se} = 0 \quad (40a)$$

and

$$\nu_{zz} \hat{z} \cdot \nabla_s \times \boldsymbol{E}_s = -i\omega Y_s \hat{z} \cdot \hat{n} \times \boldsymbol{E}_s$$

$$\kappa_{zz} \nabla_s \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s = -i\omega \tilde{Z}_s \hat{n} \cdot \bar{\bar{\epsilon}}_s \cdot \boldsymbol{E}_s \quad (40b)$$

on C. Equation (40b) is equivalent to

$$H_z = -Y_s \hat{z} \cdot \hat{n} \times \boldsymbol{E}_s \qquad E_z = \frac{\omega}{k_z} \tilde{Z}_s \hat{n} \cdot \boldsymbol{D}_s. \quad (41)$$

Similarly, from (37b), by noting that $\langle \tilde{\boldsymbol{H}}_{se}, \mathscr{L}_e^o \cdot \delta \boldsymbol{E}_s \rangle = \langle \delta \boldsymbol{E}_s, -\mathscr{L}_h^o \cdot \tilde{\boldsymbol{H}}_{se} \rangle$, and using its definition from (32), we conclude that

$$\mathscr{L}_h \cdot \boldsymbol{H}_{se} + k_{ze}^2 \mathscr{B} \cdot \boldsymbol{H}_{se} = 0 \quad (42a)$$

and

$$\kappa_{zz} \hat{z} \cdot \nabla_s \times \tilde{\boldsymbol{H}}_s = -i\omega \tilde{Z}_s \hat{z} \cdot \hat{n} \times \tilde{\boldsymbol{H}}_s$$

$$\nu_{zz} \nabla_s \cdot \bar{\bar{\mu}}_s^t \cdot \tilde{\boldsymbol{H}}_s = -i\omega Y_s \hat{n} \cdot \bar{\bar{\mu}}_s^t \cdot \tilde{\boldsymbol{H}}_s \quad (42b)$$

on C. Equation (42b) is equivalent to

$$\tilde{E}_z = \tilde{Z}_s \hat{z} \cdot \hat{n} \times \tilde{\boldsymbol{H}}_s \qquad \tilde{H}_z = \frac{\omega}{k_z} Y_s \hat{n} \cdot \tilde{\boldsymbol{B}}_s. \quad (43)$$

Note that in (41) and (43), auxiliary boundary conditions for E_z and $\tilde{H}_z$ have surfaced in addition to those assumed in (33a) and (33b). With these natural boundary conditions, to obtain a PEC at C, we let $Z_s = 0$ and $\tilde{Z}_s = 0$. Furthermore, if we were to assume an essential boundary condition for PEC where $\hat{n} \times \boldsymbol{E}_s$ or $\hat{n} \cdot \bar{\bar{\mu}}_s^t \cdot \boldsymbol{H}_s$ is zero, then the first term in (35) would vanish. For a PEC, the natural boundary condition dictates that $\tilde{Z}_s = 0$, and the second term in (35) vanishes. In such a case, we can remove the functional (35) from (34) completely. Similar statements can be made for a PMC at C.

With the variational expression (34) available, we can apply the Rayleigh–Ritz procedure to find the optimal solution as in Section IV. The resultant matrix equations are

$$\left[\bar{\bar{L}}_e - \partial \bar{\bar{L}}_e + k_z^2 \bar{\bar{B}}\right] \cdot \bar{a} = 0 \quad (44a)$$

$$\left[\bar{\bar{L}}_e^t - \partial \bar{\bar{L}}_e^t + k_z^2 \bar{\bar{B}}^t\right] \cdot \bar{b} = 0. \quad (44b)$$

The resultant equations are similar to (26a) and (26b) except for the $\partial \bar{\bar{L}}_e$ term, which is defined as

$$\left[\partial \bar{\bar{L}}_e\right]_{mn} = \langle \boldsymbol{H}_{ms}, \partial \mathscr{L}_e \cdot \boldsymbol{E}_{ns} \rangle. \quad (45)$$

If finite domain basis functions are used in (25), since (45) involves line integrals on C, only the elements touching the boundary C are involved in calculating $\partial \bar{\bar{L}}_e$. As before, we need only solve one of the two equations (44a) and (44b). Furthermore, once $\bar{a}_i$'s are found, the $\bar{b}_j$'s can be found easily.

VI. Implementation and Numerical Results

We have implemented the above theory for an inhomogeneous anisotropic dielectric waveguide. Without loss of generality, we let

$$\boldsymbol{E}_s = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \sum_{n=1}^{N} \bar{\bar{f}}_n(x,y)\cdot\bar{\alpha}_n$$

$$\boldsymbol{H}_s = \begin{pmatrix} H_x \\ H_y \end{pmatrix} = \sum_{m=1}^{N} \bar{\bar{f}}_m(x,y)\cdot\bar{\beta}_m \tag{46}$$

where $\bar{\bar{f}}_n(x,y)$ is a 2×2 matrix, and $\bar{\alpha}_n$ and $\bar{\beta}_n$ are 2×1 column vectors. In general, $\bar{\bar{f}}_n$ need not be diagonal. For simplicity, we have assumed

$$\bar{\bar{f}}_n(x,y) = \begin{pmatrix} f_n(x,y) & 0 \\ 0 & f_n(x,y) \end{pmatrix}. \tag{47}$$

The choice of $f_n(x,y)$ is such that it should be good enough to approximate the x and y components of the electric and magnetic fields. With the above choice of basis functions,

$$\left[\bar{\bar{L}}_e\right]_{mn} = \langle \bar{\bar{f}}_m, \mathscr{L}_e\cdot\bar{\bar{f}}_n\rangle \tag{48a}$$

$$\left[\bar{\bar{B}}\right]_{mn} = \langle \bar{\bar{f}}_m, \mathscr{B}\cdot\bar{\bar{f}}_n\rangle \tag{48b}$$

are 2×2 submatrices. More specifically,

$$\left[\bar{\bar{L}}_e\right]_{mn} = -\int_S dS\,\hat{z}\cdot\nabla_s\times\bar{\bar{f}}_m\nu_{zz}\nabla_s\cdot\bar{\bar{\mu}}_s^t\cdot\bar{\bar{f}}_n + \int_S dS\,\hat{z}\cdot\nabla_s\times\bar{\bar{f}}_n\kappa_{zz}\nabla_s\cdot\bar{\bar{\epsilon}}_s\cdot\bar{\bar{f}}_m - \omega^2\int_S dS\,\hat{z}\cdot\left(\bar{\bar{\epsilon}}_s\cdot\bar{\bar{f}}_n\right)\times\left(\bar{\bar{\mu}}_s^t\cdot\bar{\bar{f}}_m\right) \tag{49a}$$

$$\left[\bar{\bar{B}}\right]_{mn} = \int_S dS\,\bar{\bar{f}}_m\cdot\hat{z}\times\bar{\bar{f}}_n. \tag{49b}$$

A detailed analysis of (49) is given in [24].

We have chosen $f_n(x,y)$ to be first-order elements. They correspond to interpolating the three points of a triangle with a plane. They can also be thought of as pyramidal functions with a hexagonal base and six plane surfaces. The general mesh we use for the computation is shown in Fig. 2. This mesh is good for an arbitrarily shaped waveguide. If the waveguide is symmetric with respect to the vertical and horizontal midplanes through the waveguide, the mesh can be reduced to a quarter of its original size by imposing the necessary magnetic and electric wall boundary conditions at the midplanes. We found that results similar to those before are obtained by reducing the mesh to one quarter of its original size. The reduction in mesh size is done by a number of workers to reduce storage requirement, but this makes the program more restrictive. For a guided mode, we found that the boundary conditions on the outermost wall were not important because the field was, in general, exponentially small there.

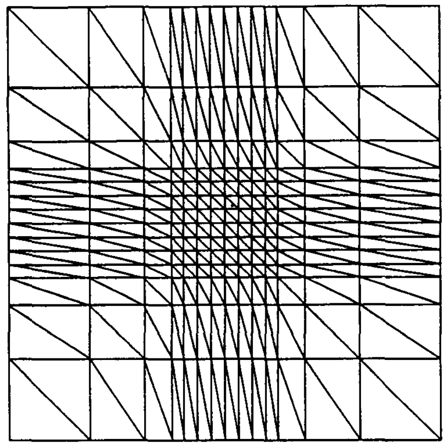

Fig. 2. A sample finite element mesh.

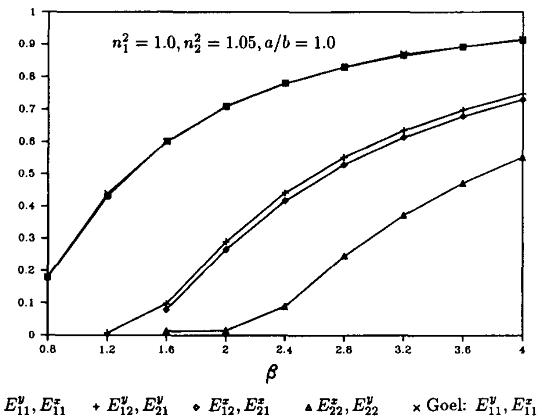

Fig. 3. Low-contrast case and the comparison with Goell's result.

We have solved the eigenequation (26a) with a standard eigenvalue solver from IMSL (International Mathematical Subroutine Library). We have compared the results generated by our code with previous results and found good agreement for both the isotropic and anisotropic cases (Figs. 3–5). We did not observe the occurrence of spurious modes.

In the following plots

$$\beta = \frac{2b}{\lambda_0}\left[\left(\frac{k_1}{k_0}\right)^2 - 1\right]^{\frac{1}{2}} \quad \text{and} \quad P = \frac{(k_z/k_0)^2 - 1}{(k_1/k_0)^2 - 1}.$$

The case considered is a rectangular dielectric waveguide with aspect ratio a/b. The simulations are done using 17×17 pyramidal functions for each E_x and E_y, corresponding to 578 unknowns. It took about 30 seconds on a CRAY-XMP/48 to solve once and for all the eigenvalues of the matrix. The first six modes are plotted, and the lowest two modes are compared with previous results.

In Fig. 3, we compare our fundamental mode with the solution of Goell [25], showing excellent agreement. Goell's solution is derived from cylindrical harmonic analysis, and

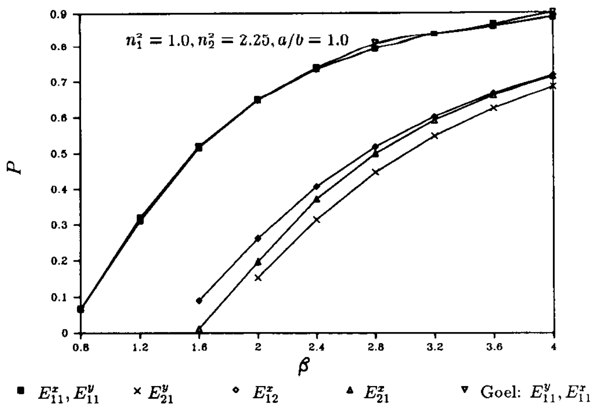

Fig. 4. High-contrast case and the comparison with Goell's result.

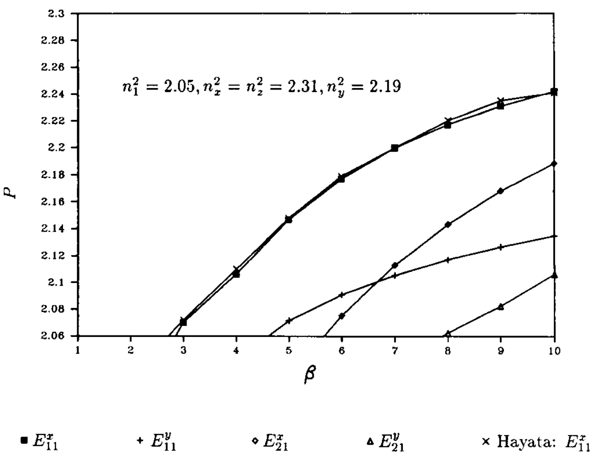

Fig. 5. Anisotropic case which is uniaxial, and the comparison with Hayata's result.

has been standard for comparison in the literature. However, a finite element solution is more versatile than a cylindrical harmonic analysis.

In Fig. 4, we display the dispersion diagram for the high-contrast case. The fundamental mode again agrees very well with Goell's result.

In Fig. 5, we display the dispersion curve for the anisotropic case, and compare the result with that of Hayata *et al.* [12]. We have rescaled β and P to agree with those defined by Hayata. The agreement is very good.

VII. Conclusions

We have developed a variational formulation for anisotropic dielectric waveguides with reflection symmetry using $\boldsymbol{E}_s$ or $\boldsymbol{H}_s$ components only. We cast our variational formulation via the use of transpose operators and shed a different light on the formulation. We show that the $\boldsymbol{E}_s$ and $\boldsymbol{H}_s$ formulations are completely equivalent to each other; hence, there is no reason to prefer one over the other. Our formulation also allows us to easily study the effect of loss and frequency dispersion. We show how the natural boundary conditions can be included in the formulation. The results of our numerical implementation also compare favorably with these results of previous formulations.

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References

[1] M. Ikeuchi, H. Sawami, and H. Niki, "Analysis of open-type dielectric waveguides by the finite-element iterative method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 234–239, Mar. 1981.
[2] N. Mabaya, P. E. Lagasse, and P. Vandenbulcke, "Finite element analysis of optical waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 600–605, June 1981.
[3] J. Katz, "Novel solution of 2-D waveguide using the finite element method," *Appl. Opt.*, vol. 21, no. 15, p. 2747, 1982.
[4] B. M. A. Rahman and J. B. Davies, "Finite-element analysis of optical and microwave waveguide problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 20–28, Jan. 1984.
[5] M. Koshiba, K. Hayata, and M. Suzuki, "Vectorial finite-element formulation without spurious modes for dielectric waveguides,' *Trans. IECE Japan*, vol. E 67, no. 4, pp. 191–196, 1984.
[6] E. Schweig and W. B. Bridges, "Computer analysis of dielectric waveguides: A finite-difference method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 531–541, May 1984.
[7] M. Koshiba, K. Hayata, and M. Suzuki, "Approximate scalar finite element analysis of anisotropic optical waveguides with off-diagonal elements in a permittivity tensor," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 587–593, June 1984.
[8] R. B. Wu and C. H. Chen, "A variational analysis of dielectric waveguides by the conformal mapping technique," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, no. 8, pp. 681–685, Aug. 1985.
[9] C. C. Su, "A surface integral equation method for homogeneous optical fibers and coupled image lines of arbitrary cross sections," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1114–1119, Nov. 1985.
[10] R. B. Wu and C. H. Chen, "A scalar variational conformal mapping technique by weakly guiding dielectric waveguides," *IEEE J. Quantum Electron.*, vol. QE-22, pp. 595–608, May 1986.
[11] K. Bierwirth, N. Schulz, and F. Arndt, "Finite-difference analysis of rectangular dielectric waveguide structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1104–1114, Nov. 1986.
[12] K. Hayata, M. Koshiba, M. Eguchi, and M. Suzuki, "Vectorial finite-element method without any spurious solutions for dielectric waveguiding problems using transverse magnetic field component," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1120–1124, Nov. 1986.
[13] C. C. Su, "A combined method for dielectric waveguides using finite-element techniques and the surface integral equations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1140–1146, Nov. 1986.
[14] T. Angkaew, M. Matsuhara, and N. Kumagai, "Finite-element analysis of waveguide modes: A novel approach that eliminates spurious modes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 117–123, 1987.
[15] W. C. Chew, "Analysis of optical and millimeter wave dielectric waveguide," *J. Electromagnetic Waves and Appl.*, to be published.
[16] L. Cairo and T. Kahan, *Variational Techniques in Electromagnetism*. New York: Gordon and Breach, 1965.
[17] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960. p. 231.
[18] C. H. Chen and C. D. Lien, "The variational formulation for non-self-adjoint electromagnetic problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 878–886, 1980.
[19] J. A. Kong, *Electromagnetic Wave Theory*. New York: Wiley, 1986.
[20] I. Stakgold, *Boundary Value Problems of Mathematical Physics*, vol. I. New York: Macmillian, 1968.

[21] E. Merzbacher, *Quantum Mechanics*. New York: Wiley, 1970.
[22] R. Wait and A. R. Mitchell, *Finite Element Analysis and Applications*. New York: Wiley, 1985.
[23] R. Harrington, *Field Computation by Moment Method*. Malabar, FL: Krieger, 1983.
[24] M. Nasir, "Finite element analysis of an inhomogeneous, anisotropic dielectric waveguide," M.S. thesis, Dept. of Elect. and Comp. Eng., Univ. of Illinois, Urbana, 1988.
[25] J. E. Goell, "A circular-harmonic computer analysis of rectangular dielectric waveguides," *Bell Syst. Tech. J.*, vol. 48, pp. 2133–2160, 1969.

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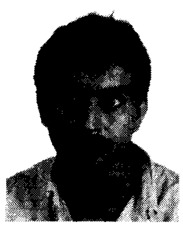

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