MAT 4170: Modern Algebra I **Name:** **Panya Sukphranee**

Skill Check 3

Due: Tuesday, 10/9/20

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1. **Let G be a group such that $a = a^{-1}$ for $a \in G$. Prove that G is Abelian.**

   Let $a, b \in G$.
$$ab = a^{-1}b^{-1} = (ba)^{-1} = ba.$$

2. **Suppose that $a, b \in G$, $|a| = 2$, $|b| = 4$, and $ab = b^3a$.**

   (a) **Prove that $bab = a$.**
$$bab = b(b^3a) = b^4a = ea = a.$$

   (b) **Prove that $ab^2 = b^2a$.**
$$ab^2 = (ab)b = (b^3a)b = b^3(ab) = b^3(b^3a) = b^4(b^2a) = b^2a.$$

   (c) **Find $|ba|$.**
   We can argue that $|ba| \neq 1$ because that would mean that $ba = e$ which can't be because b and a have different orders. So let's start with 2.
$$(ba)^2 = baba = b(b^3a)a = b^4a^2 = ee = e.$$

3. **Prove that for any group elements a and b, $|bab^{-1}| = |a|$.**

   Let $|a| = k$ and $|bab^{-1}| = n$. If $k = 1$, then $a = e$ which makes $|bab^{-1}|$ equal 1 and we're done. For other values of k,
$$(bab^{-1})^k = (bab^{-1})(bab^{-1})...(bab^{-1}) = ba^kb^{-1} = e.$$
   This means $|bab^{-1}| = n \leq k$. Now, multiplying bab^{-1} n-times gives us
$$(bab^{-1})^n = (bab^{-1})(bab^{-1})...(bab^{-1}) = ba^nb^{-1} = e.$$
   Using the last two terms, $a^n = e$, we have that $k \leq n$. It follows that $k = n$.