MAT 4170: Modern Algebra I **Name:** **Panya Sukphranee**

Skill Check 5

Due: Tuesday, 11/10/20

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1. $\alpha = (1327)(4568)$, $\beta = (157)$.

   a) $\alpha\beta = (16845)(273)$.

   b) $|\alpha| = 4$, $|\beta| = 3$, and $|\alpha\beta| = 15$.

2. a) U(5) is isomorphic to U(10).

   b) $\mathbb{Q}$ is NOT isomorphic to $\mathbb{Z}$. $\mathbb{Q}$ is countable but the operations aren't preserved.

   c) S_3 is isomorphic to D_3.

3. Show that $\phi : \mathbb{C}^* \to \mathbb{C}^*$ defined by $\phi(a + bi) = a - bi$ is an automorphism of C^*.

   1. For every complex number in $g \in \mathbb{C}^*$, we can write g as $g = a + bi$ for some $a, b \in \mathbb{R}$. By the definition of the mapping, ϕ, we can see that the element $h = a - bi$ to g. Therefore ϕ is onto.
   2. Let $g = a + bi$ and $h = c + di$ be elements in $\mathbb{C}^*$ and suppose that $\phi(g) = \phi(h)$, i.e.

      $$\phi(a + bi) = \phi(c + di).$$

      Then $a - bi = c - di$. It follows that $a = c$ and $b = d$ and so $g = h$. Therefore ϕ is injective.
   3. $\phi((a + bi)(c + di)) = \phi((ac - bd) + i(ad + bc)) = (ac - bd) - i(ad + bc)$. On the other hand,

      $$\phi(a + bi)\phi(c + di) = (a - bi)(c - di) = (ac - bd) - i(ad + bc).$$

      Thus, ϕ preserves multiplication. Since ϕ is an isomporphism from $\mathbb{C}^*$ onto itself, ϕ is an automorphism.