MAT 4170: Modern Algebra I **Name:** **Panya Sukphranee**

Skill Check 6

Due: Tuesday, 11/16/20

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1. Prove the following are homomorphisms, find its image and kernel.

   a) $\phi : \mathbb{Z} \to \mathbb{R}$, $\phi(n) = n$.
   Note that both operations are standard addition. Let $a, b \in Z$.

   $$\phi(a + b) = a + b = \phi(a) + \phi(b).$$

   We can see from the definition of ϕ that $0 \in ker\phi$. If any number $a \in ker\phi$, then

   $$a = \phi(a) = 0,$$

   implies that 0 is the only element in there. Thus,

   $$ker\phi = \{0\}.$$

   The image of ϕ is itself as a subset of $\mathbb{R}$. $Im(\phi) = \mathbb{Z} \subset \mathbb{R}$.

   b) $\phi : \mathbb{Z}_6 \to \mathbb{Z}_2$, $\phi(x) = x\ (mod 2)$.
   The mapping is onto,

   $$Im(\phi) = \{0, 1\} = \mathbb{Z}_2.$$

   Let $a, b \in \mathbb{Z}_6$.

   $$\phi(a + b) = a + b\ (mod 2).$$

   The sum on the right is understood to be $a + b$ then $mod 2$. It can be shown with the Division Algorithm that $a + b\ (mod 2)$ is equal to $a\ (mod 2) + b\ (mod 2)$. Thus,

   $$\phi(a + b) = a\ (mod 2) + b\ (mod 2) = \phi(a) + \phi(b).$$

   The kernel is the set of even numbers in $\mathbb{Z}_6$.

   $$ker\phi = \{0, 2, 4\}.$$

   c) $\phi : \mathbb{R}^* \to GL_2(\mathbb{R})$ defined by

   $$\phi(x) = \begin{bmatrix} 1 & 0 \\ 0 & x \end{bmatrix}.$$

   Let $a, b \in \mathbb{R}^*$.

   $$\phi(ab) = \begin{bmatrix} 1 & 0 \\ 0 & ab \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & b \end{bmatrix} = \phi(a)\phi(b).$$

   From the definition of ϕ,

   $$ker\phi = \{1\}.$$

   I'm not sure how to describe the image. It's the set of 2×2 diagonal matrices with $\mathbb{R}$ non-zero determinants.

2. a) Let $H_2 = \{A, C\}$. This subgroup cant be used to construct a quotient group. The only other subset that this constructs is

$$\overline{H_2} = \{B, D, E, F\}.$$

This subset is not it's own inverse and therefore has no inverse since there's only two subsets.

b) Let $H_1 = \{A, B, D\}$ and $H_2 = \{C, E, F\}$. The group table for the quotient group is

	H_1	H_2
H_1	H_1	H_2
H_2	H_2	H_1