$\boxed{\text{Prob. 15}}$ Consider the irreducible representations of the group $SO(3)$ in $T^2(\mathbb{V})$ with $\dim \mathbb{V} = 3$ as given by

$$⑨ = ⑤ \oplus ① \oplus ③$$

Write basis vectors for the ⑤, ① and ③ irreducible spaces in terms of a basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ of $\mathbb{V}$.

$\boxed{\text{Prob. 16}}$ Give a basis for $\Lambda^3(\mathbb{V})$, $\dim \mathbb{V} = 5$, in terms of a basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4, \vec{e}_5\}$ for $\mathbb{V}$. What is the dimension of $\Lambda^3(\mathbb{V})$?

$\boxed{\text{Prob. 17}}$ Construct the subspaces $P^3(\mathbb{V})$ and $\Lambda^3(\mathbb{V})$ of $T^3(\mathbb{V})$ when $\dim \mathbb{V} = 3$. What are the dimension of $P^3(\mathbb{V})$ and $\Lambda^3(\mathbb{V})$? Give explicit basis vectors for these two subspaces in terms of a basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ of $\mathbb{V}$. Convince yourself that $P^3(\mathbb{V}) \oplus \Lambda^3(\mathbb{V})$ is a proper subspace of $T^3(\mathbb{V})$.

$\boxed{\text{Prob. 18}}$ For a linear transformation $A : \mathbb{V} \to \mathbb{W}$ between the vector spaces $\mathbb{V}$ and $\mathbb{W}$, the pullback map $A^* : \Lambda^r(\mathbb{W}^*) \to \Lambda^r(\mathbb{V}^*)$ is defined by

$$(A^*\varphi)(\vec{v}_1, \ldots, \vec{v}_r) = \varphi(A\vec{v}_1, \ldots, A\vec{v}_r)$$

for an element $\varphi \in \Lambda^r(\mathbb{W}^*)$, and for any $\vec{v}_1, \ldots, \vec{v}_r \in \mathbb{V}$. Show that A^* is a linear map.

$\boxed{\text{Prob. 19}}$ Consider three arbitrary vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \mathbb{R}^3$ and an oriented frame $\vec{e}_1 \wedge \vec{e}_2 \wedge \vec{e}_3$ in the same space formed by an orthonormal basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$. Show that the volume of the parallelopiped formed by the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ is given by

$$\langle \vec{v}_1 \wedge \vec{v}_2 \wedge \vec{v}_3 , \ \vec{e}^{*1} \wedge \vec{e}^{*2} \wedge \vec{e}^{*3} \rangle ,$$

where $\{\vec{e}^{*1}, \vec{e}^{*2}, \vec{e}^{*3}\}$ is the dual basis to $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$.

$\boxed{\text{Prob 20}}$ From the result

$$\det A = \sum_{\sigma \in \Delta(n)} (\mathrm{sgn}\, \sigma)\, a_1^{\sigma(1)} \cdots\cdots a_n^{\sigma(n)}$$

derive

$$\det A = \sum_{\sigma \in \Delta(n)} (\mathrm{sgn}\, \sigma)\, a^1_{\sigma(1)} \cdots\cdots a^n_{\sigma(n)} .$$