Viscosity of Mineral Oil

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I. INTRODUCTION

The coefficient of viscosity of a fluid,

$$\eta = \frac{\frac{F_s}{A}}{\frac{\frac{dx}{dt}}{L}}$$

is derived from the concept of the Shear Modulus,

$$S = \frac{\frac{F_s}{A}}{\frac{\Delta x}{L}}$$

for solid materials. For the definition of shear modulus, the numerator and denominator are called the shearing stress and shearing strain, respectively. A fluid has a shear modulus of 0 in equilibrium state. So the analogous concept of viscosity is defined in terms of the fluids resistance to motion due to internal forces such as friction. We probed these properties of a fluid using a cylindrical apparatus, figure 2 pg 68 in Lab Manual. This allows us to refine the equation to

$$\eta = \frac{\frac{F_s}{A}}{\frac{dv}{dR}}$$

using the symmetry of the cylinder. Now,

$$v = \omega R$$
$$\Rightarrow \frac{dv}{dR} = \omega \frac{dR}{dR} + R\frac{d\omega}{dR}$$

gives us

$$\eta = \frac{dR}{R\, d\omega}\frac{F}{A}$$

Writing in terms of torque $|\tau| = F \cdot R$ and $A = 2\pi RL$,

$$\eta\, d\omega = \frac{\tau dR}{2\pi L R^3}$$

II. MATERIALS AND METHODS

Our experimenting apparatus consisted of a metal cylinder housing with inner radius, b. Inside this housing is another cylinder of radius a < b and length L_0 rotating about its center axis, concentric to the housing. The inner cylinder was rotated via a shaft connected to a pulley of radius k. A string with an attached mass was wound about the pulley.

Oil was filled in the gap between the cylinders until about 1.22cm of the inner drum was submerged. We let a 10g weight drop and marked when a terminal velocity was reached. This mark was 1.08m above the floor. Since we were dealing with rotating motions, it makes sense for the above equation to be written in terms of torque. After substitution and integrating, our equation for the coefficient of viscosity becomes,

$$\eta = \frac{b^2 - a^2}{4\pi a^2 b^2 L} \cdot \frac{\tau}{\omega_0},$$

where L is submerged depth of the rotating cylinder.

We recorded the times for the 10g weight to hit the floor from the terminal velocity point of 1.08m.

We then recorded falling times for various fill levels, times for maximum fill level for various weights, and various temperatures with various weights at maximum fill level. More detail will be provided in the Data section.

The falling weight rotates the pulley of radius k, therefore exerting a torque of

$$\tau = mgk$$

on the inner cylinder. The constant angular velocity is related to the falling distance s = 1.08 by:

$$\omega_0 = \frac{s}{kt}$$

Because the cross-section of the inner cylinder was in contact with the oil, we add a factor called ϵ to the length, so L → L + ϵ. Now,

$$\eta = \frac{(b^2 - a^2)k^2 g}{4\pi a^2 b^2 s(L + \epsilon)} \cdot mt$$

III. DATA

We measured the dimensions of our apparatus:

$$k = 18.05mm$$
$$b = 30.05mm$$
$$a = 25.00mm$$
$$L_0 = 7.59cm$$

The fraction part of η can be compactly written as some constant c. Then,

$$t = \frac{1}{mc}\eta$$

A. Finding η and ϵ by plotting t vs $\frac{1}{m}$

1. The ϵ factor

For this part of our experiment, we filled up oil between the cylinder gap until about 1.22cm of the inner cylinder was submerged. We dropped a total weight of 10g and recorded the time it took to hit the floor after passing our s=1.08m mark. We repeated this twice, totaling three data points. We then repeated this process after submerging an additional (roughly) 1 cm of cylinder in oil. Below is the data:

L [m]	time 1	time 2	time 3	avg time
0.0122	2.63	2.88	2.75	2.75
0.0210	3.87	3.75	3.91	3.84
0.0335	5.79	5.87	5.85	5.84
0.0410	6.97	6.87	6.75	6.86
0.0500	8.19	8.16	8.06	8.14
0.0600	9.34	9.44	9.37	9.38
0.0700	10.28	10.28	10.41	10.32
0.0759	11.75	11.71	11.93	11.80

We used the above data and manipulate the following equation

$$\eta = \frac{(b^2 - a^2)k^2 g}{4\pi a^2 b^2 s(L+\epsilon)} \cdot mt$$

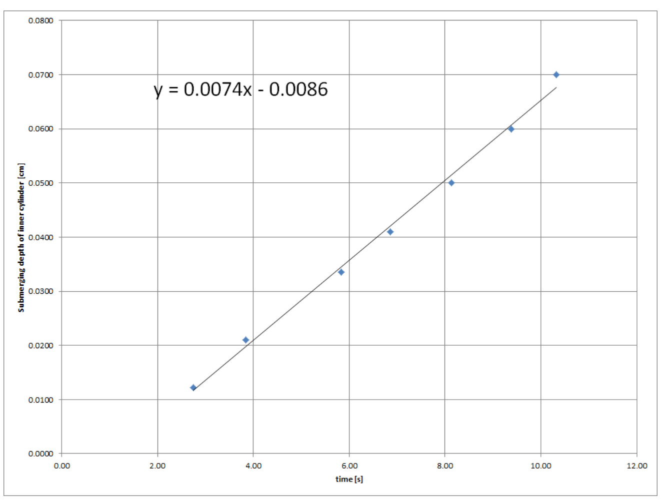
to get us

$$\begin{aligned}(L+\epsilon)\eta &= \frac{(b^2 - a^2)k^2 g}{4\pi a^2 b^2 s} \cdot mt \\ &\approx .116mt \\ L &= \frac{.116mt}{\eta} - \epsilon\end{aligned}$$

Which is a linear equation. Further, using the 10g (.01kg) weight this becomes

$$L \approx \frac{.00116}{\eta} \cdot t - \epsilon$$

(L is in [m], t is in [s]econds).



The graph of figure 1 tells us that

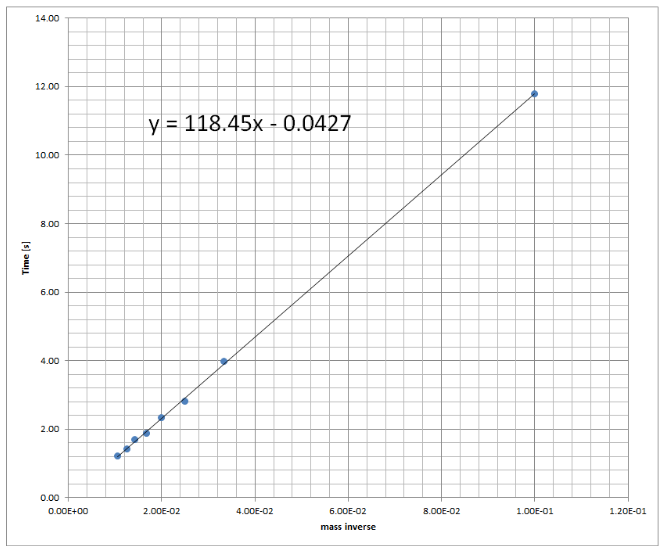
$$\begin{aligned}\epsilon &= .0086m \\ &= .86cm\end{aligned}$$

2. η coefficient

This part of the experiment was done with oil filled up to the upper part of the inner cylinder. No more than a thin layer of oil was to cover the top cross section of the inner cylinder. We started with 10g and went up to 95g, taking three time recordings for each weight. Below is our data:

mass [g]	1/mass	time 1 [s]	time 2 [s]	time 3 [s]	avg time [s]
10.00	1.00E-01	11.75	11.71	11.93	11.80
30.00	3.33E-02	4.00	3.97	4.00	3.99
50.00	2.00E-02	2.25	2.41	2.35	2.34
70.00	1.43E-02	1.72	1.68	1.69	1.70
40.00	2.50E-02	2.82	2.84	2.81	2.82
60.00	1.67E-02	1.84	1.91	1.91	1.89
80.00	1.25E-02	1.41	1.43	1.47	1.44
95.00	1.05E-02	1.22	1.22	1.19	1.21

This graph (figure 2) is a plot of $\frac{1}{m}$ vs the average time.



The slope is 118.45, therefore

$$\begin{aligned}t &= \frac{1}{mc}\eta \\ \Rightarrow \frac{\eta}{c} &= 118.45 \\ \eta &= 118.45 \cdot \frac{(b^2 - a^2)k^2 g}{4\pi a^2 b^2 s(L_0+\epsilon)} \\ &= 118.45 \cdot \frac{.116}{L_0 + \epsilon} \\ &= 118.45 \cdot \frac{.116}{.0759 + .0086} \\ &= 162.61\ Pa \cdot s\end{aligned}$$

B. Temperature Dependence of η

We will use the relationship given in (13)

$$\eta = \eta_\infty \cdot e^{\frac{\Delta E}{kT}}$$

Taking the natural log of both sides gives us a linear equation

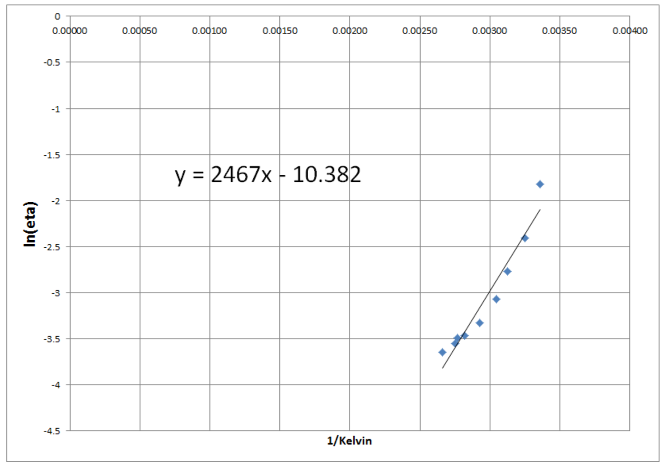
$$\begin{aligned} ln(\eta) &= \frac{\Delta E}{kT} + ln(\eta_\infty) \\ &= \frac{\Delta E}{k} \cdot \frac{1}{T} + ln(\eta_\infty) \end{aligned}$$

We conducted this experiment by dropping a 10g weight the same way as in the previous sections but with varying temperatures. We calculate η for each temperature by plugging in the average time measurements using the following equation from section A1:

$$\begin{aligned} L &= \frac{.00116}{\eta} \cdot t - \epsilon \\ \Rightarrow \eta &= \frac{.00116}{L + \epsilon} \cdot t \\ \eta &= \frac{.00116}{.0845} \cdot t \\ &= .0137t \end{aligned}$$

Here is our data table:

Temp [K]	$\frac{1}{temp}$	time	η	$ln(\eta)$
297.85000	0.00336	11.79667	0.16161	-1.82254244
307.55000	0.00325	6.56000	0.08987	-2.409368843
320.05000	0.00312	4.59000	0.06288	-2.766479422
328.05000	0.00305	3.38000	0.04631	-3.072483737
341.45000	0.00293	2.63000	0.03603	-3.3233756
361.45000	0.00277	2.22000	0.03041	-3.49285225
354.35000	0.00282	2.29000	0.03137	-3.461807629
363.35000	0.00275	2.10000	0.02877	-3.548422101
375.95000	0.00266	1.90000	0.02603	-3.64850556



Therefore,

$$\begin{aligned} \eta_\infty &= 3.05 \cdot 10^{-5}\ Pa \cdot s \\ \Delta E &= 3.40 \cdot 10^{-20} \text{ some units} \end{aligned}$$