Transmission Lines

Panya Sukphranee
Cal Poly Pomona

(Dated: October 19, 2016)

I. INTRODUCTION

I would like to start off with the derivation of Maxwell's Equations because this was part of what I went back and reviewed.

A. Maxwell's Equation

Before Maxwell, the laws governing electrodynamics were

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Using the mathematical fact that the divergence of the curl of a vector field = 0,

$$\nabla \cdot (\nabla \times \vec{B}) = \nabla \cdot (\mu_0 \vec{J})$$
$$\Rightarrow \vec{0} = \nabla \cdot \vec{J}$$

That is to say that the divergence of $\vec{J}$ is *identically* 0. Which is not true for changing currents. Another example where Ampere's Law short is the capacitor paradox (refer to Electrodynamics 4th ed., Griffiths). Maxwell used theoretical arguments to add to and make more general Ampere's original law:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$\vec{J}_d \equiv \epsilon_0 \frac{\partial \vec{E}}{\partial t}.$$

Maxwell's Equation are now complete:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

1. *A closer look at current density and charge density*

We want to break down the current density and charge density down further. We know that electric polarization $\vec{P}$ causes a bound charge density:

$$\rho_b = -\nabla \cdot \vec{P}$$

The current density as a result of this is:

$$\vec{J}_p = \frac{\partial \vec{P}}{\partial t}$$

In addition, magnetization produces a bound current density:

$$\vec{J}_b = \nabla \times \vec{M}$$

Our detailed description of charge density and current density are:

$$\begin{aligned} \vec{J} &= \vec{J}_f + \vec{J}_b + \vec{J}_p \\ &= \vec{J}_f + (\nabla \times \vec{M}) + \frac{\partial \vec{P}}{\partial t} \\ \rho &= \rho_f + \rho_b \\ &= \rho_f - \nabla \cdot \vec{P} \end{aligned}$$

With the definitions,

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$
$$\vec{H} \equiv \frac{1}{\mu_0} \vec{B} - \vec{M}$$

Maxwell's Equations in terms of free charges are:

$$\nabla \cdot \vec{D} = \rho_f$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

B. Maxwell's Equation on Our Wave Guide

Besides the Reflex Klyston, our microwave generator, there are no other sources of current or charges. So the above Maxwell's equations for our experiment becomes:

$$\nabla \cdot \vec{D} = 0$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Applying the curl to the third equation and doing some mathematics leads to what are called d'Alamberts Equations:

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\delta^2}{\delta t^2} \vec{E}$$
$$\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\delta^2}{\delta t^2} \vec{H}$$

For solutions of these equations, it can be shown (refer to Griffiths pg 428) that the z-component of $\vec{E}$ and the z-component of $\vec{H}$ *both* cannot be 0. In Griffith's text, he uses $\vec{B}$. In the case of no free charges, $\vec{B}$ and $\vec{H}$ are off by a constant. We are concerned with the case where $E_z = 0$, called TE mode. We assume that the wave guide is a perfect conductor so the following boundary equations hold:

$$\vec{B}^{\perp} = 0$$
$$\vec{E}^{||} = 0$$

The complete equations for the behavior of the TE mode are:

$$E_x = \frac{\omega \mu_0}{k^2} k_y H_1 cos(k_x x) sin(k_y y) sin(\omega t - \beta z)$$
$$E_y = -\frac{\omega \mu_0}{k^2} k_x H_1 sin(k_x x) cos(k_y y) sin(\omega t - \beta z)$$
$$E_z = 0$$
$$H_x = -\frac{\beta k_x}{k^2} H_1 sin(k_x x) cos(k_y y) sin(\omega t - \beta z)$$
$$H_y = -\frac{\beta k_y}{k^2} H_1 cos(k_x x) sin(k_y y) sin(\omega t - \beta z)$$
$$H_z = H_1 cos(k_x x) cos(k_y y) sin(\omega t - \beta z)$$

Where,

$$\omega = 2\pi\nu$$
$$\beta = \frac{2\pi}{\lambda_g}$$
$$k_x = \frac{m\pi}{a}$$
$$k_y = \frac{n\pi}{b}$$
$$\beta^2 = \omega^2 \mu_0 \epsilon_0 - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2.$$

Further arguments and mathematics gives us equations 47-52 in the lab manual. I won't include them here but want to state that the significance of these equation is that the electric field components have the same dependence on β and z as the magnetic field components; in particular H_z. As a result, the potential we measure have the same dependence.

1. *A discussion on propogating potentials*

I checked out the Microwave Principles book to supplement the lab manual. Equation 1-14 of Microwave Principles gives us a general mathematical equation for a propogating potential of a transmission line with a load.

$$v(z,t) = \frac{V_r}{2}\left[\left(1 + \frac{Z_0}{Z_r}\right) sin(\omega t + \beta z) + \left(1 - \frac{Z_0}{Z_r}\right) sin(\omega t - \beta z)\right]$$

Where Z_0, Z_r are the characteristic impedance (of the wave guide) and impedance of the load, respectively. Notice that the potential of a transmission line is represented by the superposition of traveling waves. One of which is the reflected wave. From this we can see that the maximum and minimum possible values for the amplitude of the standing waves are:

$$V_{max} = \frac{V_r}{2}\left[\left|1 + \frac{Z_0}{Z_r}\right| + \left|1 - \frac{Z_0}{Z_r}\right|\right]$$
$$V_{min} = \frac{V_r}{2}\left[\left|1 + \frac{Z_0}{Z_r}\right| - \left|1 - \frac{Z_0}{Z_r}\right|\right]$$

Now,

$$VSWR \equiv S = \frac{V_{max}}{V_{min}} = \frac{1 + |\frac{Z_r - Z_0}{Z_r + Z_0}|}{1 - |\frac{Z_r - Z_0}{Z_r + Z_0}|}$$

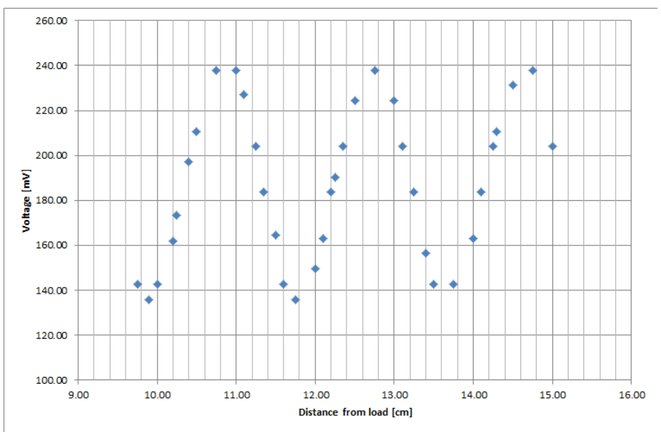
II. METHODS, MATERIALS, AND PROCEDURES

We assembled the the klystron, Faraday rotator, attenuator, frequency meter, and slotted waveguide in this order. We turned on the klystron and slid the detector along its guide and saw that the oscilloscope reading fluctuating. We chose a position where the fluctuations peaked. The klystron's reflector voltage was adjusted to get us the highest mode reading of $150.0mV$. We adjusted the frequency meter to see that the highest klystron mode corresponded to $11.1125GHz$. In free space the wavelength of this mode is:

$$\begin{aligned}\lambda &= \frac{3 \cdot 10^8}{11.1125 \cdot 10^9} \\ &= 2.70 \cdot cm\end{aligned}$$

A. Open End Load

We took 35 data points for the open ended wave guide. The lowest and highest voltage readings were 136mV and 238mV, respectively. Refer to page the data table on the last page. Below is the plot of our data. Since there are no nodes, the open end configuration is a resistive load. The reflecting wave amplitude is less than the incident wave so the resulting standing wave has zero nodes.



The VSWR value is 1.75. We were able to record over 2 periods giving us the following values for $\frac{\lambda_g}{2}$ by taking differences of the highest and lowest plot values:

$$\begin{aligned}\frac{\lambda_g}{2} &= 1.85, 2, 1.75, 2, 2, 1.75 \\ avg(\lambda_g) &= 3.78cm \\ \frac{\lambda_g}{2} &= 1.89cm\end{aligned}$$

Thus

$$\begin{aligned}\beta &= \frac{2\pi}{\lambda_g} \\ &= \frac{2\pi}{3.78} \\ &= 3.32\frac{Rad}{cm}\end{aligned}$$

$$\begin{aligned}Z_0 &= \frac{3.78}{2.70}377\Omega \\ &= 527.8\Omega\end{aligned}$$

I chose the nearest anti-node close to 13cm to be 12.75cm. $\frac{12.74}{3.78} = 3.37$ wavelengths away, equivalently .37 wavelengths, from the load.

B. Short Circuit

We attached a metal plate to end of the waveguide and recorded data (on table, last page). The lowest voltage we recorded were 12 mV and 6 mV, averaging 9 mV. The highest potential was above 300mV. So we can consider the low voltage locations as nodes.
Recall the equation:

$$v(z,t) = \frac{V_r}{2}\left[(1 + \frac{Z_0}{Z_r})sin(\omega t + \beta z) + (1 - \frac{Z_0}{Z_r})sin(\omega t - \beta z)\right]$$

With the relationship $Z_r = \frac{V_r}{I_r}$ and $V_r = 0$ for a short circuit, the above becomes (Eqn 1-18, Microwave Principles:

$$v(z,t) = \frac{I_r Z_0}{2}\left[sin(\omega t + \beta z) - sin(\omega t - \beta z)\right]$$

Observing the terms inside the brackets, we can see that the result is another wave. Taking the absolute value of which gives us a plot of "hills" with nodes equal to 0. The distance between successive nodes (or anti-nodes) are half-wavelengths.

The distance between the successive nodes were found to be 1.8cm apart, $\frac{\lambda_g}{2} = 1.8cm$. Therefore, $\lambda_g = 3.6$cm. Using

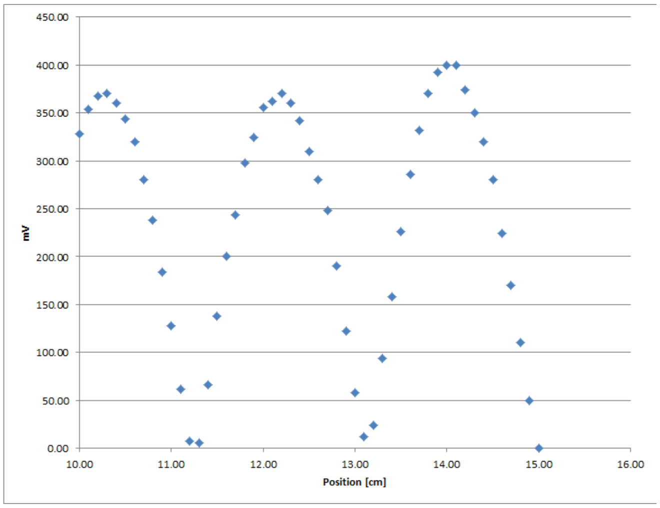
$$\begin{aligned}Z_0 &= \frac{3.6}{2.7}\,377\,\Omega \\ &= 502.67\,\Omega \\ \beta &= 1.75\frac{Rad}{cm}\end{aligned}$$

For a short circuit, VSWR $\rightarrow \infty$.

C. X190A Termination

This load is described as a match load. For this load, we found the lowest and highest voltage reading were 213.6mV and 216.8 mV, respectively. This gives us $VSWR = 1.01$. Recalling:

$$\begin{aligned}VSWR &= \frac{V_{max}}{V_{min}} \\ &= \frac{1 + |\frac{Z_r - Z_0}{Z_r + Z_0}|}{1 - |\frac{Z_r - Z_0}{Z_r + Z_0}|},\end{aligned}$$



A matched load should have $Z_0 \approx Z_r$, therefore,

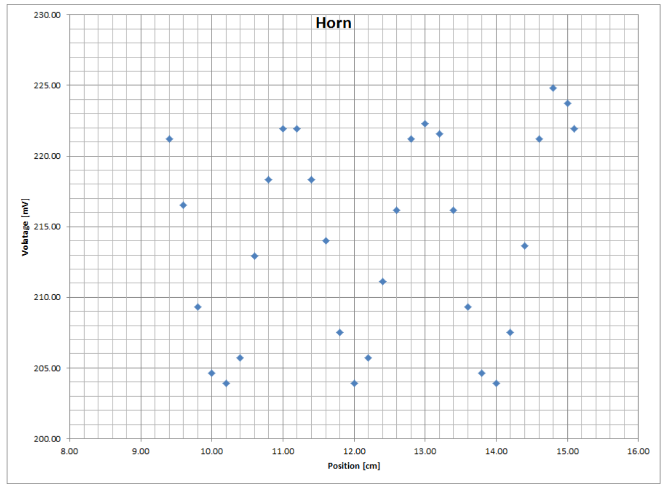
$$VSWR \approx 1$$

Just as measured.

D. Horn

We increased the gain to a 18.0mV scale. Half-wavelengths, $\frac{\lambda_g}{2}$ for this configuration was 1.9cm.

VSWR $= \frac{224.8}{203.9} = 1.10$.



At 13.0cm, we have an extrema thats effectively .44 wavelengths from the load.