5/8/17 M444

Thm A^{-1} exist iff $\det A \neq 0$

($\Rightarrow$) A^{-1} exists $\to$ $AA^{-1} = \mathbb{I}$

$\Rightarrow$ $\det(AA^{-1}) = 1$

$\Rightarrow$ $\det A \cdot \det A^{-1} = 1$

$\Rightarrow$ $\det A \neq 0$

($\Leftarrow$) suppose ~~ff~~ $\det A \neq 0$,

choose basis $\{\vec{e}_1, \dots, \vec{e}_n\}$

$0 \neq \chi \in \Lambda^n(V^*)$ // on dimensional

$\chi(e_1, \dots, e_n) \neq 0$

$\det(A) \cdot \chi(e_1, \dots, e_n) \neq 0$

pullbac $\Rightarrow$ $(A^*\chi)(e_1, \dots, e_n) \neq 0$

$\chi(Ae_1, \dots, Ae_n) \neq 0$

Claim $Ae_1, \dots, Ae_n$ Lin. Indep.

Proof by contrapositive: Supp $v_i = Ae_i$ is Lin Dep.

WLOG suppose $v_1 = \alpha_2 v_2 + \dots + \alpha_n v_n$ not all $\alpha = 0$

Then $\chi(v_1, \dots, v_n) = \chi(\alpha_2 v_2 + \dots + \alpha_n v_n, v_2, \dots, v_n)$

$= \sum_{i=2}^{n} \alpha_i \chi(v_i, v_2, \dots, v_n) = 0$ $\blacksquare$

$\Rightarrow \{Ae_i\}$ Lin. Indep. set.

Thus $e_1 = \beta_1^j Ae_j$ $\qquad$ $e_1 = \beta_1^j a_j^k e_k$

$\vdots$ $\qquad \Rightarrow$

$e_n = \beta_n^j Ae_j$ $\qquad$ $e_n = \beta_n^j a_j^k e_k$

OTOH $e_i = \delta_i^k e_k$ $\Rightarrow$ $\delta_i^k = \beta_i^\ell a_\ell^k$ $\Rightarrow$ $\beta A = \mathbb{1}$ $\blacksquare$

set of all Tan planes R fibers — each tan plane is a fiber

Bundles — Union of all is fiber bundle

Points in tan space give direction of advancement

each tan space has dual space

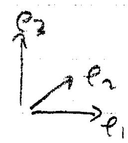
gen vect in tan space written $\vec{v}\ a^i \frac{\partial}{\partial x^i}$)

$df\left(a^i \frac{\partial}{\partial x^i}\right) = \text{number} \Rightarrow a^i\, df\left(\frac{\partial}{\partial x^i}\right)$, but $df = \frac{\partial f}{\partial x^i} dx^i$ from calc.

$dx^i\left(\frac{\partial}{\partial x^j}\right) = \delta^i_j$ $\Rightarrow$ gives rise to cotangent bundle

act on it to get volume

$dx^1 \wedge dx^2 \wedge dx^3$ is volume element



e_i not nec orth., but L.I.

$$dx'^i = \left(\frac{\partial x'^i}{\partial x^j}\right) dx^j$$

$v_1 \wedge \dots \wedge v_r$, $w_1 \wedge \dots \wedge w_r$ $\Rightarrow$ if $w_i = a_i^j v_j$, then $w_1 \wedge \dots \wedge w_r \triangleq (\det a)\, v_1 \wedge \dots \wedge v_r$

Recall $\underset{n \times n}{\det a} = \sum_{\sigma \in S_n} a_1^{\sigma(1)} \dots a_n^{\sigma(n)} \cdot \operatorname{sgn} \sigma$

$r \leq n$

$\Rightarrow w_1 \wedge \dots \wedge w_r = a_1^{\alpha_1} v_{\alpha_1} \wedge \dots \wedge a_n^{\alpha_r} v_{\alpha_r}$

$= a_1^{\alpha_1} \dots a_r^{\alpha_r}\, v_{\alpha_1} \wedge \dots \wedge v_{\alpha_r}$, $\exists$ $r!$ terms

$= \left(\sum_{\sigma \in S_r} (\operatorname{sgn} \sigma)\, a_1^{\sigma(1)} \dots a_r^{\sigma(r)}\right)(v_1 \wedge \dots \wedge v_r)$

$= (\det a)(v_1 \wedge \dots \wedge v_r)$