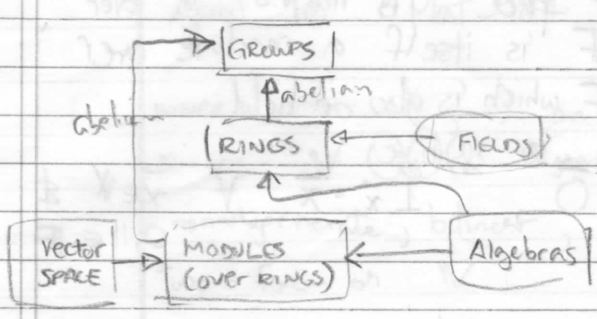
MAT444 4/5 WED.



A module M over ring R is an abelian group M under addition ($\exists\, 0 \in M : 0 + x = x + 0 = x, \ \forall x \in M$) together w/ external operation called scalar mult,

$R \times M \to M, \quad (\alpha, x) \mapsto \alpha x$ s.t.

following apply:

ⓐ $\alpha(x+y) = \alpha x + \alpha y$

ⓑ $(\alpha+\beta)x = \alpha x + \beta x$

ⓒ $(\alpha\beta)x = \alpha(\beta x) \quad \forall \alpha, \beta \in R, \ x, y \in M$

If R ring w/ident e (wrt mult), then $ex = x, \ \forall x \in M$

Some linear function on $\mathbb{V}$ $\Rightarrow$ $[G(\vec{v})](\vec{w}) = (\vec{v}, \vec{w})$

Def'd in this way, its linear & bij

clarification, linearity

$G(\alpha\vec{u} + \beta\vec{v}) = \alpha G(\vec{u}) + \beta G(\vec{v})$

$:\mathbb{V} \to \mathbb{W}$ linear, invertible

...ssociated w/ every vector ...n Hilbert space, $\exists$ canonical ...ect in dual space.

...r $\mathbb{C}$-V.S.

$(\psi, \phi) = (\phi, \psi)^*$

...ose $|\psi\rangle \in \mathcal{H}$, then corresponding $\langle\psi| \in \mathcal{H}^*$

...is Set of all square integrable function gives Hilbert space

...q. integ. : $\int_{-\infty}^{\infty} \phi^* \psi \, dx < \infty$

$\langle\phi|\psi\rangle$

...uar integrable $\int_{-\infty}^{\infty} |\psi|^2 dx < \infty$ "$\langle\psi|\psi\rangle$"

...near Map : $f: \mathbb{V} \to \mathbb{Z}$

...KNOW WHAT IT IS ALREADY

BILINEAR MAP:

$f: \mathbb{V} \times \mathbb{W} \to \mathbb{Z}$ // lin in both args

If $\mathbb{Z}$ is $\mathbb{F}$, the map is reffered to as functions.

☆ r-linear maps

$f: \mathbb{V}_1 \times \mathbb{V}_2 \times \ldots \times \mathbb{V}_r \to \mathbb{Z}$

$f(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_r)$

$\vec{v} = v^i \vec{e}_i = v'^i \vec{e}'_i$

$\vec{e}_i = a_i^j \vec{e}_j$	$\vec{e}'_i = (a^{-1})_i^j$

Lin operator //PHYS

Lin Trans //MATH

$A\vec{v} = \vec{w}$

$(w^1\ w^2\ w^3) = (v^1\ v^2\ v^3)\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$

suppose Frame $\begin{pmatrix} \vec{e}_1 \\ \vdots \\ \vec{e}_n \end{pmatrix}$

$\vec{v} = v^i \vec{e}_i$

$A(\vec{v}) = A(v^i \vec{e}_i)$

Linear Vector Space: A lin v.s. V over a field $\mathbb{F}$ is itself a module over a ring $\mathbb{F}$ which is also a field.

In addition to (a) (b) (c)

$0 \cdot \vec{x} = \vec{O}$, $1\vec{x} = \vec{x}$ $\forall$ $\vec{x} \in V$ & $0, 1 \in \mathbb{F}$

Algebra A is a module over RING R w/ identity:

1. A is a RING
2. scalar mult $(\alpha, x) \mapsto \alpha x$ satisfying
   $\alpha(xy) = (\alpha x)y = x(\alpha y)$
   $\forall$ $\alpha \in R$ $x, y \in A$