$\boxed{\text{P14.2}}$ Supplement

We have established that, supposing

$$N|\nu\rangle = \nu|\nu\rangle \quad , \; N = a^\dagger a \quad , \; \{a, a^\dagger\} = 1$$
$$\{a, a\} = \{a^\dagger, a^\dagger\} = 0$$

$\nu \geq 0$;

If $\nu \neq 0$, $1-\nu \geq 0$ or $\nu \leq 1$ and

$a|\nu\rangle$ is an eigenvector of N with eigenvalue $1-\nu$;

if $\nu \neq 1$, $a^\dagger|\nu\rangle$ is an eigenvector of N with eigenvalue $1-\nu$.

Also

$$a|0\rangle = 0 \quad , \; a^\dagger|1\rangle = 0$$

~~The only~~ So $0 \leq \nu \leq 1$

Suppose $\nu = \alpha$, $0 < \alpha < 1$

We have

$$a|\alpha\rangle = \sqrt{\alpha}\,|1-\alpha\rangle$$

so

$$a^2|\alpha\rangle = aa|\alpha\rangle = \sqrt{\alpha}\, a|1-\alpha\rangle = \sqrt{\alpha}\sqrt{1-\alpha}\,|\overset{1-(1-\alpha)}{\alpha}\rangle$$

this contradicts the fact that $\{a, a\} = 0$

if $|\alpha\rangle$ is a non-zero state.

$\therefore$ the eigenvalues $0 < \alpha < 1$ must be ruled out.

$\therefore$ the only eigenvalue of $N = a^\dagger a$ are 0 and 1.