NaI Detector Calibration - Revised

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1 Introduction

We calibrate a NaI detector that is used to detect photon emissions from radioactive samples. The detector interacts with photons via a NaI crystal. The NaI crystal is a doped semiconductor and therefore contains many available energy states. The many states allows it to interact with many wavelengths. Interaction comes in two forms: absorption and scattering. Each excites the electrons within the crystal and creates electron-hole pairs. When electrons fall into the hole, a photon is emitted. This photon is accelerated through a high potential and is mapped to some channel that's proportional to its energy. Because of variations in the circuitry, we need to calibrate the set-up by identifying channels to energies.

2 Calibration

We took the counts for $Cs^{137}, Co^{60}, Na^{22}$. The data is in the form of counts vs channel number. We uploaded the data file into a curve fitting program to get information for the photopeaks. The *Energy* column contains known values that we'll be comparing our predicted values with.

We did two (2) runs for the above samples [1]. Below are tables of our output.

Sample	**Peak Channel**	**Energy [keV]**	**Sigma**	**FWHM**	**Predicted Energy [keV]**	**Energy Difference**
Cs-137	330.6704	661.64	14.9710	0.0754	667.8490	6.2090
Co-60	557.5333	1173.24	18.1934	0.0543	1172.0745	1.1625
Co-60	628.8501	1332.50	19.7737	0.0524	1330.5832	1.9178
Na-22	257.8917	511.00	12.8830	0.0832	506.0911	4.9089
Na-22	604.3673	1274.50	19.3609	0.0533	1276.1678	1.6678
Bi-207	**286.3304**		**13.8429**	**0.0805**	**569.2989**	
	513.9488		**18.5177**	**0.0600**	**1075.2036**	

Table 1: Run 1

Full Width Half Maximum (FWHM) is the resolution of our photopeak, given by the following equation. Our σ is extracted from the photopeaks using the curve fitting program.

$$FWHM = 2\sqrt{ln(2)}\frac{\sigma}{C_0}$$

Our data is acceptable for $6\% \leq FWHM \leq 8\%$. We plot the Peak Channels vs Energy and use Excel's best fit line feature to get an equation for extrapolating the energy for Bi^{207}.

[1] Sample images from our curve fitting are attached to the report

Sample	Peak Channel	Energy [keV]	Sigma	FWHM	Predicted Energy [keV]	Energy Difference
Cs-137	328.4305	661.64	14.6209	0.0741	663.0000	1.3600
Co-60	560.5236	1173.24	17.9300	0.0533	1173.4656	0.2286
Co-60	630.7313	1332.50	19.9828	0.0528	1327.8804	4.6206
Na-22	258.6757	511.00	12.9384	0.0833	509.5813	1.4221
Na-22	608.4709	1274.50	20.2853	0.0555	1278.9209	4.4209
Bi-207	**286.3304**		**13.8429**	**0.0805**	**570.4051**	
Bi-207	**513.9488**		**18.5177**	**0.0600**	**1071.0290**	

Table 2: Run 2

We use Excel's *Best Line Fit* feature to get the following equations for Run 1 and Run 2, respectively.

$$E = 2.2226C - 67.099 \tag{1}$$

$$E = 2.1994C - 59.35 \tag{2}$$

Column **Predicted Energy** are the results from plugging in the Channel Numbers into the best fit line equations.

3 Error Analysis

We determine the uncertainty in our best fit line by first taking the difference between predicted and known energies, column **Energy Difference**. For simplicity, we take the average of the errors (ΔE) to be our uncertainty.
Run 1:

$$\Delta \bar{E}_1 = 3.1732 \tag{3}$$

Run 2:

$$\Delta \bar{E}_2 = 2.4104 \tag{4}$$

Bi^{207}	Run 1	Run 2
Peak 1	569.2989 ± 3.1732	570.4051 ± 2.4104
Peak 2	1075.2036 ± 3.1732	1071.0290 ± 2.4104

Table 3: Bi^{207}

We can see that the amplifier drift is not within our uncertainty.

3.1 Using Standard Deviation

We can also take the uncertainty to be the standard deviation of these values. Because we have a few values, the standard deviation is calculated using an unbiased estimator for the variance (σ^2) [2]

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}}$$

It gets us: $\Delta \bar{E}_1 = 2.2425$ and $\Delta \bar{E}_2 = 1.9854$

[2]STA 341 - Applied Statistics

4 Compton Edge

Compton edge and backscattering are marked in Fig 1 to be at channel 100 and 221, respectively. Using the following equation, we find the Compton Edge energy by using E_0 to be the energy of the photopeak.

$$
\begin{aligned}
K &= \frac{2E_0^2}{2E_0 + m_0c^2} \\
&= \frac{2 \times 662keV}{2\ 662keV + .511MeV} \\
&\approx 477keV
\end{aligned}
$$

Channel 100 corresponds to $\sim 160.59keV$. Then $160.59 + 477 \approx 637$ keV. Which is not far off from the photopeak energy.

4.1 Derivation

A photon with initial energy $E_{\gamma_i} = h\nu_i$ travelling in the $\hat{x}$ direction scatters of an electron at rest ($E_{e_i} = m_ec^2$). After the scatter, the photon has energy $E_{\gamma_f} = h\nu_f$ and is travelling at an angle θ relative to the original $\hat{x}$ direction. The electron has energy given by $E_{e_f} = \sqrt{p_e^2c^2 + m_e^2c^4}$ and has scattered at a different angle. The conservation of energy tells us

$$E_{\gamma_i} + E_{e_i} = E_{\gamma_f} + E_{e_f} \tag{5}$$

$$E_{\gamma_i} + m_ec^2 = \sqrt{p_e^2c^2 + m_e^2c^4} + E_{\gamma_f} \tag{6}$$

Rearranging and squaring both sides gives

$$(E_{\gamma_i} - E_{\gamma_f} + m_ec^2)^2 = p_e^2c^2 + m_e^2c^4.$$

We will use the conservation of momentum to write the $p_e^2c^2$ factor in terms of the photon energies. The conservation of momentum tells us

$$
\begin{aligned}
\vec{p_{\gamma_i}} &= \vec{p_{\gamma_f}} + \vec{p_{e_f}} \\
(\vec{p_{\gamma_i}} - \vec{p_{\gamma_f}})^2 &= \vec{p_{e_f}}^2 \\
p_{\gamma_i}^2 + p_{\gamma_f}^2 - 2p_{\gamma_i}p_{\gamma_f}\cos\theta &= p_{e_f}^2
\end{aligned}
$$

where θ is the angle between the initial photon direction $\hat{x}$ and the final photon direction. Multiplying the above equation by c^2 and using the relation E = pc, we can rewrite it as

$$E_{\gamma_i}^2 + E_{\gamma_f}^2 - 2E_{\gamma_i}E_{\gamma_f}\cos\theta = p_e^2c^2$$

Now, we can plug this into our final expression for the conservation of energy above and expand the squared brackets on the left side at the same time:

$$E_{\gamma_i}^2 + E_{\gamma_f}^2 + m_e^2c^4 + 2E_{\gamma_i}m_ec^2 - 2E_{\gamma_f}m_ec^2 - 2E_{\gamma_i}E_{\gamma_f} = E_{\gamma_i}^2 + E_{\gamma_f}^2 - 2E_{\gamma_i}E_{\gamma_f}\cos\theta + m_e^2c^4$$

Cancelling off similar terms on both sides gives us

$$E_{\gamma_i} m_e c^2 - E_{\gamma_f} m_e c^2 - E_{\gamma_i} E_{\gamma_f} = E_{\gamma_I} E_{\gamma_f} \cos\theta$$
$$E_{\gamma_f} = \frac{E_{\gamma_i}}{1 + \frac{E_{\gamma_i}}{m_e c^2}(1 - \cos\theta)}$$

$\theta = \pi$ and multiply the numerator and denominator by $m_e c^2$ gives

$$E_{\gamma_f} = \frac{m_e c^2 E_{\gamma_i}}{m_e c^2 + 2E_{\gamma_i}}$$

Somethings wrong with the above equation :(

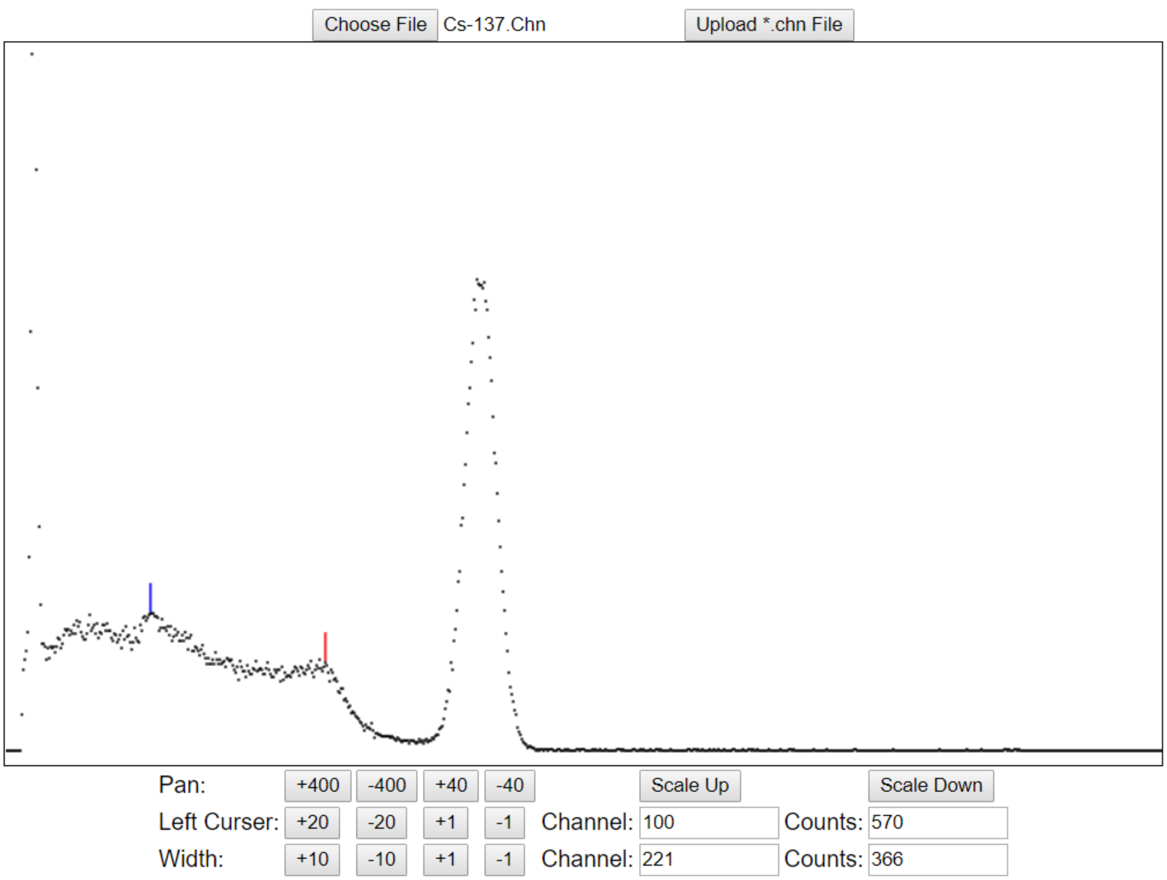

Figure 1: Compton Edge

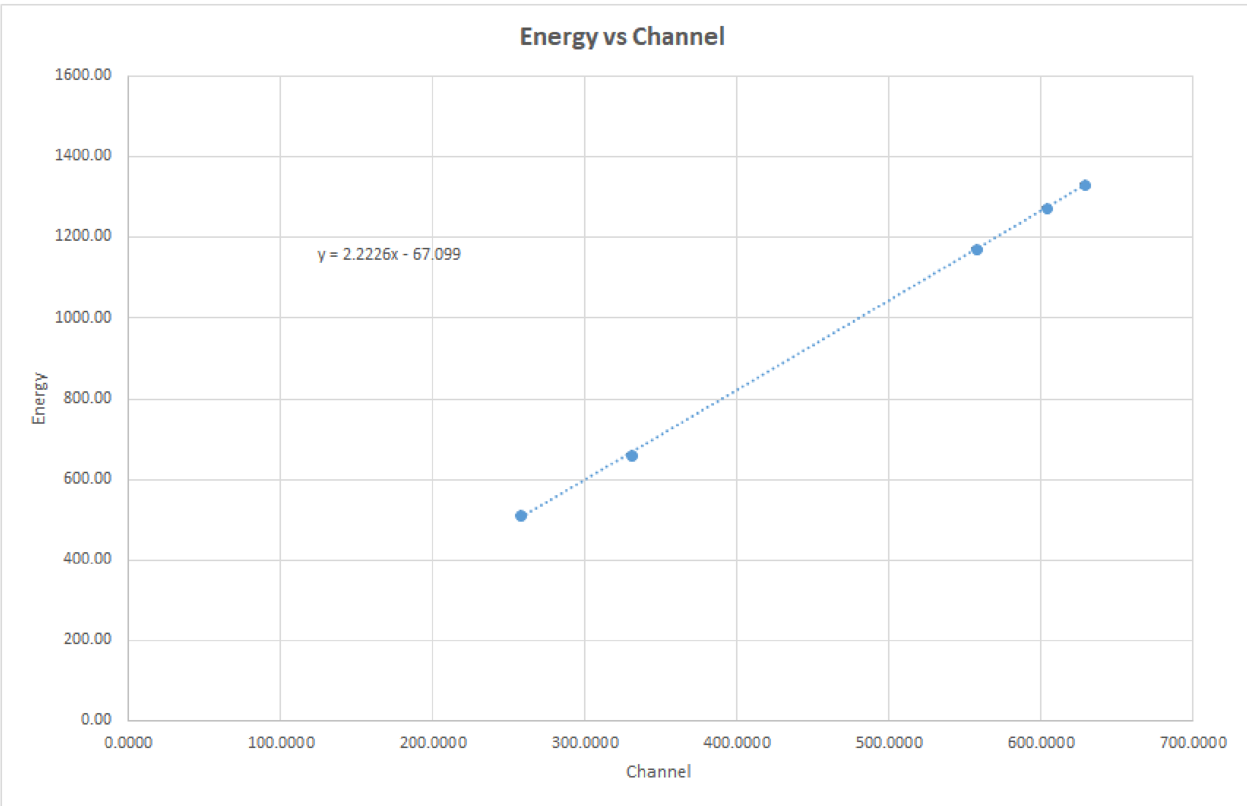

Figure 2: Run 1

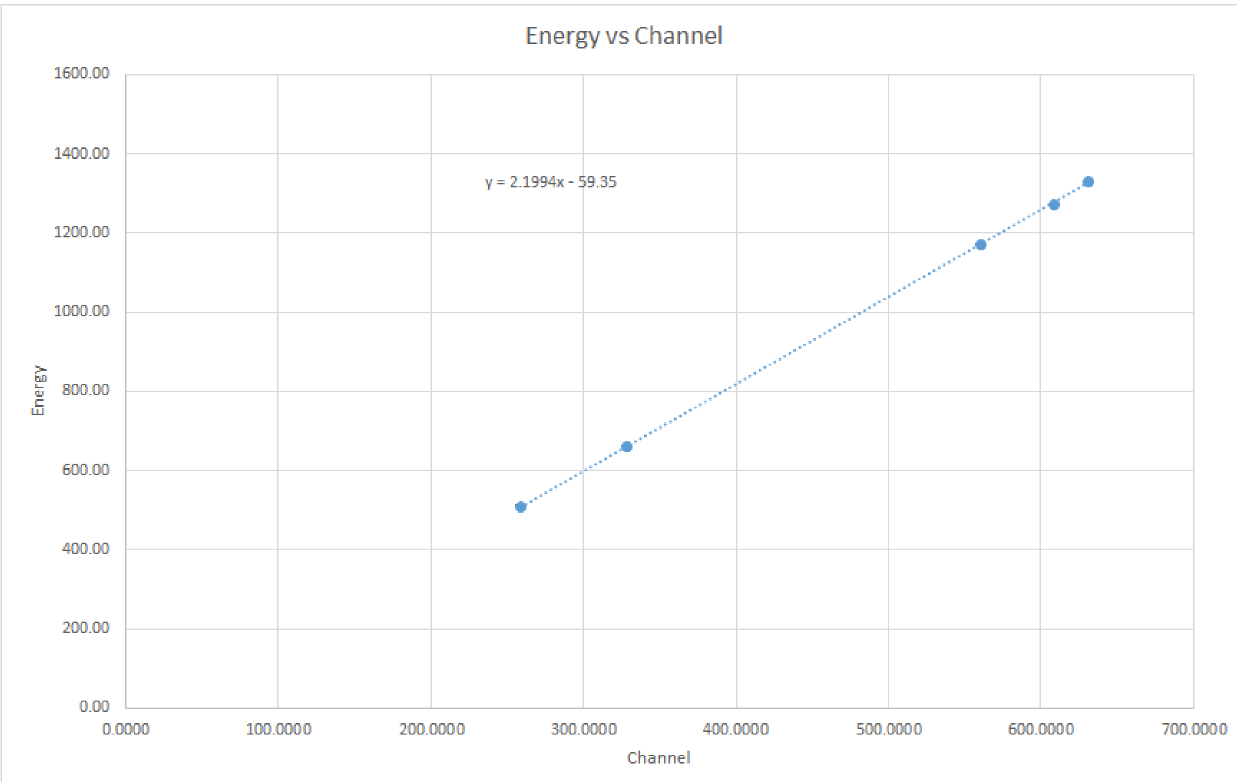

Figure 3: Run 2