Chapter 2 Problems

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A Mathematical Introduction to Robotic Manipulation

1) Let $a, b, c, \in \mathbb{R}^3$ with the standard basis $\{vece1, vece2, vece3\}$. Using Einstein summation notation,

(a)

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = (a^i \vec{e_i}) \cdot ({\epsilon^l}_{jk} b^j c^k) \vec{e_l} \tag{1}$$
$$= \delta_{il} a^i {\epsilon^l}_{jk} b^j c^k \tag{2}$$
$$= \sum_i {\epsilon^i}_{jk} a^i b^j c^k \tag{3}$$
$$= \epsilon_{ijk} a^i b^j c^k \tag{4}$$
$$= \epsilon_{kij} a^i b^j c^k \tag{5}$$
$$= (\vec{a} \times \vec{b})_k c^k \tag{6}$$
$$= (\vec{a} \times \vec{b}) \cdot \vec{c} \tag{7}$$

(b)

$$\vec{a} \times (\vec{b} \times \vec{c}) = {\epsilon^i}_{jk} a^j ({\epsilon^k}_{lm} b^l c^m) \vec{e_i} \tag{8}$$
$$= ({\epsilon^i}_{jk} {\epsilon^k}_{lm} a^j b^l c^m) \vec{e_i} \tag{9}$$
$$= \sum_i (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})(a^j b^l c^m) \vec{e_i} \tag{10}$$
$$= \sum_j (a^j b^i c^j) \vec{e_i} - (a^j b^j c^i) \vec{e_i} \tag{11}$$
$$= (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c} \tag{12}$$

2) Let $g, h \in SE(3)$, $\bar{g} = \begin{bmatrix} R_g & p_g \\ 0 & 1 \end{bmatrix}$ and $\bar{h} = \begin{bmatrix} R_h & p_h \\ 0 & 1 \end{bmatrix}$

Closure

$$\bar{g}\bar{h} = \begin{bmatrix} R_g & p_g \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_h & p_h \\ 0 & 1 \end{bmatrix} \tag{13}$$
$$= \begin{bmatrix} R_g R_h & R_g p_h + p_g \\ 0 & 1 \end{bmatrix} \tag{14}$$

Therefore, $gh = (\mathrm{R}_g R_h, R_g p_h + p_g) \in SE(3)$

Inverse Exists

Associativity