APRIL 24 MAT444

Talking about symm & antisymm tens.

Suppose $T \in T^r(V)$, V has basis $\{\vec{e}_1, \dots, \vec{e}_n\}$

$\Rightarrow$ basis for r-rank tens $\vec{e}_{i_1} \otimes \dots \otimes \vec{e}_{i_r}$

$\Rightarrow$ $T = T^{i_1 \dots i_r} \vec{e}_{i_1} \otimes \dots \otimes \vec{e}_{i_r}$ // dim $= n^r$

Thm: A tens $T \in T^r(V)$ is symm (antisymm) iff its components are sym (or antisymm) wrt all indices.

ORIG Def T symm if $\sigma(T) = T$, $\sigma \in S_r$

anti if $\sigma(T) = (\operatorname{sgn} \sigma) T$, $\sigma \in S_r$

ex) $T^2(V)$, $\dim V = 2$, $\{\vec{e}_1, \vec{e}_2\}$ basis of V

$T^{ij} \vec{e}_i \otimes \vec{e}_j$, 4 basis vects : $\vec{e}_1 \otimes \vec{e}_1$, $\vec{e}_2 \otimes \vec{e}_2$, $\vec{e}_1 \otimes \vec{e}_2$, $\vec{e}_2 \otimes \vec{e}_1$

can make symmetric bases:

$e_1 \otimes e_1$, $e_2 \otimes e_2$, $e_1 \otimes e_2 + e_2 \otimes e_1$

" " ANT-SYMM :

$e_1 \otimes e_2 - e_2 \otimes e_1$

ORIG 4-DIM space broken down into two subspaces.

HOW TO MAKE SYMM (ANT-SYMM) tens out of any tensor?

$\Rightarrow$ intro symm (anti-symm) op.

Symmetrizer $S_r(T) = \frac{1}{r!} \sum_{\sigma \in S_r} \sigma T$

Anti-symm $A_r(T) = \frac{1}{r!} \sum_{\sigma \in S_r} (\operatorname{sgn} \sigma) \sigma T$

$P^r(V) \subset T^r(V)$ s.s. of symm. tens

$\Lambda^r(V) \subset T^r(V)$ " " anti-symm " "

wave-func on bosons completely symm

ferm antysym

<u>Thm</u>: $S_r(T^r(V)) = P^r(V)$

$A_r(T^r(V)) = \Lambda^r(V)$

ex) for 2D case $\sigma T = T^{i_{\sigma(1)} i_{\sigma(2)}} e_{i_1} \otimes e_{i_2}$

$$S_2(T) = \frac{1}{2}\left(T^{i_1 i_2} e_1 \otimes e_2 + T^{i_2 i_1} e_1 \otimes e_2\right)$$

$$= \frac{1}{2}\left(T^{i_1 i_2} + T^{i_2 i_1}\right) e_1 \otimes e_2$$

$$\Rightarrow S = \begin{pmatrix} \overset{T^{11}\,=}{S^{11}} & S^{12} \\ S^{12} & \underset{=\,T^{22}}{S^{22}} \end{pmatrix}$$

$$A_2(T) = \underbrace{\frac{1}{2}\left(T^{i_1 i_2} - T^{i_2 i_1}\right)}_{A^{ij}} e_1 \otimes e_2$$

$$(\Rightarrow) A = \begin{pmatrix} 0 & A^{12} \\ -A^{12} & 0 \end{pmatrix}$$

$T^3(V)$, $\dim V = 3$

$T = T^{i_1 i_2 i_3}\, e_{i_1} \otimes e_{i_2} \otimes e_{i_3}$

~~$S_3(T) = \frac{1}{3!}(T^{123} + T^2$~~ $S_3(T) = \frac{1}{3!}\left(T^{ijk} + T^{jki} + T^{kij} + T^{ikj} + T^{kji} + T^{jik}\right) e_i \otimes e_j \otimes e_k$

$A_3(T) = \frac{1}{3!}\left(T^{ijk} + T^{jki} + T^{kij} - T^{ikj} - T^{kji} - T^{jik}\right) e_i \otimes e_j \otimes e_k$

any two same indices $\rightarrow 0$

In QM, total angular mmt j = half or whole ints

$j = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$

Dim $= 2j+1 = 2, 3, 4, 5, \ldots$