4/3 MAT 444 MONDAY

Lin Op $A: \mathbb{V} \to \mathbb{W}$ (n, m)

$\{e_1, \dots, e_n\}$, $\{d_1, \dots d_m\}$

$\vec{v} = v^i e_i \in \mathbb{V}$, $A(\vec{v}) = v^i A(\vec{e}_i)$

$A(\vec{e}_i) = a_i^j \vec{d}_j$ (n equations), (a_i^j) n×m matrix

$1 \leq i \leq n$, $1 \leq j \leq m$

$A: \mathbb{V} \to \mathbb{V}$, (a_i^j) n×n

$\{\vec{e}_i\} \to \{\vec{e}_i'\}$

$\vec{e}_i = s_i^j \vec{e}_j'$, $\cancel{(s^{-1})}$ $\vec{e}_i' = (s^{-1})_i^j \vec{e}_j$

$A(\vec{e}_i') = (a')_i^j \vec{e}_j'$

$A(\vec{e}_i) = a_i^j e_j$

$$A(\vec{e}_i') = A((s^{-1})_i^l \vec{e}_l) = (s^{-1})_i^l A(\vec{e}_l) = (s^{-1})_i^l a_l^k \vec{e}_k = (s^{-1})_i^l a_l^k s_k^j \vec{e}_j'$$

$$(a')_i^j \vec{e}_j' = [(s^{-1})_i^l a_l^k s_k^j] \vec{e}_j'$$

$$(a')_1^2 = (s^{-1})_1^l a_l^k s_k^2 \qquad a' = s^{-1} a s$$

some linear function on $\mathbb{V}$ $\Rightarrow [G(\vec{v})](\vec{w}) = (\vec{v}, \vec{w}$

Def'd in this way, its

clarification, linearity

$G(\alpha\vec{u} + \beta\vec{v}) = \alpha G(\vec{u})$

$\vec{v} = v^i \vec{e}_i = v'^i \vec{e}'_i$

$\vec{e}_i = a_i^j \vec{e}_j$	$\vec{e}'_i = (a^{-1})_i^j$

Lin operator //PHYS
Lin Trans //MATH

$A\vec{v} = \vec{w}$

$(w^1\ w^2\ w^3) = (v^1\ v^2\ v^3)\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$

suppose Frame $\begin{pmatrix} \vec{e}_1 \\ \vdots \\ \vec{e}_n \end{pmatrix}$

$\vec{v} = v^i \vec{e}_i$
$A(\vec{v}) = A(v^i \vec{e}_i)$

Tensorial transf rule under Δ of basis

$(a')_i^j = (S^{-1})_i^l a_l^k S_k^j$ // (1,1) tens

j trans as S — a_{ij} (0,2) tens
i trans as S^{-1} — a'^{ij} (2,0) tens

Suppose we have T^i_{jk}, vals depend on basis

How is T'^i_{jk} related

i is contravariant $\Rightarrow$ transform as S

$T'^i_{jk} = S^i_l (S^{-1})_j^m (S^{-1})_k^n T^l_{mn}$

Now lets make arbitrary index

$T^{i_1 \ldots i_r}_{j_1 \ldots j_s}$ (r-indices) (s-indices), (r,s) - tensor

ex) 3-dim $\rightarrow 3^{r+s}$ components

$$T'^{i_1 \dots i_r}_{j_1 \dots j_s} = S^{i_1}_{k_1} S^{i_2}_{k_2} \dots S^{i_r}_{k_r} (S^{-1})^{l_1}_{j_1} \dots (S^{-1})^{l_s}_{j_s} T^{k_1 \dots k_r}_{l_1 \dots l_s}$$

Transf that doesnt Δ length is called orthog.

$SS^T = \mathbb{I}$? Refer to audio,

Levi-Civita Tens.

ε^i_{jk} def'd s.t. $= 0$ if any two indices are the same

indices 1,2,3, so $\exists$ $3! = 6$ perms

three of which odd/even.

$\varepsilon^i_{jk} = \pm 1$ even/odd.

Antisymmetric between any two arrangments

"Completely Antisymm Levi-Civita"

Think Abt dim of tensor & dim of space tensor made of.

$: \mathbb{V} \to \mathbb{W}$ linear, invertible

sociated w every vector

Hilbert space, $\exists$ canonical

ct in dual space.

$\mathbb{C}$-V.S.

$(\psi, \phi) = (\phi, \psi)^*$

e $|\psi\rangle \in \mathcal{H}$, then corresponding

$\langle \psi | \in \mathcal{H}^*$

Set of all square integrable

nction gives Hilbert space

integ. : $\int_{-\infty}^{\infty} \phi^* \psi \, dx < \infty$

$\parallel$

$\langle \phi | \psi \rangle$

integrable $\int_{-\infty}^{\infty} |\psi|^2 dx < \infty$

$\parallel$

$\langle \psi | \psi \rangle$

Map : $f: \mathbb{V} \to \mathbb{Z}$

NOW WHAT IT IS ALREADY

BILINEAR MAP :

$f: \mathbb{V} \times \mathbb{W} \to \mathbb{Z}$ // lin in both args

if $\mathbb{Z}$ is $\mathbb{F}$, the map is reffered to as functions.

r-linear maps

$f: \mathbb{V}_1 \times \mathbb{V}_2 \times \dots \times \mathbb{V}_r \to \mathbb{Z}$

$f(\vec{v}_1, \vec{v}_2, \dots, \vec{v}_r)$

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Lin operator //PHYS

Lin Trans //MATH

$A\vec{v} = \vec{\omega}$

$(\omega^1\ \omega^2\ \omega^3) = (v^1\ v^2\ v^3)\begin{pmatrix} \cdots \\ \vdots \end{pmatrix}$

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