APRIL 21 MAT444

$T \in T^r(V) = V \otimes \ldots \otimes V$ //r-times

$T = T^{i_1 \cdots i_r} \vec{e}_{i_1} \otimes \ldots \otimes \vec{e}_{i_r}$

$\dim T^r(V) = n^r$ $\{e_1, \ldots, e_n\}$ basc for V

antisymm tens aka alternating

$\sigma \in S(r)$

$\vec{v}_{i_1} \otimes \ldots \otimes \vec{v}_{i_r}$ //monomial

$\sigma(\vec{v}_{i_1} \otimes \ldots \otimes \vec{v}_{i_r}) =$

$\vec{v}_{\sigma^{-1}(i_1)} \otimes \ldots \otimes \vec{v}_{\sigma^{-1}(}$

ex) $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$

$(\sigma T)(\vec{v}^{*1}, \ldots, \vec{v}^{*r}) =$

$T(\vec{v}^{*\sigma(1)}, \ldots, \vec{v}^{*\sigma(r)})$ definition of operation

$(\sigma T) = T^{i_1 \cdots i_r} \sigma(\vec{e}_{i_1} \otimes \ldots \otimes \vec{e}_{i_r})$

$= T^{i_1 \cdots i_r} \vec{e}_{i_{\sigma^{-1}(1)}} \otimes \ldots \otimes \vec{e}_{i_{\sigma^{-1}(r)}}$

$= T^{i_{\sigma(1)} \cdots i_{\sigma(r)}} \vec{e}_{i_1} \otimes \ldots \otimes \vec{e}_{i_r}$

ex) $T = \cancel{T^{ijk} e_i \otimes e_j \otimes e_k}$

$T = T^{i_1 i_2 i_3} e_{i_1} \otimes e_{i_2} \otimes e_{i_3}$

$\sigma T = T^{i_1 i_2 i_3} e_{i_3} \otimes e_{i_1} \otimes e_{i_2}$

$= T^{i_2 i_3 i_1} e_{i_1} \otimes e_{i_2} \otimes e_{i_3}$

$\cancel{\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}}$

$\cancel{\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}}$

$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$; $\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

Symm Tens : $\sigma T = T \quad \forall\ \sigma \in S_r$

ANTI Symm Tens : $\sigma T = (\text{sgn}\ \sigma) T \quad \forall\ \sigma \in S_r$

$\hookrightarrow \pm 1$ (even/odd)

Both for vector spaces (i.e. subspaces of $T^r(V)$)

e.x.) Suppose $\dim V = 2$,

$T^{ij}\ \vec{e}_i \otimes \vec{e}_j \Rightarrow e_1 \otimes e_1,\ e_1 \otimes e_2,\ e_2 \otimes e_1,\ e_2 \otimes e_2$

spans $T^2(V)$

for this particular example, Tensors are either sym or antisymm.

symmetric $\quad e_1 \otimes e_1,\ e_2 \otimes e_2,\ e_1 \otimes e_2 + e_2 \otimes e_1$ // 3 dim subspace

antisym $\quad e_1 \otimes e_2 - e_2 \otimes e_1$

Direct Sum of above V.S.'s. = whole space

SIDEBAR TO PHYS $\quad |\uparrow\rangle, |\downarrow\rangle$ spin-up/down electron

$e_1 \otimes e_1 = |\uparrow\uparrow\rangle, \quad e_2 \otimes e_2 = |\downarrow\downarrow\rangle$

$e_1 \otimes e_2 + e_2 \otimes e_1 = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

$e_1 \otimes e_2 - e_2 \otimes e_1 = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ then $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ to normalize

$\langle\uparrow|\uparrow\rangle = 1 = \langle\downarrow|\downarrow\rangle, \ \langle\uparrow|\downarrow\rangle = 0$

S = ANGULAR MMT	m_s
1	1
1	0
1	-1
0	0

In phys $\underset{\text{Dim}}{4} = 3 \oplus 1$ irreducible subspaces

Total ANGULAR mmt spin & orbital ($\mathbb{Z}$)

$(\frac{1}{2})$ (l)

~~$\mathbb{R}$ of~~ $\mathcal{H}$

$$\begin{array}{c|c} l=2 & m_l \\ l=2 & 2 \\ & 1 \\ & 0 \\ & 1 \\ & 2 \end{array}$$

ORBITAL angular mmt

5dim V.S.

any lin-combo of $\mathbb{P}$ $|2\ m_l\rangle$ has orbital ang mmt $= 2$

$|\frac{1}{2}, m_s\rangle$ Total space tensored togethered