MAT 444 HW Problems

Prob. 8 Show that the canonical isomorphism $G: \mathbb{V} \to \mathbb{V}^*$ induced by an inner product $(\cdot\,,\cdot)$ in $\mathbb{V}$ and defined by

$$[G(\vec{v})](\vec{w}) = (\vec{v}, \vec{w}) \quad \text{for all } \vec{v}, \vec{w} \in \mathbb{V}$$

is a linear map.

Prob. 9 Show that any linear map between 2 vector spaces of the same dimension is surjective (onto).

Prob. 10 Show that $G: \mathbb{V} \to \mathbb{V}^*$ as defined in Prob 8 is in fact an isomorphism, that is, one-to-one and onto. (bijective).

Prob. 11 Suppose that $G: \mathbb{V} \to \mathbb{V}^*$ as defined in Prob. 8 is bijective, Show that the inner product must be non-degenerate.