M444

April 10 Mat 444

INNER PROD $(,)$ on a $\mathbb{R}$-v.s. $\mathbb{V}$ is a symm, non-degenerate, bilinear, $\mathbb{R}$, two-form on $\mathbb{V}$

Symmetry: $(\vec{u},\vec{v}) = (\vec{v},\vec{u})$

non-deg: $(\vec{u},\vec{v}) = 0 \;\forall\; \vec{v} \in \mathbb{V}$
$\Rightarrow \vec{u} = 0$

bi-linear: $(\alpha\vec{u}+\beta\vec{v},\vec{w}) = \alpha(\vec{u},\vec{w}) + \beta(\vec{v},\vec{w})$

— and —

$(\vec{w},\alpha\vec{u}+\beta\vec{v}) = \alpha(\vec{w},\vec{u}) + \beta(\vec{w},\vec{v})$

Inner prod induces ~~bij~~ isomorphism onto dual sp.

intro canonical isomorphism

$G: \mathbb{V} \to \mathbb{V}^*$

let $\vec{v} \in \mathbb{V}$,

$G(\vec{v}) \in \mathbb{V}^*$

some linear function on $\mathbb{V}$

$\Rightarrow [G(\vec{v})](\vec{w}) = (\vec{v},\vec{w})$

Def'd in this way, it's linear & bij

clarification, linearity

$G(\alpha\vec{u}+\beta\vec{v}) = \alpha G(\vec{u}) + \beta G(\vec{v})$

~~$A: \mathbb{V} \to \mathbb{W}$~~ linear, invertible

associated w every vector in Hilbert space, $\exists$ canonical vect in dual space

for $\mathbb{C}$-v.s.

$(\psi,\phi) = (\phi,\psi)^*$

Suppose $|\psi\rangle \in \mathcal{H}$, then corresponding $\langle\psi| \in \mathcal{H}^*$

~~$\mathcal{H}$ is~~ Set of all square integrable function gives Hilbert space

sq. integ.: $\int_{-\infty}^{\infty} \phi^* \psi \, dx < \infty$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad /\!/$

$\quad\quad\quad\quad\quad\quad\quad\quad\langle\phi|\psi\rangle$

square integrable $\int_{-\infty}^{\infty} |\psi|^2 dx < \infty$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad /\!/$

$\quad\quad\quad\quad\quad\quad\quad\quad\langle\psi|\psi\rangle$

Linear Map: $f: \mathbb{V} \to \mathbb{Z}$

// KNOW WHAT IT IS ALREADY

BILINEAR MAP:

$f: \mathbb{V} \times \mathbb{W} \to \mathbb{Z}$ // lin in both args

if $\mathbb{Z}$ is $\mathbb{F}$, the map is reffered to as functions.

r-linear maps

$f: \mathbb{V}_1 \times \mathbb{V}_2 \times \ldots \times \mathbb{V}_r \to \mathbb{Z}$

$f(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_r)$

$\mathcal{L}(\mathbb{V}_1, \dots, \mathbb{V}_r ; \mathbb{Z}) \equiv$

set of all r-linear function

on $\prod_{i=1}^{r} \mathbb{V}_i$

so $f \in \mathcal{L}(\mathbb{V}_1, \dots, \mathbb{V}_r : \mathbb{Z})$ is a

func

$f : \prod_{i=1}^{r} \mathbb{V}_i \to \mathbb{Z}$

To turn into V.S., need to

impose linear struct.

$(\oplus)$ f, g given

$(f+g)(\vec{v}_1, \dots, \vec{v}_r) =$

$f(\vec{v}_1, \dots, \vec{v}_r) + g(\vec{v}_1, \dots, \vec{v}_r)$

$(\alpha f)(\vec{v}_1, \dots, \vec{v}_r) = (\alpha) f(\vec{v}_1, \dots, \vec{v}_r)$

now identify 0-element

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matrices $n \times m$ isomorph

to $\mathcal{L}(\mathbb{V} : \mathbb{W})$

$\dim = n$ $\dim = m$