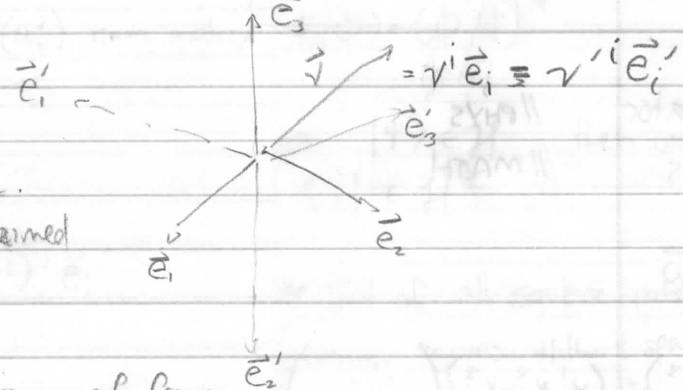
MAT444 LECTURE 2 3/29 WEDN.

notion of length not inherent in V.S.; must intro metric $\Rightarrow$ metric space,

WHEN Dealing w/ stuff, intro O.N. FRAME.

$\vec{v}$ has diff comp. wrt to ~~per~~ new frame.
HOW ARE PRIME/UNPRIMED COORDS RELATED?



$\Rightarrow$ NEED TO KNOW Relation of frames.

$$\begin{pmatrix}\vec{e}_1\\ \vec{e}_2\\ \vec{e}_3\end{pmatrix} = \begin{pmatrix}a_1^1 & a_1^2 & a_1^3\\ a_2^1 & a_2^2 & a_2^3\\ a_3^1 & a_3^2 & a_3^3\end{pmatrix}\begin{pmatrix}\vec{e}'_1\\ \vec{e}'_2\\ \vec{e}'_3\end{pmatrix}$$

Represents linear op.

Recall A is called orthog if
$AA^T = A^TA = I$
ORTHOG : det $= \pm 1$

column index summed over

WRITE $\vec{e}_i = a_i^j \vec{e}'_j$. matrix (a) specifies relation between frames.

$\vec{e} = a \cdot \vec{e}'$ // matrix eqn

$v' = v \cdot a$

$v^i \vec{e}_i = v^i a_i^j \vec{e}'_j = (v^j a_j^i) \vec{e}'_i = (v'^i) \vec{e}'_i$

$(\Rightarrow)\ v'^i = v^j a_j^i = a_j^i v^j$

Row index summed over

$\vec{e}' = a^{-1} \vec{e}$,

NOW stick in indices

$$\boxed{\vec{e}'_i = (a^{-1})_i^j \vec{e}_j}$$

$$(v'^1\ v'^2\ v'^3) = (v^1\ v^2\ v^3)\begin{pmatrix}a_1^1 & a_1^2 & a_1^3\\ \vdots & & \end{pmatrix}$$

$v' \cdot a^{-1} = v$

we assume the trans matrix are invertible

$\vec{v} = v^i \vec{e}_i = v'^i \vec{e}'_i$	
$\vec{e}_i = a_i^j \vec{e}'_j$	$\vec{e}'_i = (a^{-1})_i^j$

Lin operator //PHYS

Lin Trans //MATH

$A\vec{v} = \vec{w}$

$$(w^1\ w^2\ w^3) = (v^1\ v^2\ v^3)\begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$$

Suppose Frame $\begin{pmatrix} \vec{e}_1 \\ \vdots \\ \vec{e}_n \end{pmatrix}$

$\vec{v} = v^i \vec{e}_i$

$A(\vec{v}) = A(v^i \vec{e}_i)$