Lab 5

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The Nevada test site is found to have the following isotopes $U^{238}, U^{235}, Th^{232}, K^{40}$ and Cs^{137} with activities: 280.023, 177.762, 260.824, 265.366, and 255.271 $\frac{decay}{s}$ respectively. The Ge detector we used has an efficiency obeying the power law in the ≈ 300-1200keV region according to $\epsilon(E) = .0235E^{-0.587}$. The resolution of the detector was calculated to be FWHM = .00194 at 662keV.

I. INTRODUCTION

In this experiment, we use data from a high resolution Germanium Scintillation Detector to analyze a soil sample from a nuclear test site in Nevada to determine what isotopes exist there. Isotopes on earth can come from natural events like cosmic rays bombarding the atmosphere and nuclei excited at the beginning of earth's formation or from man made events such as nuclear bomb detonation. Two isotopes with long half-life's that were produced during earth's formation are ^{232}Th and ^{238}U. They decay into various daughter nuclei emitting photons in the process. Daughter nuclei can continue to decay into other nuclei until it reaches a steady energy state.

Isotopes like U^{238} have long half-lives on the order of billions of years so there are decays going on all around us. We analyize a soil sample from a Nevada site to see what kind of radioactive isotopes exist in the soil. We begin by calibrating our detector with known isotopes. Then analyze photopeaks emitted from the soil sample to determine isotopes and their activites.

II. THEORY

A. Calibration

Calibration of the Ge detector can be broken down into two parts. The easy part is calibration of the horizontal axis. We use photopeaks from $Cs^{137}, Bi^{207}, Na^{22}, Co^{60}$ and their known energies to find a channel number to energy relationship. The photopeak energy used for Bi^{207} comes from the NaI experiment. We keep an eye on the σ for photopeak curves to make sure that they're consistent with other readings. Otherwise, we could be looking at two overlapping curves.

To calibrate the vertical axis, we start off by considering the following relationship between count rate, activity, yield, and efficiency.

$$\frac{counts}{time} = A \times Y \times \epsilon(E) \tag{1}$$

where, A is activity, Y is the yield of the particular gamma energy from a decay, and $\epsilon(E)$ is the efficiency of a particular energy. Y can be found from literature[4] so the above equation becomes

$$\frac{counts}{time} \times \frac{1}{Y} = A \times \epsilon(E) \tag{2}$$

By choosing to analyze radiation from the ^{232}Th and ^{238}U series, we are able to hold A constant since the series is in secular equilibrium, meaning A is the same for all isotopes in the series. We can see this from equation 11 of Natural Radiation[4]:

$$\frac{Act_B}{Act_A} = \frac{\tau_A}{\tau_A - \tau_B}(1 - (\frac{1}{2})^{(\frac{\tau_A - \tau_B}{\tau_A \tau_B})t})$$

Refering to the ^{232}Th and ^{238}U tables in Natural Radiation[4], we can see that the parent half-life's of ^{232}Th and ^{238}U are much larger than their daughter nuclei, ($\tau_A >> \tau_B$). Moreover, t equals the life of earth so the $\frac{1}{2}$ term goes to zero. Therefore, the activity of the isotopes within each series is the same.

Now we have the efficiency as a function of energy. This obeys a power law for a certain region[6],

$$\epsilon \propto \beta E^b \tag{3}$$

The last step is to calculate the coefficient β in Eqn 3. Eqn 1 can be written as

$$\frac{counts}{time} = A \times Y \times \beta E^b \tag{4}$$

We examine the 1460keV emission data for a sample of 3117g of KCl and compare it to the theoretical decay to approximate β. In particular, K^{40} decaying via electron capture to Ar^{40m} emits the 1460keV photon. Electron capture is likely to happen 10.72% of the time, and decays into Ar^{40m} 99.53%. Therefore, the yield for the 1460keV emission is 10.67%. The 1460keV emission falls into the power law region for efficiency, so we calculate the theoretical value for this emission to determine β.

III. MATERIALS AND METHODS

A. Calibrating the Horizontal Axis

We use the Ge curve fitting program[1] to extract photopeaks of Cs^{137}, Bi^{207}, Na^{22}, and Co^{60} from the file CsNaCoBi.cnf. Photopeaks are extracted starting from the right by panning all the way to the 7000 channel range. We gather photopeak data for the first 7 peaks going from right to left (Table II). We plot the data in Excel and use it's best fit line feature to get an equation (Fig 1).

B. Calibrating the Vertical Axis

We used Ge fit to extract as many photopeaks as possible and tabulate them in Table I. Bold entries are used for plotting in an attempt to get a power law relationship. Excel's best fit curve for these two series' are just for comparison. The exponential factor, b, we use is from the Decay Fit program[3], Fig 4. We use the same boldfaced entries for the Decay Fit program.

Peak Center	**Sigma**	**Area**	**Predicted [keV]**	**Known [keV]**	**Isotope**	**Yield**	**Series**
381.6125	3.172	7330.939	85.486	84.40	Th-228	0.0122	Th-232
1153.8538	**2.623**	**3624.135**	**239.316**	**238.00**	**Pb-212**	**0.436**	**Th-232**
1652.9528	**3.128**	**778.065**	**338.737**	**338.30**	**Ac-228**	**0.113**	**Th-232**
2521.6646	6.745	1965.584	511.784	510.77	Tl-208	0.226	Th-232
2884.9977	3.546	1392.149	584.160	583.20	Tl-208	0.845	Th-232
4532.9687	**4.213**	**966.993**	**912.436**	**911.20**	**Ac-228**	**0.266**	**Th-232**
1438.3123	**2.743**	**1117.279**	**295.980**	**295.20**	**Pb-214**	**0.184**	**U-238**
1723.0536	**3.139**	**1823.679**	**352.701**	**351.90**	**Pb-214**	**0.356**	**U-238**
3016.1414	**3.737**	**1583.410**	**610.284**	**609.30**	**Bi-214**	**0.455**	**U-238**
5583.7487	**4.963**	**439.729**	**1121.751**	**1120.30**	**Bi-214**	**0.149**	**U-238**

TABLE I: ^{232}Th and ^{238}U Series

1. *Finding β, the power law coefficient*

The number of K nuclei is given by

$$N_K = \frac{M_{KCl}}{mm_{KCl}} \times 6.022 \times 10^{23}$$

where mm_{KCl}, M_{KCl} are the molar mass and mass of KCl sample, respectively. The number of K^{40} nuclei is given by

$$N_{K^{40}} = 0.0117\% \times N_k$$

then the 1460keV emission per unit time is given by

$$N_{1460keV} = 10.67\% \times N_{K^{40}}$$

So,

$$\beta = N_{K^{40}} \times \lambda_{K^{40}}$$

Then

$$\beta = \frac{area_{1460}}{time} \cdot \frac{1}{N_{K^{40}} \cdot \lambda_{K40} \cdot Y \cdot E^b}$$

IV. DATA

The last row of Table II seems to relatively high σ so we use the double peak script and tabulate it in Table III. The difference is less than the resolution of the detector so we just keep that row.

Peak Center	**Sigma**	**Area**	**Energy [keV]**	**FWHM**
6649.4897	5.289	1824.9013	1332.50	0.001324
6358.2231	5.075	2478.1556	1274.00	0.001329
5849.6236	5.139	2191.6451	1173.24	0.001462
5299.1994	5.043	3869.9517	1071.03	0.001584
3279.2123	4.217	5042.0238	661.64	0.002141
2817.1452	3.817	10220.4344	570.41	0.002256
2522.0904	7.940 *	12480.4547	511.00	0.005242

TABLE II: Calibration Cs, Na, Co, Bi

Peak Center	**Sigma**	**Area**	**Energy**	**FWHM**
2518.7875	3.199	4848.637	511.21	0.002115
2525.3251	3.2022	5029.032	512.51	0.002111

TABLE III: Potential Double Peak

Figure 1 shows us a linear relationship between energy and channel number given by $E = 0.1992C + 9.4685$.

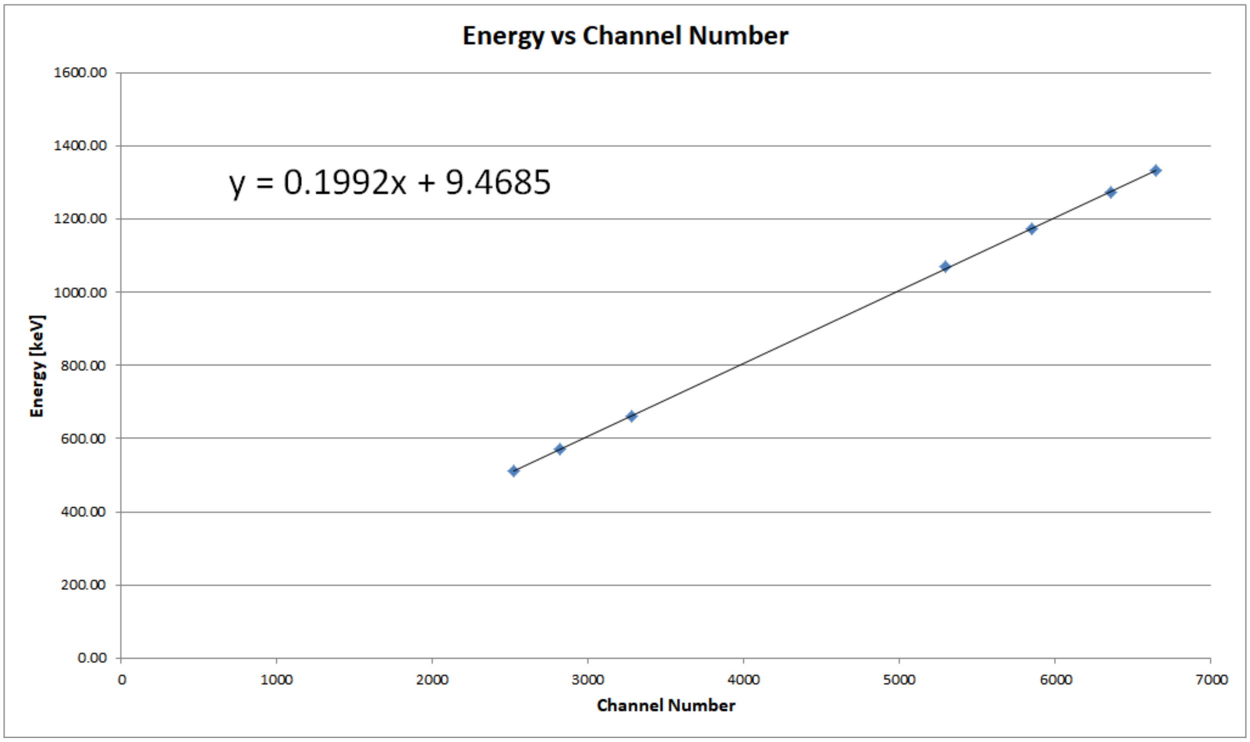

FIG. 1: Energy vs Channel Number

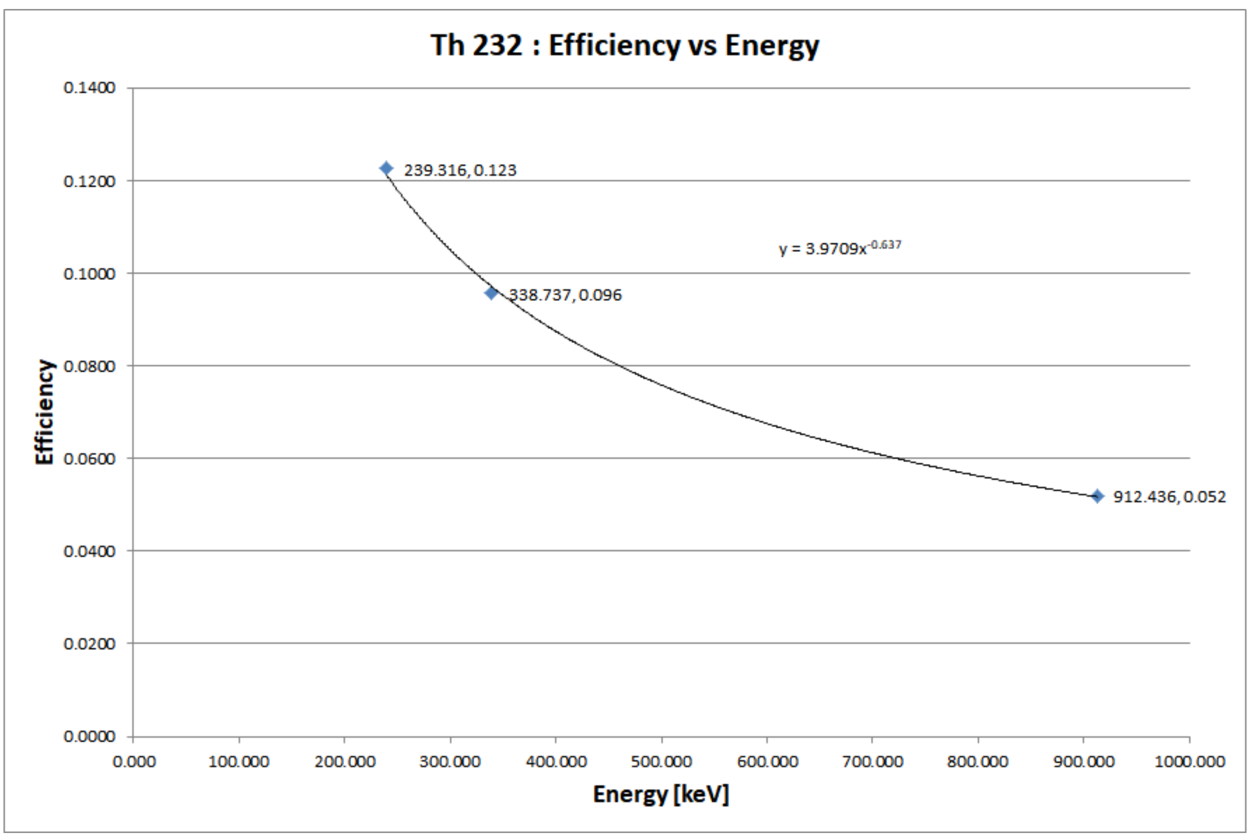

FIG. 2: Thorium Series - Efficiency

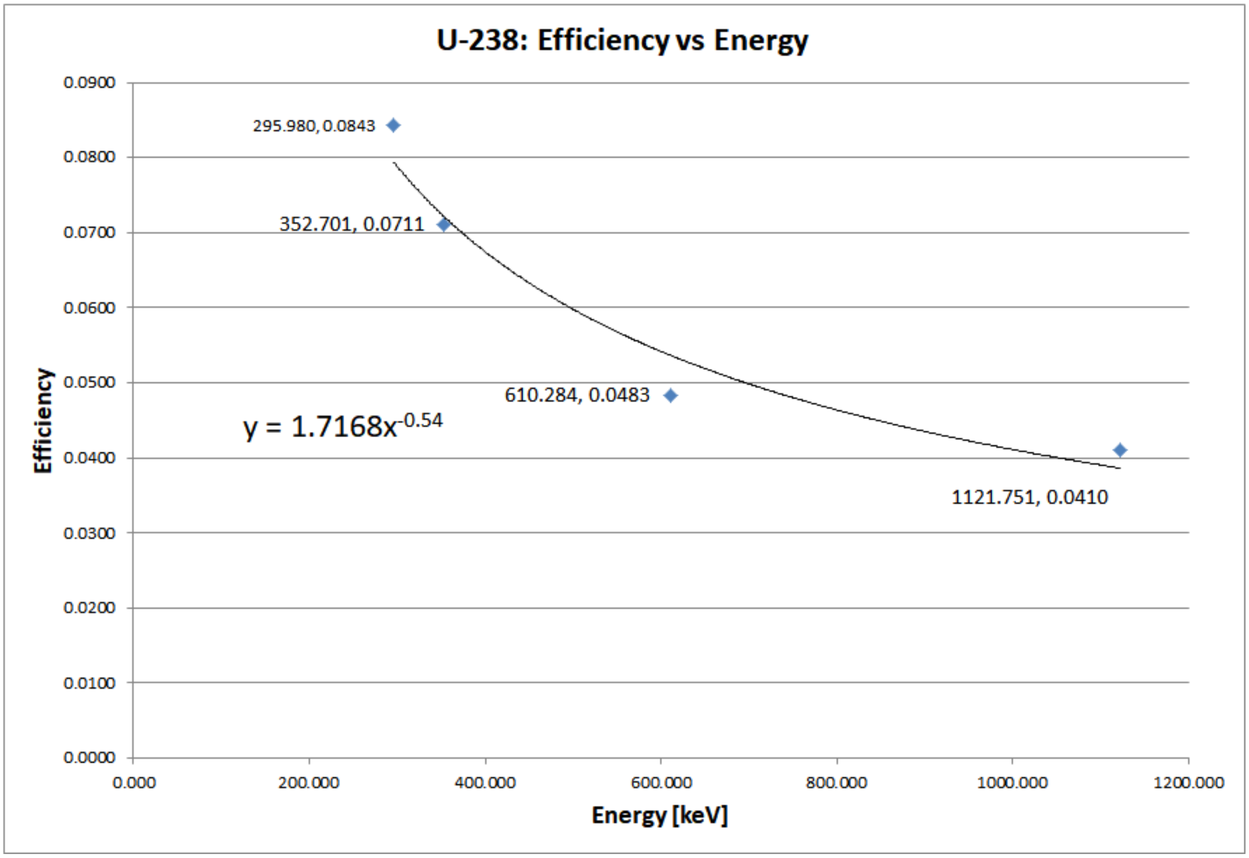

FIG. 3: Uranium Series - Efficiency

The coefficient b for Eqn 3 is taken to be -0.587. This is comparable to the ^{238}U curve fit exponent. The theoretical decay rate is $N_{K^{40}} \times \lambda_{K^{40}}$, $\lambda_{K^{40}}$ is the decay constant at 1460keV. Fig 5 shows us that the counts over the 20hr reading is 13,932.0174 arbitrary units. Eqn 4 applied to $K^{40}_{1460keV}$ becomes

$$\frac{counts}{time} = A \times Y \times \beta E^b$$
$$\frac{13932.0174}{72,000s} = \frac{3117g \cdot KCl}{74.55\frac{g}{mol}} \times 6.022 \cdot 10^{23}\frac{nuclei}{mol}$$
$$\times 0.000117 \times 0.1067 \times \beta \times (1460keV)^{-0.587}$$
$$\beta = 0.0237$$

$$\boxed{\frac{cts}{time} = AY(0.0237)E^{-0.587}} \tag{5}$$

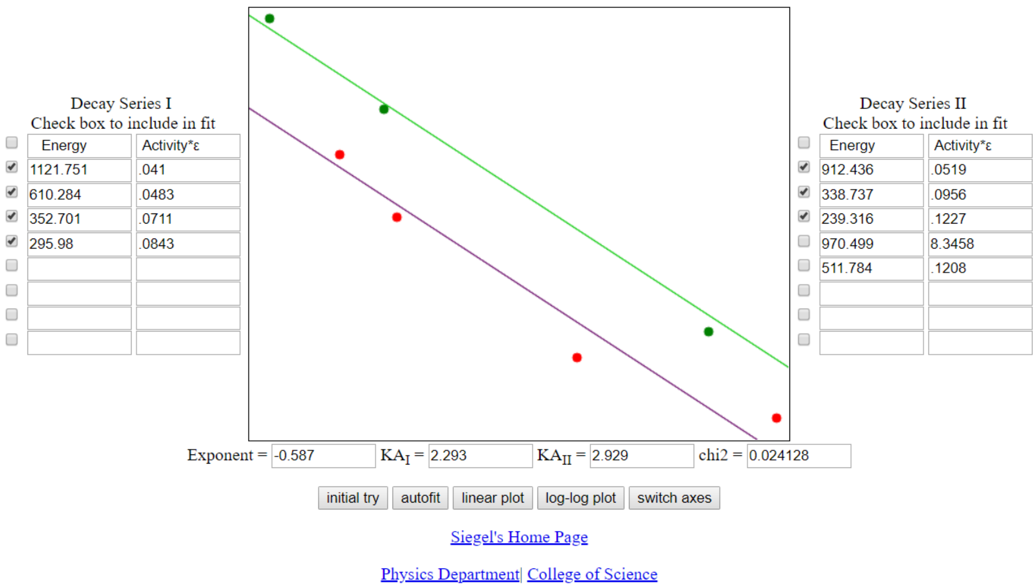

FIG. 4: Power Law Curve Fitting

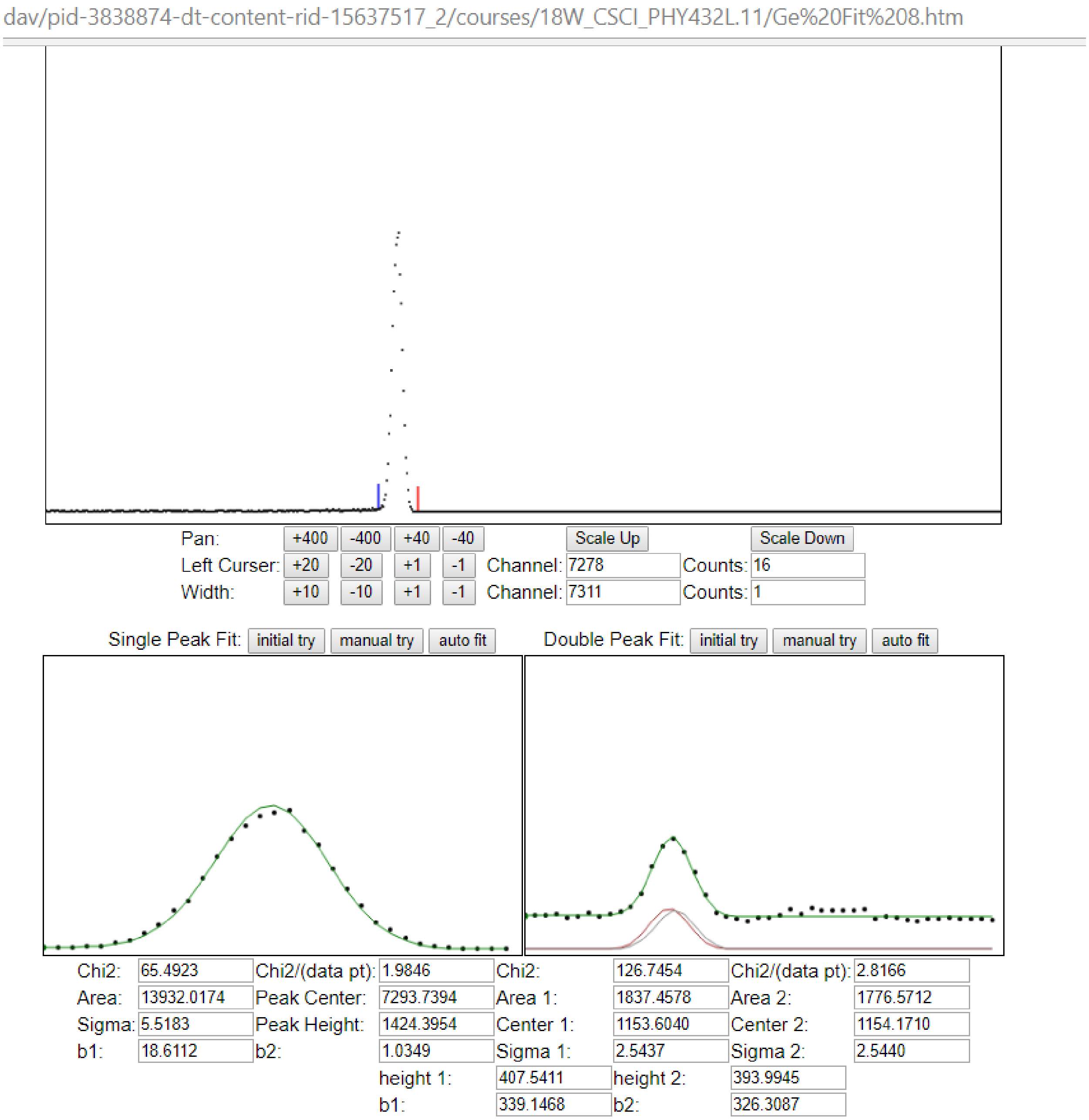

FIG. 5: K^{40} counts - 20hr = 72,000s

Peak Center	Sigma	Area	Energy	Possible Isotopes	Series	
344.1055	2.1449	8768.9555	78.0143	Fr-223	U-235	
382.2728	3.6043	5331.4764	85.6172	Th-231	U-235	*
382.2728	3.6043	5331.4764	85.6172	Th-228	Th-232	*
408.6803	3.2080	2936.2836	90.8776	Th-234	U-238	
423.5025	4.2673	3431.5779	93.8302	Th-234	U-238	
890.6261	3.2947	2208.0552	186.8812	Ra-226	U-238	
1007.0778	2.5151	1167.1069	210.0784	Ac-228	Th-232	
1154.9585	2.5022	11786.2713	239.5362	Pb-212	Th-232	
1313.5772	3.8468	1037.4833	271.1331	Rn-219	U-235	
1350.1210	2.7037	552.5240	278.4126	Tl-208	Th-232	
1439.1945	2.8223	3795.7228	296.1560	Tl-210	U-238	
1463.5441	3.0137	808.3647	301.0065	Pb-212	Th-232	*
1463.5441	3.0137	808.3647	301.0065	Th-227	U-235	*
1463.5441	3.0137	808.3647	301.0065	Pa-231	U-235	*
1655.6881	2.9422	2215.2375	339.2816	Ac-228	Th-232	*
1655.6881	2.9422	2215.2375	339.2816	Ra-223	U-235	*
1724.1006	2.9724	6006.4568	352.9093	Pb-214	U-238	
2282.3853	2.8223	617.7095	464.1197	Ac-228	Th-232	
2522.4890	5.3114	2474.1463	511.9483	Tl-208	Th-232	
2886.2345	3.4950	3449.4313	584.4064	Tl-208	Th-232	
3017.4613	3.5878	4517.2772	610.5468	Bi-214	U-238	
3280.4704	3.8073	1131.8285	662.9382	Cs-137	Cs-137	
3610.0568	4.1149	782.8170	728.5918	Bi-212	Th-232	
3950.2833	3.7207	356.3178	796.3649	Ac-228	Th-232	*
3950.2833	3.7207	356.3178	796.3649	Tl-210	U-238	*
4280.1291	4.4788	400.4585	862.0702	Tl-208	Th-232	
4534.1427	4.4974	2203.4469	912.6697	Ac-228	Th-232	
4824.3210	4.2630	1245.1981	970.4732	Ac-228	Th-232	
5584.9348	4.8569	1004.4748	1121.9875	Bi-214	U-238	
6176.6017	4.9094	400.4168	1239.8476	Bi-214	U-238	
6877.7241	5.6587	264.8774	1379.5111	Bi-214	U-238	
7294.8791	5.4441	10642.5367	1462.6084	K-40	K-40	

V. CALCULATIONS

A. Energy vs Channel Uncertainty

I took the uncertainty to be the average of the absolute value difference between the predicted and known energies.

$$\Delta_{rel} = \frac{|E_{known} - E_{predicted}|}{E_{known}}$$

Take the average of the absolute uncertainties to be our uncertainty for energy vs channel. $\bar{\Delta} = 1.8083$

Peak Center	Sigma	Area	Energy Known	Predicted	Relative Uncertainty
6649.4897	5.2887	1824.9013	1,332.5	1334.0	1.5468
6358.2231	5.0748	2478.1556	1,274.5	1276.0	1.5265
5849.6236	5.1389	2191.6451	1,173.2	1174.7	1.4735
5299.1994	5.043	3869.9517	1,071.0	1065.1	5.9610
3279.2123	4.2165	5042.0238	661.6	662.7	1.0476
2817.1452	3.817	10220.4344	570.4	570.6	0.2338
2522.0904	7.9403	12480.4547	511.0	511.9	0.8689

TABLE V: Uncertainty of Channel vs Energy

B. FWHM of 662keV

$$\begin{aligned} FWHM &= 2\sqrt{ln(2)}\frac{\sigma}{C_0} \\ &= 2\sqrt{ln(2)}\frac{3.817}{2817.1452} \\ &= 0.00194 \end{aligned}$$

C. Activity in the Soil

Table IV is a tabulation of photopeaks extracted from the soil matched up with the possible isotopes from the lab manual[4]. The peaks are consolidated into ^{238}U, ^{235}U, and^{232}Th series in the last pages attached. The first table is scratchwork and the second table deletes the values from small peaks that I have doubts about. The series activities are taken to be the averages of the last column because the decay is assumed to be in secular equilibrium.

$$U^{238} = 280.023\frac{decay}{s}$$
$$Th^{232} = 260.824\frac{decay}{s}$$
$$U^{235} = 177.762\frac{decay}{s}$$

Using Eqn 5, the activites of K^{40} and Cs^{137} are tabulated:

Peak Center	Sigma	Area [counts 20hr]	Energy [keV]	Isotope	Yield	Activity [decay/sec]
7294.8791	5.4441	10642.5367	1462.6084	K-40	0.1067	265.3654
3280.4704	3.8073	1131.8285	662.9382	Cs-137	0.0634	255.2713

TABLE VI: Activity: K-40 and Cs-137

VI. CONCLUSION

I didn't like how I needed to throw out some data points in the power law curve fitting part. The photopeaks were pretty clear but deviated from the curve significantly. When I was looking at the activity data in the last part, I decided to throw away data points in the 400 channel range and under because these were probably backscatter peaks. The decay series energies for some isotopes were so close together that it was difficult to tell which decay it belonged to. For example, Pb-212, Th-227, and Pa-231 all have decays in the 300keV range. Based on the tabulated activity value in the last part, I just deleted values that were much off from the other ones. I would like to understand how the curve fitting software works so a better decision can be made for throwing out data points.

Peak Center	U-238	Area	Energy		Yield	Activity		
3017.4613	3.5878	4517.2772	610.5468	Bi-214	0.455	251.1895	-	
5584.9348	4.8569	1004.4748	1121.9875	Bi-214	0.149	243.7894	-	
6176.6017	4.9094	400.4168	1239.8476	Bi-214	0.058	263.3723	-	
6877.7241	5.6587	264.8774	1379.5111	Bi-214	0.040	271.0253	-	
1724.1006	2.9724	6006.4568	352.9093	Pb-214	0.356	309.4324	-	
890.6261	3.2947	2208.0552	186.8812	Ra-226	0.036	776.6828	**	
~~408.6803~~	~~3.2080~~	~~2936.2836~~	~~90.8776~~	~~Th-234~~	~~0.021~~	~~1140.123~~	*	400 range
~~423.5025~~	~~4.2673~~	~~3431.5779~~	~~93.8302~~	~~Th-234~~	~~0.021~~	~~1377.079~~	-	400 range
1439.1945	2.8223	3795.7228	296.1560	Pb-214	0.184	341.3331	-	
~~3950.2833~~	~~3.7207~~	~~356.3178~~	~~796.3649~~	~~Tl-210~~	~~1.000~~	~~10.53682~~	**	Small peak

Peak Center	Th-232	Area	Energy		Yield			
1007.0778	2.5151	1167.1069	210.0784	Ac-228	0.039	406.8515	**	Small peak
1655.6881	2.9422	2215.2375	339.2816	Ac-228	0.113	351.3176	*	
2282.3853	2.8223	617.7095	464.1197	Ac-228	0.044	299.6624	-	
~~3950.2833~~	~~3.7207~~	~~356.3178~~	~~796.3649~~	~~Ac-228~~	~~0.044~~	~~241.6701~~	**	Small peak
4534.1427	4.4974	2203.4469	912.6697	Ac-228	0.266	265.3654	-	
4824.3210	4.2630	1245.1981	970.4732	Ac-228	0.162	255.2713	-	
3610.0568	4.1149	782.8170	728.5918	Bi-212	0.067	330.397	*	
1154.9585	2.5022	11786.2713	239.5362	Pb-212	0.436	394.9117	-	
1463.5441	3.0137	808.3647	301.0065	Pb-212	0.033	404.3003	**	Small peak
~~382.2728~~	~~3.6043~~	~~5331.4764~~	~~85.6172~~	~~Th-228~~	~~0.012~~	~~3489.958~~	-	400 Range
~~1350.1210~~	~~2.7037~~	~~552.5240~~	~~278.4126~~	~~Tl-208~~	~~0.063~~	~~139.7247~~	**	Small peak
2522.4890	5.3114	2474.1463	511.9483	Tl-208	0.226	249.7766	-	
2886.2345	3.4950	3449.4313	584.4064	Tl-208	0.845	100.6634	*	
4280.1291	4.4788	400.4585	862.0702	Tl-208	0.124	100.0503	-	

Peak Center	U-235	Area	Energy		Yield			
344.1055	2.1449	8768.9555	78.0143	Fr-223	0.091	728.6718	-	
1463.5441	3.0137	808.3647	301.0065	Pa-231	0.025	546.7057	*	
1655.6881	2.9422	2215.2375	339.2816	Ra-223	0.028	1397.848	*	
1313.5772	3.8468	1037.4833	271.1331	Rn-219	0.108	150.9227	-	
1463.5441	3.0137	808.3647	301.0065	Th-227	0.066	204.6005	*	
~~382.2728~~	~~3.6043~~	~~5331.4764~~	~~85.6172~~	~~Th-231~~	~~0.030~~	~~1419.25~~	*	400 Range

Peak Center	U-238	Area	Energy		Yield	Activity	
3017.4613	3.5878	4517.2772	610.5468	Bi-214	0.455	251.1895	280.02368
5584.9348	4.8569	1004.4748	1121.9875	Bi-214	0.149	243.7894	
6176.6017	4.9094	400.4168	1239.8476	Bi-214	0.058	263.3723	
6877.7241	5.6587	264.8774	1379.5111	Bi-214	0.040	271.0253	
1724.1006	2.9724	6006.4568	352.9093	Pb-214	0.356	309.4324	
1439.1945	2.8223	3795.7228	296.1560	Pb-214	0.184	341.3331	

Peak Center	Th-232	Area	Energy		Yield	260.824	
1154.9585	2.5022	11786.2713	239.5362	Pb-212	0.436	394.9117	
1655.6881	2.9422	2215.2375	339.2816	Ac-228	0.113	351.3176	*
2282.3853	2.8223	617.7095	464.1197	Ac-228	0.044	299.6624	
2522.4890	5.3114	2474.1463	511.9483	Tl-208	0.226	249.7766	
2886.2345	3.4950	3449.4313	584.4064	Tl-208	0.845	100.6634	
3610.0568	4.1149	782.8170	728.5918	Bi-212	0.067	330.397	
4280.1291	4.4788	400.4585	862.0702	Tl-208	0.124	100.0503	
4534.1427	4.4974	2203.4469	912.6697	Ac-228	0.266	265.3654	
4824.3210	4.2630	1245.1981	970.4732	Ac-228	0.162	255.2713	

Peak Center	U-235	Area	Energy		Yield		
1313.5772	3.8468	1037.4833	271.1331	Rn-219	0.108	150.9227	
1463.5441	3.0137	808.3647	301.0065	Th-227	0.066	204.6005	*

REFERENCES

[1]Peter Seigel, Germanium Curve Fit Program, `https://www.cpp.edu/~pbsiegel/javascript/gefit8.html`

[2]Peter Seigel, Lab Manual, *Experiment 5* `https://www.cpp.edu/~pbsiegel/phy432/labman/exp5w16.pdf`

[3]Peter Seigel, Lab Manual, *Decay Fit* `http://www.cpp.edu/~pbsiegel/javascript/soilfit.html`

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[5]Los Alamos National Laboratory, Passive Nondestructive Assay Manual - PANDA, *Gamma Ray Detectors* `http://www.lanl.gov/orgs/n/n1/panda/00326398.pdf`

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