MAY 12th

exterior - antisymm

<u>Thm</u> Given v.s. $\mathbb{V}$,
Suppose $\{\vec{v}_\alpha, \vec{w}_\alpha\}$, $\{\vec{v}_\alpha', \vec{w}_\alpha'\}$ are two sets of vectors in $\mathbb{V}$.
$(1 \le \alpha \le k)$

If $\{\vec{v}_\alpha, \vec{w}_\alpha\}$ is linearly indep set, and

$$\sum^{k} v_\alpha \wedge w_\alpha = \sum v_\alpha' \wedge w_\alpha'$$

then $\{v_\alpha', w_\alpha'\}$ is L.I. & v_α', w_α' are lin combo's of (v_α, w_α)

ex of 1-form

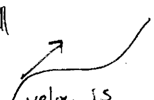
Force field $F_x(x,y,z)\,dx + F_y(x,y,z)\,dy + F_z(x,y,z)\,dz$

$$\vec{F}(x,y,z) = F_x \vec{e}_1 + F_y \vec{e}_2 + F_z \vec{e}_3$$
$$d\vec{l} = dx\,\vec{e}_1 + dy\,\vec{e}_2 + dz\,\vec{e}_3$$

$$\Rightarrow \int_C \underbrace{\vec{F} \cdot d\vec{l}}_{\text{DIFF}} = W_{12}$$

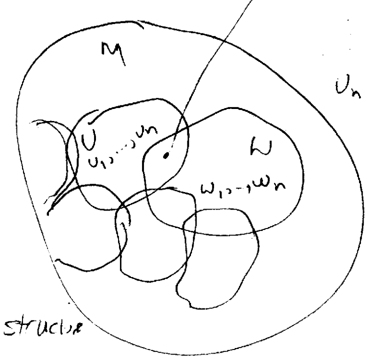
DIFF 1-form, i.e. depends on dx, dy, dz

Recall



velo. is tan vect.

dimension const. at everywhere in manifold.



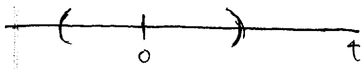
must exist
$u_1 = u_1(w_1, \dots, w_n)$

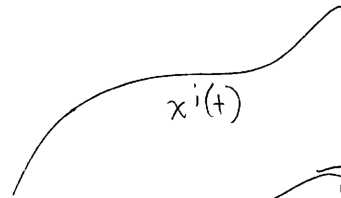
$u_n = u_n(w_1, \dots, w_n)$

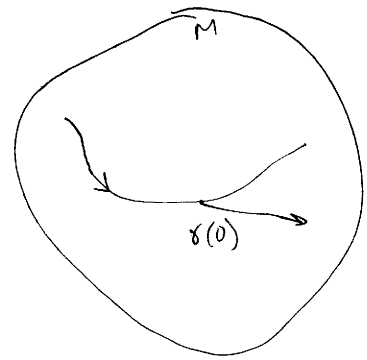
conversely also

recall diffeomorphism
$\bigcup$ of all charts ~~manifold~~ give diff ~~str~~ structure

Path $\quad \gamma : \mathbb{R} \to M$







WHAT IS velocity?

$v^i = \dfrac{dx^i}{dt}$ in elementary physics

$= \dfrac{d}{dt}\left(x^i \circ \gamma\right)\Big|_{t=0}$

function on manifold $\quad x^i : M \to \mathbb{R}$

$= \dot{x}^i(0)$

$T_p(M)$

let $\quad f : M \to \mathbb{R} \quad , \quad f(x^1, \dots, x^n)$

$$\frac{\partial f}{\partial x^i} \underbrace{\frac{dx^i}{dt}}_{\to v^i}\Big|_{t=0}$$

$\dfrac{\partial f}{\partial x^i} v^i \qquad$ write as $(\partial_i f)(v^i)$

coords of 1-form $\qquad (v^i \partial_i)(f) = \vec{v}$

pairing $\quad \langle \vec{v}, df \rangle \qquad df = \dfrac{\partial f}{\partial x^i} dx^i + \dots$

diff of every func is vect $\in$ cot