$\boxed{P26.1}$ Define $J_\pm = J_x \pm i J_y$,

then $[J_x, J_y] = i J_z \Rightarrow [J_+, J_-] = 2J_z$

Let $|Ajm\rangle \equiv |j_1 j_2 (j_{12}) j_3; jm\rangle$

$|Bjm\rangle \equiv |j_1 j_3 (j_{13}) j_2; jm\rangle$.

Then $C_m \equiv \langle Ajm|Bjm\rangle$

$$= \frac{1}{2m}\langle Ajm|2J_z|Bjm\rangle = \frac{1}{2m}\langle Ajm|[J_+, J_-]|Bjm\rangle.$$

Now let $m = j$. Then

$$\langle Ajj|[J_+, J_-]|Bjj\rangle = \langle Ajj|J_+J_-|Bjj\rangle - \underbrace{\langle Ajj|J_-J_+|Bjj\rangle}_{=0}$$

$$= \langle Ajj|J_+J_-|Bjj\rangle$$

But $J_-|Bjj\rangle = \sqrt{j(j+1)-j(j-1)}\,|Bj,j-1\rangle = \sqrt{2j}\,|Bj,j-1\rangle$,

thus $\langle Ajj|J_+ = \sqrt{2j}\,\langle Aj,j-1|$

$$\therefore\ C_j = \langle Ajj|Bjj\rangle = \frac{1}{2j}\sqrt{2j}\sqrt{2j}\,\langle Aj,j-1|Bj,j-1\rangle$$

$$= \langle Aj,j-1|Bj,j-1\rangle = C_{j-1}$$

For $m < j$

$$C_m = \frac{1}{2m}\left[\langle Ajm|J_+J_-|Bjm\rangle - \langle Ajm|J_-J_+|Bjm\rangle\right]$$

$$= \frac{1}{2m}\left[\{j(j+1)-m(m-1)\}C_{m-1} - \{j(j+1)-m(m+1)\}C_{m+1}\right],$$ ie.

C_m satisfies the recursion relation

$$\boxed{2m\,C_m = \{j(j+1)-m(m-1)\}C_{m-1} - \{j(j+1)-m(m+1)\}C_{m+1}}$$

This equation is satisfied identically for C_m = constant (independent of m). We already know that $C_j = C_{j-1}$. Hence

C_m = constant for $m = j, j-1, \dots, -j$.

$\boxed{\text{Prob. 26.1}}$ cont'd

$$|j_1 j_2 (j_{12}) j_3 ; j m\rangle = |j_{12} j_3 ; j m\rangle$$

$$= \sum_{m_{12}\, m_3} |j_{12} m_{12} ; j_3 m_3\rangle \langle j_{12} m_{12} ; j_3 m_3 | j m\rangle$$

$\downarrow$ write as C.G. coefficient

$$\langle j_{12} j_3 ; m_{12} m_3 | j m\rangle$$

$$= \sum_{m_{12}\, m_3} |j_{12} m_{12}\rangle |j_3 m_3\rangle \langle j_{12} j_3 ; m_{12} m_3 | j m\rangle$$

$$= \sum_{\substack{m_{12}\, m_3 \\ m_1\, m_2}} |j_1 m_1 ; j_2 m_2\rangle \langle j_1 m_1 ; j_2 m_2 | j_{12} m_{12}\rangle\, |j_3 m_3\rangle \times \langle j_{12} j_3 ; m_{12} m_3 | j m\rangle$$

$$= \sum_{\substack{m_{12}\, m_3 \\ m_1\, m_2}} |j_1 m_1\rangle |j_2 m_2\rangle |j_3 m_3\rangle \times \langle j_1 j_2 ; m_1 m_2 | j_{12} m_{12}\rangle \langle j_{12} j_3 ; m_{12} m_3 | j m\rangle$$
