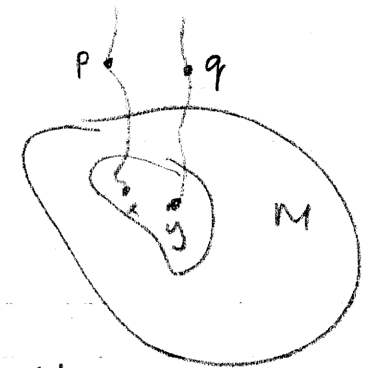
5/17 M444



$\pi(p) = x$

$\pi(q) = y$

$\pi : E \to M$

Section $s : M \to E$

$\pi \circ s : M \to M = id$

can make section into v.s.

e.g.) $\vec{E}(x) + \vec{E}_2(x) \underset{def}{=} (\vec{E}_1 + \vec{E}_2)(x)$ $\underset{\text{manifold}}{\overset{\text{in general}}{\Rightarrow}}$ $(S_1 + S_2)(x) = S_1(x) + S_2(x)$

$f(x) S \in \Gamma(E)$

recall module

spc of all sections $\to$ $\Gamma(E)$ is a C^∞-module, every element in Γ can be mult by C^∞ func & is closed

WE HAVE MANIFOLD M,

TAN SPC @ $x \in M$ $T_x(M)$

COT SPC @ $x \in M$ $T_x^*(M)$

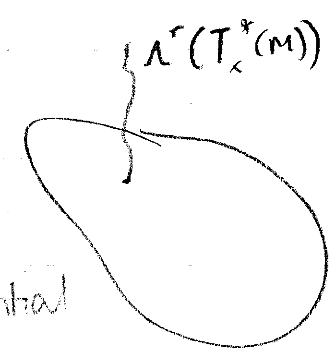
if $\dim M = n$, $\dim T_x(M) = n = \dim T_x^*(M)$

Bundle of exterior r-forms

$M \xrightarrow[\text{@ } x \in M]{\text{cot spc}} T_x^*(M)$, consider $\Lambda^r(T_x^*(M))$

lets BUNDLE TOGETHER: $\bigsqcup_{x \in M} \Lambda^r(T_x^*(M))$ write as $\Lambda^r(M^*)$

$\Lambda^r(M^*) \xrightarrow{\text{smooth sections on } \Lambda^r(M^*)} \Gamma(\Lambda^r(M^*)) = A^r(M)$



NOW Defferential Form,

a diff form is an element in $A^r(M)$

<u>Def</u> The elements of the C^∞-module, $A^r(M)$ are called exterior differential r-forms.

OR An exterior diff r-form on M is a smooth anti-symmetric covariant tensor field of order r [a $(0,r)$-type tensor field] on M

ex) $M = \mathbb{R}^3$, $p \in M$, $T_p^*(M)$

$$\Lambda^1(T_p^*(M)) = T_p^*(M)$$

$\{dx, dy, dz\}$ can be considered basis in cot spc.

$\Lambda^2(T_p^* M)$: $\{dx \wedge dy, dy \wedge dz, dz \wedge dx\}$

$\Lambda^3(T^*_p M)$ $dx \wedge dy \wedge dz$

ex) 1-form $\omega^1 = f_1(x,y,z)\,dx + f_2(x,y,z)\,dy + f_3(x,y,z)\,dz$

2-form $\omega^2 = g_1(x,y,z)\,dx \wedge dy + g_2(x,y,z)\,dy \wedge dz + g_3(x,y,z)\,dz \wedge dx$

$\omega^3 = h(x,y,z)\,dx \wedge dy \wedge dz$

$$A(M) = \oplus \sum_{r=0}^{n} A^r(M) = A^0(M) \oplus \cdots \oplus A^n(M)$$

ex) $(\omega^1 \oplus \omega^2) \wedge (\phi^2 \oplus \phi^3)$

$= \omega^1 \wedge \phi^2 \oplus \omega^2 \wedge \phi^2 \oplus \omega^1 \wedge \phi^3 \oplus \omega^2 \wedge \phi^3$

ω^r ξ^s forms

$\omega^r(x)$ $\xi^s(x)$ tensors

$(\omega^r \wedge \xi^s)(x) \equiv \omega^r(x) \wedge \xi^r(x)$

$\wedge : A^r(M^*) \times A^s(M^*) \to A^{r+s}(M^*)$

BASIS vect of r-form $dx^{i_1} \wedge \cdots \wedge dx^{i_r}$

$\omega = a_{i_1 \cdots i_r}(x^1, \ldots, x^n)\, dx^{i_1} \wedge \cdots \wedge dx^{i_r}$

$$\left\langle \frac{\partial}{\partial x^{i_1}} \wedge \dots \wedge \frac{\partial}{\partial x^{i_r}}, dx^{j_1} \wedge \dots \wedge dx^{j_r} \right\rangle = \delta^{j_1 \dots j_r}_{i_1 \dots i_r}$$

exterior derivative : d

<u>Thm</u> Let M be an n-dim manifold.

Then $\exists$ a <u>unique</u> map

$d : A(M) \to A(M)$ s.t.

$d(A^r(M)) \subseteq A^{r+1}(M)$

called exterior derivative which follows

1) $\forall\ \omega_1, \omega_2 \in A(M)$

$$d(\omega_1 + \omega_2) = d(\omega_1) + d(\omega_2)$$

2) suppose $\omega_1 \in A^r(M)$, then $\forall\ \omega_2 \in A(M)$

$$d(\omega_1 \wedge \omega_2) = d(\omega_1) \wedge \omega_2 + (-1)^r \omega_1 \wedge d(\omega_2)$$

note $d^2 = 0$ always