ECON 512
Homework 2

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September 25, 2018

Problem 1

- The demand for product A is $\frac{e}{1+2e}$.
- The demand for product B is $\frac{e}{1+2e}$.
- The demand for outside option is $\frac{1}{1+2e}$.

Problem 2

```
>> clear;
>> close all;
>> p=[1;1]; % initial guessing
>> v=[2;2];
>> fVal=betrand(p,v);
>> iJac=inv(myJac('betrand',p,v));
>> maxit = 100;
>> tol = 1e-6;
>> tic
>> for iter = 1:maxit
fnorm = norm(fVal);
fprintf('iter %d: q(1) = %f, q(2) = %f, norm(f(x)) = %.8f\n', iter, ...
p(1), p(2), norm(fVal));
if norm(fVal) < tol % convergence criteria
break
end
d = - (iJac * fVal);
p = p+d;
fOld = fVal;
fVal = betrand(p,v);
u = iJac*(fVal - fOld);
iJac = iJac + ( (d - u) * (d'*iJac) )/ (d'*u);
end
```

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```
iter 1: q(1) = 1.000000, q(2) = 1.000000, norm(f(x)) = 0.59724897
iter 2: q(1) = 1.656495, q(2) = 1.656495, norm(f(x)) = 0.06375194
iter 3: q(1) = 1.593177, q(2) = 1.593177, norm(f(x)) = 0.00631719
iter 4: q(1) = 1.598886, q(2) = 1.598886, norm(f(x)) = 0.00006158
iter 5: q(1) = 1.598942, q(2) = 1.598942, norm(f(x)) = 0.00000006
>>toc
Elapsed time is 0.046751 seconds.
```

Problem 3

```
>> clear; close all;
>> v=[2;2];
>> f1=@(p) 1-p(1)*(1-exp(v(1)-p(1))/(1+exp(v(1)-p(1))+exp(v(2)-p(2))));
>> f2=@(p) 1-p(2)*(1-exp(v(2)-p(2))/(1+exp(v(1)-p(1))+exp(v(2)-p(2))));
>> p=[1;1];
>> pold=[0.5;0.5];
>> f1Old=f1(pold);
>> f2Old=f2(pold);
>> tol = 1e-8;
>> maxit = 100;
>> pNew=zeros(2,1);
>> tic
>>for iter =1:maxit
fVal1=f1(p);
fVal2=f2(p);
fprintf('iter %d: p(1) = %.8f, p(2) = %.8f,f1(p) = %.8f\n, f2(p)=%.8f \n',...
iter, p(1,1),p(2,1), fVal1,fVal2);
if norm([fVal1;fVal2]) < tol
 break
else
pNew(1,1) = p(1,1) - ( (p(1,1) - pold(1,1)) / (fVal1 - f1Old)...
)* fVal1;
pNew(2,1) = p(2,1) - ( (p(2,1) - pold(2,1)) / (fVal2 - f2Old)...
)* fVal2;
pold = p;
p = pNew;
f1Old = fVal1;
f2Old = fVal2;
end
end
iter 1: p(1) = 1.00000000, p(2) = 1.00000000,f1(p) = 0.42231880
, f2(p)=0.42231880
iter 2: p(1) = 1.69784157, p(2) = 1.69784157,f1(p) = -0.07801513
, f2(p)=-0.07801513
iter 3: p(1) = 1.58902984, p(2) = 1.58902984,f1(p) = 0.00767532
, f2(p)=0.00767532
iter 4: p(1) = 1.59877614, p(2) = 1.59877614,f1(p) = 0.00012854
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```
, f2(p)=0.00012854
iter 5: p(1) = 1.59894214, p(2) = 1.59894214,f1(p) = -0.00000022
, f2(p)=-0.00000022
iter 6: p(1) = 1.59894186, p(2) = 1.59894186,f1(p) = 0.00000000
, f2(p)=0.00000000
>>toc
Elapsed time is 0.045289 seconds.
```

- **Guass-Sidel Method is faster because Broyden's Method has one more loop.**

Problem 4

```
>> clear; close all;
>> v=[2;2];
>> f1=@(p) (1-exp(v(1)-p(1))/(1+exp(v(1)-p(1))+exp(v(2)-p(2))));
>> f2=@(p) (1-exp(v(2)-p(2))/(1+exp(v(1)-p(1))+exp(v(2)-p(2))));
>> p=[1;1];
>> pnew=zeros(2,1);
>> maxit = 100;
>> tol = 1e-6;
>>tic
>>for iter =1:maxit
fVal1=f1(p);
fVal2=f2(p);
fprintf('iter %d: p(1) = %.8f, p(2) = %.8f,f1(p) = %.8f\n, f2(p)=%.8f \n', ...
iter, p(1,1),p(2,1), fVal1, fVal2);
pnew(1,1)=1/fVal1;
% p1=[pnew(1,1),p(2,1)];
%pnew(2,1)=1/f2(p1);
pnew(2,1)=1/fVal2;
if norm(p-pnew) < tol
break
else
p=pnew;
end
end
iter 1: p(1) = 1.00000000, p(2) = 1.00000000,f1(p) = 0.57768120
, f2(p)=0.57768120
iter 2: p(1) = 1.73105858, p(2) = 1.73105858,f1(p) = 0.63823011
, f2(p)=0.63823011
iter 3: p(1) = 1.56683301, p(2) = 1.56683301,f1(p) = 0.62242097
, f2(p)=0.62242097
iter 4: p(1) = 1.60662966, p(2) = 1.60662966,f1(p) = 0.62613731
, f2(p)=0.62613731
iter 5: p(1) = 1.59709378, p(2) = 1.59709378,f1(p) = 0.62524005
, f2(p)=0.62524005
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```
iter 6: p(1) = 1.59938571, p(2) = 1.59938571,f1(p) = 0.62545532
, f2(p)=0.62545532
iter 7: p(1) = 1.59883524, p(2) = 1.59883524,f1(p) = 0.62540359
, f2(p)=0.62540359
iter 8: p(1) = 1.59896747, p(2) = 1.59896747,f1(p) = 0.62541601
, f2(p)=0.62541601
iter 9: p(1) = 1.59893571, p(2) = 1.59893571,f1(p) = 0.62541303
, f2(p)=0.62541303
iter 10: p(1) = 1.59894334, p(2) = 1.59894334,f1(p) = 0.62541375
, f2(p)=0.62541375
iter 11: p(1) = 1.59894151, p(2) = 1.59894151,f1(p) = 0.62541358
, f2(p)=0.62541358
>>toc
Elapsed time is 0.034829 seconds.
```

Problem 5

Use Gauss-Sidel Method:

```
>> clear; close all;
>> vB=0:.2:3;
>> numB=length(vB);
>> maxit=100;
>> p=zeros(2,numB);
>> tol = 1e-6;
>>tic
>>for i=1:numB
p1=[0.5;1];
pold=[0.2;0.5];
pnew=zeros(2,1);
v=[2,vB(i)];
f1=@(p) 1-p(1)*(1-exp(v(1)-p(1))/(1+exp(v(1)-p(1))+exp(v(2)-p(2))));
f2=@(p) 1-p(2)*(1-exp(v(2)-p(2))/(1+exp(v(1)-p(1))+exp(v(2)-p(2))));
f1Old=f1(pold);
f2Old=f2(pold);
for j=1:maxit
fVal1=f1(p1);
fVal2=f2(p1);
if norm([fVal1;fVal2]) < tol
p(1,i)=p1(1,1);
p(2,i)=p1(2,1);
break
else
pNew(1,1) = p1(1,1) - ( (p1(1,1) - pold(1,1)) / (fVal1 - f1Old)...
)* fVal1;
pNew(2,1) = p1(2,1) - ( (p1(2,1) - pold(2,1)) / (fVal2 - f2Old)...
)* fVal2;
```

```
pold = p1;
p1 = pNew;
f1Old = fVal1;
f2Old = fVal2;
end
end
end
>>toc
Elapsed time is 6.24496 seconds.
>> plot(vB(:),p(1,:),vB(:),p(2,:))
>> legend('A price','B Price')
```