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A Real-life Application of the Two-Phase Simplex Method

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Abstract

It is a crucial part for every company to make decisions on a regular basis about how to allocate resources effectively and efficiently to various activities in order to achieve organizational goals. The task of deciding the optimum plan for distributing goods at the lowest cost possible is a case in point. In today's highly competitive business environment, lowering transportation costs is critical. This research focuses on the use of linear programming with the help of Two-Phase Simplex Method to assist managers in a Malaysian trading company in determining the best transportation plan for transporting polymer from six supply plants to six demand destinations at the lowest possible cost, while satisfying the supply and demand needs.

Introduction

For any business, the effective and efficient movement of products or services from points of supply to points of demand is critical. Transporting finished goods to market at the lowest possible cost results in significant cost savings and, as a result, increases the company's profit. As a result, the company aims to optimize their product distribution plan in terms of transportation costs. In a highly competitive business environment, product conversion costs and ex-factory prices are nearly identical everywhere, leaving the delivered price to vary depending on the distance between consumers and suppliers.

The transportation problem is defined as the selection of the quantity to be supplied, the delivery location, and the most appropriate and cost-effective mode of transportation. The goal is to keep the cost of shipping goods from one location to another as low as possible, so that each arrival area's needs are met while each supply location runs at full capacity.

Improvements to a company's transportation plan have a significant impact on its bottom line. According to research, a 5% reduction in transportation costs has the same impact as a 30% increase in sales. It is a method of long-term cost reduction. Furthermore, improvements in transportation typically result in higher levels of service.

The cost of distribution typically ranges from 9% to 14% of sales. It includes, among other things, all logistics-related costs such as warehousing, personnel, and transportation. The cost of transportation alone accounts for nearly half of the total distribution cost. The goal is to get the right amount of products to the right place, at the right time, and in the right condition, all while contributing the most to the company.

In light of this, the goal of this research is to develop a network representation, mathematical formulation, create a python code to implement the Two-Phase Simplex Method, as well as analyse the case of a Malaysian trading company's transportation problem. The minimum shipping cost of poly vinyl chloride polymer from six supply points to six demand destinations is calculated using linear programming and verified using a spreadsheet. [2]

Transportation Problem with Linear Programming

The goal of distributing goods from any supply point to any demand destination at the lowest overall distribution cost achievable is fundamental to the transportation problem. Each supply point has a specific supply capacity, and each destination has a specific demand that must be met. The cost of transporting goods from one supply location to another is proportional to the quantity delivered. Indeed, the transportation problem is viewed as a linear programming problem that may be addressed using the simplex method.

Linear programming is a powerful problem-solving tool that can help managers make better decisions. The primary technique is to create a mathematical model that describes the problem as a linear programming model and then analyse it using a spreadsheet. Any linear programming model includes decision variables that represent the decisions to be taken, constraints that indicate the limitations on the feasible values of these decision variables, and an objective function that expresses the problem's overall measure of performance. [2]

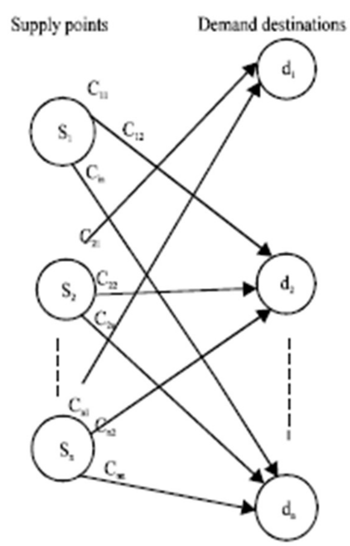
Linear Programming Model

Network Representation

The representation of a network: The Linear Programming model is designed to minimize shipping costs and meet each demand while not surpassing the polymer-producing petrochemical plants' maximum capacity.

The following information is stated in general:

- A set of n supply points from which product is shipped. Supply point i can supply at most s_i units.
- A set of m demand destinations to which the product is shipped. Demand destination j must receive at least d_j units of the shipped product.
- Each plant produced at supply point i and shipped to demand destination j incur a variable cost of c_{ij}. [1]



Mathematical Model

A linear programming mathematical model is formulated based on the network shown above. Let there be n supply points and m demand destinations. Let x_{ij} represent the number of units sent from supply point i to demand destination j. The following is a representation of the formulation of a transportation problem with the goal of minimizing transportation costs: [1]

$$\text{Min} \sum_{i=1}^{n}\sum_{j=1}^{m} cij * xij$$

The above is subjected to two constraints:
Supply constraints:

$$\sum_{j=1}^{n} xij \leq si \qquad (i = 1, 2, \ldots, n)$$

Demand constraints:

$$\sum_{j=1}^{m} xij \leq dj \qquad (j = 1, 2, \ldots, n)$$

And since x_{ij} must be non-negative

$$xij \geq 0 \qquad (i = 1, 2, \ldots, n; \qquad j = 1, 2, \ldots, n)$$

The objective function would be:

$$\begin{aligned} &\mathit{Minimize\ the\ shipping\ cost} \\ &\qquad = c11 * x11 + c12 * x12 + \ldots + c1m * x1m + c21 * x21 \\ &\qquad + c22 * x22 + \ldots + c2m * x2m + \ldots + cn1 * xn1 + cn2 \\ &\qquad * xn2 + \ldots + cnm * xnm \end{aligned}$$

Subject to the supply constraint:

- Supply point 1, S_1: $x_{11}+x_{12}+...+x_{1n}$
- Supply point 2, S_2: $x_{21}+x_{22}+...+x_{2n}$
- Supply point n, S_n: $x_{n1}+x_{n2}+...+x_{nn}$

And subject to demand constraint:

- Demand destination 1, d_1: $x_{11}+x_{21}+...+x_{n1}$
- Demand destination 2, d_2: $x_{12}+x_{22}+...+x_{n2}$
- Demand destination n, d_n: $x_{1n}+x_{2n}+...+x_{nn}$

The Simplex Algorithm

The Full Tableau Implementation

Summarizing the mechanics of the full tableau implementation.

1. A typical iteration starts with the tableau associated with a basis matrix B and the corresponding basic feasible solution x.
2. Examine the reduced costs in the zeroth-row o f t h e tableau. If they are all nonnegative, the current basic feasible solution is optimal, and the algorithm terminates; else, choose some j for which $c_j' < 0$.
3. Consider the vector $u = B^{-1}A_j$, which is the j^{th} column (the pivot column) of the tableau. If no component of u is positive, the optimal cost is $-\infty$, and the algorithm terminates.
4. For each i for which u_i is positive, compute the ratio $x_{B(i)}/u_i$. Let l be the index of a row that corresponds to the smallest ratio. The column $A_{B(l)}$ exits the basis and the column A_j enters the basis.
5. Add to each row of the tableau a constant multiple of the l^{th} row (the pivot row) so that u_l (the pivot element) becomes one and all other entries of the pivot column become zero. [3][4]

Pivoting Rules

Bland's rule:

1. Find the smallest j for which the reduced cost cj’ is negative and have the column Aj enter the basis.
2. Out of all variables xi that are tied in the test for choosing an exiting variable, select the one with the smallest value of i.

Lexicographic pivoting rule:

1. Choose an entering column Aj arbitrarily, as long as its reduced cost cj’ is negative. Let u = B-1Aj be the jth column of the tableau.
2. For each i with ui > 0, divide the ith row of the tableau (including the entry in the zeroth column) by ui and choose the lexicographically smallest row. If row l is lexicographically smallest, then the lth basic variable xB(l) exits the basis. [4]

The Two-Phase Simplex Method

We can summarize a complete algorithm for linear programming problems in standard form.

Phase I:

1. By multiplying some of the constraints by - 1, change the problem so that $b>0$.
2. Introduce artificial variables $y_1, \ldots, y_m$, if necessary, and apply the simplex method to the auxiliary problem with cost $\sum y_i$, $i=1, \ldots, m$.
3. If the optimal cost in the auxiliary problem is positive, the original problem is infeasible and the algorithm terminates.
4. If the optimal cost in the auxiliary problem is zero, a feasible solution to the original problem has been found. If no artificial variable is in the final basis, the artificial variables and the corresponding columns are eliminated, and a feasible basis for the original problem is available.
5. If the lth basic variable is an artificial one, examine the lth entry of the columns $B^{-1}A_j$, $j = 1, \ldots, n$. If all of these entries are zero, the lth row represents a redundant constraint and is eliminated. Otherwise, if the lth entry of the jth column is nonzero, apply a change of basis (with this entry serving as the pivot element): the lth basic variable exits and x_j enters the basis. Repeat this operation until all artificial variables are driven out of the basis.

Phase II:

1. Let the final basis and tableau obtained from Phase I be the initial basis and tableau for Phase II.
2. Compute the reduced costs of all variables for this initial basis, using the cost coefficients of the original problem.
3. Apply the simplex method to the original problem. [4]

Pseudo-code

```
Input vectors A, b, c in standard form
Specify if the problem is Min/Max

Class SimplexAlgorithm:
    Changing to a Min problem if the problem is Max
    Changing array elements into float to get decimals values
    Initializing m, n
    Checking the condition for infeasibility (any b<0)
    Creating Basis (Identity Matrix) and Adding Artificial Variables to A
    Initializing the variables in the basis
    Calculating the cost and reduced cost
    Creating a dataframe for the tableau

    Phase I begins
    While (any reduced cost < 0):
        Using Bland's rule to determine the variable entering
        Finding the Ratios to determine which variable is exiting
        Checking for Lexicographically positive rows
        Checking for Unbounded LP condition
        Performing Row operations (Updates the whole tableau)

    Checking for feasibility condition (auxiliary problem cost is zero)
        Checking if artificial variables are still present in the basis
            Driving out the artificial variables if any
    Determining the initial basic variables for Phase II

    Removing all the artificial variable columns from the tableau
    Computing the updated cost (-c_B x_B)
    Computing the updated reduced cost (c_j-c_B B^-1 A_j)

    Phase II begins
    While (any reduced cost < 0):
        Using Bland's rule to determine the variable entering
        Finding the Ratios to determine which variable is exiting
        Checking for Lexicographically positive rows
        Checking for Unbounded LP condition
        Performing Row operations (Updates the whole tableau)

    Print optimal cost
    Print x_B
```

Case Study

A trading company, wholly-owned subsidiary of a Malaysian petrochemical company is involved in buying and selling poly vinyl chloride polymer. The polymer is produced by six petrochemical plants in Malaysia and is exported to six destinations namely China, the Middle East, Europe, South East Asia, South Korea and Russia. The available production capacity of each polymer producing petrochemical plant and the demand of the customers are indicated in the Tables below. Each plant has a fixed capacity per annum that shall be distributed to the customers. Similarly, each destination has a fixed demand per annum that must be fulfilled from the various plants.

The shipping costs from the polymer producing petrochemical plants to the various destinations are shown in below. The unit cost of shipping varies as a result of differences in among others distance and also currencies exchange rates. Hence, allocating the production capacities to the various demand destinations in the optimal way to minimize total cost of shipping is crucial for the trading company. [1]

Plant	Production Capacity (Thousand ton per annum)
P1	110
P2	75
P3	95
P4	125
P5	60
P6	140

Production capacity of each Plant

Distribution	Shipment quantity (Thousand ton per annum)
China (D1)	200
Middle East (D2)	90
Europe (D3)	40
South East Asia (D4)	45
South Korea (D5)	110
Russia (D6)	70

Demand of each Customer

Plant	China	Middle East	South East Asia	Europe	South Korea	Russia
P1	200	300	100	600	250	500
P2	400	350	150	650	300	400
P3	300	250	150	600	150	350
P4	500	400	200	700	450	450
P5	100	200	300	650	200	300
P6	600	450	250	550	150	500

Shipping cost from Plants to Destinations

The goal of this case study's transportation problem is to determine the best allocation of production capacity to each demand destination in order to save shipping costs. Six polymer manufacturing sites and six demand destinations are being considered. As a result, i = 1, 2, 3, 4, 5, 6 and j = 1, 2, 3, 4, 5, 6 are equal.
Let x_{ij} denote the amount (ton per year) to be shipped from polymer manufacturing units i to demand destinations j.
Let c_{ij} be the unit cost of shipping from polymer manufacturing units i to demand destinations j (per ton).

The objective function is stated mathematically as follows:

Minimize shipping cost =
$C_{11}x_{11} + C_{12}x_{12} + C_{13}x_{13} + C_{14}x_{14} + C_{15}x_{15} + C_{16}x_{16} +$
$C_{21}x_{61} + C_{22}x_{22} + C_{23}x_{23} + C_{24}x_{24} + C_{25}x_{25} + C_{26}x_{26} +$
$C_{31}x_{61} + C_{32}x_{32} + C_{33}x_{33} + C_{34}x_{34} + C_{35}x_{35} + C_{36}x_{36} +$
$C_{41}x_{61} + C_{42}x_{42} + C_{43}x_{43} + C_{44}x_{44} + C_{45}x_{45} + C_{46}x_{46} +$
$C_{51}x_{61} + C_{52}x_{52} + C_{53}x_{53} + C_{54}x_{54} + C_{55}x_{55} + C_{56}x_{56} +$
$C_{61}x_{61} + C_{62}x_{62} + C_{63}x_{63} + C_{64}x_{64} + C_{65}x_{65} + C_{66}x_{66}$

The trading firm is constrained by two factors. To begin with, any plant's overall supply cannot exceed the plant's capacity. Second, each destination will receive the amount of polymer required to match its demand.
As a result, the following constraints are stated as Linear Programming constraints:

Supply Constraint:
P_1: $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} \leq 110$
P_2: $x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} \leq 75$
P_3: $x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} \leq 95$
P_4: $x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} \leq 125$
P_5: $x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} \leq 60$
P_6: $x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} \leq 140$

Demand Constraint:
D_1: $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 200$
D_2: $x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 90$
D_3: $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 40$
D_4: $x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} = 45$
D_5: $x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 110$
D_6: $x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 70$

And since x_{ij} must be non-negative,
$x_{ij} \geq 0$, i = 1, 2, 3, 4, 5, 6 and j = 1, 2, 3, 4, 5, 6

The following Linear Programming formulation in Standard form is obtained by combining the objective function, supply constraint, demand constraint, and sign restriction:

Minimize shipping cost =

$200x_{11} + 300x_{12} + 100x_{13} + 600x_{14} + 250x_{15} + 500x_{16} +$
$400x_{61} + 350x_{22} + 150x_{23} + 650x_{24} + 300x_{25} + 400x_{26} +$
$300x_{61} + 250x_{32} + 150x_{33} + 600x_{34} + 150x_{35} + 350x_{36} +$
$500x_{61} + 400x_{42} + 200x_{43} + 700x_{44} + 450x_{45} + 450x_{46} +$
$100x_{61} + 200x_{52} + 300x_{53} + 650x_{54} + 200x_{55} + 300x_{56} +$
$600x_{61} + 450x_{62} + 250x_{63} + 550x_{64} + 150x_{65} + 500x_{66}$

Subject to:

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + s_1 = 110$
$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + s_2 = 75$
$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + s_3 = 95$
$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + s_4 = 125$
$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} + s_5 = 60$
$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} + s_6 = 140$
$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 200$
$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 90$
$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 40$
$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} = 45$
$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 110$
$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 70$
$x_{ij} \geq 0$, i = 1, 2, 3, 4, 5, 6 and j = 1, 2, 3, 4, 5, 6

Constraints in the form of a Spreadsheet:

	Destinations								
Unit Cost	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**			**Capacity**
P1	x1	x2	x3	x4	x5	x6	0	≤	110
P2	x7	x8	x9	x10	x11	x12	0	≤	75
P3	x13	x14	x15	x16	x17	x18	0	≤	95
P4	x19	x20	x21	x22	x23	x24	0	≤	125
P5	x25	x26	x27	x28	x29	x30	0	≤	60
P6	x31	x32	x33	x34	x35	x36	0	≤	140
Total	0	0	0	0	0	0			
	=	=	=	=	=	=			**Total Cost**
Demand	200	90	40	45	110	70			**?**

Results

The above formulated LP is fed into the Two-Phase Simplex Algorithm Python code and the following results appear.

After performing Phase I:

```
END OF PHASE I
Final resulting tableau is
      cost    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15   16   17   18   19   20   21   22   23   24   25   26   27   28   29   30   31   32   33   34   35   36   37   38   39   40   41   42   43   44   45   46   47   48   49   50   51   52   53   54
cost   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0
25    60.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  1.0  1.0  1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
21    35.0 -1.0 -1.0  0.0  0.0  0.0  0.0 -1.0 -1.0  0.0  0.0  0.0  0.0 -1.0 -1.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  1.0  1.0  1.0  0.0  0.0  1.0  1.0  1.0  1.0  0.0  0.0  1.0  1.0  1.0  1.0  0.0  0.0  0.0  1.0  1.0  1.0  0.0  0.0  0.0  1.0  1.0  1.0 -1.0 -1.0  0.0  0.0  0.0  0.0
17    45.0 -1.0 -1.0 -1.0 -1.0  0.0  0.0 -1.0 -1.0 -1.0 -1.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  1.0  0.0  0.0  0.0  0.0  1.0  1.0  0.0  0.0  0.0  0.0  1.0  1.0  0.0  0.0  0.0  0.0  1.0  1.0  0.0  0.0  1.0  1.0  1.0  1.0  0.0  0.0  1.0  1.0  1.0  1.0 -1.0 -1.0 -1.0 -1.0  0.0  0.0
11    65.0  1.0  1.0  1.0  1.0  1.0  0.0  1.0  1.0  1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0 -1.0  0.0  0.0  0.0  0.0  0.0 -1.0  0.0  0.0  0.0  0.0  0.0 -1.0  0.0  0.0  0.0  0.0  0.0 -1.0  0.0  0.0 -1.0 -1.0 -1.0 -1.0  0.0  0.0 -1.0 -1.0 -1.0 -1.0  1.0  1.0  1.0  1.0  1.0  0.0
37    50.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0
12    10.0 -1.0 -1.0 -1.0 -1.0 -1.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  1.0  1.0  1.0  1.0  1.0  0.0  1.0  1.0  1.0  1.0  1.0 -1.0 -1.0 -1.0 -1.0 -1.0  0.0
15     5.0  1.0  1.0  1.0  0.0  0.0  0.0  1.0  1.0  1.0  0.0  0.0  0.0  1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0 -1.0 -1.0 -1.0  0.0  0.0  0.0 -1.0 -1.0 -1.0  0.0  0.0  0.0 -1.0 -1.0 -1.0  0.0  0.0  0.0 -1.0 -1.0 -1.0  0.0  0.0  0.0 -1.0 -1.0 -1.0  1.0  1.0  1.0  0.0  0.0  0.0
19     0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0 -1.0 -1.0 -1.0 -1.0 -1.0  0.0 -1.0 -1.0 -1.0 -1.0 -1.0  0.0  0.0  0.0  0.0 -1.0 -1.0  0.0  0.0  0.0  0.0 -1.0 -1.0  1.0  0.0  0.0  0.0  0.0  0.0
31   140.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  1.0  1.0  1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0
6     60.0  1.0  1.0  1.0  1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 -1.0 -1.0 -1.0 -1.0 -1.0  0.0 -1.0 -1.0 -1.0 -1.0 -1.0  1.0  1.0  1.0  1.0  1.0  1.0
20    90.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0
16    45.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0

The initial basis for PHASE II is:
[25, 21, 17, 11, 37, 12, 15, 19, 31, 6, 20, 16]
```

After performing Phase 2:

```
END OF PHASE II
Final resulting tableau is
           cost    1      2      3      4      5      6    7    8    9   10    11   12   13   14     15    16   17    18    19   20   21   22     23   24   25     26     27     28     29     30     31     32     33   34   35     36     37    38     39   40     41     42
cost -142500.0  0.0  150.0  150.0  150.0  200.0  300.0  0.0  0.0  0.0  0.0  50.0  0.0  0.0  0.0  100.0  50.0  0.0  50.0  50.0  0.0  0.0  0.0  150.0  0.0  0.0  150.0  450.0  300.0  250.0  200.0  300.0  200.0  200.0  0.0  0.0  200.0  250.0  50.0  150.0  0.0  350.0  150.0
25         60.0  0.0    0.0    0.0    0.0    0.0    0.0  0.0  0.0  0.0  0.0   0.0  0.0  0.0  0.0    0.0   0.0  0.0   0.0   0.0  0.0  0.0  0.0    0.0  0.0  1.0    1.0    1.0    1.0    1.0    1.0    0.0    0.0    0.0  0.0  0.0    0.0    0.0   0.0    0.0  0.0    1.0    0.0
7          30.0  0.0   -1.0   -1.0   -1.0   -1.0   -1.0  1.0  0.0  0.0  0.0   0.0  0.0  1.0  0.0    0.0   0.0  0.0   0.0   1.0  0.0  0.0  0.0    0.0  0.0  0.0   -1.0   -1.0   -1.0   -1.0   -1.0    1.0    0.0    0.0  0.0  0.0    0.0   -1.0   0.0    0.0  0.0   -1.0    0.0
17         15.0  0.0    0.0    0.0    1.0    1.0    0.0  0.0  0.0  0.0  1.0   1.0  0.0  0.0  0.0    0.0   1.0  1.0   0.0   0.0  0.0  0.0  1.0    1.0  0.0  0.0    0.0    0.0    1.0    1.0    0.0   -1.0   -1.0   -1.0  0.0  0.0   -1.0    0.0   0.0    0.0  0.0    0.0   -1.0
35         95.0  0.0    0.0    0.0   -1.0    0.0    0.0  0.0  0.0  0.0 -1.0   0.0  0.0  0.0  0.0    0.0  -1.0  0.0   0.0   0.0  0.0  0.0 -1.0    0.0  0.0  0.0    0.0    0.0   -1.0    0.0    0.0    1.0    1.0    1.0  0.0  1.0    1.0    0.0   0.0    0.0  0.0    0.0    1.0
40         50.0  0.0    0.0    0.0    0.0    0.0    0.0  0.0  0.0  0.0  0.0   0.0  0.0  0.0  0.0    0.0   0.0  0.0   0.0   0.0  0.0  0.0  0.0    0.0  0.0  0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0  0.0  0.0    0.0    1.0   1.0    1.0  1.0    1.0    1.0
8           5.0  0.0    1.0    0.0    1.0    1.0    1.0  0.0  1.0  0.0  1.0   1.0  1.0 -1.0  0.0   -1.0   0.0  0.0   0.0  -1.0  0.0 -1.0  0.0    0.0  0.0  0.0    1.0    0.0    1.0    1.0    1.0   -1.0    0.0   -1.0  0.0  0.0    0.0    1.0   1.0    0.0  0.0    1.0    0.0
1         110.0  1.0    1.0    1.0    1.0    1.0    1.0  0.0  0.0  0.0  0.0   0.0  0.0  0.0  0.0    0.0   0.0  0.0   0.0   0.0  0.0  0.0  0.0    0.0  0.0  0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0  0.0  0.0    0.0    1.0   0.0    0.0  0.0    0.0    0.0
14         80.0  0.0    0.0    0.0   -1.0   -1.0    0.0  0.0  0.0  0.0 -1.0  -1.0  0.0  1.0  1.0    1.0   0.0  0.0   1.0   0.0  0.0  0.0 -1.0   -1.0  0.0  0.0    0.0    0.0   -1.0   -1.0    0.0    1.0    1.0    1.0  0.0  0.0    1.0    0.0   0.0    1.0  0.0    0.0    1.0
34         45.0  0.0    0.0    0.0    1.0    0.0    0.0  0.0  0.0  0.0  1.0   0.0  0.0  0.0  0.0    0.0   1.0  0.0   0.0   0.0  0.0  0.0  1.0    0.0  0.0  0.0    0.0    0.0    1.0    0.0    0.0    0.0    0.0    0.0  1.0  0.0    0.0    0.0   0.0    0.0  0.0    0.0    0.0
24         70.0  0.0    0.0    0.0    0.0    0.0    1.0  0.0  0.0  0.0  0.0   0.0  1.0  0.0  0.0    0.0   0.0  0.0   1.0   0.0  0.0  0.0  0.0    0.0  1.0  0.0    0.0    0.0    0.0    0.0    1.0    0.0    0.0    0.0  0.0  0.0    1.0    0.0   0.0    0.0  0.0    0.0    0.0
9          40.0  0.0    0.0    1.0    0.0    0.0    0.0  0.0  0.0  1.0  0.0   0.0  0.0  0.0  0.0    1.0   0.0  0.0   0.0   0.0  0.0  1.0  0.0    0.0  0.0  0.0    0.0    1.0    0.0    0.0    0.0    0.0    0.0    1.0  0.0  0.0    0.0    0.0   0.0    0.0  0.0    0.0    0.0
20          5.0  0.0    0.0    0.0    0.0    0.0   -1.0  0.0  0.0  0.0  0.0   0.0 -1.0  0.0  0.0    0.0   0.0  0.0  -1.0   1.0  1.0  1.0  1.0    1.0  0.0  0.0    0.0    0.0    0.0    0.0   -1.0    0.0    0.0    0.0  0.0  0.0   -1.0   -1.0  -1.0   -1.0  0.0   -1.0   -1.0

The optimal cost is 142500.0
The BFS is : x* = [110.   0.   0.   0.   0.   0.  30.   5.  40.   0.   0.   0.   0.  80.   0.   0.  15.   0.   0.   5.   0.   0.   0.  70.  60.   0.   0.   0.   0.   0.   0.   0.   0.  45.  95.   0.   0.   0.   0.  50.   0.   0.]
```

We get an optimal cost of ***142500***.

And the values of the variables x_{ij} are given in the table below.

	Destinations								
Unit Cost	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**			**Capacity**
P1	110	0	0	0	0	0	110	≤	110
P2	30	5	40	0	0	0	75	≤	75
P3	0	80	0	0	15	0	95	≤	95
P4	0	5	0	0	0	70	75	≤	125
P5	60	0	0	0	0	0	60	≤	60
P6	0	0	0	45	95	0	140	≤	140
Total	200	90	40	45	110	70			
	=	=	=	=	=	=			**Total Cost**
Demand	200	90	40	45	110	70			**142500**

Discussion

The case study focuses on the distribution of polymer from Malaysia's four polymer manufacturing plants to four demand destinations: China, the Middle East, Southeast Asia, and Europe. The goal is to figure out which distribution strategy will result in the lowest total transportation cost to the trading business responsible with delivering the polymer. It is constrained in two ways. Each supply source has a set manufacturing capability, and each demand destination has a set polymer quantity. Furthermore, the transport unit cost from the plants to the various destinations varies. An effective solution was found using Linear Programming and a spreadsheet to satisfy the goal of lowering the cost of carrying the polymer from the facility to the market.

According to the findings, a 200,000 ton/year supply for the Chinese market should be sourced from Plant 1 (110,000 ton/year), Plant 2 (30,000 ton/year) and Plant 5 (60,000 ton/year). The 90,000 ton/year demand for the Middle East must be met by three plants: Plant 2 (5,000 ton/year), Plant 3 (80,000 ton/year), and Plant 4 (5,000 ton/year). Plant 2 (40,000 tons per year) should be used to deliver polymer to the Southeast Asian market. Plant 6 will meet the 45,000 ton/annum demand for the European market. Two plants must meet the 110,000 ton/year demand for South Korean market: Plant 3 (15,000 ton/year), and Plant 6 (95,000 ton/year). And finally, Plant 4 must meet the 70,000 ton/year demand for the Russian market.

The entire shipping cost as a result is 142,500 million. This technique reduces the overall cost of moving polymer from six polymer manufacturing plants to six demand destinations.

Conclusion

In a case study of a transportation problem faced by a Malaysian trading company, this study highlighted the application of Linear Programming. The best plan and solution for reducing total transportation costs were devised and analysed. Engineers and managers can use linear programming as an alternative decision tool to ensure that their operations are carried out effectively and efficiently at the lowest cost possible, maximizing the company's profit.

References

[1] Zulkipli Ghazali, M. Amin Abd Majid and Mohd Shazwani, 2012. Optimal Solution of Transportation Problem Using Linear Programming: A Case of a Malaysian Trading Company. Journal of Applied Sciences, 12: 2430-2435.

[2] Reeb, J. and S. Leavengood, 2002. Transportation problem: A special case for linear programming problems. Operations Research, Oregon State University.

[3] Reeb, J. and S. Leavengood, 1998. Using the simplex method to solve linear programming maximization problems. Operation Research, Oregon State University.

[4] Dimitris Bertsimas & John N. Tsitsiklis 1997. Introduction to Linear Optimization.