

CHAPTER 9

9.1 The flop counts for the tridiagonal algorithm in Fig. 9.6 can be summarized as

	Mult/Div	Add/Subtr	Total
Forward elimination	$3(n-1)$	$2(n-1)$	$5(n-1)$
Back substitution	$2n-1$	$n-1$	$3n-2$
Total	$5n-4$	$3n-3$	$8n-7$

Thus, as n increases, the effort is much, much less than for a full matrix solved with Gauss elimination which is proportional to n^3 .

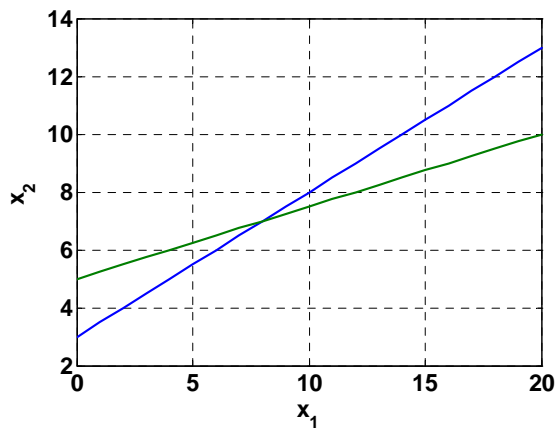
9.2 The equations can be expressed in a format that is compatible with graphing x_2 versus x_1 :

$$x_2 = \frac{18+3x_1}{6} = 3+0.5x_1$$

$$x_2 = \frac{40+2x_1}{8} = 5+0.25x_1$$

which can be plotted as

```
a11=3;a12=-6;b1=-18;
a21=-2;a22=8;b2=40;
x1=[0:20];x21=(b1-a11*x1)/a12;x22=(b2-a21*x1)/a22;
plot(x1,x21,x1,x22,'--'),grid
xlabel('x_1'),ylabel('x_2')
```



Thus, the solution is $x_1 = 8$, $x_2 = 7$. The solution can be checked by substituting it back into the equations to give

$$3(8) - 6(7) = 24 - 42 = -18$$

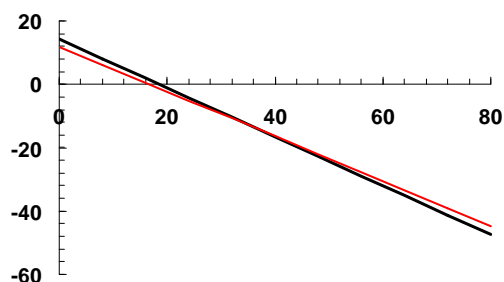
$$-2(8) + 8(7) = -16 + 56 = 40$$

9.3 (a) The equations can be rearranged into a format for plotting x_2 versus x_1 :

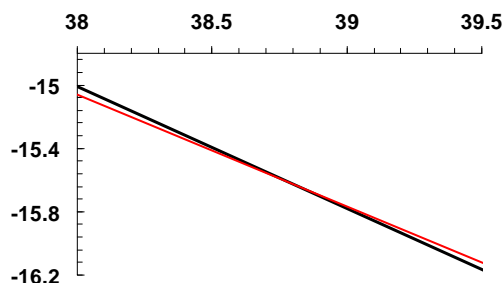
$$x_2 = 14.25 - 0.77x_1$$

$$x_2 = 11.76471 - \frac{1.2}{1.7}x_1$$

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If you zoom in, it appears that there is a root at about $(38.7, -15.6)$.



The results can be checked by substituting them back into the original equations:

$$0.77(38.7) - 15.6 = 14.2 \cong 14.25$$

$$1.2(38.7) + 1.7(-15.6) = 19.92 \cong 20$$

(b) The plot suggests that the system may be ill-conditioned because the slopes are so similar.

(c) The determinant can be computed as

$$D = 0.77(1.7) - 1(1.2) = 0.11$$

which is relatively small. Note that if the system is normalized first by dividing each equation by the largest coefficient,

$$0.77x_1 + x_2 = 14.25$$

$$0.705882x_1 + x_2 = 11.76471$$

the determinant is even smaller

$$D = 0.77(1) - 1(0.705882) = 0.064$$

9.4 (a) The determinant can be computed as:

$$A_1 = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1(0) - 1(1) = -1$$

$$A_2 = \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} = 2(0) - 1(3) = -3$$

$$A_3 = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = 2(1) - 1(3) = -1$$

$$D = 0(-1) - 2(-3) + 5(-1) = 1$$

(b) Cramer's rule

$$x_1 = \frac{\begin{vmatrix} 1 & 2 & 5 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix}}{D} = \frac{-2}{1} = -2$$

$$x_2 = \frac{\begin{vmatrix} 0 & 1 & 5 \\ 2 & 1 & 1 \\ 3 & 2 & 0 \end{vmatrix}}{D} = \frac{8}{1} = 8$$

$$x_3 = \frac{\begin{vmatrix} 0 & 2 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{vmatrix}}{D} = \frac{-3}{1} = -3$$

(c) Pivoting is necessary, so switch the first and third rows,

$$3x_1 + x_2 = 2$$

$$2x_1 + x_2 + x_3 = 1$$

$$2x_2 + 5x_3 = 1$$

Multiply pivot row 1 by $2/3$ and subtract the result from the second row to eliminate the a_{21} term. Note that because $a_{31} = 0$, it does not have to be eliminated

$$3x_1 + x_2 = 2$$

$$0.33333x_2 + x_3 = -0.33333$$

$$2x_2 + 5x_3 = 1$$

Pivoting is necessary so switch the second and third row,

$$3x_1 + x_2 = 2$$

$$2x_2 + 5x_3 = 1$$

$$0.33333x_2 + x_3 = -0.33333$$

Multiply pivot row 2 by $0.33333/2$ and subtract the result from the third row to eliminate the a_{32} term.

$$3x_1 + x_2 = 2$$

$$2x_2 + 5x_3 = 1$$

$$+0.16667x_3 = -0.5$$

The solution can then be obtained by back substitution

$$x_3 = \frac{-0.5}{0.16667} = -3$$

$$x_2 = \frac{1 - 5(-3)}{2} = 8$$

$$x_1 = \frac{2 - 0(-3) - 1(8)}{3} = -2$$

The results can be checked by substituting them back into the original equations:

$$2(8) + 5(-3) = 1$$

$$2(-2) + 8 - 3 = 1$$

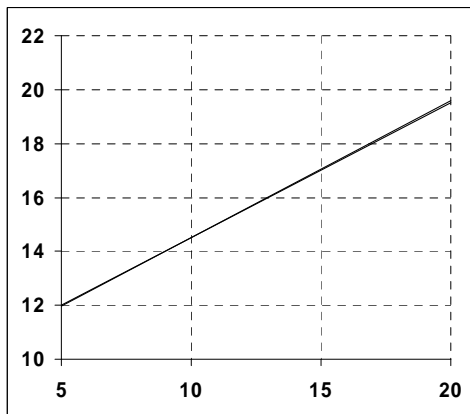
$$3(-2) + 8 = 2$$

9.5 (a) The equations can be expressed in a format that is compatible with graphing x_2 versus x_1 :

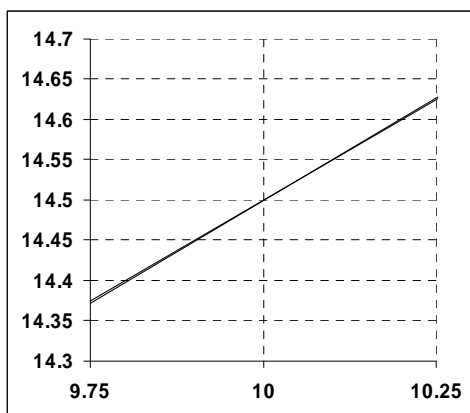
$$x_2 = 0.5x_1 + 9.5$$

$$x_2 = 0.51x_1 + 9.4$$

The resulting plot indicates that the intersection of the lines is difficult to detect:



Only when the plot is zoomed is it at all possible to discern that solution seems to lie at about $x_1 = 14.5$ and $x_2 = 10$.



(b) The determinant can be computed as

$$\begin{vmatrix} 0.5 & -1 \\ 1.02 & -2 \end{vmatrix} = 0.5(-2) - (-1)(1.02) = 0.02$$

which is close to zero.

(c) Because the lines have very similar slopes and the determinant is so small, you would expect that the system would be ill-conditioned

(d) Multiply the first equation by $1.02/0.5$ and subtract the result from the second equation to eliminate the x_1 term from the second equation,

$$\begin{aligned} 0.5x_1 - x_2 &= -9.5 \\ 0.04x_2 &= 0.58 \end{aligned}$$

The second equation can be solved for

$$x_2 = \frac{0.58}{0.04} = 14.5$$

This result can be substituted into the first equation which can be solved for

$$x_1 = \frac{-9.5 + 14.5}{0.5} = 10$$

(e) Multiply the first equation by $1.02/0.52$ and subtract the result from the second equation to eliminate the x_1 term from the second equation,

$$\begin{aligned} 0.52x_1 - x_2 &= -9.5 \\ -0.03846x_2 &= -0.16538 \end{aligned}$$

The second equation can be solved for

$$x_2 = \frac{-0.16538}{-0.03846} = 4.3$$

This result can be substituted into the first equation which can be solved for

$$x_1 = \frac{-9.5 + 4.3}{0.52} = -10$$

Interpretation: The fact that a slight change in one of the coefficients results in a radically different solution illustrates that this system is very ill-conditioned.

9.6 (a) Multiply the first equation by $-3/10$ and subtract the result from the second equation to eliminate the x_1 term from the second equation. Then, multiply the first equation by $1/10$ and subtract the result from the third equation to eliminate the x_1 term from the third equation.

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 27 \\ -4.4x_2 + 1.7x_3 &= -53.4 \\ 0.8x_2 + 6.1x_3 &= -24.2 \end{aligned}$$

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Multiply the second equation by $0.8/(-4.4)$ and subtract the result from the third equation to eliminate the x_2 term from the third equation,

$$\begin{array}{rcl} 10x_1 + 2x_2 & -x_3 & = 27 \\ -4.4x_2 & +1.7x_3 & = -53.4 \\ 6.409091x_3 & & = -33.9091 \end{array}$$

Back substitution can then be used to determine the unknowns

$$\begin{aligned} x_3 &= \frac{-33.9091}{6.409091} = -5.29078 \\ x_2 &= \frac{(-53.4 - 1.7(-5.29078))}{-4.4} = 10.0922 \\ x_1 &= \frac{(27 - 5.29078 - 2(10.0922))}{10} = 0.152482 \end{aligned}$$

(b) Check:

$$\begin{aligned} 10(0.152482) + 2(10.0922) - (-5.29078) &= 27 \\ -3(0.152482) - 5(10.0922) + 2(-5.29078) &= -61.5 \\ 0.152482 + 10.0922 + 5(-5.29078) &= -21.5 \end{aligned}$$

9.7 (a) Pivoting is necessary, so switch the first and third rows,

$$\begin{array}{rcl} -8x_1 + x_2 - 2x_3 & = & -20 \\ -3x_1 - x_2 + 7x_3 & = & -34 \\ 2x_1 - 6x_2 - x_3 & = & -38 \end{array}$$

Multiply the first equation by $-3/(-8)$ and subtract the result from the second equation to eliminate the a_{21} term from the second equation. Then, multiply the first equation by $2/(-8)$ and subtract the result from the third equation to eliminate the a_{31} term from the third equation.

$$\begin{array}{rcl} -8x_1 & +x_2 & -2x_3 = -20 \\ -1.375x_2 + 7.75x_3 & = & -26.5 \\ -5.75x_2 - 1.5x_3 & = & -43 \end{array}$$

Pivoting is necessary so switch the second and third row,

$$\begin{array}{rcl} -8x_1 & +x_2 & -2x_3 = -20 \\ -5.75x_2 - 1.5x_3 & = & -43 \\ -1.375x_2 + 7.75x_3 & = & -26.5 \end{array}$$

Multiply pivot row 2 by $-1.375/(-5.75)$ and subtract the result from the third row to eliminate the a_{32} term.

$$\begin{array}{rcl} -8x_1 & +x_2 & -2x_3 = -20 \\ -5.75x_2 & & -1.5x_3 = -43 \\ 8.108696x_3 & = & -16.21739 \end{array}$$

At this point, the determinant can be computed as

$$D = -8 \times -5.75 \times 8.108696 \times (-1)^2 = 373$$

The solution can then be obtained by back substitution

$$\begin{aligned} x_3 &= \frac{-16.21739}{8.108696} = -2 \\ x_2 &= \frac{-43 + 1.5(-2)}{-5.75} = 8 \\ x_1 &= \frac{-20 + 2(-2) - 1(8)}{-8} = 4 \end{aligned}$$

(b) Check:

$$\begin{aligned} 2(4) - 6(8) - (-2) &= -38 \\ -3(4) - (8) + 7(-2) &= -34 \\ -8(4) + (8) - 2(-2) &= -20 \end{aligned}$$

9.8 Multiply the first equation by $-0.4/0.8$ and subtract the result from the second equation to eliminate the x_1 term from the second equation.

$$\begin{bmatrix} 0.8 & -0.4 & \\ & 0.6 & -0.4 \\ & -0.4 & 0.8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 41 \\ 45.5 \\ 105 \end{bmatrix}$$

Multiply pivot row 2 by $-0.4/0.6$ and subtract the result from the third row to eliminate the x_2 term.

$$\begin{bmatrix} 0.8 & -0.4 & \\ & 0.6 & -0.4 \\ & & 0.533333 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 41 \\ 45.5 \\ 135.3333 \end{bmatrix}$$

The solution can then be obtained by back substitution

$$\begin{aligned} x_3 &= \frac{135.3333}{0.533333} = 253.75 \\ x_2 &= \frac{45.5 - (-0.4)253.75}{0.6} = 245 \\ x_1 &= \frac{41 - (-0.4)245}{0.8} = 173.75 \end{aligned}$$

(b) Check:

$$\begin{aligned} 0.8(173.75) - 0.4(245) &= 41 \\ -0.4(173.75) + 0.8(245) - 0.4(253.75) &= 25 \\ -0.4(245) + 0.8(253.75) &= 105 \end{aligned}$$

9.9 Mass balances can be written for each of the reactors as

$$200 - Q_{13}c_1 - Q_{12}c_1 + Q_{21}c_2 = 0$$

$$Q_{12}c_1 - Q_{21}c_2 - Q_{23}c_2 = 0$$

$$500 + Q_{13}c_1 + Q_{23}c_2 - Q_{33}c_3 = 0$$

Values for the flows can be substituted and the system of equations can be written in matrix form as

$$\begin{bmatrix} 130 & -30 & 0 \\ -90 & 90 & 0 \\ -40 & -60 & 120 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 200 \\ 0 \\ 500 \end{bmatrix}$$

The solution can then be developed using MATLAB,

```
>> A=[130 -30 0;-90 90 0;-40 -60 120];
>> B=[200;0;500];
>> C=A\B
```

```
C =
    2.0000
    2.0000
    5.8333
```

9.10 Let x_i = the volume taken from pit i . Therefore, the following system of equations must hold

$$0.52x_1 + 0.20x_2 + 0.25x_3 = 4800$$

$$0.30x_1 + 0.50x_2 + 0.20x_3 = 5800$$

$$0.18x_1 + 0.30x_2 + 0.55x_3 = 5700$$

MATLAB can be used to solve this system of equations for

```
>> A=[0.55 0.2 0.25;0.3 0.5 0.2;0.18 0.3 0.55];
>> b=[4800;5800;5700];
>> x=A\b
```

```
x =
    1.0e+003 *
    3.7263
    7.2991
    5.1628
```

Therefore, we take $x_1 = 3726.3$, $x_2 = 7299.1$, and $x_3 = 5162.8 \text{ m}^3$ from pits 1, 2 and 3 respectively.

9.11 Let c_i = component i . Therefore, the following system of equations must hold

$$15c_1 + 17c_2 + 19c_3 = 2120$$

$$0.25c_1 + 0.33c_2 + 0.42c_3 = 43.4$$

$$1.0c_1 + 1.2c_2 + 1.6c_3 = 164$$

The solution can be developed with MATLAB:

```
A=[15 17 19;0.25 0.33 0.42;1 1.2 1.6];
b=[2120;43.4;164];
```

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$$c = A \setminus b$$

$$c = \begin{bmatrix} 20.0000 \\ 40.0000 \\ 60.0000 \end{bmatrix}$$

Therefore, $c_1 = 20$, $c_2 = 40$, and $c_3 = 60$.

9.12 Centered differences (recall Chap. 4) can be substituted for the derivatives to give

$$0 = D \frac{c_{i-1} - 2c_i + c_{i+1}}{\Delta x^2} - U \frac{c_{i+1} - c_{i-1}}{2\Delta x} - kc_i$$

collecting terms yields

$$-(D + 0.5U\Delta x)c_{i-1} + (2D + k\Delta x^2)c_i - (D - 0.5U\Delta x)c_{i+1} = 0$$

Assuming $\Delta x = 1$ and substituting the parameters gives

$$-2.5c_{i-1} + 4.2c_i - 1.5c_{i+1} = 0$$

For the first interior node ($i = 1$),

$$4.2c_1 - 1.5c_2 = 200$$

For the last interior node ($i = 9$)

$$-2.5c_8 + 4.2c_9 = 15$$

These and the equations for the other interior nodes can be assembled in matrix form as

$$\begin{bmatrix} 4.2 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.5 & 4.2 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.5 & 4.2 & -1.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.5 & 4.2 & -1.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2.5 & 4.2 & -1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.5 & 4.2 & -1.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2.5 & 4.2 & -1.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2.5 & 4.2 & -1.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2.5 & 4.2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{bmatrix} = \begin{bmatrix} 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 15 \end{bmatrix}$$

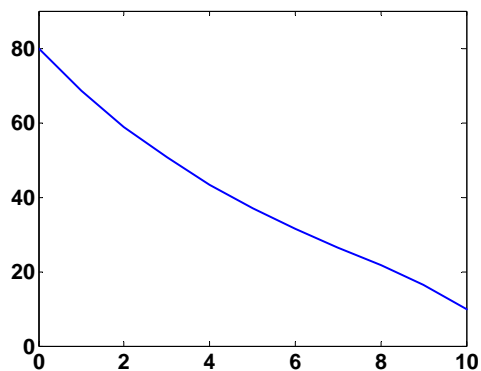
The following script generates and plots the solution:

```
A=[4.2 -1.5 0 0 0 0 0 0 0
    -2.5 4.2 -1.5 0 0 0 0 0 0
    0 -2.5 4.2 -1.5 0 0 0 0 0
    0 0 -2.5 4.2 -1.5 0 0 0 0
    0 0 0 -2.5 4.2 -1.5 0 0 0
    0 0 0 0 -2.5 4.2 -1.5 0 0
    0 0 0 0 0 -2.5 4.2 -1.5 0
    0 0 0 0 0 0 -2.5 4.2 -1.5]
```

```

0 0 0 0 0 0 0 -2.5 4.2];
b=[200 0 0 0 0 0 0 0 0 15]';
c=A\b;
c=[80 c' 10];
x=0:1:10;
plot(x,c)

```



9.13 For the first stage, the mass balance can be written as

$$F_1 y_{\text{in}} + F_2 x_2 = F_2 x_1 + F_1 x_1$$

Substituting $x = Ky$ and rearranging gives

$$-\left(1 + \frac{F_2}{F_1} K\right) y_1 + \frac{F_2}{F_1} K y_2 = -y_{\text{in}}$$

Using a similar approach, the equation for the last stage is

$$y_4 - \left(1 + \frac{F_2}{F_1} K\right) y_5 = -\frac{F_2}{F_1} x_{\text{in}}$$

For interior stages,

$$y_{i-1} - \left(1 + \frac{F_2}{F_1} K\right) y_i + \frac{F_2}{F_1} K y_{i+1} = 0$$

These equations can be used to develop the following system,

$$\begin{bmatrix} 11 & -10 & 0 & 0 & 0 \\ -1 & 11 & -10 & 0 & 0 \\ 0 & -1 & 11 & -10 & 0 \\ 0 & 0 & -1 & 11 & -10 \\ 0 & 0 & 0 & -1 & 11 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The solution can be developed in a number of ways. For example, using MATLAB,

```

>> format short g
>> A=[11 -10 0 0 0;-1 11 -10 0 0;0 -1 11 -10 0;0 0 -1 11 -10;0 0 0 -1 11];
>> B=[0.1;0;0;0;0];

```

```
>> Y=A\B
Y =
    0.00999999
    0.00099999
    9.99e-005
    9.9e-006
    9e-007
```

Note that the corresponding values of X can be computed as

```
>> X=5*Y
X =
    0.05
    0.00499995
    0.00049995
    4.95e-005
    4.5e-006
```

Therefore, $y_{\text{out}} = 0.0000009$ and $x_{\text{out}} = 0.05$.

9.14 Assuming a unit flow for Q_1 , the simultaneous equations can be written in matrix form as

$$\begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & -2 & 3 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \begin{Bmatrix} Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$

These equations can then be solved with MATLAB,

```
>> A=[-2 1 2 0 0 0;
0 0 -2 1 2 0;
0 0 0 0 -2 3;
1 1 0 0 0 0;
0 1 -1 -1 0 0;
0 0 0 1 -1 -1];
>> B=[0 0 0 1 0 0]';
>> Q=A\B
```

```
Q =
    0.5059
    0.4941
    0.2588
    0.2353
    0.1412
    0.0941
```

9.15 The solution can be generated with MATLAB,

```
>> A=[1 0 0 0 0 0 0 0 1 0;
0 0 1 0 0 0 0 1 0 0;
0 1 0 3/5 0 0 0 0 0 0;
-1 0 0 -4/5 0 0 0 0 0 0;
0 -1 0 0 0 0 3/5 0 0 0;
0 0 0 0 -1 0 -4/5 0 0 0;
0 0 -1 -3/5 0 1 0 0 0 0;
0 0 0 4/5 1 0 0 0 0 0;
```

```

0 0 0 0 0 -1 -3/5 0 0 0;
0 0 0 0 0 0 4/5 0 0 1];
>> B=[0 0 -74 0 0 24 0 0 0 0]';
>> x=A\B

```

```

x =
    37.3333
   -46.0000
    74.0000
   -46.6667
    37.3333
    46.0000
   -76.6667
   -74.0000
   -37.3333
    61.3333

```

Therefore, in kN

$AB = 37.3333$	$BC = -46$	$AD = 74$	$BD = -46.6667$	$CD = 37.3333$
$DE = 46$	$CE = -76.6667$	$A_x = -74$	$A_y = -37.3333$	$E_y = 61.3333$

9.16

```

function x=pentadol(A,b)
% pentadol: pentadiagonal system solver banded system
%   x=pentadol(A,b):
%       Solve a pentadiagonal system Ax=b
% input:
%   A = pentadiagonal matrix
%   b = right hand side vector
% output:
%   x = solution vector

% Error checks
[m,n]=size(A);
if m~=n,error('Matrix must be square');end
if length(b)~=m,error('Matrix and vector must have the same number of
rows');end
x=zeros(n,1);

% Extract bands
d=[0;0;diag(A,-2)];
e=[0;diag(A,-1)];
f=diag(A);
g=diag(A,1);
h=diag(A,2);
delta=zeros(n,1);
epsilon=zeros(n-1,1);
gamma=zeros(n-2,1);
alpha=zeros(n,1);
c=zeros(n,1);
z=zeros(n,1);

% Decomposition
delta(1)=f(1);
epsilon(1)=g(1)/delta(1);
gamma(1)=h(1)/delta(1);
alpha(2)=e(2);
delta(2)=f(2)-alpha(2)*epsilon(1);
epsilon(2)=(g(2)-alpha(2)*gamma(1))/delta(2);

```

```

gamma(2)=h(2)/delta(2);
for k=3:n-2
    alpha(k)=e(k)-d(k)*epsilon(k-2);
    delta(k)=f(k)-d(k)*gamma(k-2)-alpha(k)*epsilon(k-1);
    epsilon(k)=(g(k)-alpha(k)*gamma(k-1))/delta(k);
    gamma(k)=h(k)/delta(k);
end
alpha(n-1)=e(n-1)-d(n-1)*epsilon(n-3);
delta(n-1)=f(n-1)-d(n-1)*gamma(n-3)-alpha(n-1)*epsilon(n-2);
epsilon(n-1)=(g(n-1)-alpha(n-1)*gamma(n-2))/delta(n-1);
alpha(n)=e(n)-d(n)*epsilon(n-2);
delta(n)=f(n)-d(n)*gamma(n-2)-alpha(n)*epsilon(n-1);
% Forward substitution
c(1)=b(1)/delta(1);
c(2)=(b(2)-alpha(2)*c(1))/delta(2);
for k=3:n
    c(k)=(b(k)-d(k)*c(k-2)-alpha(k)*c(k-1))/delta(k);
end
% Back substitution
x(n)=c(n);
x(n-1)=c(n-1)-epsilon(n-1)*x(n);
for k=n-2:-1:1
    x(k)=c(k)-epsilon(k)*x(k+1)-gamma(k)*x(k+2);
end

```

Test of function:

```

>> A=[8 -2 -1 0 0
-2 9 -4 -1 0
-1 3 7 -1 -2
0 -4 -2 12 -5
0 0 -7 -3 15];
>> b=[5 2 1 1 5]';
>> x=pentasol(A,b)

```

```

x =
    0.7993
    0.5721
    0.2503
    0.5491
    0.5599

```

9.17 Here is the M-file function based on Fig. 9.5 to implement Gauss elimination with partial pivoting

```

function [x, D] = GaussPivotNew(A, b, tol)
% GaussPivotNew: Gauss elimination pivoting
% [x, D] = GaussPivotNew(A,b,tol): Gauss elimination with pivoting.
% input:
%   A = coefficient matrix
%   b = right hand side vector
%   tol = tolerance for detecting "near zero"
% output:
%   x = solution vector
%   D = determinant

[m,n]=size(A);
if m~=n, error('Matrix A must be square'); end
nb=n+1;
Aug=[A b];
npiv=0;
% forward elimination

```

```

for k = 1:n-1
    % partial pivoting
    [big,i]=max(abs(Aug(k:n,k)));
    ipr=i+k-1;
    if ipr~=k
        npiv=npiv+1;
        Aug([k,ipr],:)=Aug([ipr,k],:);
    end
    absakk=abs(Aug(k,k));
    if abs(Aug(k,k))<=tol
        D=0;
        error('Singular or near singular system')
    end
    for i = k+1:n
        factor=Aug(i,k)/Aug(k,k);
        Aug(i,k:nb)=Aug(i,k:nb)-factor*Aug(k,k:nb);
    end
end
for i = 1:n
    if abs(Aug(i,i))<=tol
        D=0;
        error('Singular or near singular system')
    end
end
% back substitution
x=zeros(n,1);
x(n)=Aug(n,nb)/Aug(n,n);
for i = n-1:-1:1
    x(i)=(Aug(i,nb)-Aug(i,i+1:n)*x(i+1:n))/Aug(i,i);
end
D=(-1)^npiv;
for i=1:n
    D=D*Aug(i,i);
end

```

Here is a script to solve Prob. 9.5 for the two cases of tol:

```

clear; clc; format short g
A=[0.5 -1;1.02 -2];
b=[-9.5;-18.8];
disp('Solution and determinant calculated with GaussPivotNew:')
[x, D] = GaussPivotNew(A,b,1e-5)
disp('Determinant calculated with det:')
D=det(A)

```

The resulting output is

Solution and determinant calculated with GaussPivotNew:

```

x =
    10
   14.5
D =
    0.02

```

Determinant calculated with det:

```

D =
    0.02

```