

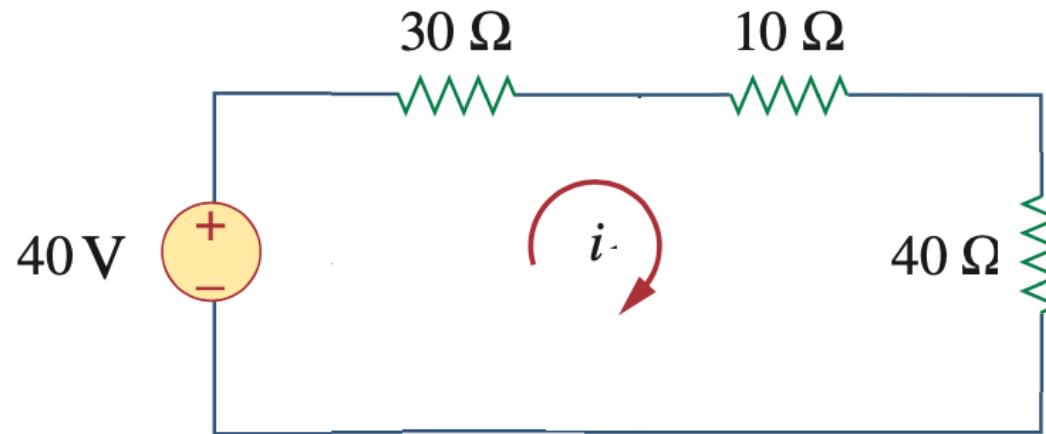
# Lecture 21

## Phasors – 7 of 9

AC power

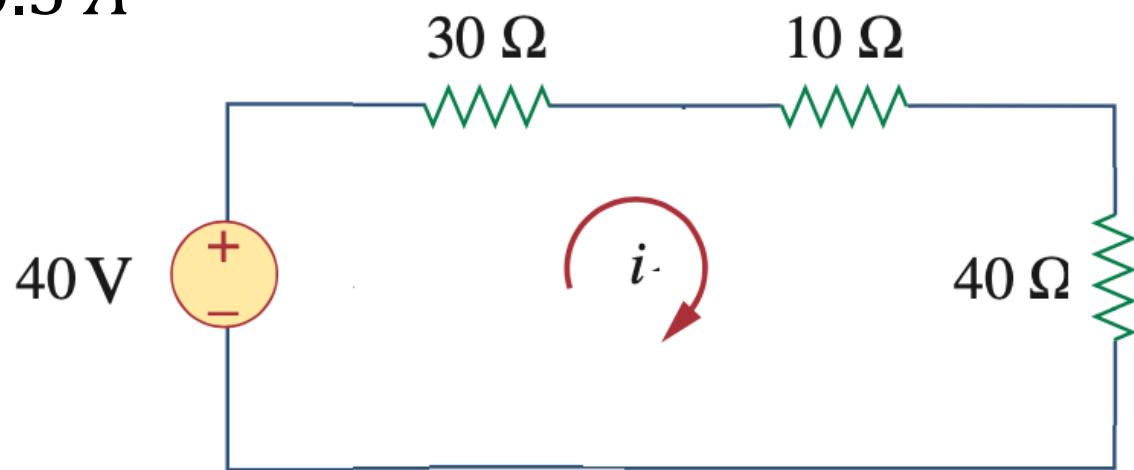
# Power – DC Resistive Circuits

- General expression:  $P = v i$



- Solve for clockwise current:  $i = \frac{40V}{80\Omega} = 0.5A$

- With  $i = \frac{40 \text{ V}}{80 \Omega} = 0.5 \text{ A}$



- Using  $v = R i$  and  $P = v i$

$$v_{30} = 15 \text{ V}$$

$$v_{10} = 5 \text{ V}$$

$$v_{40} = 20 \text{ V}$$

$$i_{source} = -0.5 \text{ A}$$

$$P_{30} = 7.5 \text{ W}$$

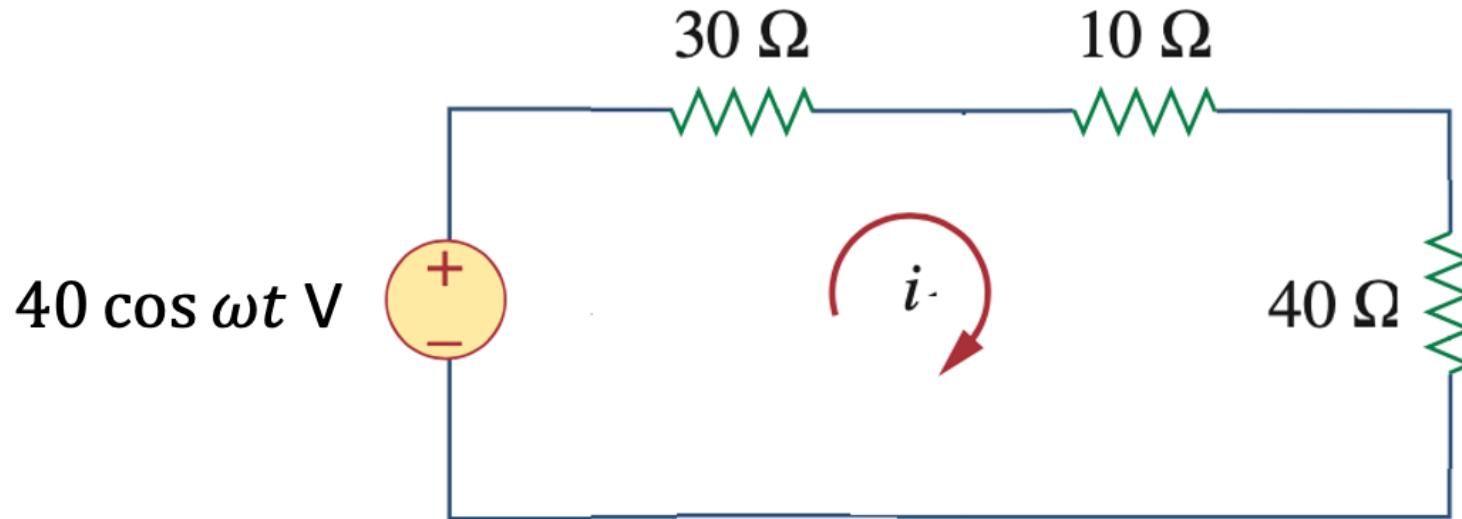
$$\mathbf{P_{10} = 2.5 \text{ W}}$$

$$P_{40} = 10 \text{ W}$$

$$P_{source} = -20 \text{ W}$$

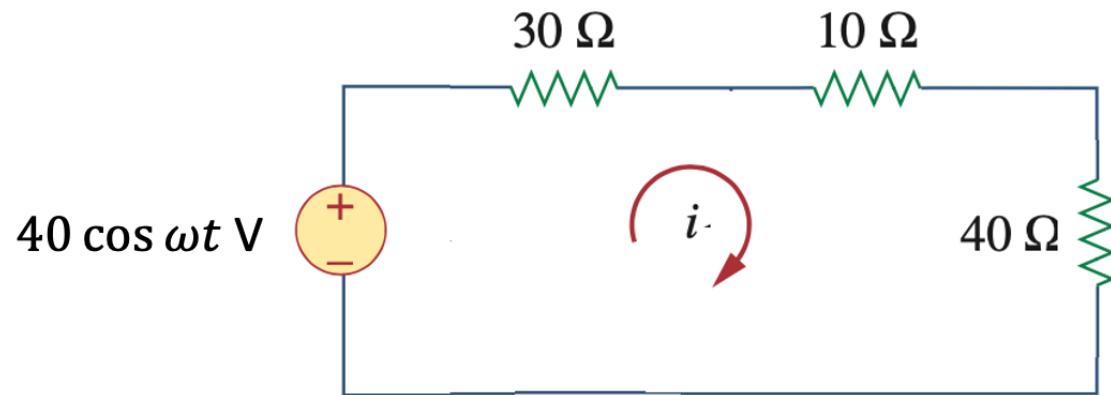
Power  
sums to  
zero

# Sinusoidal Source



- Using phasors, clockwise current phasor is  $\mathbf{I} = \frac{40}{80} = 0.5$  so  $i(t) = 0.5 \cos \omega t$  A

- With  $i(t) = 0.5 \cos \omega t$



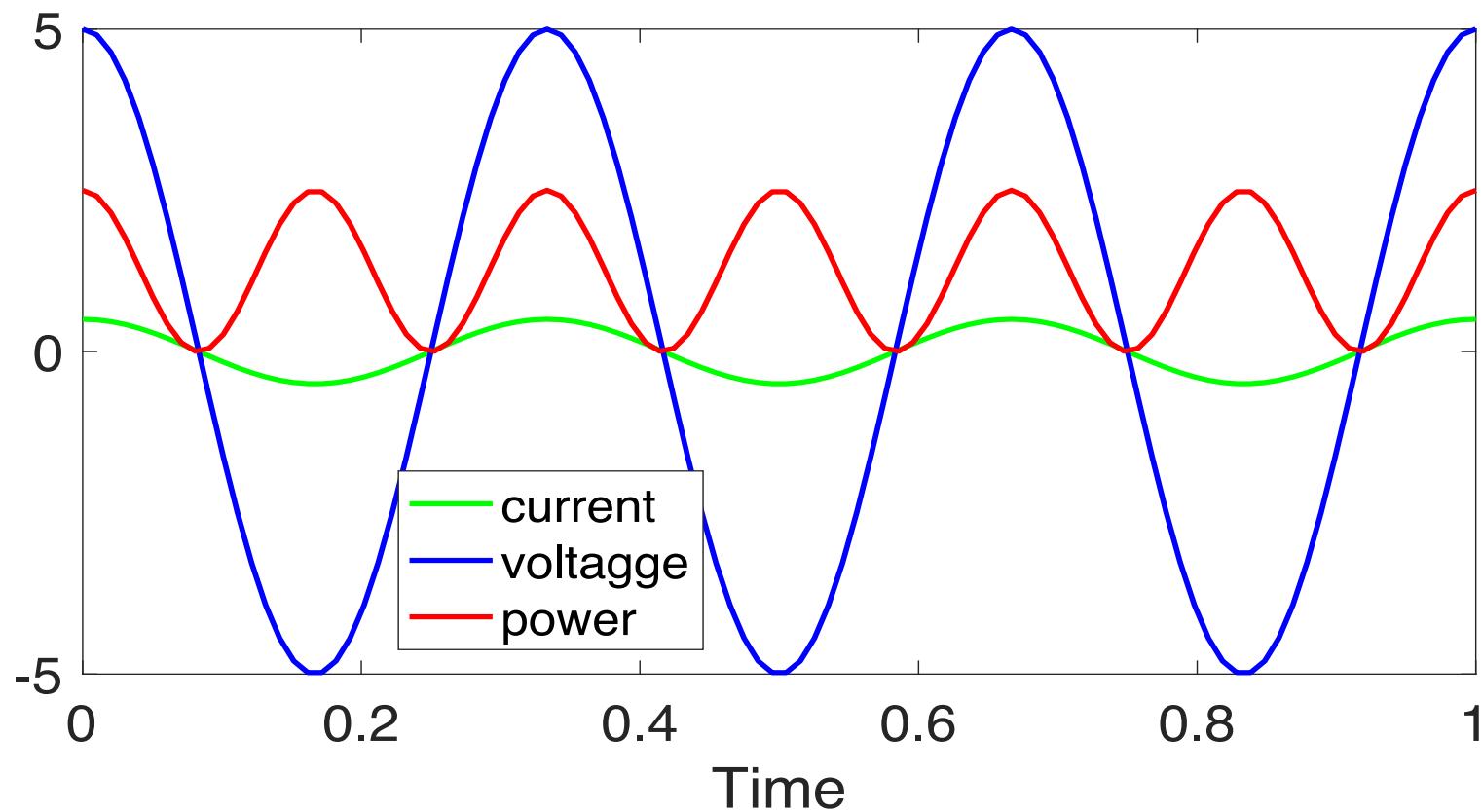
- Consider the  $10 \Omega$  resistor:

$$i_{10}(t) = 0.5 \cos \omega t \text{ A}$$

$$v_{10}(t) = 10 i_{10}(t) = 5 \cos \omega t \text{ V}$$

$$\begin{aligned}
 p_{10}(t) &= v_{10}(t) i_{10}(t) = 2.5 \cos^2 \omega t \text{ W} \\
 &= 1.25 + 1.25 \cos 2\omega t \text{ W}
 \end{aligned}$$

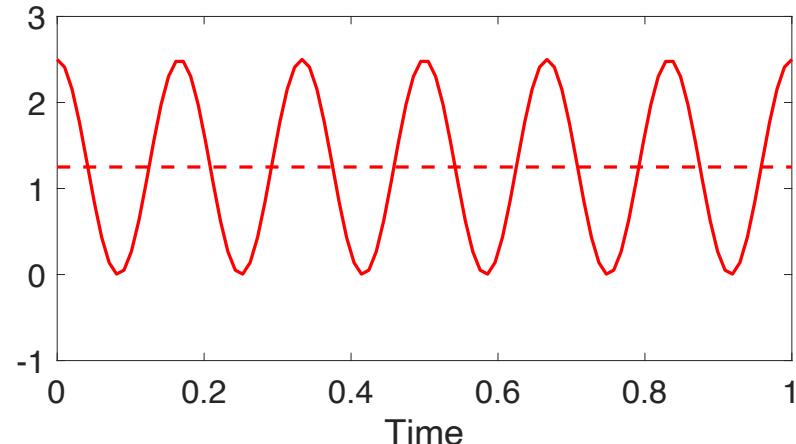
- Graphically:



- To simplify, keep just the average value
  - Since periodic, average over 1 period

$$P = \frac{1}{T} \int_t^{t+T} v(t)i(t)dt$$

- For our example



$$P_{4\Omega} = \frac{1}{T} \int_0^T (1.25 + 1.25 \cos 2\omega t)dt = \mathbf{1.25 \text{ W}}$$

5 volts, 0.5 amps, but not 2.5 watts

- In general, for AC and resistors  $V = I R$

$$v(t) = V \cos \omega t \quad i(t) = I \cos \omega t$$

$$P(t) = v(t) i(t) = V I \cos^2 \omega t$$

$$= \frac{VI}{2} + \frac{VI}{2} \cos 2\omega t = \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} + \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} \cos 2\omega t$$

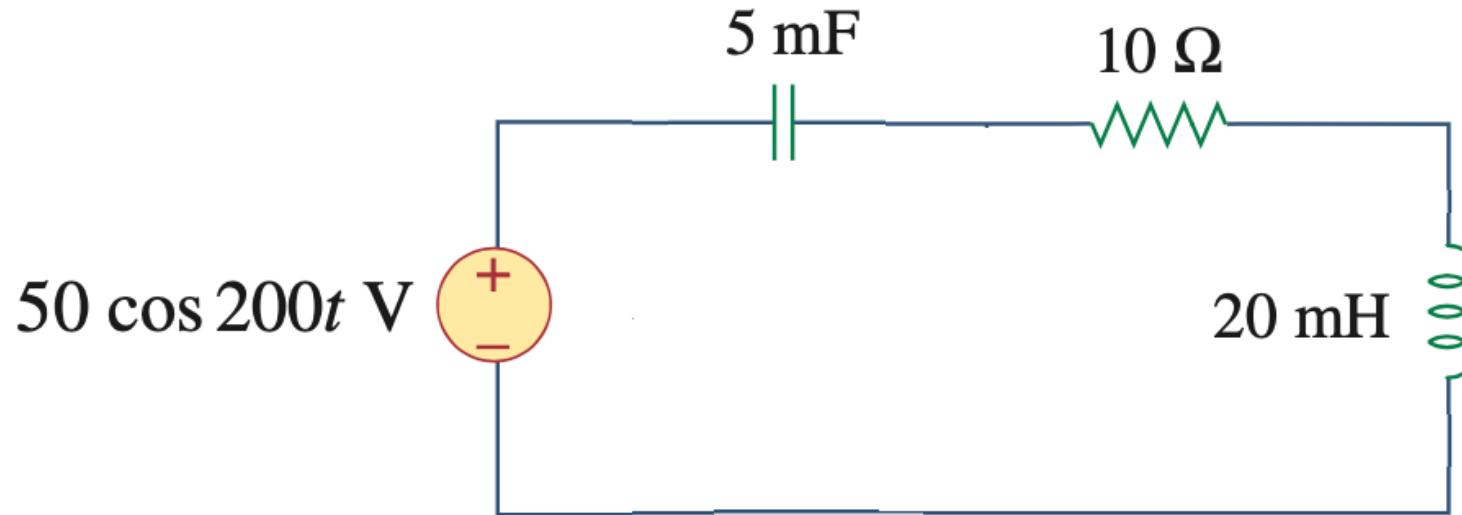
$$P_{average} = \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} = V_{RMS} I_{RMS}$$



What the DMM reads!

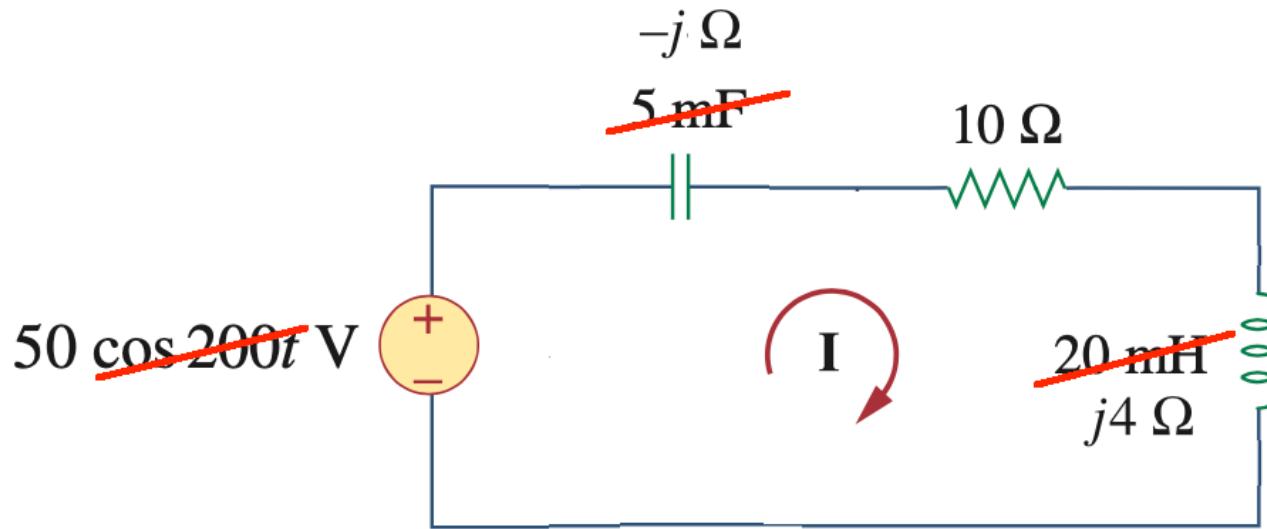
# What about an RLC Load?

- Consider



- Solve for the current and voltages using phasors

- Convert to phasors and impedances

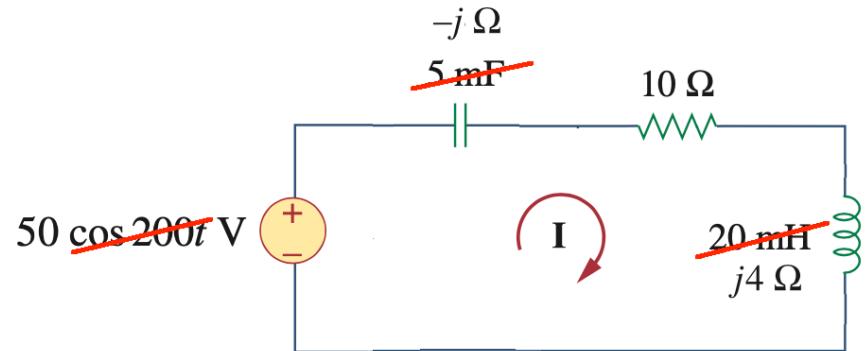


- The clockwise current is

$$I = \frac{50}{-j1+10+j4} = \frac{50}{10+j3} = 4.59 - j1.38$$

$$i(t) = 4.79 \cos(200t - 16.7^\circ) \text{ A}$$

- Resistor:



$$\mathbf{I} = 4.59 - j1.38$$

$$i(t) = 4.79 \cos(200t - 16.7^\circ) \text{ A}$$

$$\mathbf{V} = 10 \quad \mathbf{I} = 45.9 - j13.8$$

$$v_R(t) = 47.9 \cos(200t - 16.7^\circ) \text{ V}$$

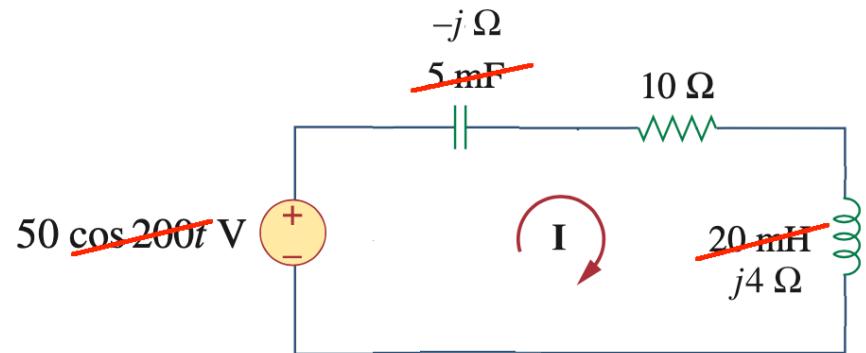
$$p(t) = v(t) i(t)$$

$$= 229 \cos^2(200t - 16.7^\circ) \text{ W}$$

$$= 115 + 115 \cos(400t - 33.4^\circ) \text{ W}$$

So  $P_{average} = 115 \text{ W}$

- Inductor:



$$\mathbf{I} = 4.59 - j1.38$$

$$i(t) = 4.79 \cos(200t - 16.7^\circ) \text{ A}$$

$$\mathbf{V} = j4 \quad \mathbf{I} = 5.50 + j18.3$$

$$v_L(t) = 19.2 \cos(200t + 73.3^\circ) \text{ V}$$

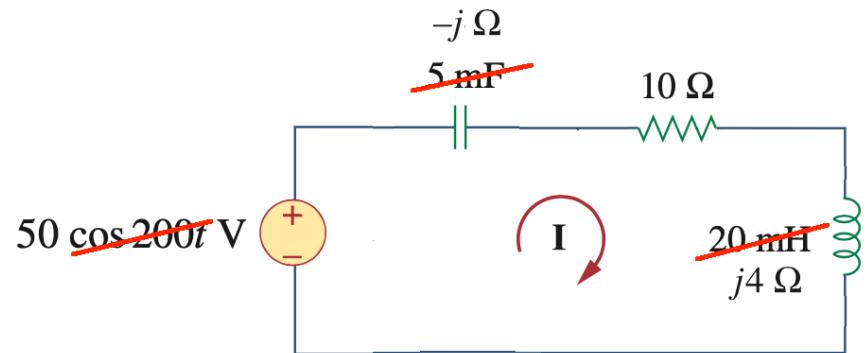
$$p(t) = v(t) i(t)$$

$$= 92.0 \cos(200t - 16.7^\circ) \cos(200t + 73.3^\circ) \text{ W}$$

$$= 46.0 \cos(400t + 56.6^\circ) \text{ W}$$

So  $P_{average} = 0$

- Capacitor:



$$\mathbf{I} = 4.59 - j1.38$$

$$i(t) = 4.79 \cos(200t - 16.7^\circ) \text{ A}$$

$$\mathbf{V} = -j1 \mathbf{I} = -1.38 - j4.59$$

$$v_C(t) = 4.79 \cos(200t - 107^\circ) \text{ V}$$

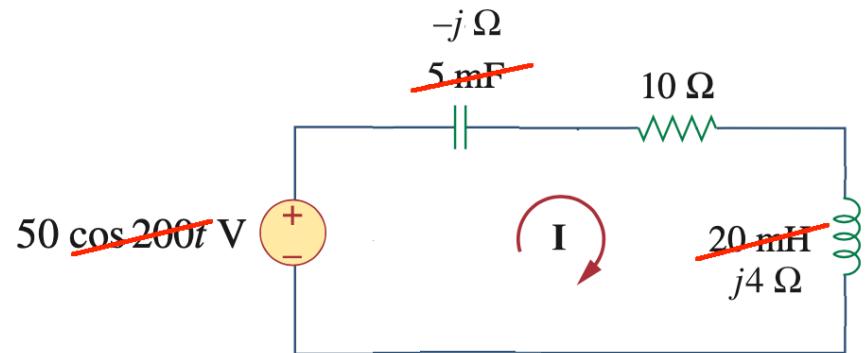
$$p(t) = v(t) i(t)$$

$$= 22.9 \cos(200t - 16.7^\circ) \cos(200t - 107^\circ) \text{ W}$$

$$= 11.5 \cos(400t - 124^\circ) \text{ W}$$

So  $P_{average} = 0$

- The source:



$$\mathbf{I} = 4.59 - j1.38$$

$$i(t) = 4.79 \cos(200t - 16.7^\circ) \text{ A}$$

$$\mathbf{V} = -50$$

$$V_s(t) = 50 \cos 200t \text{ V}$$

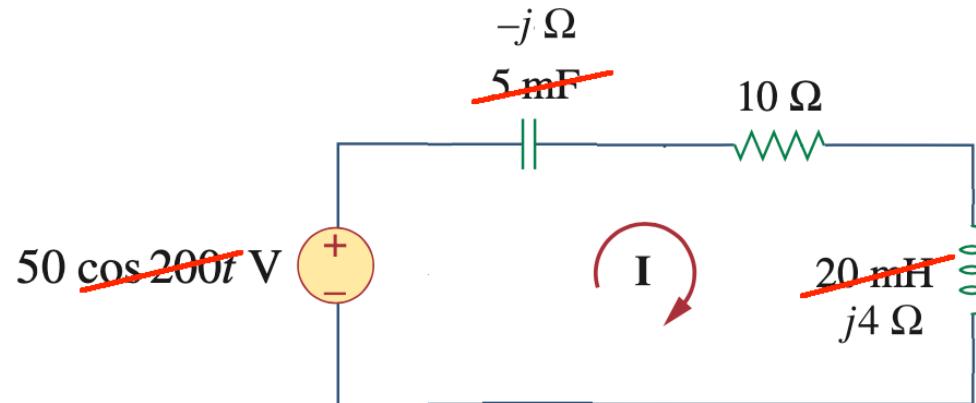
$$p(t) = V_s(t) i(t)$$

$$= -239 \cos 200t \cos(200t - 16.7^\circ) \text{ W}$$

$$= -115 - 120 \cos(400t - 16.7^\circ) \text{ W}$$

So  $P_{average} = -115 \text{ W}$

- Summary:



Real power

$P = 115$  Watts



$$P_R(t) = 115 + 115 \cos(400t - 33.4^\circ) \text{ W}$$

$$P_L(t) = 46.0 \cos(400t + 56.6^\circ) \text{ W}$$

$$P_C(t) = 11.5 \cos(400t - 124^\circ) \text{ W}$$

$$P_S(t) = -115 - 120 \cos(400t - 16.7^\circ) \text{ W}$$

Reactive power  
 $Q = 34.5$  VARs



Time varying portions sum to zero

Complex Power  
 $S = P + jQ$

# Complex Power

- Can get these quantities directly from phasors

$$S = P + jQ = \frac{\mathbf{V} \mathbf{I}^*}{2} = \begin{cases} \frac{(\mathbf{Z} \mathbf{I}) \mathbf{I}^*}{2} = \frac{\mathbf{Z} \mathbf{I} \mathbf{I}^*}{2} = \frac{|\mathbf{I}|^2}{2} \mathbf{Z} \\ \frac{\mathbf{V} (\mathbf{V}/\mathbf{Z})^*}{2} = \frac{\mathbf{V} \mathbf{V}^*}{2 \mathbf{Z}^*} = \frac{|\mathbf{V}|^2}{2 \mathbf{Z}^*} \end{cases}$$

- Our example

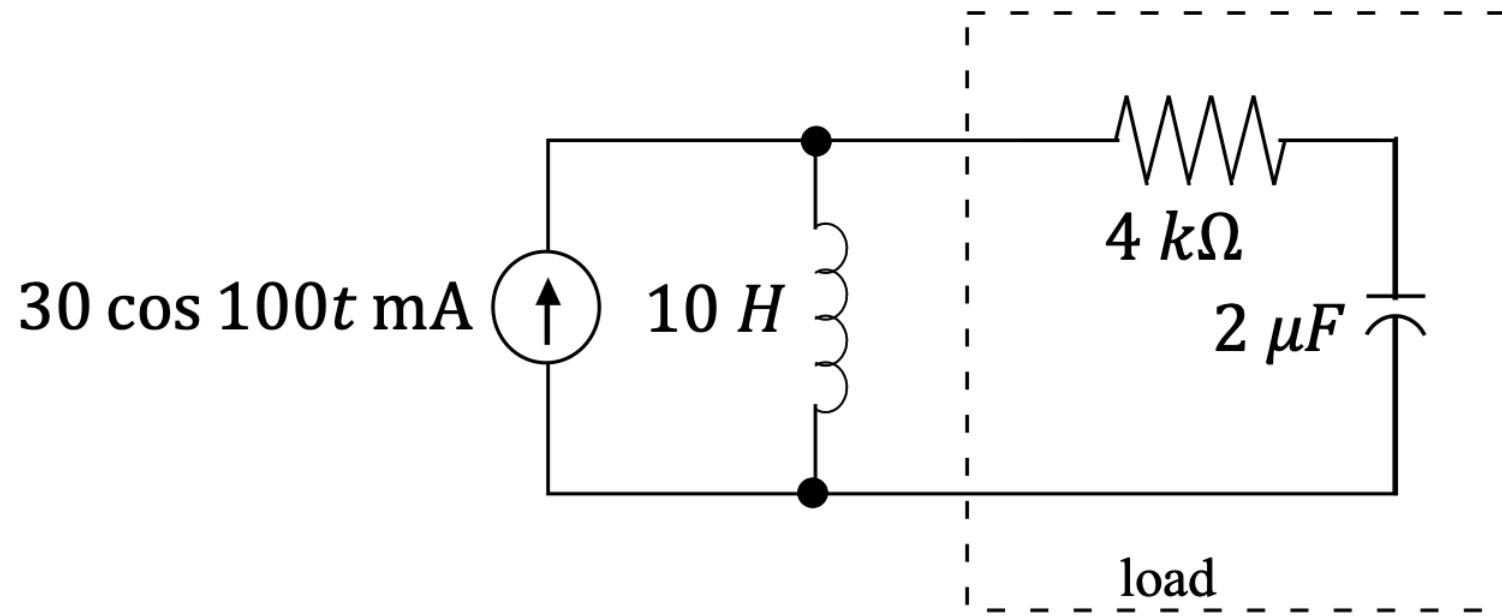
$$S = \frac{50 (4.59 - j1.38)^*}{2} = 115 + j34.5$$

- Consider a load  $\mathbf{Z} = |Z|e^{j\theta}$  and that we have the phasor relationship  $\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}$ 
  - The angle between the current and voltage phasors (vectors) is  $\theta$
- Further, recall the vector dot product so that the average power is

$$\begin{aligned}
 P &= \frac{\mathbf{V} \mathbf{I}^*}{2} = \frac{|\mathbf{V}| |\mathbf{I}| \cos \theta}{2} = \frac{|\mathbf{V}|}{\sqrt{2}} \frac{|\mathbf{I}|}{\sqrt{2}} \cos \theta \\
 &= V_{RMS} I_{RMS} \cos \theta
 \end{aligned}$$

 “power factor”

**Example problem:** Find the real and reactive powers absorbed by the load in this circuit.



*56.25 mW, 56.25 mVARs*