

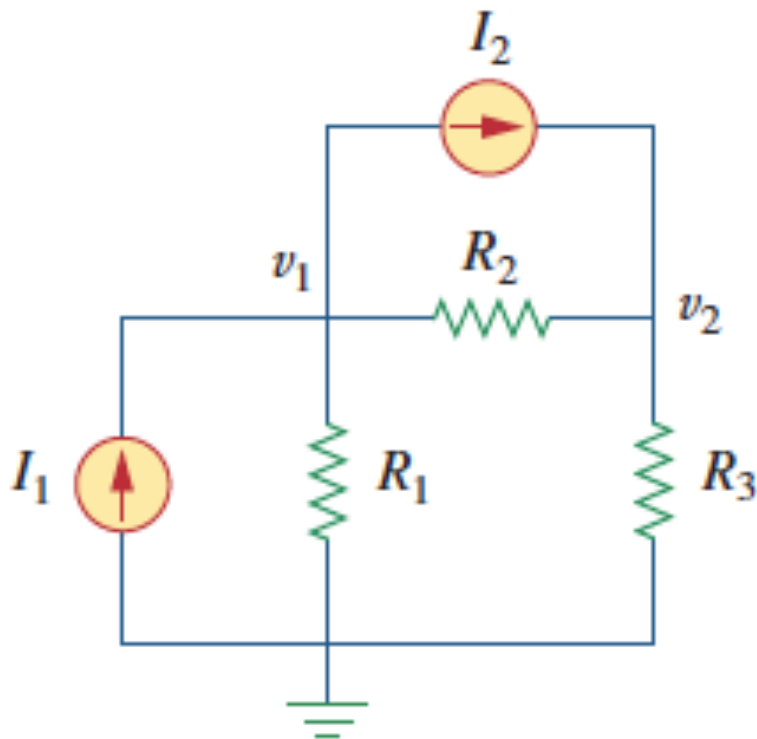
Lecture 10

Node Analysis – 3 of 7

vector form

Matrix-Vector Form

- Reconsider the initial simple circuit:
- The node equations were:



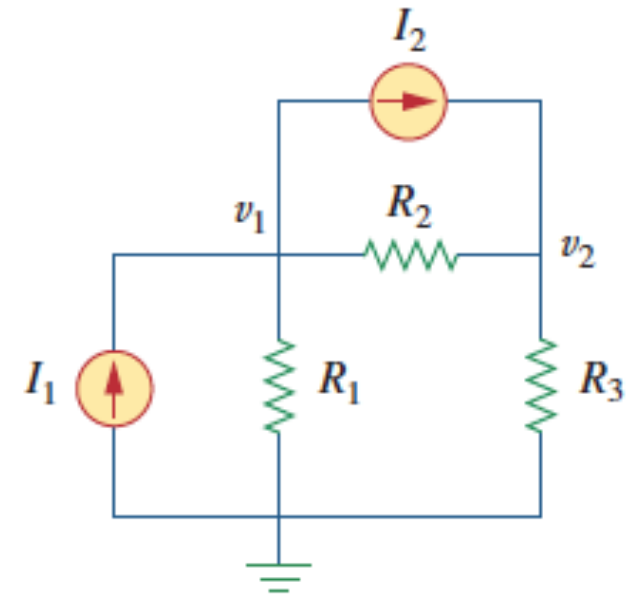
$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} - I_1 + I_2 = 0$$

$$\frac{v_2}{R_3} + \frac{v_2 - v_1}{R_2} - I_2 = 0$$

- Grouping terms

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_1 - \left(\frac{1}{R_2}\right) v_2 = I_1 - I_2$$

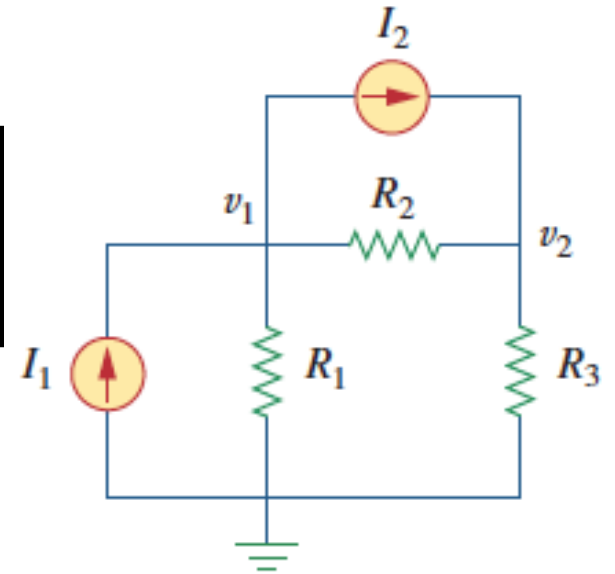
$$-\left(\frac{1}{R_2}\right) v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_2 = I_2$$



- Or, in vector/matrix form

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$



- General node result: $G v = I$
 - G is matrix of conductances (reciprocals of R 's)
 - Diagonals – sum of those connected to a node
 - Off diagonals – negative of those between nodes
 - v = vector of unknown node voltages
 - I = vector of currents into the nodes
 - Solving, $v = G^{-1} I$

- Solving symbolically:

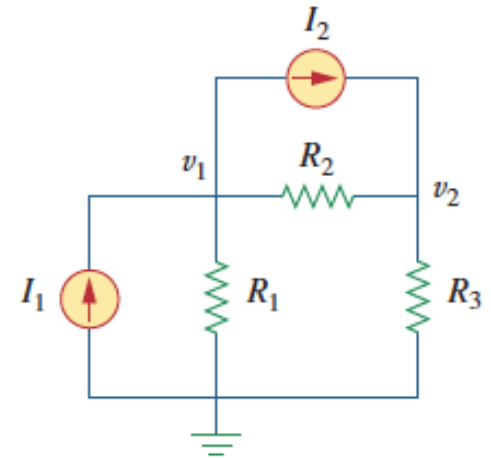
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix}^{-1} \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_1 + \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix} I_2$$

$$= \begin{bmatrix} \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \\ \frac{R_1 R_3}{R_1 + R_2 + R_3} \end{bmatrix} I_1 + \begin{bmatrix} -\frac{R_1 R_2}{R_1 + R_2 + R_3} \\ \frac{R_2 R_3}{R_1 + R_2 + R_3} \end{bmatrix} I_2$$

- Further observations:

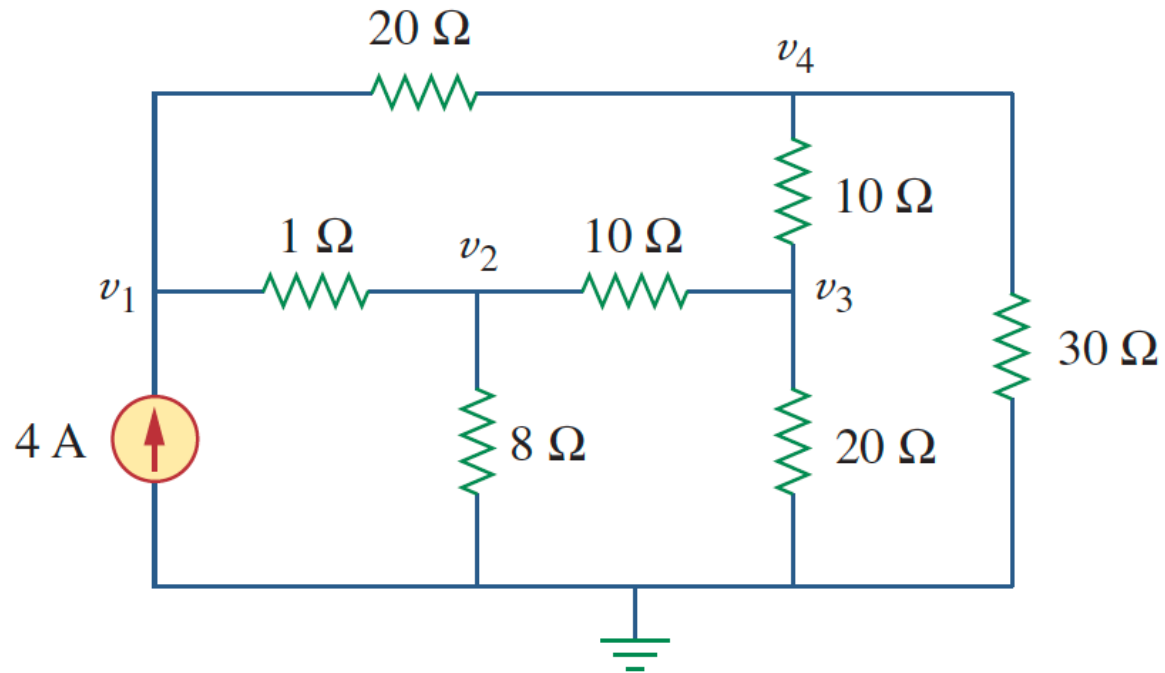
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \\ \frac{R_1 R_3}{R_1 + R_2 + R_3} \end{bmatrix} I_1 + \begin{bmatrix} -\frac{R_1 R_2}{R_1 + R_2 + R_3} \\ \frac{R_2 R_3}{R_1 + R_2 + R_3} \end{bmatrix} I_2$$



Linearity: for each input, the output is proportional to that input

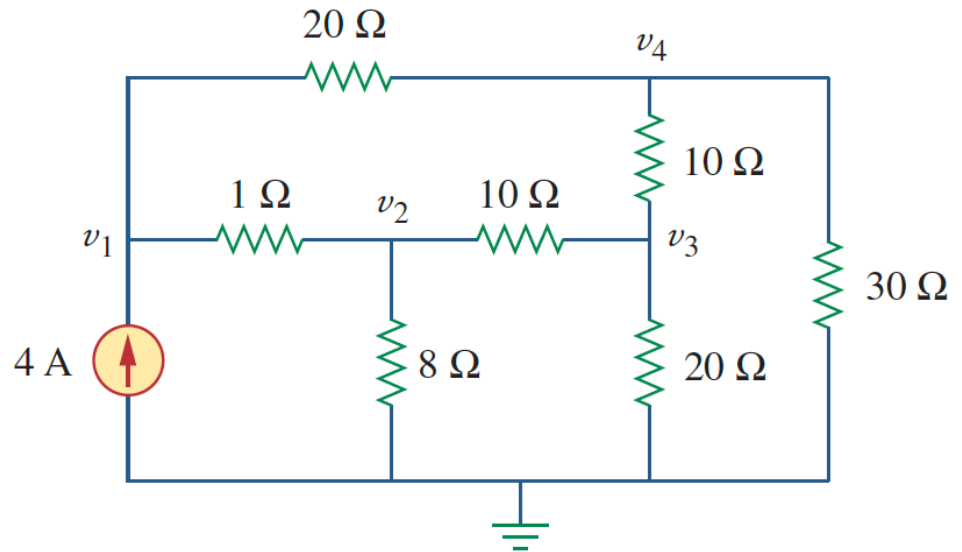
Superposition: the output due to multiple inputs is the sum of the responses due to each individual input

Example (details on next slide)



Using MatLab:

Set up the matrix
of conductances

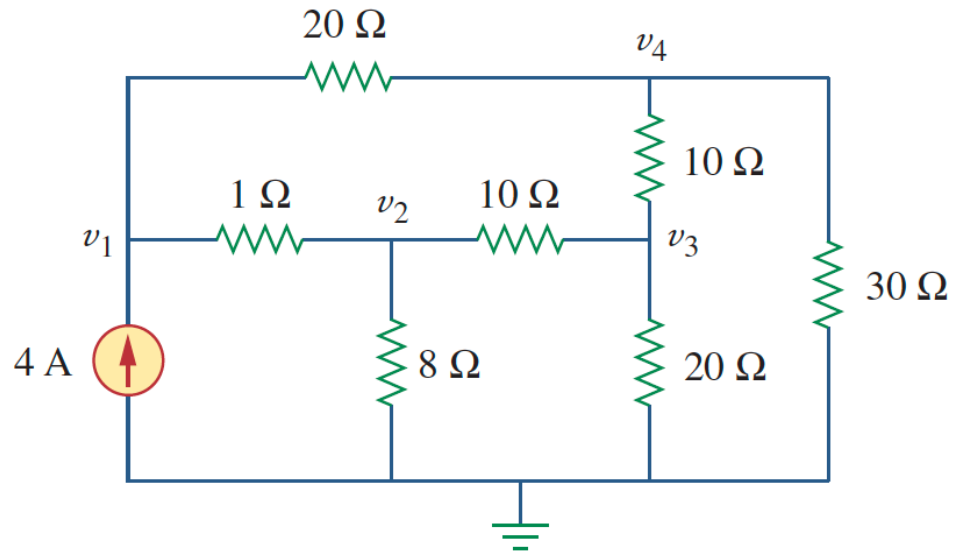


```
>> G = [ 1+1/20, -1, 0, -1/20;  
-1, 1+1/8+1/10, -1/10, 0;  
0, -1/10, 1/10+1/10+1/20, -1/10;  
-1/20, 0, -1/10, 1/10+1/20+1/30 ]
```

G =

1.0500	-1.0000	0	-0.0500
-1.0000	1.2250	-0.1000	0
0	-0.1000	0.2500	-0.1000
-0.0500	0	-0.1000	0.1833

Set up currents
and solve:



```
>> I = [ 4 ; 0 ; 0 ; 0 ]
```

```
I =
```

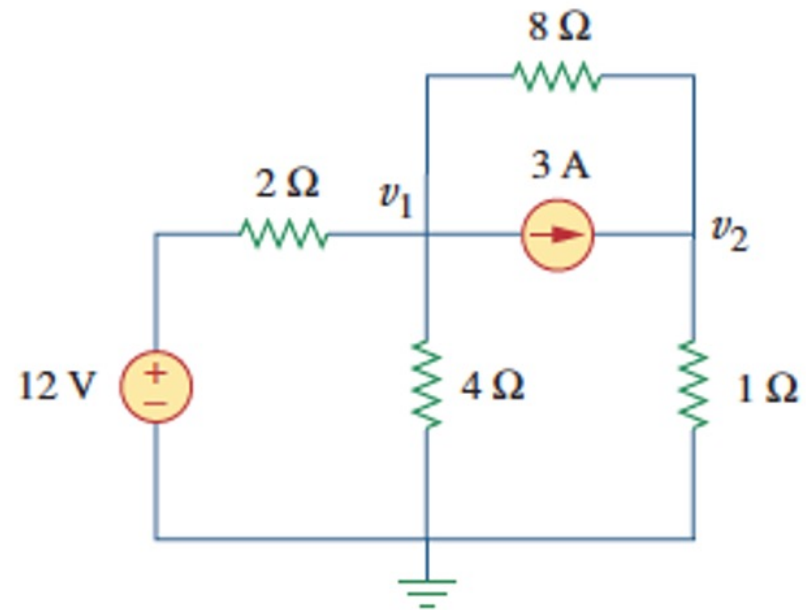
```
4
0
0
0
```

```
>> G\I
```

```
ans =
```

```
25.5247
22.0480
14.8420
15.0569
```

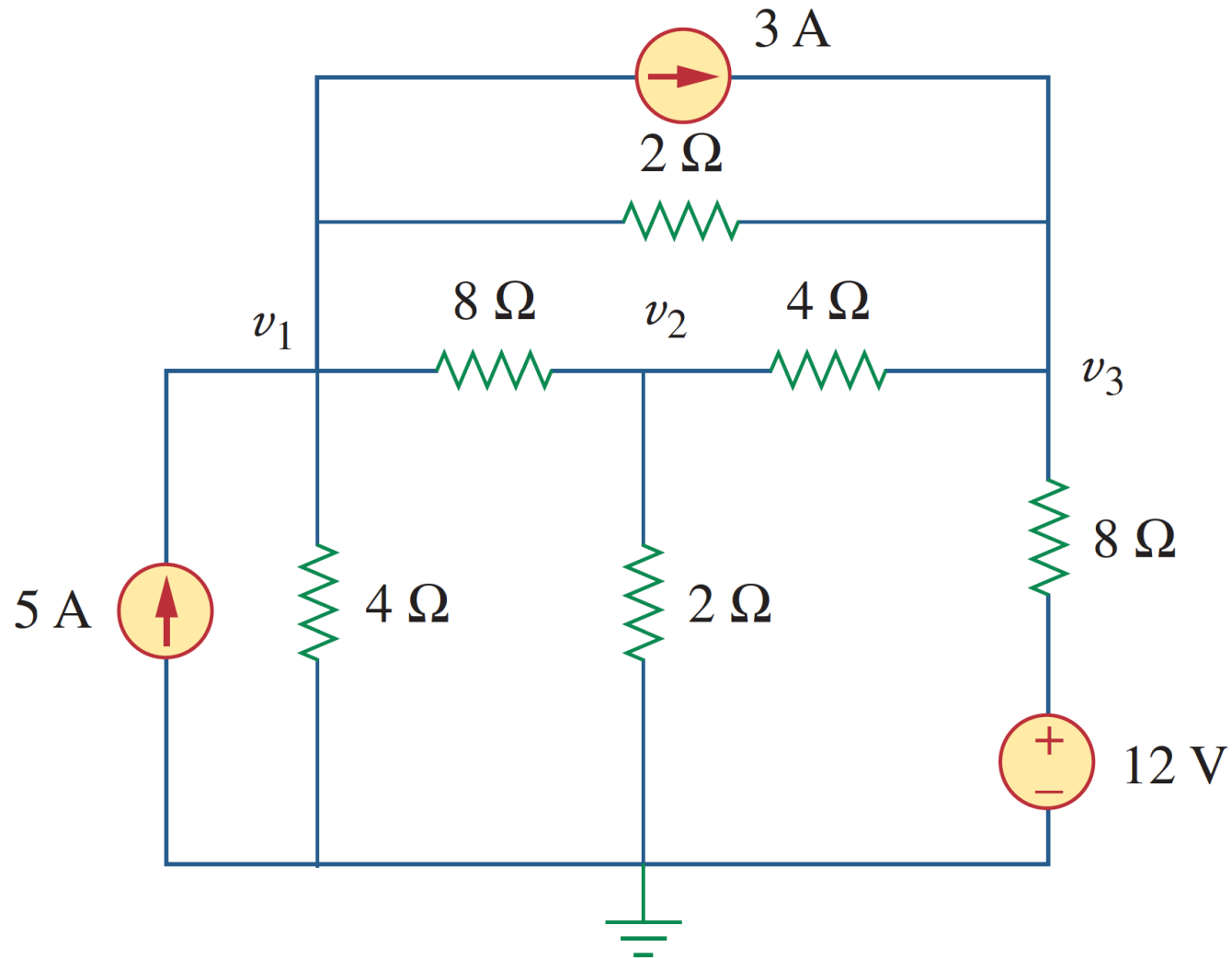
- Extension to branches with voltage sources



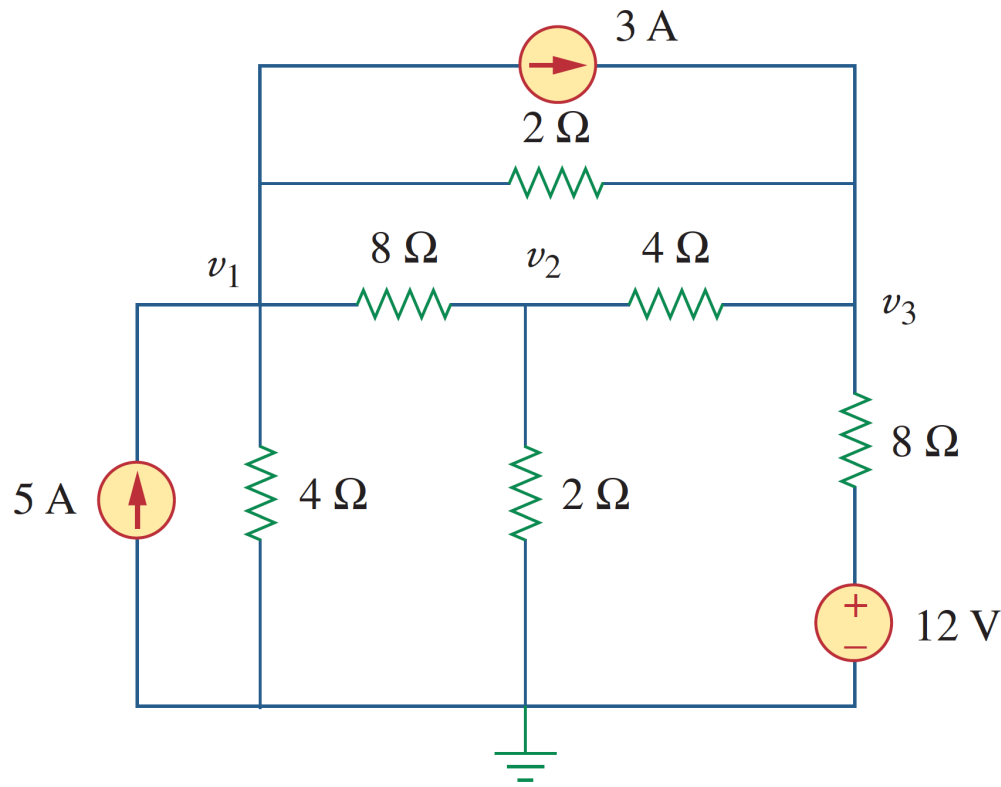
- Current **into** node due to source/resistor

$$\begin{bmatrix} \frac{1}{2} + \frac{1}{4} + \frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{1}{1} + \frac{1}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{12}{2} - 3 \\ 3 \end{bmatrix}$$

Example (see next slide)

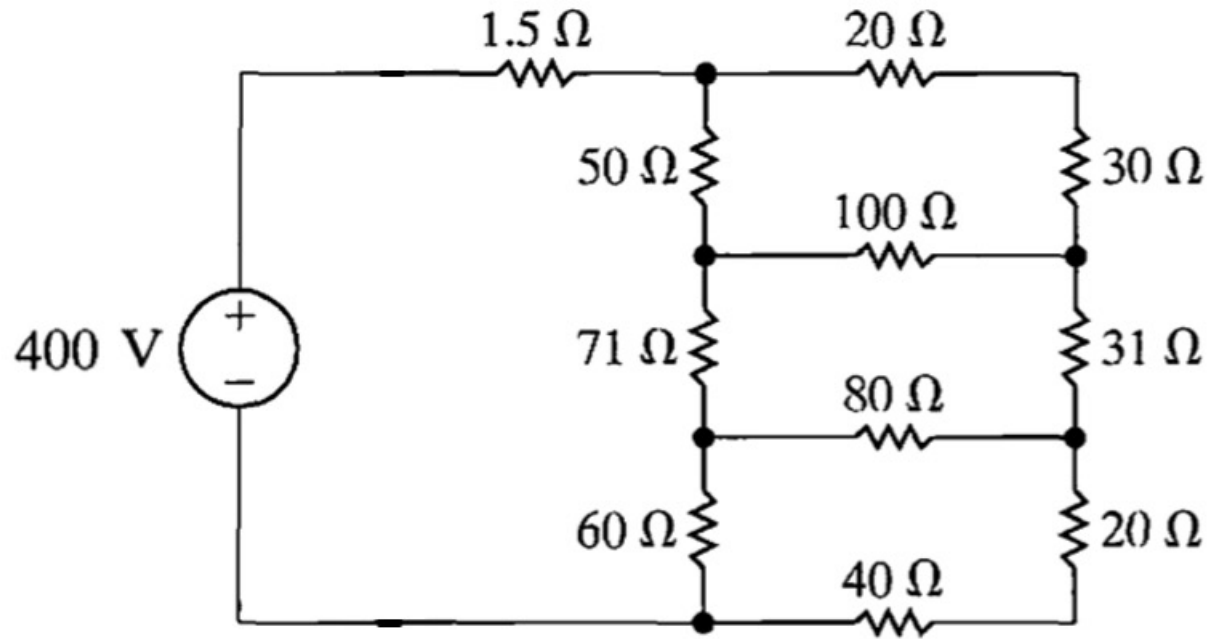


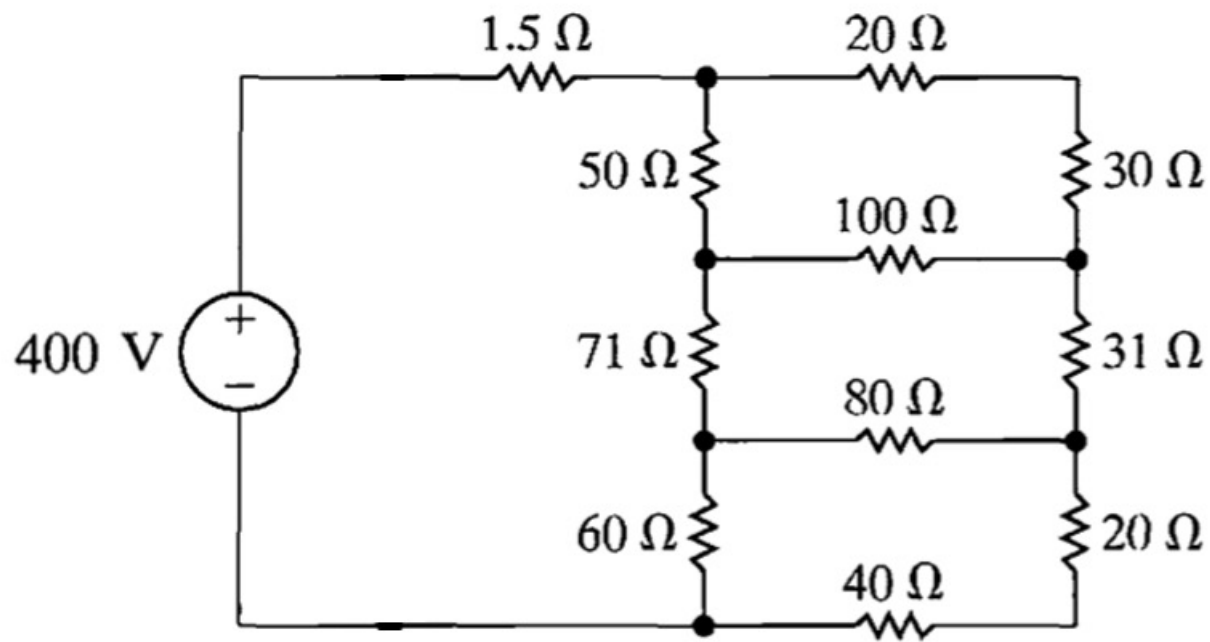
```
>> A = [ 1/4+1/8+1/2, -1/8,      -1/2
        -1/8,      1/8+1/2+1/4, -1/4
        -1/2,      -1/4,      1/2+1/4+1/8 ];
>> b = [ 5-3; 0; 3+12/8 ];
```



```
>> v = A\b
v =
    1.0000e+01
    4.9333e+00
    1.2267e+01
```

Example: find the power of the $31\ \Omega$ resistor



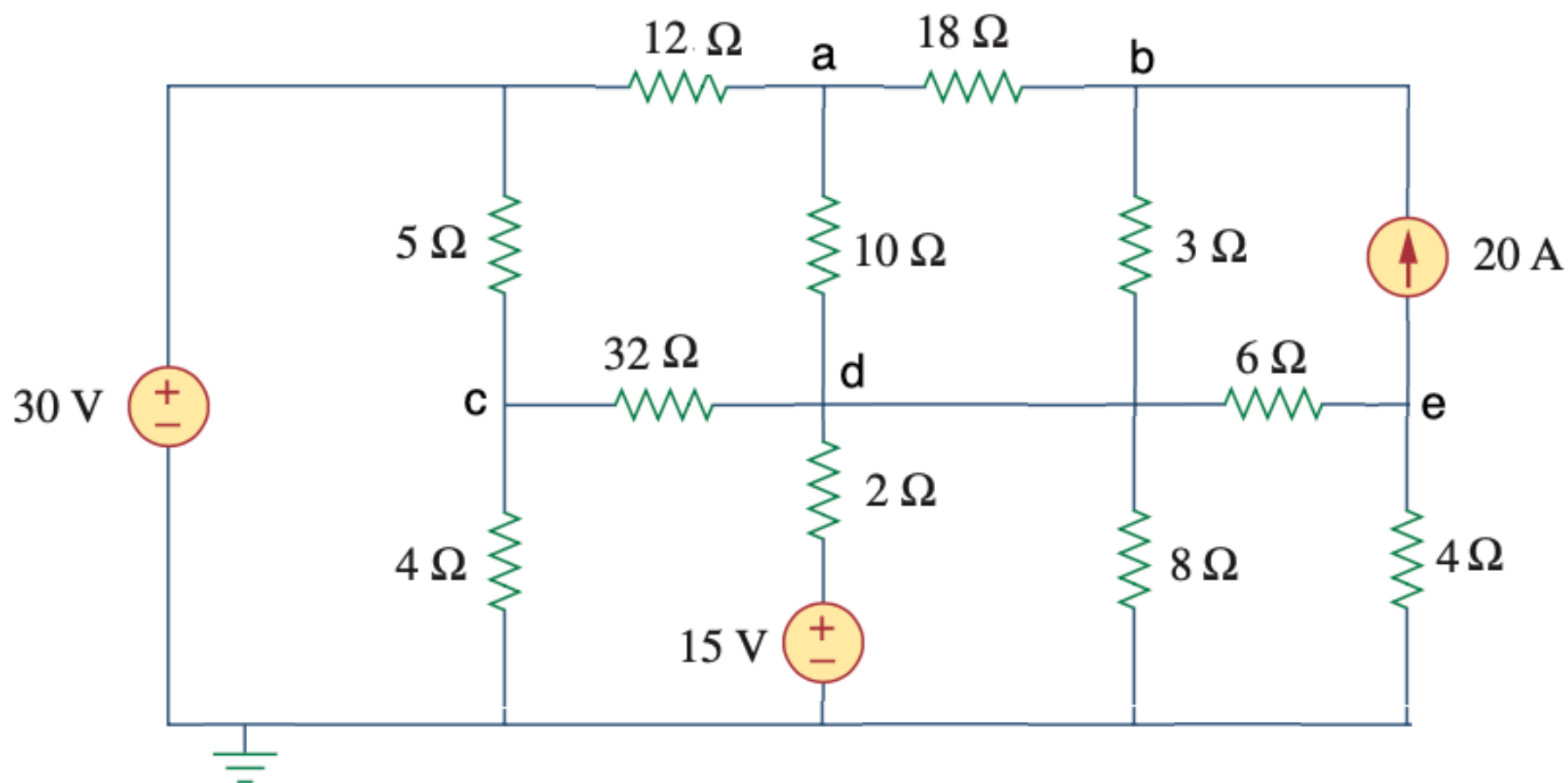


$$v_{31} = 93 \text{ V}$$

$$P_{31} = 279 \text{ W}$$

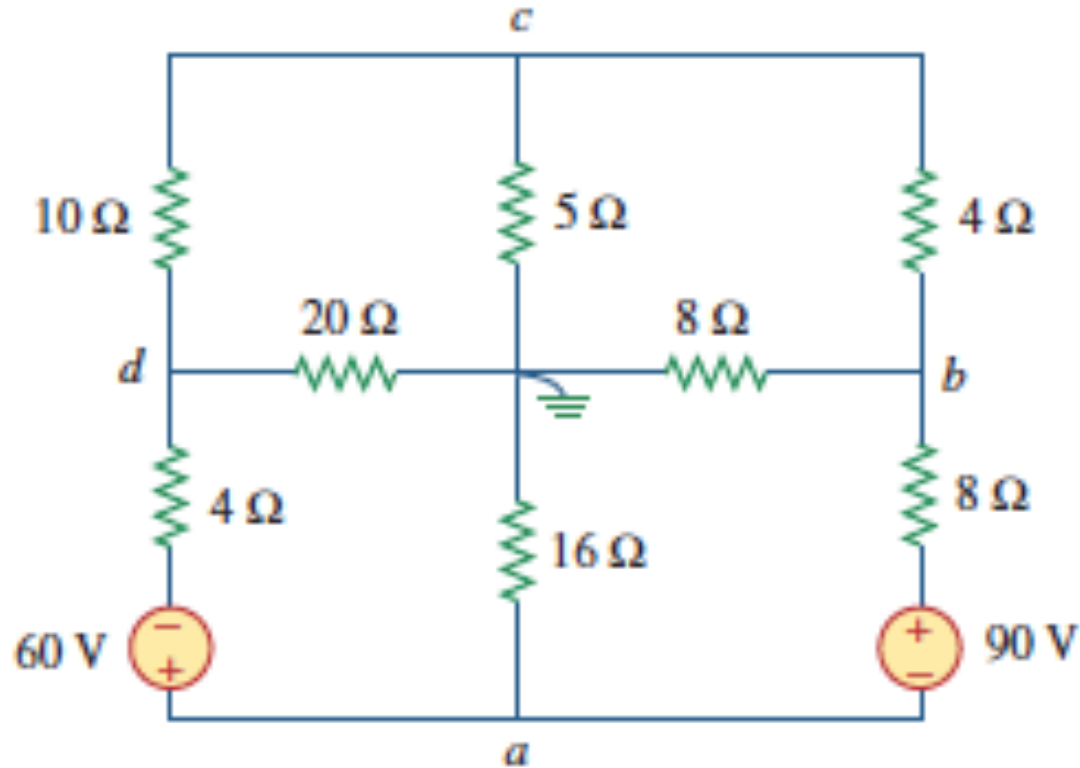
39.4 V

Practice problem: find v_a



Practice problem: find a, b, c, d

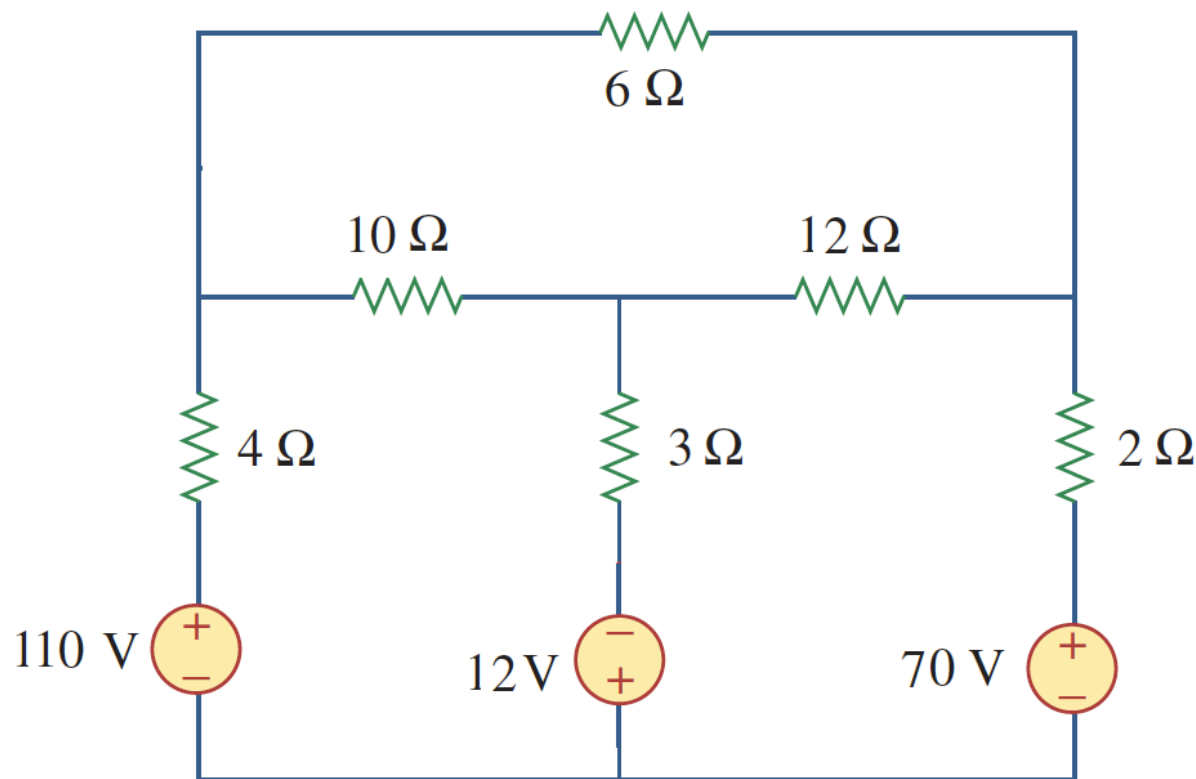
$$-\frac{95}{9}, \frac{185}{9}, \frac{25}{18}, -\frac{175}{4} \text{ V}$$



Practice problem: find the power dissipated in the $10\ \Omega$ resistor

$$v_{10} = 60\text{ V}$$

$$P_{31} = 360\text{ W}$$



Practice problem: find i

$$i = \frac{75}{221} A$$

