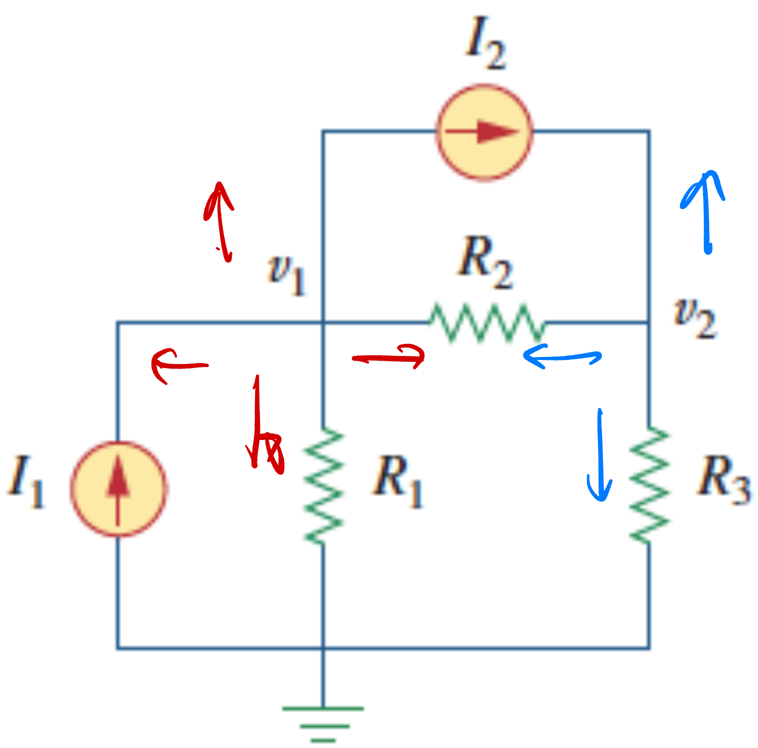
Lecture 10
Node Analysis – 3 of 7

vector form

Matrix-Vector Form

- Reconsider the initial simple circuit:



- The node equations were:

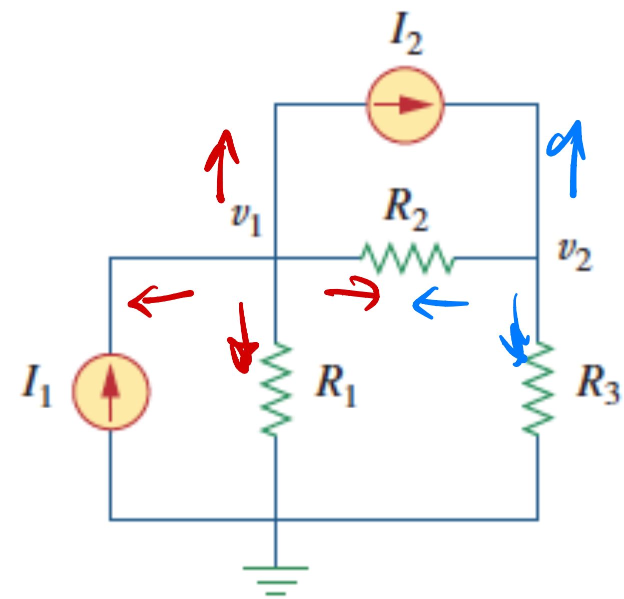
$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} - I_1 + I_2 = 0$$

$$\frac{v_2}{R_3} + \frac{v_2 - v_1}{R_2} - I_2 = 0$$

- Grouping terms

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_1-\left(\frac{1}{R_2}\right)v_2=I_1-I_2$$

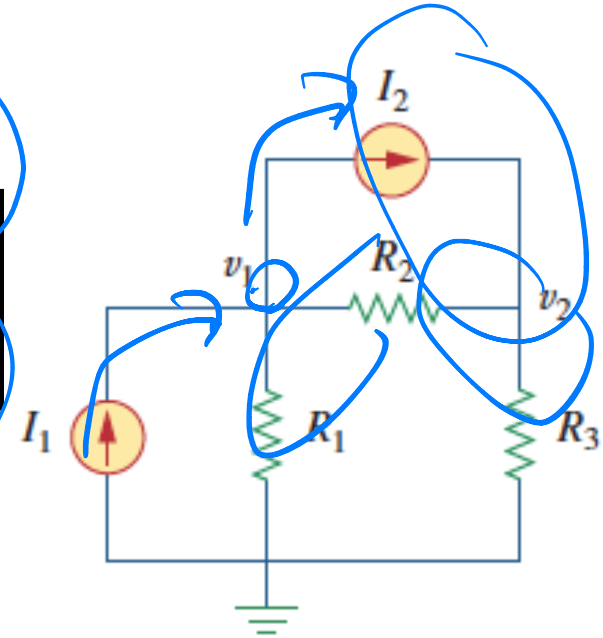
$$-\left(\frac{1}{R_2}\right)v_1+\left(\frac{1}{R_2}+\frac{1}{R_3}\right)v_2=I_2$$



- Or, in vector/matrix form

$$\begin{bmatrix}\frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2}\\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3}\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}I_1-I_2\\ I_2\end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$



- General node result: $\mathrm{G}\,\mathrm{v} = \mathrm{I}$
  - G is matrix of conductances (reciprocals of R’s)
    - Diagonals – sum of those connected to a node
    - Off diagonals – negative of those between nodes
  - v = vector of unknown node voltages
  - I = vector of currents into the nodes
  - Solving, $\mathrm{v} = \mathrm{G}^{-1}\,\mathrm{I}$

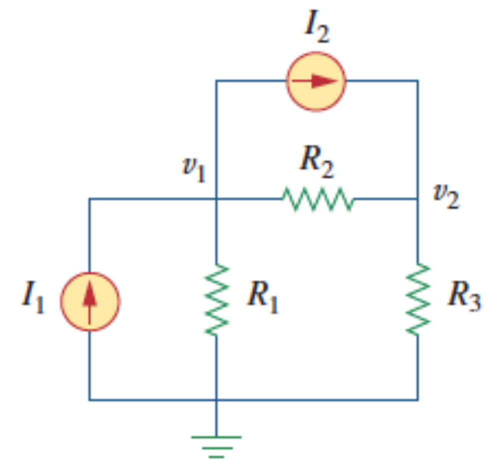
- Solving symbolically:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix}^{-1} \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_1 + \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix} I_2$$

$$= \begin{bmatrix} \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \\ \frac{R_1R_3}{R_1+R_2+R_3} \end{bmatrix} I_1 + \begin{bmatrix} -\frac{R_1R_2}{R_1+R_2+R_3} \\ \frac{R_2R_3}{R_1+R_2+R_3} \end{bmatrix} I_2$$

- Further observations:



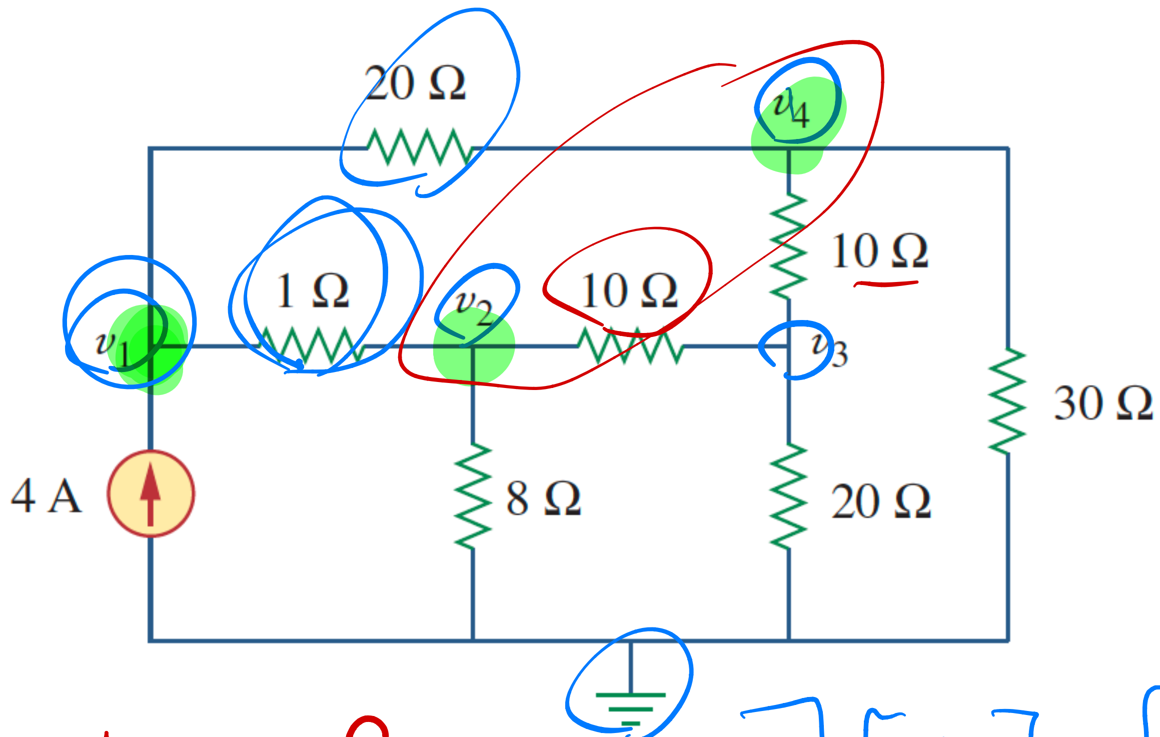
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \dfrac{R_1(R_2+R_3)}{R_1+R_2+R_3} \\ \\ \dfrac{R_1R_3}{R_1+R_2+R_3} \end{bmatrix} I_1 + \begin{bmatrix} -\dfrac{R_1R_2}{R_1+R_2+R_3} \\ \\ \dfrac{R_2R_3}{R_1+R_2+R_3} \end{bmatrix} I_2$$

Linearity: for each input, the output is proportional to that input

Superposition: the output due to multiple inputs is the sum of the responses due to each individual input

1- label nodes

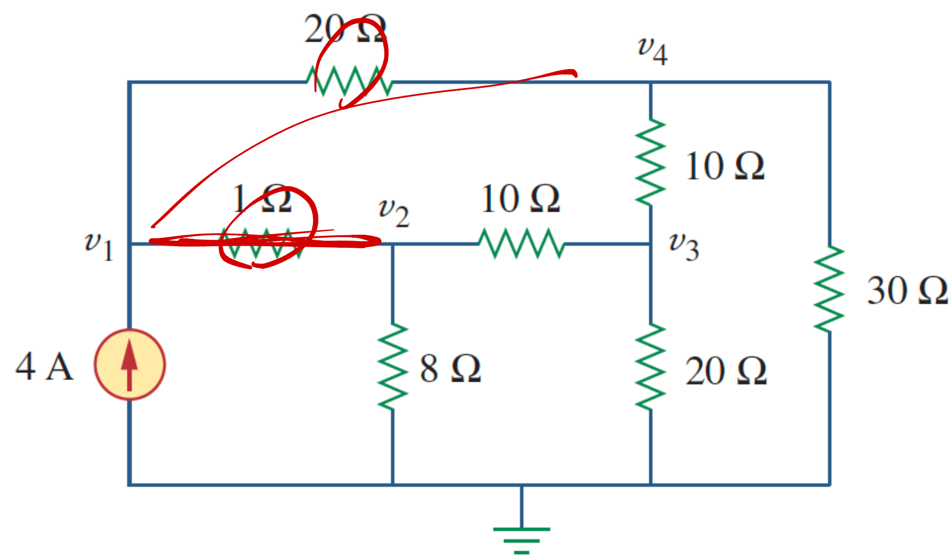
Example (details on next slide)



$$\begin{bmatrix} 1.05 & -1 & 0 & -.05 \\ -1 & 1.225 & -0.1 & 0 \\ 0 & -0.1 & 0.25 & -0.1 \\ -.05 & 0 & -0.1 & 0.1833 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

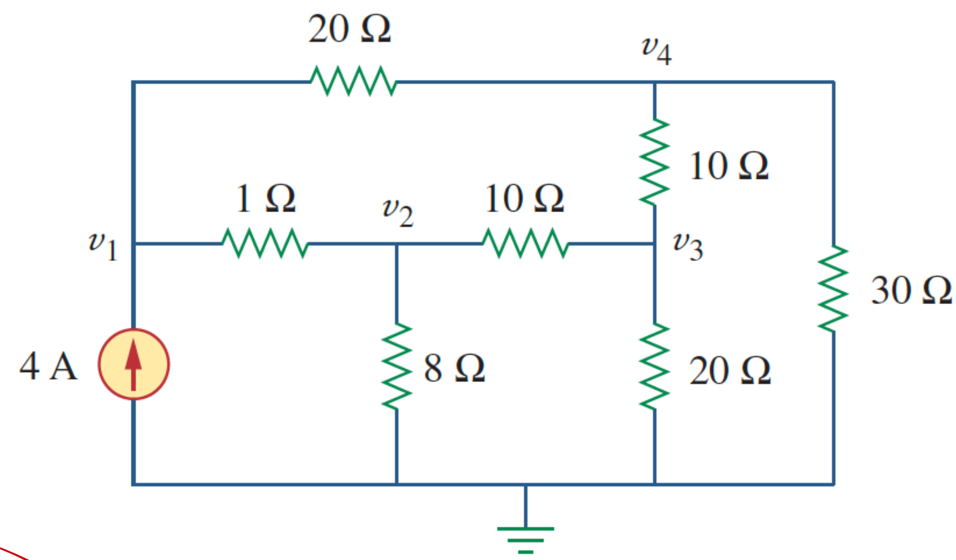
Using MatLab:

Set up the matrix of conductances




```
>> G = [ 1+1/20, -1, 0, -1/20;
-1, 1+1/8+1/10, -1/10, 0;
0, -1/10, 1/10+1/10+1/20, -1/10;
-1/20, 0, -1/10, 1/10+1/20+1/30 ]

G =

    1.0500   -1.0000         0   -0.0500
   -1.0000    1.2250   -0.1000         0
         0   -0.1000    0.2500   -0.1000
   -0.0500         0   -0.1000    0.1833
```

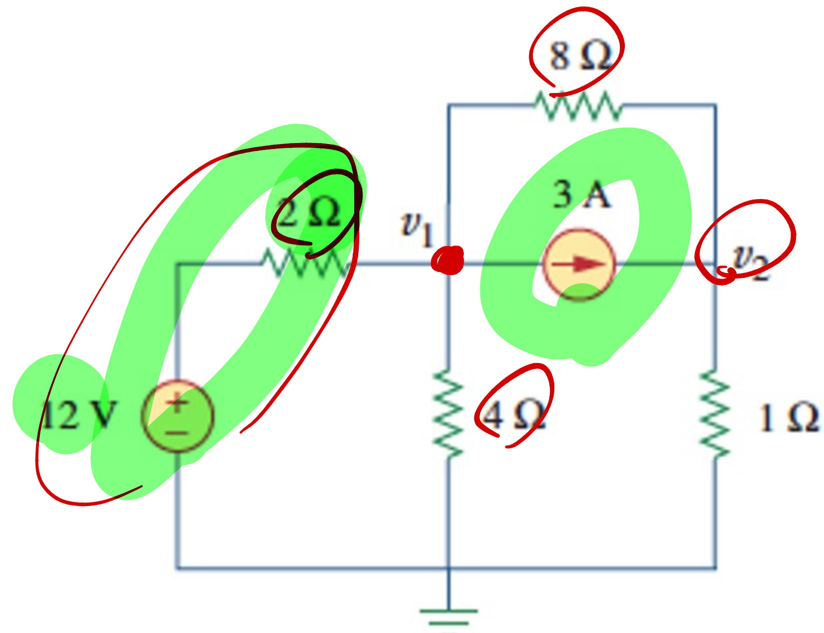
Set up currents and solve:




```
>> I = [ 4 ; 0 ; 0 ; 0 ]

I =

     4
     0
     0
     0
```

```
>> G\I

ans =

   25.5247
   22.0480
   14.8420
   15.0569
```

- Extension to branches with voltage sources



  - Current into node due to source/resistor

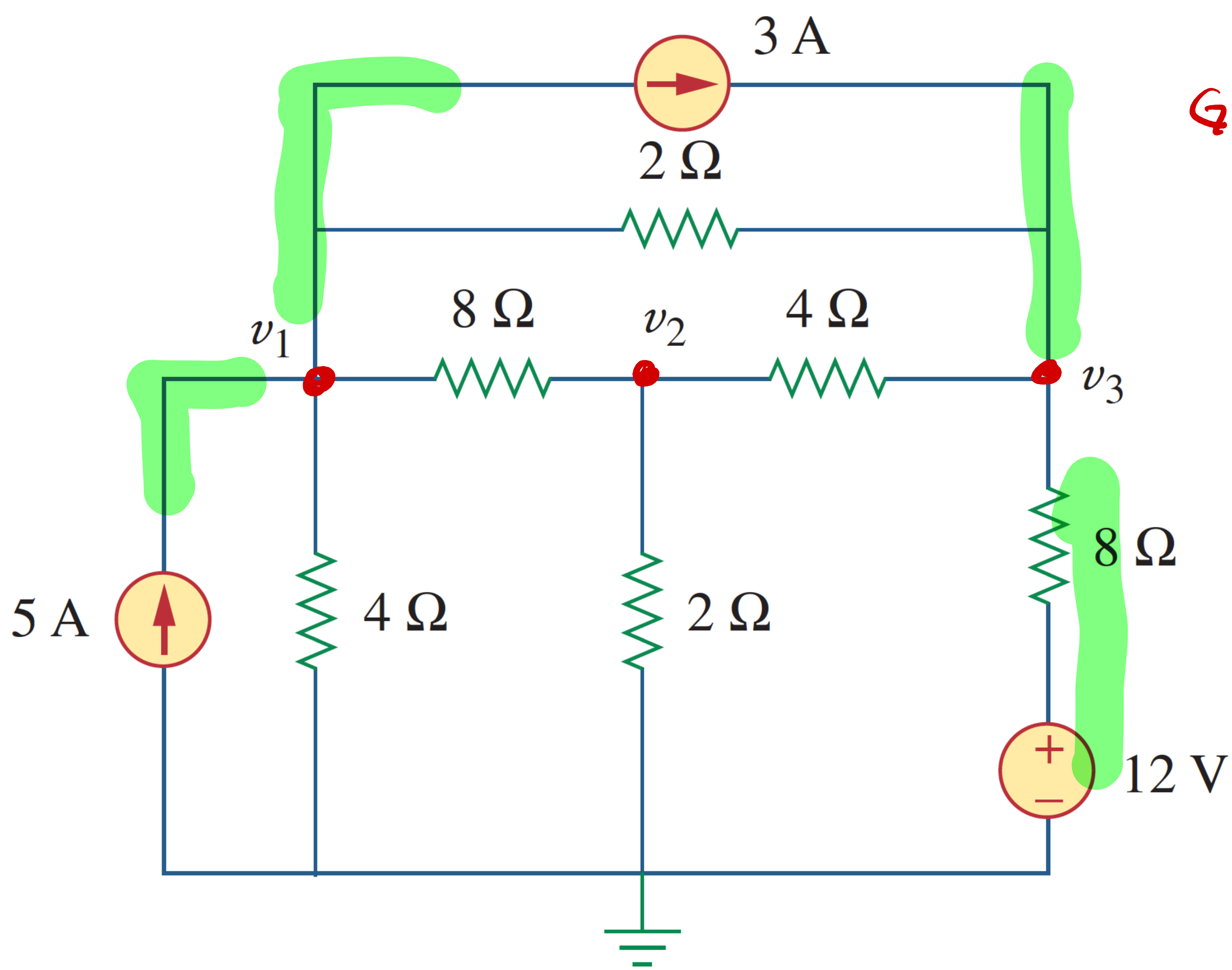
$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{1}{1}+\frac{1}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{12}{2} - 3 \\ 3 \end{bmatrix}$$

1- label ✓

2- write $\underline{G}\,\underline{v} = \underline{I}$

$\underline{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

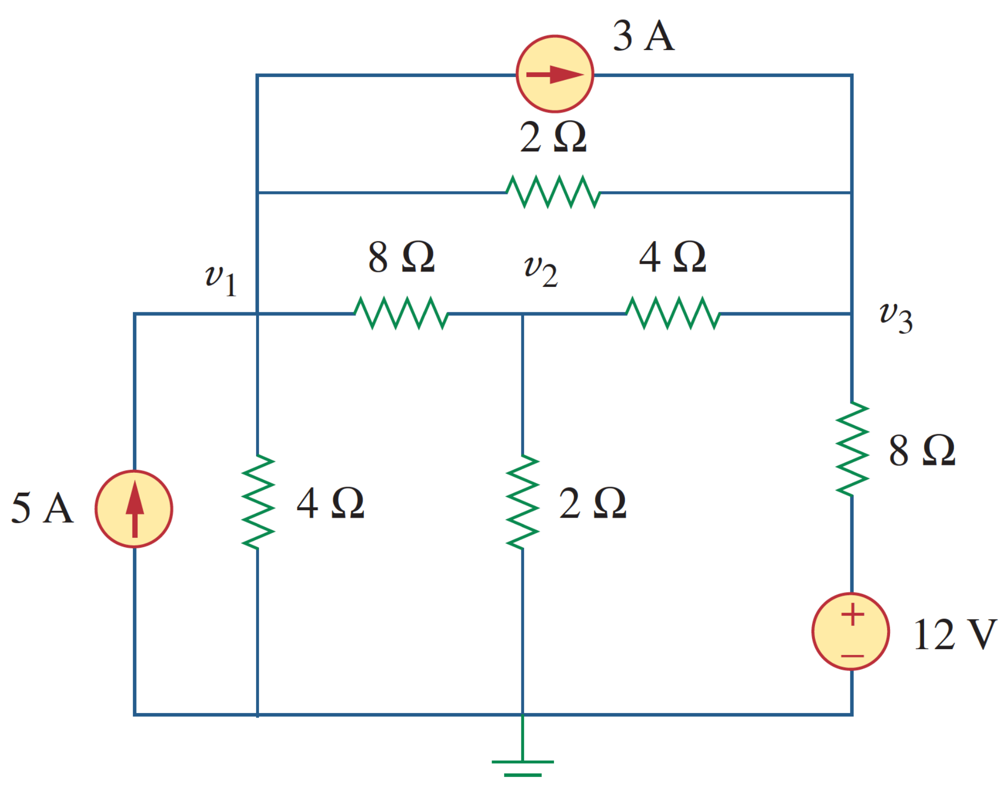
Example (see next slide)



$\underline{G} = \begin{bmatrix} 3 \times 3 \end{bmatrix}$

$\underline{I} = \begin{bmatrix} 2 \\ 0 \\ 3 + \frac{12}{8} \end{bmatrix}$

```
>> A = [ 1/4+1/8+1/2, -1/8,              -1/2
         -1/8,           1/8+1/2+1/4, -1/4
         -1/2,           -1/4,           1/2+1/4+1/8 ];
>> b = [ 5-3; 0; 3+12/8 ];
```




```
>> v = A\b
v =
   1.0000e+01
   4.9333e+00
   1.2267e+01
```

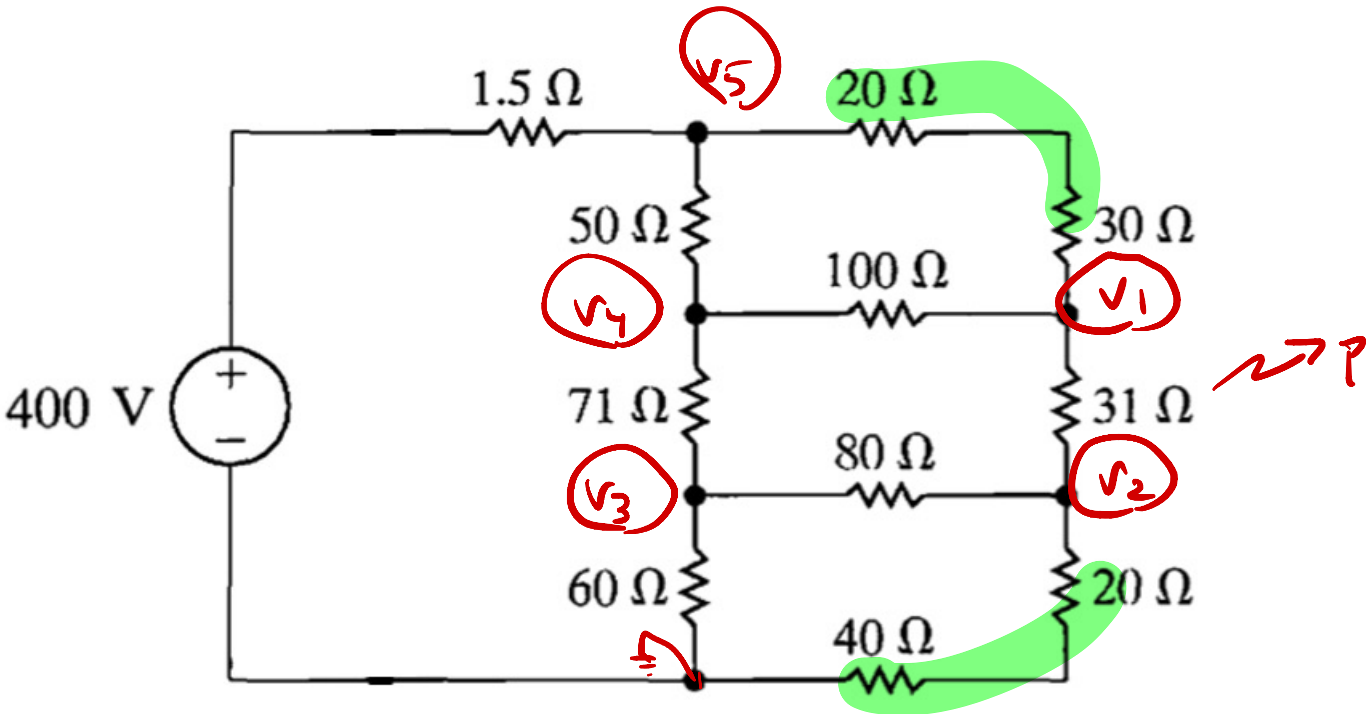
Example: find the power of the 31 Ω resistor

1- $P = \frac{v^2}{31}$

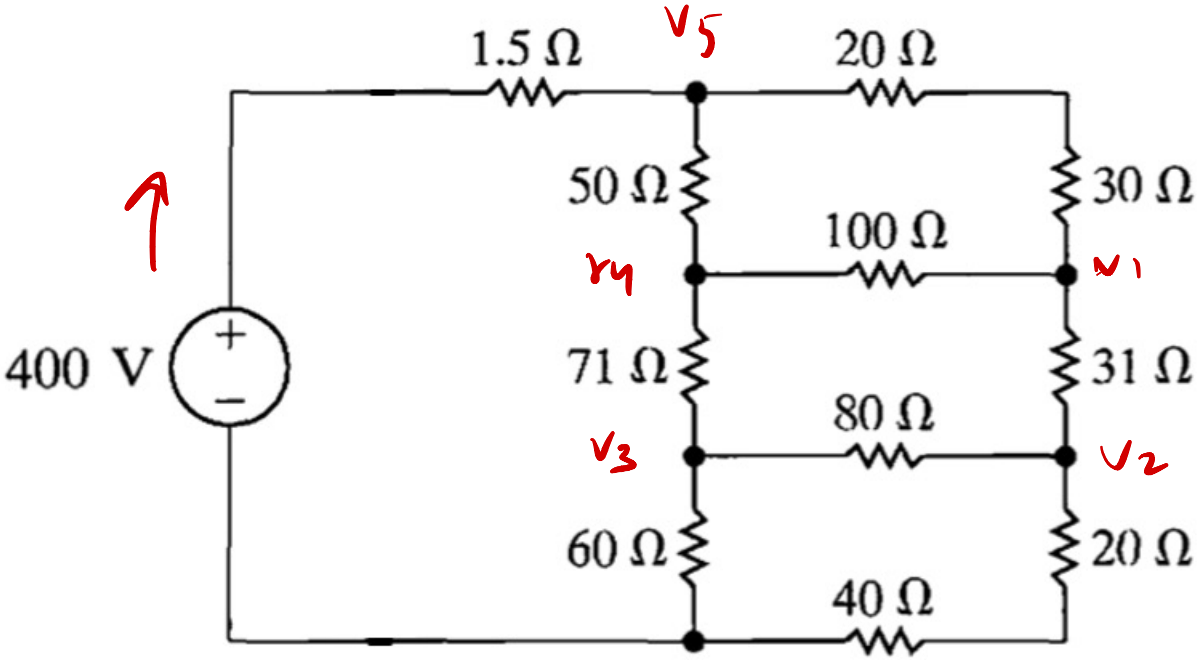
2- label node

3- $P = \frac{(V_1 - V_2)^2}{31}$

4- node anal.



$$\left[\begin{array}{c|c|c|c|c}
\frac{1}{50} + \frac{1}{100} + \frac{1}{31} & -\frac{1}{31} & 0 & -\frac{1}{100} & -\frac{1}{50} \\
\hline
-\frac{1}{31} & \frac{1}{31} + \frac{1}{60} + \frac{1}{80} & -\frac{1}{80} & 0 & 0 \\
 & & & &
\end{array}\right.$$



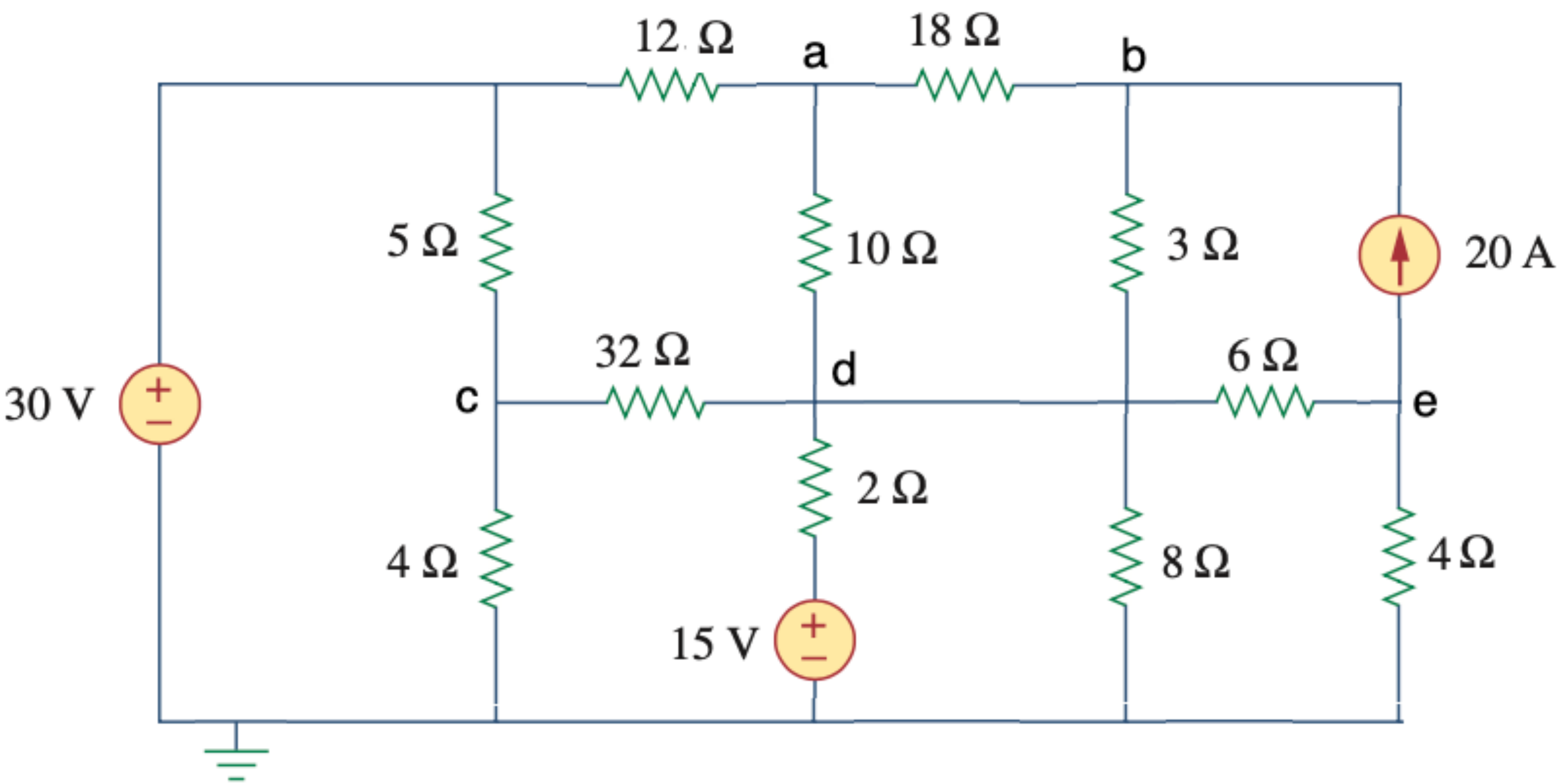
$$I = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{400}{1.5} \end{bmatrix}$$

$v_{31} = 93\ V$

$P_{31} = 279\ W$

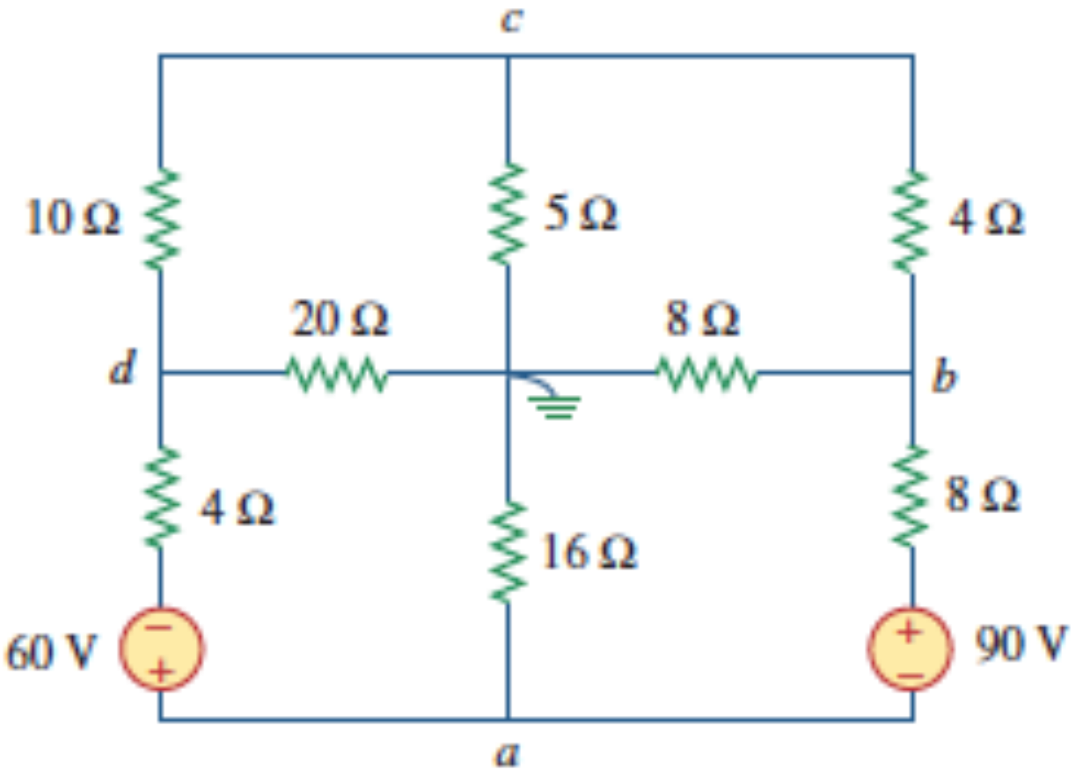
$39.4\ V$

Practice problem: find v_a



Practice problem: find a, b, c, d

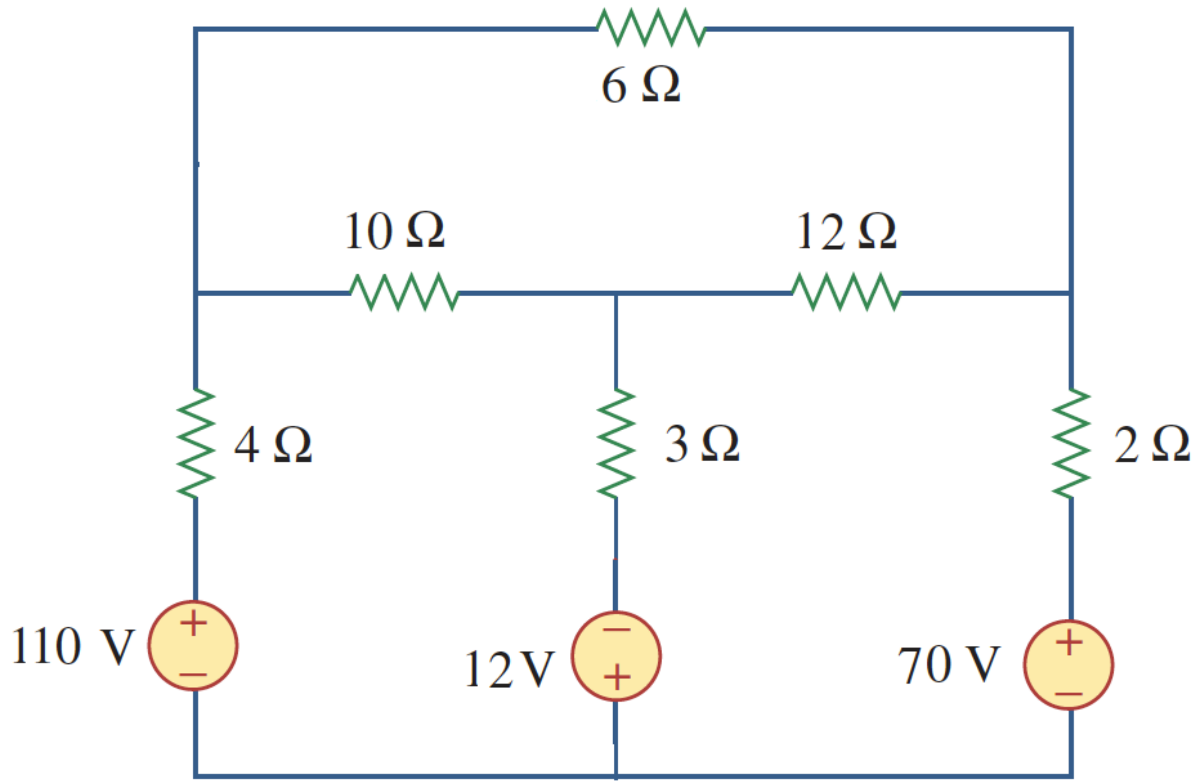
$$-\frac{95}{9}, \frac{185}{9}, \frac{25}{18}, -\frac{175}{4}\ V$$



Practice problem: find the power dissipated in the $10\ \Omega$ resistor

$v_{10} = 60\ V$

$P_{31} = 360\ W$



Practice problem: find i

$$i = \frac{75}{221} A$$