

Lecture 18

Phasors – 4 of 9

using phasors;
variation with frequency

Phasor Review

- Extend sinusoidal voltages/currents to phasors (complex)
- Convert components (R,L,C) to impedances
- Solve the problem using Ohm's Law, KVL/KCL, ...
- Convert back

$$V_s \cos(\omega t + \phi) \Rightarrow \mathbf{V} = V_s e^{j\phi}$$

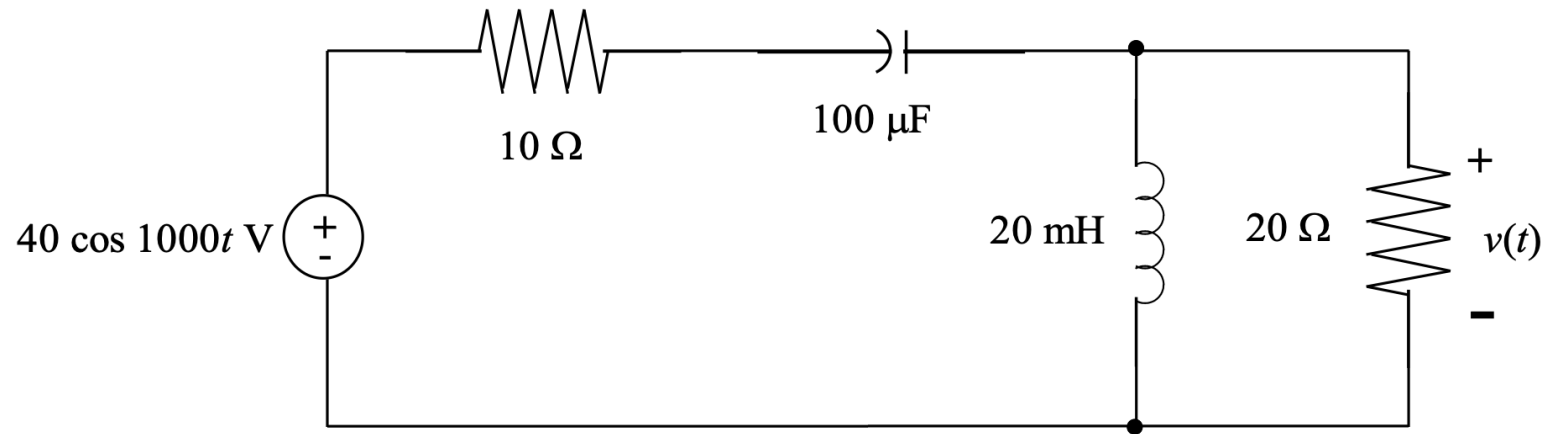
$$I_s \cos(\omega t + \phi) \Rightarrow \mathbf{I} = I_s e^{j\phi}$$

$$Z = \begin{cases} R & \text{resistor} \\ j\omega L & \text{inductor} \\ \frac{1}{j\omega C} = -j \frac{1}{\omega C} & \text{capacitor} \end{cases}$$

$$B \angle \theta \Rightarrow B \cos(\omega t + \theta)$$

Common Usage

- Find the voltage $v(t)$

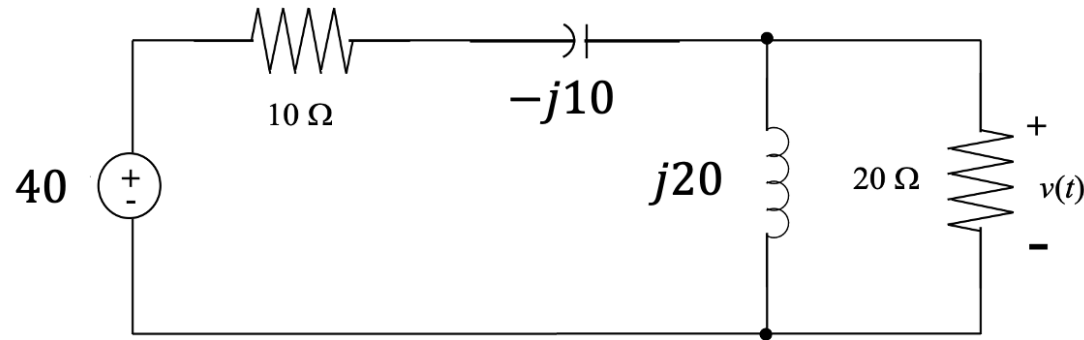


- Convert:

$$40 \cos 1000 t \rightarrow 40$$

$$20 \text{ mH} \rightarrow j\omega L = j20$$

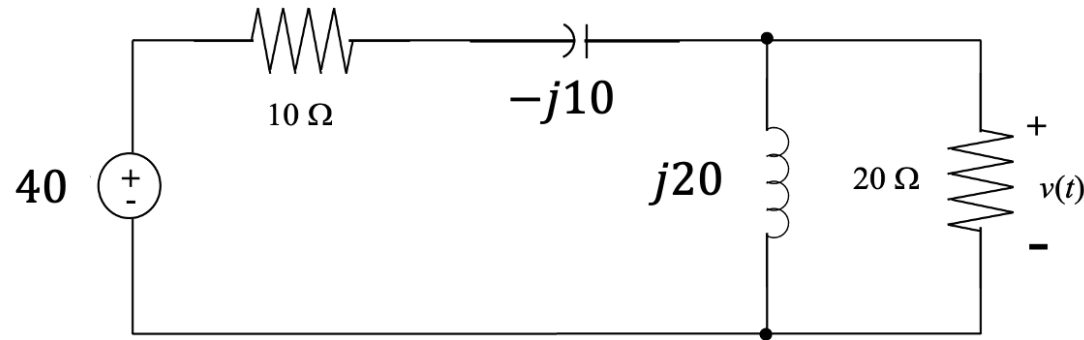
$$100 \mu\text{F} \rightarrow -j \frac{1}{\omega C} = -j10$$



- Solve: series/parallel combining, voltage division

$$Z_s = 10 - j10 = 10(1 - j) \qquad Z_p = \frac{(20)(j20)}{20 + j20} = \frac{j20}{1 + j}$$

$$V = 40 \frac{Z_p}{Z_s + Z_p}$$



- Simplify and convert to polar form

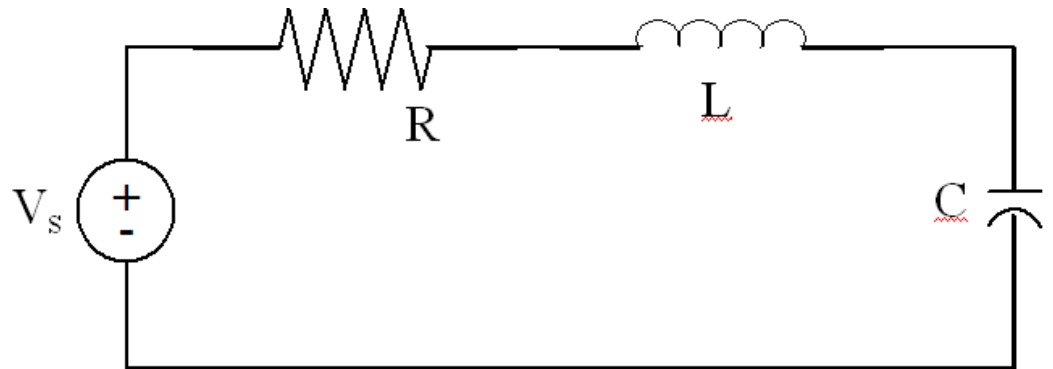
$$V = 40 \frac{\frac{j20}{1+j}}{10(1-j) + \frac{j20}{1+j}} = \frac{40(1+j)}{2} = 28.3 \angle 45^\circ$$

- Convert to a time function

$$v(t) = 28.3 \cos(1000t + 45^\circ) \text{ V}$$

Consider Variation with Frequency

Consider voltage division:



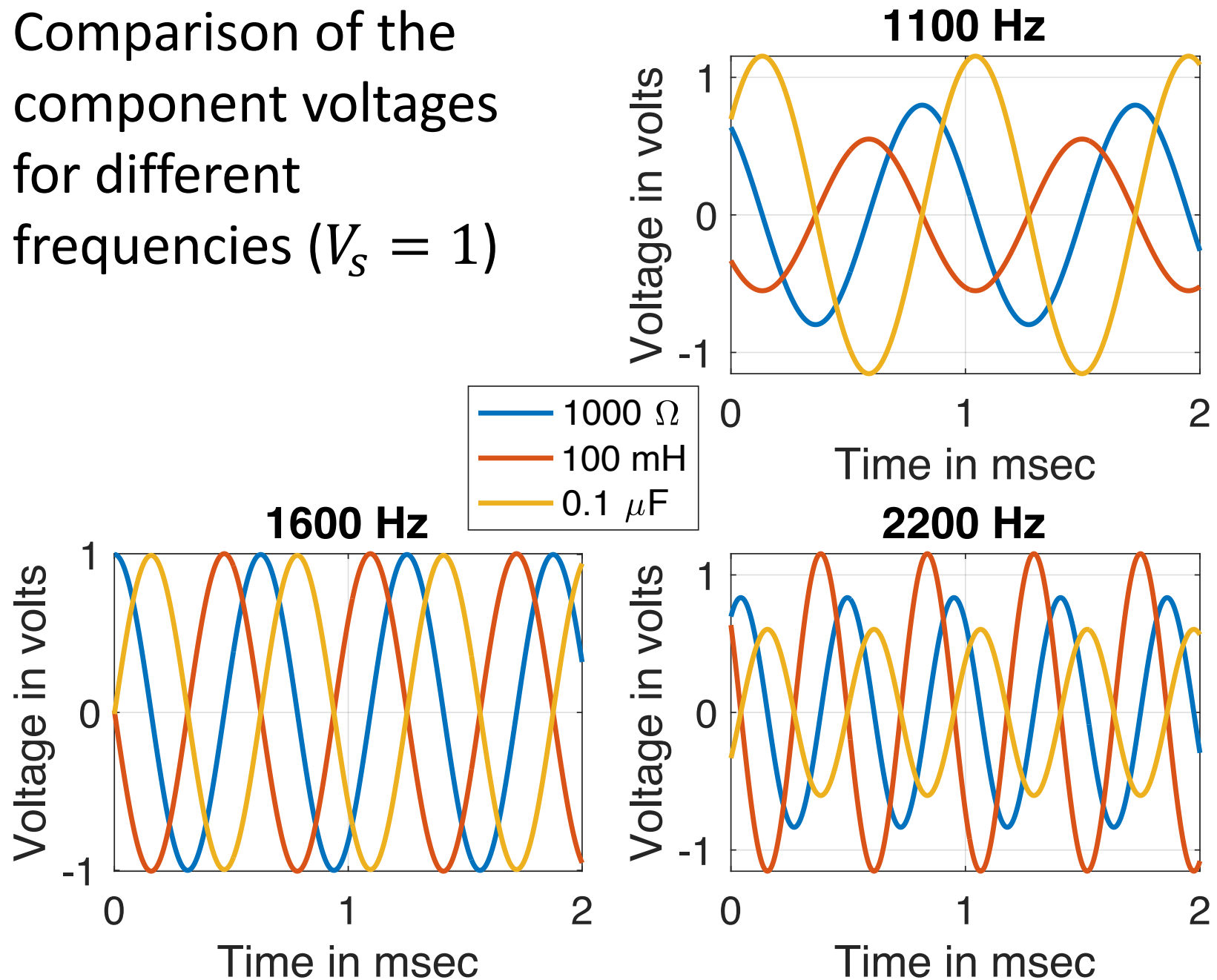
$$V_R = \frac{RV_s}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC} V_s$$

Similarly

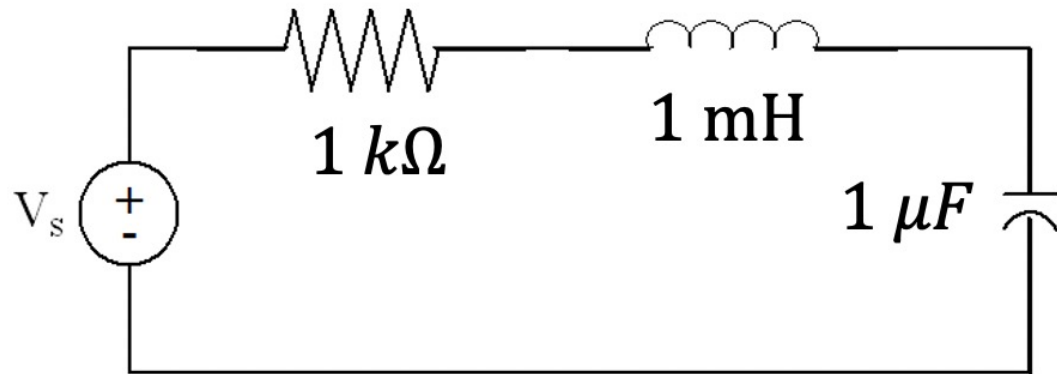
$$V_L = \frac{-\omega^2 LC}{1 - \omega^2 LC + j\omega RC} V_s$$

$$V_C = \frac{1}{1 - \omega^2 LC + j\omega RC} V_s$$

- Comparison of the component voltages for different frequencies ($V_s = 1$)



Consider combined impedance variation

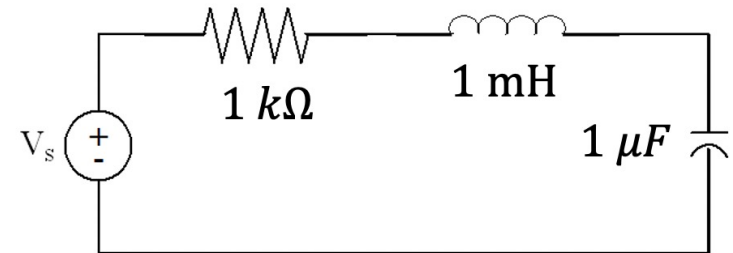


$$\begin{aligned} Z &= R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right) \\ &= 1000 + j\left(\frac{\omega}{100} - \frac{10^6}{\omega}\right) \end{aligned}$$

Question – at what frequency does this “appear” purely resistive?

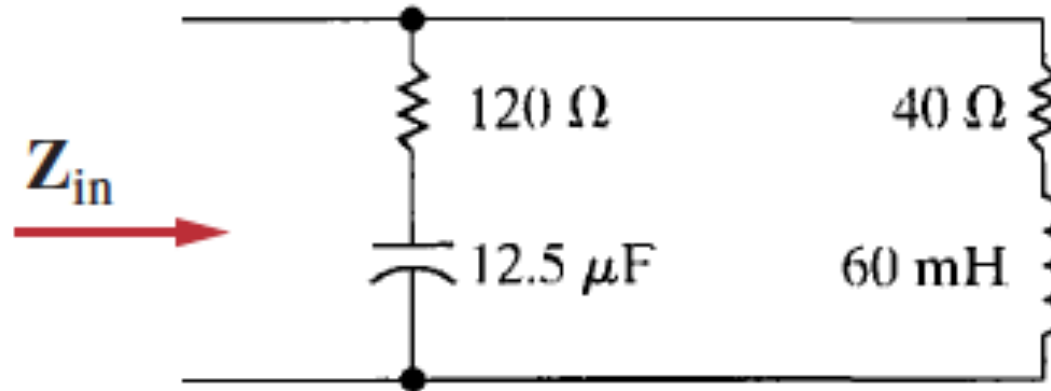
How might we graph impedance vs frequency?

$$Z = 1000 + j \left(\frac{\omega}{100} - \frac{10^6}{\omega} \right)$$



- Real/imag vs frequency?
- Real vs imag?
- See recording on website

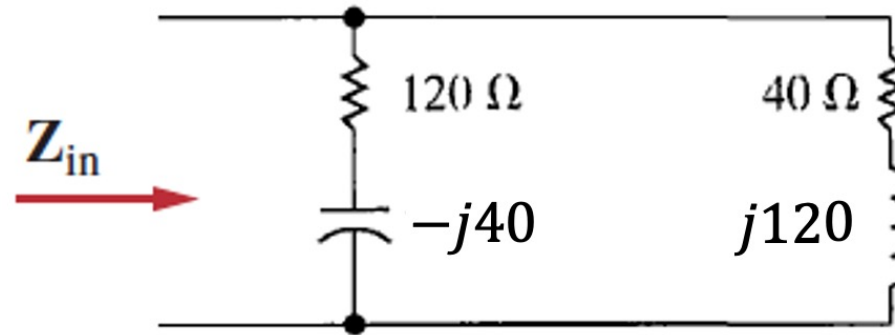
Example: find Z_{in} if $\omega = 2000$ rad/sec



- Convert:

$$60\ \text{mH} \rightarrow j\omega L = j120$$

$$12.5\ \mu\text{F} \rightarrow -j \frac{1}{\omega C} = -j40$$



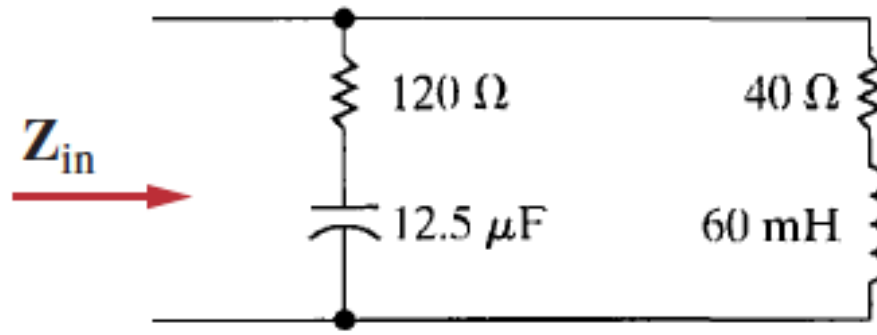
- Solve: series/parallel combining

$$Z_{s,left} = 120 - j40 = 40(3 - j) \qquad Z_{s,right} = 40 + j120 = 40(1 + j3)$$

$$\begin{aligned} Z_{in} &= \frac{40(3 - j)40(1 + j3)}{40(3 - j) + 40(1 + j3)} = \frac{40(6 + j8)}{4 + j2} = \frac{40(3 + j4)}{2 + j} \\ &= \frac{40(3 + j4)(2 - j)}{5} = 80 + j40 \end{aligned}$$

80 ohm resistor in series with 20 mH inductor !!

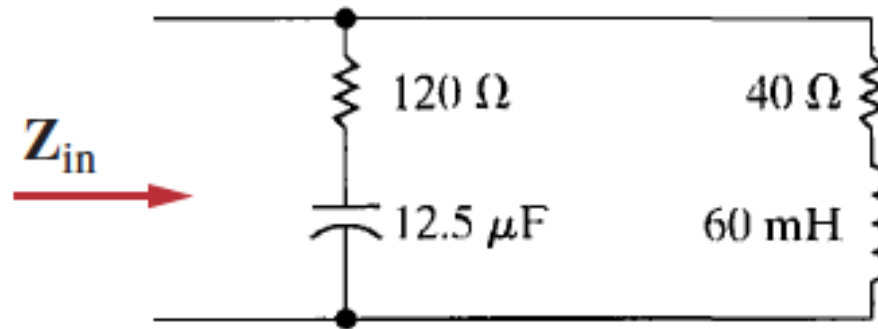
Example: How does Z_{in} vary with frequency? Is it ever purely real?



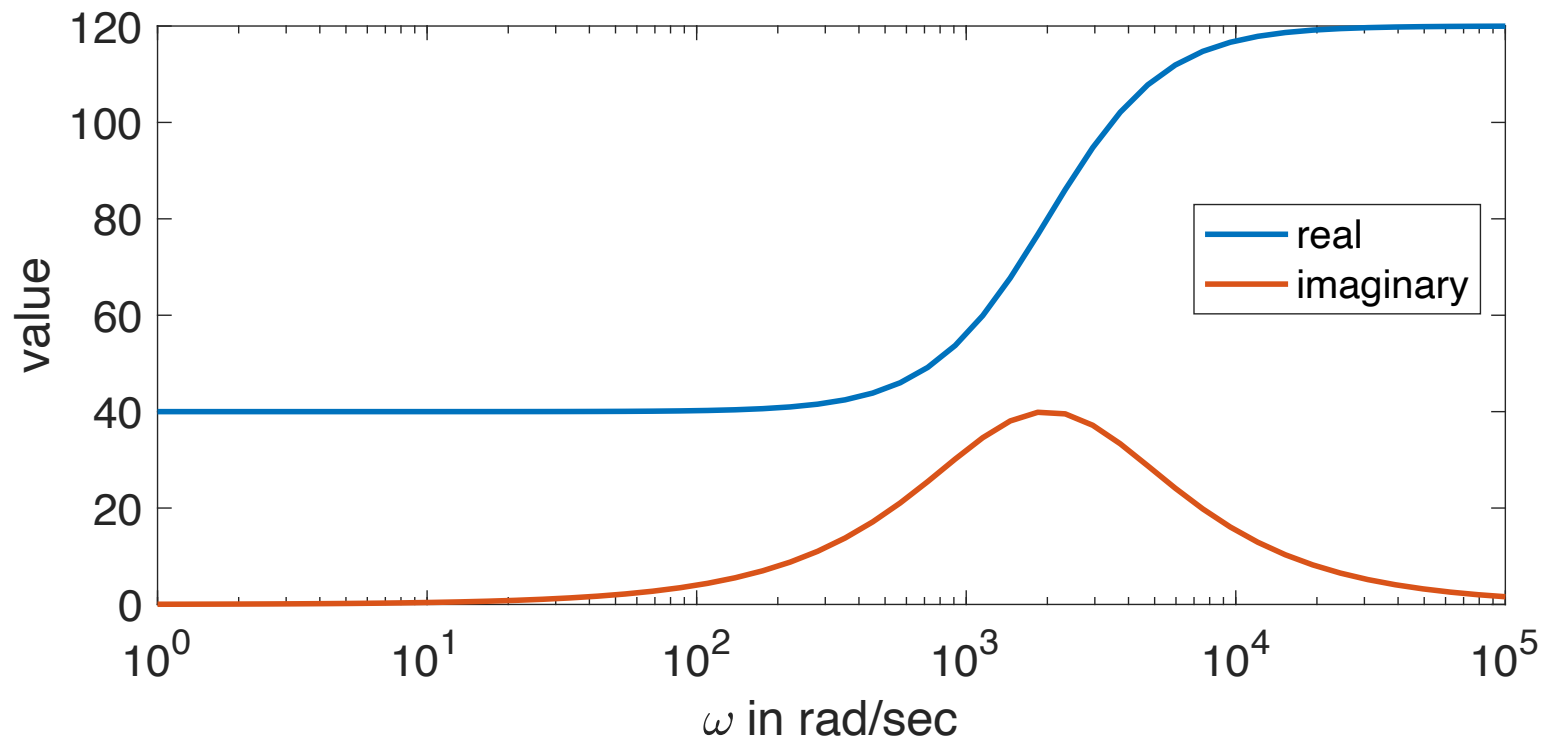
- Solve: series/parallel combining

$$Z_{s,left} = 120 - j \frac{80,000}{\omega} \qquad Z_{s,right} = 40 + j60\omega$$

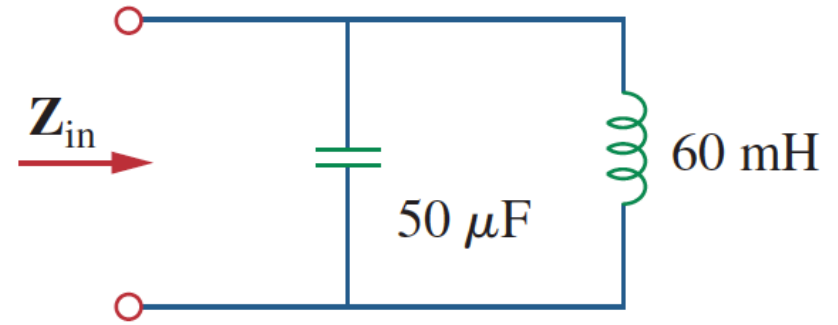
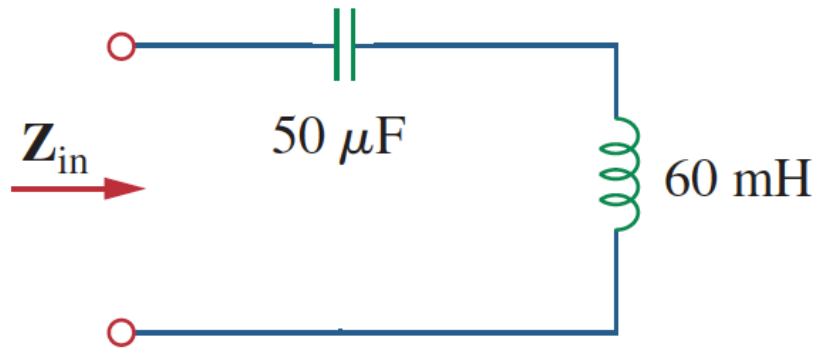
$$Z_{in} = \frac{Z_{s,left} \times Z_{s,right}}{Z_{s,left} + Z_{s,right}} = \frac{(120 - j \frac{80,000}{\omega})(40 + j60\omega)}{120 - j \frac{80,000}{\omega} + 40 + j60\omega} = \dots$$



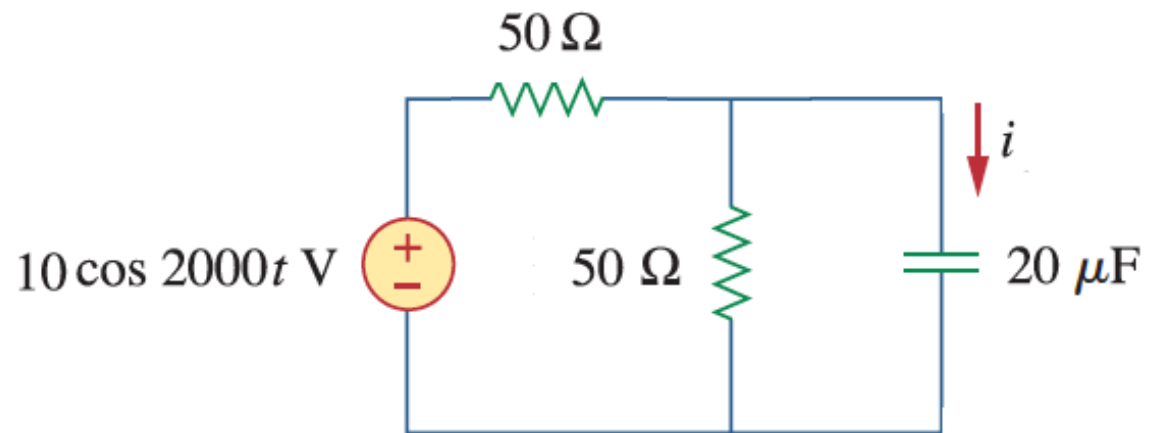
$$Z_{in} = \frac{120\omega^2 + 160 \times 10^6}{\omega^2 + 4 \times 10^6} + j \frac{160,000\omega}{\omega^2 + 4 \times 10^6}$$



Example: How do these vary with frequency?

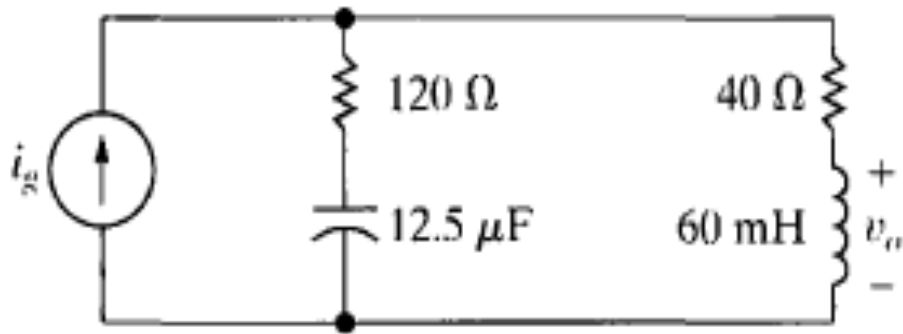


- **Example:** find $i(t)$



$$141 \cos(2000t + 45^\circ) \text{ mA}$$

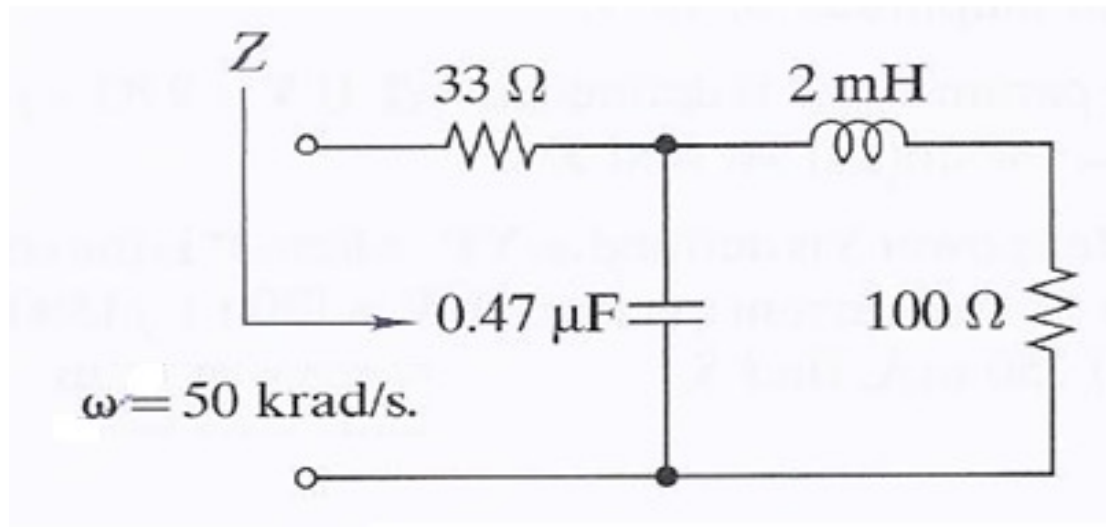
Example: find $v_o(t)$ if $i_g(t) = 500 \cos 2000 t$ mA



$$1.59 \cos(2000t - 17.0^\circ) \text{ V}$$

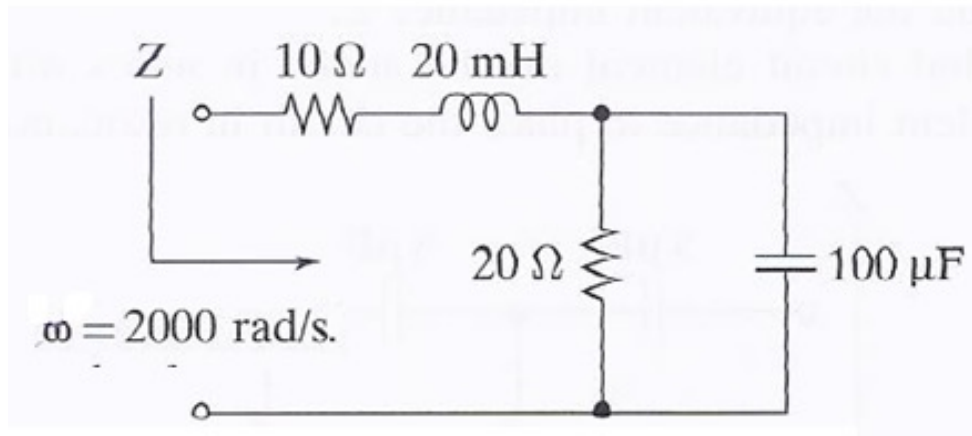
Example: find Z

$$46.4 - j50.4;$$
$$46.4 \, \Omega \quad 0.397 \, \mu F$$

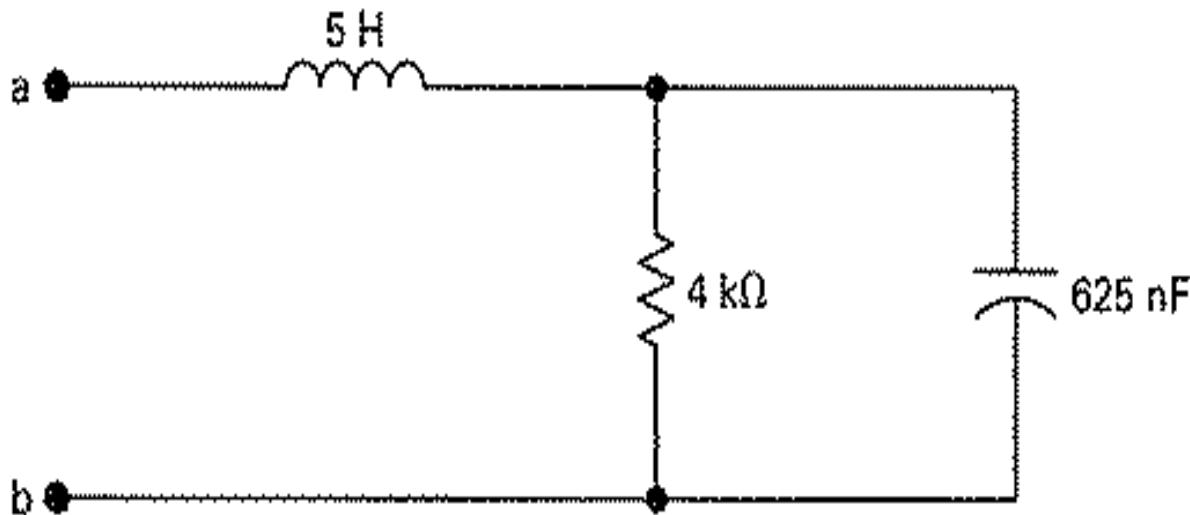


Practice problem: find Z

$11.2 + j35.3$;
 $11.2\ \Omega$ 17.6 mH



Practice problem: at what frequency does this circuit seem purely resistive? What is the resistance?



$$400 \frac{\text{rad}}{\text{sec}}; 2 \text{ k}\Omega$$

- **Practice problem:** consider the parallel connection of a $220\ \Omega$ resistor, a $0.5\ \mu\text{F}$ capacitor, and a $5\ \text{mH}$ inductor.
 - What is the equivalent impedance of this circuit at $1000\ \text{Hz}$?
 - At $5000\ \text{Hz}$?
 - At what frequency is the impedance purely real?

$11.6 + j49.2\ \Omega$; $1.42 - j17.6\ \Omega$; $1.59\ \text{kHz}$

Practice problem: Find the time expression for $v_o(t)$.

Note that $\sin \omega t = \cos(\omega t - 90^\circ)$

$17.1 \cos 200t \text{ V}$

