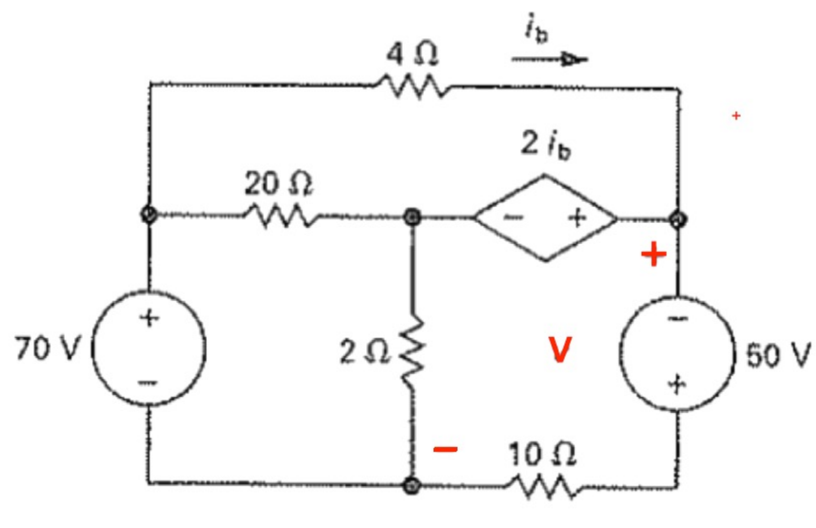
Lecture 8
Node Analysis – 1 of 7

basic concepts

General Methods to Analyze Circuits

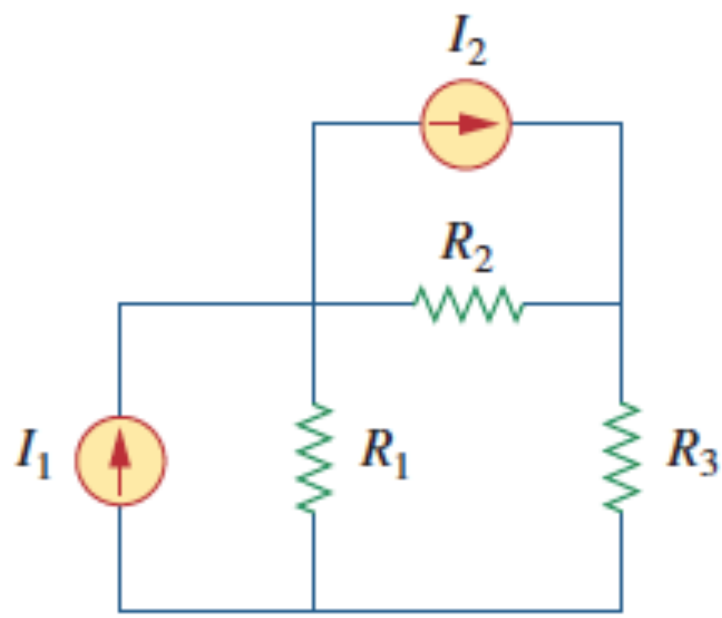
- What to do first?
  - KVL?
  - KCL?
  - Ohm's Law?



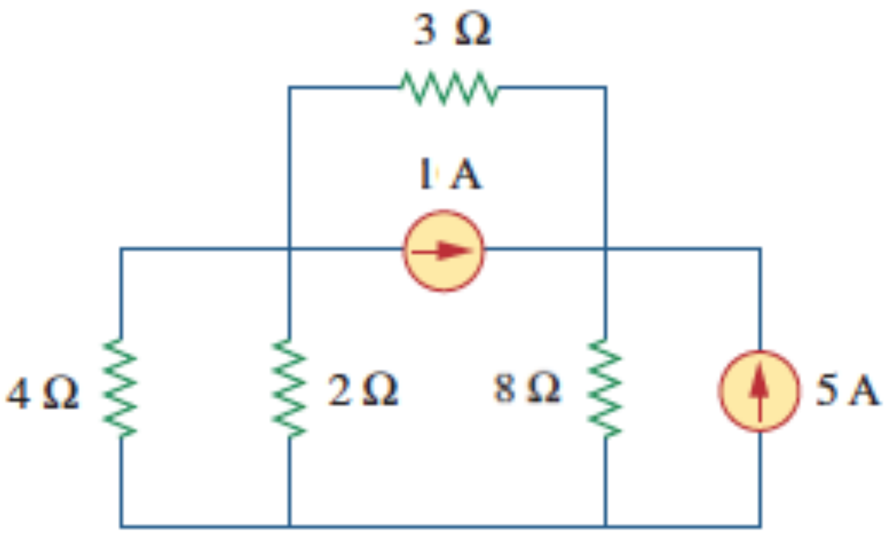
- We need a more direct approach:
  - Nodal analysis (KCL based)
  - Mesh analysis (KVL based, **end of semester**)
- Review appendix A (also online)

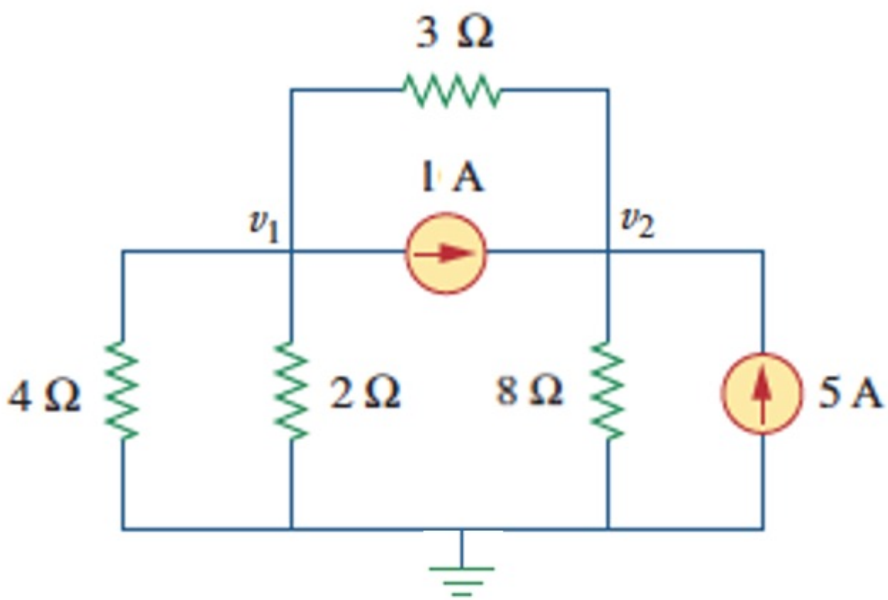
Node Analysis

- Consider a circuit with current sources and resistors only
- Identify nodes
  - Select one as "ground"
  - Label others
  - Write KCL on these other nodes
  - Use Ohm's Law for current in the resistive branches
  - Solve resulting equations



Example (details repeated on next slide)





$$\frac{v_1}{2} + \frac{v_1}{4} + \frac{v_1 - v_2}{3} + 1 = 0$$
$$\frac{v_2}{8} + \frac{v_2 - v_1}{3} = 1 + 5$$

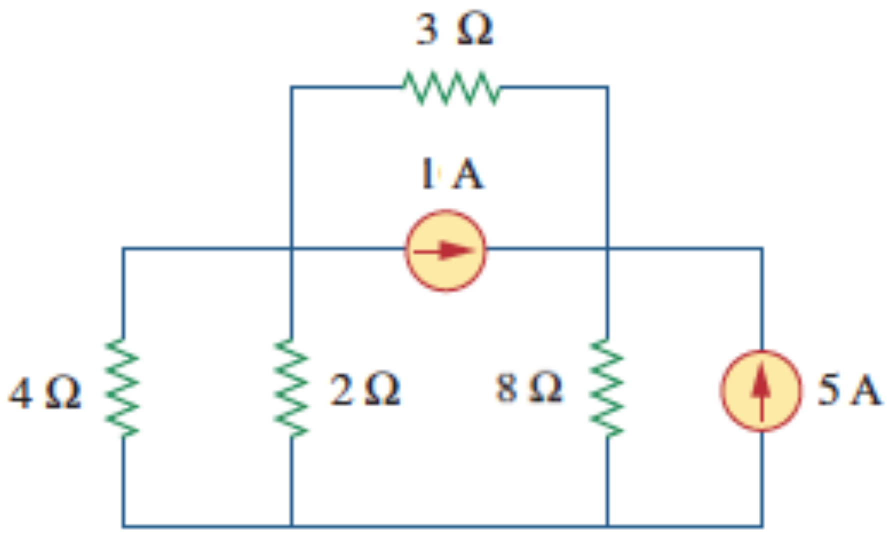
$$6v_1 + 3v_1 + 4v_1 - 4v_2 = -12$$
$$3v_2 + 8v_2 - 8v_1 = 144$$

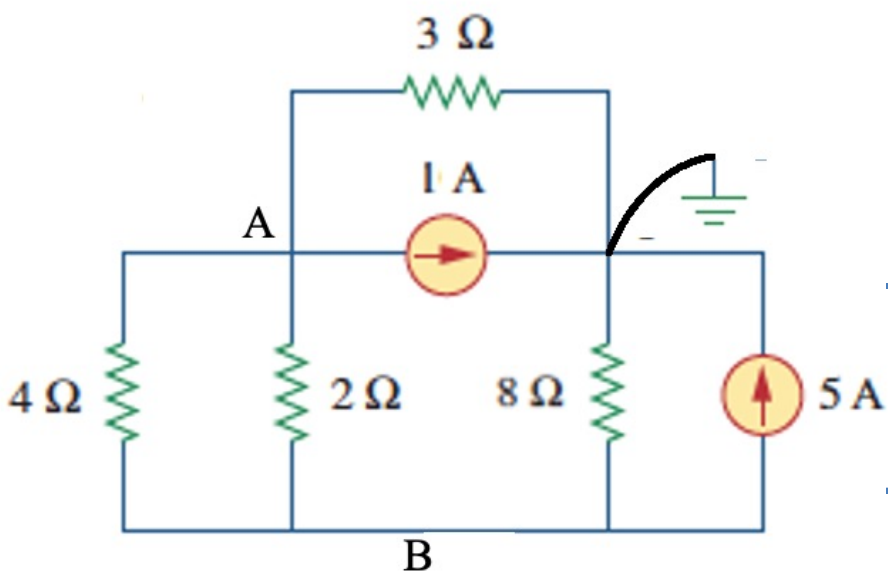
$$13v_1 - 4v_2 = -12$$
$$-8v_1 + 11v_2 = 144$$

$$v_1 = \frac{\begin{vmatrix} -12 & -4 \\ 144 & 11 \end{vmatrix}}{\begin{vmatrix} 13 & -4 \\ -8 & 11 \end{vmatrix}} = \frac{-132 + 576}{143 - 32} = \frac{444}{111} = 4 \text{ volts}$$

$$v_2 = \frac{\begin{vmatrix} 13 & -12 \\ -8 & 144 \end{vmatrix}}{\begin{vmatrix} 13 & -4 \\ -8 & 11 \end{vmatrix}} = \frac{1872 - 96}{111} = \frac{1776}{111} = 16 \text{ volts}$$

Example (same circuit, change ground to top right, details on next slide)





$$\frac{A-B}{2}+\frac{A-B}{4}+\frac{A}{3}+1 = 0$$
$$\frac{B-A}{4}+\frac{B-A}{2}+\frac{B}{8}+5 = 0$$

$$6A-6B+3A-3B+4A = -12$$
$$2B-2A+4B-4A+B = -40$$

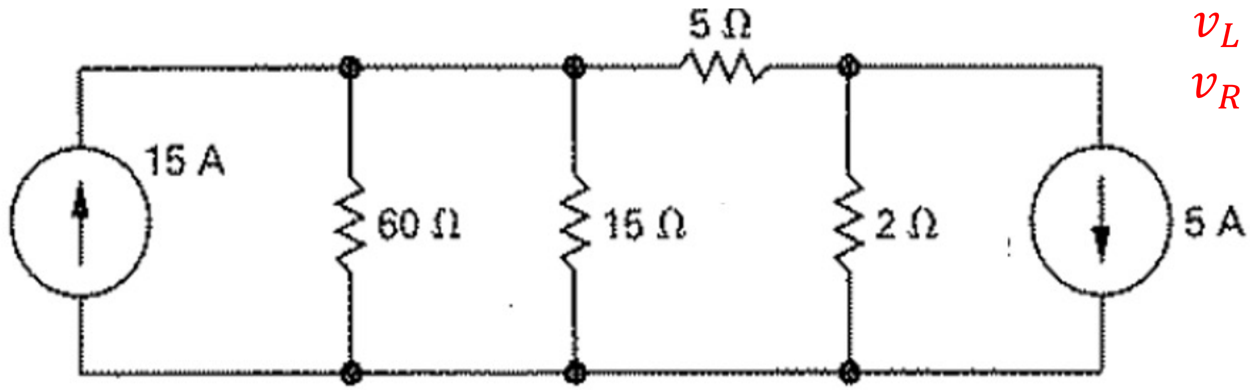
$$13A-9B = -12$$
$$-6A+7B = -40$$

$$A = \frac{\begin{vmatrix}-12 & -9\\ -40 & 7\end{vmatrix}}{\begin{vmatrix}13 & -9\\ -6 & 7\end{vmatrix}} = \frac{-84-360}{91-54} = \frac{-444}{37} = -12\text{ volts}$$

$$B = \frac{\begin{vmatrix}13 & -12\\ -6 & -40\end{vmatrix}}{\begin{vmatrix}13 & -9\\ -6 & 7\end{vmatrix}} = \frac{-520-72}{35} = \frac{-592}{37} = -16\text{ volts}$$

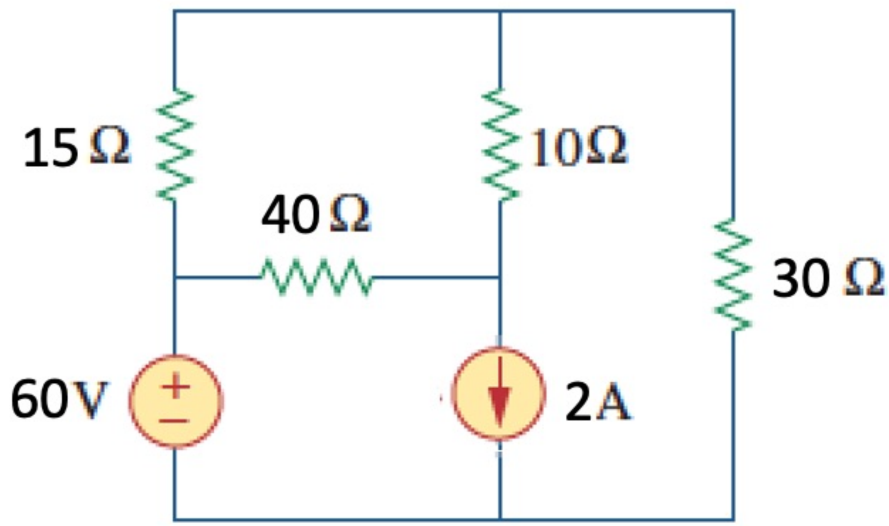
Example:

$v_L = 60\ V$
$v_R = 10\ V$



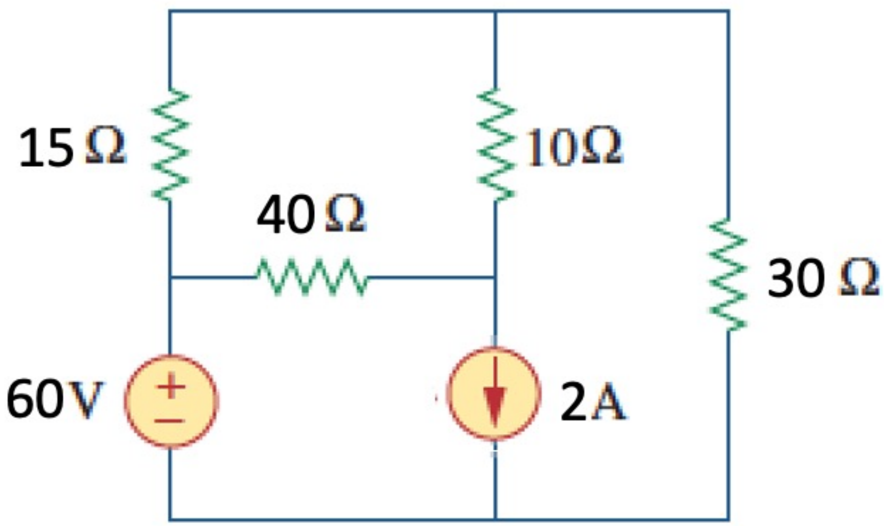
Extension #1 – a V-only branch

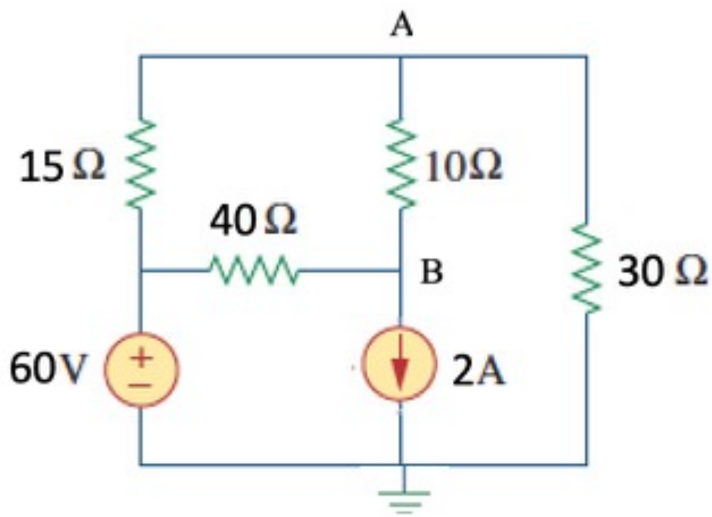
- Consider:



  - **IF** connected to ground, it's just one less node voltage to worry about

Example (details on next slide)





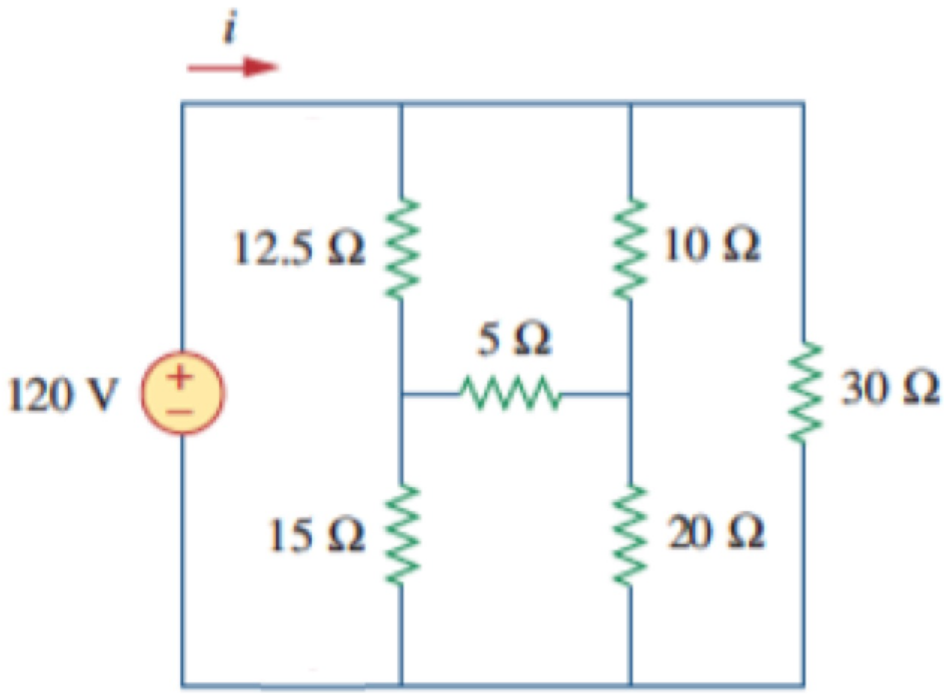
$$\frac{A-60}{15}+\frac{A-B}{10}+\frac{A}{30} = 0$$
$$\frac{B-60}{40}+\frac{B-A}{10}+2 = 0$$

$$6A-3B = 120$$
$$-4A+5B = -20$$

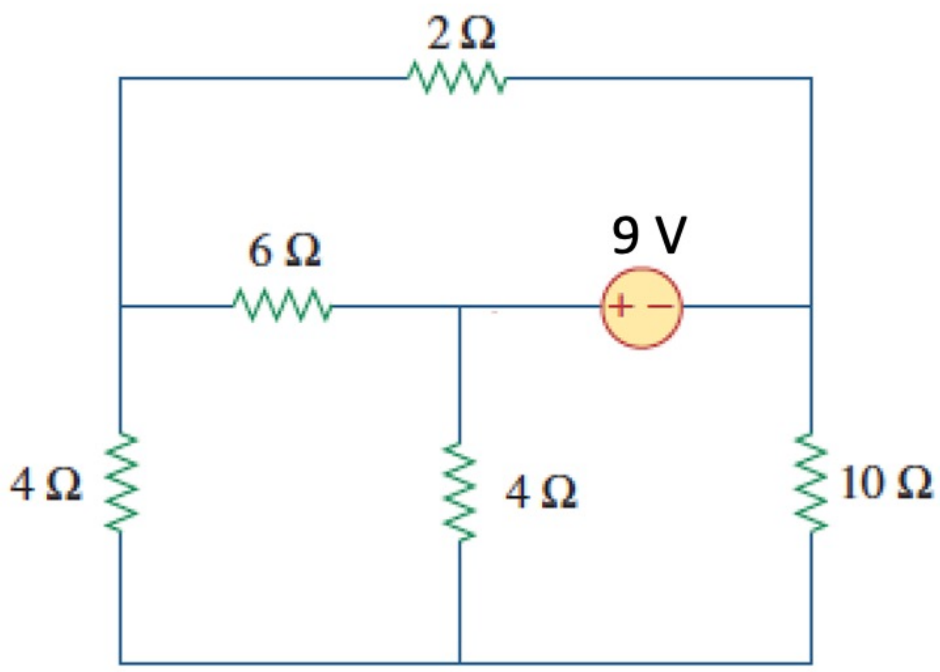
$$A = 30 \text{ volts}$$
$$B = 20 \text{ volts}$$

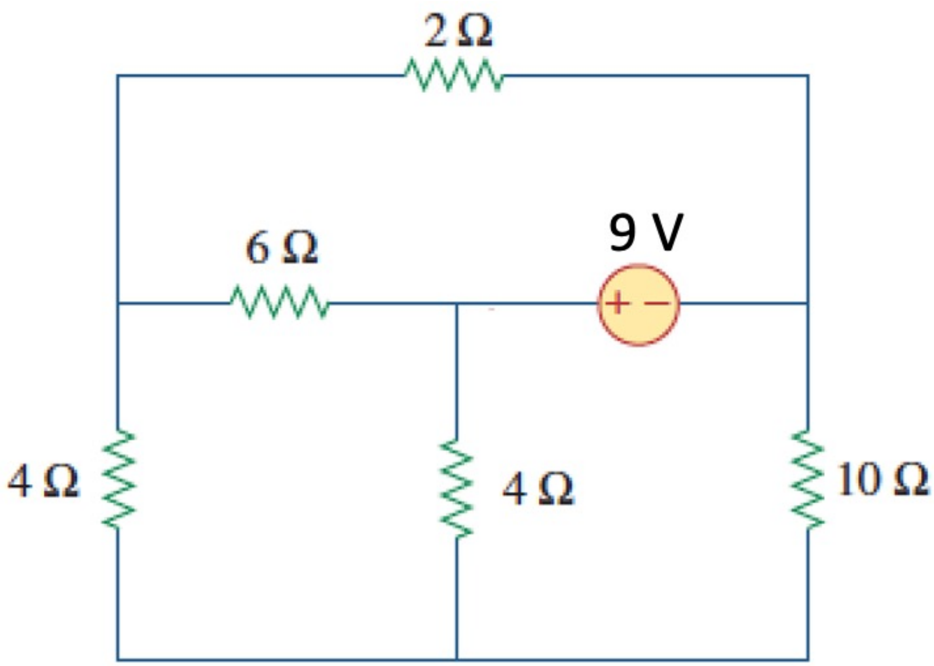
$$v_L = 70.8\ V$$
$$v_R = 74.8\ V$$
$$i = 12.5\ A$$

Example from Delta-Wye transformations



Example: recall that we can place ground as needed; put it on the right and solve for left and bottom

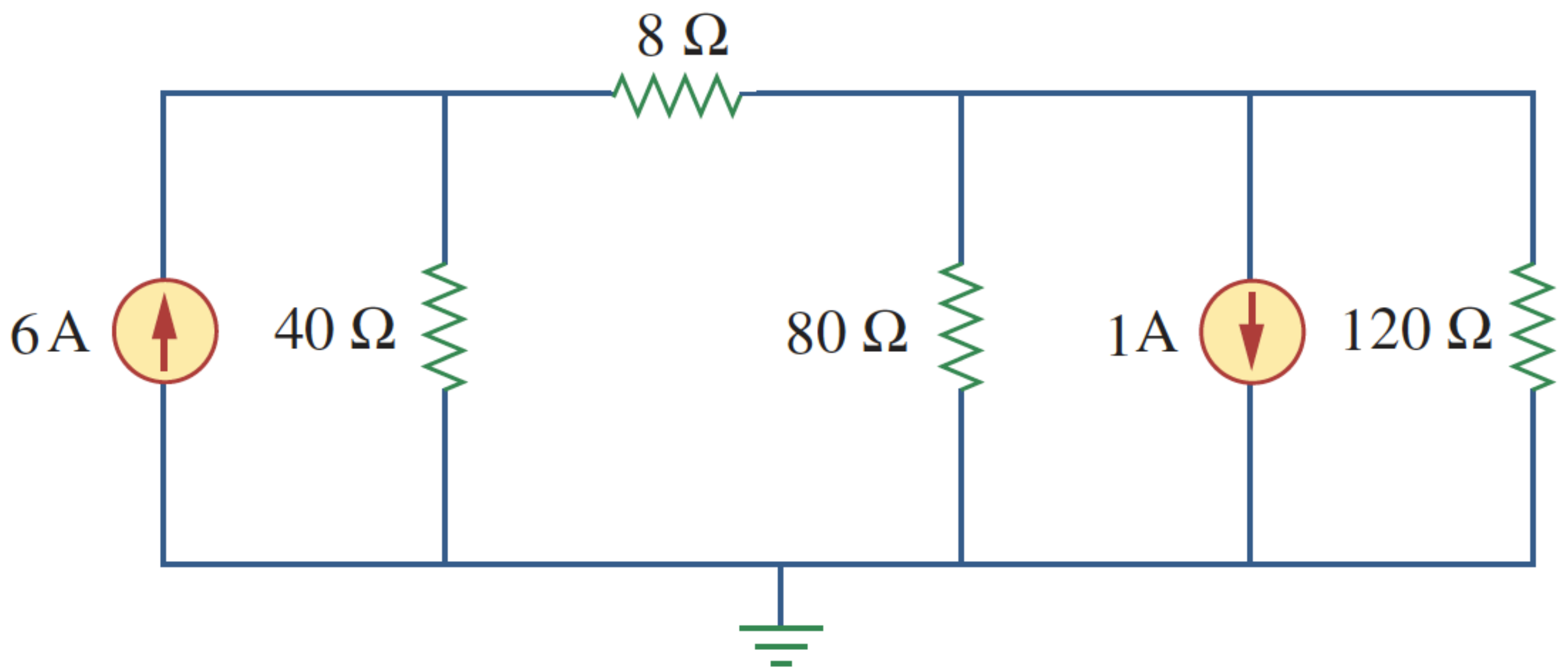




$$v_L = 3\ V$$
$$v_B = 5\ V$$

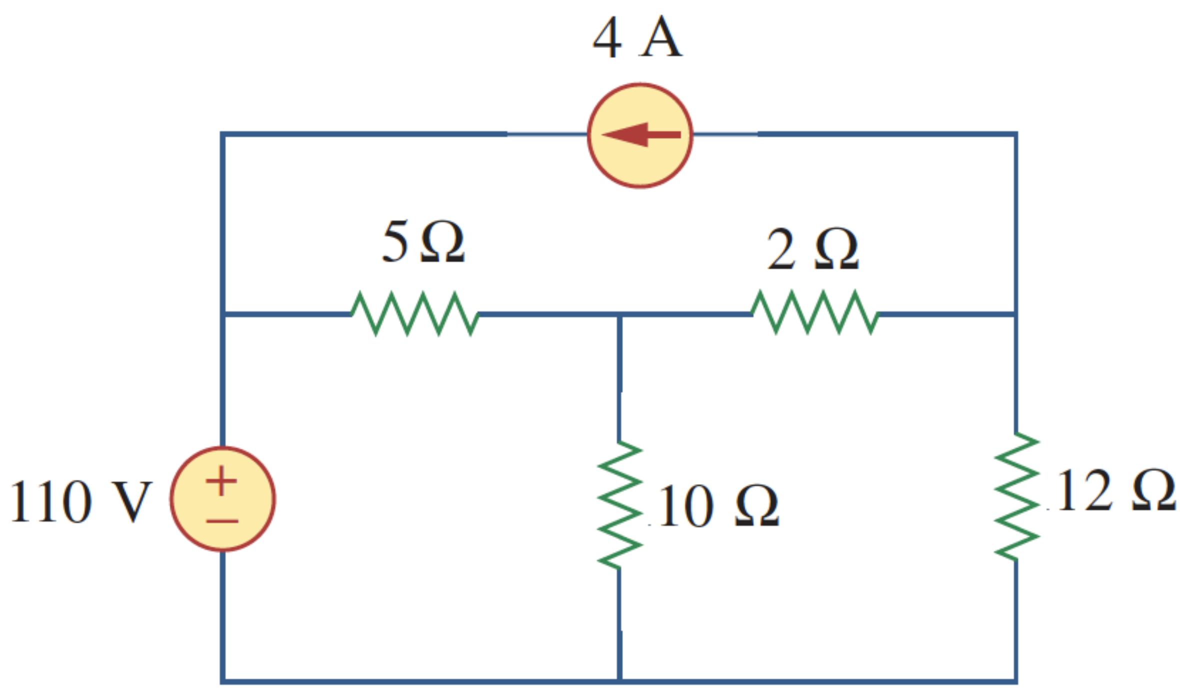
$120\ V, 96\ V$

Practice problem: find the nodes voltages to the left and right of the 8 Ω resistor



$50\ V, 36\ V$

Practice problem: assuming ground on the bottom, find the nodes voltages to the left and right of the 2 Ω resistor



$70\ V, 60\ V$

Practice problem: find the nodes voltages to the left and right of the $10\ k\Omega$ resistor, ground at the bottom