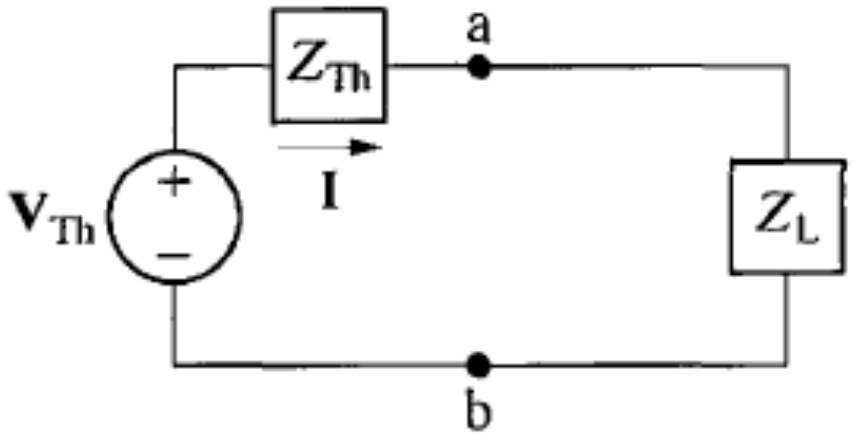
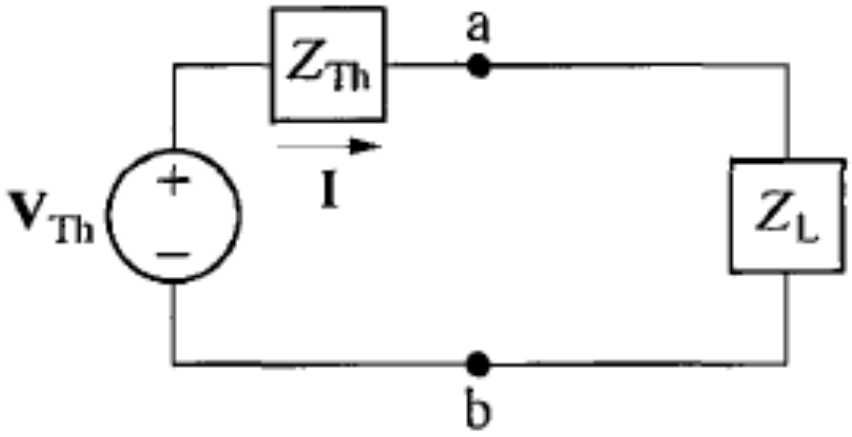
Lecture 29
Theorems – 6 of 6

max power with phasors

Maximum AC Power

- Given a phasor Thevenin model, how do we get maximum power to Z_L?

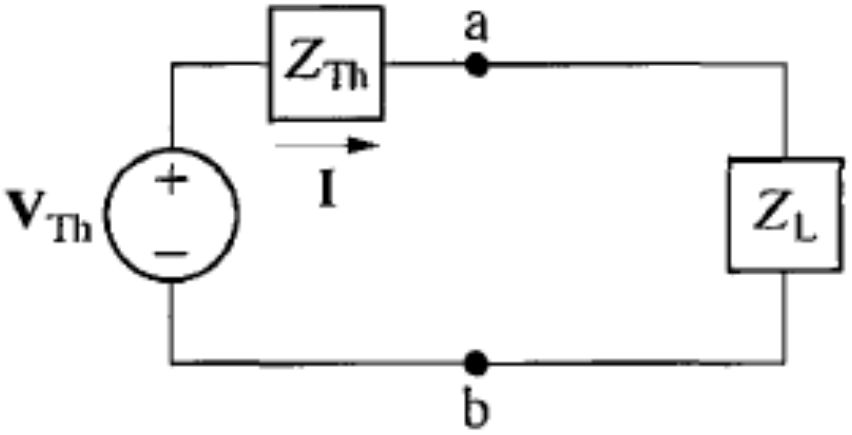




- As previously seen (lecture 21), for sinusoidal sources and RLC circuits, power is

$$\text{Complex power} \qquad S = \frac{|\mathbf{I}|^2}{2}\mathrm{Z_L}$$

$$\text{Real power} \qquad P = \frac{|\mathbf{I}|^2}{2}\mathrm{R_L}$$



- For our problem

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathrm{Z_{Th}} + \mathrm{Z_L}} = \frac{\mathbf{V}_{Th}}{(\mathrm{R_{Th}} + \mathrm{R_L}) + j(\mathrm{X_{Th}} + \mathrm{X_L})}$$

- So

$$P = \frac{|\mathbf{I}|^2}{2}\mathrm{R_L} = \frac{1}{2}\frac{|\mathbf{V}_{Th}|^2 \mathrm{R_L}}{(\mathrm{R_{Th}} + \mathrm{R_L})^2 + (\mathrm{X_{Th}} + \mathrm{X_L})^2}$$

$$P = \frac{1}{2}\frac{|\mathbf{V}_{\mathrm{Th}}|^2 \mathrm{R_L}}{(\mathrm{R_{Th}} + \mathrm{R_L})^2 + (\mathrm{X_{Th}} + \mathrm{X_L})^2}$$

- We can optimize this using calculus
- How depends upon which parameters we can change

$$P = \frac{1}{2}\frac{|\mathbf{V}_{\mathrm{Th}}|^2 \mathrm{R_L}}{(\mathrm{R_{Th}} + \mathrm{R_L})^2 + (\mathrm{X_{Th}} + \mathrm{X_L})^2}$$

- Case 1 (unusual): both R_L and X_L are free to choose

$$\frac{\partial P}{\partial X_L} = 0 \qquad \frac{\partial P}{\partial R_L} = 0$$

$$Z_L = Z_{\mathrm{Th}}{}^*$$

$$P_{max} = \frac{|\mathbf{V}_{\mathrm{Th}}|^2}{8R_{Th}}$$

$$P = \frac{1}{2}\frac{|\mathbf{V}_{\mathrm{Th}}|^2 \mathrm{R_L}}{(\mathrm{R_{Th}} + \mathrm{R_L})^2 + (\mathrm{X_{Th}} + \mathrm{X_L})^2}$$

- Case 2: X_L is fixed, but R_L is free to choose

$$\frac{\partial P}{\partial R_L} = 0$$

$$\mathrm{R_L} = \sqrt{\mathrm{R_{Th}}^2 + (\mathrm{X_{Th}} + \mathrm{X_L})^2}$$

$$P_{max} = \cdots$$

$$P = \frac{1}{2}\frac{|\mathbf{V}_{\mathrm{Th}}|^2 \mathrm{R_L}}{(\mathrm{R_{Th}} + \mathrm{R_L})^2 + (\mathrm{X_{Th}} + \mathrm{X_L})^2}$$

- Case 3: R_L is fixed, but X_L is free to choose

$$\frac{\partial P}{\partial X_L} = 0$$

$$\mathrm{X_L} = -\mathrm{X_{Th}}$$

$$P_{max} = \cdots$$

$$P = \frac{1}{2}\frac{|\mathbf{V}_{\mathrm{Th}}|^2 \mathrm{R_L}}{(\mathrm{R_{Th}} + \mathrm{R_L})^2 + (\mathrm{X_{Th}} + \mathrm{X_L})^2}$$

- Case 4: X_L is 0, but R_L is free to choose

$$\frac{\partial P}{\partial R_L} = 0$$

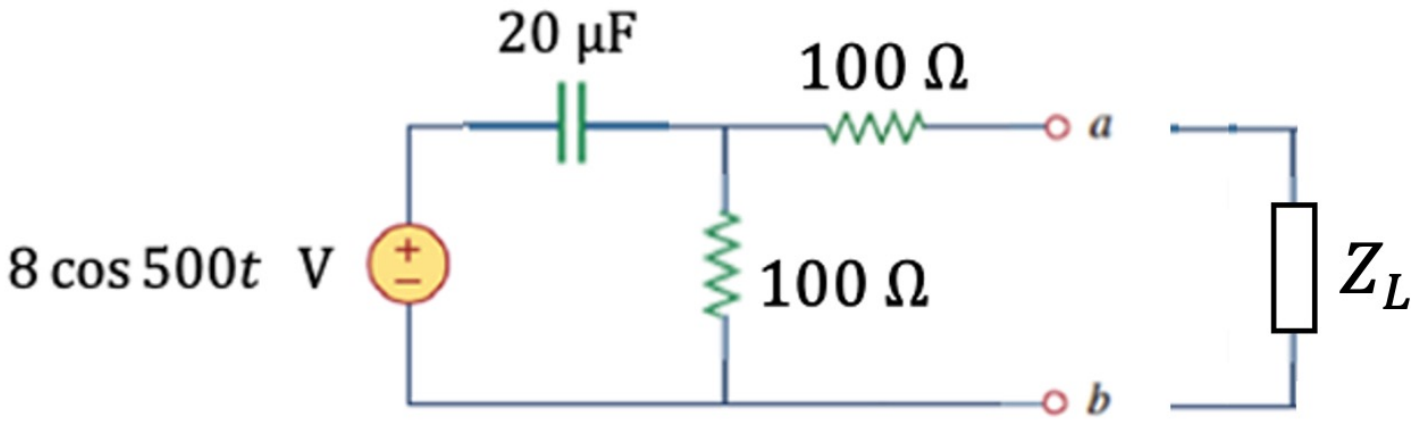
$$\mathrm{R_L} = \sqrt{\mathrm{R_{Th}}^2 + \mathrm{X_{Th}}^2}$$

$$P_{max} = \cdots$$

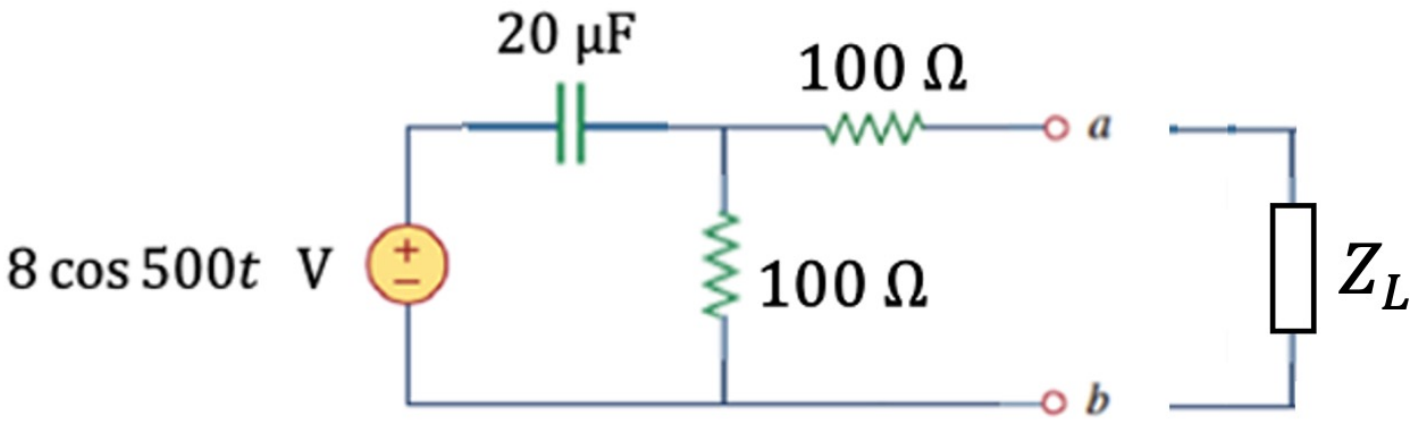
$$P = \frac{1}{2}\frac{|\mathbf{V}_{\mathrm{Th}}|^2 \mathrm{R_L}}{(\mathrm{R_{Th}} + \mathrm{R_L})^2 + (\mathrm{X_{Th}} + \mathrm{X_L})^2}$$

- Other scenarios:
  - Fixed angle on Z_L
  - Limits on $\mathrm{R_L}$ and $\mathrm{X_L}$

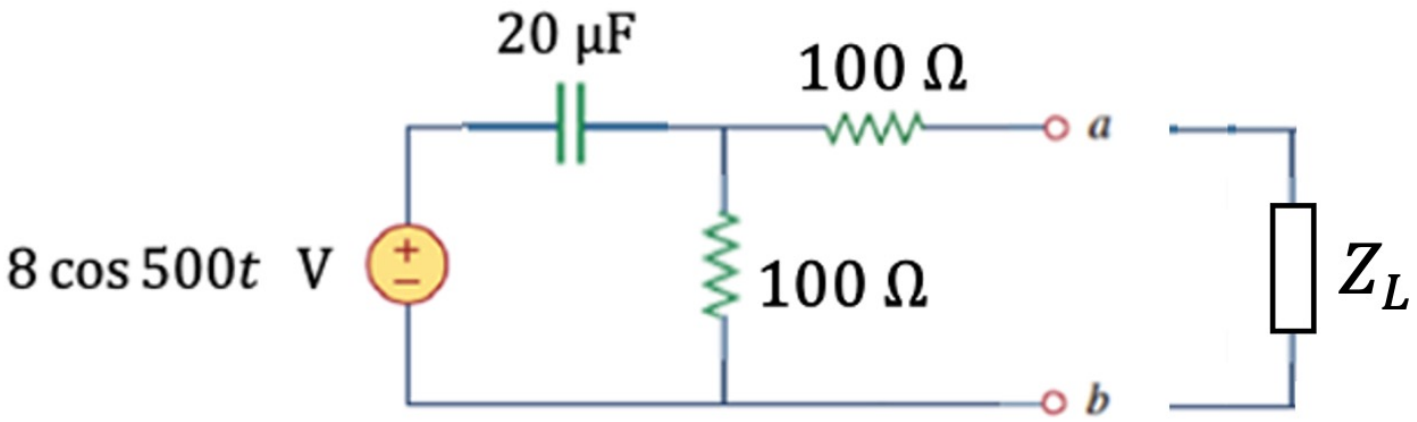
Example: find Z_L to maximize the power transfer



If Z_L is free to choose or must be resistive only

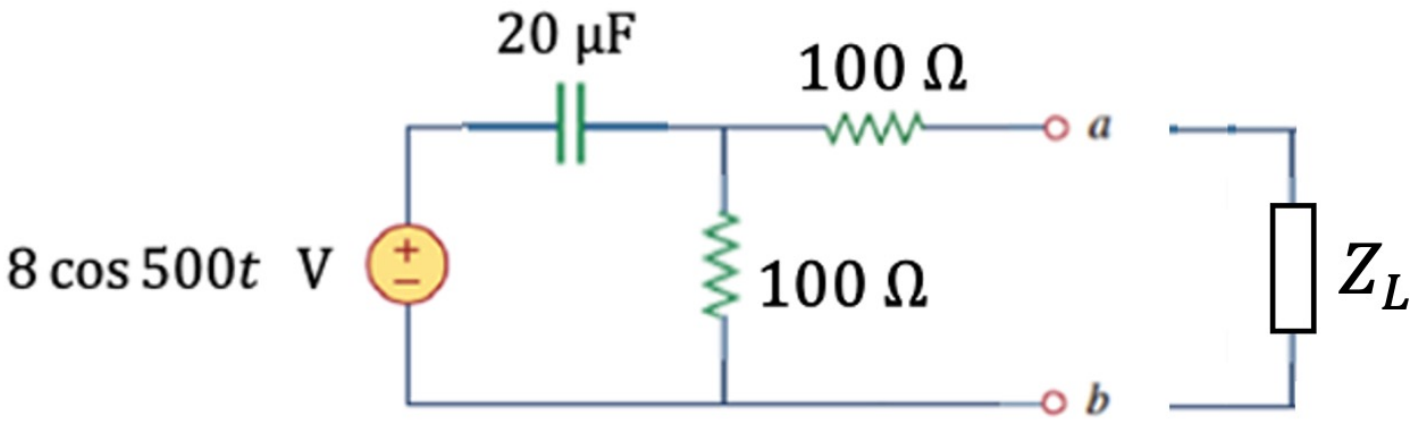


$V_{Th} = 4\sqrt{2}\ \cos(500t + 45°)\ V$ and $Z_{Th} = 150 - j50\ \Omega$,



- If $Z_L = R_L + jX_L$

$Z_L = 150 + j50\ \Omega = 150\ \Omega, 0.1\ H$, and $P = 26.7\ mW$



- If $Z_L = R_L$

$Z_L = 158\ \Omega$ and $P = 26.0\ mW$