Lecture 14
# Node Analysis – 7 of 7

op amp circuit analysis

(an example of circuit approximation)
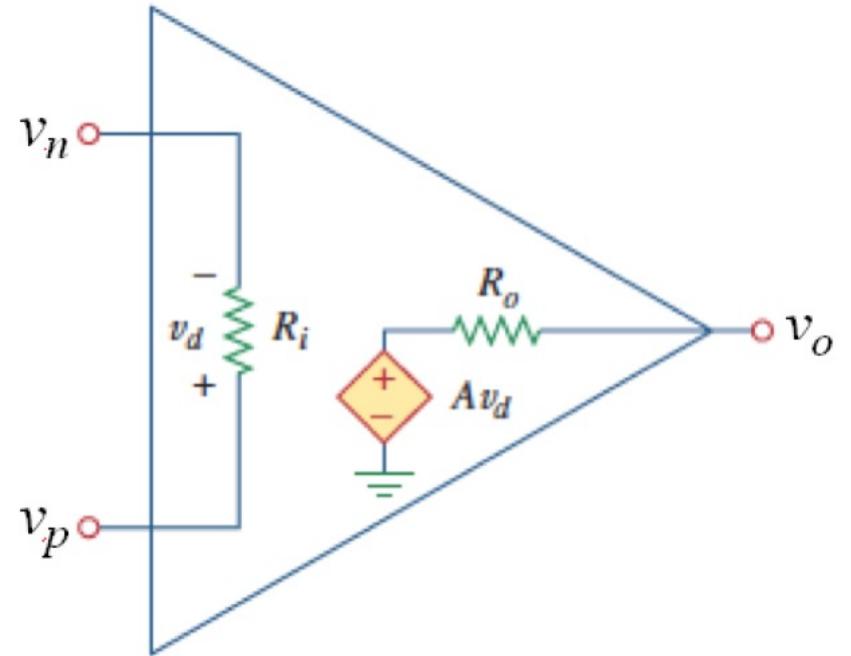
Op Amps

- A 3-terminal device
  - Inverting input v_n
  - Non-inverting input v_p
 - Output v_o
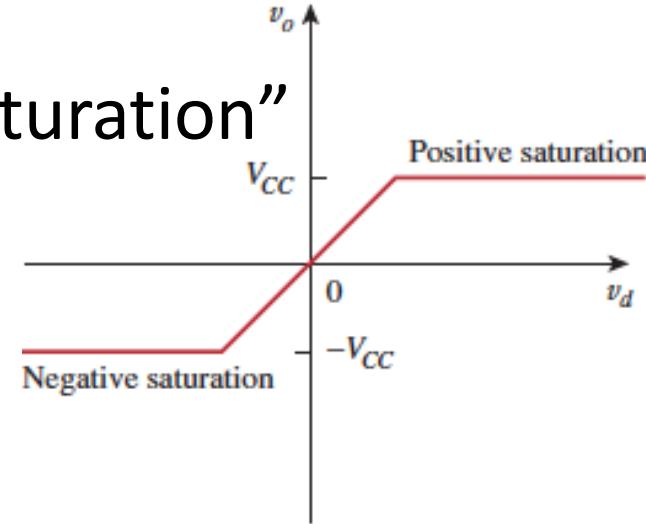
- Is a powered device, not unlike gates in ELE 201/202
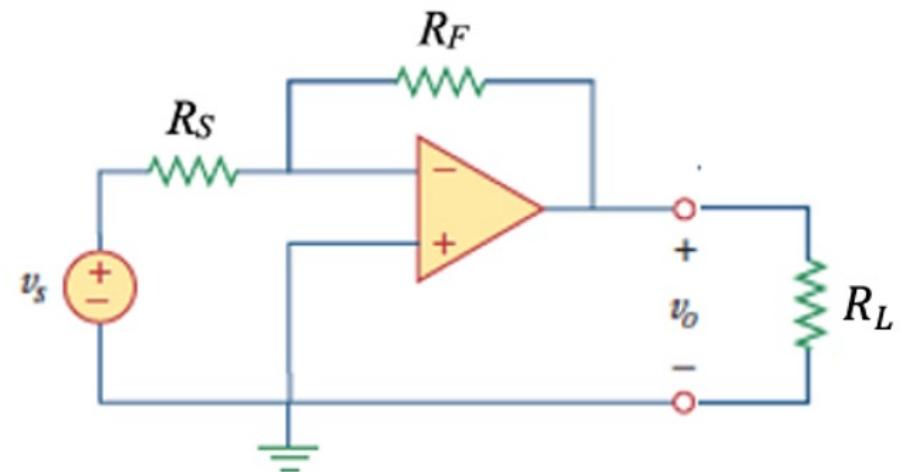
- Nominally behaves like a voltage dependent voltage source:
  - Control voltage is $v_d = v_p - v_n$
  - "Open loop" gain A is large
  - Two resistors:
    - "Input resistance" R_i is large
   - "Output resistance" R_o is small
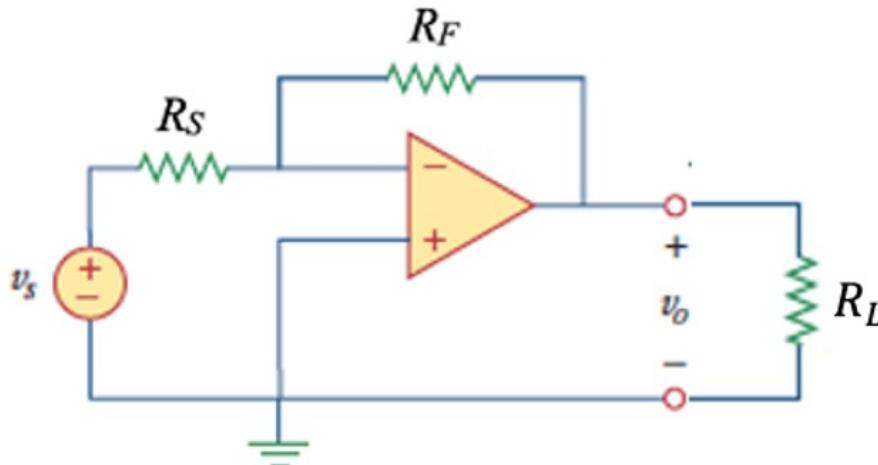
- Experiences open loop "saturation"
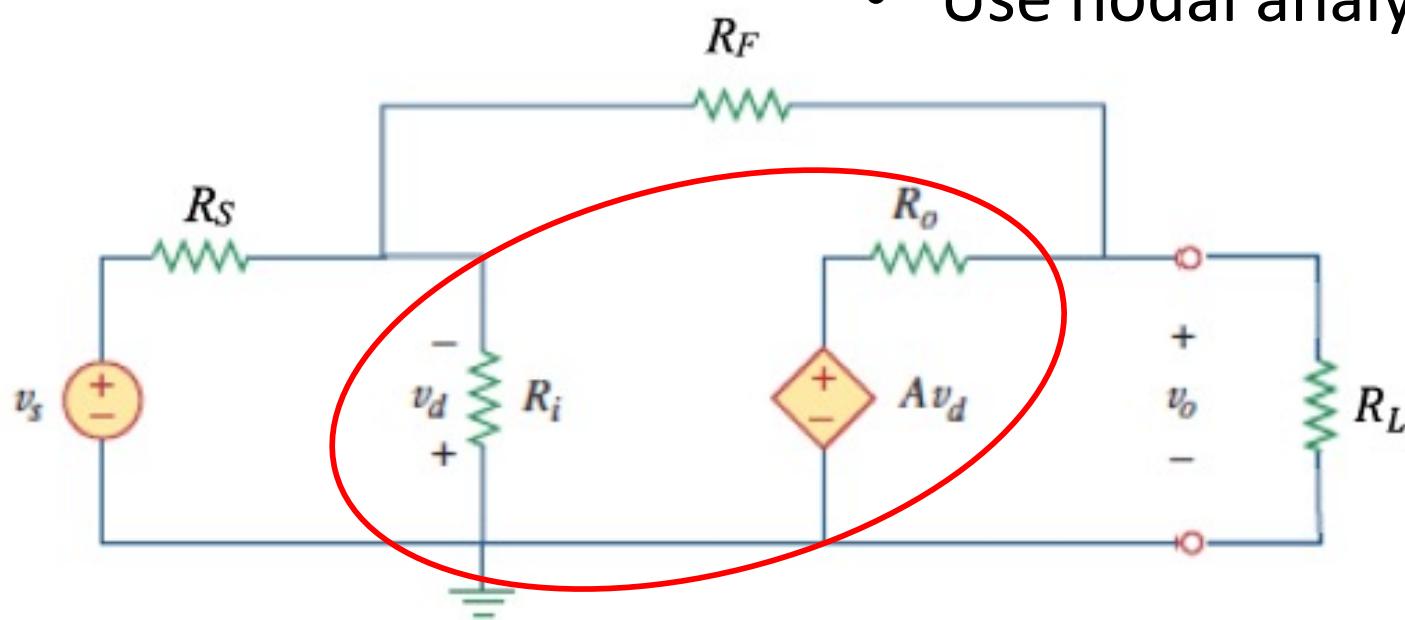

  - Usually configured with "negative feedback"
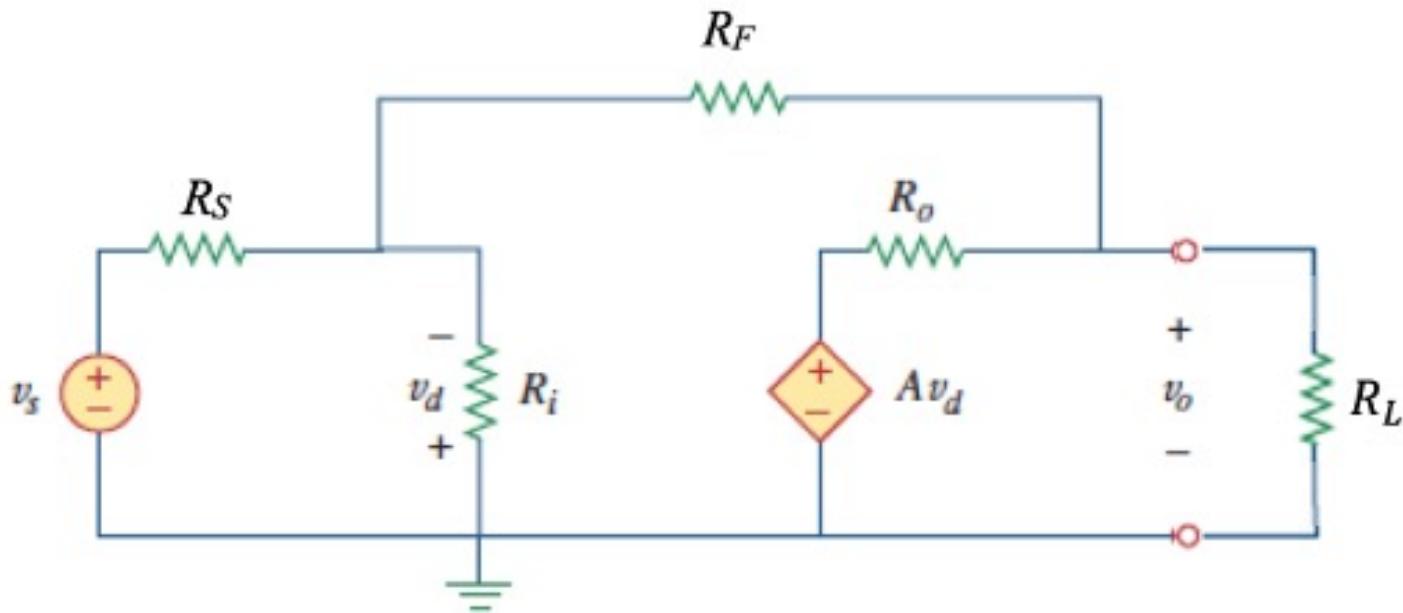
# Sample Analysis

- Circuit with "source", "feedback", and "load" resistors

- Substitute the simple model for the op amp

- Use nodal analysis
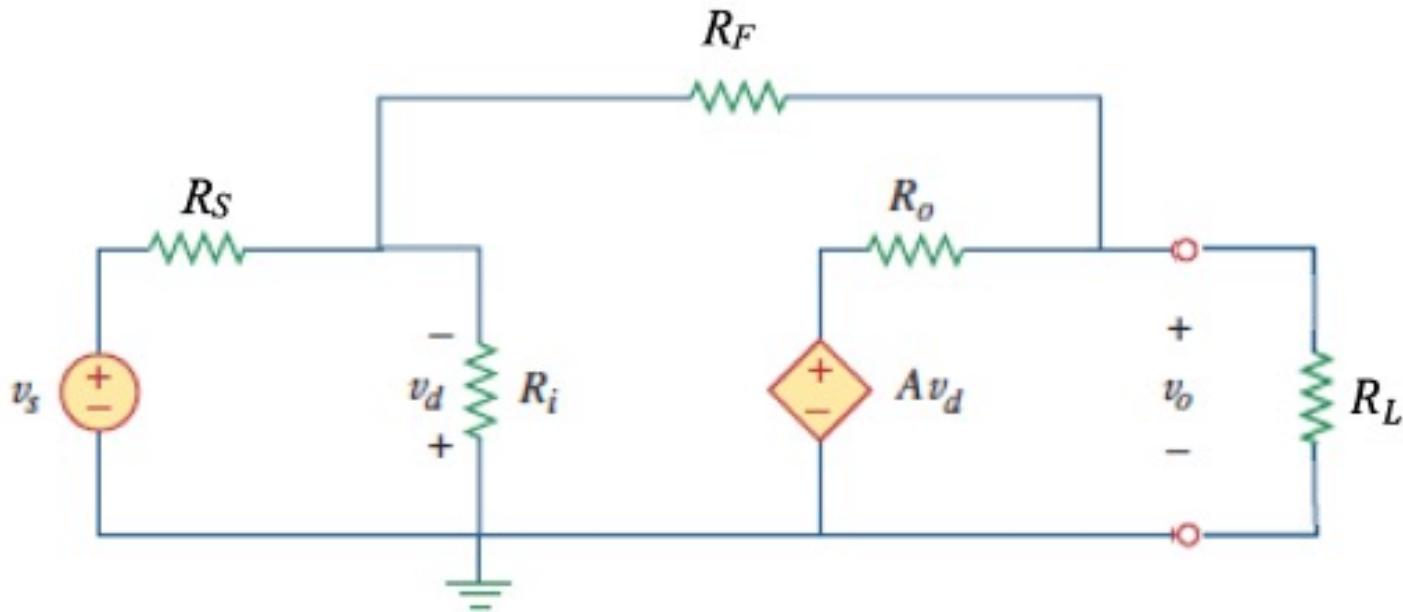
- **Nodes**: ground, v_o , and $v_1 = -v_d$

$$\frac{v_1}{R_i} + \frac{v_1 - v_S}{R_S} + \frac{v_1 - v_o}{R_F} = 0$$
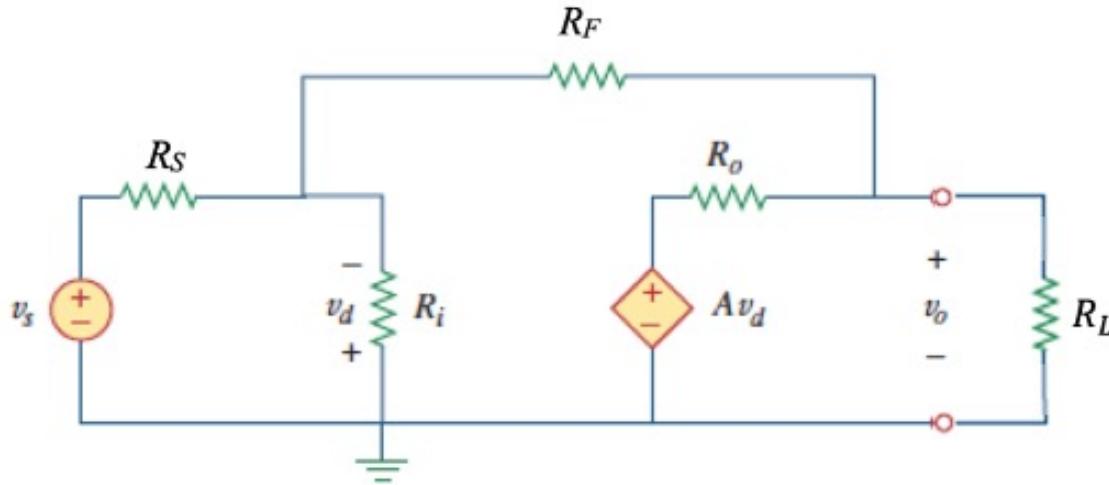
$$\frac{v_o + Av_1}{R_o} + \frac{v_o - v_1}{R_F} + \frac{v_o}{R_L} = 0$$

- After some tedious, symbolic algebra:

$$v_o = -\frac{R_i R_L (R_F A - R_o)}{R_L (R_S + R_i)(R_F + R_o) + (A + 1)R_S R_i R_L + R_o (R_S R_i + R_F R_i + R_S R_F)} v_s$$

- Note linearity, gain term is negative ("inverting amplifier")

- External components:
  - $R_S = 10\ k\Omega$
  - $R_F = 20\ k\Omega$
  - $R_L = 40\ k\Omega$

- Nominal model parameters:
  - $R_i = 2\ M\Omega$
  - $R_o = 50\ \Omega$
  - $A = 200{,}000$

$$v_o = -1.99997\ v_S = -2.00\ v_S$$
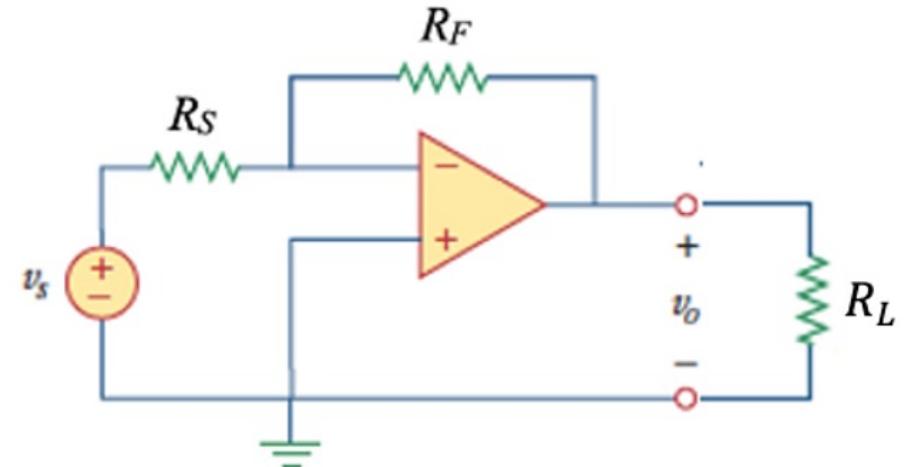
Sensitivity Analysis

For $R_S = 10\ k\Omega$,
$R_F = 20\ k\Omega$, and
$R_L = 40\ k\Omega$
we have $v_0 = -2.00\ v_s$

– How do the op amp's internal parameters $(R_i, R_o, \text{and } A)$ impact this result?

– What happens when we change the circuit's components $(R_S, R_F, \text{and } R_L)$?

Sensitivity to internal values:

R_S	R_F	R_L	R_i	R_o	A	gain
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	$200,000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	**$1\ \text{M}\Omega$**	$50\ \Omega$	$200,000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	**$4\ \text{M}\Omega$**	$50\ \Omega$	$200,000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	**$25\ \Omega$**	$200,000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	**$100\ \Omega$**	$200,000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	**$100,000$**	**-1.99994**
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	**$400,000$**	**-1.99998**
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	**$50,000$**	**-1.99988**
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	**$20,000$**	**-1.99970**
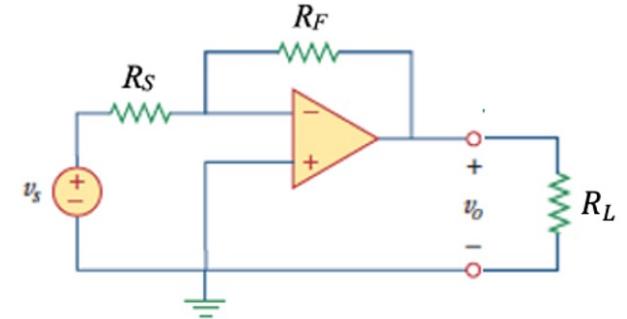
Seems insensitive!

Sensitivity to circuit component values:

R_S	R_F	R_L	R_i	R_o	A	gain
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	-1.99997
$10\ k\Omega$	$20\ k\Omega$	**$20\ k\Omega$**	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	-1.99997
$10\ k\Omega$	$20\ k\Omega$	**$80\ k\Omega$**	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	-1.99997
$10\ k\Omega$	**$10\ k\Omega$**	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-0.99999**
$10\ k\Omega$	**$40\ k\Omega$**	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-3.99990**
$10\ k\Omega$	**$100\ k\Omega$**	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-9.99945**
$5\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-3.99990**
$20\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-0.99999**
$40\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-0.499996**
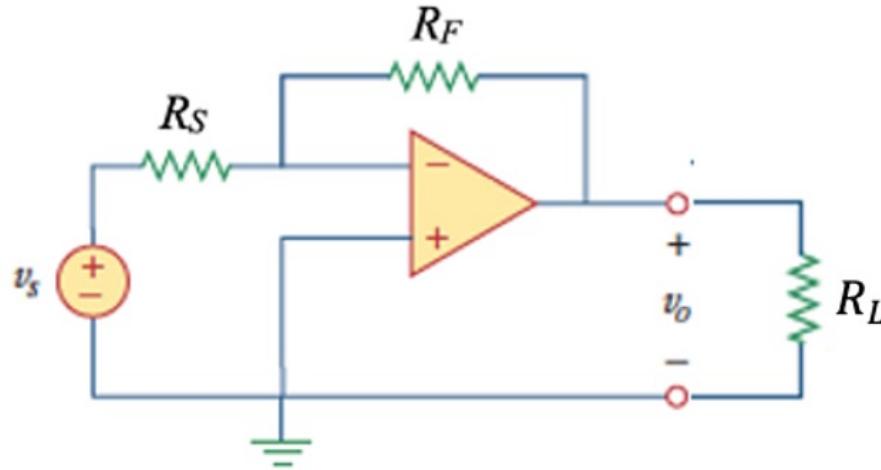
**Only depends upon source and feedback resistors!**

Asymptotics

- Consider the gain term when A is large, R_i is large, and R_o is small

$$\frac{v_o}{v_s}$$

$$= -\frac{R_i R_L (R_F A - R_o)}{R_L (R_S + R_i)(R_F + R_o) + (A + 1)R_S R_i R_L + R_o(R_S R_i + R_F R_i + R_S R_F)}$$

$$\approx -\frac{R_i R_L R_F A}{R_L R_i R_F + A R_S R_i R_L + R_o R_S R_i + R_o R_F R_i}$$

$$\approx -\frac{R_L R_F A}{R_L R_F + A R_S R_L + R_o R_S + R_o R_F} \approx -\frac{R_F}{R_S}$$

- So:

$$\frac{v_o}{v_s} \approx -\frac{R_F}{R_S}$$

- Simularly :
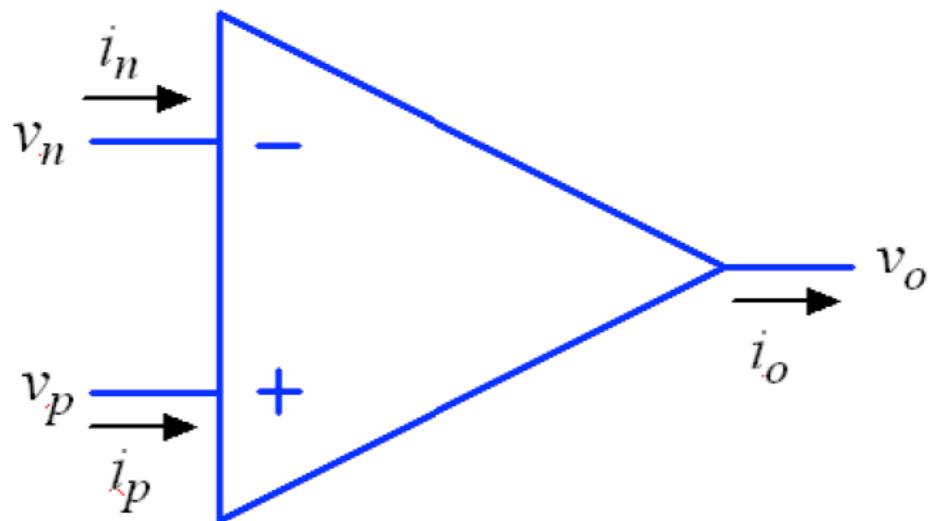
$$\frac{v_d}{v_s} \approx 0 \qquad \frac{i_d}{v_s} \approx 0$$

More general model:
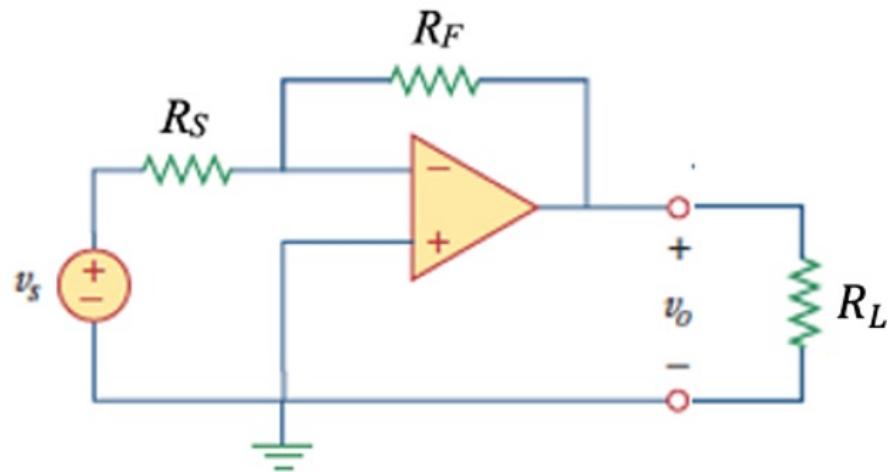
- $v_d \approx 0$
  - we have a "virtual short" across the inputs
    $$v_p = v_n$$

- $i_d \approx 0$
  - the inputs are open circuits
    $$i_p = i_n = 0$$

- And $i_o = ?$
 - The device generates whatever current is needed
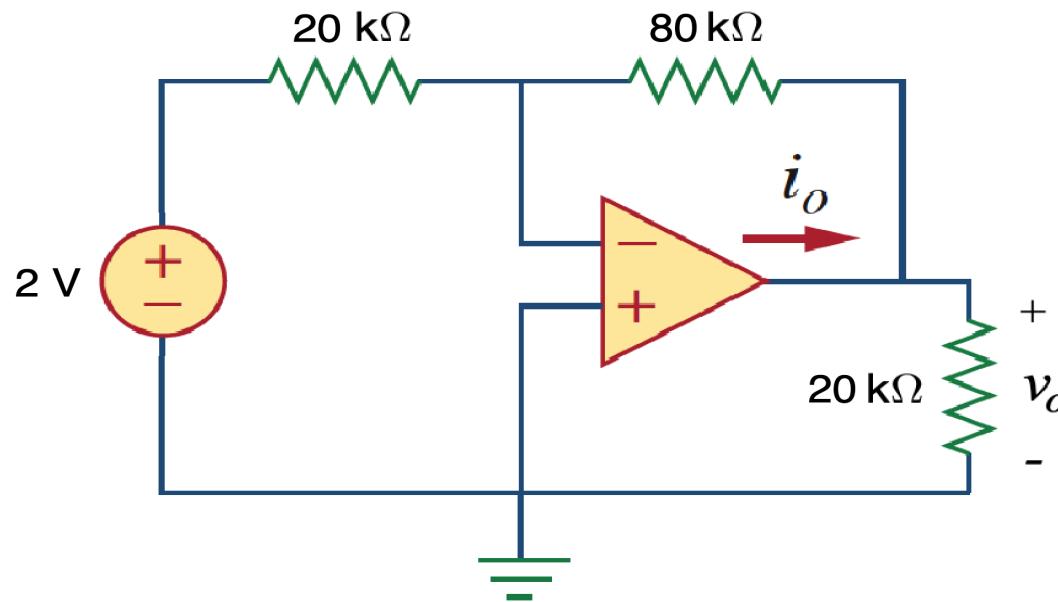
Using the Simple Model

Example: find v_o and i_o

$$v_o = -8\,V, i_0 = -0.5\,mA$$
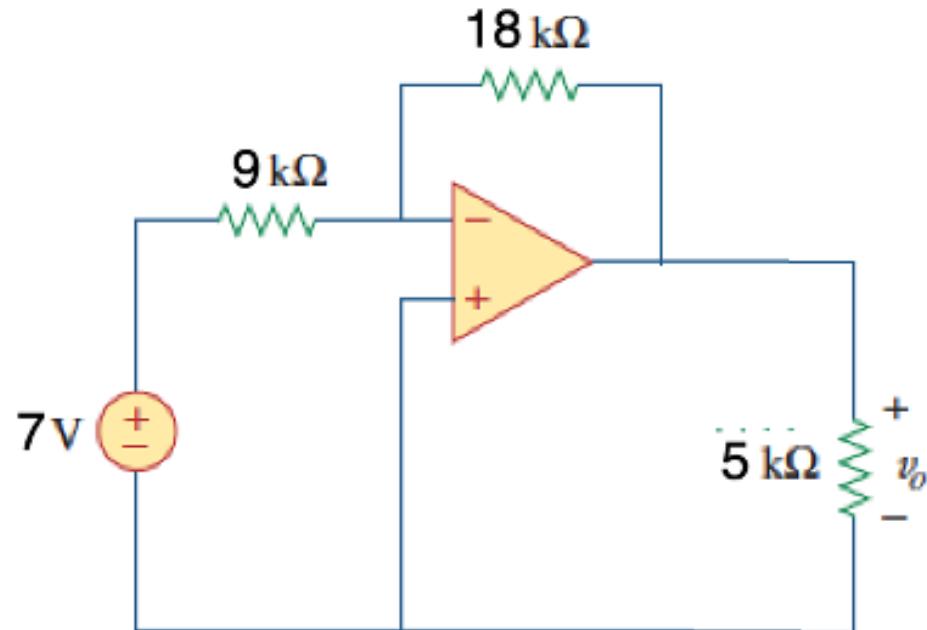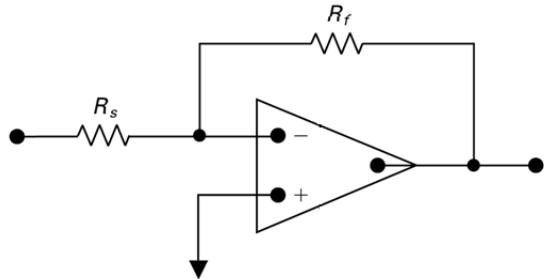
Example: find v_o

$$v_o = -14\,V$$

18 kΩ

9 kΩ

7V

5 kΩ v_o
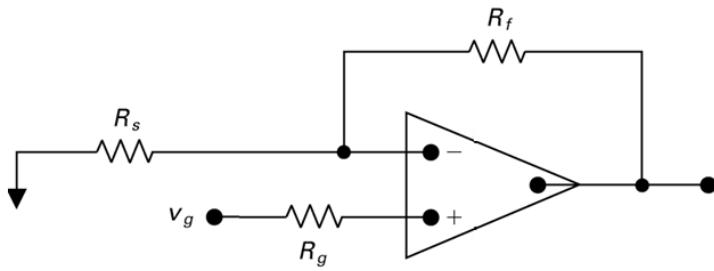
Other Op Amp Uses

- ## Inverting amplifier

$$v_o = -\frac{R_F}{R_S} v_s$$

- ## Difference amplifier
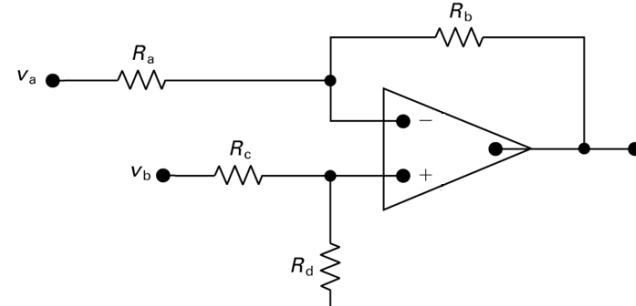
$$v_o = \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right)v_2 - \frac{R_2}{R_1}v_1$$

- ## Non-inverting amplifier

$$v_o = \left(1 + \frac{R_F}{R_1}\right)v_s$$

- ## Summing amplifier

$$v_o = -\left(\frac{R_F}{R_1}v_1 + \frac{R_F}{R_2}v_2 + \frac{R_F}{R_3}v_3\right)$$

Practice problem: find i