

# Lecture 17

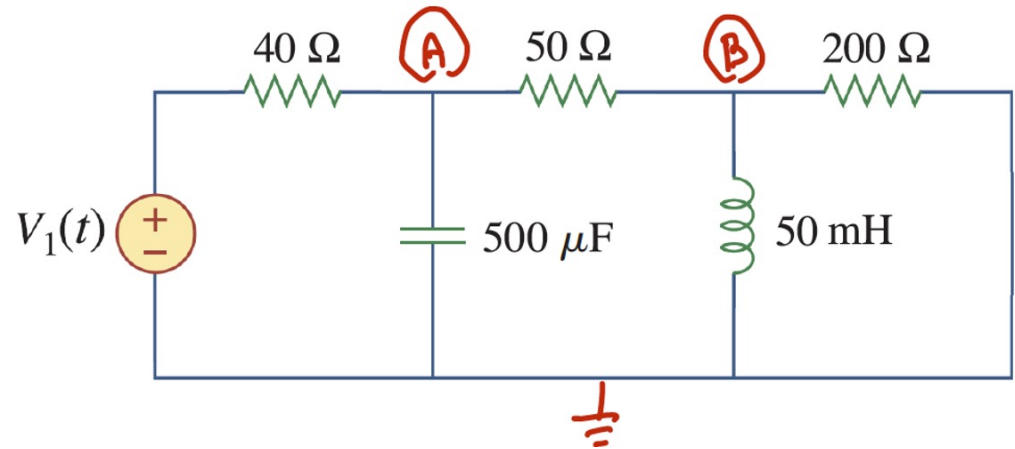
## Phasors – 3 of 9

how phasors help

# Reconsider our Example

- Use a phasor model for the source (and again, just watch)

$$V_1(t) = \mathbf{V}e^{j\omega t}$$

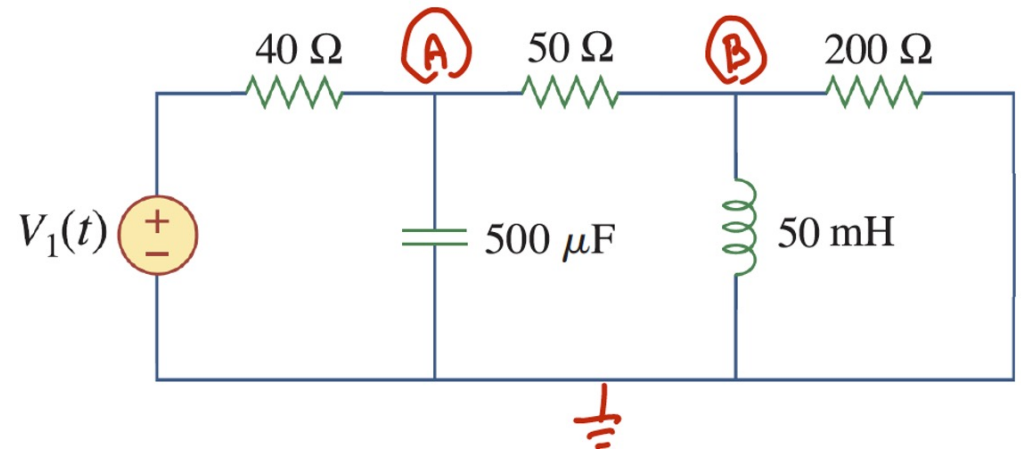


$$\frac{A(t) - V_1(t)}{40} + i_C(t) + \frac{A(t) - B(t)}{50} = 0$$

$$\frac{B(t)}{200} + i_L(t) + \frac{B(t) - A(t)}{50} = 0$$

- If our forcing function is

$$V_1(t) = \mathbf{V}e^{j\omega t}$$



- Then the particular (steady-state) solution is also a complex exponential

**Table 2.1** Method of Undetermined Coefficients

Term in $r(x)$	Choice for $y_p(x)$
$ke^{\gamma x}$	$Ce^{\gamma x}$
$kx^n$ ( $n = 0, 1, \dots$ )	$K_n x^n + K_{n-1} x^{n-1} + \dots + K_1 x + K_0$
$k \cos \omega x$	$\left\{ K \cos \omega x + M \sin \omega x \right\} = \mathbf{K \cos (\omega t + \theta)}$
$k \sin \omega x$	

- Phasor notation:

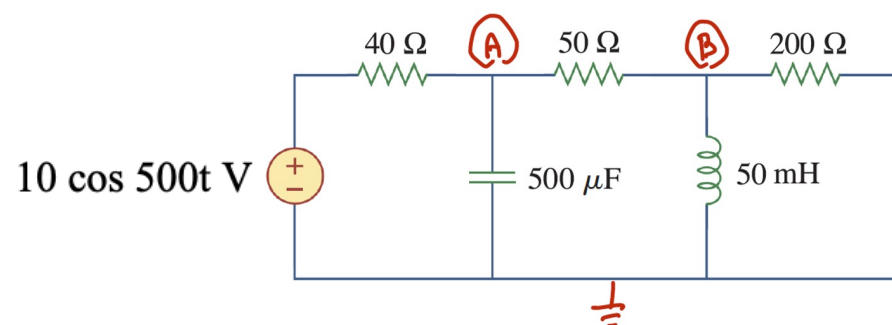
$$\begin{array}{lll}
 A(t) \Rightarrow \mathbf{A}e^{j500t} & B(t) \Rightarrow \mathbf{B}e^{j500t} & V_1(t) \Rightarrow 10e^{j500t} \\
 \mathbf{A} = Ae^{j\phi} & \mathbf{B} = Be^{j\theta} & \mathbf{V}_1 = 10
 \end{array}$$

- Consider the capacitor and inductor currents with this general model; if the voltage is  $v(t) = \mathbf{V}e^{j\omega t}$  then

$$i_C(t) = C \frac{dv}{dt} = C \frac{d}{dt} (\mathbf{V}e^{j\omega t}) = j\omega C \mathbf{V}e^{j\omega t} = j\frac{1}{4} \mathbf{V}e^{j\omega t}$$

$$i_L(t) = \frac{1}{L} \int^t v(u) du = \frac{1}{L} \int^t \mathbf{V}e^{j\omega u} du = \frac{\mathbf{V}e^{j\omega t}}{j\omega L} = \frac{\mathbf{V}e^{j\omega t}}{j25}$$

- Back to the original node equations

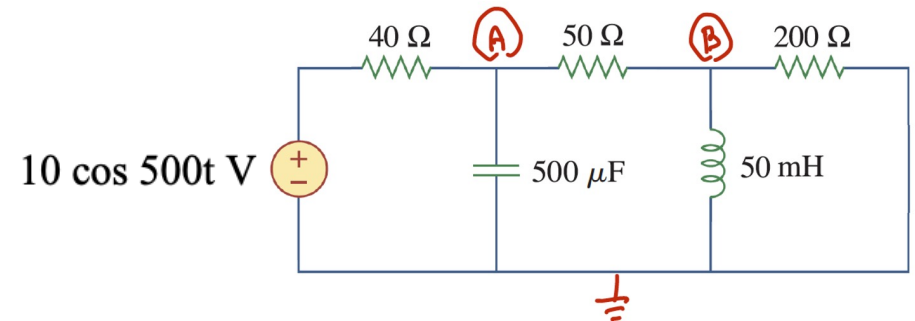


$$\frac{\mathbf{A}e^{j500t} - 10e^{j500t}}{40} + j\frac{1}{4}\mathbf{A}e^{j500t} + \frac{\mathbf{A}e^{j500t} - \mathbf{B}e^{j500t}}{50} = 0$$

$$\frac{\mathbf{B}e^{j500t}}{200} + \frac{\mathbf{A}e^{j500t}}{j25} + \frac{\mathbf{B}e^{j500t} - \mathbf{A}e^{j500t}}{50} = 0$$

- Can cancel the common  $e^{j500t}$  term

- Result is a set of simultaneous equations with complex coefficients



$$\frac{\mathbf{A} - 10}{40} + j \frac{\mathbf{A}}{4} + \frac{\mathbf{A} - \mathbf{B}}{50} = 0$$

$$\frac{\mathbf{B}}{200} + \frac{\mathbf{A}}{j25} + \frac{\mathbf{B} - \mathbf{A}}{50} = 0$$

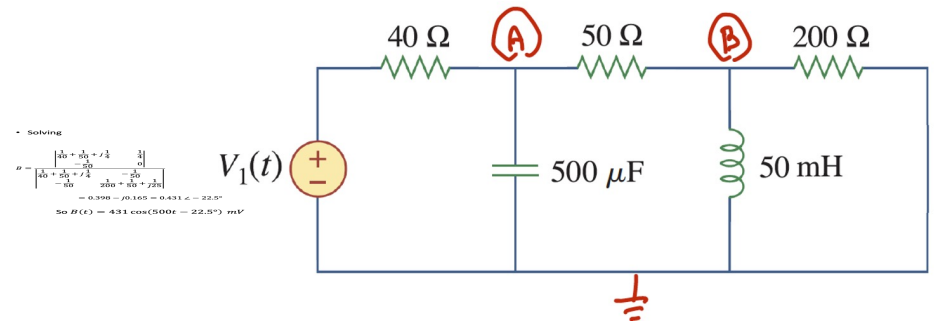
- Solve using various methods (Cramer, matrix inverse, etc)

- Solving

$$B = \frac{\begin{vmatrix} \frac{1}{40} + \frac{1}{50} + j\frac{1}{4} & \frac{1}{4} \\ -\frac{1}{50} & 0 \end{vmatrix}}{\begin{vmatrix} \frac{1}{40} + \frac{1}{50} + j\frac{1}{4} & -\frac{1}{50} \\ -\frac{1}{50} & \frac{1}{200} + \frac{1}{50} + \frac{1}{j25} \end{vmatrix}}$$

$$= 0.398 - j0.165 = 0.431 \angle -22.5^\circ$$

$$\text{So } B(t) = 431 \cos(500t - 22.5^\circ) \text{ mV}$$



**Same as last time !!**

# So How to Use Phasors?

- Extend sinusoidal voltages/currents to phasors (complex)
- Convert components (R,L,C) to impedances
- Solve the problem using Ohm's Law, KVL/KCL, ...
- Convert back

$$V_s \cos(\omega t + \phi) \Rightarrow \mathbf{V} = V_s e^{j\phi}$$

$$I_s \cos(\omega t + \phi) \Rightarrow \mathbf{I} = I_s e^{j\phi}$$

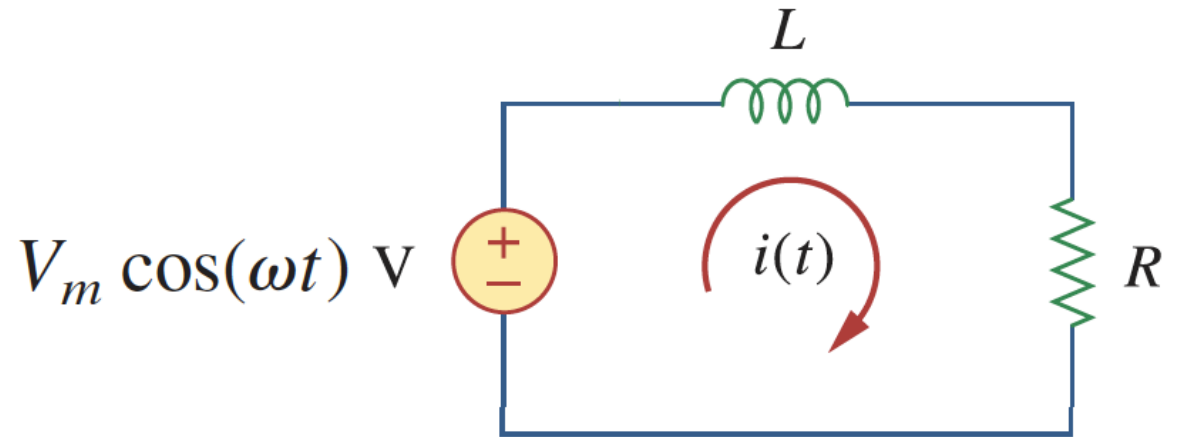
$$Z = \begin{cases} R & \text{resistor} \\ j\omega L & \text{inductor} \\ \frac{1}{j\omega C} = -j \frac{1}{\omega C} & \text{capacitor} \end{cases}$$

$$B \angle \theta \Rightarrow B \cos(\omega t + \theta)$$



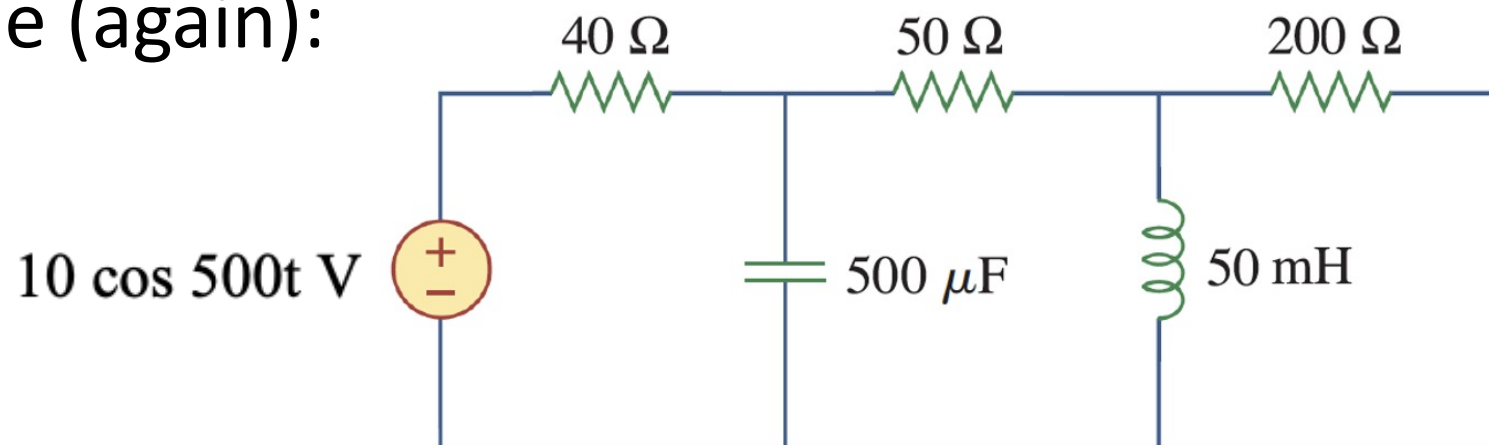
## Example:

- Convert problem
- Combine in series
- Apply Ohm's Law
- Convert back

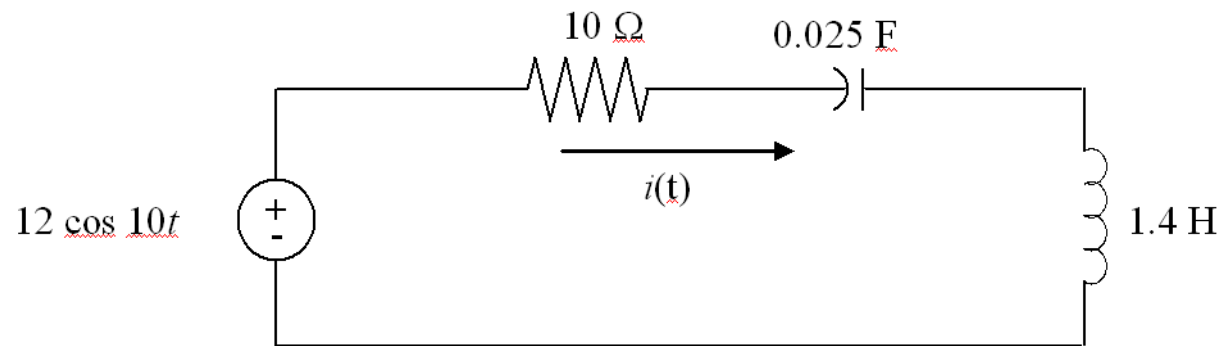


$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R}\right)$$

Example (again):



**Example:** find the current  $i(t)$  (answer on next slides)

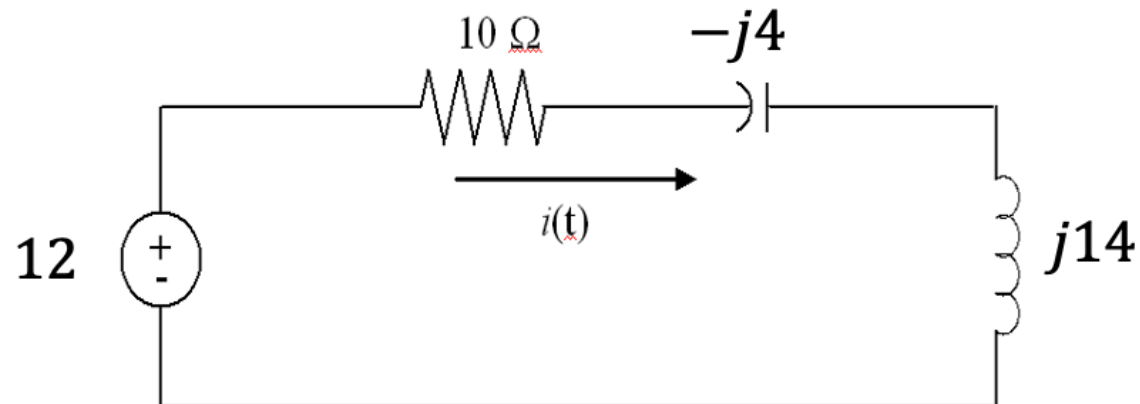


- **Step 1:** Convert to phasors and impedances

$$12 \cos 10t \rightarrow 12$$

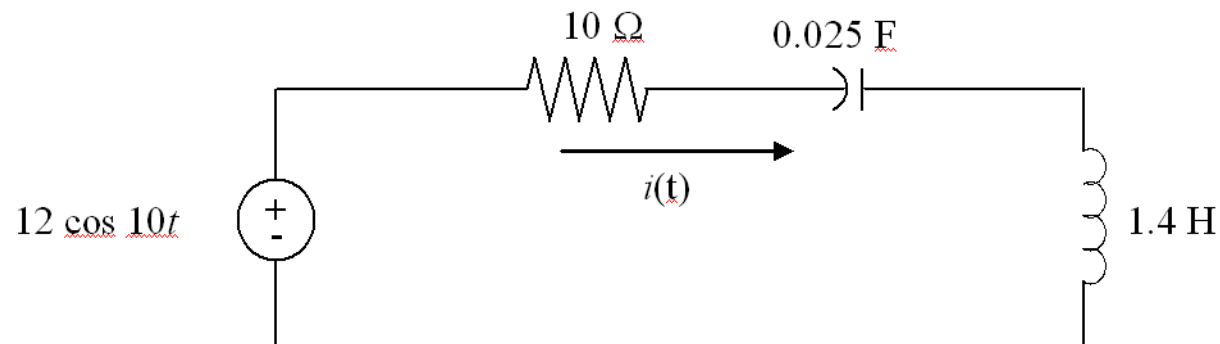
$$1.4 \text{ H} \rightarrow j\omega L = j(10)(1.4) = j14$$

$$0.025 \text{ F} \rightarrow -j \frac{1}{\omega C} = -j \frac{1}{(10)(0.025)} = -j4$$



- **Step 2:** Solve
  - Find the current by series impedance

$$I = \frac{12}{10 - j4 + j14} = \frac{12}{10 + j10} = \frac{12}{10\sqrt{2} \angle 45^\circ} = 0.848 \angle -45^\circ$$

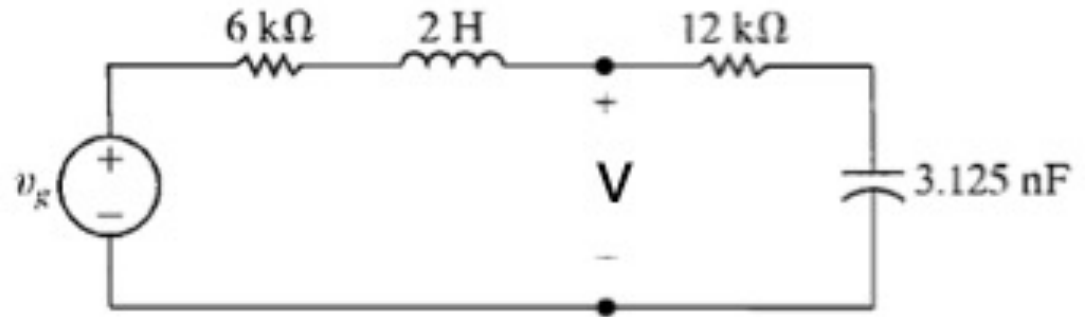


- **Step 3:** Convert back to a time function

$$i(t) = 0.848 \cos(10t - 45^\circ) \text{ A}$$

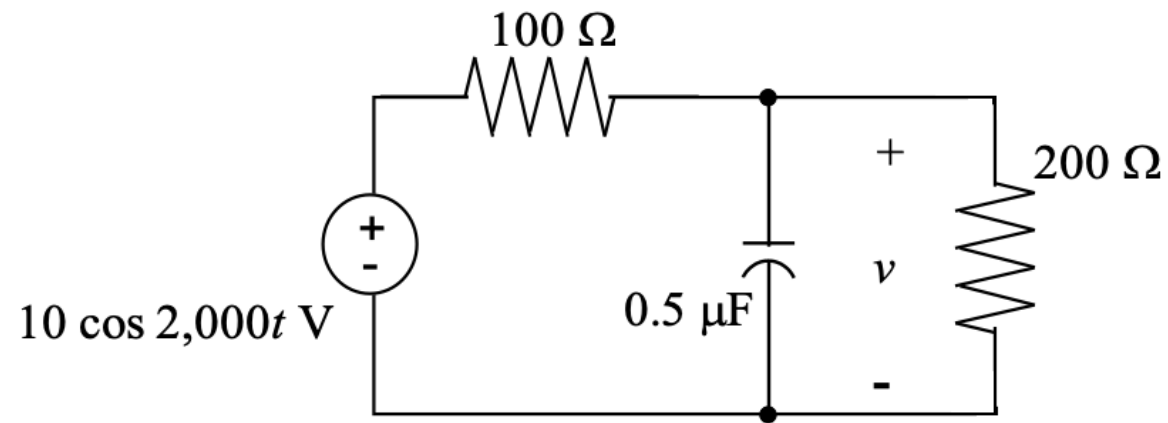
**Example:** find  $V(t)$   
by voltage division

$$v_g(t) = 75 \cos 20,000t \text{ V}$$



$$50 \cos(20,000t - 106^\circ) \text{ V}$$

**Example:** find  $v$

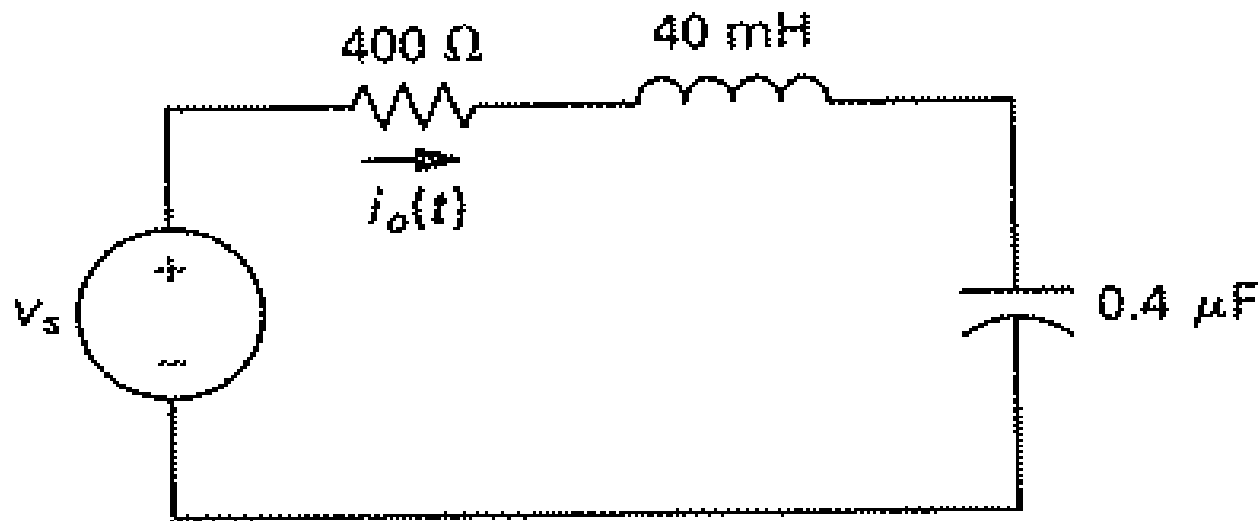




$$6.65 \cos(2000t - 3.81^\circ) \text{ V}$$

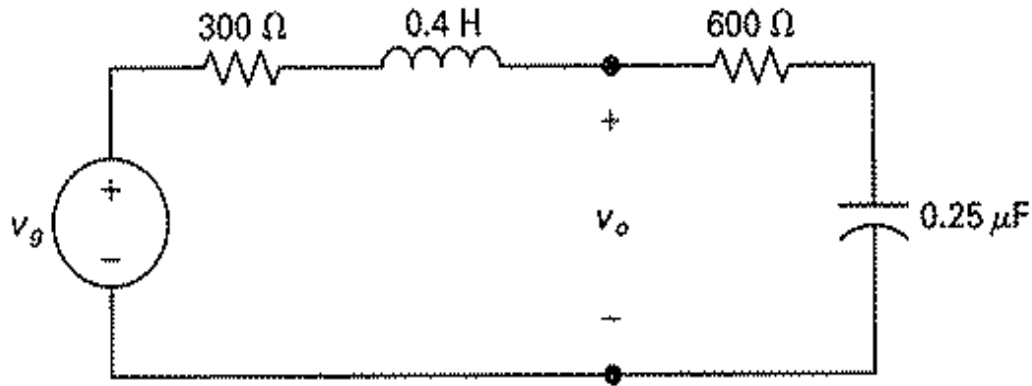
**Practice problem:** find the current if

$$v_s(t) = 750 \cos 5000t$$



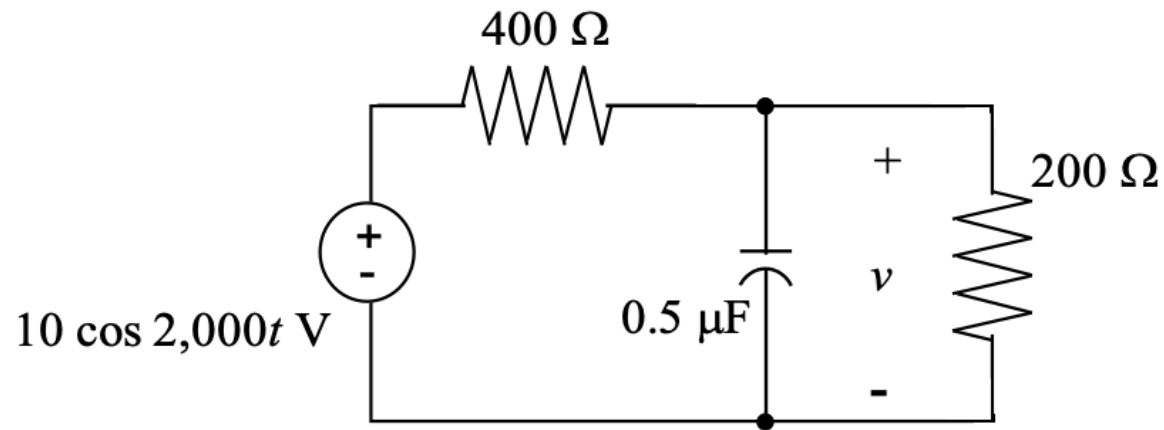
$$1.5 \cos(5000t + 36.9^\circ)\text{ A}$$

**Practice problem:** if  $v_g(t) = 75 \cos 5000t$  V, find the time expression for  $v_o$



$$130 \cos(5000t - 93.4^\circ) \text{ V}$$

**Practice problem:** find  $v$



$$3.30 \cos(2000t - 7.60^\circ) \text{ V}$$