Lecture 18
Phasors – 4 of 9

using phasors;

variation with frequency

Phasor Review

- Extend sinusoidal voltages/currents to phasors (complex)

$$V_s \cos(\omega t + \phi) \Rightarrow \mathbf{V} = V_s e^{j\phi}$$

$$I_s \cos(\omega t + \phi) \Rightarrow \mathbf{I} = I_s e^{j\phi}$$

- Convert components (R,L,C) to <u>impedances</u>

$$Z = \begin{cases} R & \text{resistor} \\ j\omega L & \text{inductor} \\ \dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C} & \text{capacitor} \end{cases}$$

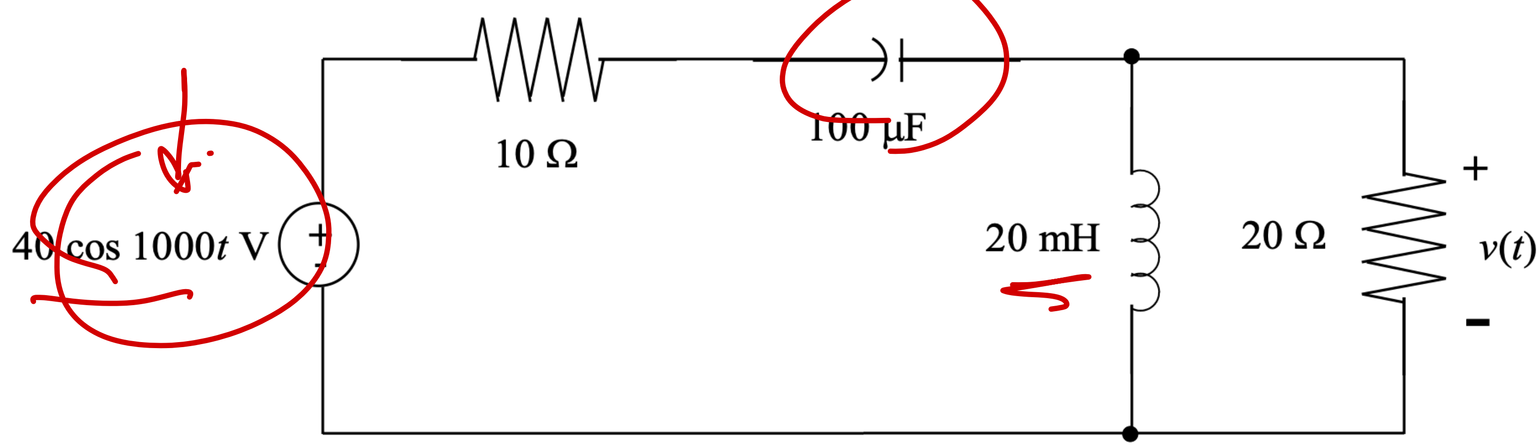
- Solve the problem using Ohm's Law, KVL/KCL, …

- Convert back

$$B\angle\theta \Rightarrow B\cos(\omega t + \theta)$$

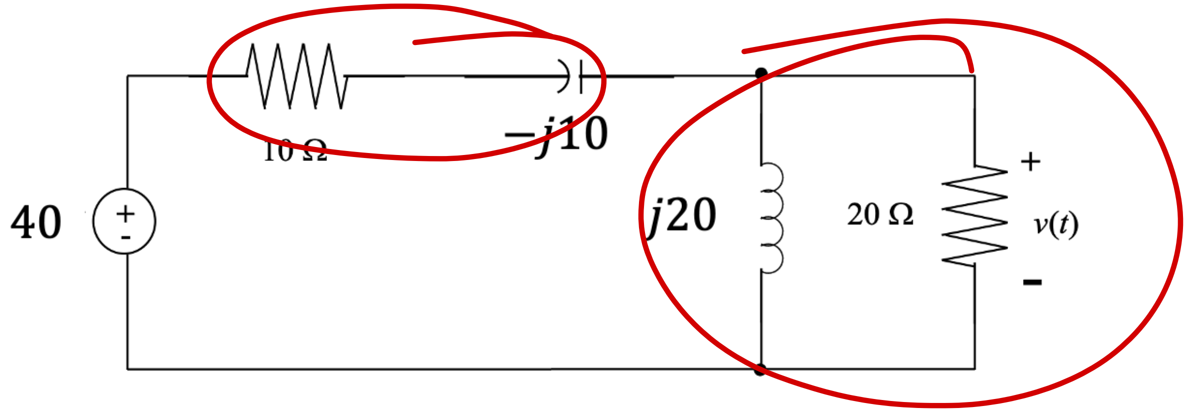
Common Usage

- Find the voltage $v(t)$



- Convert:

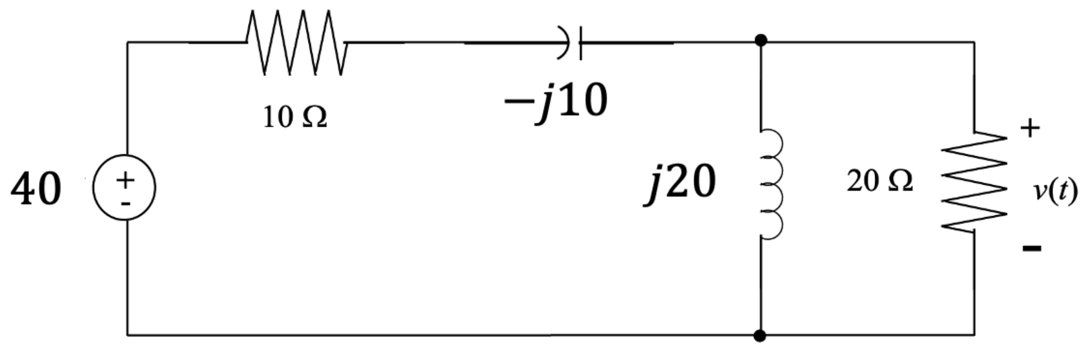
$$40 \cos 1000\, t \rightarrow 40$$
$$20\ mH \rightarrow j\omega L = j20$$
$$100\ \mu F \rightarrow -j\frac{1}{\omega C} = -j10$$



- Solve: series/parallel combining, voltage division

$$Z_s = 10 - j10 = 10(1 - j) \qquad Z_p = \frac{(20)(j20)}{20 + j20} = \frac{j20}{1 + j}$$

$$V = 40\,\frac{Z_p}{Z_s + Z_p}$$



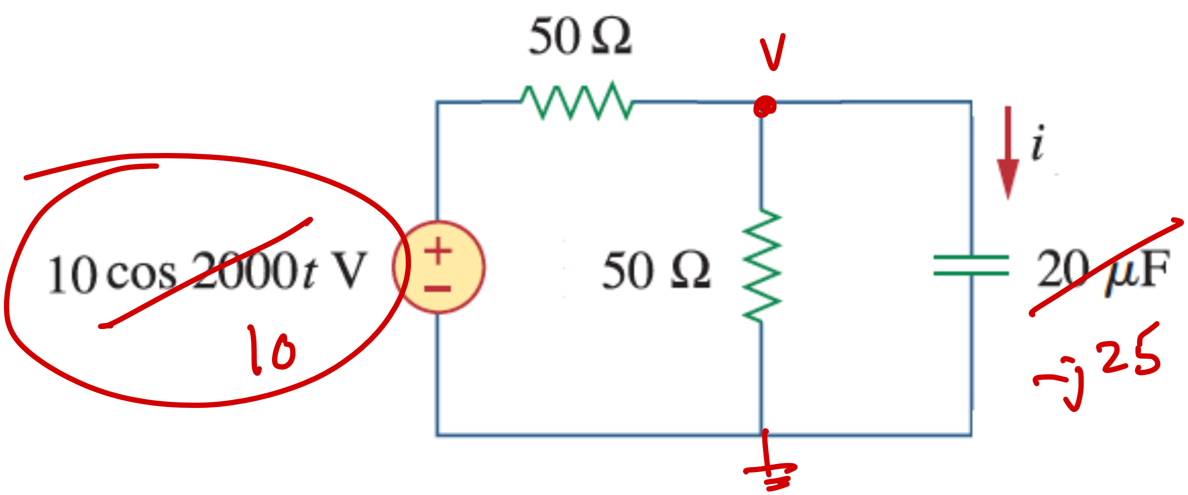
- Simplify and convert to polar form

$$V = 40\,\frac{\frac{j20}{1+j}}{10(1-j)+\frac{j20}{1+j}} = \frac{40(1+j)}{2} = 28.3\,\angle 45°$$

- Convert to a time function

$$v(t) = 28.3\cos(1000t + 45°)\ V$$

- **Example:** find $i(t)$



1- convert to phasor/impedance

$$Z_c = -j\frac{1}{\omega C} = -j\frac{10^6}{2000 \cdot 20}$$

$$= -j25$$

2- node

$$v = \frac{5}{1+j}$$

3- ohm's law

$$I = \frac{V}{-j25}$$

$$= \frac{5}{(1+j)(-j25)}$$

$$50\left(\frac{v-10}{50} + \frac{v}{50} + \frac{v}{-j25} = 0\right)$$

$$v - 10 + v + j2v = 0$$

$$v(2+j2) = 10$$

$$v = \frac{10}{2(1+j)} = \frac{5}{1+j}$$

$$I = \frac{5}{(1+j)(-j25)}$$

$$= \frac{5\angle 0^\circ}{\sqrt{2}\angle 45^\circ \quad 25\angle -90^\circ}$$

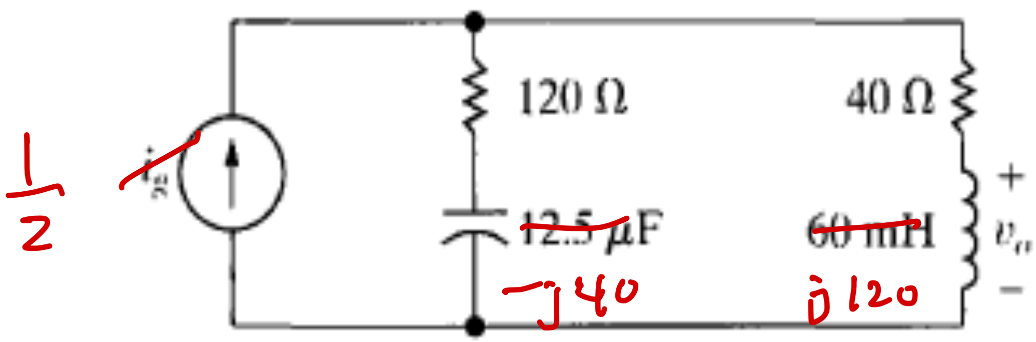
$$= \frac{5}{25\sqrt{2}} \angle 0-45+90$$

$$= \frac{1}{5\sqrt{2}} \angle 45^\circ$$

$$\frac{\sqrt{2}}{10} = \frac{1.414}{10} = .141$$

$141\cos(2000t + 45^\circ)\ mA$

Example: find $v_o(t)$ if $i_g(t) = 500 \cos 2000\, t$ mA

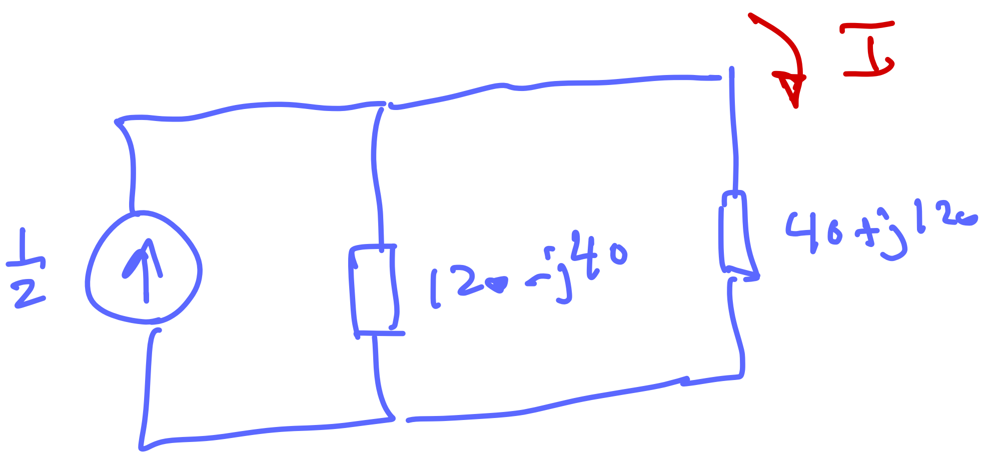


1 - convert

$$Z_L = \frac{60}{1000} \cdot j\,2000 = j120$$

$$Z_C = -j\,\frac{10^6}{2000 \cdot 12.5} = -j40$$

2- series comb.



3- current div

$$I = \frac{1}{2} \cdot \frac{120 - j40}{160 + j80}$$

4- ohm's law

$$v_o = j120 \cdot I$$

$42.4 \cos(2000t + 45°)$ V

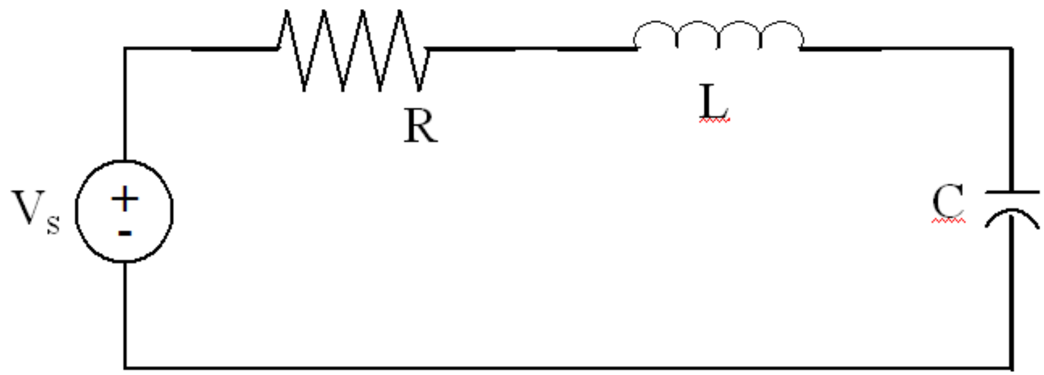
$$V_0 = \underbrace{j120 \cdot \frac{1}{2}}_{60\angle 90°} \cdot \frac{120 - j40}{160 + j80} \qquad \frac{126.5\angle -18.43°}{178.9\angle 26.57°}$$

$$60 \cdot \frac{126.5}{178.9} = 42.4$$

$$90 + (-18.43) - (26.57) = 45$$

Consider Variation with Frequency

Consider voltage division:



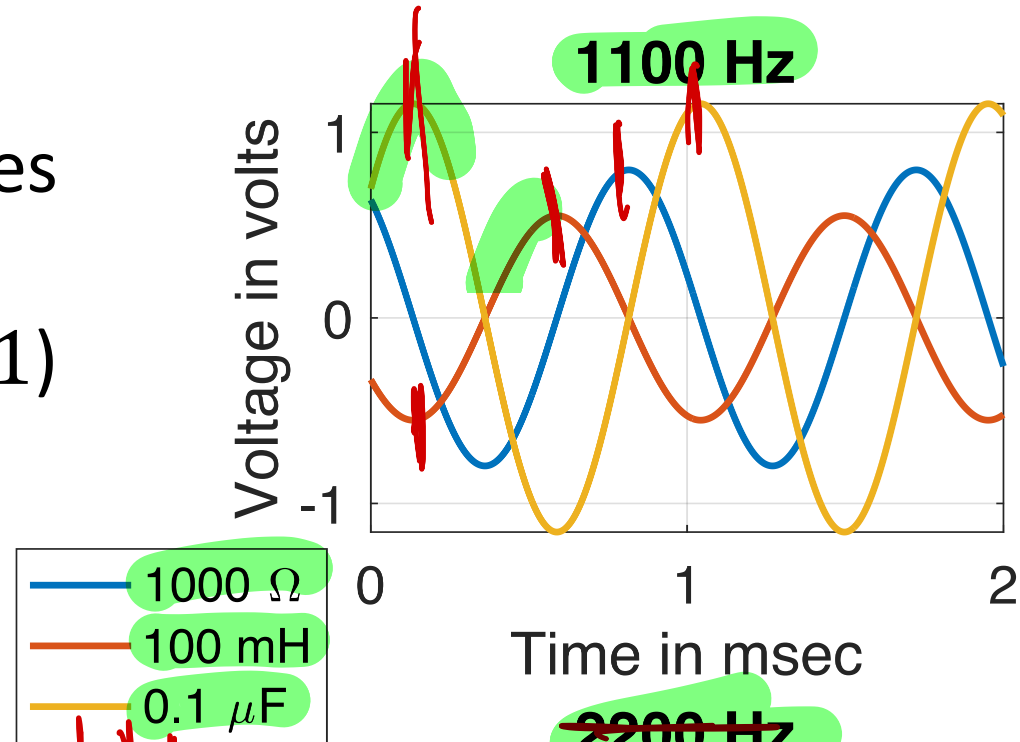
$$\boldsymbol{V}_R = \frac{R\boldsymbol{V}_s}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}\boldsymbol{V}_s$$

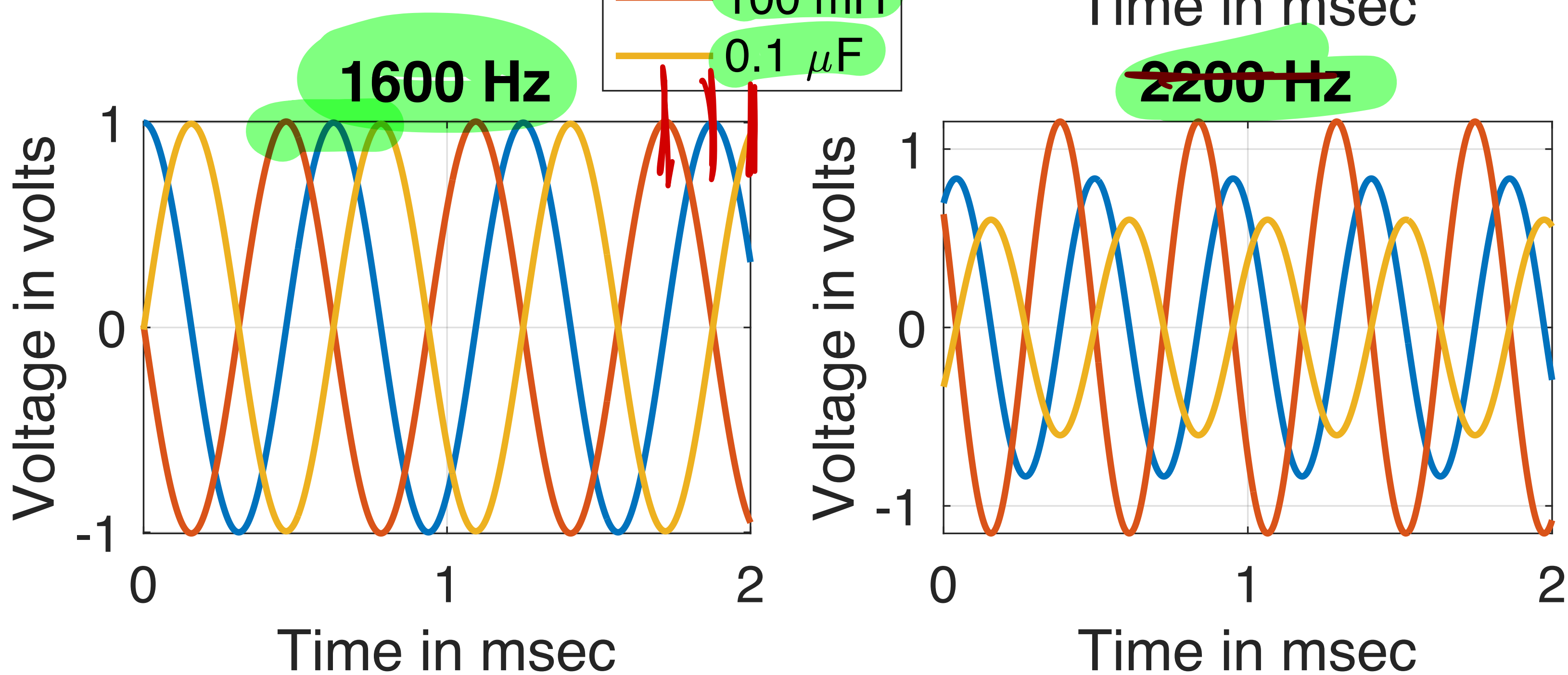
Similarly

$$\boldsymbol{V}_L = \frac{-\omega^2 LC}{1 - \omega^2 LC + j\omega RC}\boldsymbol{V}_s$$

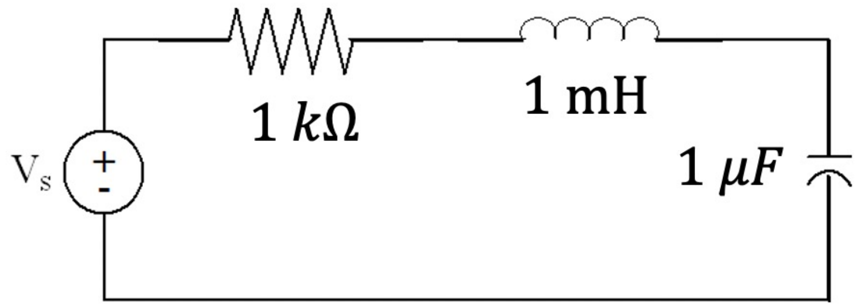
$$\boldsymbol{V}_C = \frac{1}{1 - \omega^2 LC + j\omega RC}\boldsymbol{V}_s$$

- Comparison of the component voltages for different frequencies ($V_s = 1$)





Consider combined impedance variation



$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

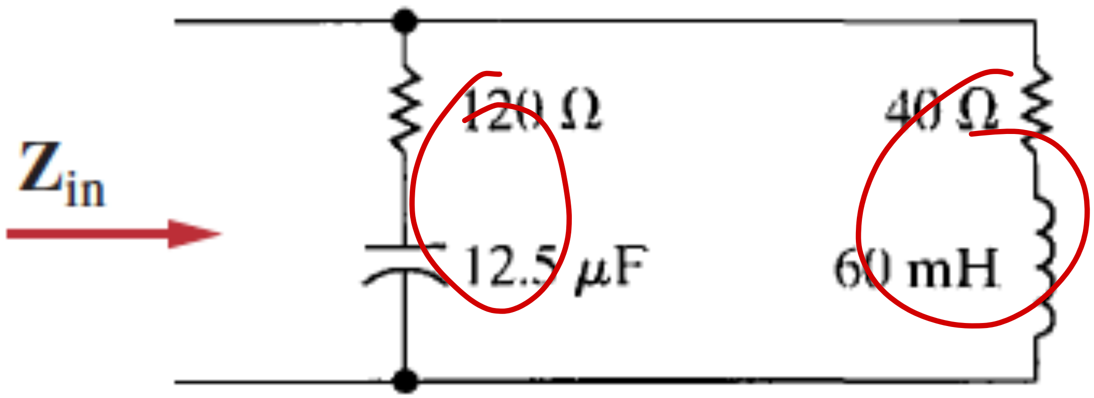
$$= 1000 + j\left(\frac{\omega}{100} - \frac{10^6}{\omega}\right)$$

$\omega = 10{,}000$

Questions:

- At what frequency does this “appear” purely resistive?
- What happens at small frequency? Large frequency?

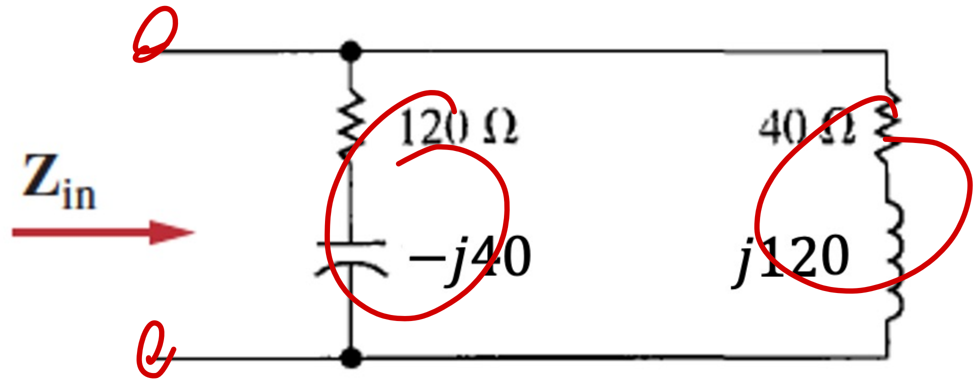
Example: find Z_{in} if $\omega = 2000$ rad/sec



- Convert:

$$60\ mH \rightarrow j\omega L = j120$$

$$12.5\ \mu F \rightarrow -j\frac{1}{\omega C} = -j40$$



- Solve: series/parallel combining

$$Z_{s,left} = 120 - j40 = 40(3 - j) \qquad Z_{s,right} = 40 + j120 = 40(1 + j3)$$

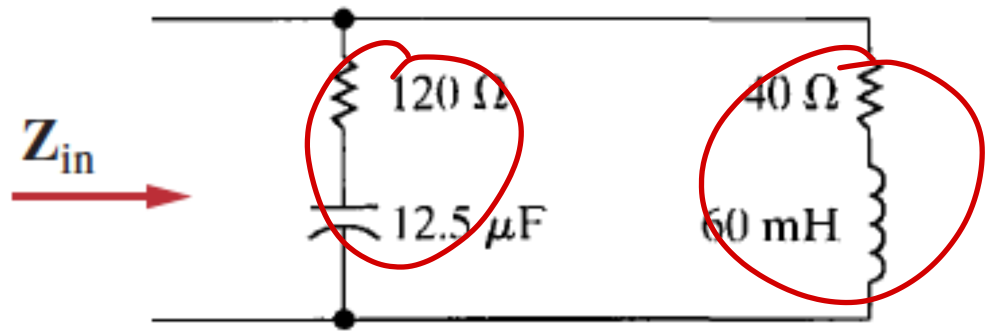
$$Z_{in} = \frac{40(3-j)40(1+j3)}{40(3-j) + 40(1+j3)} = \frac{40(6+j8)}{4+j2} = \frac{40(3+j4)}{2+j}$$

$$= \frac{40(3+j4)(2-j)}{5} = \boxed{80 + j40}$$

80 ohm resistor in series with 20 mH inductor !!

$\omega = 2000$ rad/sec

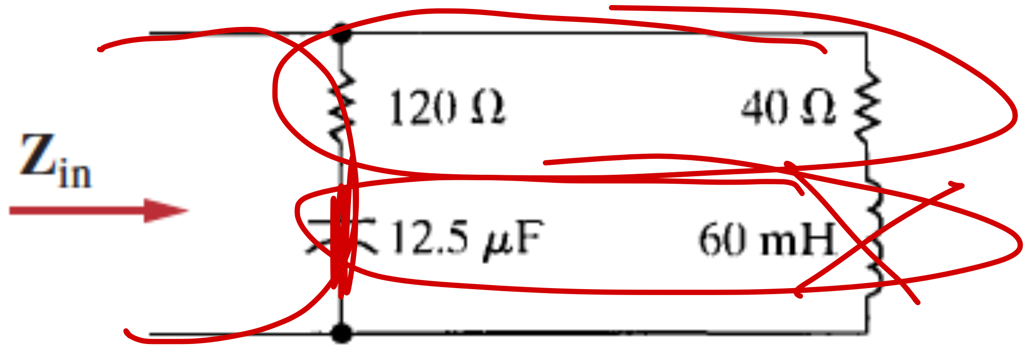
Example: How does Z_{in} vary with frequency? Is it ever purely real?



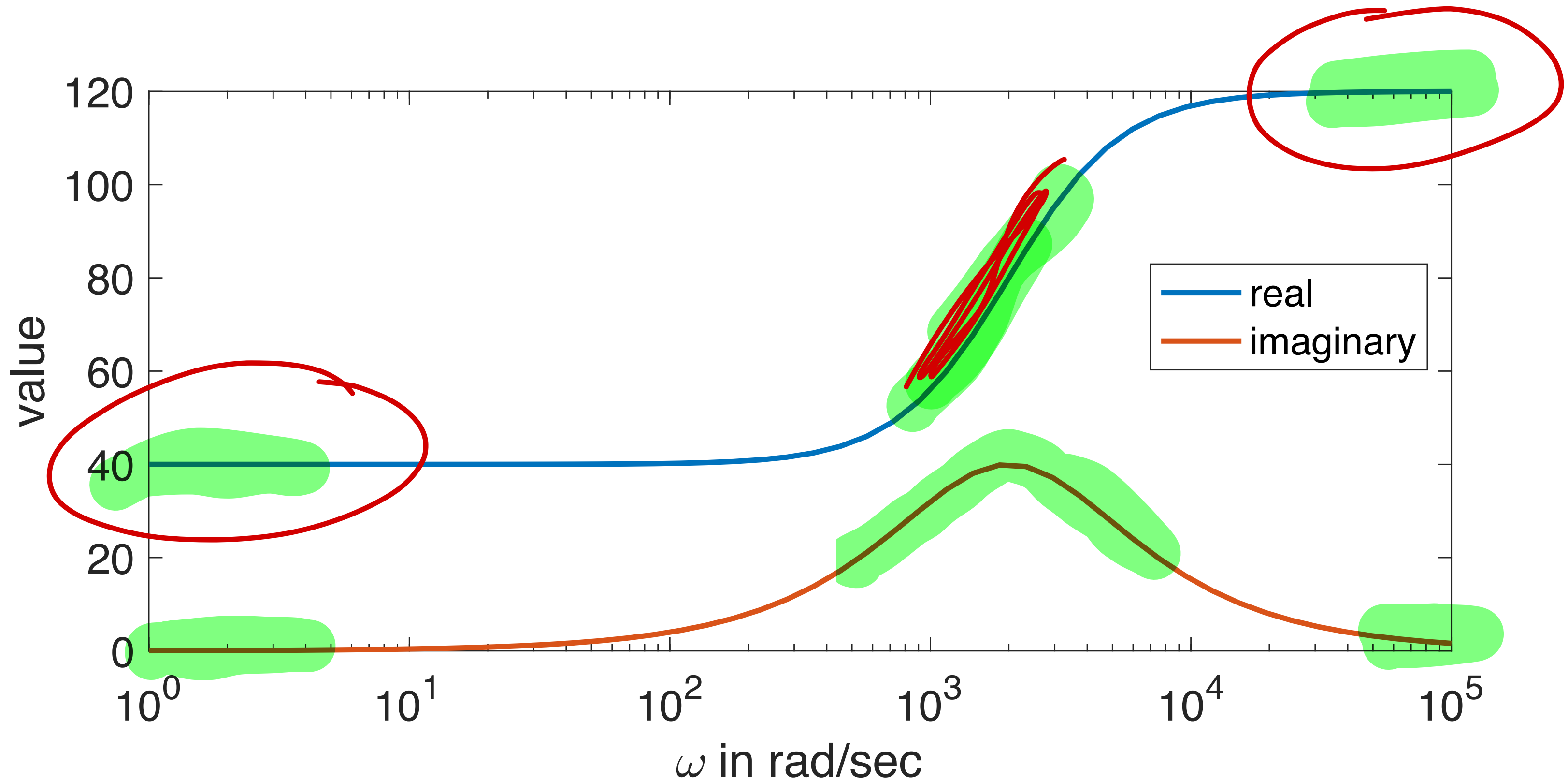
- Solve: series/parallel combining

$$Z_{s,left} = 120 - j\frac{80{,}000}{\omega} \qquad Z_{s,right} = 40 + j60\omega$$

$$Z_{in} = \frac{Z_{s,left} \times Z_{s,right}}{Z_{s,left} + Z_{s,right}} = \frac{\left(120 - j\frac{80{,}000}{\omega}\right)(40 + j60\omega)}{120 - j\frac{80{,}000}{\omega} + 40 + j60\omega} = \cdots$$

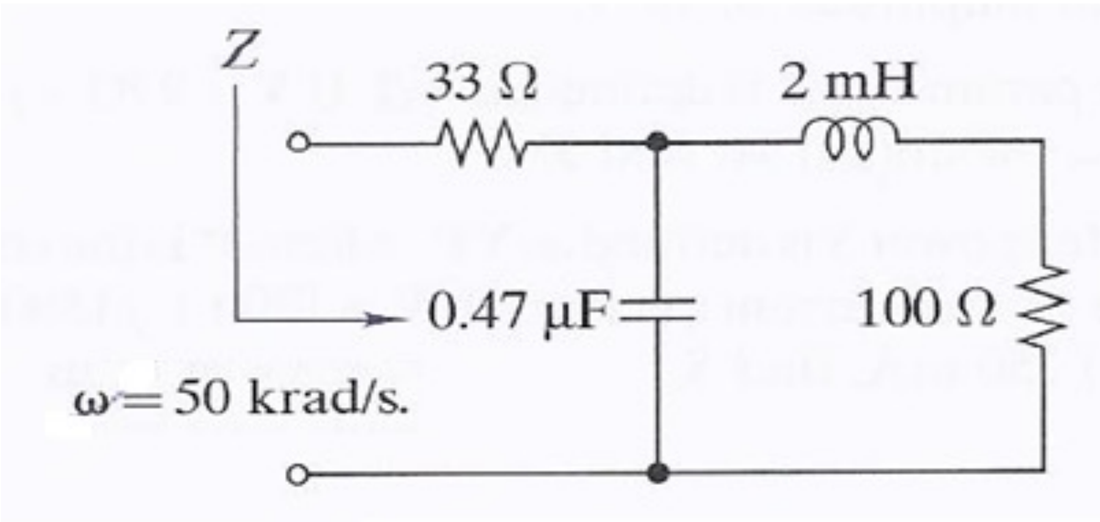


$$Z_{in} = \frac{120\omega^2 + 160{\times}10^6}{\omega^2 + 4{\times}10^6} + j\,\frac{160{,}000\omega}{\omega^2 + 4{\times}10^6}$$



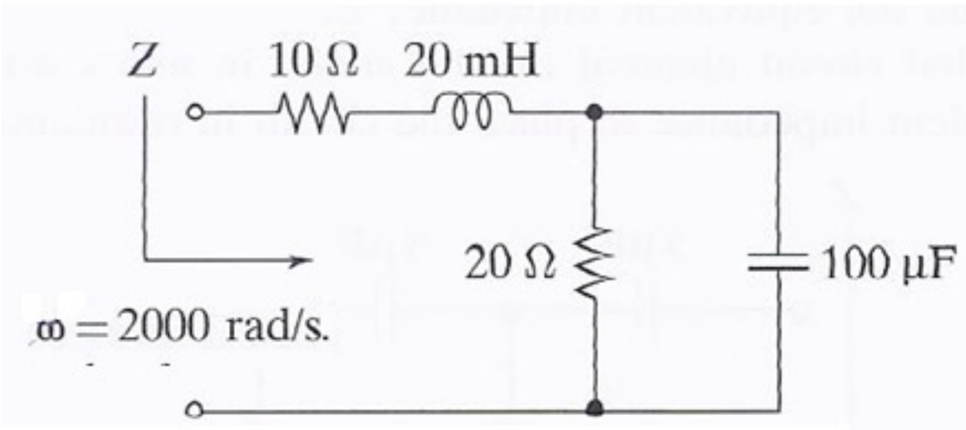
Practice problem: find Z

$46.4 - j50.4;$
$46.4\ \Omega\ \ 0.397\ \mu F$



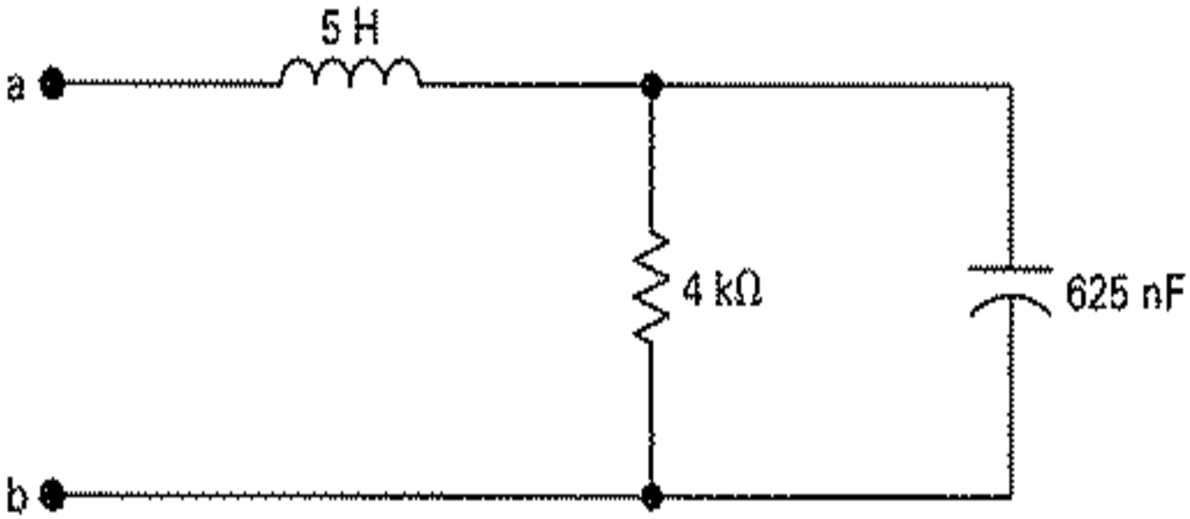
Practice problem: find Z

$11.2 + j35.3;$
$11.2\ \Omega\ \ 17.6\ mH$



Practice problem: at what frequency does this circuit seem purely resistive? What is the resistance?

$$400\,\frac{rad}{sec};\,2\ k\Omega$$



- **Practice problem:** consider the parallel connection of a 220 Ω resistor, a 0.5 μF capacitor, and a 5 mH inductor.
  - What is the equivalent impedance of this circuit at 1000 Hz?
  - At 5000 Hz?
  - At what frequency is the impedance purely real?

$11.6 + j49.2\ \Omega;\ 1.42 - j17.6\ \Omega;\ 1.59\ kHz$

Practice problem: Find the time expression for $v_o(t)$.
Note that $\sin \omega t = \cos(\omega t - 90°)$

$17.1 \cos 200t \ V$