Lecture 18
Phasors – 4 of 9

using phasors;

variation with frequency

Phasor Review

- Extend sinusoidal voltages/currents to phasors (complex)

$$V_s \cos(\omega t + \phi) \Rightarrow \mathbf{V} = V_s e^{j\phi}$$

$$I_s \cos(\omega t + \phi) \Rightarrow \mathbf{I} = I_s e^{j\phi}$$

- Convert components (R,L,C) to <u>impedances</u>

$$Z = \begin{cases} R & \text{resistor} \\ j\omega L & \text{inductor} \\ \dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C} & \text{capacitor} \end{cases}$$

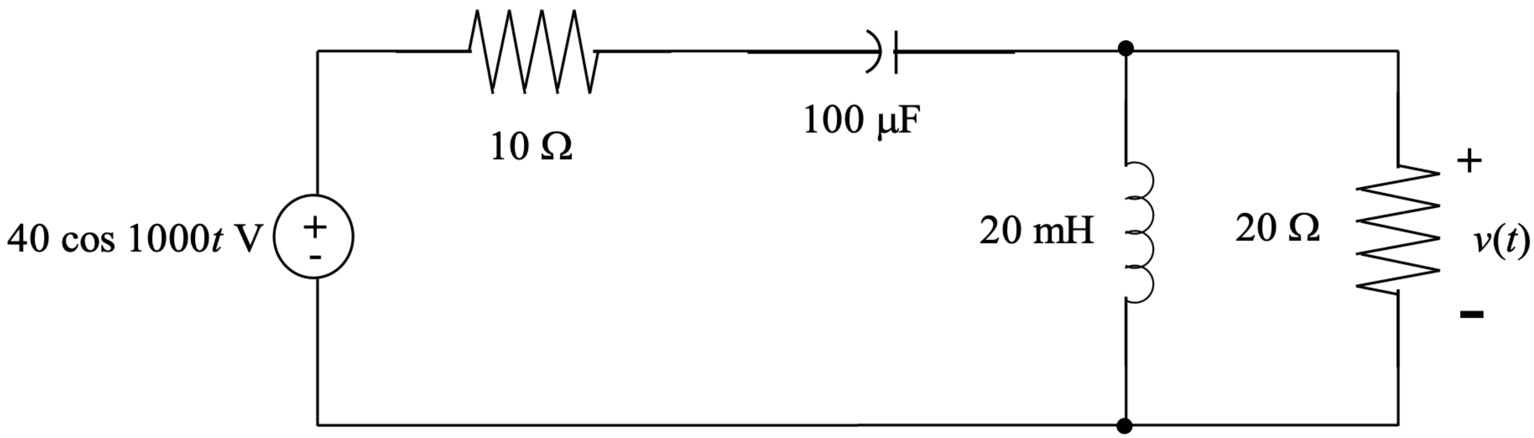
- Solve the problem using Ohm's Law, KVL/KCL, …

- Convert back

$$B\angle\theta \Rightarrow B\cos(\omega t + \theta)$$

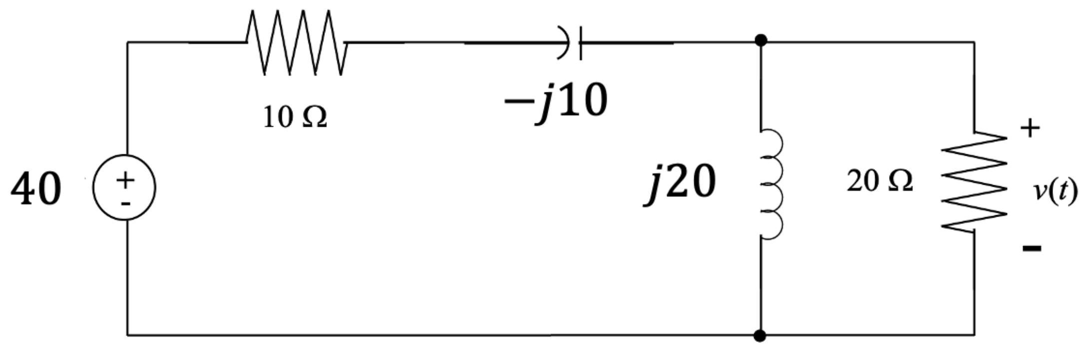
Common Usage

- Find the voltage *v*(*t*)



- Convert:

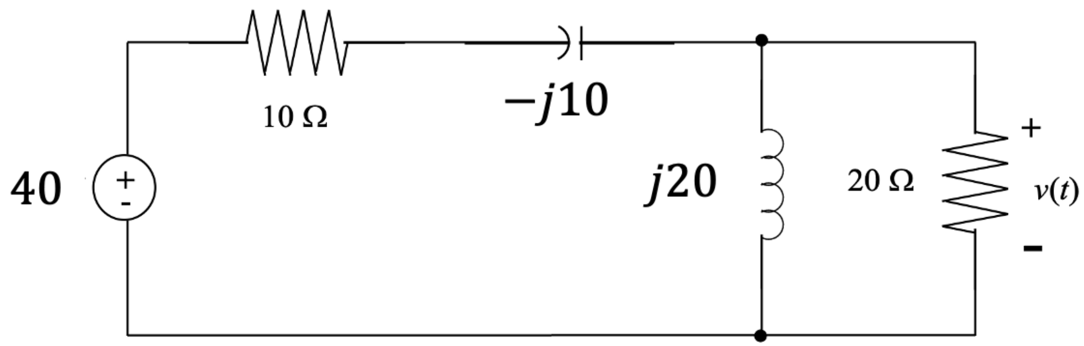
$$40 \cos 1000\, t \;\rightarrow 40$$
$$20\, mH \rightarrow j\omega L = j20$$
$$100\, \mu F \rightarrow -j\,\frac{1}{\omega C} = -j10$$



- Solve: series/parallel combining, voltage division

$$Z_s = 10 - j10 = 10(1 - j) \qquad Z_p = \frac{(20)(j20)}{20 + j20} = \frac{j20}{1 + j}$$

$$V = 40\ \frac{Z_p}{Z_s + Z_p}$$



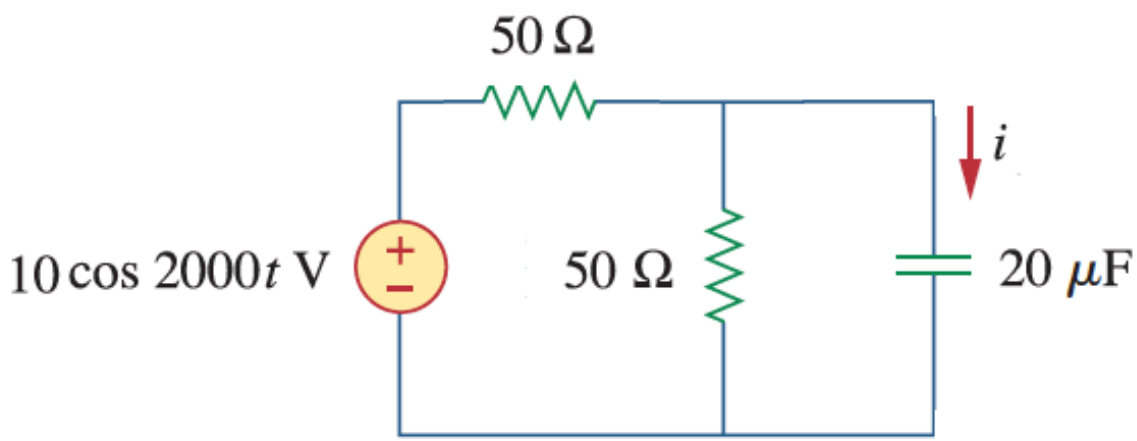
- Simplify and convert to polar form

$$V = 40\,\frac{\dfrac{j20}{1+j}}{10(1-j)+\dfrac{j20}{1+j}} = \frac{40(1+j)}{2} = 28.3\ \angle 45°$$

- Convert to a time function

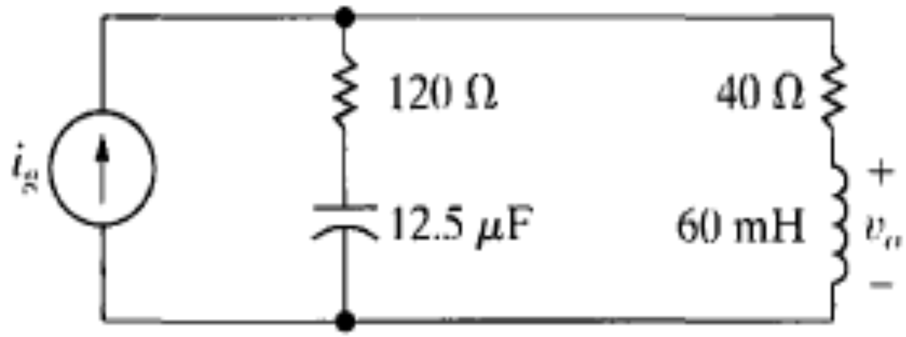
$$v(t) = \ 28.3\cos(1000t + 45°)\ V$$

- **Example:** find $i(t)$



$141\cos(2000t + 45°)\ mA$

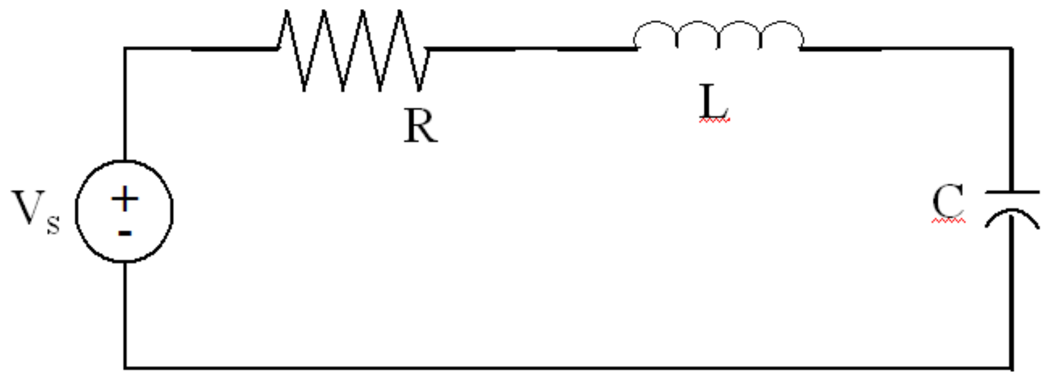
Example: find $v_o(t)$ if $i_g(t)$ = 500 cos 2000 t mA



$42.4\cos(2000t + 45°)$ V

Consider Variation with Frequency

Consider voltage division:



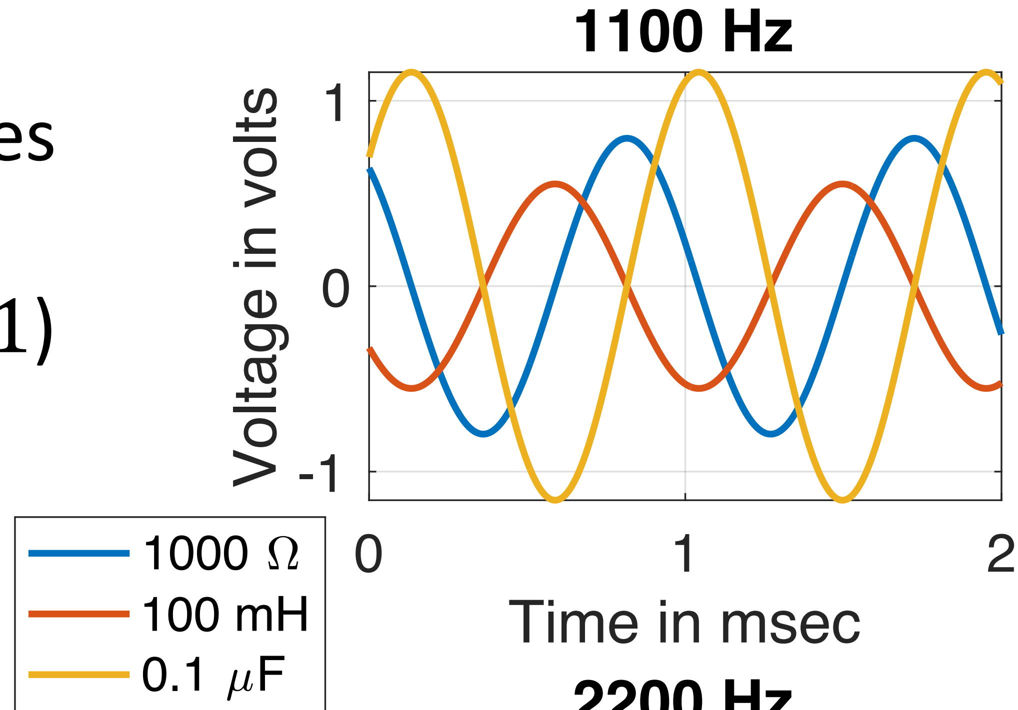
$$\boldsymbol{V}_R = \frac{R\boldsymbol{V}_s}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}\boldsymbol{V}_s$$

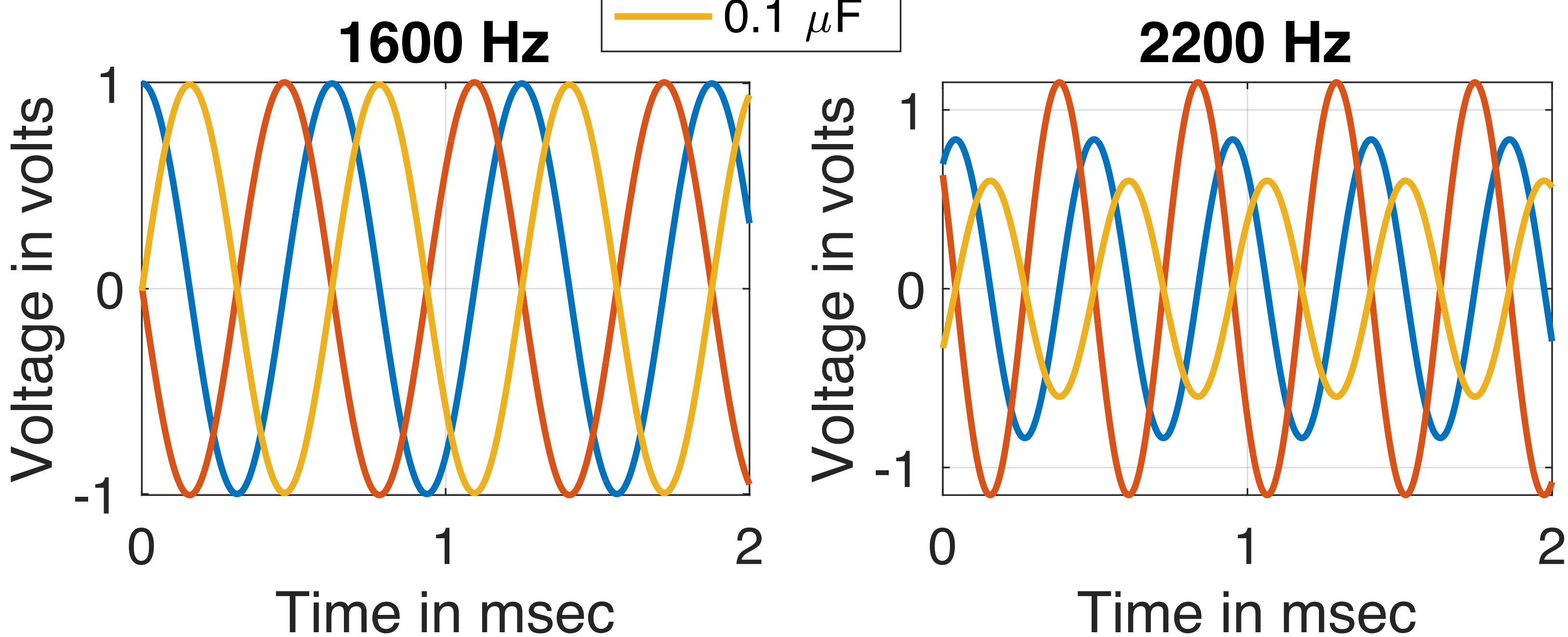
Similarly

$$\boldsymbol{V}_L = \frac{-\omega^2 LC}{1 - \omega^2 LC + j\omega RC}\boldsymbol{V}_s$$

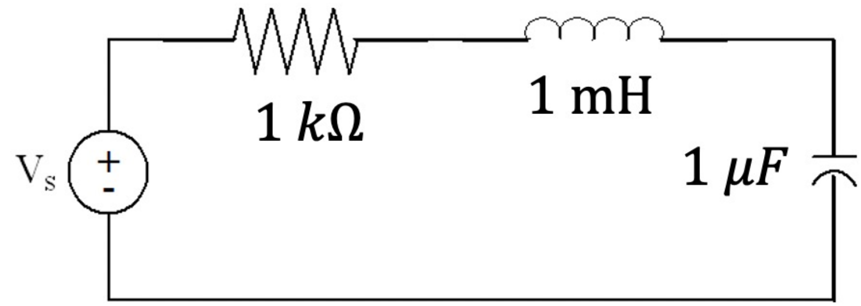
$$\boldsymbol{V}_C = \frac{1}{1 - \omega^2 LC + j\omega RC}\boldsymbol{V}_s$$

- Comparison of the component voltages for different frequencies ($V_s = 1$)





Consider combined impedance variation



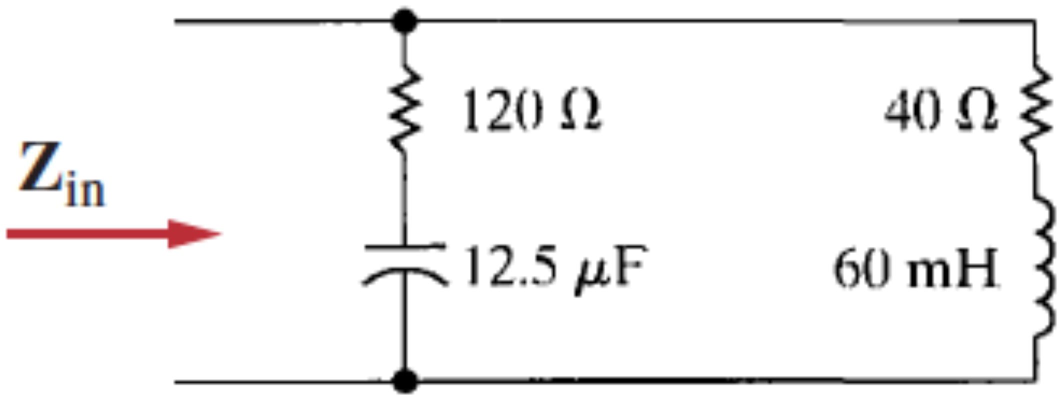
$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= 1000 + j\left(\frac{\omega}{100} - \frac{10^6}{\omega}\right)$$

Questions:

- At what frequency does this “appear” purely resistive?
- What happens at small frequency? Large frequency?

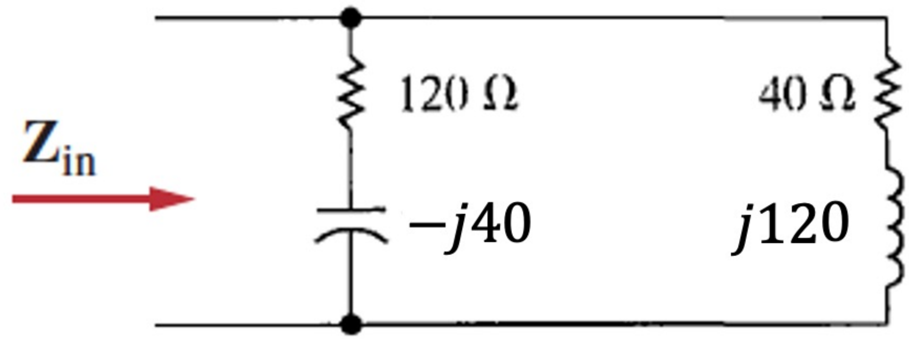
Example: find Z_{in} if $\omega = 2000$ rad/sec



- Convert:

$$60\ mH \rightarrow j\omega L = j120$$

$$12.5\ \mu F \rightarrow -j\frac{1}{\omega C} = -j40$$



- Solve: series/parallel combining

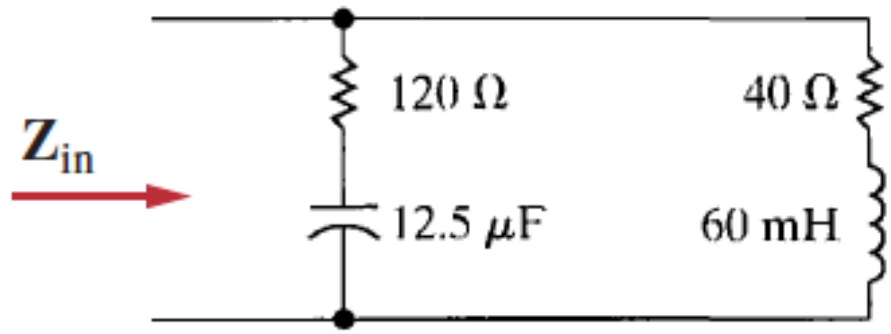
$$Z_{s,left} = 120 - j40 = 40(3 - j) \qquad Z_{s,right} = 40 + j120 = 40(1 + j3)$$

$$Z_{in} = \frac{40(3 - j)40(1 + j3)}{40(3 - j) + 40(1 + j3)} = \frac{40(6 + j8)}{4 + j2} = \frac{40(3 + j4)}{2 + j}$$

$$= \frac{40(3 + j4)(2 - j)}{5} = 80 + j40$$

80 ohm resistor in series with 20 mH inductor !!

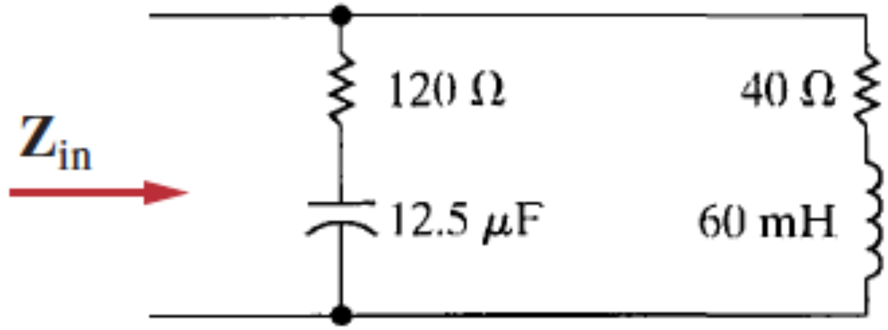
Example: How does Z_{in} vary with frequency? Is it ever purely real?



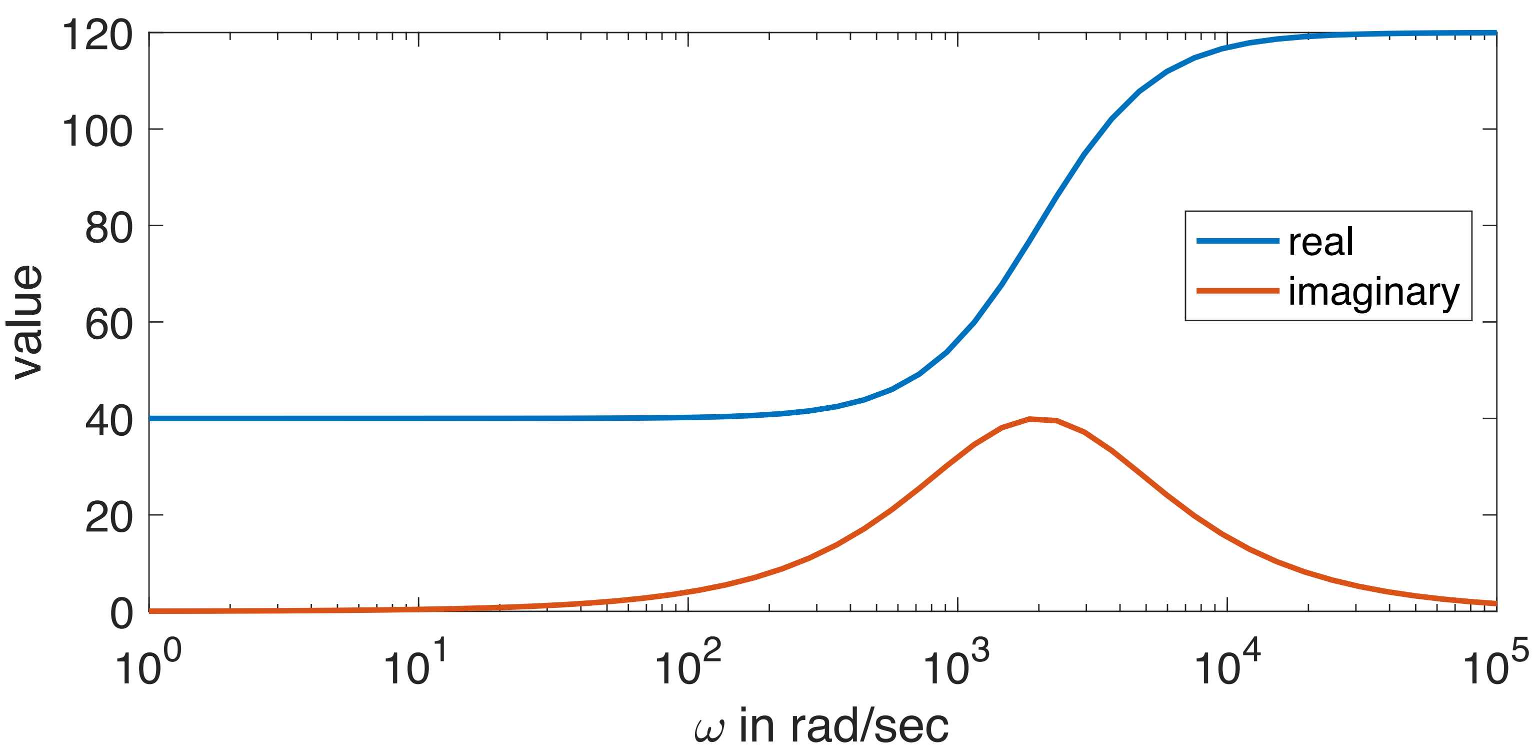
- Solve: series/parallel combining

$$Z_{s,left} = 120 - j\frac{80{,}000}{\omega} \qquad Z_{s,right} = 40 + j60\omega$$

$$Z_{in} = \frac{Z_{s,left} \times Z_{s,right}}{Z_{s,left} + Z_{s,right}} = \frac{(120 - j\frac{80{,}000}{\omega})(40 + j60\omega)}{120 - j\frac{80{,}000}{\omega} + 40 + j60\omega} = \cdots$$

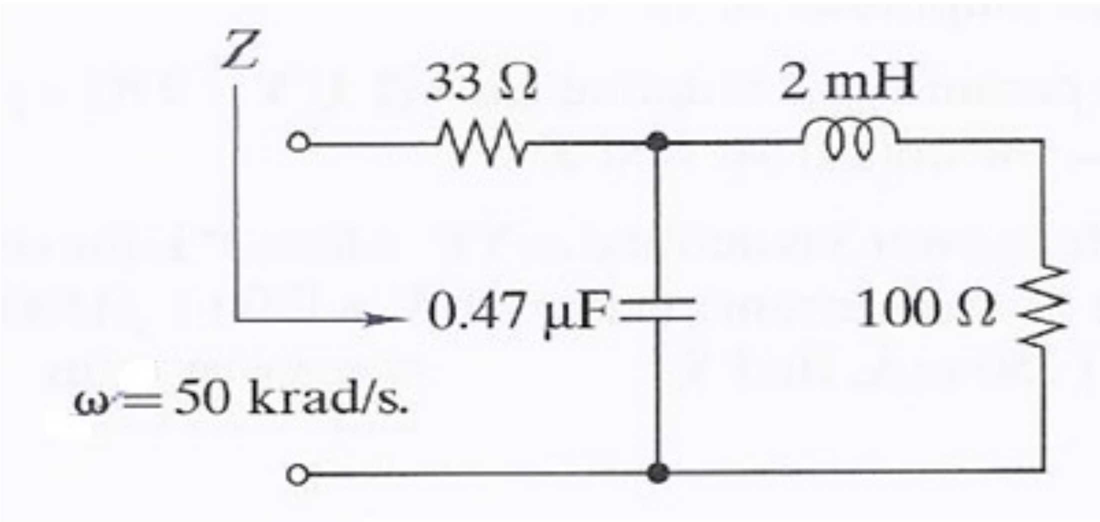


$$Z_{in} = \frac{120\omega^2 + 160\times10^6}{\omega^2 + 4\times10^6} + j\,\frac{160{,}000\omega}{\omega^2 + 4\times10^6}$$



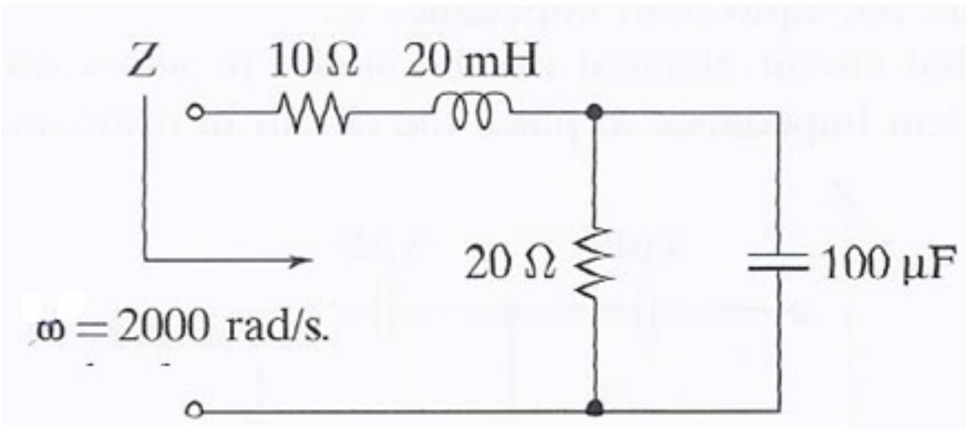
Practice problem: find Z

$46.4 - j50.4;$
$46.4\ \Omega\ \ 0.397\ \mu F$



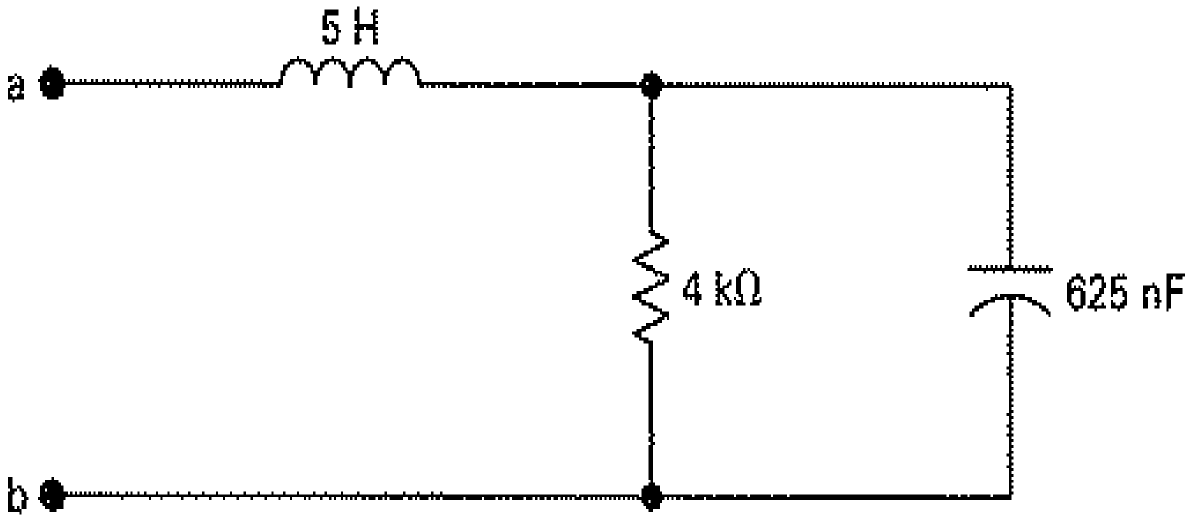
Practice problem: find Z

$11.2 + j35.3;$
$11.2\ \Omega \quad 17.6\ mH$



Practice problem: at what frequency does this circuit seem purely resistive? What is the resistance?

$$400\,\frac{rad}{sec};\,2\;k\Omega$$



- **Practice problem:** consider the parallel connection of a 220 Ω resistor, a 0.5 μF capacitor, and a 5 mH inductor.
  - What is the equivalent impedance of this circuit at 1000 Hz?
  - At 5000 Hz?
  - At what frequency is the impedance purely real?

$11.6 + j49.2\ \Omega;\ 1.42 - j17.6\ \Omega;\ 1.59\ kHz$

Practice problem: Find the time expression for $v_o(t)$.

Note that $\sin \omega t = \cos(\omega t - 90°)$

$17.1 \cos 200t\ V$