

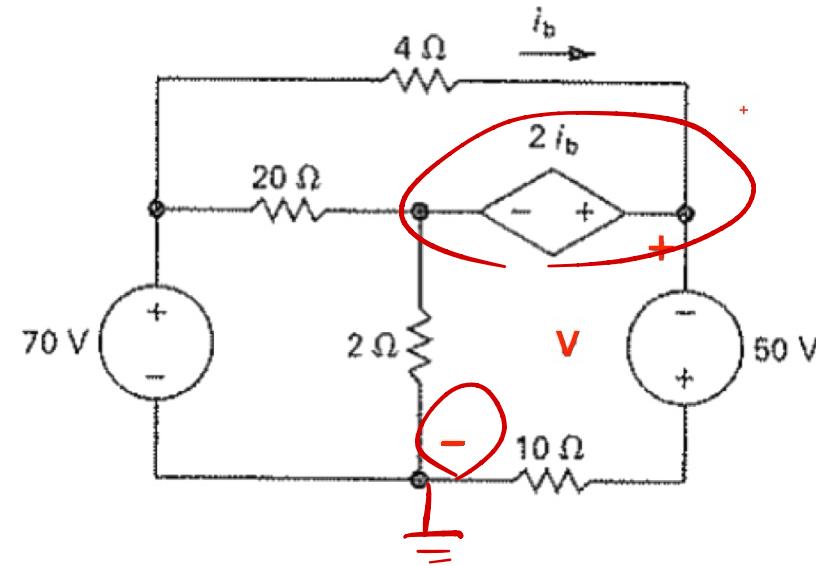
Lecture 8

Node Analysis – 1 of 7

basic concepts

General Methods to Analyze Circuits

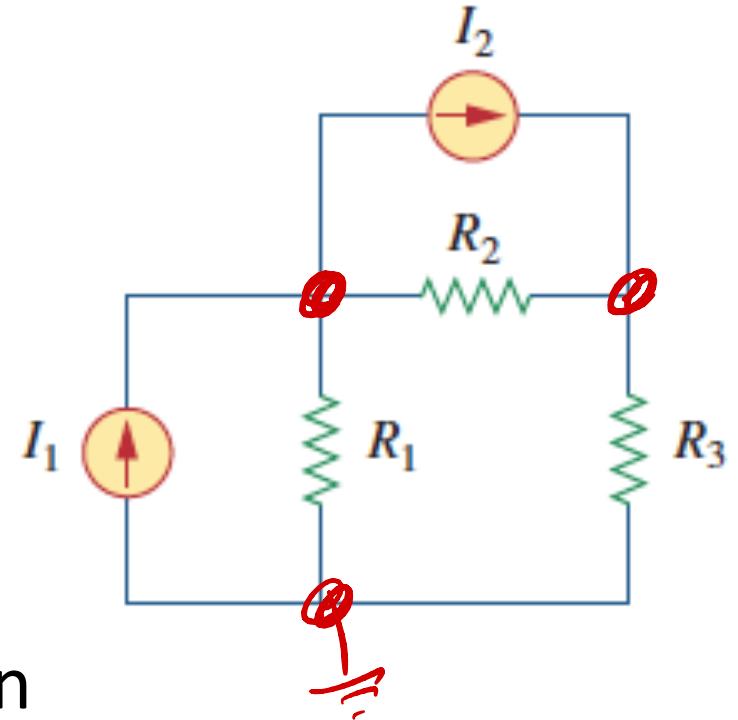
- What to do first?
 - KVL?
 - KCL?
 - Ohm's Law?



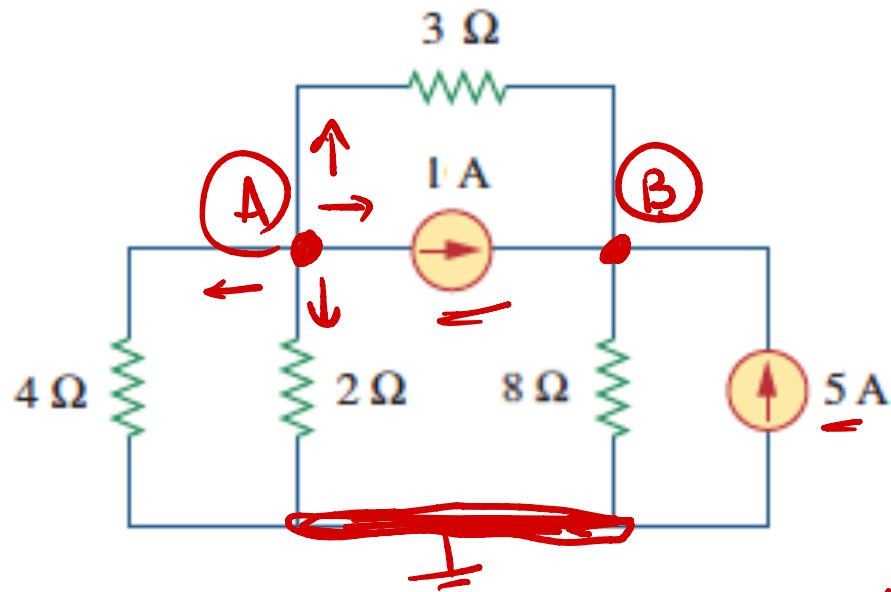
- We need a more direct approach:
 - Nodal analysis (KCL based)
 - Mesh analysis (KVL based, **end of semester**)
- Review appendix A (also online)

Node Analysis

- Consider a circuit with current sources and resistors only
- Identify nodes
 - Select one as “ground”
 - Label others
 - Write KCL on these other nodes
 - Use Ohm’s Law for current in the resistive branches
 - Solve resulting equations



Example (details repeated on next slide)



KCL

A:

$$1 + \frac{A}{2} + \frac{A}{8} - \frac{A-B}{3} = 0$$

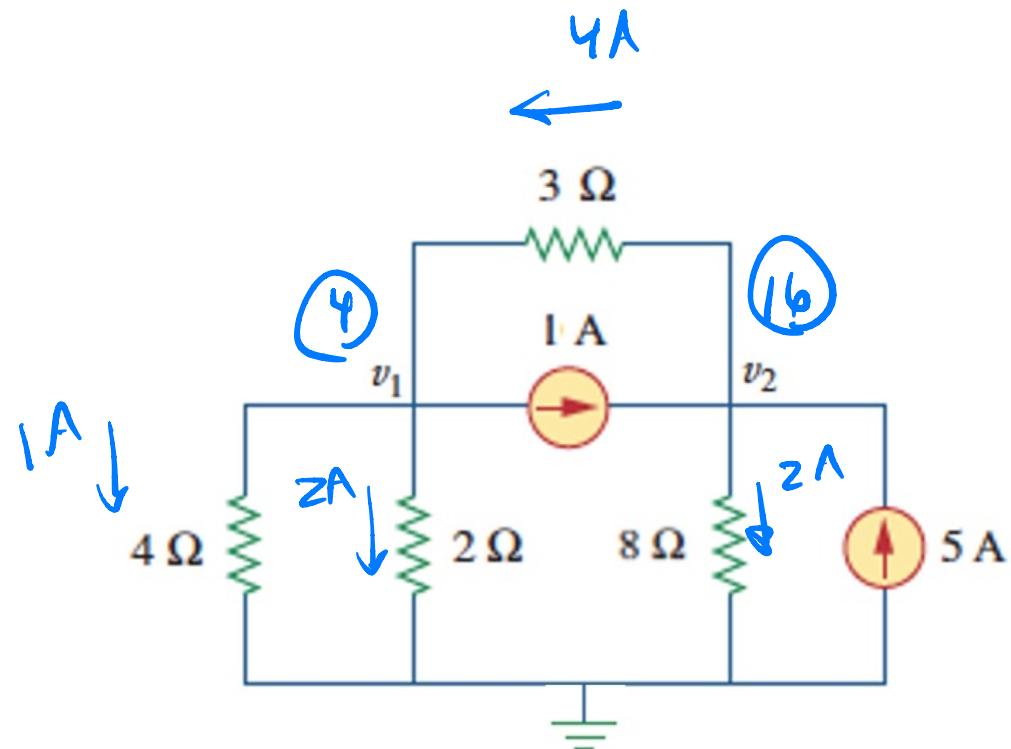
B: $14 = 0$

$$A\left(\frac{1}{2} + \frac{1}{8} + \frac{1}{3}\right) + B\left(-\frac{1}{3}\right) = -1$$

$$\frac{A-B}{3} + 1 + 5 = \frac{B}{8}$$

$$A\left(-\frac{1}{3}\right) + B\left(\frac{1}{8} + \frac{1}{3}\right) = 6$$

$$\frac{B}{8} - 5 - 1 + \frac{B-A}{3} = 0$$



$$\begin{aligned} \frac{v_1}{2} + \frac{v_1}{4} + \frac{v_1 - v_2}{3} + 1 &= 0 \\ \frac{v_2}{8} + \frac{v_2 - v_1}{3} &= 1 + 5 \end{aligned}$$

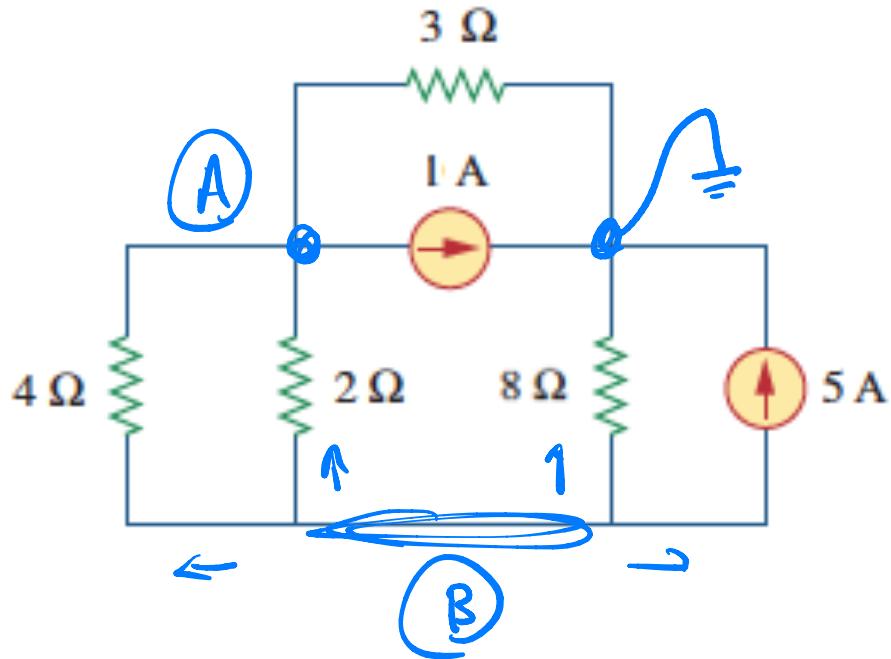
$$\begin{aligned} 6v_1 + 3v_1 + 4v_1 - 4v_2 &= -12 \\ 3v_2 + 8v_2 - 8v_1 &= 144 \end{aligned}$$

$$\begin{aligned} 13v_1 - 4v_2 &= -12 \\ -8v_1 + 11v_2 &= 144 \end{aligned}$$

$$v_1 = \frac{\begin{vmatrix} -12 & -4 \\ 144 & 11 \end{vmatrix}}{\begin{vmatrix} 13 & -4 \\ -8 & 11 \end{vmatrix}} = \frac{-132 + 576}{143 - 32} = \frac{444}{111} = 4 \text{ volts}$$

$$v_2 = \frac{\begin{vmatrix} 13 & -12 \\ -8 & 144 \end{vmatrix}}{\begin{vmatrix} 13 & -4 \\ -8 & 11 \end{vmatrix}} = \frac{1872 - 96}{111} = \frac{1776}{111} = 16 \text{ volts}$$

Example (same circuit, change ground to top right, details on next slide)



KCL for A:

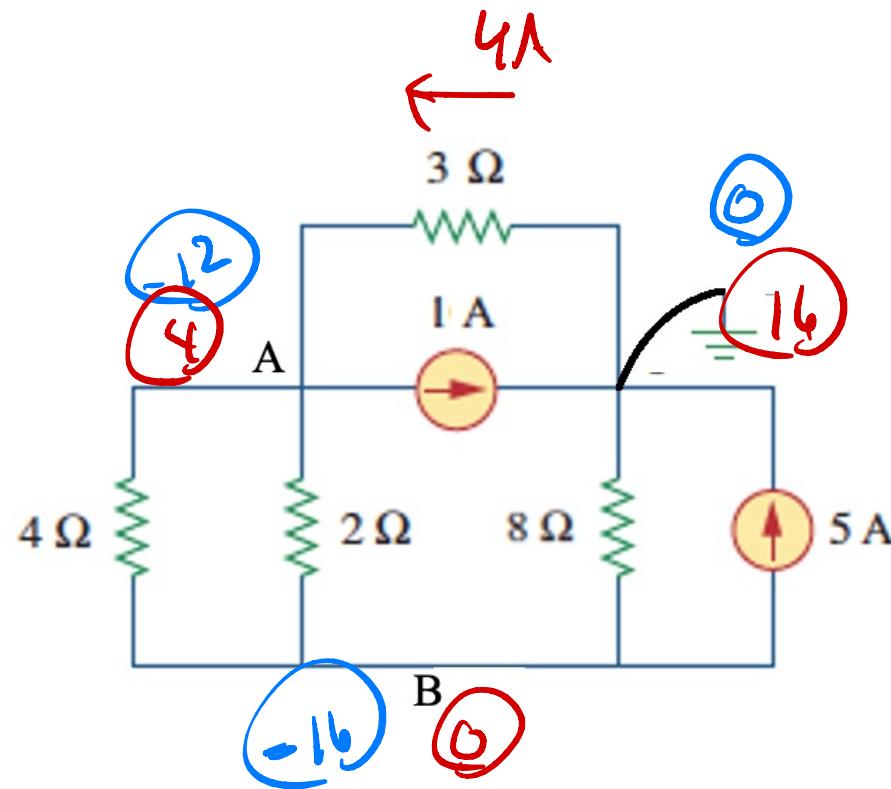
$$1 + \frac{A}{3} + \frac{A-B}{2} + \frac{A-B}{4} = 0$$

KCL at 0:

$$5 + \frac{B}{8} + \frac{B-A}{2} + \frac{B-A}{4} = 0$$

$$A\left(\frac{1}{3} + \frac{1}{2} + \frac{1}{4}\right) - B\left(\frac{1}{2} + \frac{1}{4}\right) = -1$$

$$A\left(-\frac{1}{2} - \frac{1}{4}\right) + B\left(\frac{1}{8} + \frac{1}{2} + \frac{1}{2}\right) = -5$$



$$\begin{aligned}\frac{A-B}{2} + \frac{A-B}{4} + \frac{A}{3} + 1 &= 0 \\ \frac{B-A}{4} + \frac{B-A}{2} + \frac{B}{8} + 5 &= 0\end{aligned}$$

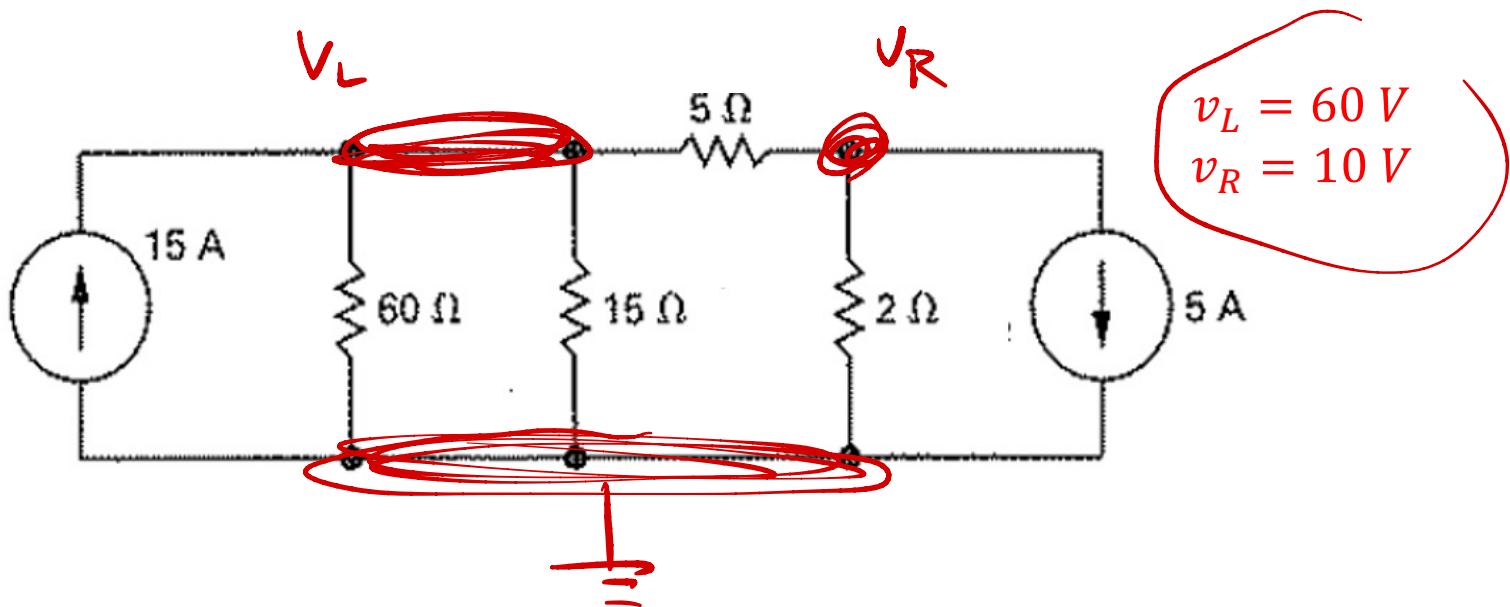
$$\begin{aligned}6A - 6B + 3A - 3B + 4A &= -12 \\ 2B - 2A + 4B - 4A + B &= -40\end{aligned}$$

$$\begin{aligned}13A - 9B &= -12 \\ -6A + 7B &= -40\end{aligned}$$

$$A = \frac{\begin{vmatrix} -12 & -9 \\ -40 & 7 \end{vmatrix}}{\begin{vmatrix} 13 & -9 \\ -6 & 7 \end{vmatrix}} = \frac{-84 - 360}{91 - 54} = \frac{-444}{37} = -12 \text{ volts}$$

$$B = \frac{\begin{vmatrix} 13 & -12 \\ -6 & -40 \end{vmatrix}}{\begin{vmatrix} 13 & -9 \\ -6 & 7 \end{vmatrix}} = \frac{-520 - 72}{35} = \frac{-592}{37} = -16 \text{ volts}$$

Example:



KCL at v_L :

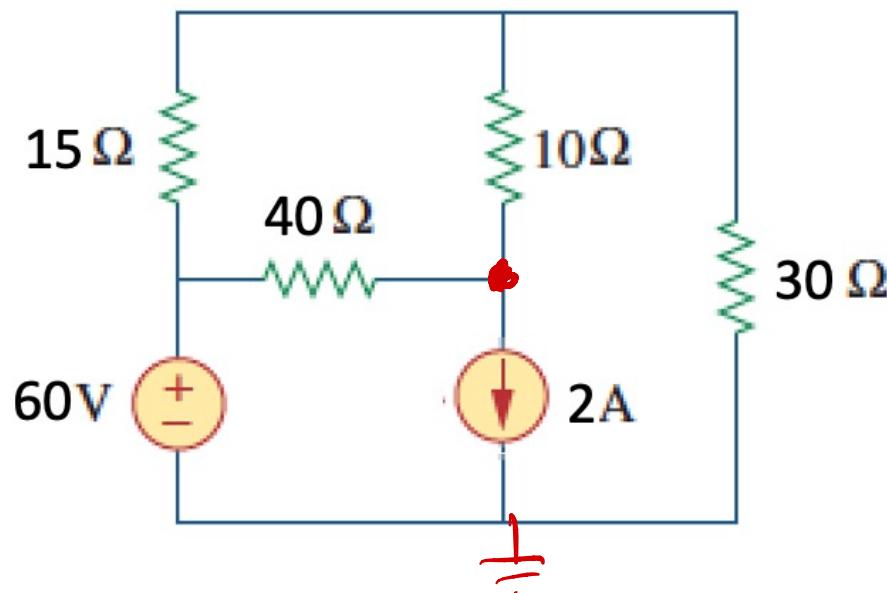
$$-15 + \frac{v_L}{60} + \frac{v_L}{15} + \frac{v_L - v_R}{5} = 0$$

KCL at v_R :

$$+5 + \frac{v_R}{2} + \frac{v_R - v_L}{5} = 0$$

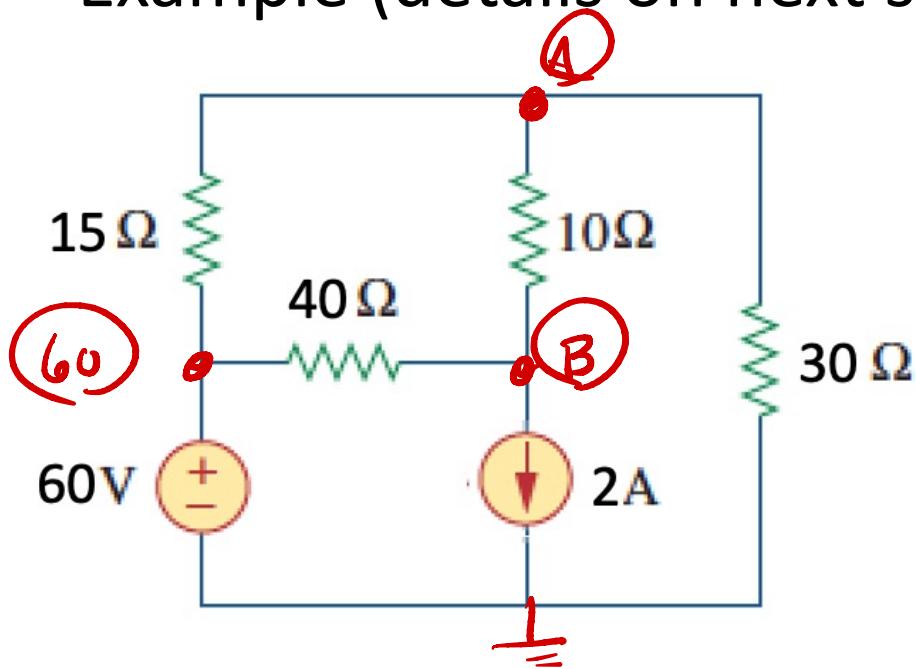
Extension #1 – a V-only branch

- Consider:



- **IF** connected to ground, it's just one less node voltage to worry about

Example (details on next slide)



+ 1 labelled nodes
picking ground

\Rightarrow - RCL:

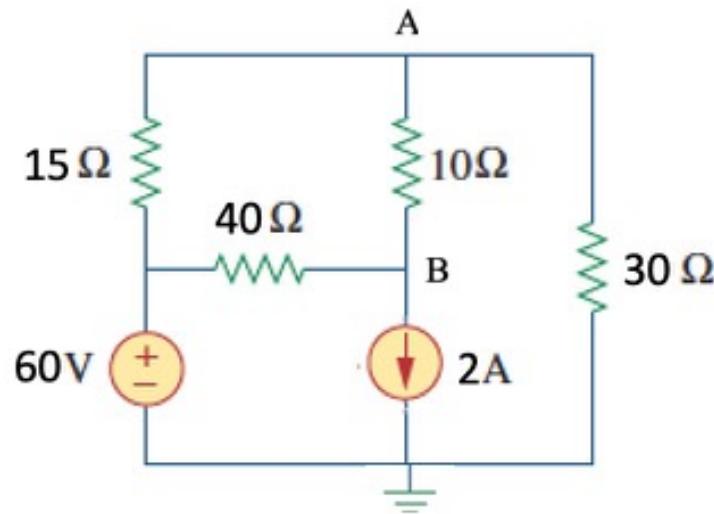
$$\begin{aligned} \underline{A}: & \quad 30 \left(\frac{A - 60}{15} + \frac{A - B}{10} + \frac{A}{30} = 0 \right) \\ \underline{B}: & \quad 40 \left(2 + \frac{B - A}{10} + \frac{B - 60}{40} = 0 \right) \end{aligned}$$

$$2\underline{A} - 120 + 3\underline{A} - 3\underline{B} + \underline{A} = 0$$

$$80 + 4\underline{B} - 4\underline{A} + \underline{B} - 60 = 0$$

$$6\underline{A} - 3\underline{B} = 120$$

$$-4\underline{A} + 5\underline{B} = -20$$



$$\frac{A - 60}{15} + \frac{A - B}{10} + \frac{A}{30} = 0$$

$$\frac{B - 60}{40} + \frac{B - A}{10} + 2 = 0$$

$$6A - 3B = 120$$

$$-4A + 5B = -20$$

$$A = 30 \text{ volts}$$

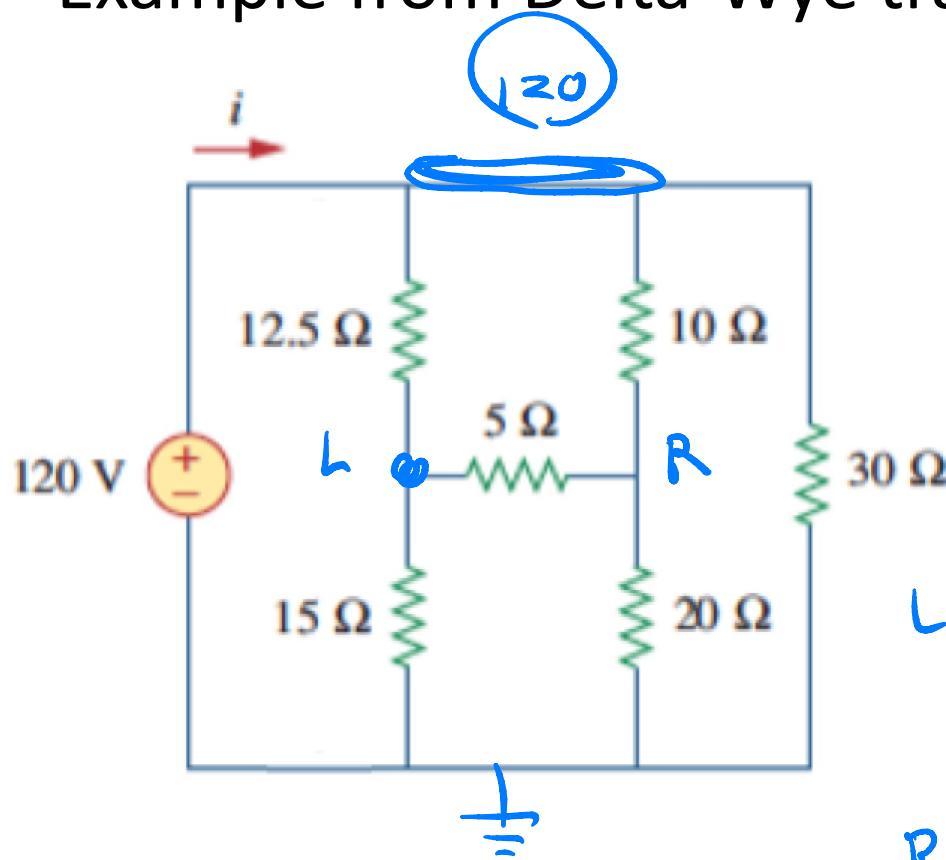
$$B = 20 \text{ volts}$$

Example from Delta-Wye transformations

$$v_L = 70.8 \text{ V}$$

$$v_R = 74.8 \text{ V}$$

$$i = 12.5 \text{ A}$$



$$\{ = \frac{120 - L}{12.5} + \frac{120 - R}{10} + \frac{120}{30}$$

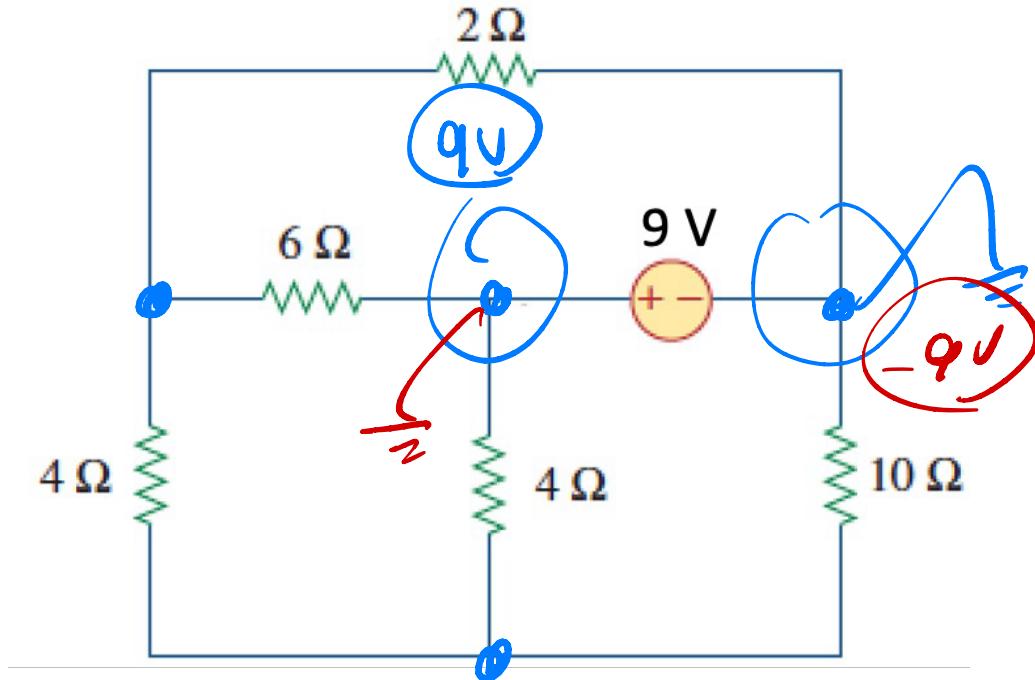
L:

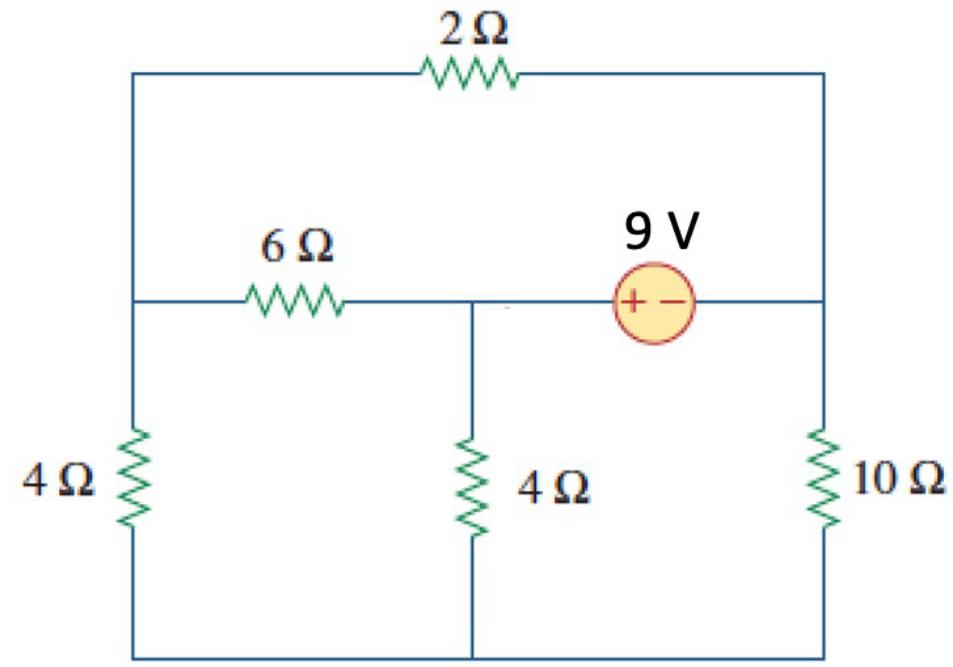
$$\frac{L - 120}{12.5} + \frac{L - R}{5} + \frac{L}{15} = 0$$

R:

$$\frac{R - 120}{10} + \frac{R - L}{5} + \frac{R}{20} = 0$$

Example: recall that we can place ground as needed; put it on the right and solve for left and bottom

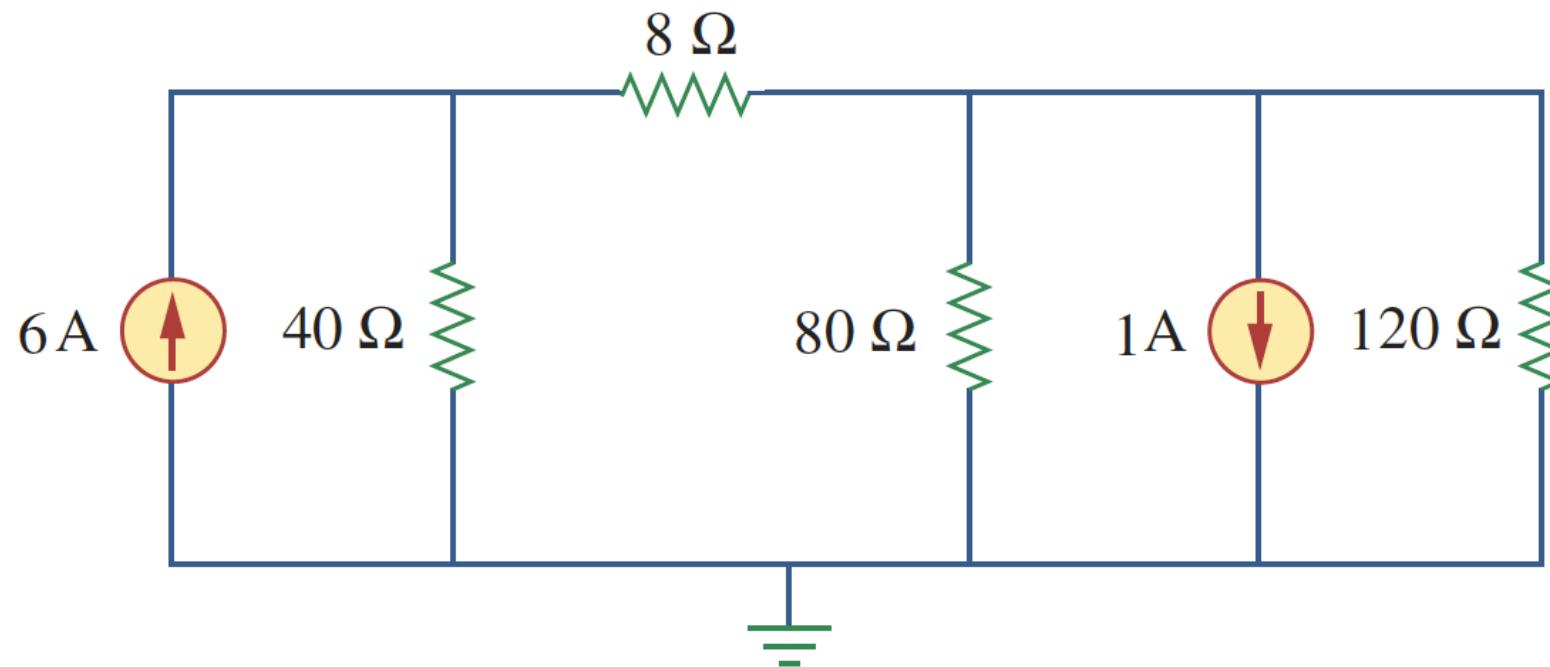




$$v_L = 3 \text{ V}$$
$$v_B = 5 \text{ V}$$

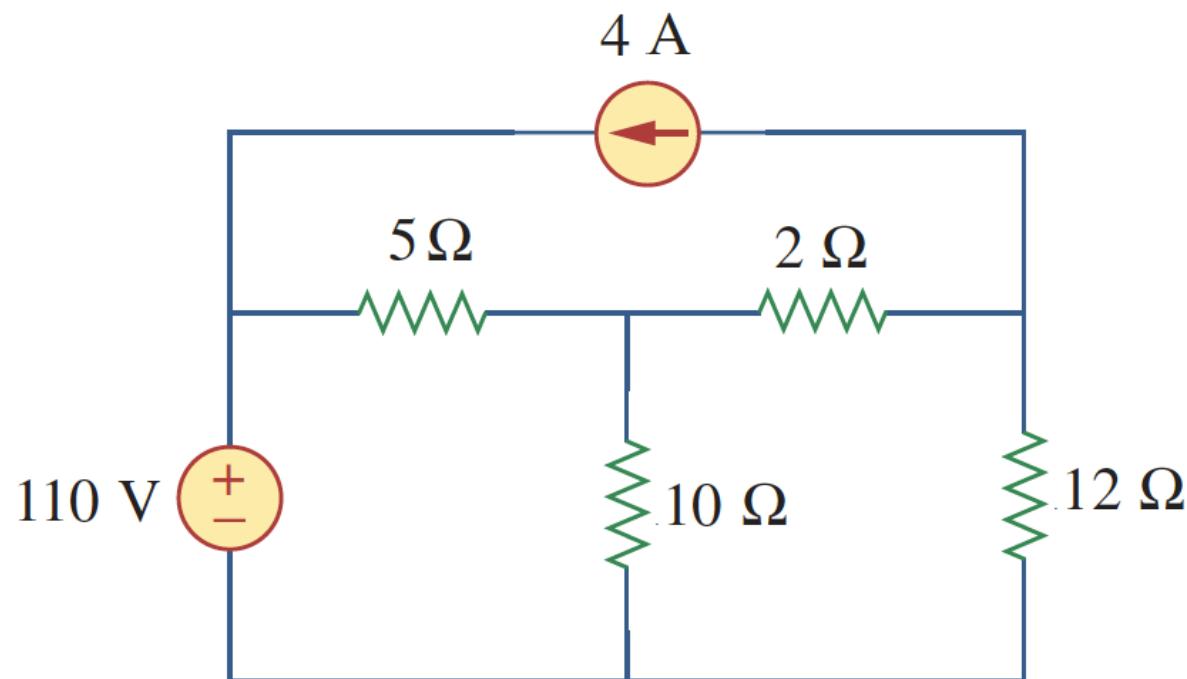
120 V, 96 V

Practice problem: find the nodes voltages to the left and right of the $8\ \Omega$ resistor



50 V, 36 V

Practice problem: assuming ground on the bottom, find the nodes voltages to the left and right of the $2\ \Omega$ resistor



70 V, 60 V

Practice problem: find the nodes voltages to the left and right of the $10 \text{ k}\Omega$ resistor, ground at the bottom

