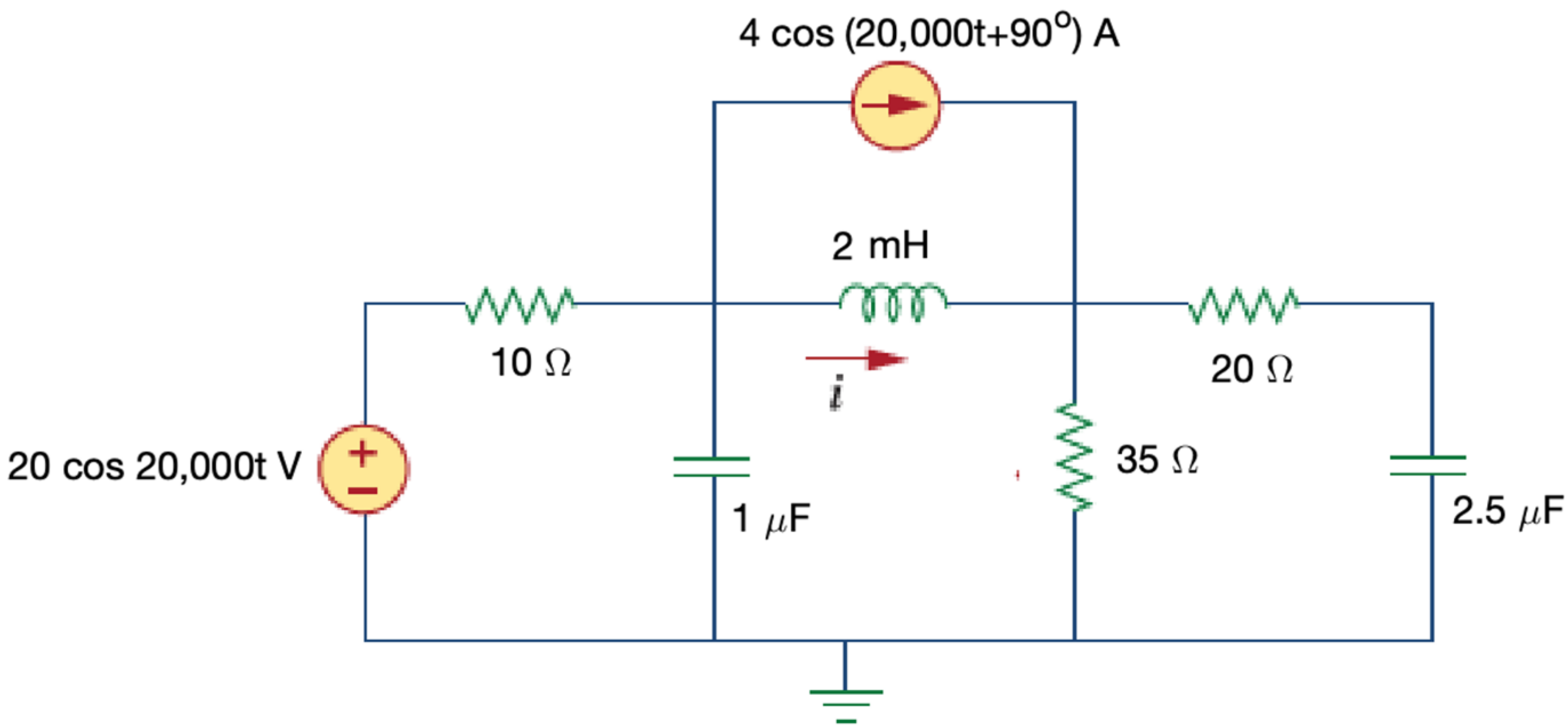
Lecture 20
Phasors – 6 of 9

more examples

Where Are We?

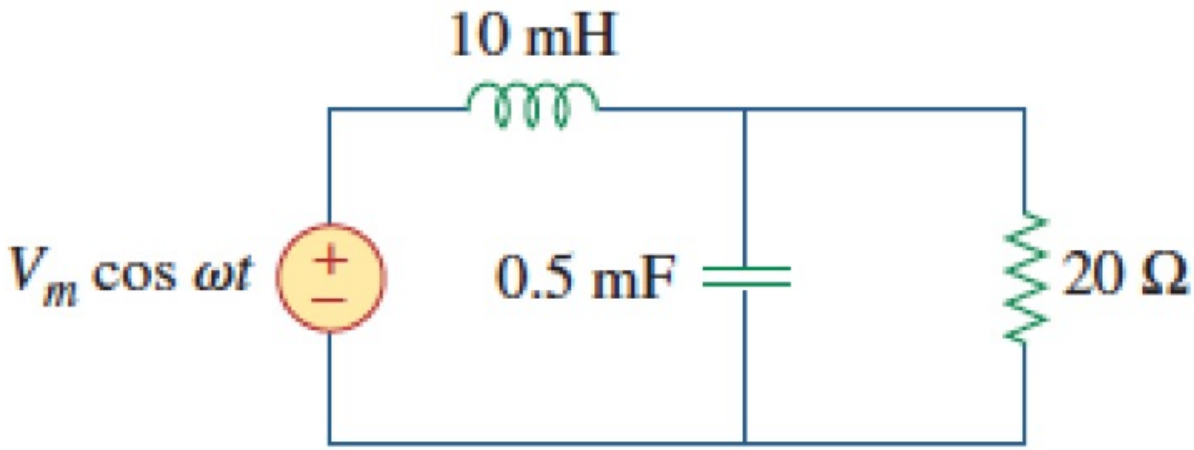
- What we know how to solve: find $i(t)$:



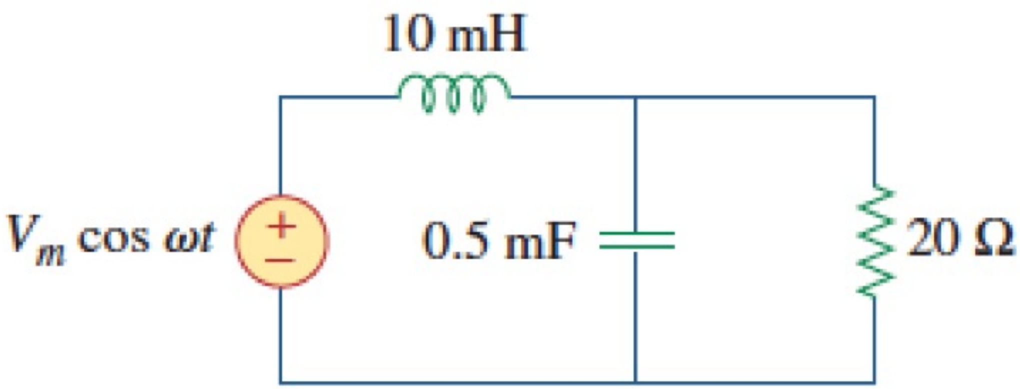
$1.91\cos(20{,}000t - 123°)\ A$

Other Question Types

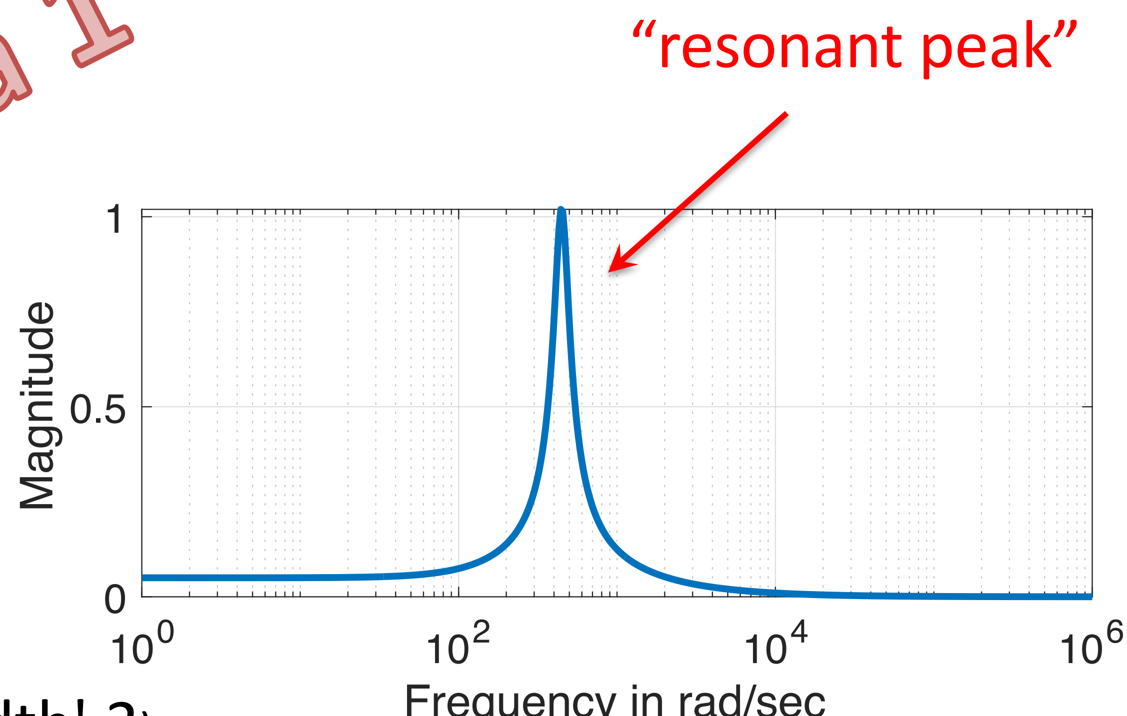
Sample: for what frequency is the source current the largest?



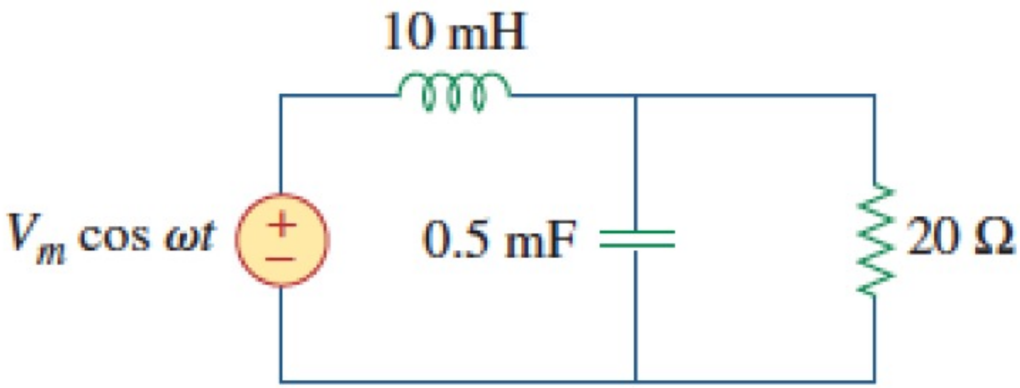
– Method?
  - Numerical calculation
  - Analysis (paper and symbolic)



Idea 1 – compute it




```
om = logspace(0,6,500);
ZC = 1./(1j*om*0.5e-3);
ZP = 20*ZC./(20+ZC);
I0 = 1./(1j*om*10e-3+ZP);
semilogx(om,(abs(I0)),'linewidth',3)
```

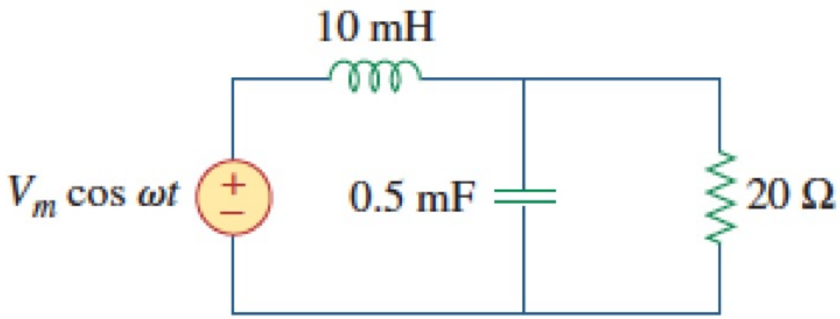


Idea 2 – analyze it

- Equivalent impedance $Z = Z_L + R||Z_C$
- Source current $I_S = -\frac{V_m}{Z}$
- By "largest" we mean largest in magnitude

$$|I_S| = \left|\frac{V_m}{Z}\right| = \frac{|V_m|}{|Z|} = \frac{V_m}{|Z|}$$

- So largest is the same as finding minimum $|Z|$



$$Z = Z_L + R||Z_C = \frac{j\omega}{100} + \frac{(20)\left(-j\,\frac{200}{\omega}\right)}{20 - j\,\frac{200}{\omega}} = \cdots$$

and

$$|Z| = \sqrt{\frac{\omega^4 - 390{,}000\omega^2 + 40\times10^9}{10{,}000(\omega^2 + 10{,}000)}}$$

To minimize $|Z|$, we set its ω derivative to zero; for math simplicity, let's work with the square of $|Z|$

$$|Z|^2 = \frac{\omega^4 - 390{,}000\omega^2 + 40\times10^9}{10{,}000(\omega^2 + 10{,}000)} = \frac{N(\omega)}{D(\omega)}$$

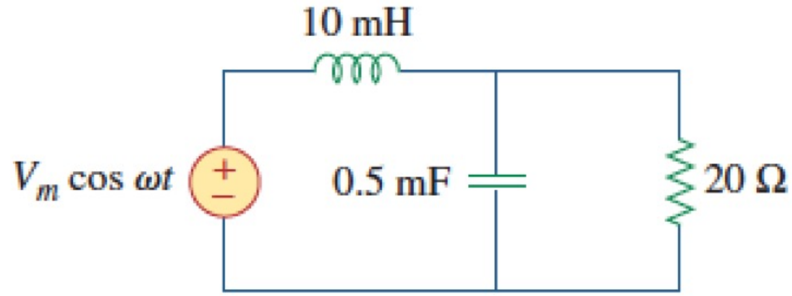
And we want

$$\frac{\partial|Z|^2}{\partial\omega} = \frac{\omega(\omega^4 + 20{,}000\,\omega^2 - 439\times10^8)}{5{,}000(\omega^2 + 10{,}000)^2} = 0$$

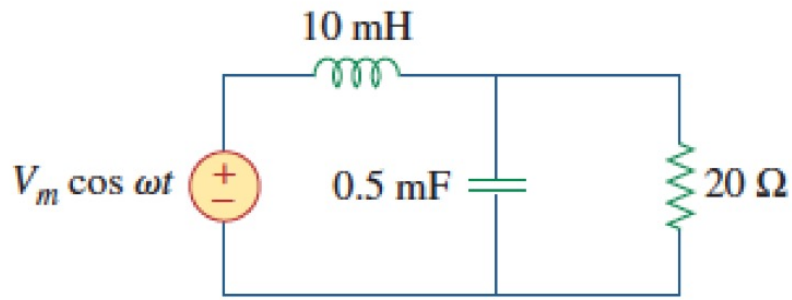
or

$$\omega^4 + 20{,}000\,\omega^2 - 439\times10^8 = 0 \rightarrow \omega = 447$$

```
>> syms w real
>> zc = 1/(1j*w*5e-4)

zc =

-2000i/w

>> zp = 20*zc/(20+zc)

zp =

40000i/(w*(2000i/w - 20))

>> z = 1j*w*1e-2 + zp

z =

(w*1i)/100 + 40000i/(w*(2000i/w - 20))
```

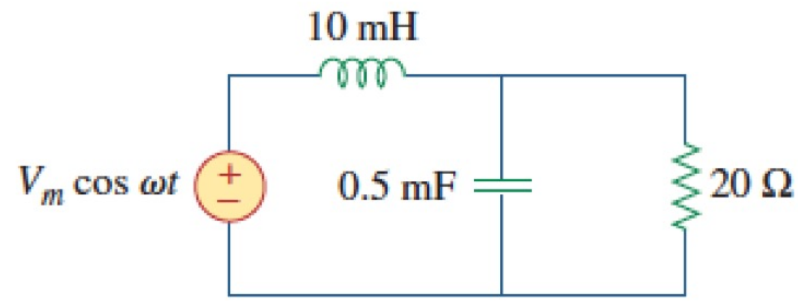


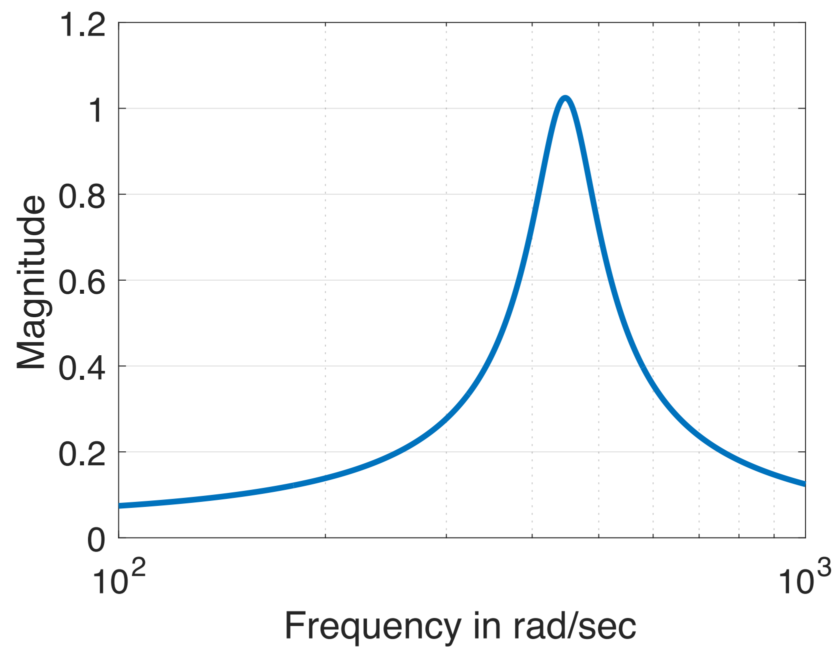
Or use a tool



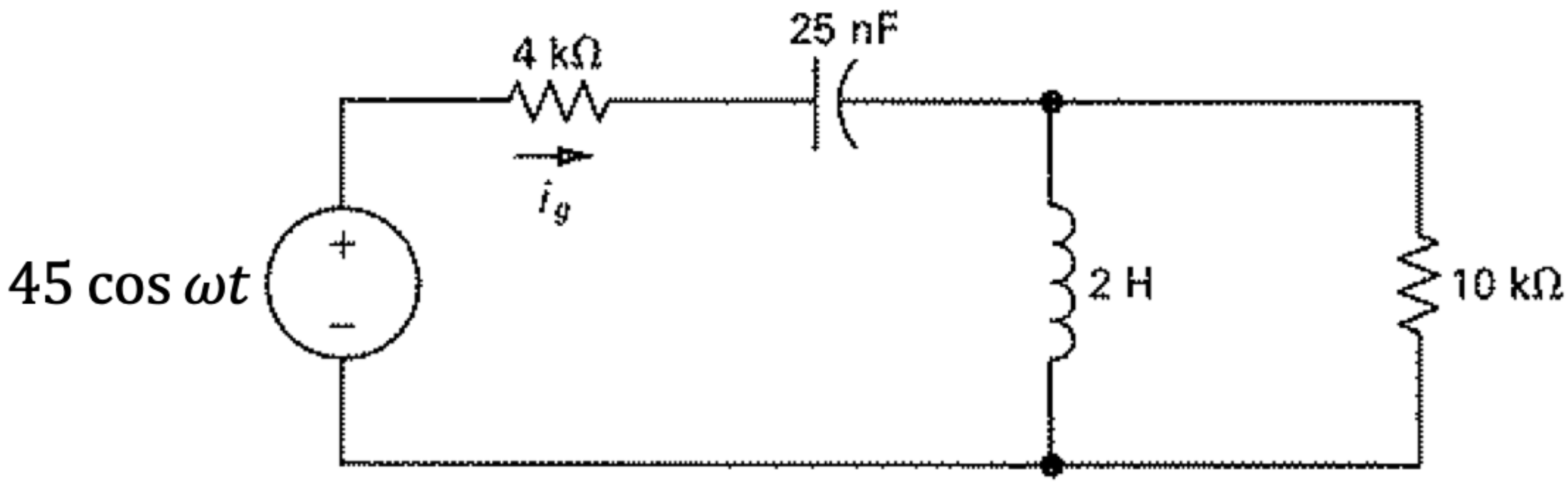

```
>> H = 1/z;
>> aH = sqrt( real(H)^2 + imag(H)^2 );
>> pretty(aH)
    / /  w           800000       \2                            \
    | | --- - ------------------- |                             |
    | | 100     / 4000000       \ |                             |
    | |       w | ------- + 400 | |                             |
    | |         |     2         | |                             |
    | \         \    w          / /        6400000000000000     |
sqrt| ------------------------------ + ------------------------ |
    |               #1                  4    / 4000000       \2 |
    |                                  w  #1 | ------- + 400 |  |
    |                                        |     2         |  |
    \                                        \    w          /  /

where

         /    6400000000000000     /  w           800000       \2 \2
   #1 == | --------------------- + | --- - ------------------- |  |
         |  4 / 4000000       \2   | 100     / 4000000       \ |  |
         | w  | ------- + 400 |    |       w | ------- + 400 | |  |
         |    |     2         |    |         |     2         | |  |
         \    \    w          /    \         \    w          / /  /
```

```
>> daH = diff(aH,w);
>> solve(daH,w)

ans =

  (20000*110^(1/2) - 10000)^(1/2)
 -(20000*110^(1/2) - 10000)^(1/2)

>> eval(ans(1))

ans =

  446.9472
```





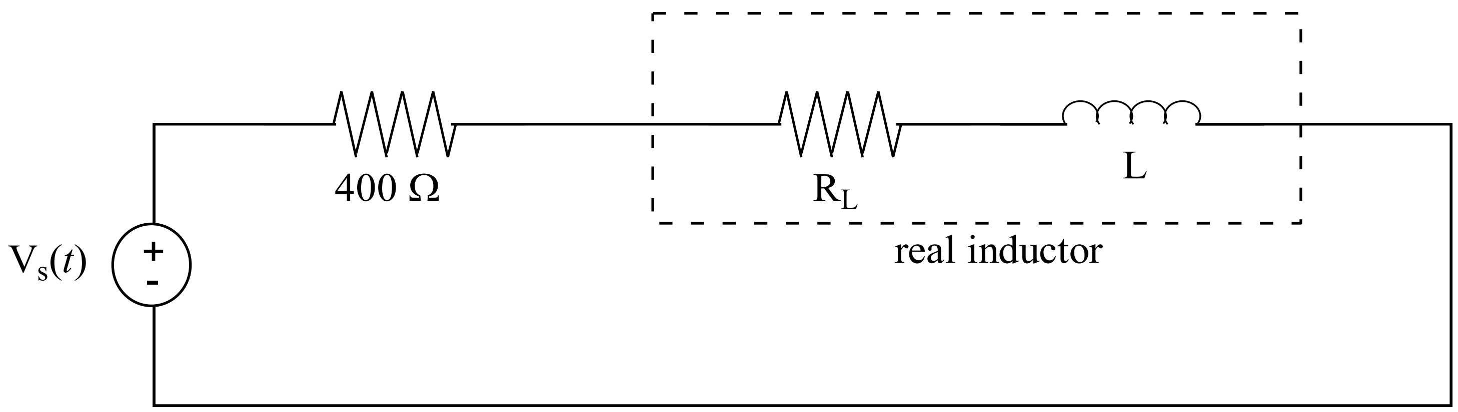
Example: At what frequency ω is i_g in phase with the voltage source?



10^4 rad/sec

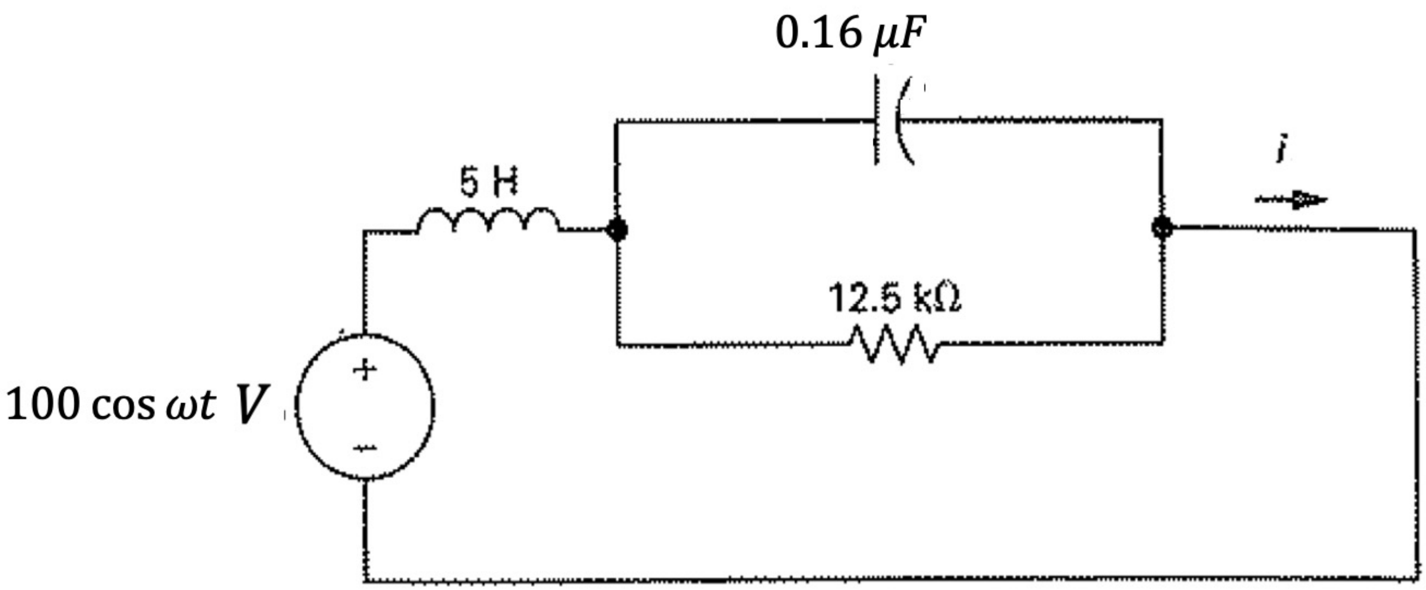
10^4 rad/sec

Example: We model a real inductor as shown with a series parasitic resistance R_L. To measure its parameters, R_L and L, we build the circuit shown (with a 60 Hz source) and use an AC voltmeter to measure the amplitudes of the component voltages. Given $|V_S| = 120\ V, |V_R| = 100\ V, |V_L| = 30$, find R_L and L.



$70\ \Omega, 259\ mH$

Practice problem: At what frequency does the current i have the largest magnitude? What is that magnitude?



$$1120\frac{rad}{sec};43.8\;mA$$