ELE 215 – Exercise 4 – The Notch Filter

Objectives

- To better compare theory and practice for Lab 7.

Notes

- Work with your Lab 7 partner on this exercise; only 1 of you needs to upload it.

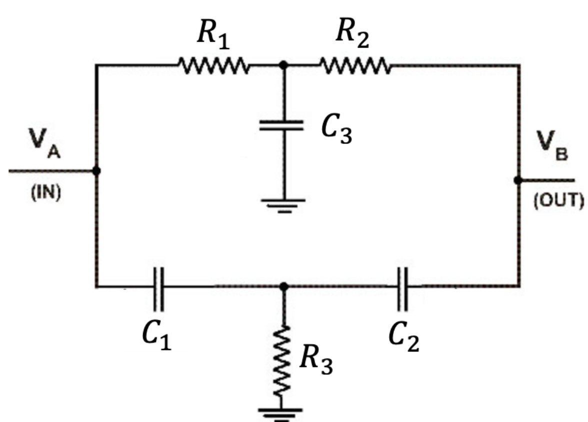
Background

It's very likely that your theoretical and experimental data from Lab 7 did not match too well; certainly not as well as you should have seen in Labs 5 and 6. The theoretical transfer function in the lab instruction relies on the resistors and capacitors being perfectly matched in value and experimentally you had some small variation. For this exercise you are to recompute the Bode plot assuming <u>non-identical</u> values for the resistors and capacitors, comparing this better approximation to your experimental data.

Procedure

1. <u>Relabel the circuit diagram:</u>

   We start by relabeling the resistor and capacitors as different values



2. <u>Write the node equations</u>:

   Label the three nodes: V_a at the top of capacitor C_3, V_c at the top of resistor R_3, and the output node already labeled V_B. Write 3 phasor node equations using V_A as the input source. We know from ELE 212 how to solve this problem for a fixed frequency, finding the magnitude and phase angle for the output; however, for a Bode plot we want to compute these for a range of frequencies. Further, in Recitation 8 I presented an example of using a loop in MatLab to successively solve such node equations. Do this for the range of frequencies from your Lab 7 results and use this as a better theoretical Bode plot.

3. <u>Generate a new Bode Plot</u>:

   Modify your Lab 7 script to do the following:
   - Generate the more accurate Bode plot from above using your actual resistor and capacitor values; this should appear as a solid line.
   - Replot your 9 pairs of experimental magnitude and phase values using clearly visible symbols (NO connecting line).
   - Make the Bode plot pretty by adding legends, etc.

   Hopefully the experimental data is a better fit to this new theoretical curve.

4. <u>Submission</u>:

   a) Recopy your component values and experimental data onto the Summay Page for Exercise 4. This is the first page of your submission

   b) Print the Bode plot (magnitude and phase) as a single page in pdf; this is the second page in your submission. This page should read normally, and NOT be rotated.

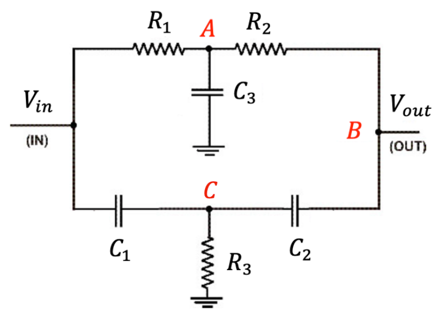
   c) Upload these 2 pages as a single pdf document to the ELE 215 Brighspace site. it is due by 5 PM on Wednesday Apr 15 via Brightspace; late submissions will not be accepted

   d) Please use the following convention for the filename, substituting the last 3 digits of your HW ID number:

Exercise_4_789.pdf

Solution

Let's derive the general expression for the transfer function of this filter. Allowing the individual components to be of distinct values (as shown below),



consider nodal analysis with internal nodes A and C and desired node V_{out} or B. The 3 equations in matrix form are

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+j\omega C_3 & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2}+j\omega C_2 & -j\omega C_2 \\ 0 & -j\omega C_2 & \frac{1}{R_3}+j\omega C_1+j\omega C_2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{V_{in}}{R_1} \\ 0 \\ j\omega C_1 V_{in} \end{bmatrix}$$

with solution

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+j\omega C_3 & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2}+j\omega C_2 & -j\omega C_2 \\ 0 & -j\omega C_2 & \frac{1}{R_3}+j\omega C_1+j\omega C_2 \end{bmatrix}^{-1} \begin{bmatrix} \frac{V_{in}}{R_1} \\ 0 \\ j\omega C_1 V_{in} \end{bmatrix}$$

Using MatLab we can just evaluate this in a loop and be done with it.

In general, this is

$$\begin{bmatrix} A \\ \\ B \\ \\ C \end{bmatrix} = \begin{bmatrix} * & * & * \\ \\ a & * & b \\ \\ * & * & * \end{bmatrix} \begin{bmatrix} \frac{V_{in}}{R_1} \\ \\ 0 \\ \\ j\omega C_1 V_{in} \end{bmatrix}$$

in which we have written many elements of the inverse using * as we do not care about these entries. Specifically, our output is

$$V_{out} = B = a\frac{V_{in}}{R_1} + bj\omega C_1 V_{in} = \left(\frac{a}{R_1} + j2\pi f\ b\ C_1\right) V_{in}$$

Let's simplify our matrix to

$$M = \begin{bmatrix} d & e & 0 \\ f & g & h \\ 0 & j & k \end{bmatrix}$$

Then its determinant is

$$\Delta = \det(M) = dgk - efk - dhj$$

Or

$$\begin{aligned} \Delta = \det(M) = &\left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C_3\right)\left(\frac{1}{R_2} + j\omega C_2\right)\left(\frac{1}{R_3} + j\omega C_1 + j\omega C_2\right) \\ &- \frac{1}{{R_2}^2}\left(\frac{1}{R_3} + j\omega C_1 + j\omega C_2\right) + \omega^2 {C_2}^2\left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C_3\right) \end{aligned}$$

Why bother any more….I don't need the theory