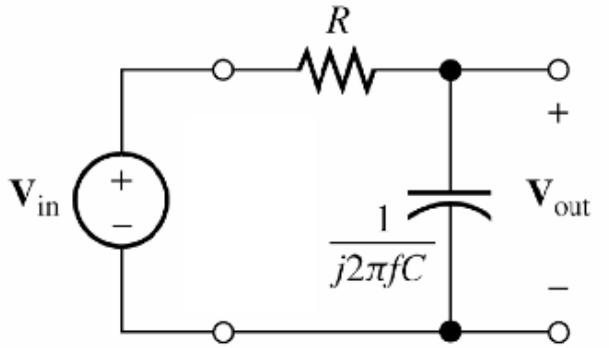
ELE 215
Linear Circuits Laboratory

Recitation 7

Bode plots I

Example of Voltage versus Frequency

- Consider the RC circuit



- Using phasors:

$$\boldsymbol{V}_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}\,\boldsymbol{V}_{in} = \underbrace{\frac{1}{1 + j\omega RC}}\,\boldsymbol{V}_{in}$$

Describes change in amplitude and phase

- Define the “transfer function” or “frequency response” as the phasor input-to-output ratio

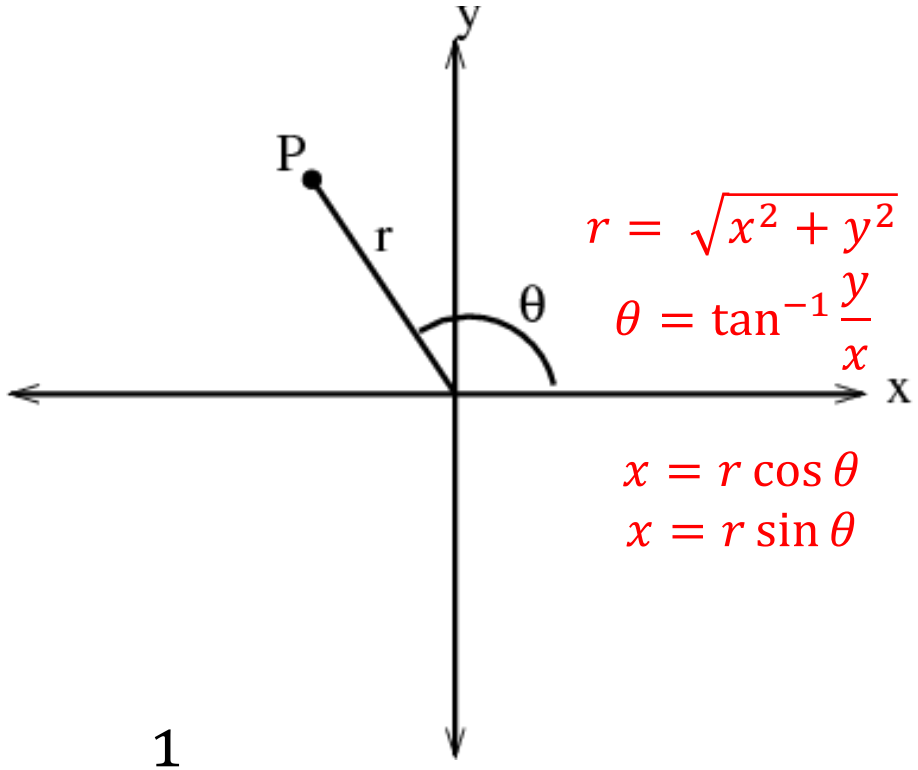
$$H(\omega) = \frac{\boldsymbol{V}_{out}}{\boldsymbol{V}_{in}} = \frac{1}{1 + j\omega RC}$$

or

$$H(f) = \frac{1}{1 + j\ 2\pi f\ RC}$$

- Comments:
  - It describes how the circuit changes a unit amplitude, zero phase input to the output
  - It is often written in terms of f, Hertz
  - It is a complex valued function of frequency
- Question – how to visualize??

- We use the “polar coordinate” representation
  - x = real part
  - y = imaginary part
  - r = magnitude
  - θ = phase angle



$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\theta$$

$$x = r\sin\theta$$

$$H(f) = \frac{1}{1 + j\,0.2\,\pi\,f}$$

- Magnitude $|H(f)| = \dfrac{1}{\sqrt{1+0.04\,\pi^2\,f^2}}$

- Phase $\theta(f) = -\tan^{-1}(0.2\,\pi\,f)$

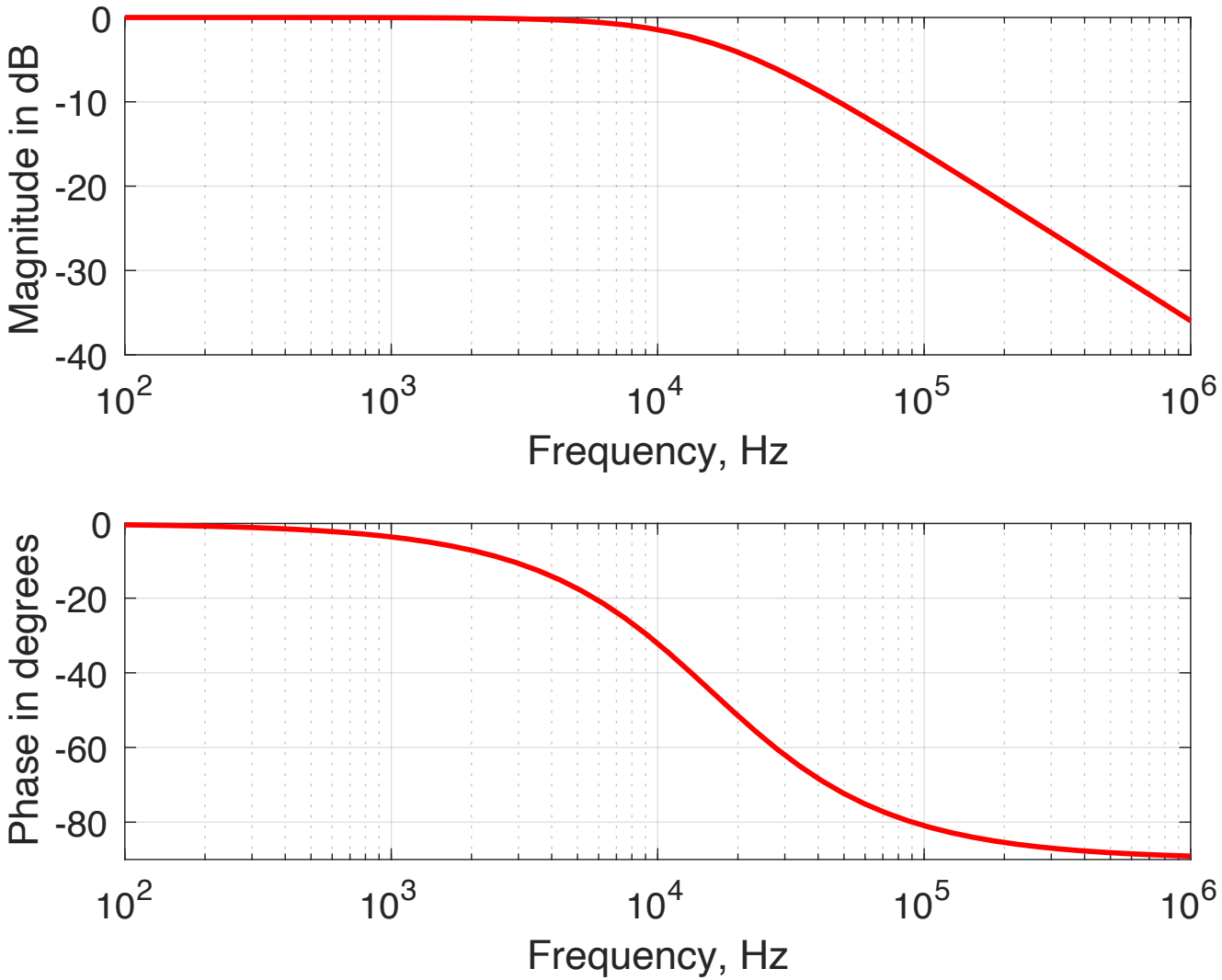
- Bode plot: plot of the transfer function using
  - 2 stacked plots, magnitude and phase vs frequency
  - Log base 10 scale for frequency
  - Linear scale for phase in degrees
  - Decibel scale for magnitude (dB), $20\ \log_{10} mag$

    - Note: decibel scale

mag	dB	mag	dB
1	0		
2	3	0.5	-3
10	20	0.1	-20
100	40	0.01	-40

- Sample Bode plot for $H(f) = \frac{1}{1+j\ 2\pi f\ RC}$

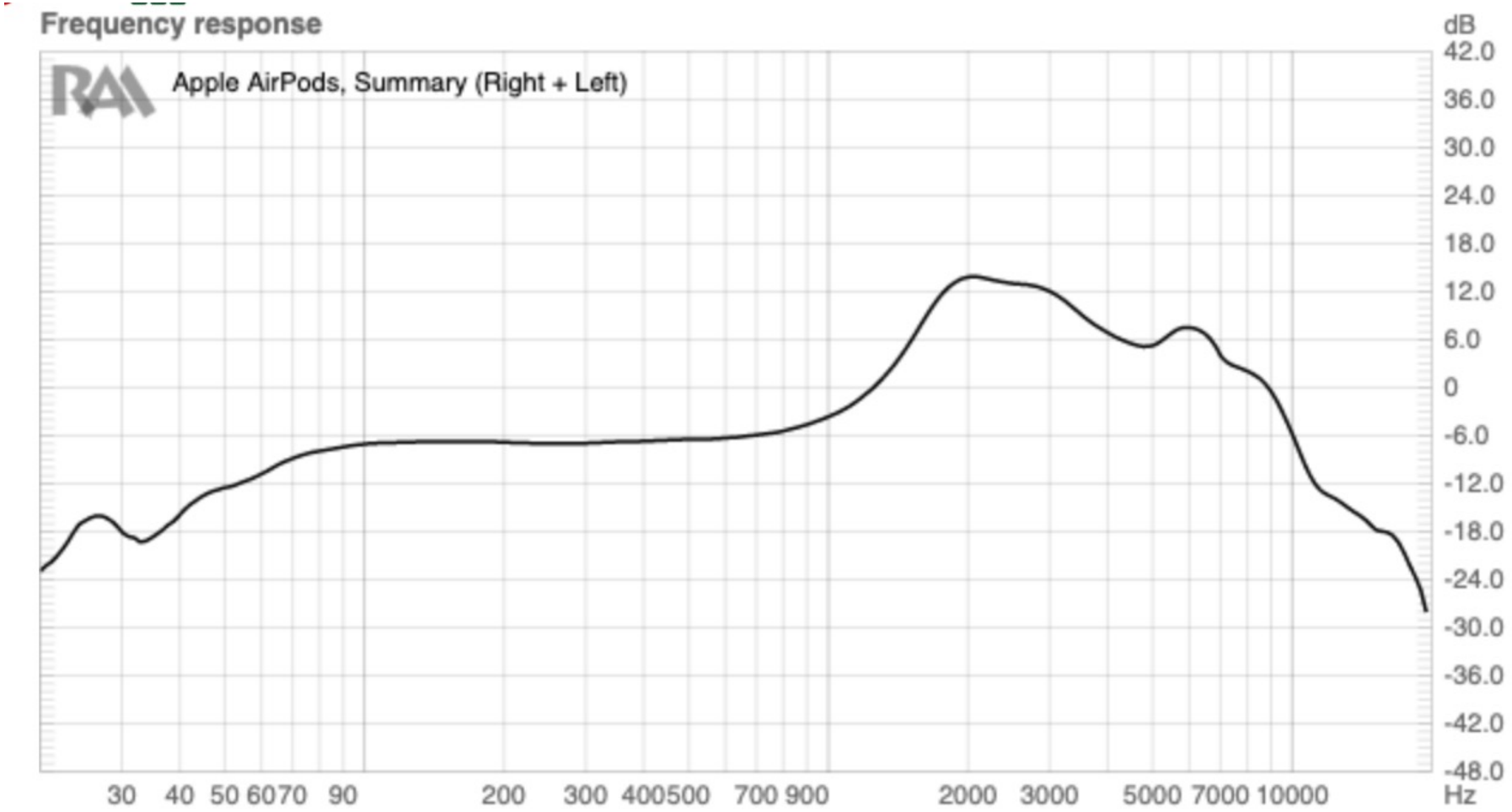
$R = 10\ k\ \Omega$
$C = 0.001\ \mu F$



- Full MatLab code:

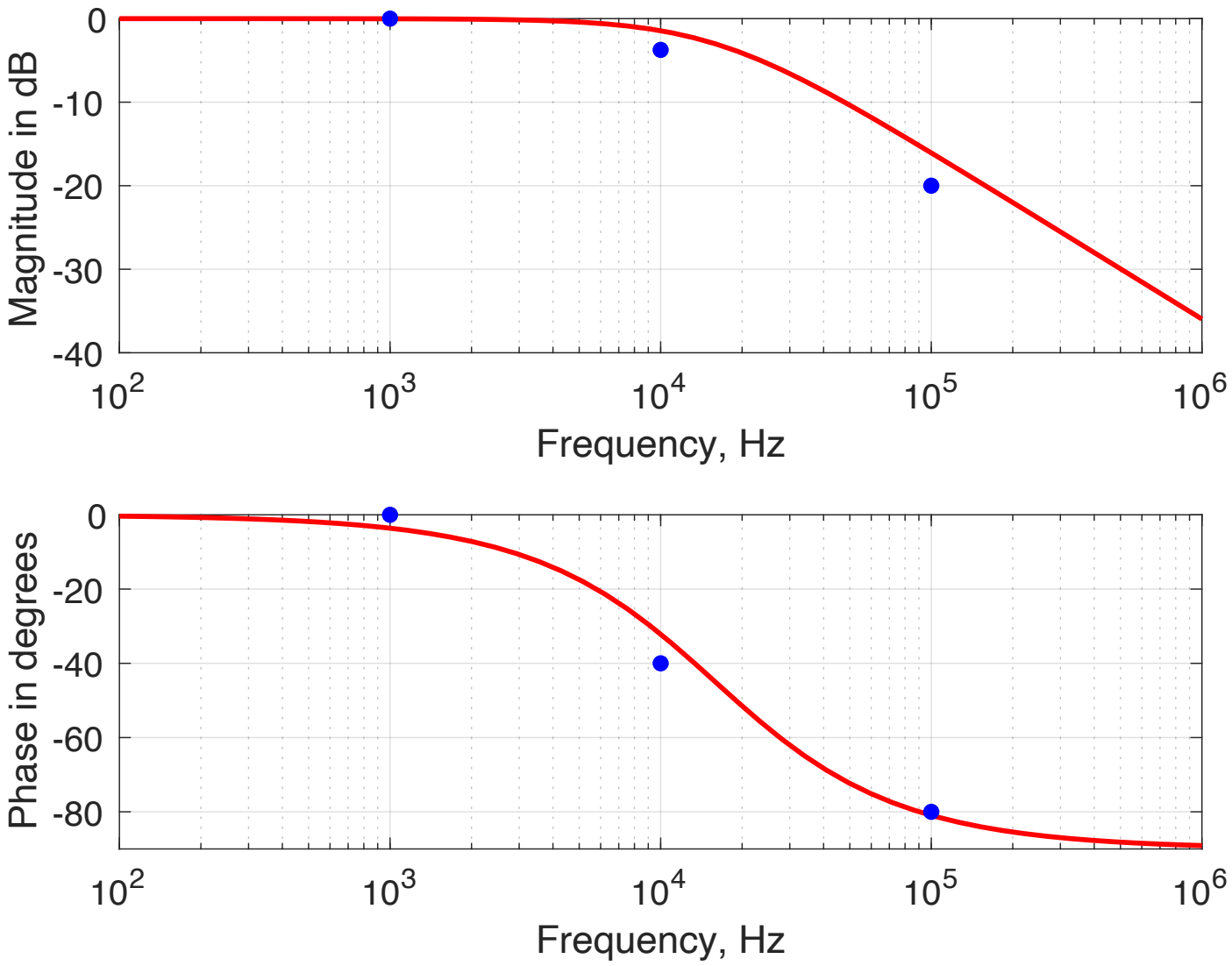
```
subplot(211)
semilogx(f,20*log10(abs(H)),'r-','linewidth',2)
ylim([-40 0])
xlabel('Frequency, Hz')
ylabel('Magnitude in dB')
set(gca,'fontsize',14)
grid on
subplot(212)
semilogx(f,180/pi*angle(H),'r-','linewidth',2)
ylim([-90 0])
xlabel('Frequency, Hz')
ylabel('Phase in degrees')
set(gca,'fontsize',14)
grid on
```

- Such visuals are common
  - Airpods: “-7.1 to +13.9 dB on 100 Hz to 10 kHz”



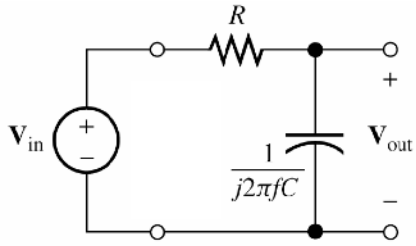
- Adding measured data – unconnected dots

```
fm = [ 1000 10000 100000 ];
am = [ 1 .65 .1 ];
pm = [ 0 -40 -80 ];

hold on
semilogx(fm,20*log10(am),'bo','markersize',6,...
'markerfacecolor','b')
hold off

hold on
semilogx(fm,pm,'bo','markersize',6,...
'markerfacecolor','b')
hold off
```

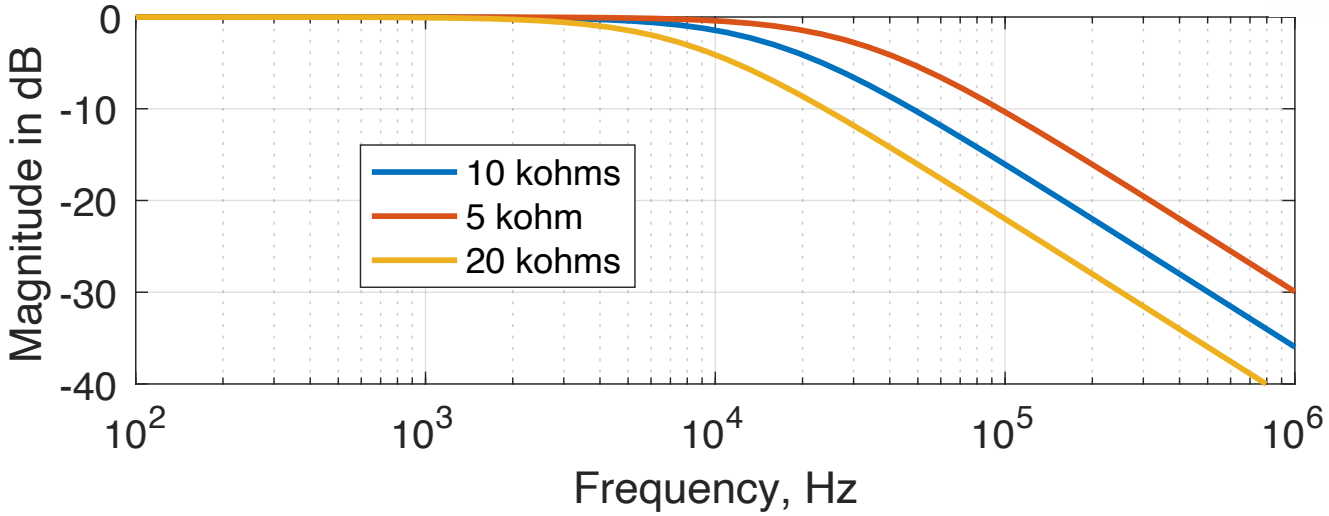

0
-10
-20
-30
-40
Magnitude in dB
10^2
10^3
10^4
10^5
10^6
Frequency, Hz
0
-20
-40
-60
-80
Phase in degrees
10^2
10^3
10^4
10^5
10^6
Frequency, Hz

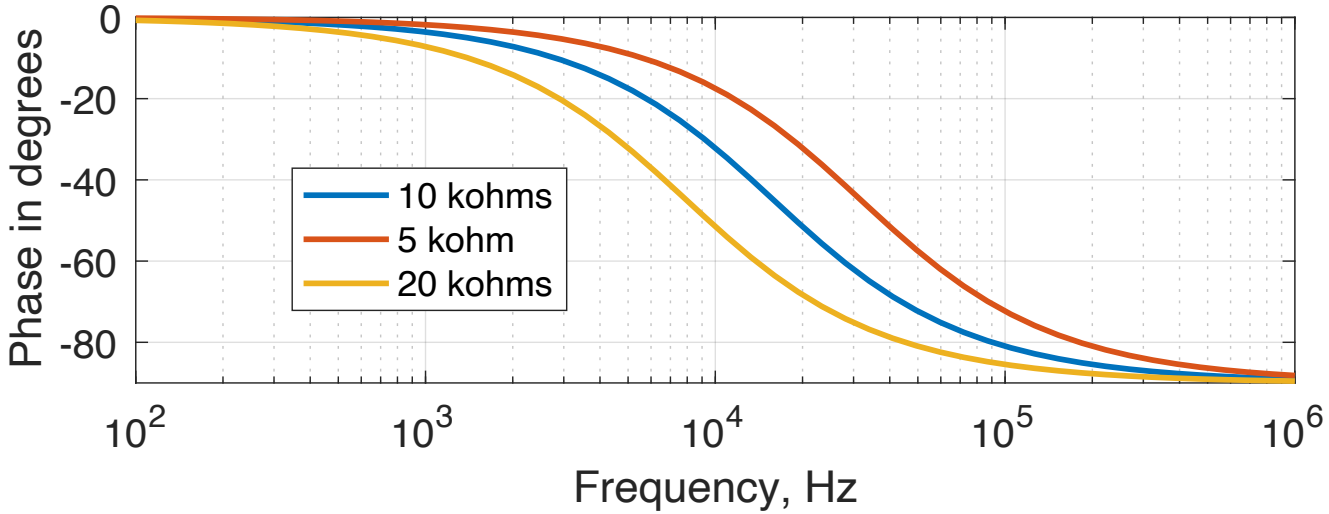
Utility of Bode Plots

- Similar plots for similar circuits:



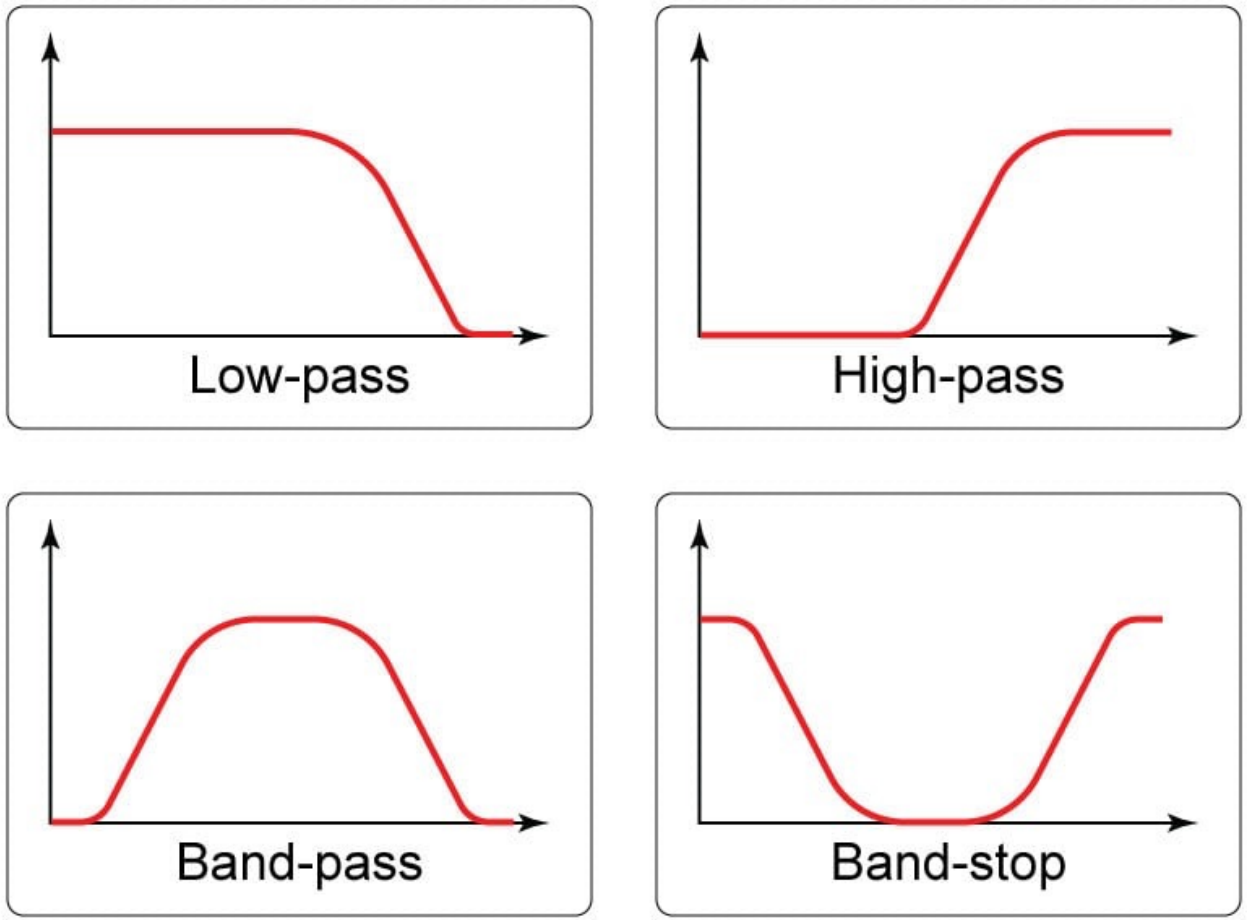
$C = 0.001\ \mu F$



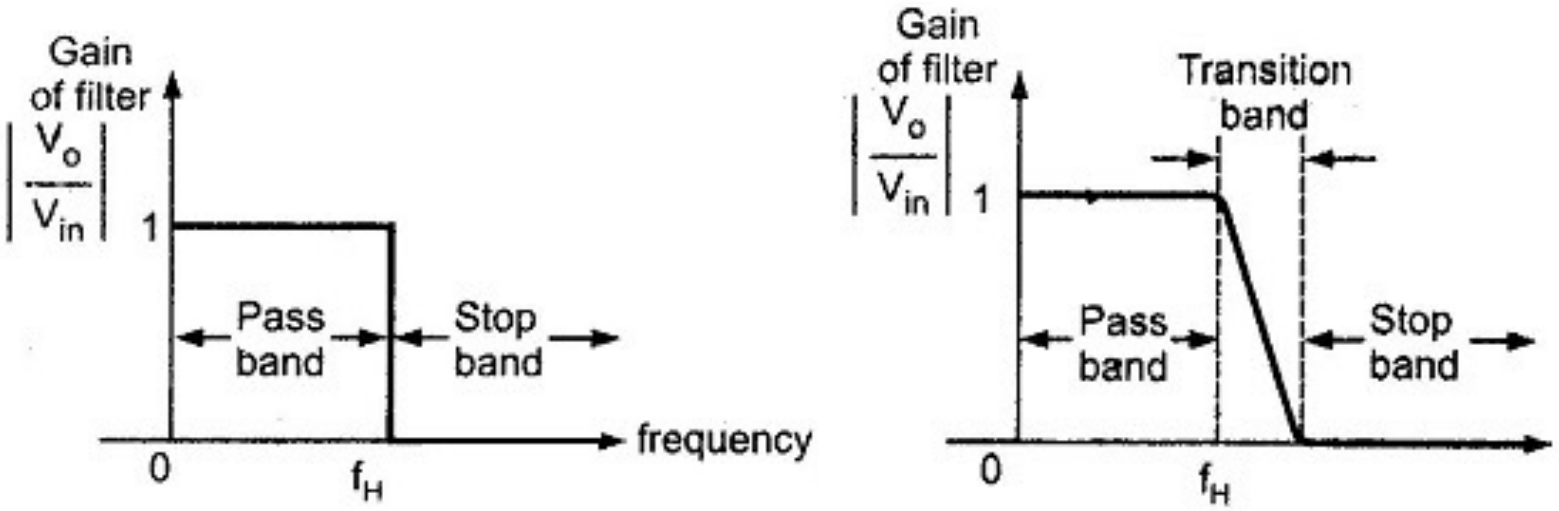


Types of Responses – "Filters"

- Qualitative descriptors of the sensitivity of the magnitude to frequency



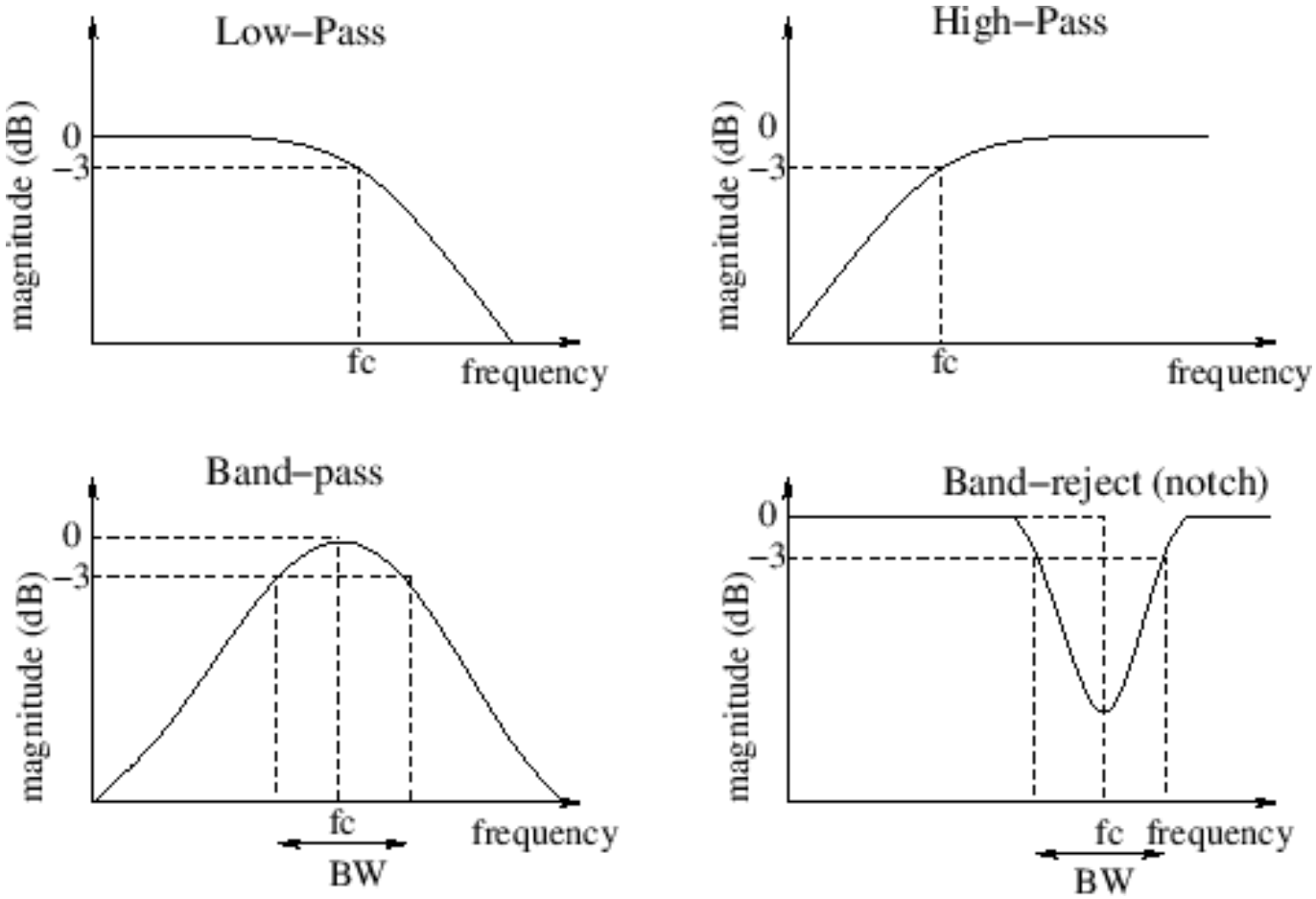
- Terminology
  - Idealized “brick wall” filter vs actual
  - Passband, transition band, and stopband concepts



  - More complex circuits can have “steeper” transitions

– Passband edge(s) usually defined at “3 dB frequency when the magnitude is reduced by (one-half power)

$$-3\ \text{dB} = 0.707 = \frac{1}{\sqrt{2}}$$



- For the RC lowpass

$$H(f) = \frac{1}{1+j\ 2\pi f\ RC}$$

- At the break frequency the magnitude is

$$|H(f_b)| = \frac{1}{\sqrt{1+(2\pi f_b RC)^2}} = \frac{1}{\sqrt{2}}$$

solving

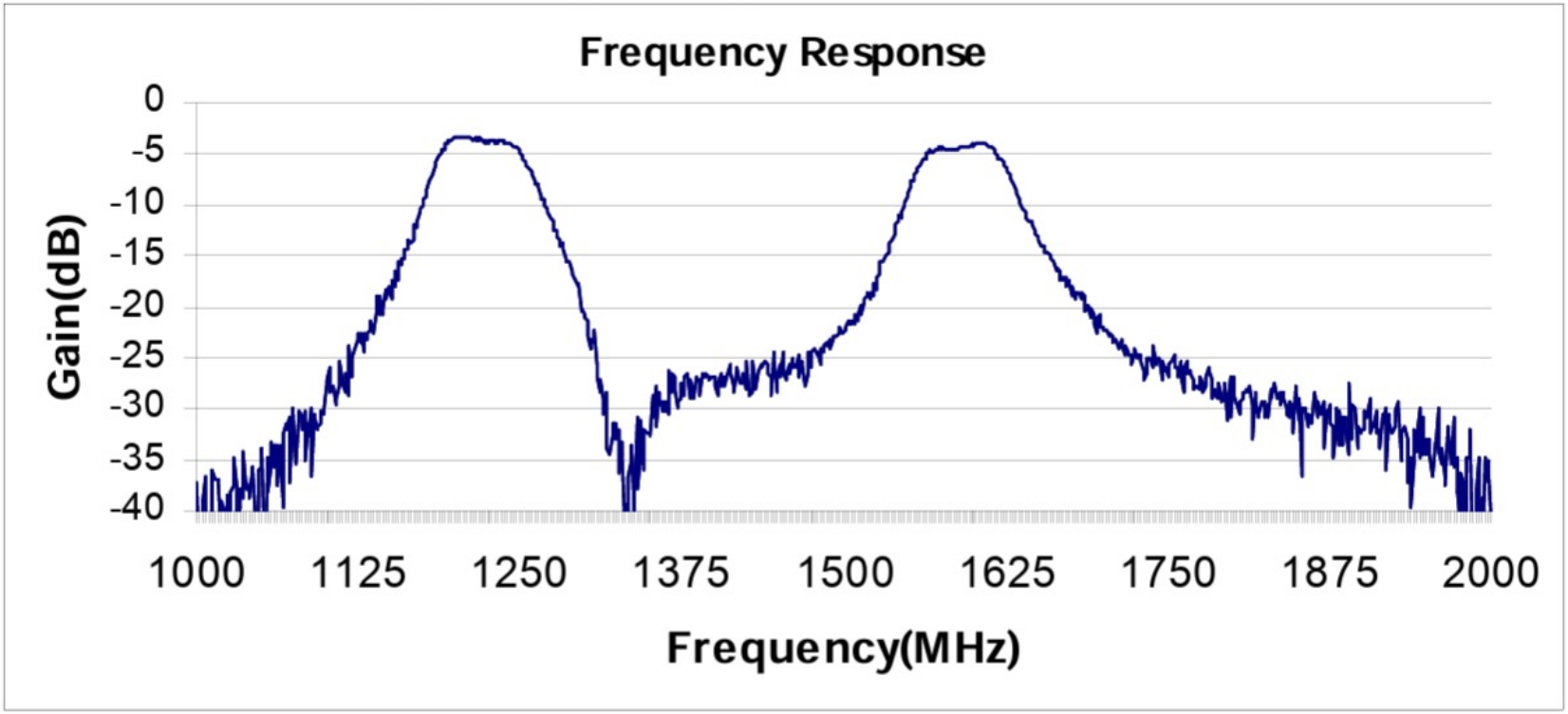
$$1 + (2\pi f_b RC)^2 = 2$$

$$(2\pi f_b RC)^2 = 1$$

$$f_b = \frac{1}{2\pi\ RC}$$

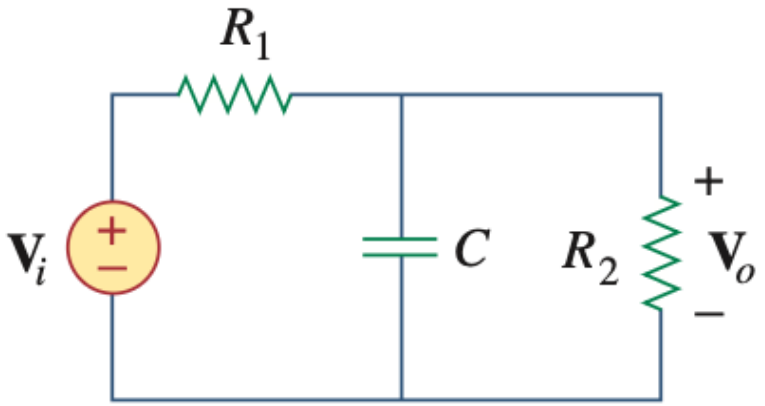
- Example: a passive BPF for GPS receivers

L1L2F Passive Filter



Second Circuit Example

- By voltage division



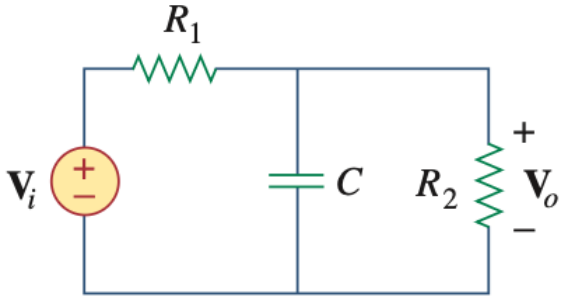
$$\boldsymbol{V}_o = \frac{\frac{R_2\frac{1}{j\omega C}}{R_2+\frac{1}{j\omega C}}}{R_1+\frac{R_2\frac{1}{j\omega C}}{R_2+\frac{1}{j\omega C}}}\,\boldsymbol{V}_i$$

Or

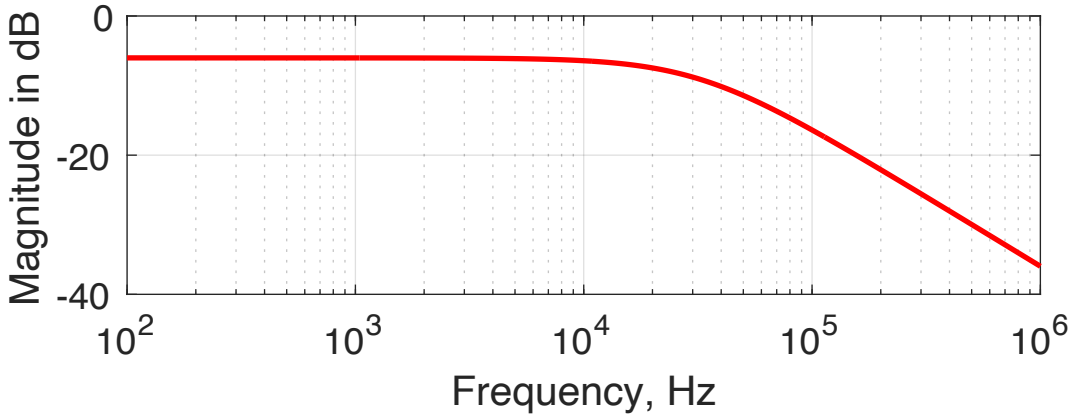
$$H(\omega) = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C}$$

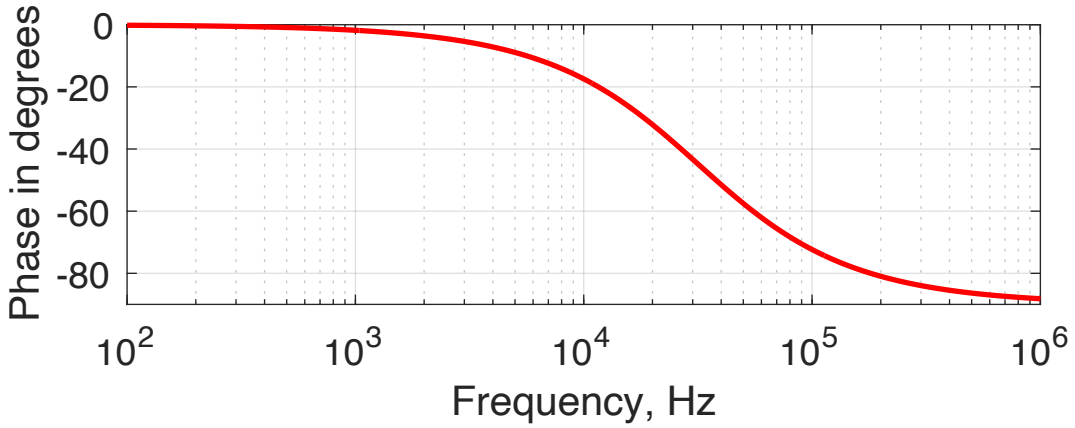
- Example:

$R_1 = R_2 = 10\ k\Omega,$
$C = 0.001\ \mu F$



- Notes:
  - Max is now 0.5 (−6 dB)
  - Break point f_b is at − 9 dB





- For this second R-RC lowpass

$$H(f) = \frac{R_2}{R_1 + R_2 + j2\pi f\ R_1 R_2 C}$$

- Magnitude at f_b satisfies

$$|H(f_b)| = \frac{R_2}{\sqrt{(R_1 + R_2)^2 + (2\pi f_b\ R_1 R_2 C)^2}} = \frac{1}{\sqrt{2}}\ \frac{R_2}{R_1 + R_2}$$

Or

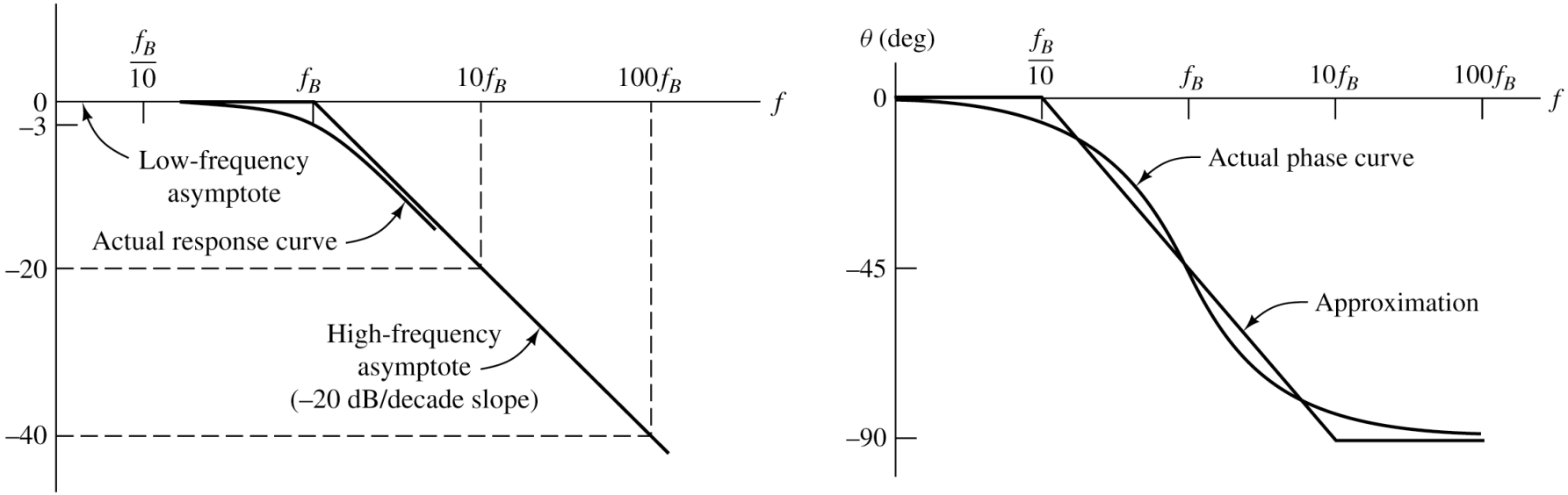
$$f_b\ = \frac{R_1 + R_2}{2\pi R_1 R_2}$$

- Conclusion: f_b varies with the circuit

What Range of Frequencies to Show?

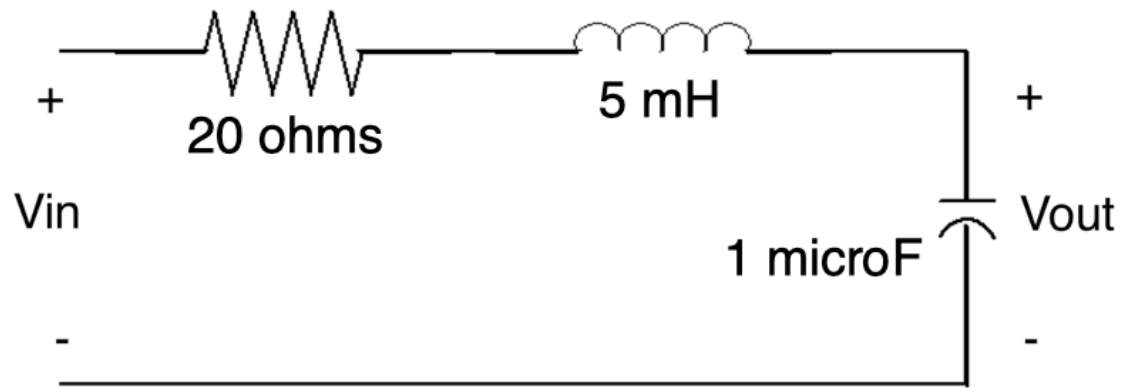
- Our simple RC lowpass filter has:

$$H(f) = \frac{1}{1 + j\,2\pi f\,RC} = \frac{1}{1 + j\,2\pi\,\frac{f}{f_B}}$$



  - "break frequency" for this circuit is $f_B = \frac{1}{RC}$
  - +/- one decade around "f_B" seems to show it all

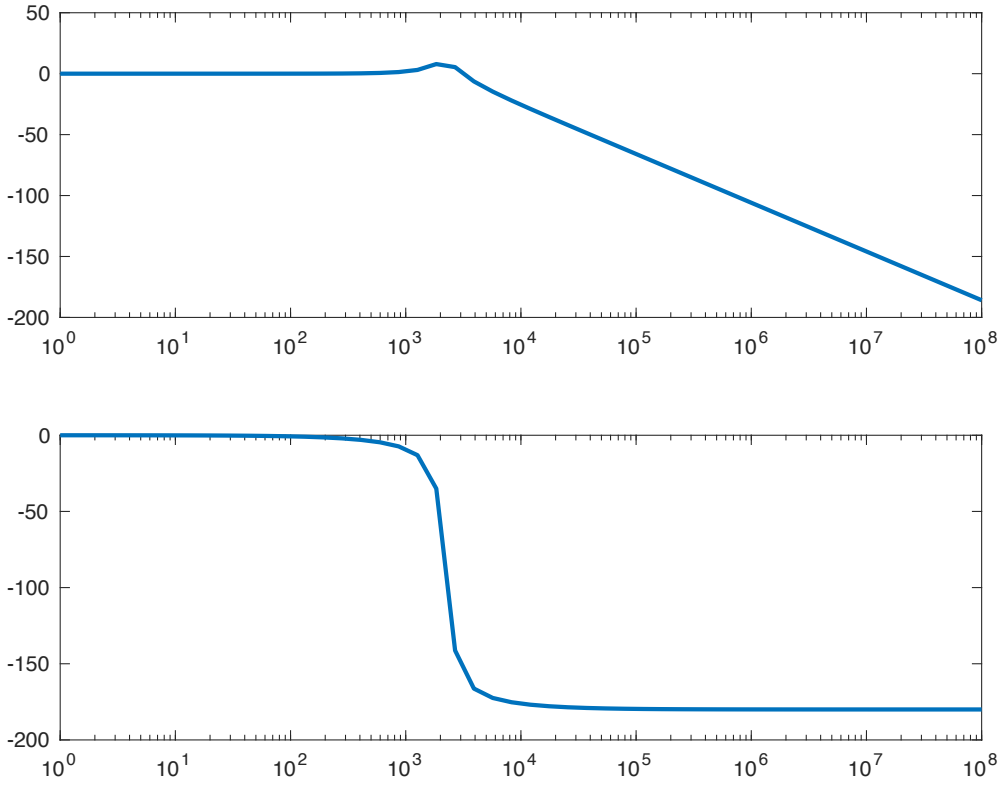
- Example – RLC series circuit – solve by voltage division



$$H(\omega) = \frac{Z_C}{R + Z_L + Z_C} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{(1 - \omega^2 LC) + j\omega RC}$$

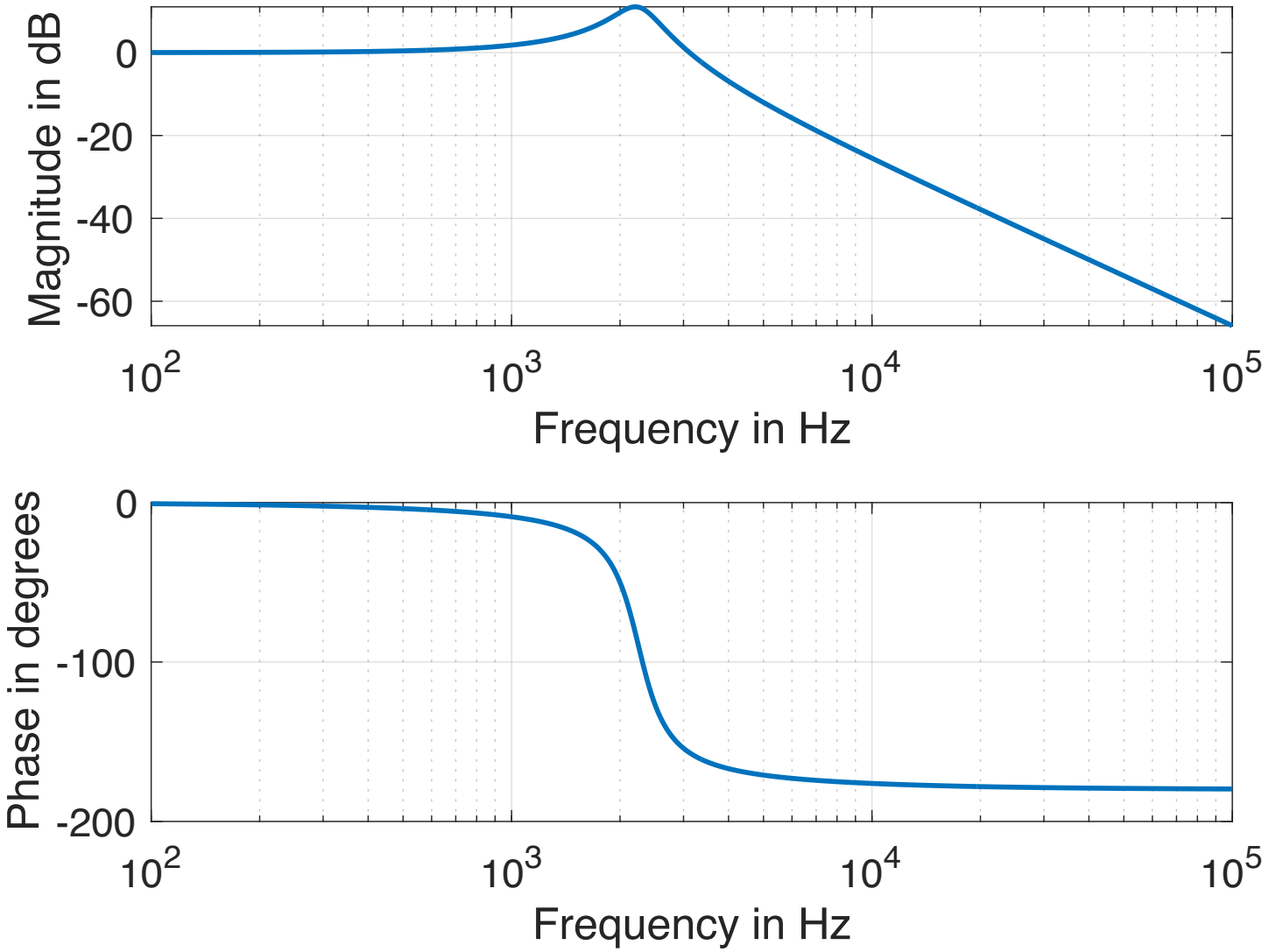
  - What range for frequency? Harder to see from algebraic form; math tools can help
  - Note – no need to simplify algebra if computing

- Try plotting with f = logspace(0,8)

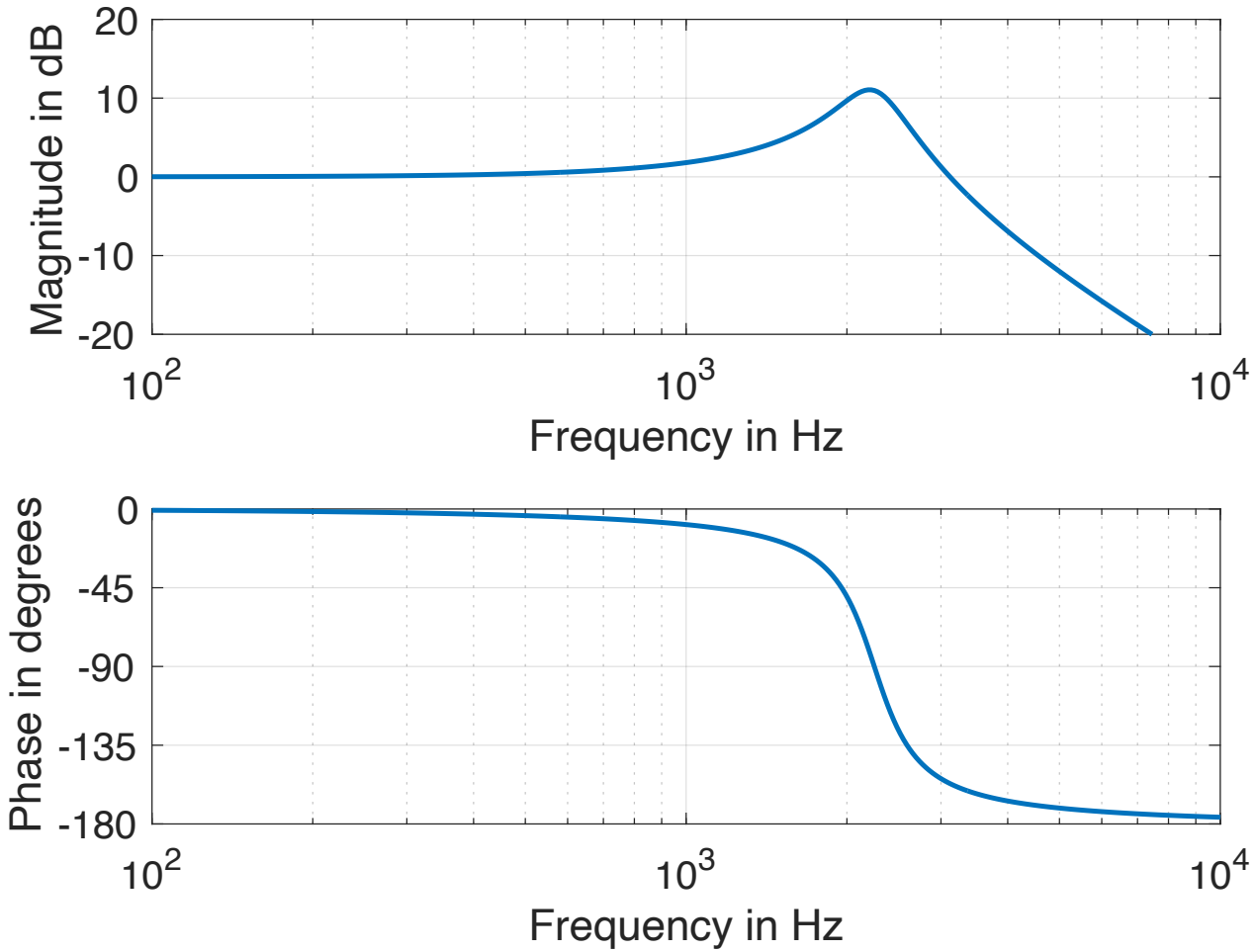


- Notice:
  - Frequency range of 10^2 to 10^5 sufficient

- Decreased range for f; up computation to 500 points; add legends and grid
- Result:



- Limit vertical scales, adjust ticks on phase
- Result:



```
f = logspace(2,5,500);
om = 2*pi*f;
ZL = 1j*om*5e-3;
ZC = -1j./om/1e-6;
H = ZC./(ZC+ZL+20);
figure(1)
subplot(211)
semilogx(f,20*log10(abs(H)),'linewidth',2)
ylim([-20 20])
Xlim([1e2 1e4])
grid on
xlabel('Frequency in Hz')
ylabel('Magnitude in dB')
set(gca,'fontsize',16)
subplot(212)
semilogx(f,180/pi*angle(H),'linewidth',2)
ylim([-180 0])
Xlim([1e2 1e4])
grid on
xlabel('Frequency in Hz')
ylabel('Phase in degrees')
set(gca,'fontsize',16)
```

Note all the pretty stuff

Bode Plot “Rules”

- 2 stacked plots: magnitude and phase versus frequency
  - Lines for theory
  - Symbols for experimental data (e.g. circles)
- dB scale for magnitude, 20 log10 (abs(H))
  - Limit to about 40 dB of range (or less)
- Degrees scale for phase
  - Limit to multiples of 90 as needed
- Frequency on a log base 10 scale (e.g. semilogx in MatLab)
  - Limit to range with “interesting” curves
- Clear annotations with large fonts

Specifics for next week

- Continue work on Exercise 1 (programming) and Exercise 3 (simulation)

- Lab 5 – 100 points (pairs)
  - Instructions posted on ELE 215 website
  - Summary sheets available in lab rooms and on website
  - Due by 5 PM Wednesday Mar 25