ELE 215
Linear Circuits Laboratory

Recitation 4

MatLab uses in ELE 212 and 215

Simultaneous Equations

- Imagine the case of two equations in two unknowns

$$\begin{aligned} 3x + 2y &= 12 \\ 5x - 3y &= 1 \end{aligned}$$

How do we solve for x and y?

- A variety of methods exist (resources on website):
  - Cramer's rule
  - Substitution
  - Gaussian elimination
  - Matrix inversion

- Cramer's rule:

$$\begin{aligned} 3x + 2y &= 12 \\ 5x - 3y &= 1 \end{aligned}$$

  - Ratios of determinants of coefficients

$$x = \frac{\begin{vmatrix} 12 & 2 \\ 1 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 5 & -3 \end{vmatrix}} = \frac{-36 - 2}{-9 - 10} = \frac{-38}{-19} \quad \rightarrow \quad \boldsymbol{x = 2}$$

$$y = \frac{\begin{vmatrix} 3 & 12 \\ 5 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 5 & -3 \end{vmatrix}} = \frac{3 - 60}{-9 - 10} = \frac{-57}{-19} \quad \rightarrow \quad \boldsymbol{y = 3}$$

- Substitution:

$$\begin{aligned} 3x + 2y &= 12 \\ 5x - 3y &= 1 \end{aligned}$$

  - Solve for one variable in terms of another

$$x = \frac{12 - 2y}{3}$$

  - Then substitute, and solve

$$5\left(\frac{12 - 2y}{3}\right) - 3y = 20 - \frac{19}{3}y = 1$$

$$-\frac{19}{3}y = -19 \quad \rightarrow \quad \boldsymbol{y = 3} \quad \rightarrow \quad \boldsymbol{x} = \frac{12 - 2 * 3}{3} \boldsymbol{= 2}$$

- Gaussian elimination:

$$\begin{aligned} 3x + 2y &= 12 \\ 5x - 3y &= 1 \end{aligned}$$

  - Use row operations to get rid of a variable

$$\begin{aligned} 3 &* (3x + 2y = 12) \\ +2 &* (5x - 3y = 1) \end{aligned} \quad \rightarrow \quad 19\,x = 38 \quad \rightarrow \quad \boldsymbol{x = 2}$$

  - Followed by substitution

$$3 * 2 + 2y = 12 \quad \rightarrow \quad \boldsymbol{y = 3}$$

- Matrix inversion:

$$\begin{aligned} 3x + 2y &= 12 \\ 5x - 3y &= 1 \end{aligned}$$

  - Put in matrix/vector form

$$\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \end{bmatrix}$$

  - Then

$$\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 1 \end{bmatrix} = -\frac{1}{19} \begin{bmatrix} -3 & -2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 12 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{2} \\ \mathbf{3} \end{bmatrix}$$

More Generally

- All these methods work for n equations in n unknowns
- Beyond this usual situation we have:
  - Overdetermined

$$\begin{aligned} 3x + 2y &= 12 \\ 5x - 3y &= 1 \\ x + 3y &= 11 \end{aligned} \qquad \begin{aligned} 3x + 2y &= 12 \\ 5x - 3y &= 1 \\ x + 3y &= 30 \end{aligned}$$

  - Underdetermined

$$\begin{aligned} 3x + 2y - 4z &= 12 \\ 5x - 3y + 5z &= 1 \end{aligned}$$

- Solution types:
  - non-singular (in other words, just right)
    - one unique solution

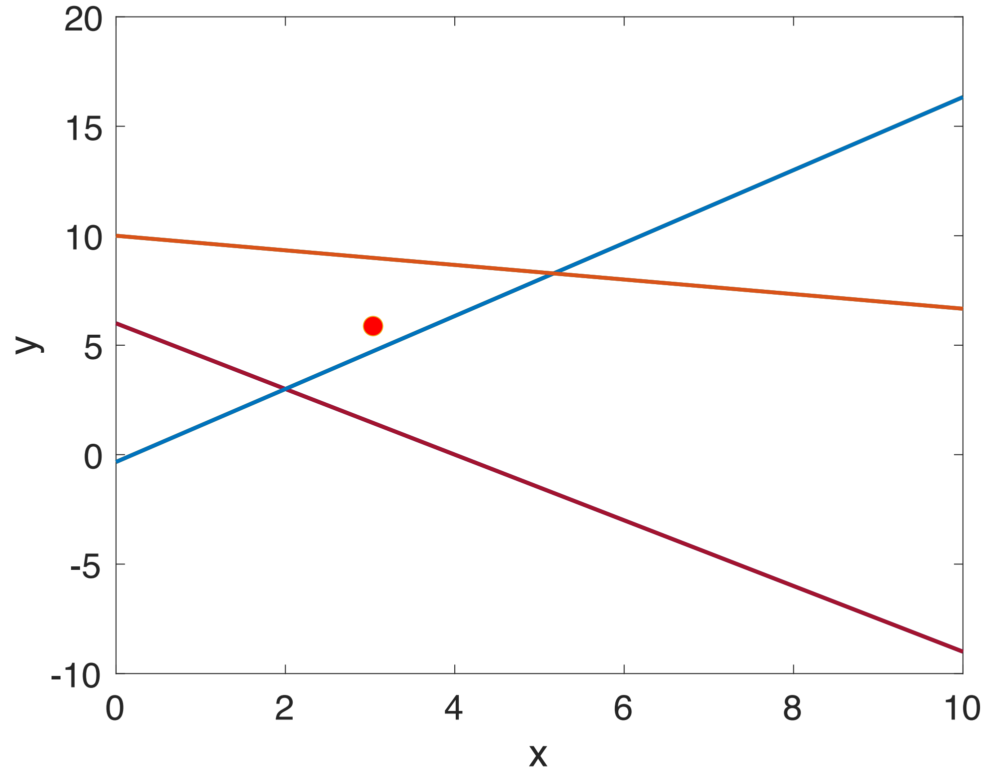
$$\begin{aligned} 3x + 2y &= 12 \\ 5x - 3y &= 1 \end{aligned} \rightarrow \boldsymbol{x = 2, y = 3}$$

  - Singular, underdetermined – many solutions

$$\begin{aligned} 3x + 2y - 4z &= 12 \\ 5x - 3y + 5z &= 1 \end{aligned} \rightarrow \boldsymbol{x = 2 + \frac{2z}{19}, y = 3 + \frac{35z}{19}}$$

– Singular, overdetermined – closest fit

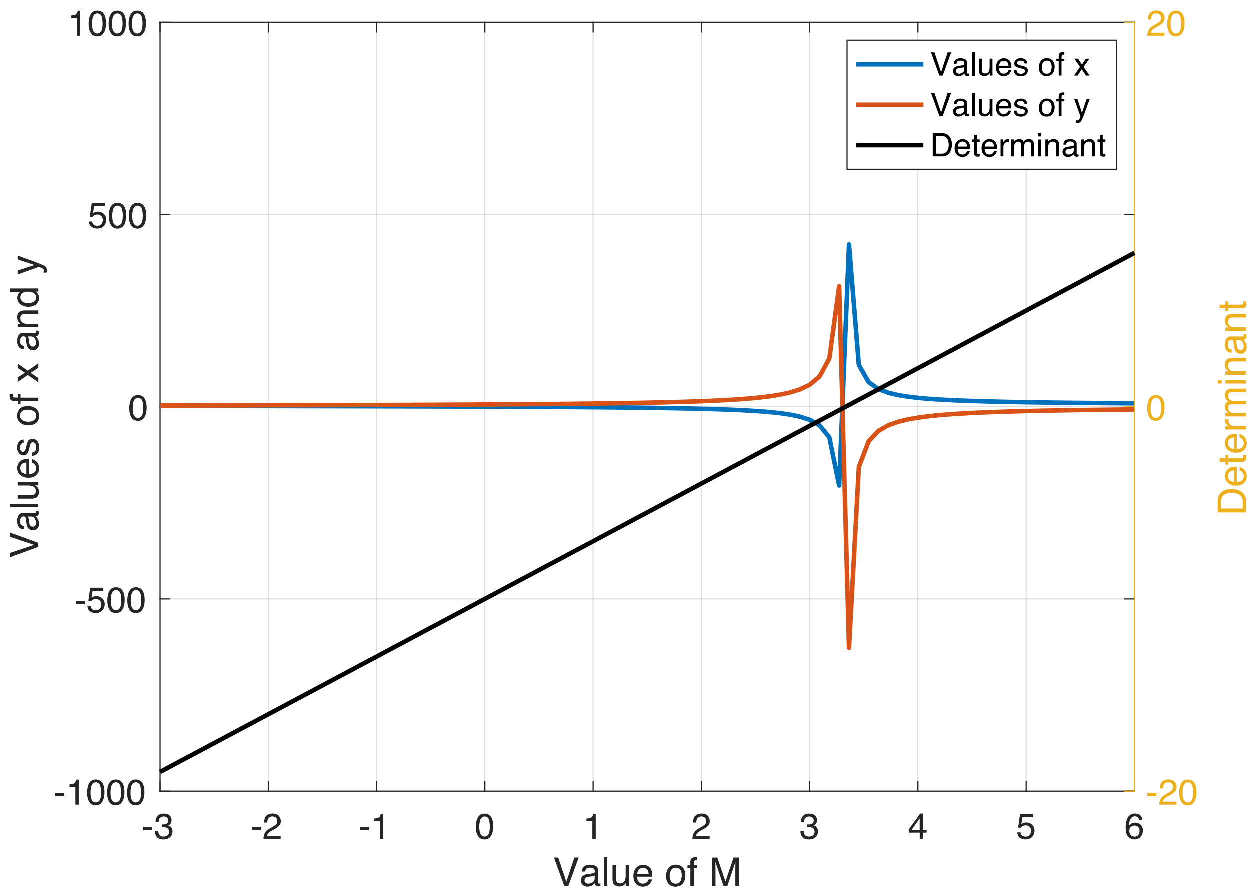
$$\begin{aligned} 3x + 2y &= 12 \\ 5x - 3y &= 1 \\ x + 3y &= 30 \end{aligned} \quad \rightarrow \quad \boldsymbol{x = 3.4, y = 5.9}$$



(MatLab does them all)

- Ill-conditioning
  - Consider how the solution varies with M

$$
\begin{aligned}
3x + y &= 12 \\
6x + My &= 1
\end{aligned}
$$



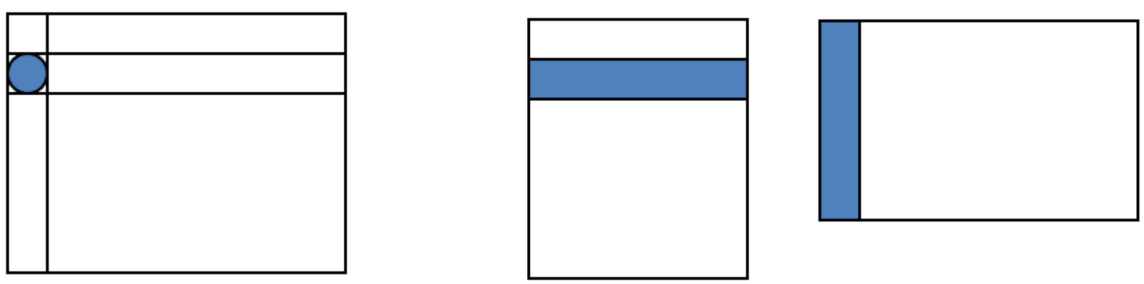
MatLab and Matrices

- Beyond "element-by-element" operations

- Definition for multiplication from linear algebra:

$F = A * B$ means

$$F(r,c) = \sum_k A(r,k)B(k,c)$$



- Notes:
  - # columns of A must equal # rows of B

$$\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3*2+2*3 \\ 5*2-3*3 \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \end{bmatrix}$$

  - Matrix multiplication does not commute $A * B \neq B * A$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 27 & 25 \\ 34 & 32 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 11 & 17 \\ 11 & 25 & 39 \\ 17 & 39 & 61 \end{bmatrix}$$

- But matrix-vector notation is perfect for simultaneous equations

$$\begin{aligned} 3x + 2y - 4z &= 12 \\ 5x - 3y + 5z &= 1 \end{aligned} \quad \rightarrow \quad A * v = b$$

With

$$A = \begin{bmatrix} 3 & 2 & -4 \\ 5 & -3 & 5 \end{bmatrix} \quad v = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad b = \begin{bmatrix} 12 \\ 1 \end{bmatrix}$$

- Solving $A * v = b$ in MatLab
  - Can use the inverse (only works if A is square)

$$v = \mathrm{inv}(A) * b$$

  - Or the “backslash” operator (always defined)

$$v = A \backslash b$$

- MatLab’s tools extend to complex elements in A and b

Specifics for next week

- Extra time to finish Lab 2
  - Note Wednesday is Monday schedule

- Continue work on Exercise 1 – MatLab programming

- Exercise 2 – 25 points (individual) – using MatLab to solve simultaneous equations (like early exercises in ELE 212, but bigger) – due by 9 AM Monday Feb 23

- Prelab 3 – 30 points (individual) – instructions and submission online (see the 215 website) – due by 9 AM Monday Feb 23