HW2_DS, Theory Problem / 20205075 박성준

Problem 1: Comparing Growth Rates (10 pts)
Arrange the following functions by growth rate (slowest growth to fastest growth). Indicate which functions grow at the same rate (i.e. f(n) = Θ(g(n)))

23n, 42n³, 2ⁿ, √n, 3ⁿ, n², log n, 2/n, 128, n log n, 2ⁿ⁺¹, n!

- $23n = O(n)$
- $42n^3 = O(n^3)$
- $\sqrt{n} = O(n^{\frac{1}{2}})$
- $n^2 = O(n^2)$
- $2^n = O(2^n)$
- $3^n = O(3^n)$
- $2^{n+1} = 2 \cdot 2^n = O(2^n)$
- $n\log n = O(n\log n)$
- $n! = O(n!)$
- $128 = O(1)$
- $\log n = O(\log n)$
- $\frac{2}{n}$ → decrease rate

slowest fastest

$n! < 3^n < 2^n = 2^{n+1} < 42n^3 < n^2 < n\log n < 23n < \sqrt{n} < \log n < 128 < \frac{2}{n}$

① prove $\log n = O(n^a)$, $a > 0$

$\frac{d}{dn} \cdot \frac{\log n}{n^a} = \frac{\frac{1}{n} \cdot n^a - an^{a-1}\log n}{n^{2a}} = \frac{1 - a\log n}{n^{a+1}}$ → $\frac{\log n}{n^a}$ is increasing for $n \geq e^{\frac{1}{a}}$

∴ $\log n = O(n^a)$

② prove $a^n = O(n!)$

let $f(n) = \frac{n!}{a^n}$ $(a > 1)$, $\frac{f(n+1)}{f(n)} = \frac{n+1}{a}$, if $n+1 > a$, $f(n)$ is increasing.

Problem 2.

a)

```
int sum = 0;
for (int i = 0; i < n; i++)
    for(int k=i; k < n; k++)
        sum++;
```

\# operations

1

$n+1$

$(n+1)+n+\cdots+2+1$ → $\frac{(n+1)(n+2)}{2}$

$(n+1)+n+\cdots+2+1$

$(n+2)^2$

→ running time $\Theta(n^2)$

b)

```
int sum = 0;
for (int i = 0; i < 23; i++)
    for (int j = 0; j < n; j++)
        sum++;
```

1

24

$23(n+1)$

$23(n+1)$

$46n+71$ → running time $\Theta(n)$

c)

```
int foo(int x,int k) {
    if (x <= k)
        return 1;   → constant
    else
        return foo(x / k, k) + 1;
}
```

if $k=1$ → not terminate

$n + \frac{n}{k} + \cdots + \frac{n}{k^c}$

↳ value is 1.

∴ $c = \log_k n$

→ running time $\Theta(\log_k n)$