PSYCH 201B

Statistical Intuitions for Social Scientists

Modeling data V

You can download these slides:
course website > Week 7 > Overview

02/17/2025

Today's Plan

1. First Half (together)
    - Review - Parameter Inference
    - Categorical Predictors (2-levels)
    - Categorical + Continuous predictor
    - Categorical x Continuous predictor
2. Second Half (on your own)
    - Notebooks 4 & 5
    - Look at notebook solutions for 1-3 if you haven't

Review: Parameter Inference

Parameter inference is proportional to nested model comparison

```python
# Or more easily
from statsmodels.stats.anova import anova_lm

anova_lm(compact_results, augmented_results)
```

✓ 0.0s

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	399.0	8.433991e+07	0.0	NaN	NaN	NaN
1	398.0	6.620874e+07	1.0	1.813117e+07	108.991715	1.030886e-22

$$F = t^2$$

$$t = \sqrt{F}$$

```python
print(augmented_results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                Balance   R-squared:                       0.215
Model:                            OLS   Adj. R-squared:                  0.213
Method:                 Least Squares   F-statistic:                     109.0
Date:                Wed, 12 Feb 2025   Prob (F-statistic):           1.03e-22
Time:                        09:16:07   Log-Likelihood:                -2970.9
No. Observations:                 400   AIC:                             5946.
Df Residuals:                     398   BIC:                             5954.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept    246.5148     33.199      7.425      0.000     181.247     311.783
Income         6.0484      0.579     10.440      0.000       4.909       7.187
==============================================================================
Omnibus:                       42.505   Durbin-Watson:                   1.951
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               20.975
Skew:                           0.384   Prob(JB):                     2.79e-05
Kurtosis:                       2.182   Cond. No.                         93.3
==============================================================================
```

Bootstrapping let's us simulate uncertainty

```python
nsim = 5000
params = []

for _ in range(nsim):

    # Generate new dataset with replacement
    new_data = df.sample(fraction=1., with_replacement=True)

    # Fit model to it
    bmodel = ols('Balance ~ Income', data=new_data.to_pandas())
    bresults = bmodel.fit()

    # Save the parameters
    params.append(bresults.params.to_numpy())

# Calculate SD, T, and CI of re-sampled distribution
params = np.array(params)
std_devs = params.std(ddof=1, axis=0)
tstats = augmented_results.params.to_numpy() / std_devs
CI_limits = np.percentile(params, [2.5, 97.5], axis=0)

# Combine into a DataFrame with original beta estimates
boot_results = pl.DataFrame(CI_limits).transpose().with_columns(
    names = np.array(['Intercept', 'Income']),
    se=std_devs,
    tstats=tstats,
    estimate=augmented_results.params.to_numpy(),
    ).select(
        col('names').alias('variable'),
        col('estimate').alias('coef'),
        col('se').alias('std err'),
        col('tstats').alias('t'),
        col('column_0').alias('[0.025'),
        col('column_1').alias('0.975]'),
    )
```

variable	coef	std err	t	[0.025	0.975]
str	f64	f64	f64	f64	f64
"Intercept"	246.514751	31.862764	7.736766	186.117461	310.232929
"Income"	6.048363	0.595855	10.150734	4.877137	7.220309

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Building the null…by permuting (shuffling)

```python
nperm = 5000
tstats = []

for _ in range(nsim):

    # Generate new dataset by shuffling rows in the DV column only
    new_data = df.with_columns(
        Balance = df['Balance'].sample(fraction=1.,
                                       with_replacement=False,
                                       shuffle=True)
        )

    # Fit model to it
    bmodel = ols('Balance ~ Income', data=new_data.to_pandas())
    bresults = bmodel.fit()

    # Save the t-stats
    tstats.append(bresults.tvalues.to_numpy())

tstats = np.array(tstats)

# Get the # of permuted t-stats >= observed t-stat
proportion = np.sum(
    np.abs(tstats) >= np.abs(augmented_results.tvalues.to_numpy()),
    axis=0) + 1

# Calculate p-value
pval = proportion / (nperm + 1)
```

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  - Effective sample size (# observations vs # params aka DF)
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Statistical significance is not practical significance

Categorical predictors

Credit data set

index	income	limit	rating	cards	age	education	gender	student	married	ethnicity	balance
1	14.89	3606	283	2	34	11	Male	No	Yes	Caucasian	333
2	106.03	6645	483	3	82	15	Female	Yes	Yes	Asian	903
3	104.59	7075	514	4	71	11	Male	No	No	Asian	580
4	148.92	9504	681	3	36	11	Female	No	No	Asian	964
5	55.88	4897	357	2	68	16	Male	No	Yes	Caucasian	331
6	80.18	8047	569	4	77	10	Male	No	No	Caucasian	1151
7	21.00	3388	259	2	37	12	Female	No	No	African American	203
8	71.41	7114	512	2	87	9	Male	No	No	Asian	872
9	15.12	3300	266	5	66	13	Female	No	No	Caucasian	279
10	71.06	6819	491	3	41	19	Female	Yes	Yes	African American	1350

variable	description
income	in thousand dollars
limit	credit limit
rating	credit rating
cards	number of credit cards
age	in years
education	years of education
gender	male or female
student	student or not
married	married or not
ethnicity	African American, Asian, Caucasian
balance	average credit card debt in dollars

Do students have a different credit card balance from non-students?

H_0: Students and non-students have the same balance.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

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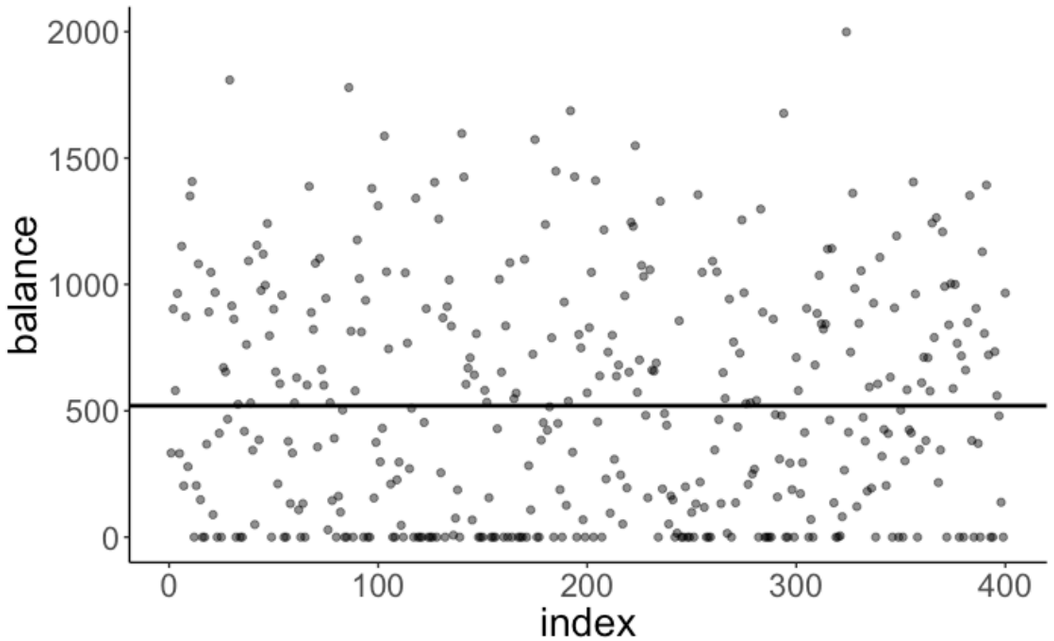
Model prediction



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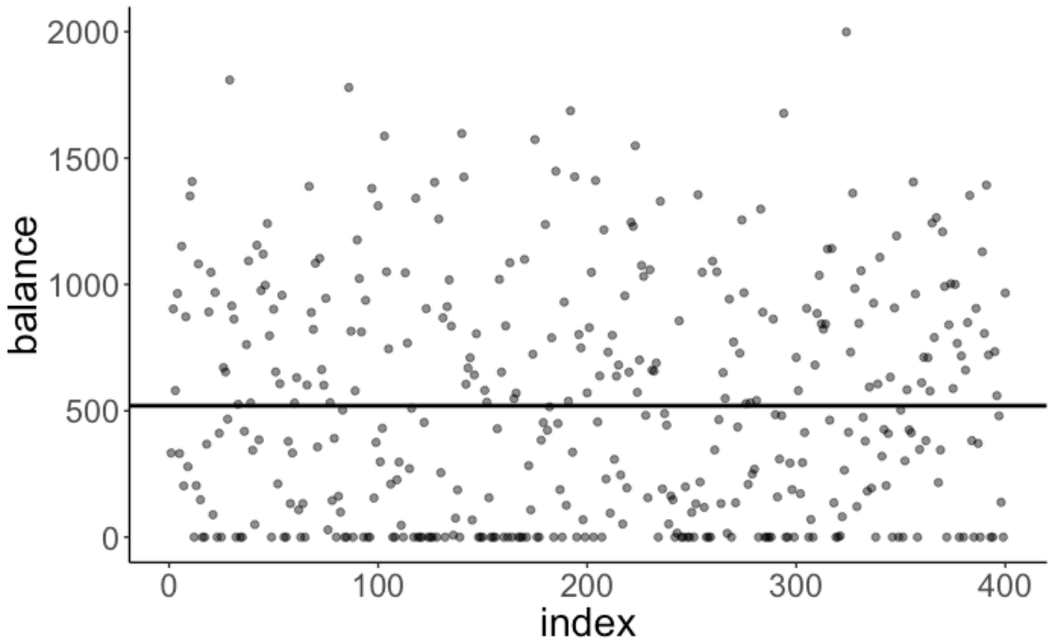

H_1: Students and non-students have different balances.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

student

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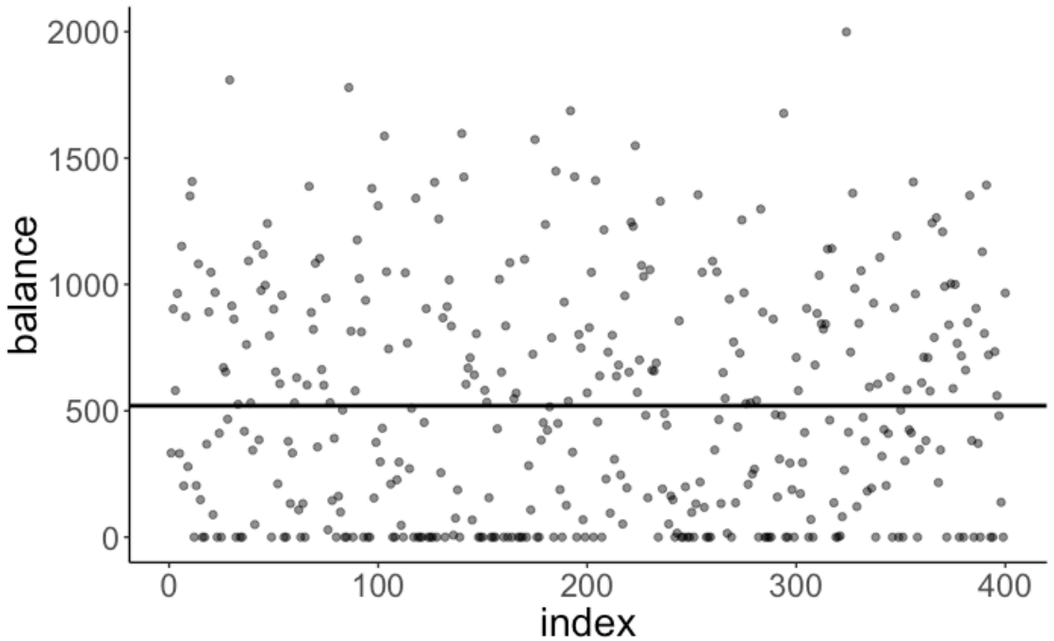

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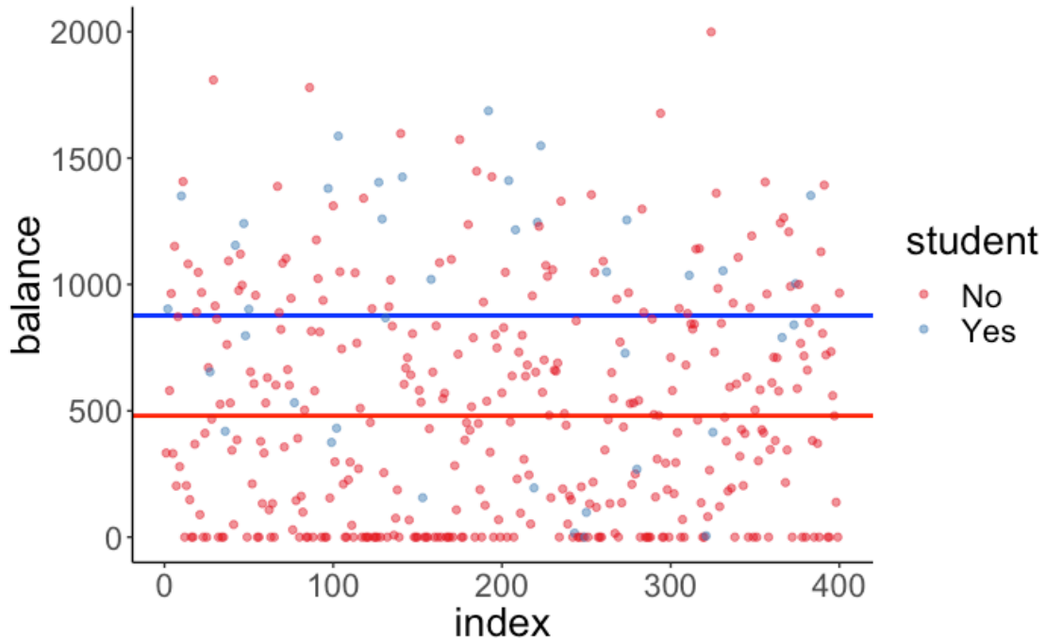

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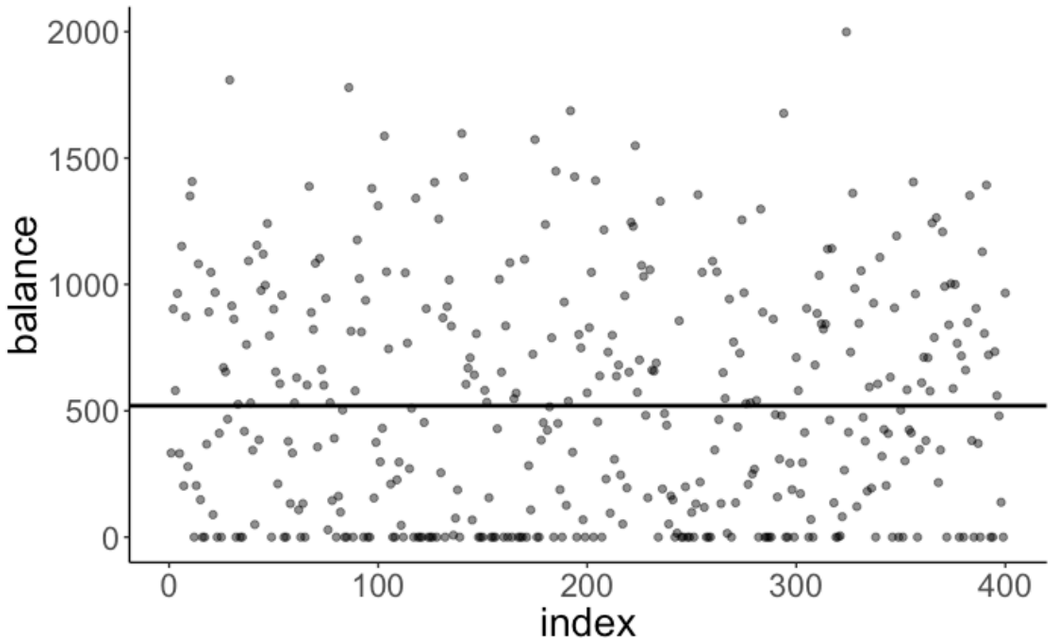

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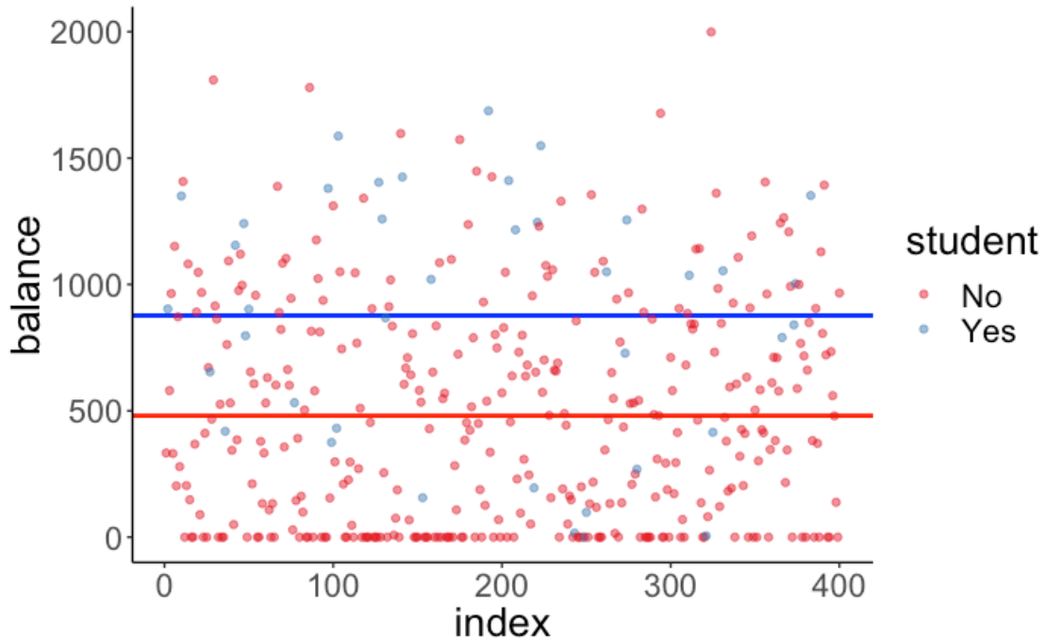

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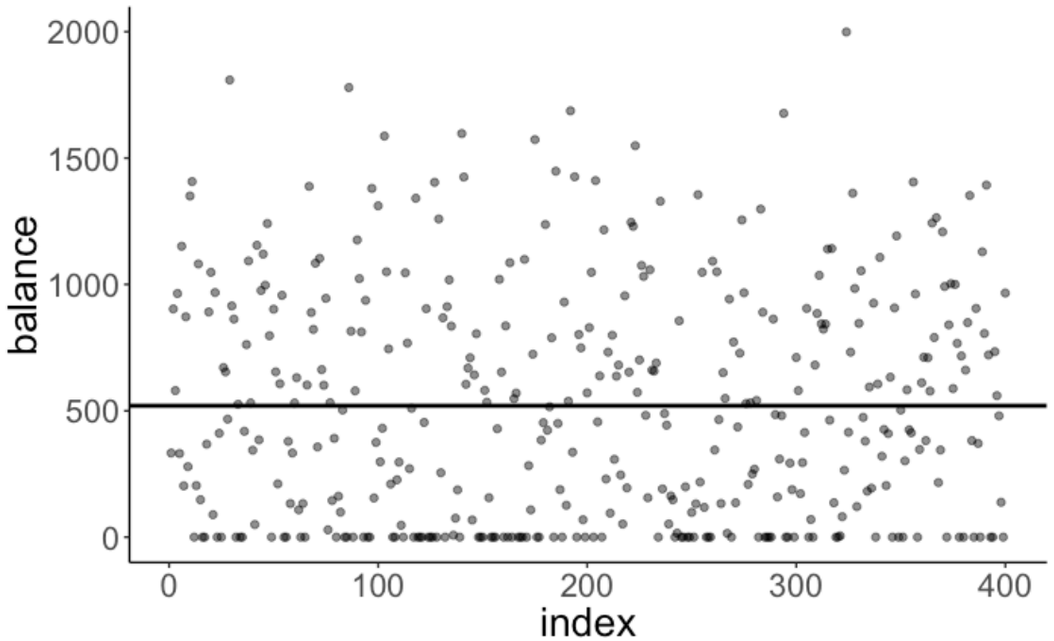

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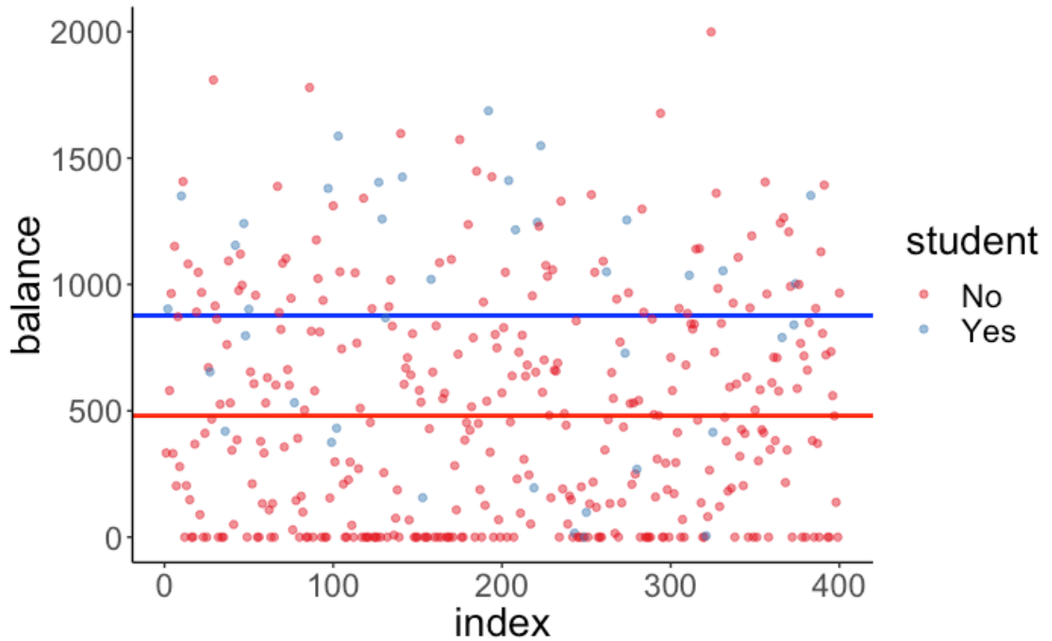

predicts two intercepts

Worth it?

```
# Compact Model
c_model = ols('Balance ~ 1', data=df.to_pandas())
c_results = c_model.fit()

# Augmented Model
# Treat "Student" as a categorical variable
a_model = ols('Balance ~ C(Student)', data=df.to_pandas())
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Interpreting parameter estimates

```
print(a_results.summary(slim=True))
```

```
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Model:                            OLS   Adj. R-squared:                  0.140
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Intercept            463.2368     78.252      5.920      0.000     307.317     619.156
C(Student)[T.Yes]    401.4474    110.664      3.628      0.001     180.944     621.951
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Interpreting parameter estimates

what do these represent?

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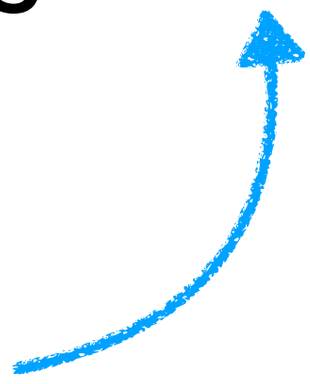
We'll dive into more detail tomorrow



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Balance	Student
i64	str
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1216	"Yes"
148	"No"
108	"No"
532	"Yes"

Data

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```
array([[1., 1.],
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Design Matrix

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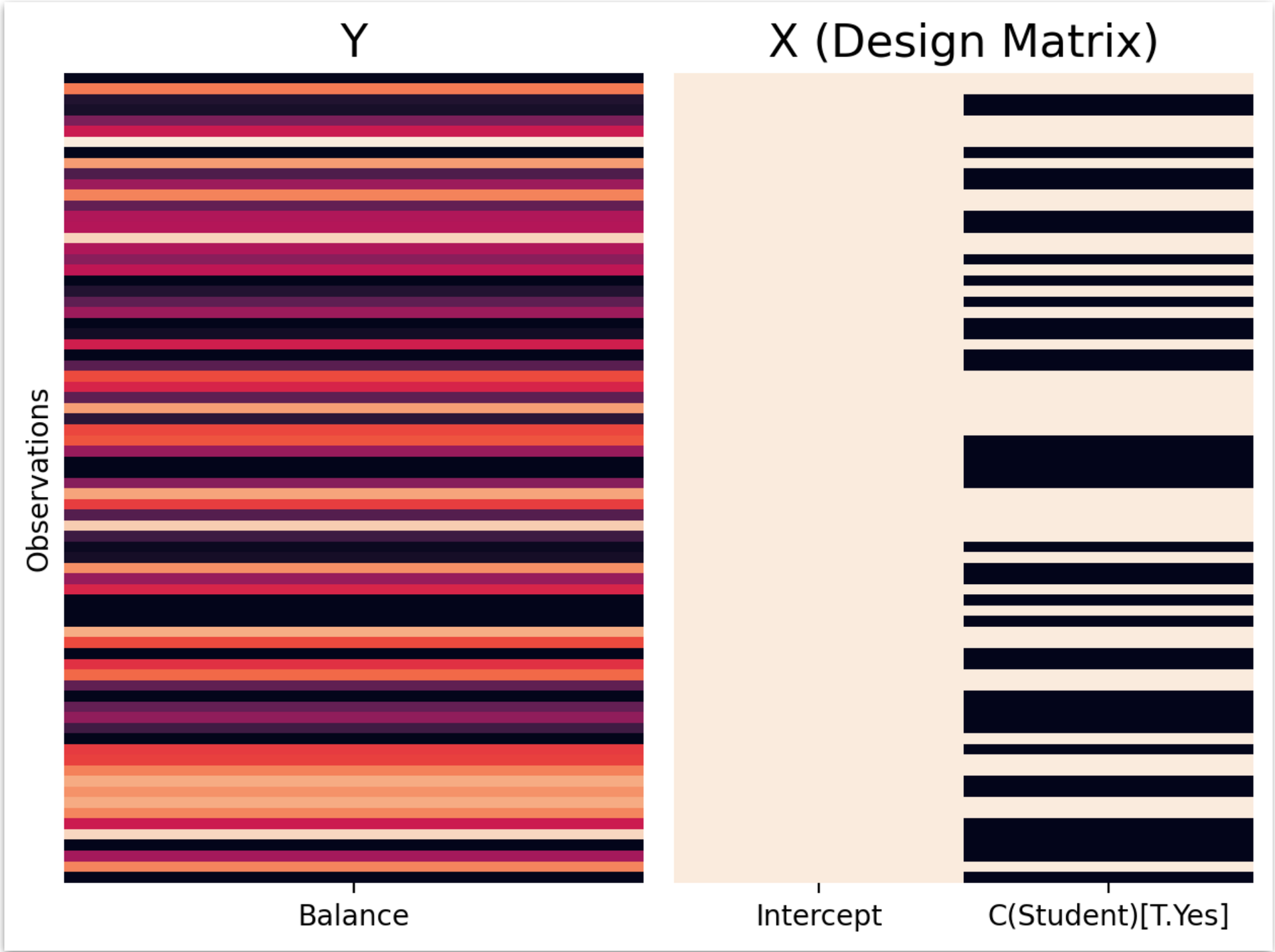

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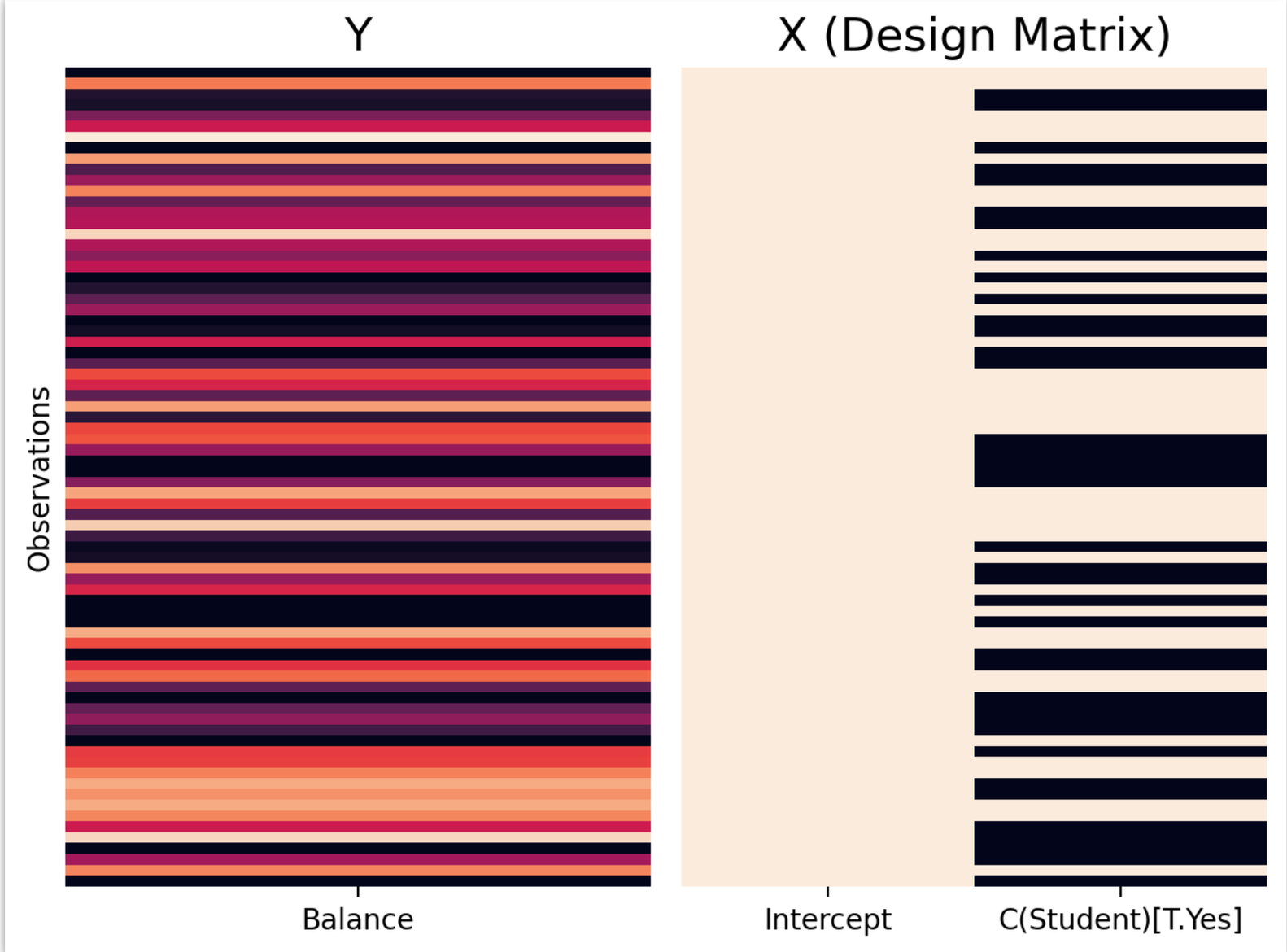

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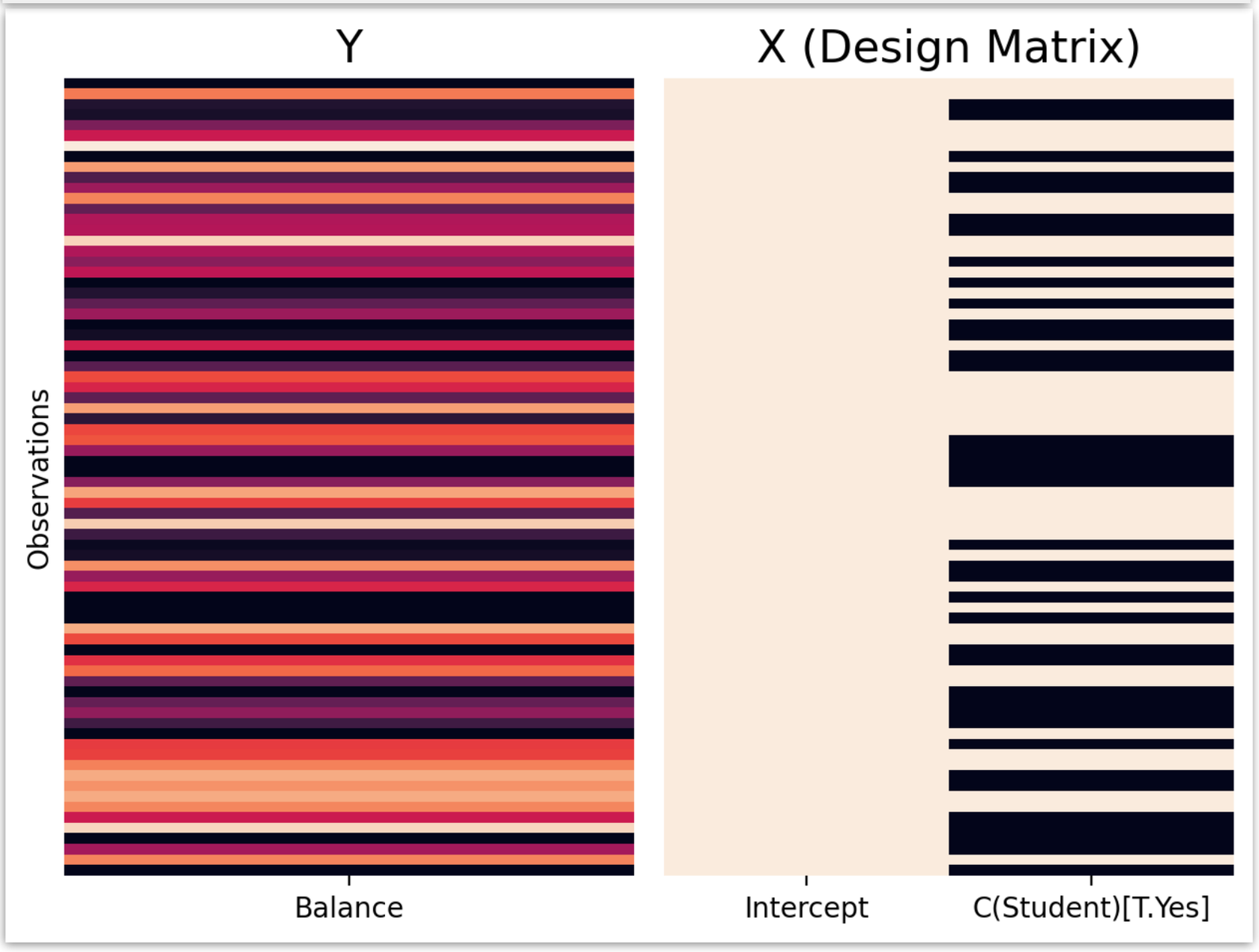

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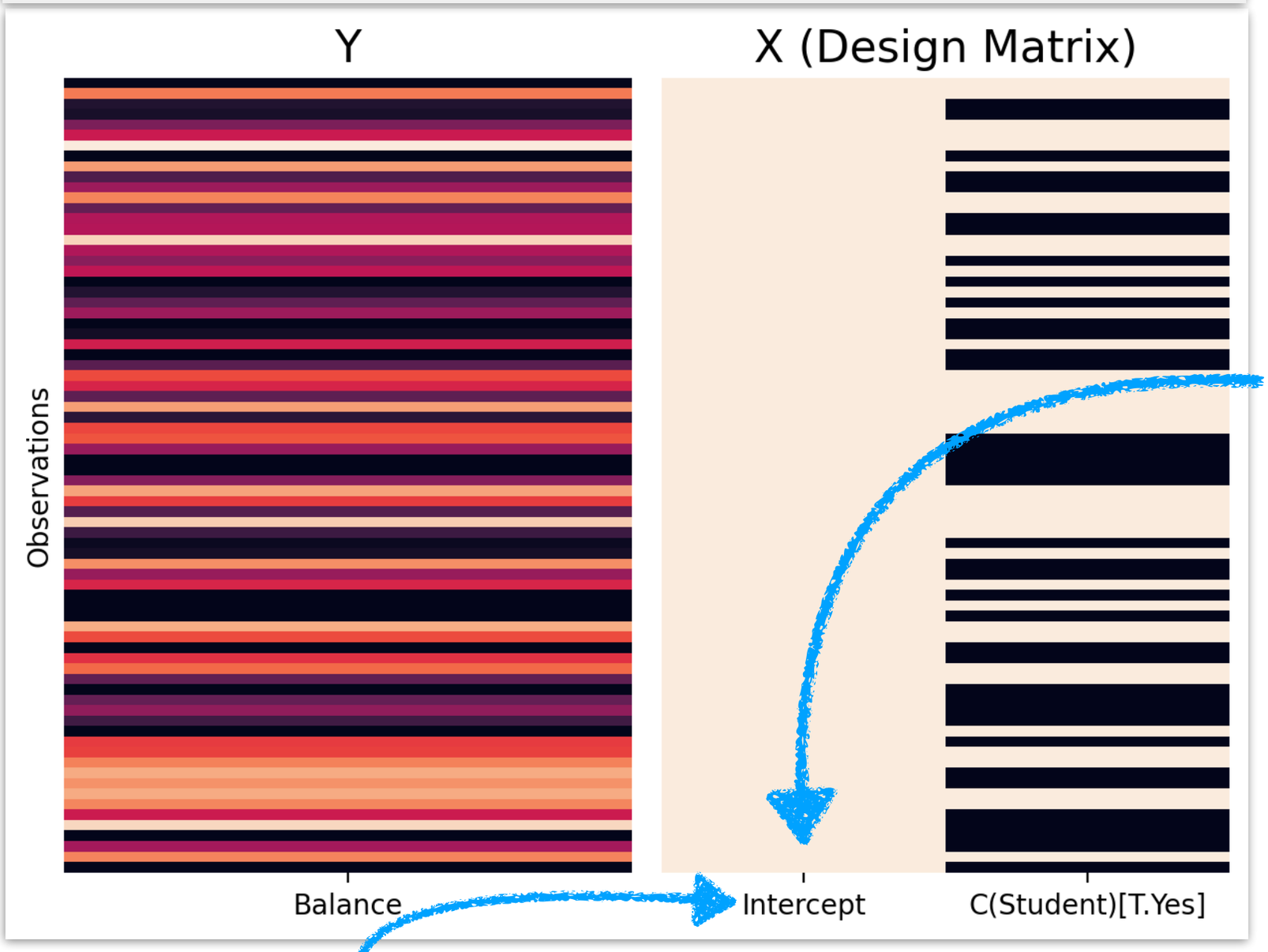

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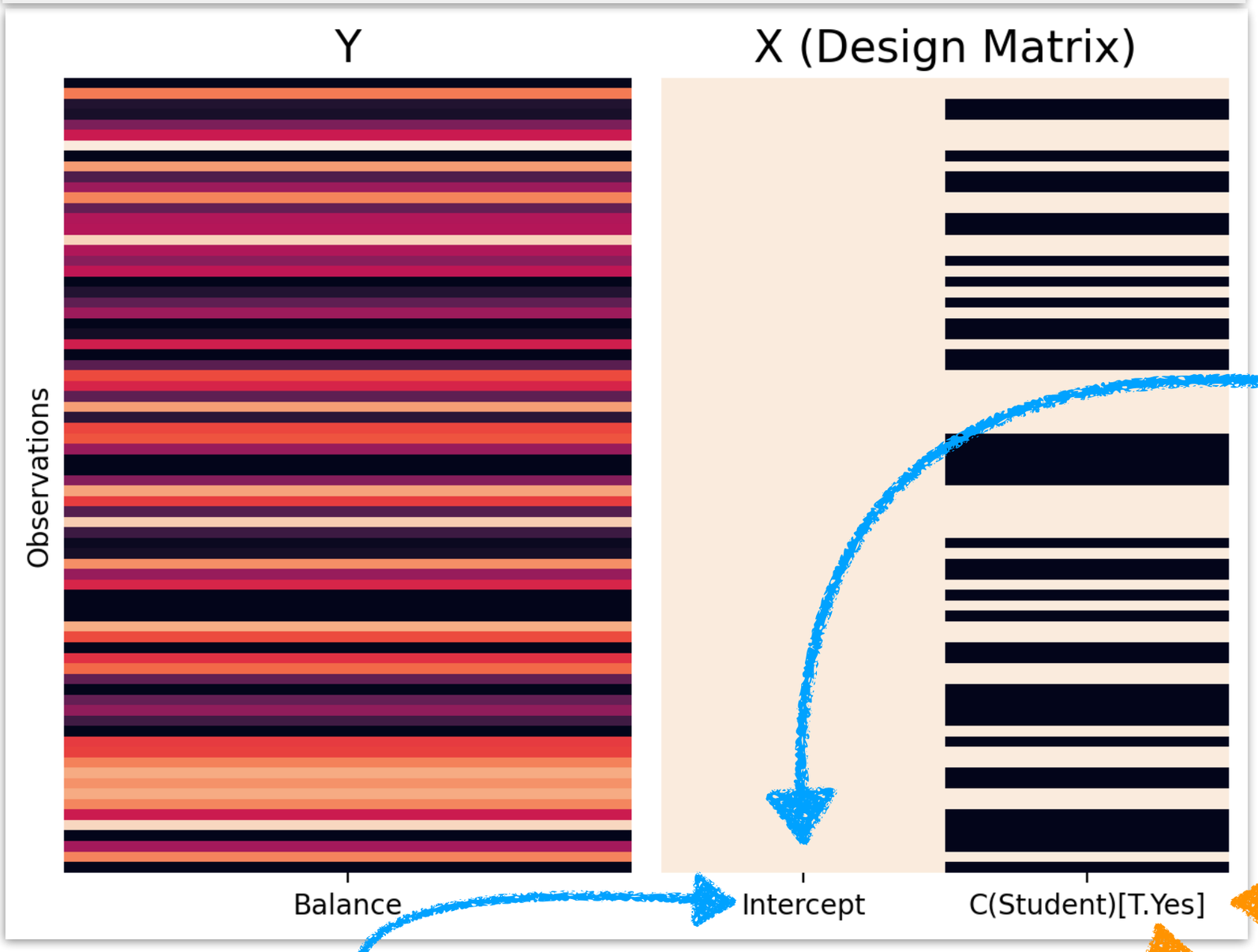

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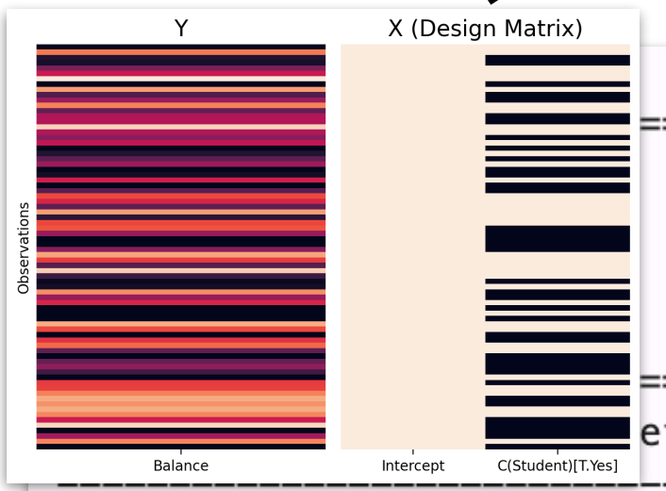

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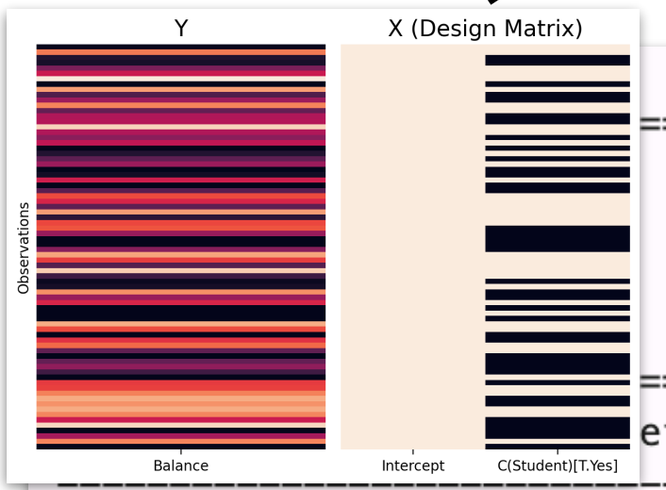
Treatment (Dummy) Coding



OLS Regression Results

Balance	R-squared:		0.151
OLS	Adj. R-squared:		0.140
76	F-statistic:		13.16
nonrobust	Prob (F-statistic):		0.000523

	ef	std err	t	P>|t|	[0.025	0.975]
Intercept	463.2368	78.252	5.920	0.000	307.317	619.156
C(Student)[T.Yes]	401.4474	110.664	3.628	0.001	180.944	621.951

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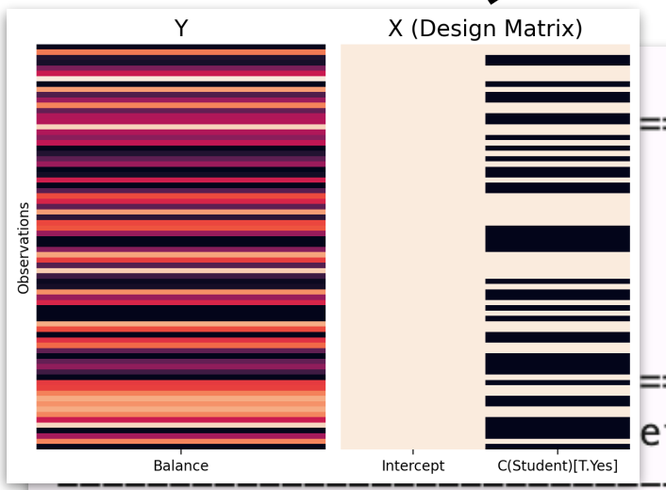

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Model:                            OLS   Adj. R-squared:                  0.140
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==============================================================================
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$$\hat{y}_i = \beta_0 + \beta_1 \cdot X_{1_i}$$

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```

$$\hat{y}_i = \beta_0 + \beta_1 \cdot X_{1_i}$$

$$\widehat{Balance}_i = 463.24 + 401.45 \cdot \text{student\_dummy}_i$$

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==============================================================================
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$$\hat{y}_i = \beta_0 + \beta_1 \cdot X_{1_i}$$

$$\widehat{Balance}_i = 463.24 + 401.45 \cdot \text{student\_dummy}_i$$

$$\widehat{Balance}_i = 463.24$$

0: Student = No

1: Student = Yes

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$$\widehat{Balance}_i = 463.24 + 401.45 \cdot \text{student\_dummy}_i$$

0: Student = No → $\widehat{Balance}_i = 463.24$

1: Student = Yes → $\widehat{Balance}_i = 463.24 + 401.45 = 864.49$

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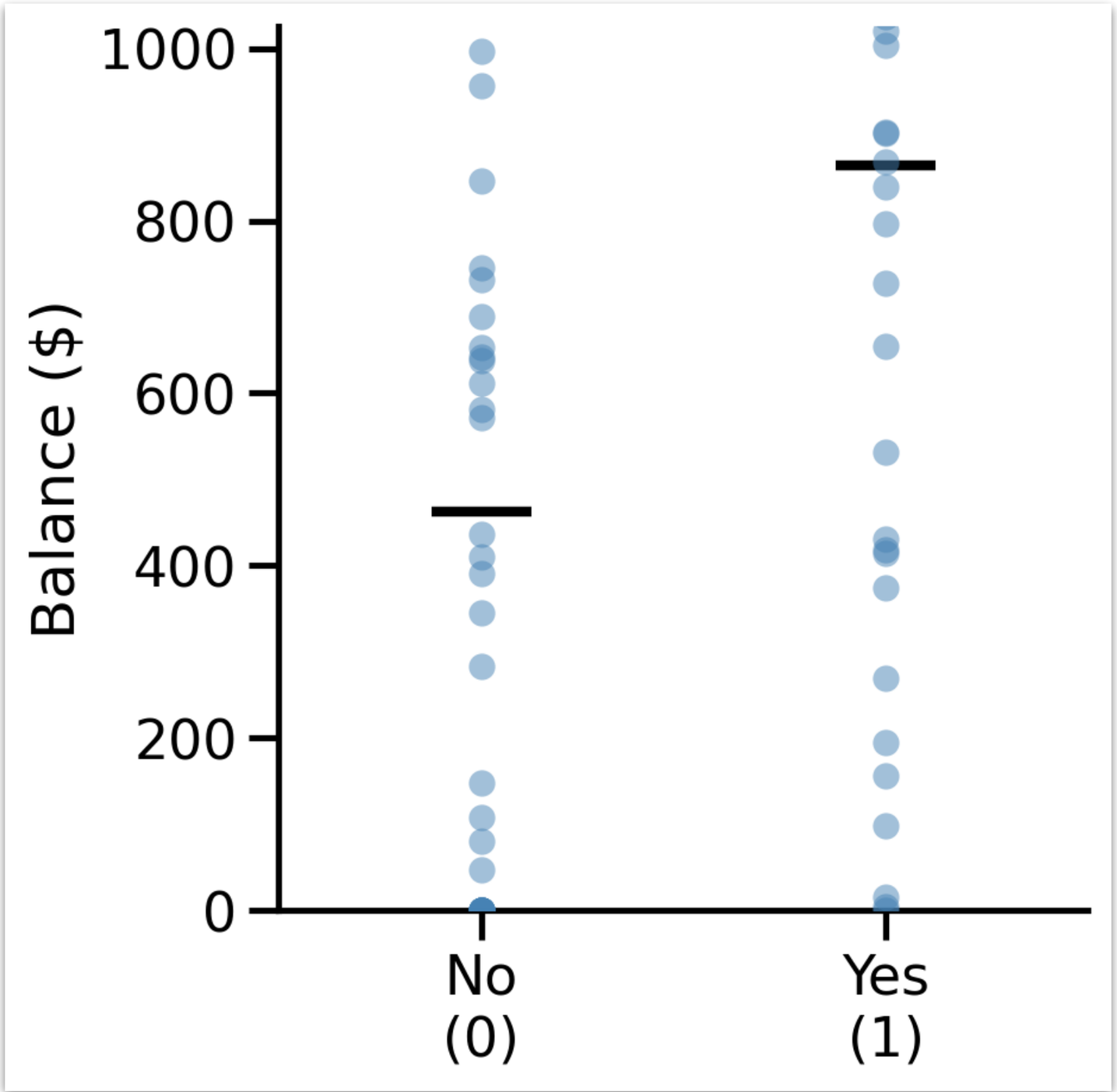
Treatment (Dummy) Coding Visually



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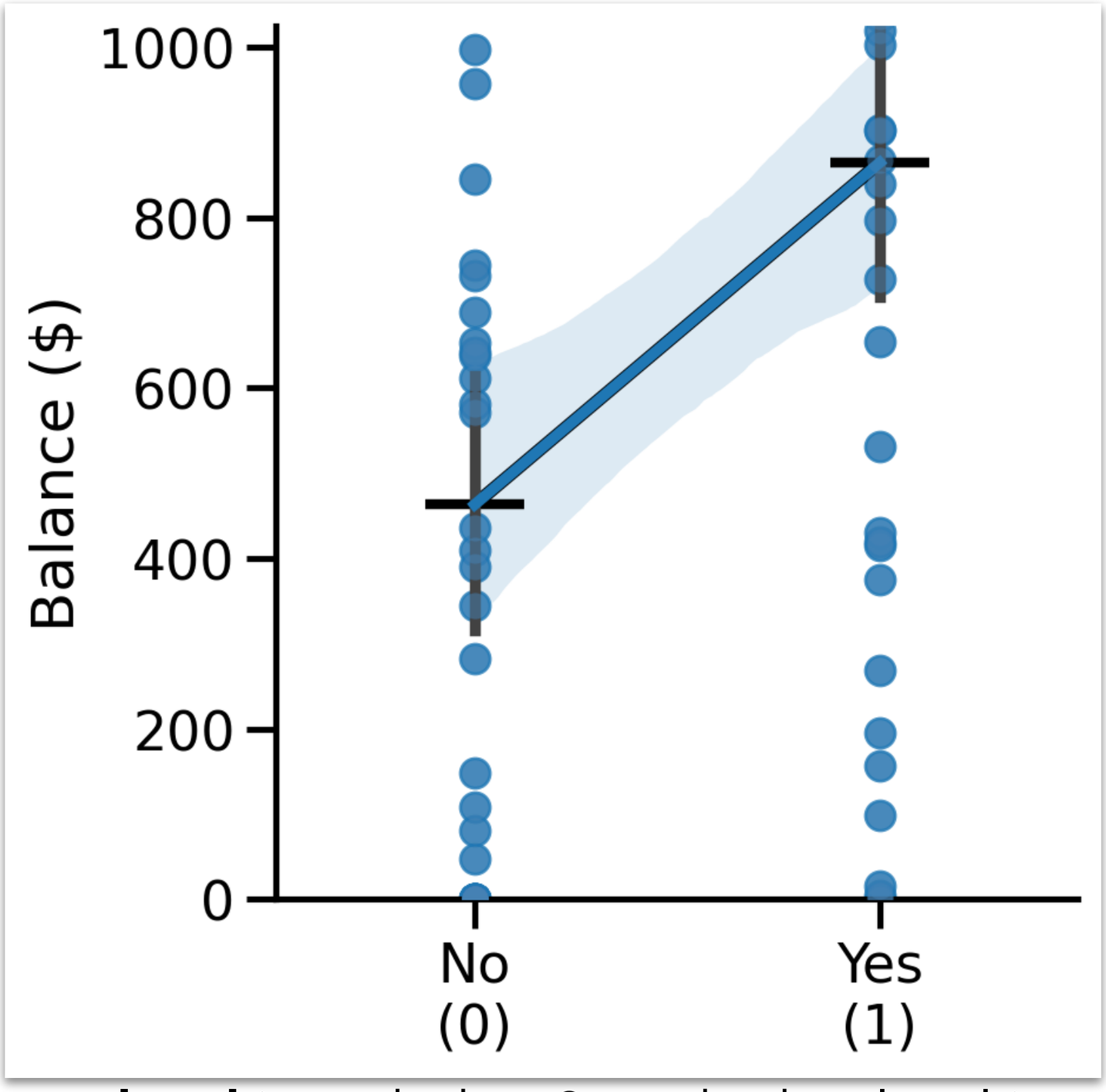

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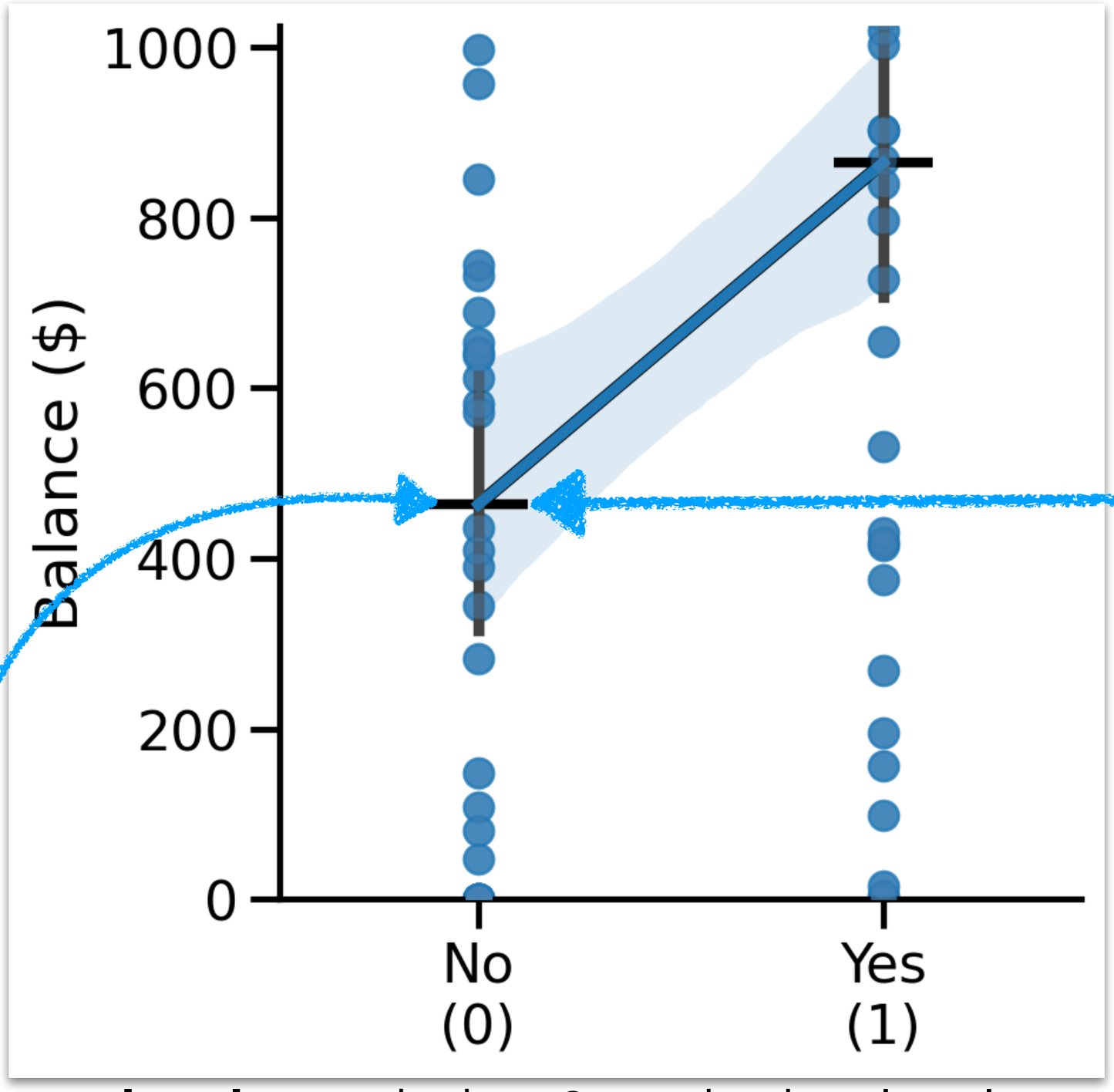

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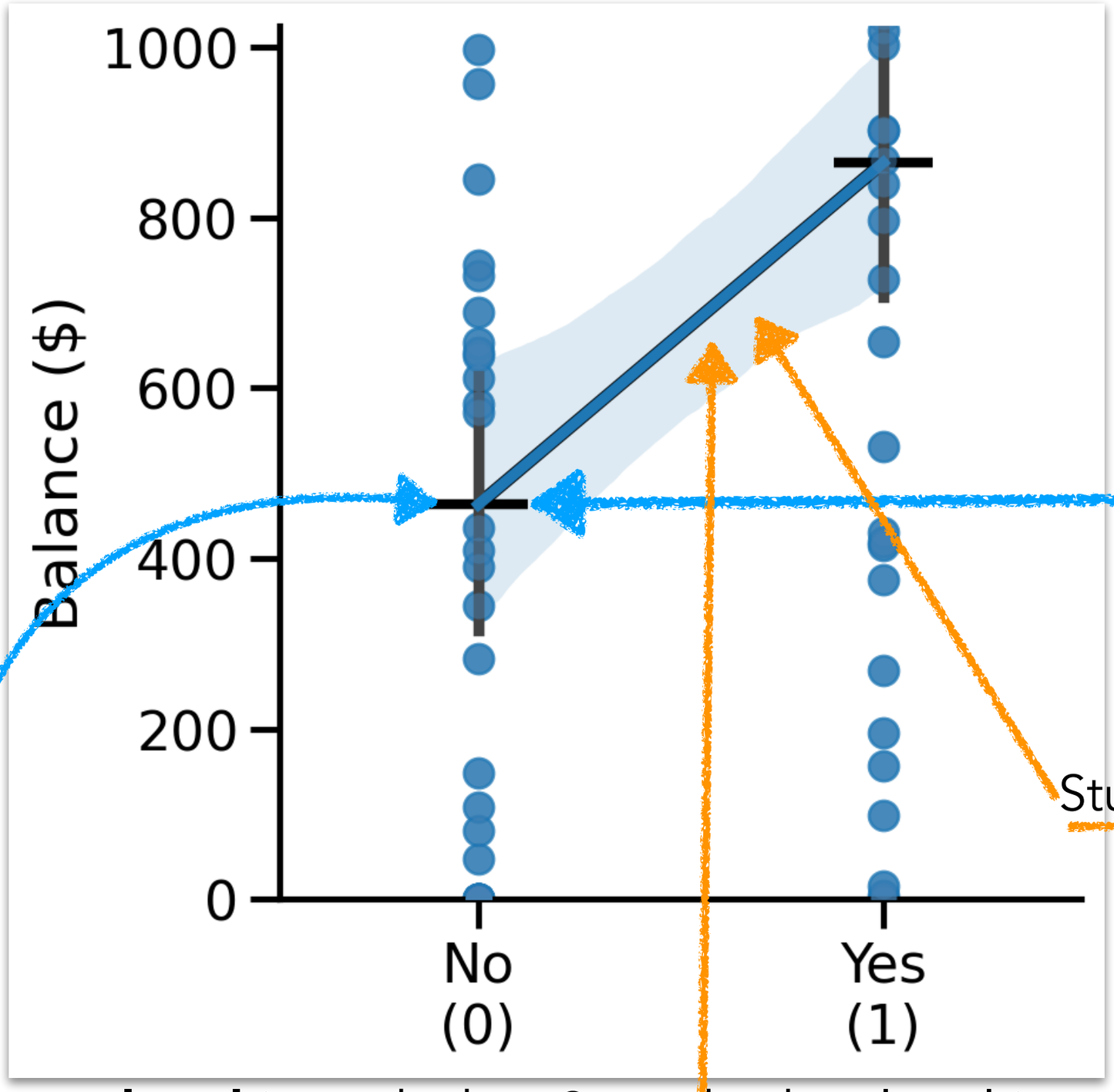

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Student (Yes) - Student (No)

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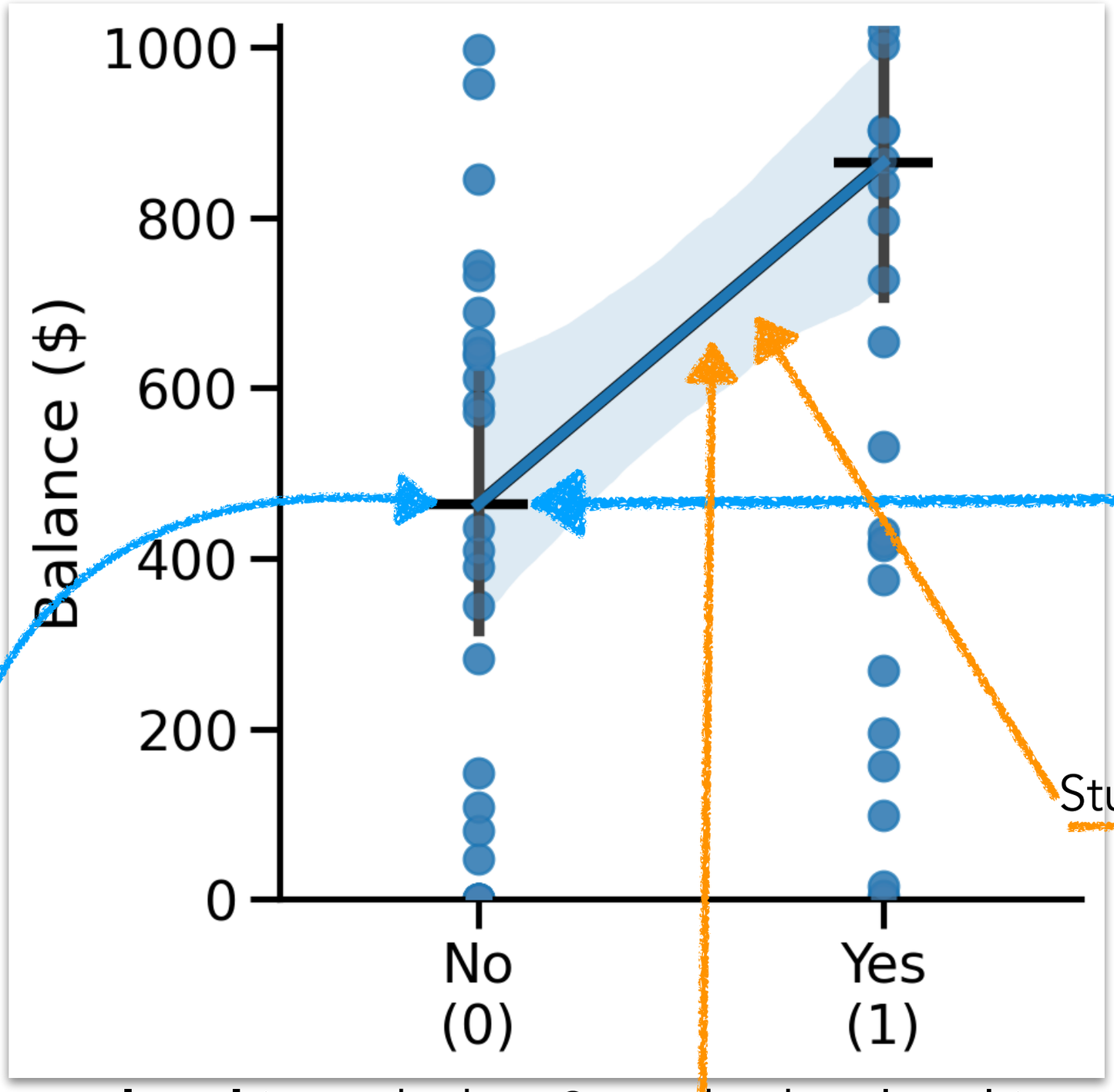
Linear model of mean differences



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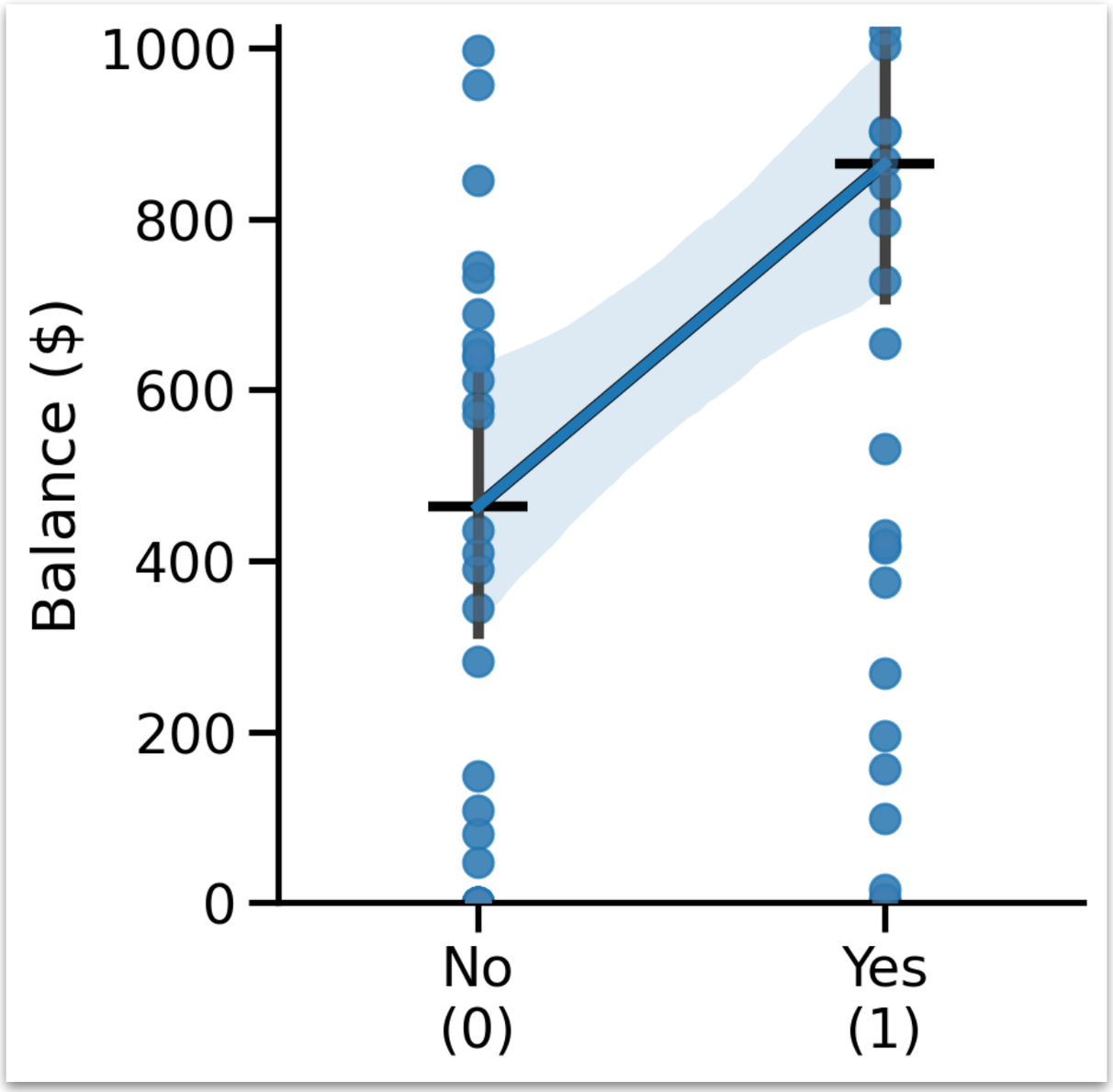
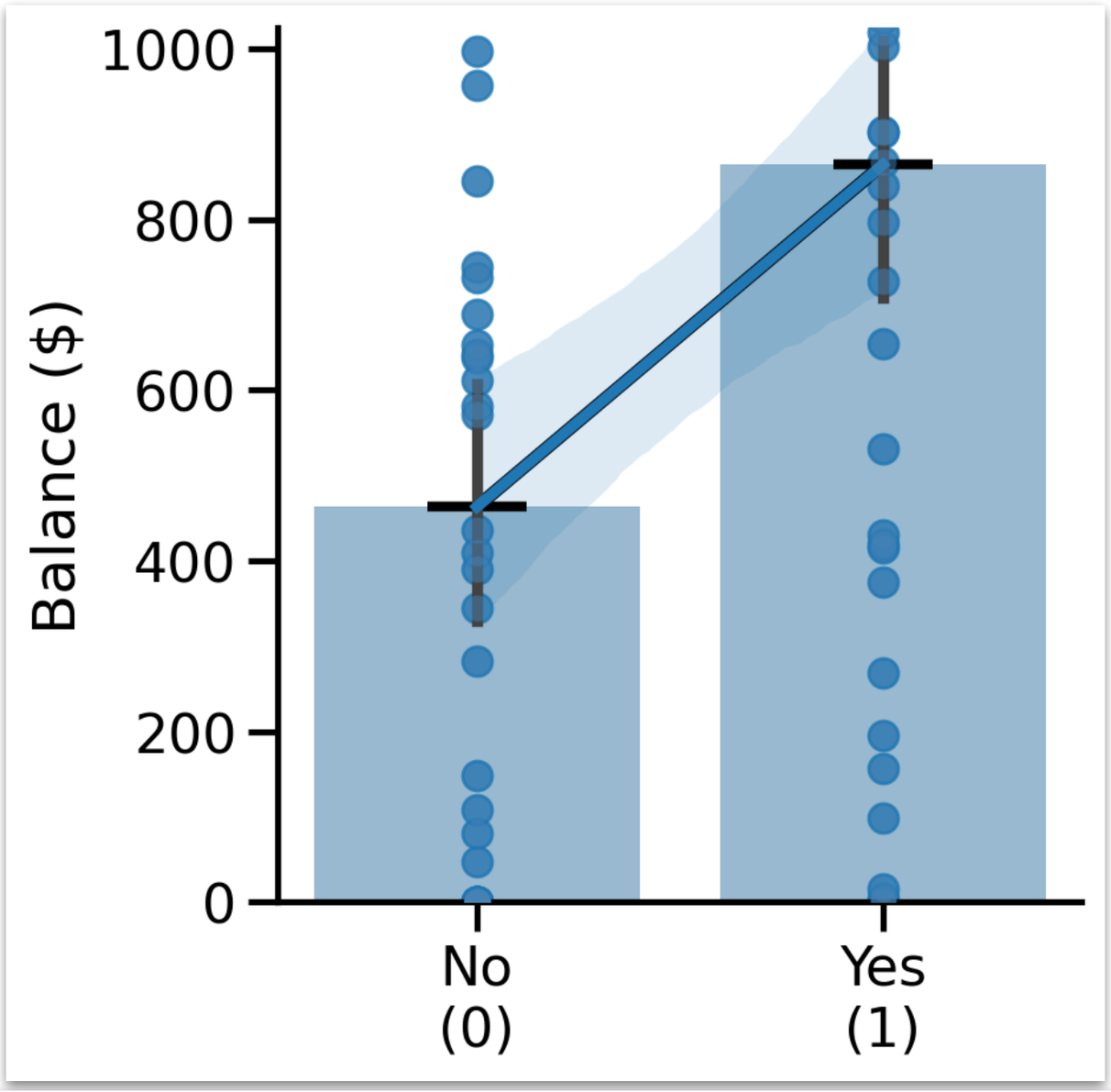


	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	75.0	2.028075e+07	0.0	NaN	NaN	NaN
1	74.0	1.721872e+07	1.0	3.062040e+06	13.159573	0.000523

Students have a significantly higher average credit card balance (*Mean* = 864.68, *SD* = 494.57) than non-students (*Mean* = 463.24, *SD* = 469.86), $F(1, 74) = 13.159$, $p < .001$.

Independent samples t-test



	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	463.2368	78.252	5.920	0.000	307.317	619.156
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Students have a significantly higher average credit card balance (*Mean* = 864.68, *SD* = 494.57) than non-students $t(74) = 3.628$, $b = 401.45$ [180.94 621.95], $p < .001$.

How does the GLM see **categorical variables**?

We represent ***k levels*** of a categorical variable
with ***k-1 parameters*** using one of many
possible **coding schemes**

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We'll meet some more tomorrow

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For 2-level categorical variables = independent-samples t-test

Categorical + continuous predictor

Credit data set

index	income	limit	rating	cards	age	education	gender	student	married	ethnicity	balance
1	14.89	3606	283	2	34	11	Male	No	Yes	Caucasian	333
2	106.03	6645	483	3	82	15	Female	Yes	Yes	Asian	903
3	104.59	7075	514	4	71	11	Male	No	No	Asian	580
4	148.92	9504	681	3	36	11	Female	No	No	Asian	964
5	55.88	4897	357	2	68	16	Male	No	Yes	Caucasian	331
6	80.18	8047	569	4	77	10	Male	No	No	Caucasian	1151
7	21.00	3388	259	2	37	12	Female	No	No	African American	203
8	71.41	7114	512	2	87	9	Male	No	No	Asian	872
9	15.12	3300	266	5	66	13	Female	No	No	Caucasian	279
10	71.06	6819	491	3	41	19	Female	Yes	Yes	African American	1350

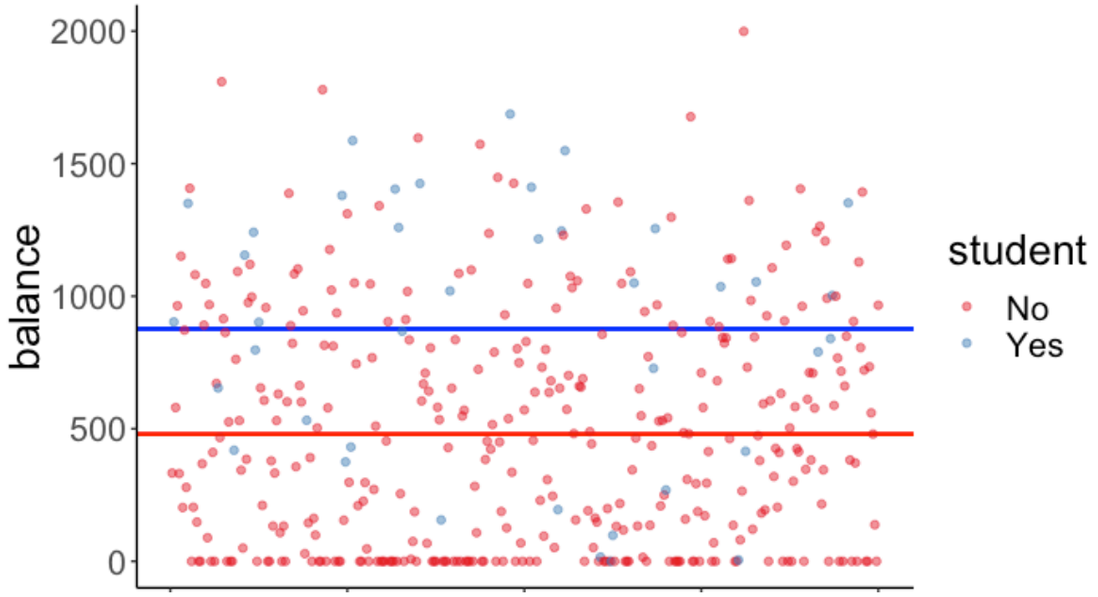
variable	description
income	in thousand dollars
limit	credit limit
rating	credit rating
cards	number of credit cards
age	in years
education	years of education
gender	male or female
student	student or not
married	married or not
ethnicity	African American, Asian, Caucasian
balance	average credit card debt in dollars

Do students have a different credit card balance from non-students, when controlling for income?

Model C_1: 1 intercept; 1 slope

$$\text{balance}_i = \beta_0 + \beta_1 \text{student\_dummy}_i$$

H_{01}: Students and non-students have difference balances, regardless of income



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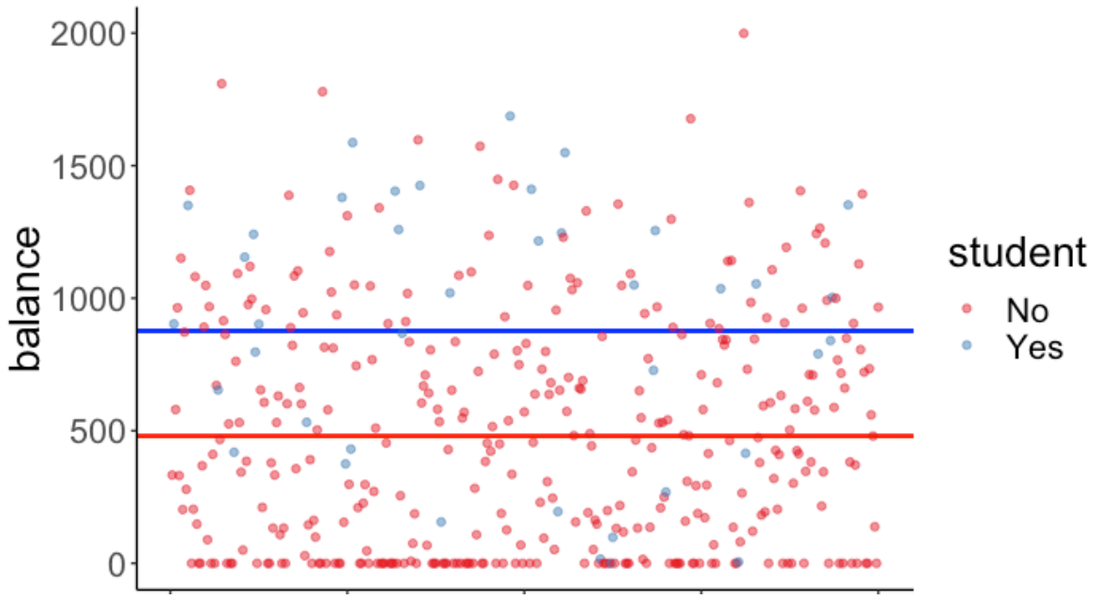
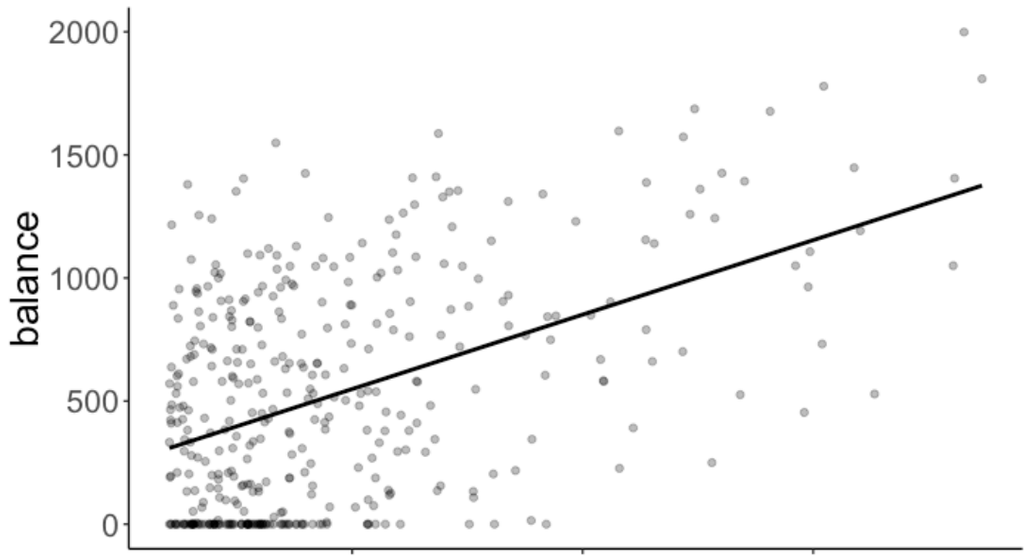


Model C_2: 1 intercept; 1 slope

$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i$

H_{02}: Students and non-students have the same balances when accounting for income



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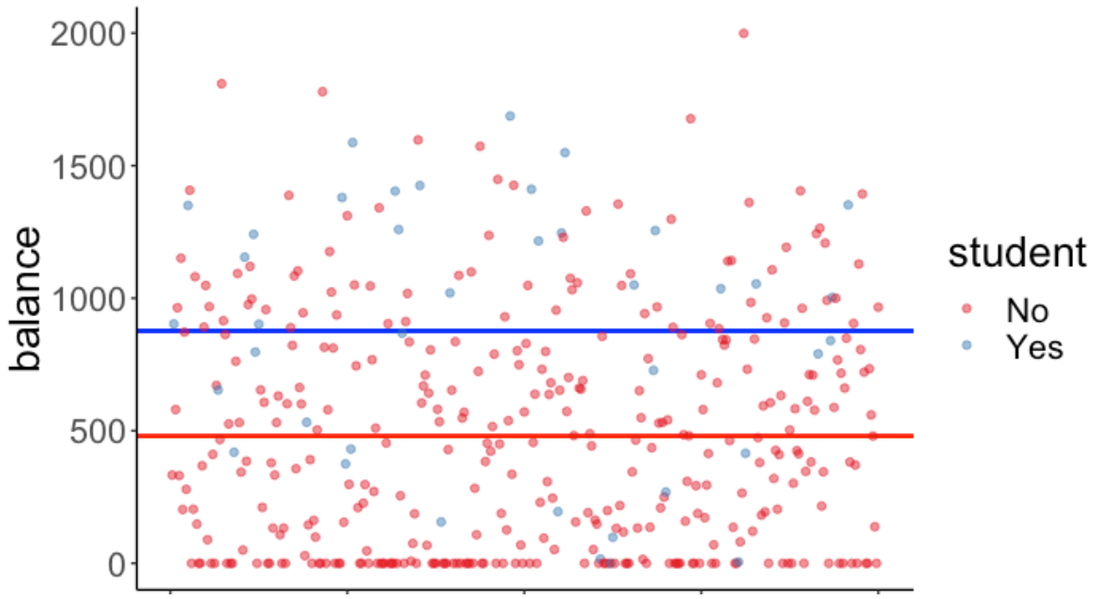

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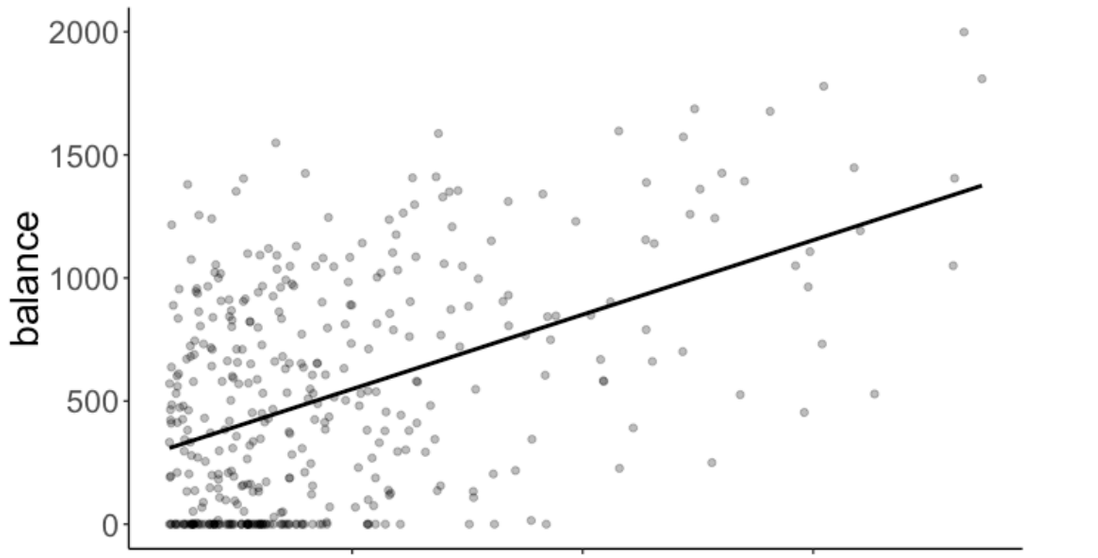

Model A: 2 intercepts; 1 slope

$\text{balance}_i = \beta_0 + \beta_1 \text{student\_dummy}_i + \beta_2 \text{income}_i$

H_{02}: Students and non-students
have different balances when
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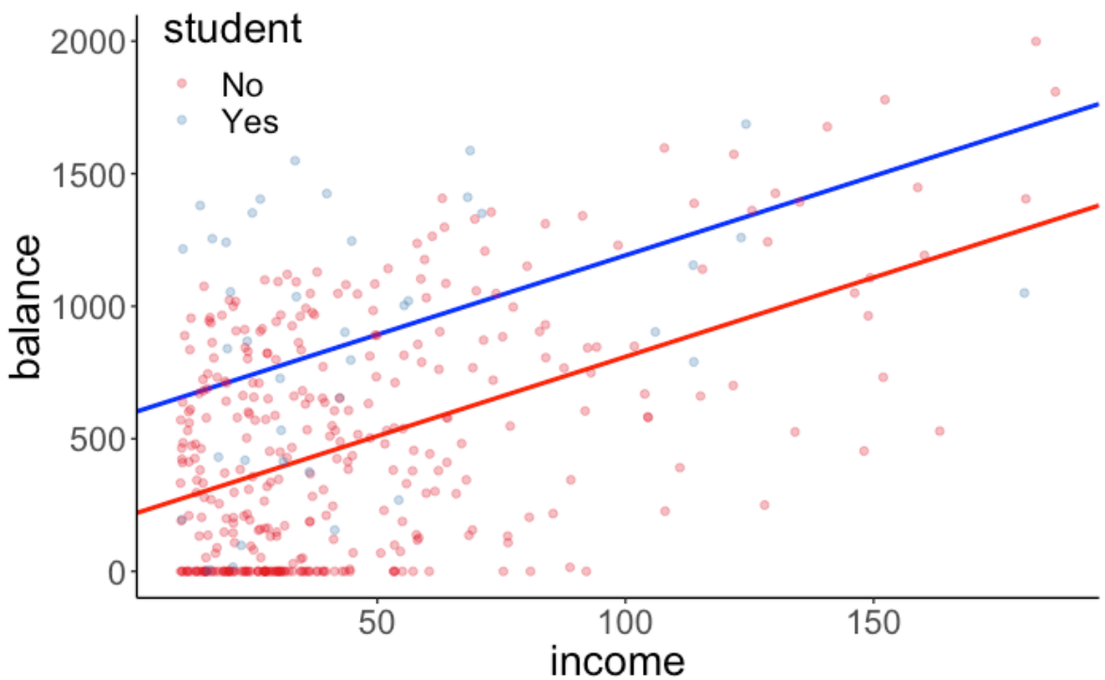

Worth it?

```python
# Student only
s_model = ols('Balance ~ C(Student)', data=df.to_pandas())
s_results = s_model.fit()

# Income only
i_model = ols('Balance ~ Income', data=df.to_pandas())
i_results = i_model.fit()

# Student + Income
si_model = ols('Balance ~ C(Student) + Income', data=df.to_pandas())
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```python
# S+I vs S
anova_lm(s_results, si_results)
```

✓ 0.0s

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	74.0	1.721872e+07	0.0	NaN	NaN	NaN
1	73.0	1.374135e+07	1.0	3.477369e+06	18.473293	0.000052

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# S+I vs I
anova_lm(i_results, si_results)
```

✓ 0.0s

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	74.0	1.685200e+07	0.0	NaN	NaN	NaN
1	73.0	1.374135e+07	1.0	3.110658e+06	16.525165	0.00012

Worth it!

Interpreting the parameter estimates

what do these represent?

OLS Regression Results

Dep. Variable:	Balance	R-squared:	0.322
Model:	OLS	Adj. R-squared:	0.304
No. Observations:	76	F-statistic:	17.37
Covariance Type:	nonrobust	Prob (F-statistic):	6.75e-07

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	207.8553	92.109	2.257	0.027	24.282	391.429
C(Student)[T.Yes]	404.6330	99.538	4.065	0.000	206.254	603.012
Income	5.5138	1.283	4.298	0.000	2.957	8.071

In GLM we interpret each estimate **assuming other parameters = 0**

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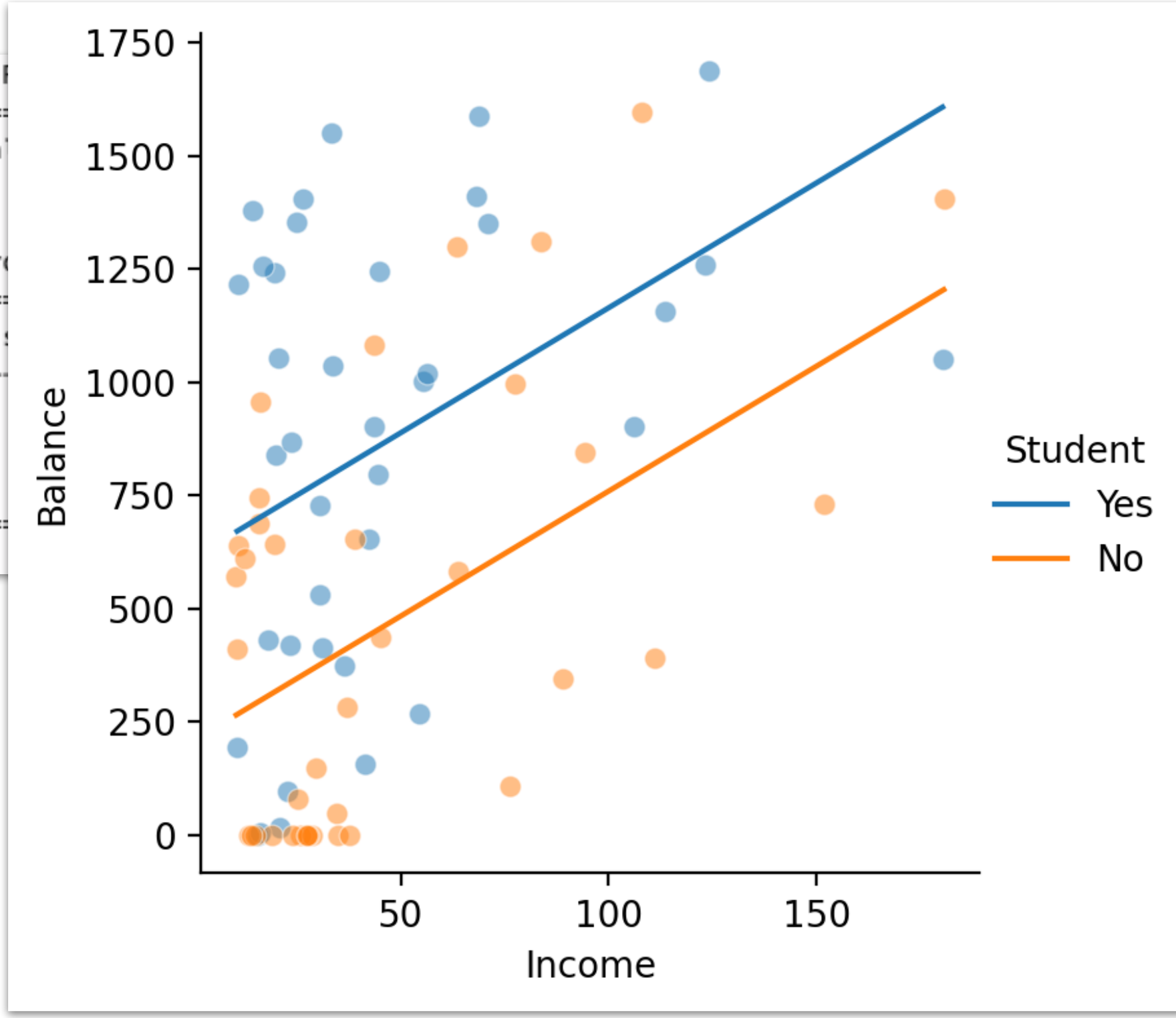
```
                            OLS R
==================================
Dep. Variable:                  Ba
Model:
No. Observations:
Covariance Type:              nonr
==================================
                     coef    s
----------------------------------
Intercept          207.8553
C(Student)[T.Yes]  404.6330
Income               5.5138
==================================
```



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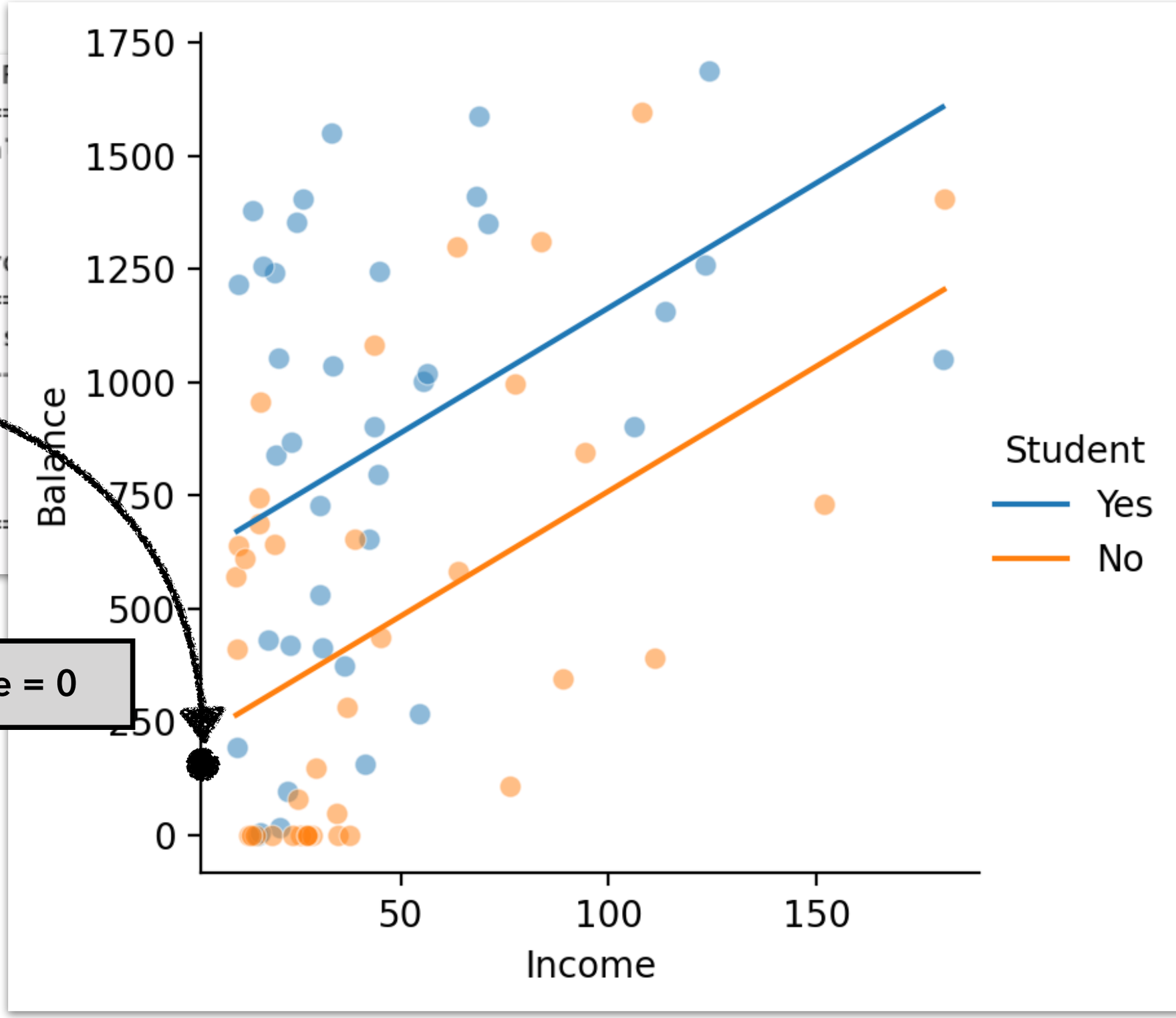
Covariance Type:              nonr
==================================
                     coef    s
----------------------------------
Intercept          207.8553
C(Student)[T.Yes]  404.6330
Income               5.5138
==================================
```

$\hat{\beta}_0$ = Student = 0 (No); Income = 0



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Dep. Variable:                  Ba
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C(Student)[T.Yes]  404.6330
Income               5.5138
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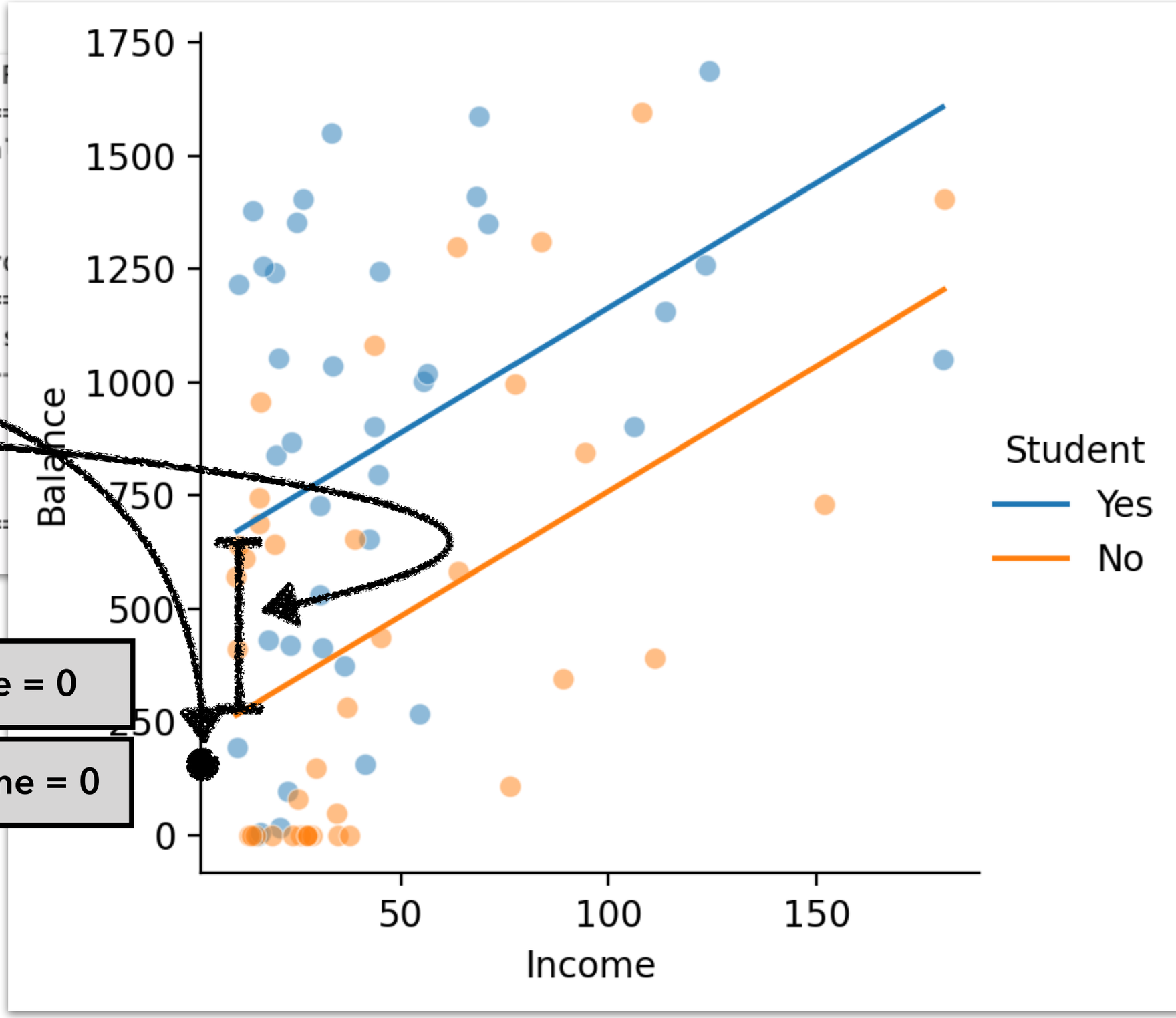

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Model:
No. Observations:
Covariance Type:              nonr
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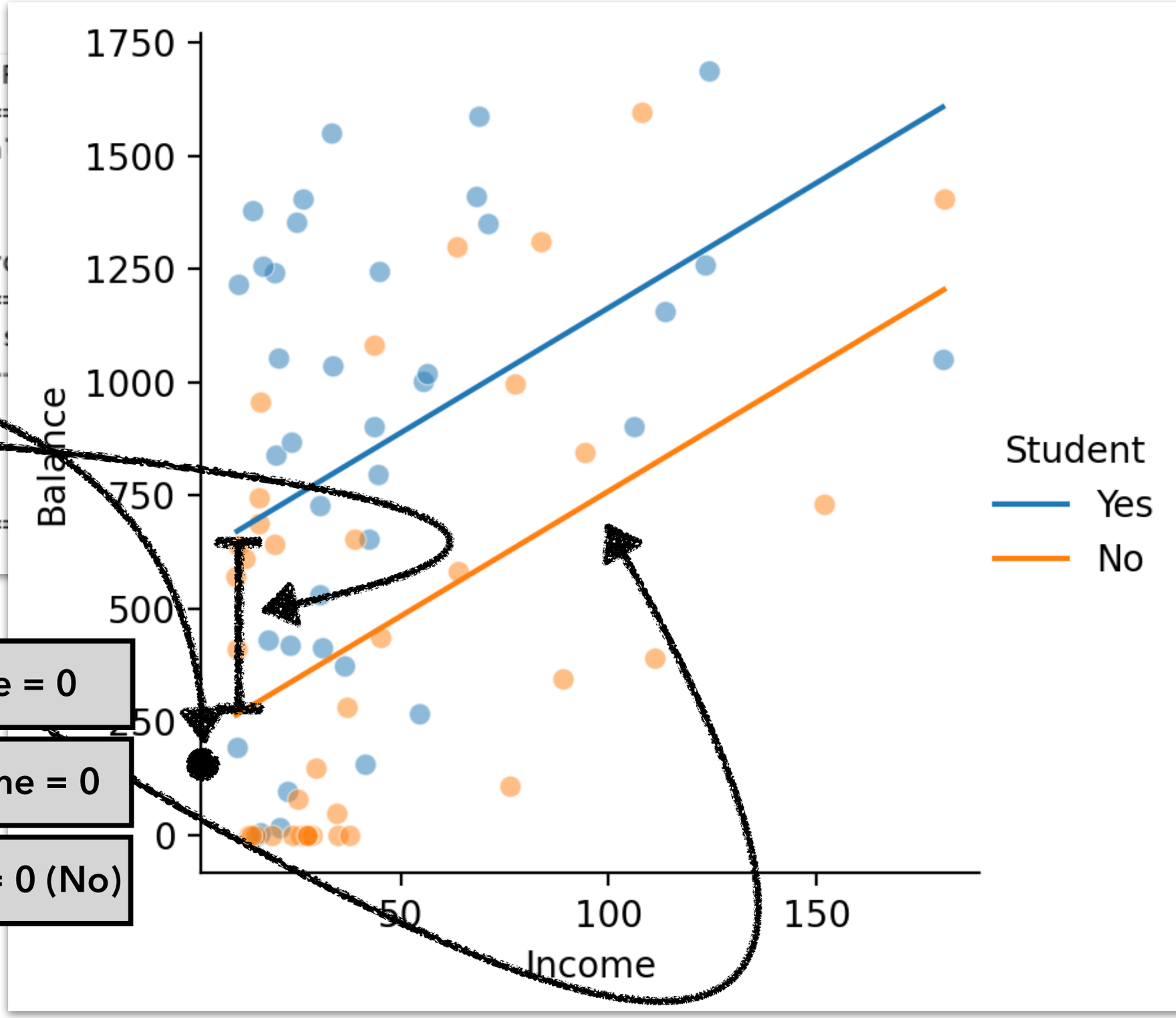
                     coef    s
----------------------------------
Intercept          207.8553
C(Student)[T.Yes]  404.6330
Income               5.5138
==================================
```

$\hat{\beta}_0$ = Student = 0 (No); Income = 0

$\hat{\beta}_1$ = Student (Yes - No); Income = 0

$\hat{\beta}_2$ = Income Slope; Student = 0 (No)



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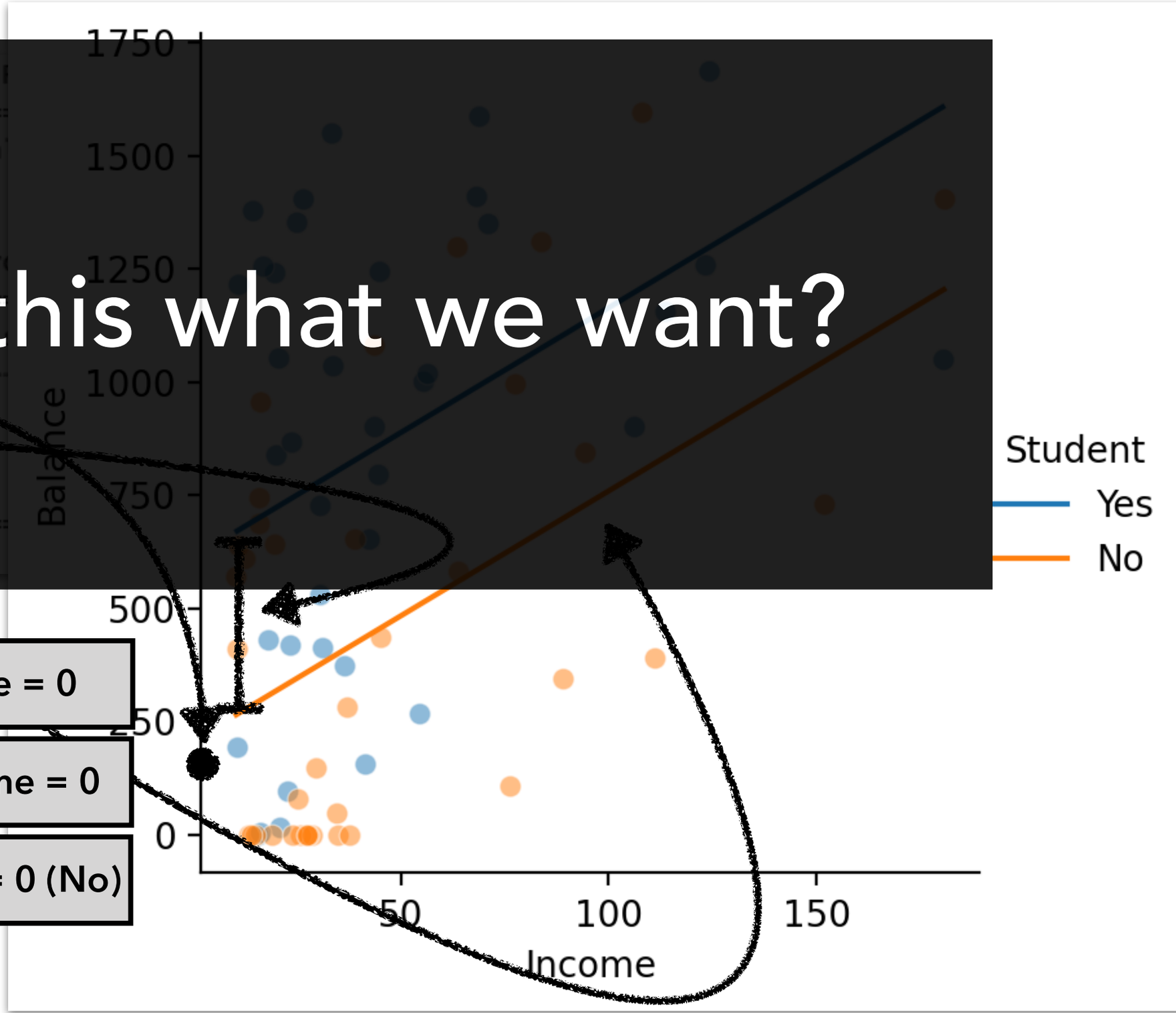
                            OLS
==============================
Dep. Variable:                 Ba
Model:
No. Observations:
Covariance Type:            nonr
==============================

------------------------------
Intercept            207.8553
C(Student)[T.Yes]    404.6330
Income                 5.5138
==============================
```

Is this what we want?



$\hat{\beta}_0$ = Student = 0 (No); Income = 0

$\hat{\beta}_1$ = Student (Yes - No); Income = 0

$\hat{\beta}_2$ = Income Slope; Student = 0 (No)

Improving interpretations with **center-ing**

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Centering Predictors:

- subtracting the mean from each observation
- allows us to interpret each parameter estimate when **other parameters = their mean(s)**

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si_model_centered = ols('Balance ~ C(Student) + center(Income)', data=df.to_pandas())
```

Improving interpretations with **center-ing**

Centering Predictors:

- subtracting the mean from each observation
- allows us to interpret each parameter estimate when **other parameters = their mean(s)**

```
si_model_centered = ols('Balance ~ C(Student) + center(Income)', data=df.to_pandas())
```

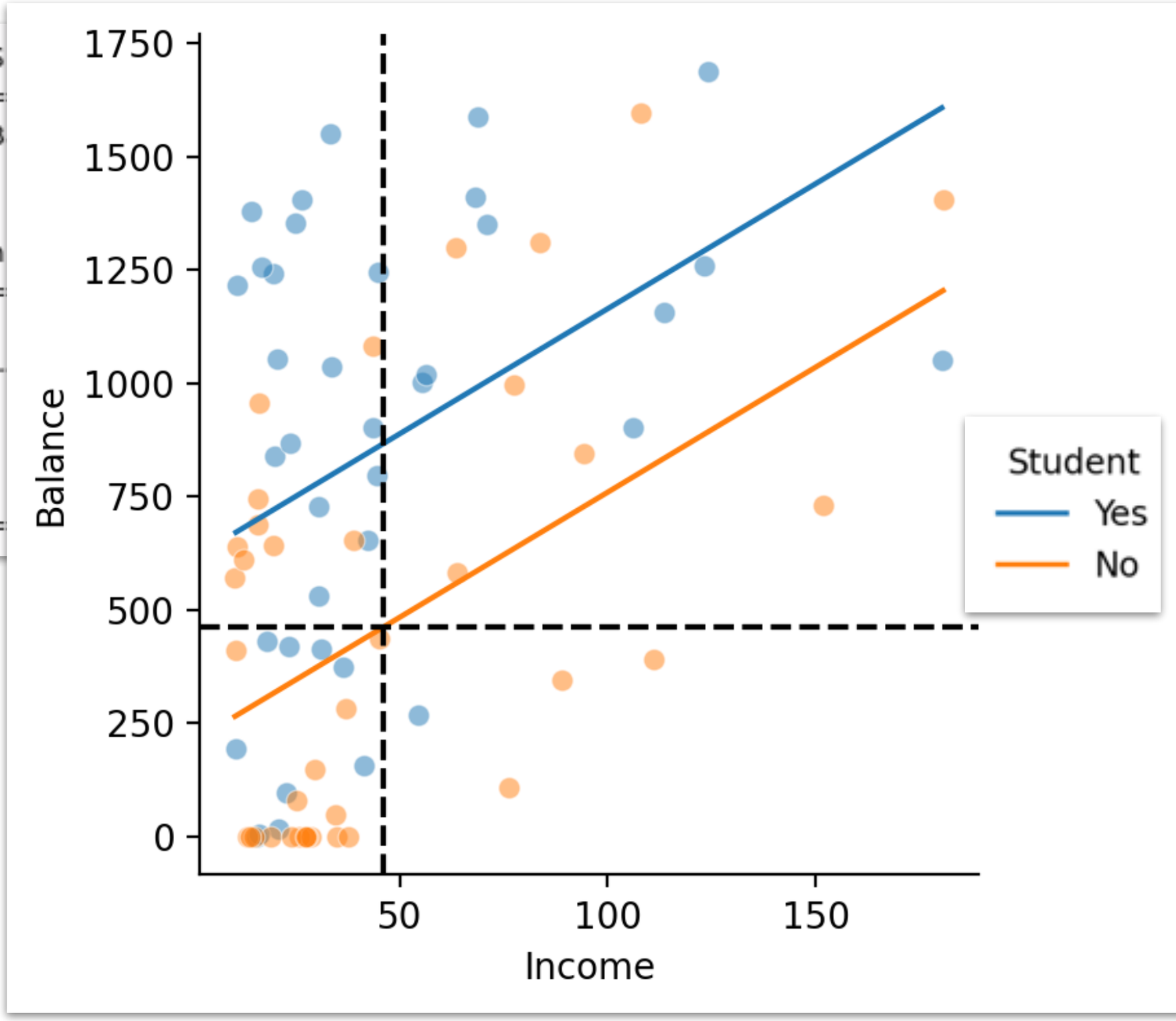
OLS Regression Results

Dep. Variable:	Balance	R-squared:	0.322
Model:	OLS	Adj. R-squared:	0.304
No. Observations:	76	F-statistic:	17.37
Covariance Type:	nonrobust	Prob (F-statistic):	6.75e-07

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	461.6440	70.383	6.559	0.000	321.371	601.917
C(Student)[T.Yes]	404.6330	99.538	4.065	0.000	206.254	603.012
center(Income)	5.5138	1.283	4.298	0.000	2.957	8.071

Centering: interpret **assuming other parameters = their mean**

```
                            OLS
==============================
Dep. Variable:                 B
Model:
No. Observations:
Covariance Type:             non
==============================
                            coef
------------------------------
Intercept                461.6440
C(Student)[T.Yes]        404.6330
center(Income)             5.5138
==============================
```



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```
                            OLS
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Dep. Variable:                 B
Model:
No. Observations:
Covariance Type:             non
==============================
                            coef
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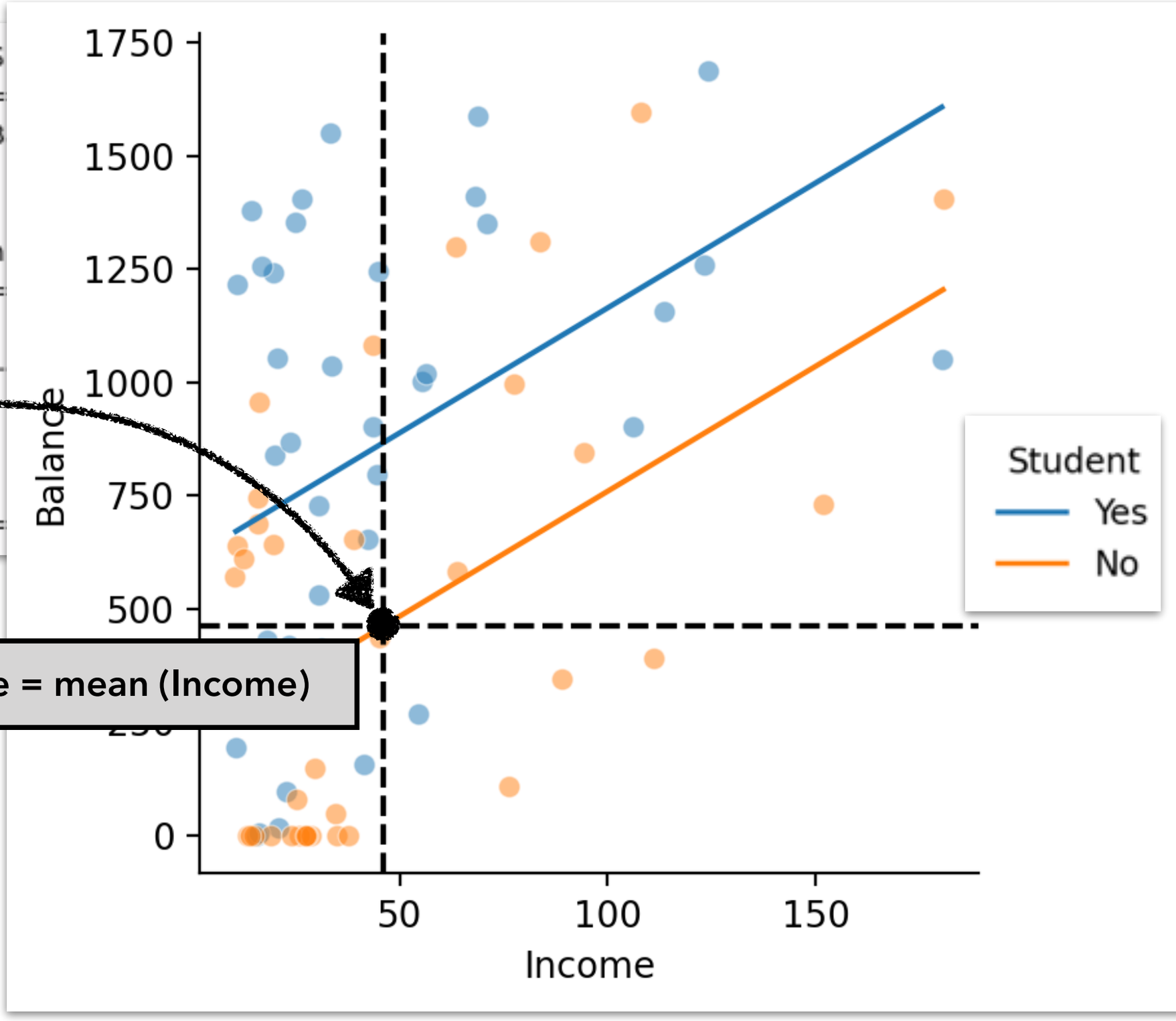

$\hat{\beta}_0$ = Student = 0 (No); Income = mean (Income)

Centering: interpret **assuming other parameters = their mean**

```
                            OLS
==============================
Dep. Variable:                 B
Model:
No. Observations:
Covariance Type:             non
==============================
                            coef
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Intercept                461.6440
C(Student)[T.Yes]        404.6330
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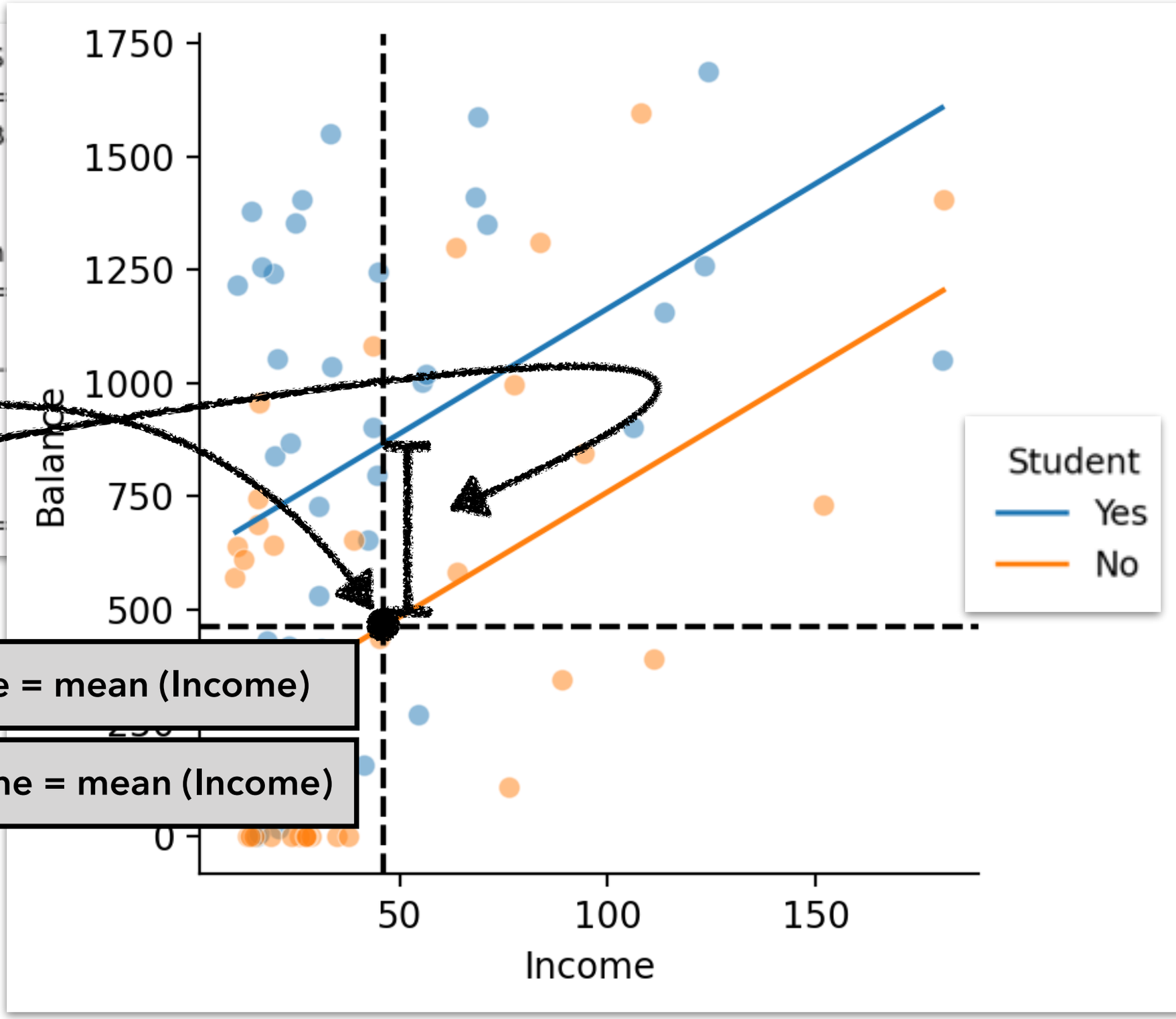

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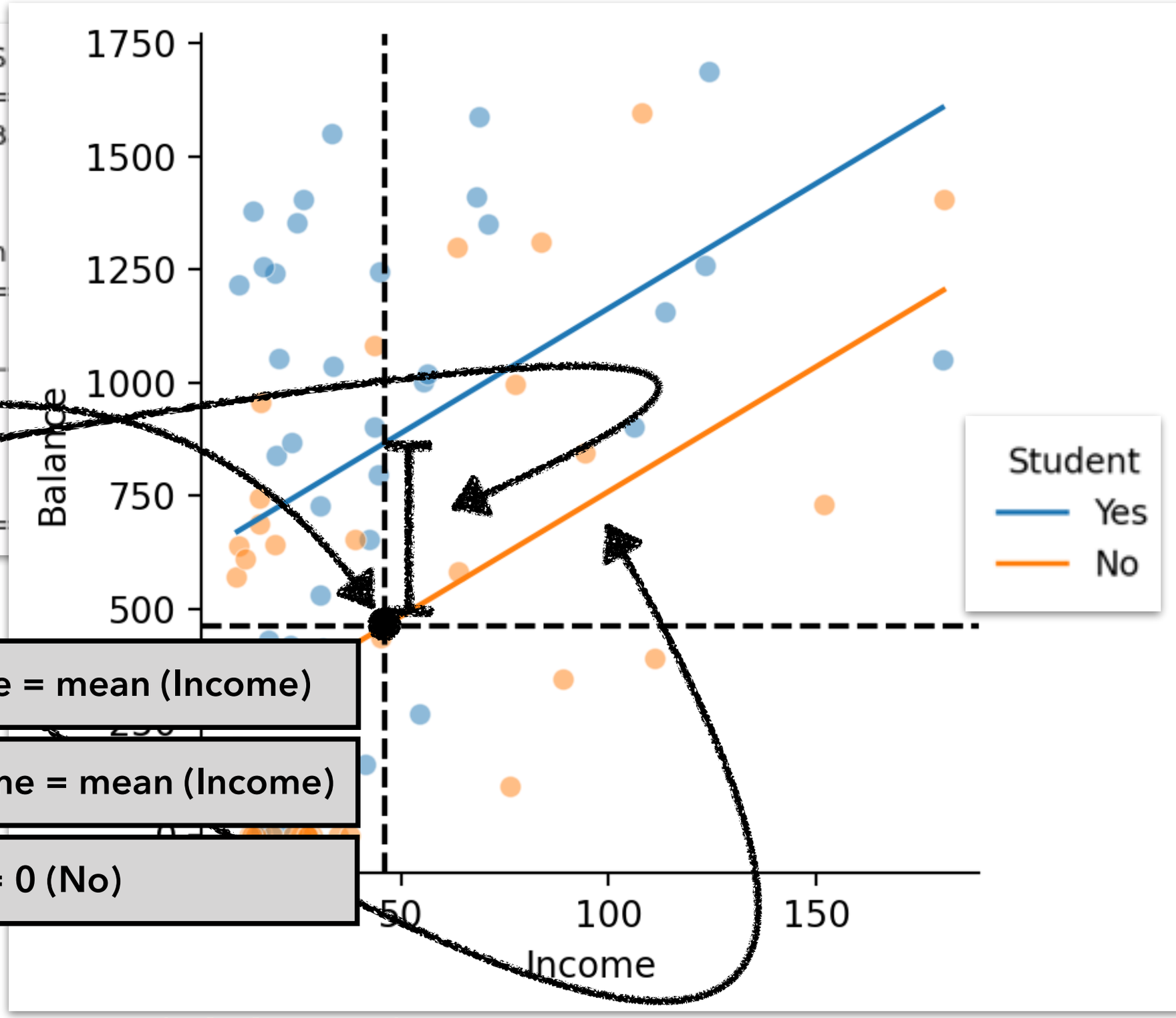

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$\hat{\beta}_1$ = Student (Yes - No); Income = mean (Income)

$\hat{\beta}_2$ = Income Slope; Student = 0 (No)

predictor we centered **did not** change

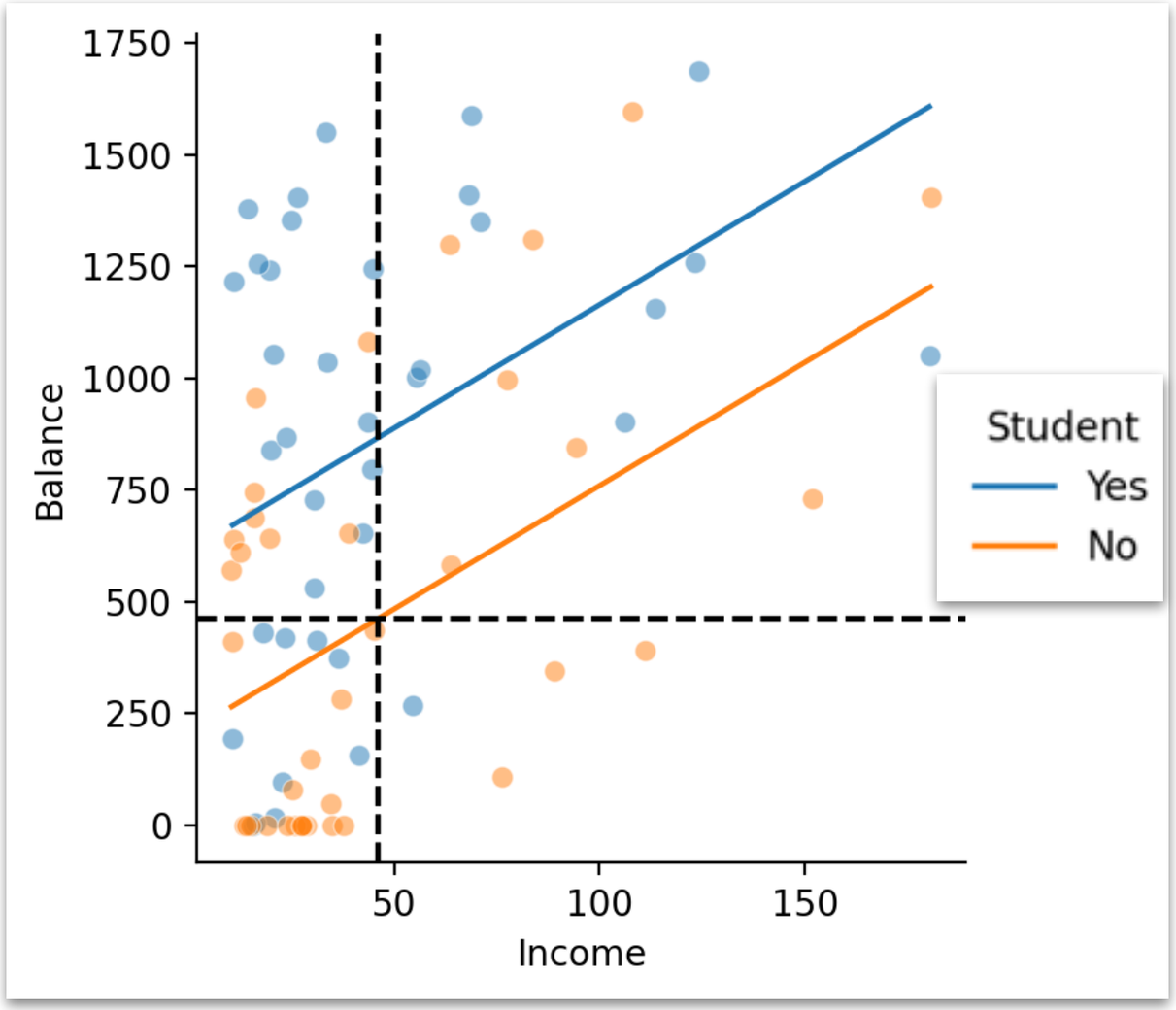
Summary: **center**-ing

```
si_model_centered = ols('Balance ~ C(Student) + center(Income)', data=df.to_pandas())
```

How: subtracting the mean from each observation

- allows us to interpret each parameter estimate when **other parameters = their mean(s)**
- **does not** change estimate of centered variable
- **does not** change model predictions
- 2-level predictors: unless other predictor = 0 is *where* you want to compare the difference **always mean center**

Reporting the results



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Intercept	461.6440	70.383	6.559	0.000	321.371	601.917
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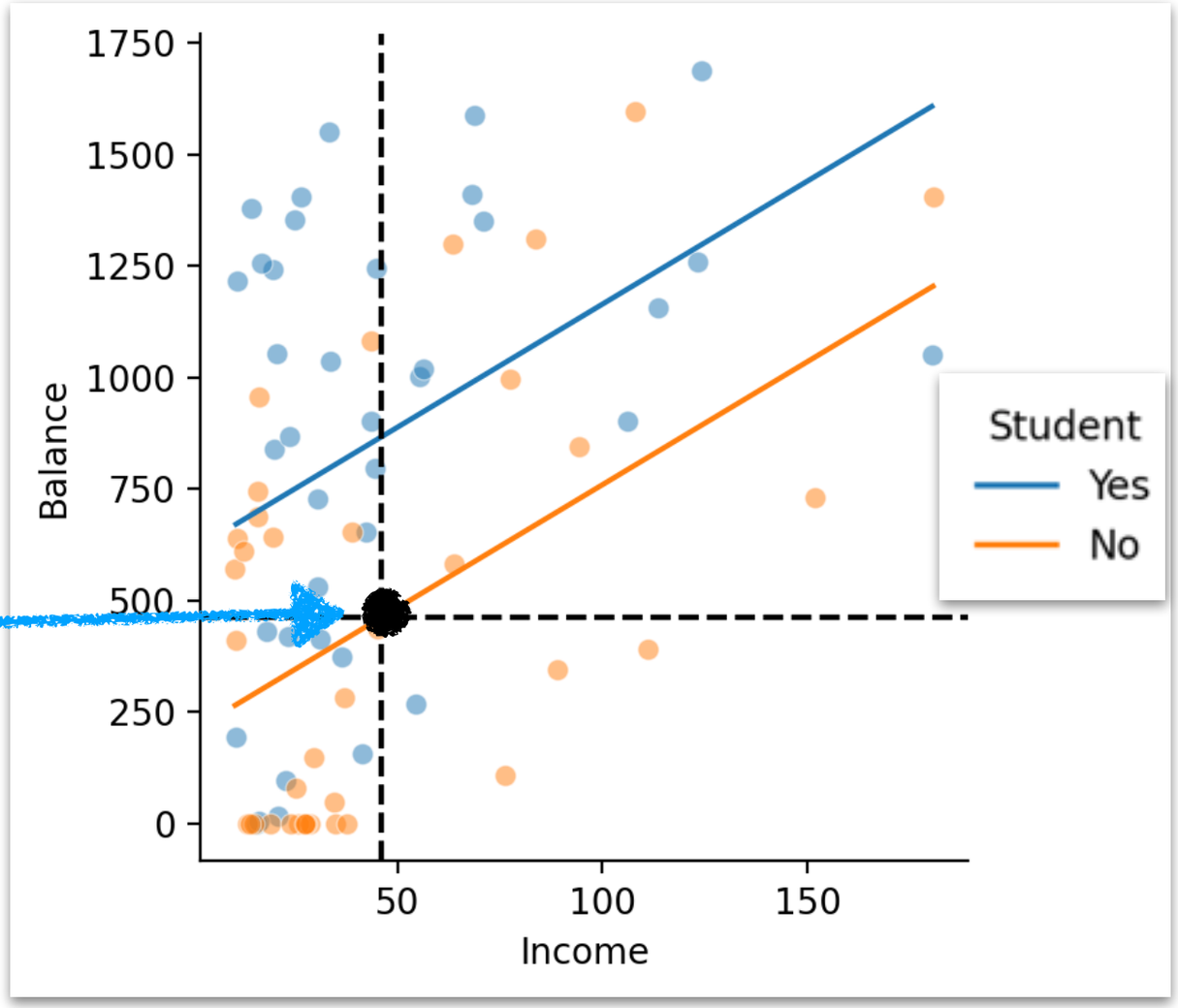
Controlling for income, students have a significantly higher average credit card balance (*Mean* = 866.28) than non-students (*Mean* = 461.64), t(75) = 99.54, *b* = 404.63 [206.25 603.012], $p < .001$.

Reporting the results

The mean for Student = No is the intercept!



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Reporting the results

We can use `.predict()` to generate adjusted mean for Student = Yes

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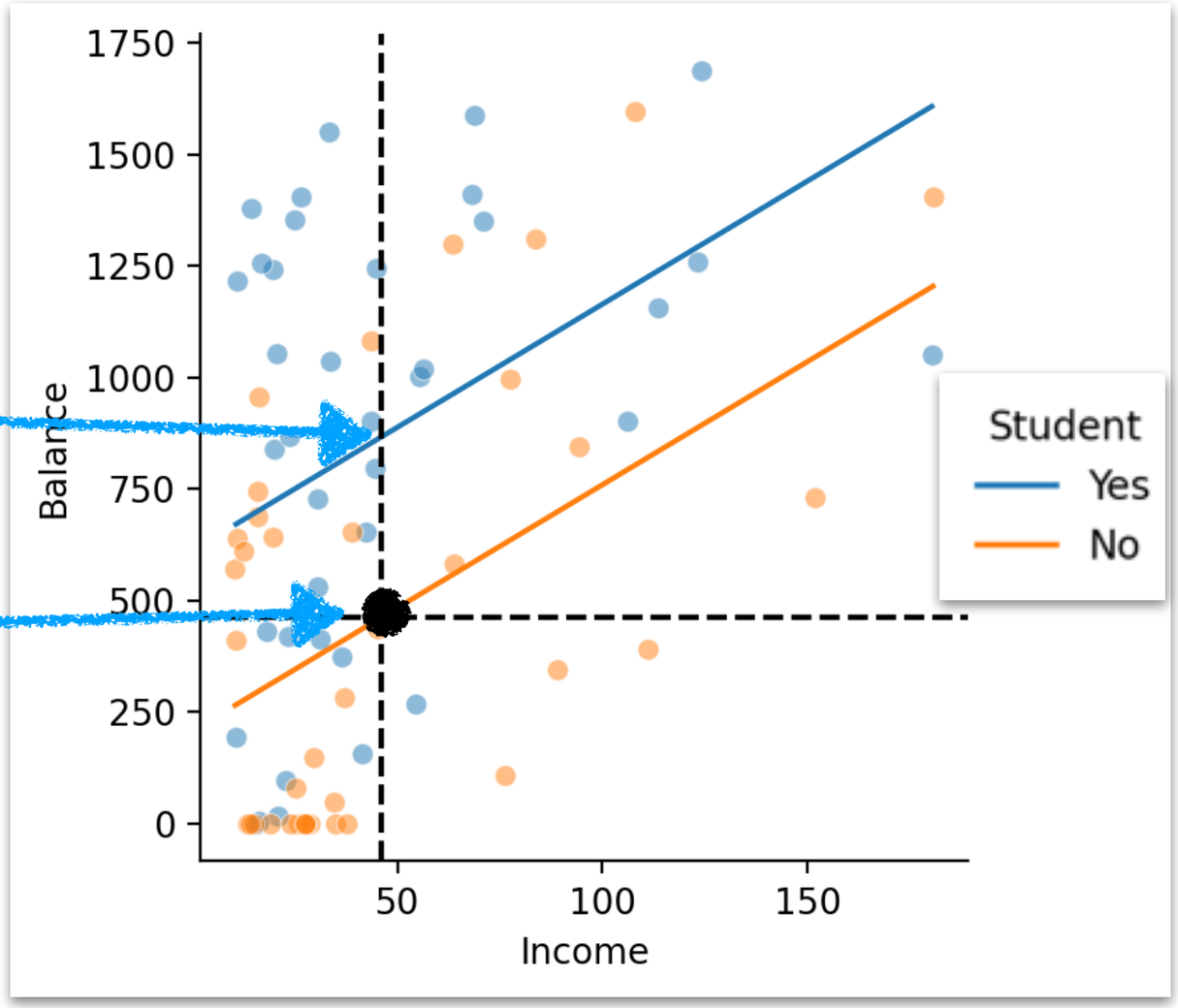

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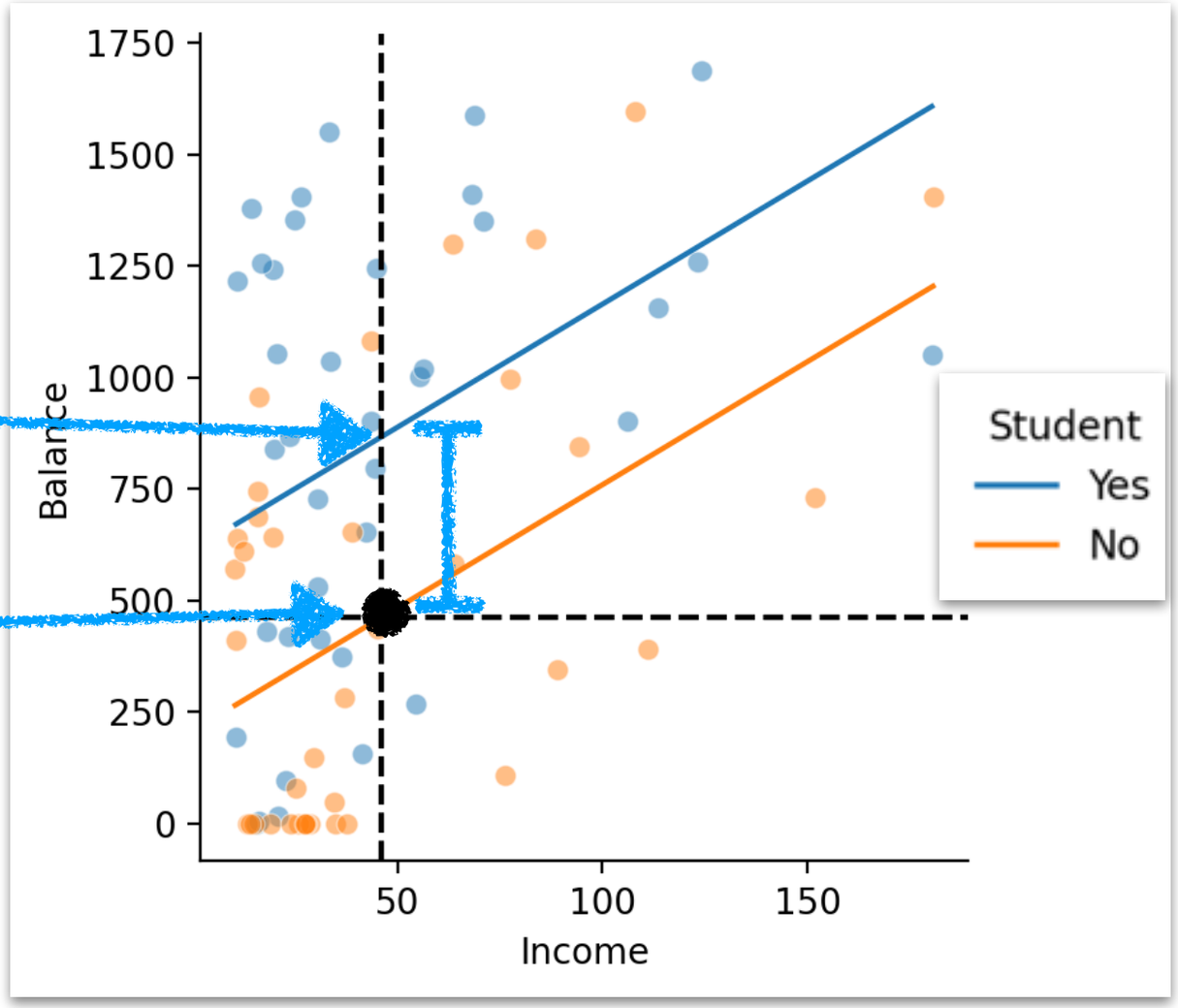

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Categorical x continuous predictor (interactions)

Is the **relationship** between income and balance different for students than it is for non-students?

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Compact Model

$$\widehat{\text{balance}}_i = b_0 + b_1 \text{income}_i + b_2 \text{student}_i$$

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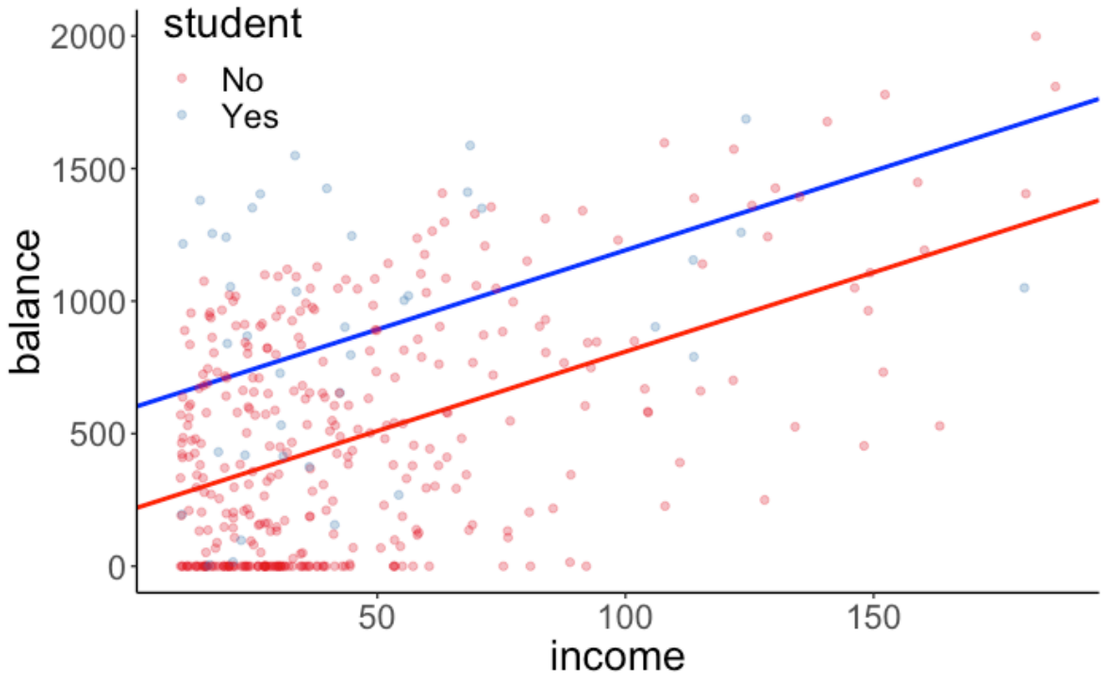
Augmented Model

$$\widehat{\text{balance}}_i = b_0 + b_1 \text{income}_i + b_2 \text{student}_i + b_3 (\text{income}_i \times \text{student}_i)$$

Model C: 2 intercepts; 1 slope

$$\text{balance}_i = \beta_0 + \beta_1 \text{student\_dummy}_i + \beta_2 \text{income}_i$$

H_0: Students and non-students have the same relationship between income and balance



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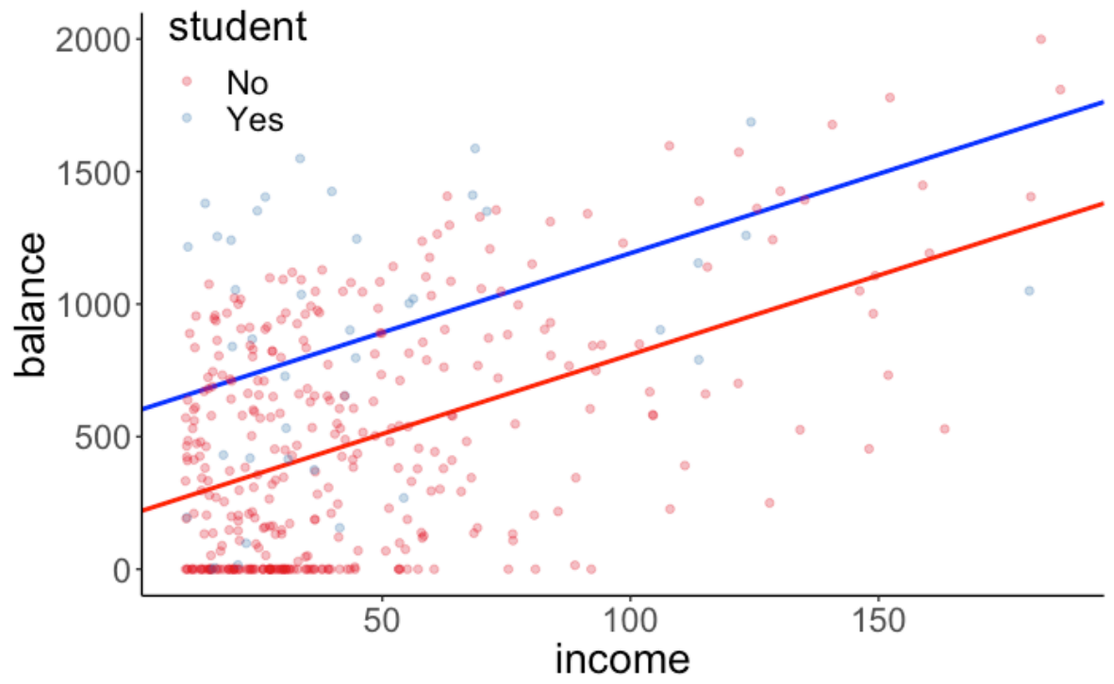
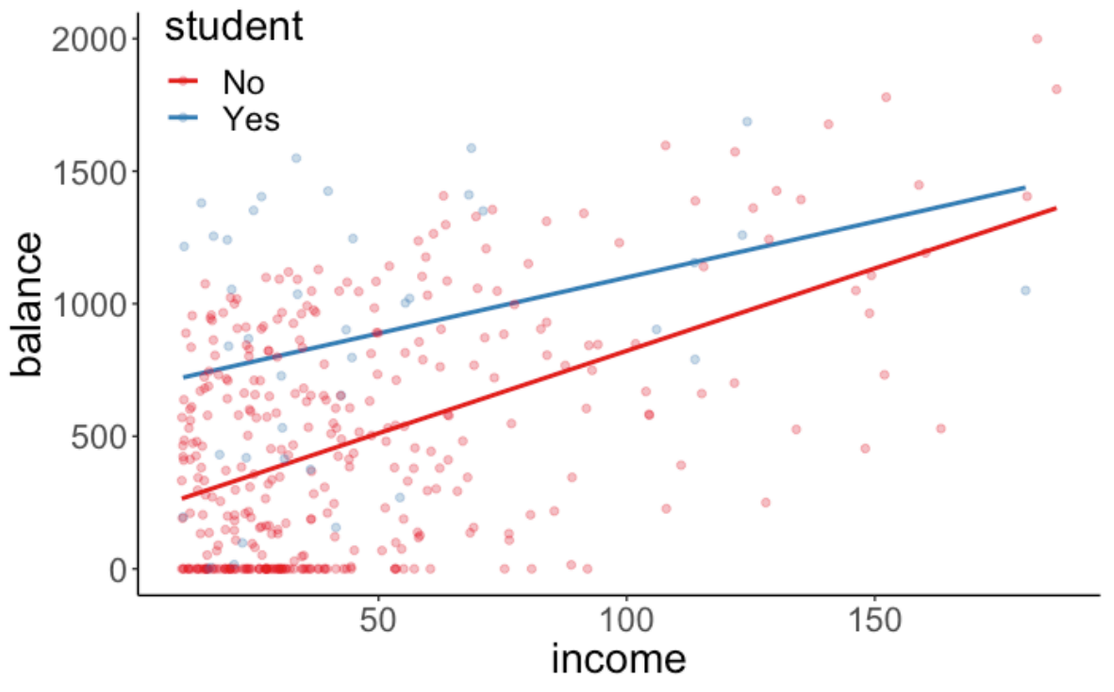


Model A: 2 intercepts; 2 slopes

$$\text{balance}_i = \beta_0 + \beta_1 \text{student\_dummy}_i + \beta_2 \text{income}_i + \beta_3 \text{student\_dummy} * \text{income}_i$$

H_A: Students and non-students have different relationships between income and balance



Worth it?

Is the **relationship** between income and balance different for students than it is for non-students?

Worth it?

Is the **relationship** between income and balance different for students than it is for non-students?

```
# These are the same!
'Balance ~ C(Student) + Income + C(Student):Income'
'Balance ~ C(Student) * Income'
```

```
six_model = ols('Balance ~ C(Student) * Income', data=df.to_pandas())
six_results = six_model.fit()

anova_lm(si_results, six_results)
```

✓ 0.0s

	df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
0	73.0	1.374135e+07	0.0	NaN	NaN	NaN
1	72.0	1.368314e+07	1.0	58201.972255	0.306256	0.581701

not worth it!

Interpreting the parameter estimates

what do these represent?

OLS Regression Results

Dep. Variable:	Balance	R-squared:	0.325
Model:	OLS	Adj. R-squared:	0.297
No. Observations:	76	F-statistic:	11.57
Covariance Type:	nonrobust	Prob (F-statistic):	2.82e-06

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	176.9471	108.096	1.637	0.106	-38.539	392.434
C(Student)[T.Yes]	470.4184	155.351	3.028	0.003	160.732	780.105
Income	6.1811	1.765	3.502	0.001	2.662	9.700
C(Student)[T.Yes]:Income	-1.4298	2.584	-0.553	0.582	-6.580	3.721

Interpreting the parameter estimates

…oops we don't care about Income = 0…
let's center!

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Un-centered

OLS Regression Results

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Centered

Doesn't change

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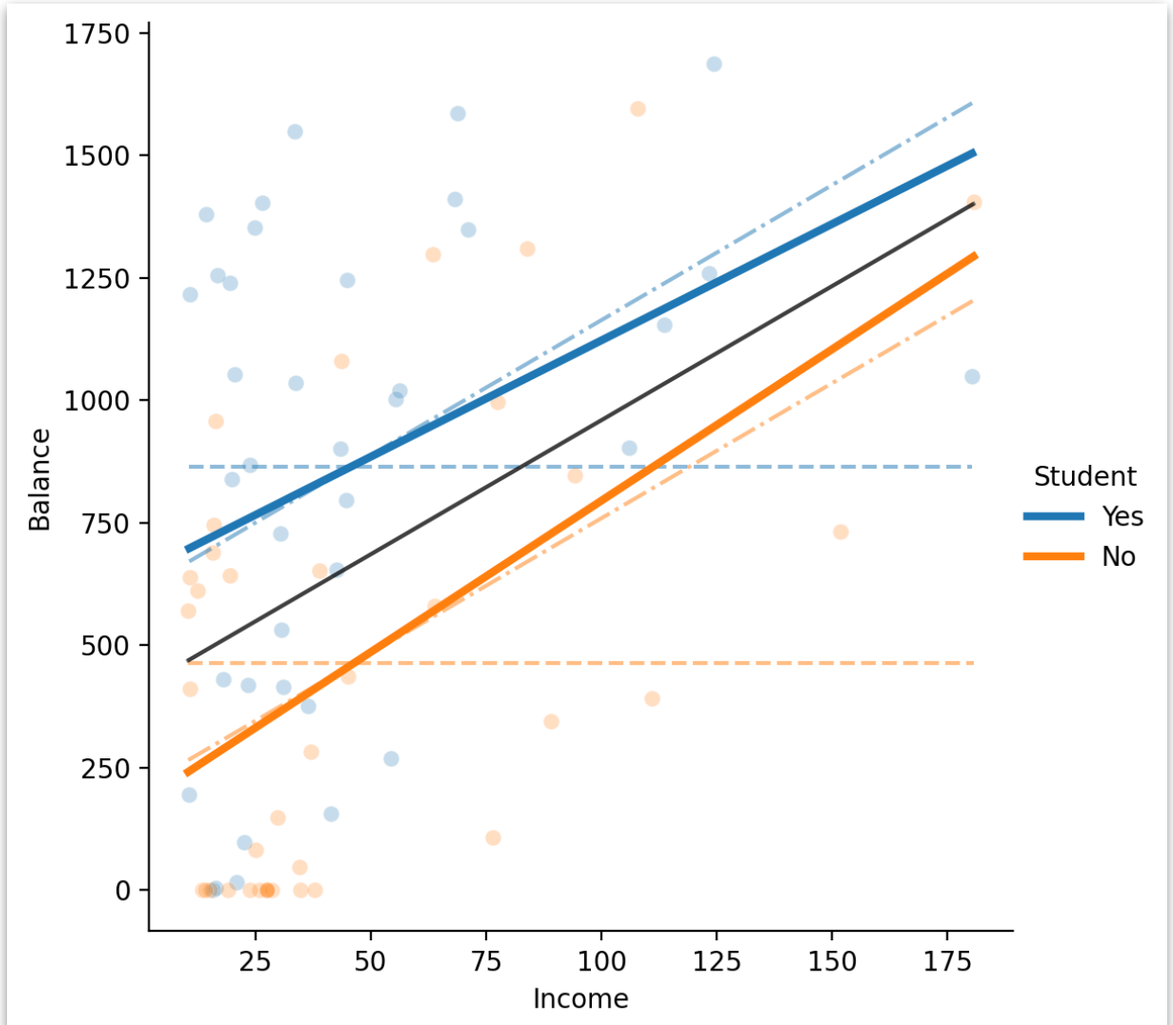
Reporting the results



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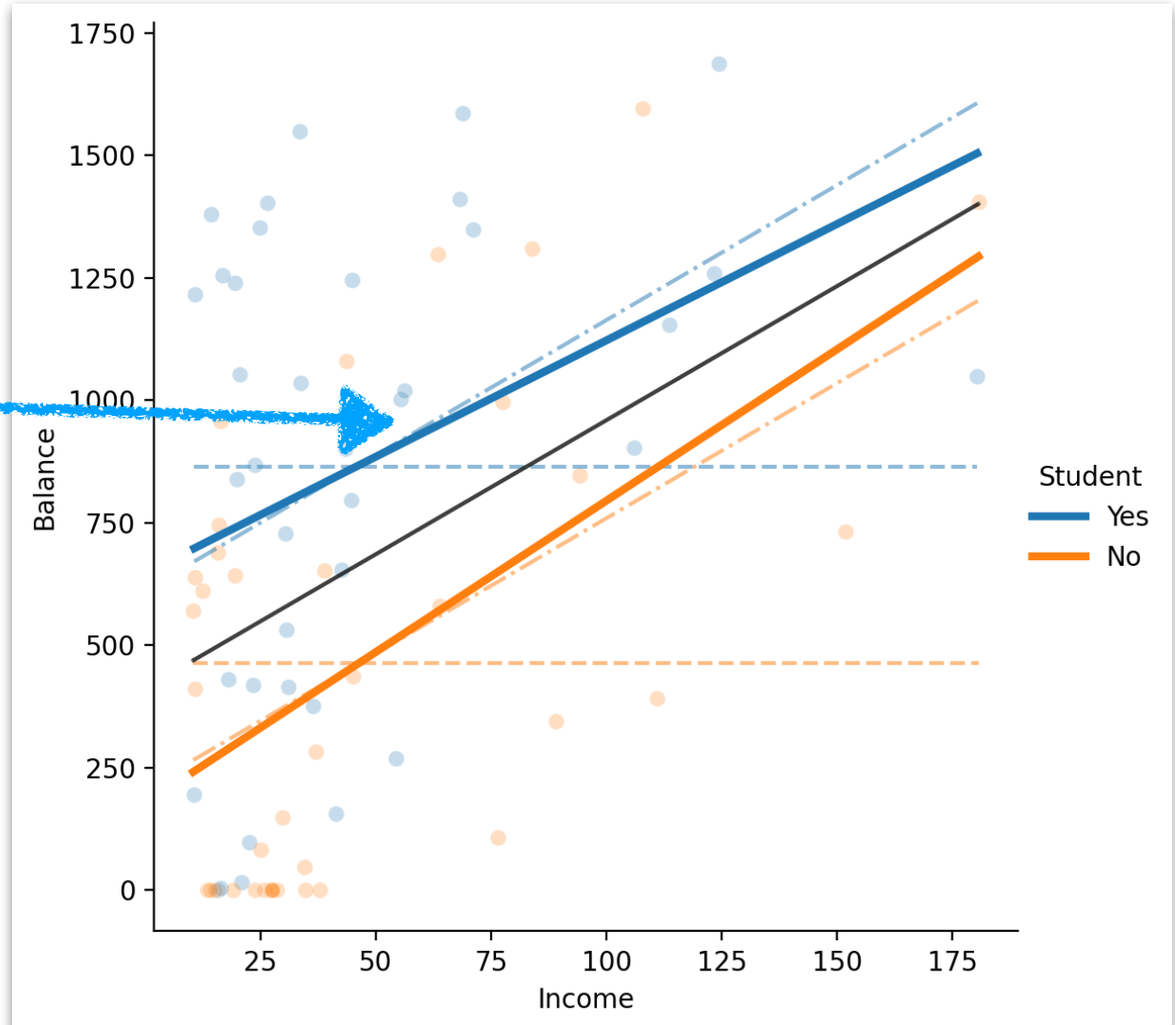
Reporting the results

We can use `.predict()` to calculate adjusted slope for Student = Yes



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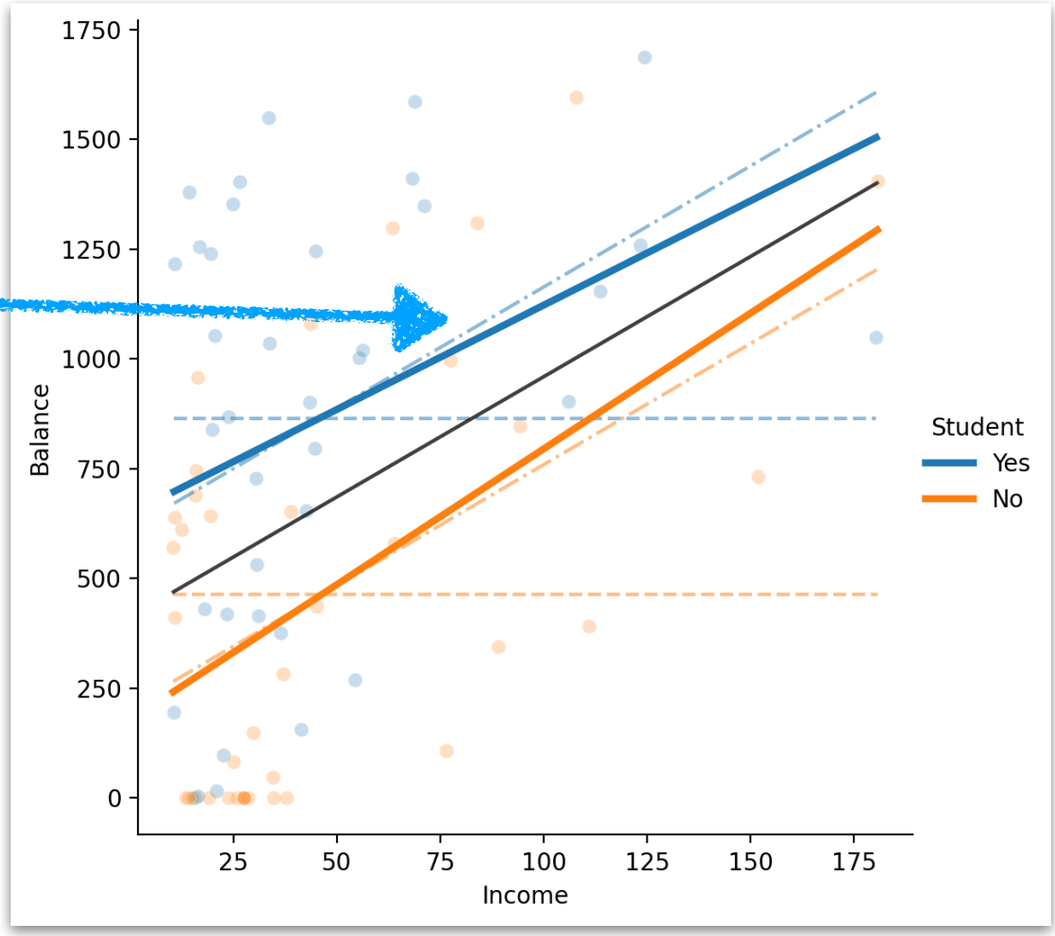

For *students*, an increase in income is associated with an increase in $4.75 of average credit card balance.

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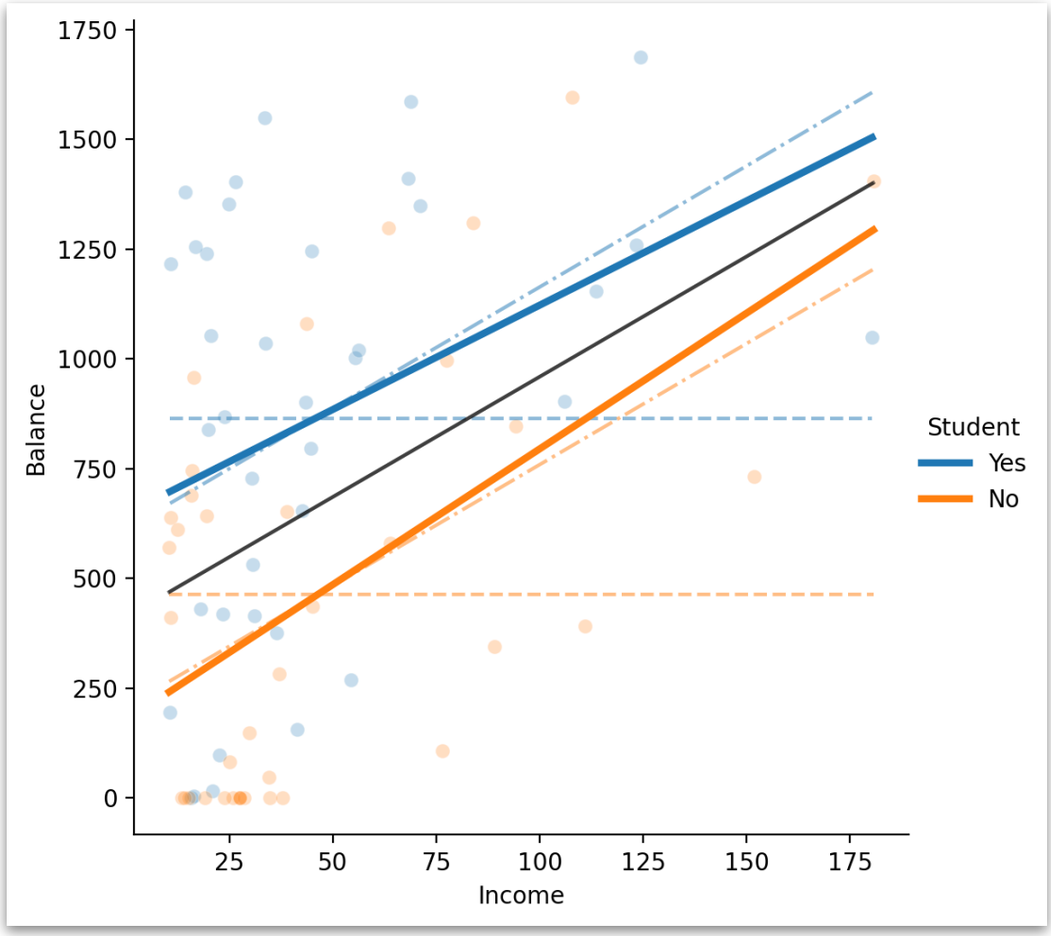
Reporting the results



For *non-students*, an increase in income is associated with an increase in $6.18 of average credit card balance.

For *students*, an increase in income is associated with an increase in $4.75 of average credit card balance.

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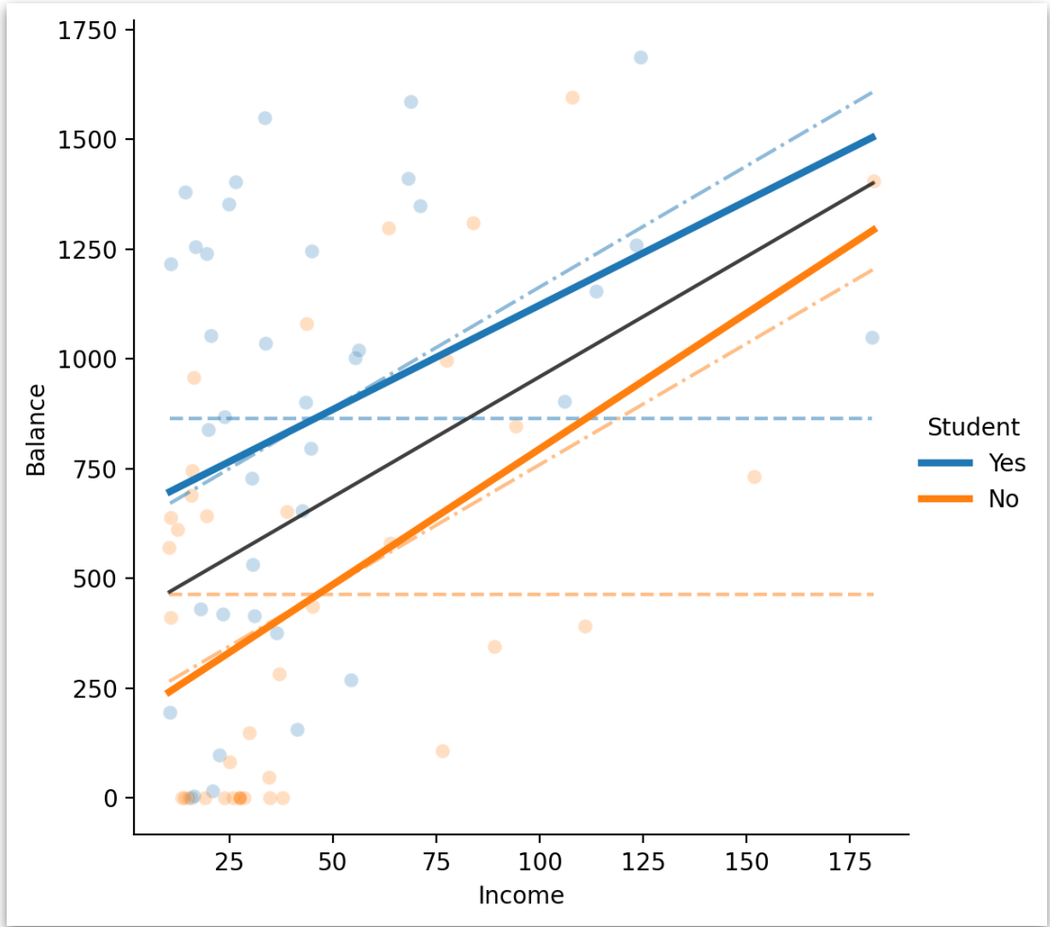
C(Student)[T.Yes]:center(Income)	-1.4298	2.584	-0.553	0.582	-6.580	3.721

Reporting the results

There is no significant difference in the relationship between income and balance for students versus non-students, $F(1, 72) = 0.33$, $p = 0.582$

df_resid	ssr	df_diff	ss_diff	F	Pr(>F)
73.0	1.374135e+07	0.0	NaN	NaN	NaN
72.0	1.368314e+07	1.0	58201.972255	0.306256	0.581701



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Your Turn

1. First Half (together)
2. **Second Half (on your own)**
    - Notebooks 4 & 5
    - And/or look at solutions for Notebooks 1-3 if you haven't
    - **Its ok if you don't finish today**
    - But please **commit** and **push** your work and **tell us** where you stopped before leaving (so we can plan for tomorrow)