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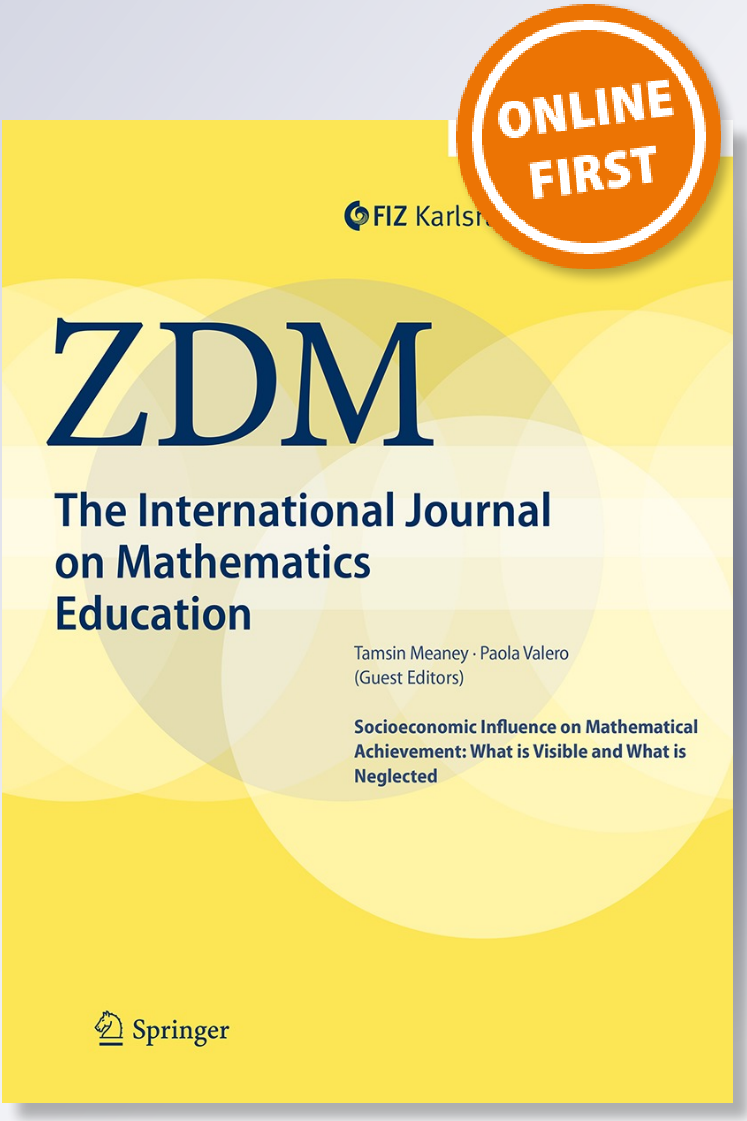

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ORIGINAL ARTICLE

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Michelle. L. Stephan

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Abstract Design research is usually motivated by university members with experience and interest in building theory and instructional designs in collaboration with one teacher. Typically, the teacher is considered as a member of the research team, with the primary responsibility of implementing instruction. However, in this chapter, I describe a Classroom Design Research project that was conducted by a team comprised mostly of classroom teachers. Their goal was to create a stable instructional unit for integer addition and subtraction that they could use to help students learn the topic with meaning. In this paper, I outline the basic tenets of Classroom Design Research, including the instructional theory of Realistic Mathematics Education and how it guided them in designing instruction. I introduce the construct of a classroom learning trajectory and elaborate on it with the integer project. Finally, I argue that Design Research is mutually beneficial for researchers and teachers. The team of teachers that participated in Design Research embarked on a unique professional development experience, one in which they engaged in practices that supported a new way of preparing their instruction. Reciprocally, the teachers' unique craft and pedagogical content knowledge shaped the integer instruction theory in unique ways that do not occur in typical Design Research.

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1 Introduction

Design Research has gained prominence in the last two decades in mathematics and science education and varies in its purposes and educational settings (Cobb et al. 2003; Corcoran et al. 2009; Daro et al. 2011). The Design Research Project described in this article was conducted in classroom settings to test and revise the third version of a mathematics instructional sequence. Classroom-based Design Research (Classroom-DR) involves a research team that includes a teacher who implements an instructional sequence that is tested and revised during the phases of design research (Cobb 2000; Confrey and Lachance 2000) Classroom-DR is typically initiated by university members with experience and interest in building theory and engineering instructional designs to be adapted by practitioners. Typically, the teacher is considered as a full member of the research team, with the primary responsibility of implementing instruction. As Gravemeijer and van Eerde (2009) argue, however, Classroom-DR can serve as an extremely powerful professional development opportunity in which teachers learn as they create hypothetical learning trajectories for their specific classroom contexts. In this article, I build upon Gravemeijer and van Eerde's claim by elaborating a Classroom-based Design Research (DR) project that was conducted by a team comprised mostly of classroom teachers. The goal of the DR project was to adapt and contribute to the stabilization of a local instruction theory for integer addition and subtraction that we could use to help students learn the topic with meaning. The research questions that guided the project were:

1. What are the mathematical practices that emerge as students interact with the newest version of the integer instructional sequence?

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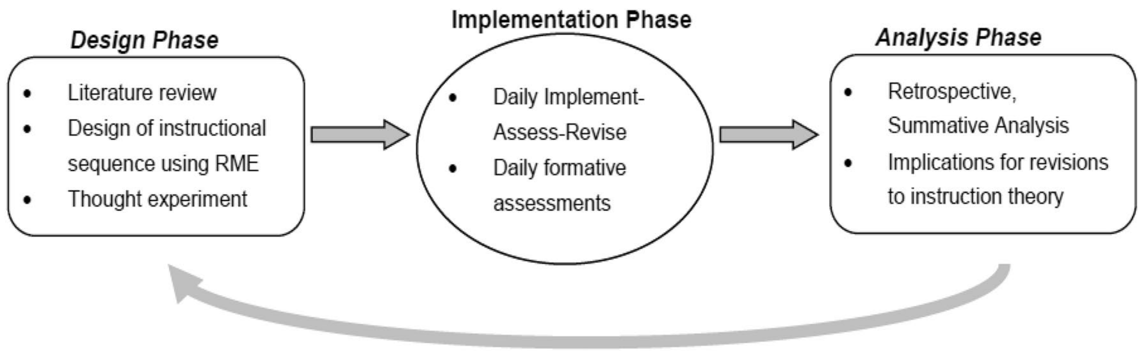


Fig. 1 The classroom design research cycle

2. How do individual students participate in and contribute to these mathematical practices?
3. What is the viability of using an empty number line as a model for students' integer addition and subtraction strategies?
4. Do students' scores on integer subtraction tasks improve significantly?

The first two questions are germane to Classroom-DR projects that use classroom learning trajectories as their primary design construct. Analyses for the first two questions document the learning of the students in the classroom and the means of supporting that learning. The third question investigates the viability of a particular inscriptional support for addition/subtraction strategies. Finally, the fourth question was important to us since integer research up to this point had shown increases in student scores in adding and subtracting except when subtracting negative numbers. While these four research questions led the research team, during the analysis of the data, a fifth research question emerged concerning the various roles that practicing teachers can play as members of the research team.

5. How do classroom teachers participate in and contribute to Design Research?

The first four research questions have been addressed in another article (Stephan and Akyuz 2012); therefore, the intent of this article is to describe Classroom Design Research, its phases of implementation and the *role that teachers play in both participating in and contributing to the theory that is built during experimentation*. I begin by describing each phase of the design process in general and elaborating the instructional design theory that was utilized to create the integer instruction theory. Incorporated within this description will be the results of our data analysis regarding roles that the teachers and researcher/teacher played in each of the phases. I conclude by reflecting back on the Classroom-DR project to elaborate the results of the experiment concerning the role that design research can play in developing teachers' practices through research. At the same time, I also argue that the Classroom-DR process is enhanced in significant ways when involving practitioners in the daily analysis and implementation process and leads to a more rich and generalizable instructional design.

2 Classroom-based design research methodology

The appeal of Design Research lies in the fact that, while much educational research is divorced from practical problems, Design Research is necessarily located in the practical settings of which it is designed to study. Although Design Research can be conducted at a range of levels (Cobb et al. 2003), the work I report here is Classroom Design Research (Cobb 2003) in which an instructional design is tested, analyzed and revised on a daily basis by several teachers in their classrooms. The main activity of researchers in this approach (Class-DR) is highly interventionist in that the researcher is proactively altering the classroom context and striving for change. The goal is not to test if the design worked but rather to explore the implementation to provide an analysis of the way in which the design was realized and the means of supporting that realization for those who might wish to adapt the design in their own contexts (Cobb 2003).

A Classroom Design Research Cycle consists of three phases (see Fig. 1), each of which will be described in turn. To begin a discussion of the first phase, *Design*, it is necessary to detail the instructional design theory that undergirds our design work. In the next section, I describe three tenets of Realistic Mathematics Education to set the foundation for the design of the instructional sequence. To be clear, RME is an instructional design theory that can be used by designers to create their instruction. Other design programs include Variation Theory (Koichu et al. 2013), Anthropological Theory of the Didactics (García and Ruiz-Higueras 2013) and frameworks from the Shell Centre in England (Burkhardt and Swan 2013).

3 Realistic mathematics education

Dissatisfaction with current integer curricular materials served as the catalyst for designing the integer instructional sequence. The textbook we were using at the time offered an exploratory approach using a "chips model". In these activities, two colored chips were used, one color signifying a positive chip and the other a negative (cf. Battista 1983; Lappan et al. 2002). Students would use these chips to model a problem like $-5—(-3)$ by placing five red colored chips on a mat and taking away three of them to see that there were two red chips remaining for an answer of -2. The difficulty arises when students have problems like $5—(-3)$ and there are no red chips to remove. Students were taught to place three *zero pairs* on the mat which were three red and three clear chips so that three red chips could be removed leaving eight positives. We found in our teaching that students merely memorized that step rather than making quantitative meaning for their actions. Additionally, the moves were modeled to them and did not arise from their own mathematizations nor was there a realistic context that would motivate students to create these strategies in meaningful ways. We, therefore, decided to work together more formally to create, test, and revise a classroom learning trajectory that aimed to support students' development of integer addition and subtraction using the instructional design theory of Realistic Mathematics Education (De Beer et al. 2015; Gravemeijer 1994; van den Heuvel-Panhuizen and Drijvers 2014) to create our classroom learning trajectory. We spent the next few years testing and revising it in our individual classrooms.

Realistic Mathematics Education has become the driving force behind the design of a variety of strong instructional programs including differential equations (Rasmussen et al. 2004), linear measurement (Stephan et al. 2003) and functions (Doorman and Gravemeijer 2008). The roots of RME are based on the idea of mathematics as a human activity (Freudenthal 1973) involving progressive mathematizations. To mathematize, "one sees, organizes, and interprets the world through and with mathematical models" (p. 189) (Fosnot and Dolk 2005). Students are guided by the teacher to reinvent mathematical ideas through mathematizing realistic contexts that are didactically rich. There are at least three heuristics that guide the design of instructional sequences from an RME perspective (van den Heuvel-Panhuizen and Drijvers 2014).

3.1 Sequences must be experientially real for students

The starting points of instructional sequences should be experientially real. Often, this heuristic is equated with situating students' activity in real-world scenarios, but Gravemeijer (1994) argues that the scenarios only need to be experienced as real. There are many contexts that have been explored for introducing integer operations such as positively and negatively charged particles (Battista 1983), the activities of patrons in a disco (Linchevski and Williams 1999), passengers on a bus (Streefland 1996), lengths of positive and negative trains (Schwarz et al. 1993/1994), LOGO turtles moving along a horizontal number line (Thompson and Dreyfus 1988), and a two-colored chips scenario (Lytle 1994; Smith 1995). Research shows that all these contexts have strengths, and when using them, students demonstrate a significantly better understanding of negative numbers. However, the contexts cited previously still had difficulty supporting students creating meaning for why the opposite of a negative "makes a positive". The realistic scenario for the Classroom-DR project built on the Western development of "debt" by giving students fictitious financial statements and asking them to compare two or more individual's net worths. While they have never filled out an actual net worth statement, students could readily interpret the results of debts and assets on a person's finances and were motivated to work within that context.

3.2 Guided reinvention/hypothetical learning trajectory

A second heuristic for guiding the design of an instructional sequence is guided reinvention. The instructional materials should be designed to encourage students' reinvention of key mathematical concepts (Freudenthal 1973). To develop a sequence of instructional activities, the designer first engages in a thought experiment to imagine a trajectory the class might invent (Gravemeijer 2004). I use the term *classroom learning trajectory* to refer to the hypothesized learning route developed by a class of students as they interact with one another and a teacher rather than an *individual learning trajectory* which is created by an individual in a one-on-one experiment with a teacher or researcher. The knowledge of the history of mathematics as well as prior research on students' understanding of the domain can be used to develop a hypothetical learning trajectory for a classroom community. Oftentimes modern students' conceptual difficulties mirror the historical struggles of the mathematics community; hence, an RME designer might include these impediments as learning goals in the trajectory. Additionally, research on individual students' learning trajectories (cf. Clements and Sarama 2004; Confrey et al. 2009; Steffe and Olive 2010; Steffe et al. 1983) can greatly inform the classroom learning trajectory.

Many researchers have interviewed, surveyed, and conducted teaching experiments as a way to document students' conceptions of integers from very early ages (Bofferding 2010) to early adolescence (Gallardo 2002; Vlassis 2008). However, at the time of our work, no individual learning trajectories had been created for integer operations. Thus, we drew heavily upon the historical development of integers to create the learning goals of our classroom trajectory. Historically, negative numbers were considered "absurd" early in their conception because mathematicians had not developed a way to understand numbers less than zero. In fact, Glaeser (1981) referred to early views of negative integers as "fictive" or imaginary in nature with dual meaning: *position* and *transformation* (Thompson and Dreyfus 1988), *state* and *operator* (Glaeser 1981; Streefland 1996) or *unary* and *binary* (Vlassis 2008). Mathematicians also questioned, in particular, why negating a negative number should result in a positive quantity. Knowing these historical difficulties, we conjectured that these two conceptions would become prominent learning goals throughout our classroom learning trajectory.

3.3 Emergent models

The third heuristic involves helping students build models of their mathematical activity. Instructional activities should be sequenced in such a way as to encourage students to transition from reasoning with models-of their informal mathematical activity to models-for mathematical reasoning, also called emergent modeling (Gravemeijer and Stephan 2002). During the transition from informal to formal, the designer should support students' modeling by introducing teacher- or student-inspired tools, such as physical devices, inscriptions, and symbols that can be used by students to organize and explain their mathematical reasoning. Research has posited two different models for integers: *neutralization* and *number line* models, both of which attempt to build the unary and binary functions of the negative sign (Stephan and Akyuz 2012). The integer instructional sequence was designed such that students' reasoning with a vertical empty number line can evolve to become a model for formal addition/subtraction strategies for two digit numbers (Stephan and Akyuz 2012). Our inspiration for the vertical empty number line came from a previous classroom-DR project that used a horizontal empty number line to support first graders' development of linear-based, arithmetical (Cobb et al. 1997).

3.4 RME and social constructivism

Even though RME is a theory of instructional design, the view that mathematics is a human activity of organizing the world mathematically is completely consistent with the tenets of social constructivism, the view of learning that serves as the basis for our analysis of data from classroom-DR. The emergent perspective (Cobb and Yackel 1996), a version of social constructivism, treats learning as *both* an individual construction as well as socially situated within the classroom dynamics. RME is well suited to classroom learning trajectories that use the emergent perspective as the basis because the designer uses hypotheses about both individual students' strategies (individual) and the mathematics that becomes taken-as-shared in the classroom community (social) to develop the hypothetical learning trajectory. Simon (1995) echoes this by arguing that learning is inherently social in nature; therefore, the teacher can make predictions about the direction of a trajectory but the trajectory itself is socially negotiated, influenced by the culture in which it is being created. As a consequence, the *realized* trajectory is co-created by the teacher and students in action and typically differs from the *anticipated* trajectory.

The socially situated nature of learning trajectories has been underplayed in recent discussions of learning trajectories. What makes this learning trajectory different from those rooted in radical constructivism is the equal attention paid to social aspects of supporting mathematical development. A classroom learning trajectory involves making conjectures about the mathematical ideas that become taken-as-shared and individuals' ways of participating in and contributing to them (i.e., the mathematical practices of the class and the diversity in students' individual reasoning). Conjectures about this emergent trajectory include the mathematical goals and tool use as students engage with the instructional tasks. Additionally, the trajectory attempts to outline the supports that the teacher may draw on to aid students along a learning route. After implementation, the designer analyzes the results of student learning to determine the actual learning trajectory that was created (i.e., the classroom mathematical practices and diverse reasoning) and revises the instructional sequence accordingly. Another classroom experiment occurs with a newly revised, hypothesized learning trajectory (HLT) and the results feedback to inform future implementations (Stephan and Cobb 2013).

4 The teachers and context

The teachers in this study had been working towards teaching practices consistent with what Lampert et al. (2010, 2013) call *ambitious teaching*. To teach ambitiously, teachers should aim to help all students, regardless of background or ability, to understand and use mathematical knowledge, and to use it to solve authentic problems. Ambitious teaching would involve identifying clear objectives, eliciting students' thinking and representing it on the board, and using student reasoning to plan instruction. To support ambitious teaching practices to be adapted successfully by teachers at large scale, Cobb and Jackson (2011, 2015) contend that the teachers must be members of a coherent instructional system which exhibits seven characteristics. The teachers and researcher/teacher involved in this study all worked at the same Florida, suburban middle school whose mathematics department had embodied five of Cobb and Jackson's seven characteristics: setting explicit goals for students' mathematical learning, creating a detailed vision of ambitious teaching, implementing instructional materials aligned with ambitious teaching practices, working as a community of learners that met every day during planning and crafting common assessments aligned with ambitious teaching practices (Stephan et al. 2012). These characteristics of the school's mathematics department had been initiated at the behest of the principal and supported by a mathematics coach with over 10 years' experience developing and implementing ambitious teaching practices.

The project team consisted of three 7th grade teachers, myself (also a teacher) and a doctoral student from the local university. We had planned together since the beginning of the year but decided to engage in a formal Classroom-DR experiment for our unit on integers. Of the three teachers, only one of them could be considered a veteran with ten years' experience as a special educator. Each of the other two teachers had taught in public school no more than 4 years, with one of the teachers in her first year. At the time, I had taught 4 years, full-time for the middle school, but had been a researcher at Purdue University Calumet for 5.5 years prior to leaving the University setting to teach public school. As such, I served as both a teacher and researcher who had a vast amount of experience in conducting Design Research (cf. Rasmussen et al. 2004; Stephan et al. 2003). I had taught 2 years with the special educator and we had begun creating the integer instructional sequence that our research team decided to investigate formally. In this way, the integer instructional sequence attempted to engage all students, including those with special educational needs (cf. Prediger and Krägeloh 2015). In the remainder of this article, I use the term *teacher* to refer to the classroom teachers and *researcher/teacher* to refer to myself to avoid confusion, even though I consider each person of the research team to be teacher researchers. The doctoral student, for her part, collected data in the form of video tape, field notes, collection of students' work, audio tape of all research meetings and audio-taped interviews with one of the teachers before and after teaching each day.

The project research questions only involved students' learning; therefore, no explicit data were collected on the teachers' roles. However, we were able to use the audio tapes of all, daily debrief sessions to document the roles of the teachers. Additionally, to prepare for a presentation at the University of Georgia on the topic of design research, our teachers filled out a questionnaire that asked them about the artifacts and practices they found most useful in Design Research. These data were then analyzed using a constant comparison method (Glaser and Strauss 1967).

5 Phases of classroom design research

The Integer Classroom-DR took place over a total of 5 years with the most formal cycle occurring in year 3. In the other cycles, we did not collect as much formal data because we did not have the resources as classroom teachers. In this section, I use the data collected from year three to discuss the activities associated with each of the three Design Research Phases. I also make reference to the work of classroom teachers as they participated as full members of the research team.

5.1 Phase one: preparation/design

The initial phase of Classroom-DR involved finding an experientially real context that was engaging and motivating for our 12–14-year-old students and hypothesizing a classroom learning trajectory. Since I had the most experience with Classroom-DR, I took the lead in this phase with the other teachers taking a secondary role that will be explained later. I began by conducting a thought experiment in which I imagined students working with various amounts of debts and assets to determine a person's net worth. I imagined that students would relate easily to debts, especially in a period of economic hardships in the United States. Furthermore, I conjectured that transactions with assets and debts (e.g., taking away a \$50 asset or taking away a \$50 debt) would be experientially real to students and could serve as the basis for making sense of symbolic manipulations, like—(+\$50) or—(−\$50). For their part, the teachers agreed that the context would be experientially real for their students and they could begin to imagine conversations rooted in this context.

The instructional activities would be designed to pique students' interest in finances, and would be sequenced so that students' reinventions would move from the more concrete situational imagery of assets and debts to the abstract symbol manipulation. I also conjectured that a vertical, empty number line might serve as a productive reasoning device to help students structure their activity with integers. Furthermore, I tried to imagine the quality of the mathematical discussions that a teacher might try to support as students solved the integer tasks. The classroom learning trajectory that served as the backbone for this study is shown in Table 1 and is organized into five categories (Stephan and Akyuz 2012): The tools, imagery, activity/taken-as-shared interest, possible topics of mathematical discourse, and possible gestures and metaphors (Rasmussen et al. 2004) that would support students' learning of integer operations. The instructional sequence is organized into five segments to delineate the proposed shifts in mathematical thinking that we attempted to support within the classroom.

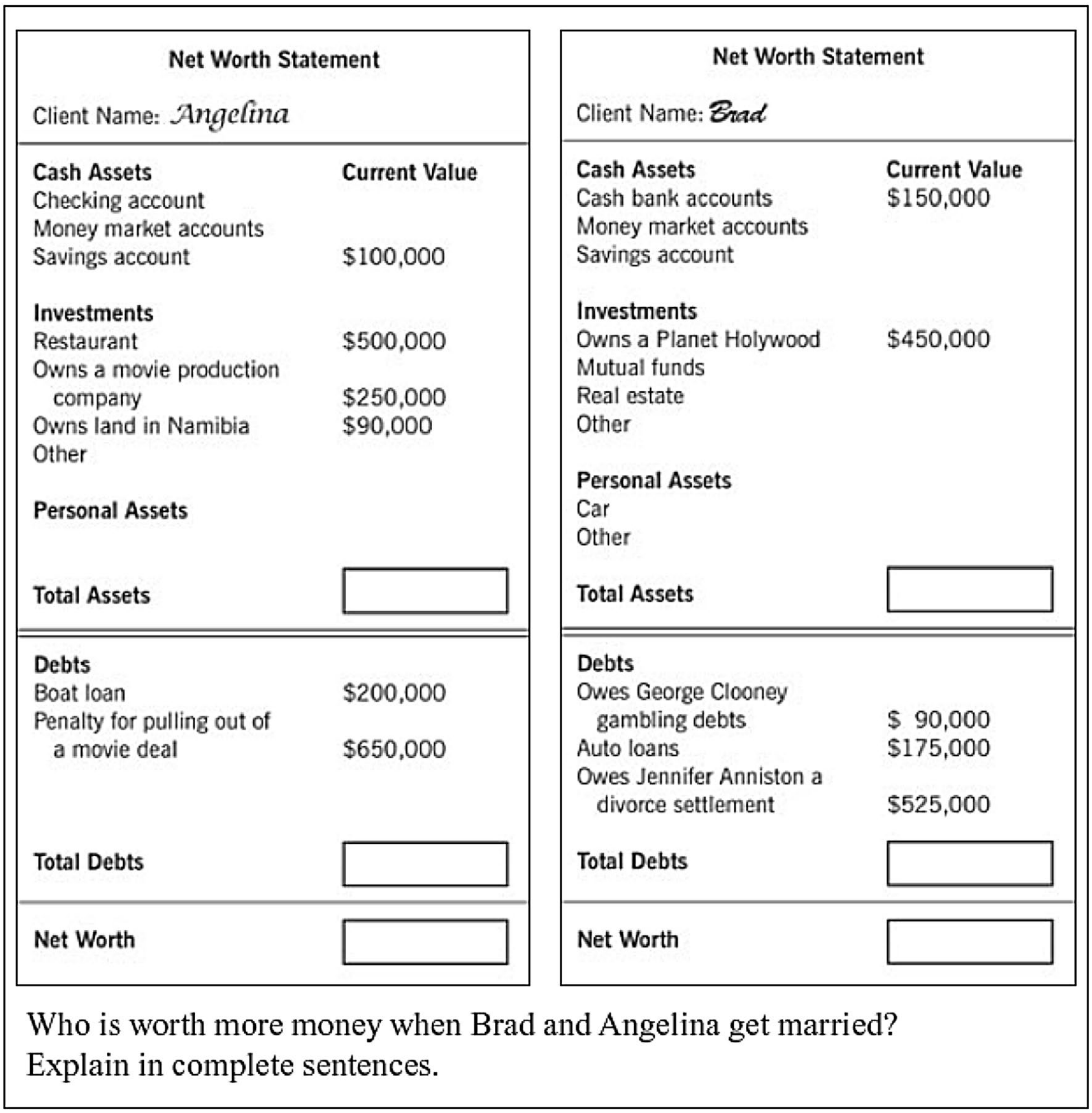
Table 1 The proposed classroom learning trajectory

Segment	Tool	Imagery	Activity/taken-as-shared interests	Possible topics of mathematical discourse	Possible gesturing
One	Net worth statement template	Assets and debts are quantities that have opposite effect on net worth	What does net worth mean	Conceptualizing that an asset is something you own and debt you owe; net worth is a relation between total assets and total debts; net worth is abstract, intangible	
Two	Net worth statements	Differences in collections of assets and collections of debts	Determining a person's net worth; comparing net worths	Inventing different strategies for determining a net worth	
Three	Symbols (+ and −)	+ means asset and − means debt	Determining net worth; transactions on net worths	Inventing different strategies for determining a net worth; creating additive inverses as mathematical objects; how do various transactions affect net worth	Arms moving up or down to indicate positive or negative affects
Four	Net worth tracker (vertical number line)	Movements along a vertical number line	Reasoning with a net worth tracker to determine results of transactions	How do various transactions affect net worth; going through zero; Inventing different strategies for determining a net worth	Up and down movement with arms
Five	Coffee spill net worth statements		Determining different transactions yielding same result; reinventing integer rules	Inventing integer rules $+(+)=+$ $-(-)=+$ $+(-)=-$ $-(+)=-$	

Before the Implementation Phase, the research team met to discuss the hypothesized classroom learning trajectory. Each person had a copy of Table 1 and we began to discuss the rationale behind it. In the next sections, I talk through only a small portion of these discussions to illustrate the types of conversations that occurred during the Design Phase.

5.1.1 Launching the trajectory

In segment *one*, the teacher would begin the sequence by grounding students' activity within the realistic context of finance by asking students if they have ever heard of Oprah Winfrey (an American celebrity) and what they think she is worth. We expected students to respond that she has a lot of cash in the bank, she owns jets, a school in Africa, Harpo Studios, and so on. We did not expect them to mention that she probably owes money, so the teacher would ask about Oprah's possible debt. Most students would respond that she has mortgages on her jets, loans from the bank to finance her studio and her school and other examples. Students would then be given an actual net worth statement template and asked to search for financial terms that they recognize. They would then be asked to describe their informal definitions of net worth.



Net Worth Statement

Client Name: *Angelina*

Cash Assets	Current Value
Checking account	
Money market accounts	
Savings account	$100,000
Investments	
Restaurant	$500,000
Owns a movie production company	$250,000
Owns land in Namibia	$90,000
Other	
Personal Assets	
Total Assets	
Debts	
Boat loan	$200,000
Penalty for pulling out of a movie deal	$650,000
Total Debts	
Net Worth	

Net Worth Statement

Client Name: *Brad*

Cash Assets	Current Value
Cash bank accounts	$150,000
Money market accounts	
Savings account	
Investments	
Owns a Planet Holywood	$450,000
Mutual funds	
Real estate	
Other	
Personal Assets	
Car	
Other	
Total Assets	
Debts	
Owes George Clooney gambling debts	$ 90,000
Auto loans	$175,000
Owes Jennifer Anniston a divorce settlement	$525,000
Total Debts	
Net Worth	

Who is worth more money when Brad and Angelina get married? Explain in complete sentences.

Fig. 2 Net worth comparison activity

Fig. 3 Two types of instructional activities for determining net worth

```
Bank Balance: -$100
Investment in Energy Efficient Fuel:
+$20,000

Organic Sweet Potato Farm: +$5000

Stock Market Loss on Mushrooms that
power cars: -$20,000

     Net Worth:$
```

```
+$20000
-$20000
+$7000
-$2000
-$5000


Net Worth
```

5.1.2 *Comparing net worths*

We conjectured that students must first understand how assets and debts work together quantitatively to form a person's net worth before they could understand how operations with integers work in this context. Segment *two* attempts to build on this finance imagery by having students compare one net worth to another (see Fig. 2). We expected most students to sum the assets, sum the debts and then attempt to take the difference between the two in some way. Topics that might arise in discourse involve discussions about the "fictive" nature of a negative net worth (Glaeser 1981); in other words, how can a person have less than $0. Students might wrestle with the idea that most assets are tangible (e.g., a motorcycle) but debts are not (e.g., a mortgage). These mathematical ideas can arise from the students themselves or with teacher questioning.

It is worth noting at this point, that when we discussed these tasks, we anticipated both the ways students might reason through the Brad and Angelina problem (individual perspective) as well as the mathematical ideas that might arise in the public discourse as supported by the teacher (social perspective). In this way, the classroom learning trajectory involves the coordination of the individual perspective (the conjectured reasoning of a variety of students) and the social perspective (the mathematical ideas that might become taken-as-shared in discourse).

As the learning trajectory continues (segment *three*), there are more tasks of a similar nature, but the symbolizing changes so that the net worth statements listed assets and debts in random order with only words and positive/negative signs (see Fig. 3).

The research team anticipated that students would create different strategies for finding net worths, such as (a) adding assets and debts one at a time until finished and (b) finding the *total* assets then *total* debts, and the *difference* between them. With any luck, some students might notice that −$20,000 and +$20,000 combine to $0. If so, we could capitalize on this contribution to introduce additive inverses. Nevertheless, the teacher could expect to lead a discussion in which students express these three strategies and decide on the efficiency of them.

5.1.3 *Modeling with a vertical empty number line*

In segment *four*, a vertical empty number would emerge as a tool that students use to record the net worths of individuals (see Fig. 4). The intent is that students begin to structure the space between two integers as a positive distance and to use an empty number line to visually and mentally structure the relationship between integer quantities. Additionally, we hypothesized that students might invent a going-through-zero strategy to aid in their structuring activity (Peled et al. 1989). Whole class discussions would focus both on the difference as well as students' strategies for finding it.

Space constraints restrict me from discussing the remaining segments in great detail, so I refer the reader to Stephan and Akyuz (2012) to read more about how the conjectured design progressed. In brief, student conversations revolved around the position of integers on the number line (*position*) versus the numbers alongside the up and down arrows (*transformation*, e.g., goes down $1,000). Instructional activities moved progressively toward reasoning with number sentences posed simply as 1,000 − (+2,000) = ?

5.1.4 *Team member roles during the design phase*

In conducting Classroom Design Research with a group of teachers, the lead designer/researcher with expertise in instructional design takes responsibility for drawing up a hypothesized classroom learning trajectory. Although I did this in isolation from the other teachers, it need not be done this way. However, with the lack of design expertise, the classroom teachers preferred that I take sole leadership with the initial learning trajectory. The other team members, for their part, read selected research articles and discussed/questioned the hypothetical learning trajectory as I presented a sketch of my ideas (Table 1) prior to our implementation. Additionally, one of the team members participated in cognitive interviews of 20 students to gauge the viability of the instructional sequence's starting point. The teachers' main activity in the design phase was to analyze and question the designer on the elements of the classroom learning trajectory as presented to them in Table 1 as well as the rationale for designed activities. During the implementation phase next, teachers continued this role but were able to participate more effectively since they were able to see how the hypothesized trajectory actually played out with their own students.

5.2 Implementation phase

The team implemented the hypothetical learning trajectory for integers in each of their own classrooms (over 150 students) over a period of 6 weeks. We met almost daily to analyze how students engaged with the tasks that day and create revisions that needed to be made immediately (mini-cycles of design, implementation and analysis of student thinking). All meetings were audio-taped to document improvements to the design that were made in real time.

Similar to the activity of researchers in this phase, the teachers chose the task(s) to be used in class, anticipated the diversity of ways in which students would engage, and imagined how the whole class discussion would ensue, a practice we named *lesson imaging* (Schoenfeld 2000). In fact, the teachers rated collaborative lesson imaging as the most important activity in which they engaged during the project. They made a distinction between lesson imaging and lesson planning, with the latter being the act of scheduling which activities to do throughout the week. Lesson imaging began with a recap of the mathematical ideas that emerged in their classes that day which often led to revisions to the instructional sequence in real time. For example, we discovered that opportunities to introduce the vertical number line emerged from students much earlier than anticipated in the HLT in Table 1. This opportunity arose for the first time as early as day 3 when students were trying to justify how a person's net worth was −400 when she had a total of $8,000 in assets and $8,400 in debts. Students argued that this person must use all $8,000 to "pay off" her debts as much as she can and then she would still be in debt $400. The teachers capitalized on these arguments and recorded them on a vertical number line. We then created new tasks posed on a vertical number line to support students' invented strategies for finding differences between two integers (Stephan and Akyuz 2012).

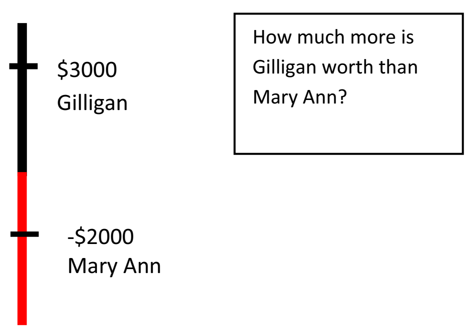


Fig. 4 A vertical empty number line task

One of the mathematical ideas that continually came up from teachers was the idea that net worth is an abstract quantity for students. For his part, once McDonald had constructed meaning for that idea, he continually pushed his students (and us) to make meaning of it as well. He would offer his own instructional activity (see Fig. 5) that helped students understand that net worth represents nothing concrete, but rather an abstract "worth" of a person.

McDonald's task deliberately contains three different interpretations of net worth that had come up in student conversations and asks students to explain which students' interpretation they agree with and why. In this way, McDonald can be seen to contribute to the *realized* classroom learning trajectory as his task was included in the revisions to the sequence. Teachers brought their knowledge of mathematics as well as pedagogical content knowledge (Shulman 1986) to bear during these daily debriefs. For example, the first year teacher, Dunham, typically questioned the rationale for certain activities and what mathematical conversations might emerge. Such a role is sensible since first year teachers find it very difficult to anticipate how students might reason through tasks. The more experienced mathematics teacher and special educator often asked questions about the learning trajectory (e.g., what does it mean that net worth is an abstract quantity?) and anticipated supportive mathematical imagery. For example, consider a conversation that occurred a few days into implementation in which the experienced teacher and

Fig. 5 McDonald's activity to assess students' meaning of net worth

Spongebob	*Mr. Krabs*
Total Assets: $14,000	*Total Assets: $16,000*
Total Debts: $14,500	*Total Debts: $15,900*

Sam Said: Spongebob is worth more because his net worth is $500.00 and Mr. Krabs' net worth is only $100.00. So, Spongebob is worth more.

Sid Said: Mr. Krabs is worth more because he paid his debts off and still has some leftover. Spongebob paid off everything he could and now has no money and still owes $500.00.

Sue Said: Spongebob is worth more because Mr. Krabs has more debt than Spongebob.

Who do you agree with and why? Write a short paragraph explaining your choice.

special educator offered the metaphor of payoff as effective for later inscriptional reasoning.

McDonald: When you say she has lower net worth, it means net worth is worse because she has assets, she can cash them all and *pay off* all debts and still be in debt… I wonder if there is anything important, mathematically speaking in *paying it off*?

Stephan: Two things I can come up with. One is when we get to integer operations and let's say we have $600,000 and pretend there is a transaction −$900,000…she goes into debt 300,000 dollars.

McDonald: Because 600,000 goes to zero on the number line, you have enough to go to zero but there is still more left over.

Star: Or I only have this much to pay off but I still owe this!

McDonald: That is all related with the number line idea we try to develop later [going through zero strategy].

The metaphor of *paying off* was introduced by McDonald as a potentially powerful, student-generated idea. The research team discussed its merit to decide if it was a line of reasoning that would be lucrative for later student reasoning. Deciding that it was, the research team capitalized on their own students who brought up *paying off*. In fact, *pay off* was such a critical student metaphor that it is now included as part of the classroom learning trajectory.

McDonald continually pushed for regular formal assessments and all teachers and researcher/teacher drew on their mathematical content knowledge to suggest real-time revisions of the instructional sequence. Consider the following conversation that occurred after McDonald questioned whether there were activities that support students' ordering integers.

McDonald: I want to be sure that when I put a problem up there, all the kids realize that zero is not same as negative numbers. Then they understand the abstract value; you can go below zero and it makes you go in more debt.

Akyuz: Are you going to make them compare negative numbers also?

McDonald: Yes… I think you cannot compare negative numbers until they conceptualize net worth as abstract value.

Stephan: Because if you cannot conceptualize anything below zero [as objects in their own right], you will have difficulty with that [ordering negatives]. I am really rethinking the order of sequence now. I think the page after this…we need a number line here. I really think so because even with *concept* of integer, forget *operations* right now, it is a big deal to order those numbers on the number line.

Dunham: I want to see order on the number line. I want to see that—4,000 is below—1,000 and I want them to see that, too.

Stephan: I think we need to have a page asking students to order a bunch of positive and negative numbers. And then we may ask how much more Juli's worth than Deanna. And then operations. This way, they can start making objects out of distances from zero and they might also start thinking about those pay off ideas again.

This excerpt shows the role that the teachers played in using their knowledge of mathematics to suggest revisions to the instructional activities and, consequently, classroom learning trajectory. An additional role that the special educator played was to question both the mathematics and the means of supporting students with special educational needs. For my part as the lead designer, I attended to the mathematical ideas that emerged in conversation to see how they aligned with what had been anticipated. Together, we made several revisions to the instructional sequence based upon our daily analyses of students' mathematical contributions in class.

5.3 Analysis/revision phase

The final phase of Classroom-DR is to analyze data that were formally collected during the implementation to determine the classroom mathematical (integer) practices and individual ways of reasoning that emerged (i.e., the *realized* classroom learning trajectory). Akyuz and Stephan took the lead in this analysis as they were highly trained in qualitative research methodology. As mentioned at the outset of this article, the learning of the class was reported in another article and was used to inform the revisions of the instructional sequence for the following school year.

6 Discussion

The original research questions for the Design team involved documenting the learning of both the community and individual students as well as the role that tools, gestures, metaphors and imagery played in that learning. We have documented the answers to those questions more fully elsewhere, but used this experiment to investigate another central question that occurred to us afterwards, *How do teachers participate in and contribute to Design Research*?

Our Classroom Design Research project, where the teachers outnumber the researchers, has important findings for design researchers. At the completion of our project, the research team presented aspects of our collaboration at a conference at the University of Georgia. Here, our teachers and researcher/teacher argued that there were three practices they learned from engaging in design research that can be effective in aiding teachers in their implementation of new or familiar instructional designs. The practices of classroom-based design research that were highly valued involved collaboration with both other teachers and researchers, lesson imaging and daily reflection. All three of these practices are inherent in design research in classroom settings and align with the type of community practices these teachers value. All teachers remarked that they would never have adopted these practices without their exposure to the design research methodology. This finding suggests the strong compatibility between design research and the practice of teaching, the strong link between theory and practice, and the importance of research that is situated in the classroom and that is relevant to the practice of teaching. Our findings are consistent with (cf. Gravemeijer and van Eerde (2009) who also claim that Classroom Design Research can be a profound professional development experience.

The teachers also argued that one of the most crucial artifacts for designing and/or choosing instructional materials was to be up-to-date on research in the area. In fact, one teacher, McDonald, routinely reads research to this day when he is preparing to teach a new mathematics topic. The artifact that was most valued by teachers was the instructional activities themselves. They remarked that if they ever lost their way in the classroom, they would always go back to the activities in the sequence to get a sense of the important mathematical ideas and direction to go. Interestingly, the absence of a teacher's manual was seen as beneficial for these teachers since it caused them to create their own lesson image, not read one from a book. Of course, the presence of the actual instructional designer can be seen as "better than the textbook." Even when adapting a commercial textbook series, the teachers reveal that they no longer look at the teacher's manual first; they wait until after they lesson image, as a way to gauge their understanding of the authors' intent *after* forming their own.

Lesson study and action research have similarities to the work we described in this article. Both of these programs are similar to our Classroom-DR experience in that teachers read research to prepare for their teaching, the experimentation is located in classroom settings, data are gathered and instruction is often altered as a result of collaborative planning and debriefing. The main differences involve the fact that Classroom-DR is not a professional development program. Our intention is to create theory that will be shared with wider audiences including the school as well as national and international audiences. Our work necessarily appeals to both the practitioner and the researcher, whereas the other programs typically involve practitioners. Another difference is that, although lesson study and action research can involve researchers, our approach necessitates that researchers will not only take part in the project but be present every day and perform more rigorous data analyses both in real time and post experiment. Finally, the scope of Classroom-DR is much longer than in traditional action research and lesson study whose focus is generally on a few lessons or class sessions.

While design research can impact teachers' professionalism, it is also enhanced by those same teachers in ways that are not possible by researchers (and a single teacher) alone. Since the team was comprised of three other teachers and a doctoral student, the instructional sequence benefited from a wide diversity of craft knowledge. Recall the input that teachers gave that led to the re-ordering of some parts of the instructional sequence. Their deep knowledge of not only the content, but also their knowledge of the best order for teaching integer concepts was instrumental in developing the final integer instruction theory. Additionally, the instruction theory was stronger in viability due to the fact that data, whether formal or informal, were being collected from over 150 students as interpreted by three different teachers, a researcher/teacher and a researcher. The instruction theory benefited from the expertise of the special educator who suggested changes to certain activities that would make them more accessible to students with special educational needs. Oftentimes, initial Classroom-DR is conducted in collaboration with one teacher in one classroom, yet working with many teachers at once draws on a wider range of input making an even stronger, viable theory much quicker.

7 Conclusion

Design research is usually motivated by university members with experience and interest in building theory and instructional designs. Typically, the teacher is considered as a member of the research team, with the primary responsibility of implementing instruction. However, in this chapter, I described a Classroom Design Research project that was conducted by a team comprised mostly of classroom teachers whose primary interests involved improving their teaching practice and students' mathematical reasoning. Their goal was to create a stable instructional unit for integer addition and subtraction that they could use to help students learn the topic with meaning. Even though I was a teacher at the time and had the same pragmatic goals, I led a Classroom Design Research on integers as a way to induct my colleagues into a different set of professional teaching practices as well as to build a theory of instruction for integer addition and subtraction. In this paper, I outlined the basic tenets of Classroom Design Research, arguing that it is comprised of numerous cycles of Design, Implementation, and Analysis/revision. I then discussed the instructional theory of Realistic Mathematics Education and how it guided the designer in the Design Phase of the research. Next, I described each of the three phases of the Research project using the integer experiment as the context. In doing so, I introduced the construct of a *classroom learning trajectory* which refers to the anticipated classroom mathematical practices that might evolve over the course of an instructional sequence as well as the ways in which the trajectory is supported in social context (e.g., imagery, tool use, discourse, gestures, teacher questioning, etc.). The classroom learning trajectory differs from mainstream learning trajectories in that it accounts for learning in social context and places importance on the collective discourse and tool use that support students' learning. Finally, I reflected back upon my discussion of the phases of Classroom Design Research taking into account the ways in which this methodology can be mutually beneficial for teachers and researchers that do not typically occur in traditional Classroom-DR teams. I claimed that the teachers embarked on a unique professional development experience, one in which they learned professional practices that supported a new way of preparing their instruction. Reciprocally, the teachers drew on their craft knowledge to contribute in meaningful ways to the enactment and stabilization of the integer instruction theory. Working with a small group of teachers rather than just one offers a unique opportunity for the researcher and resulting theory to be influenced more directly by the diversity of teacher's mathematical knowledge for teaching and over 150 students' mathematical work.

References

Battista, M. T. (1983). A complete model for operations on integers. *Arithmetic Teacher, 30*(9), 26–31.

Bofferding, L. (2010). Addition and subtraction with negatives: acknowledging the multiple meanings of the minus sign. In P. Brosnan, D. B. Erchick, & L. Flevares (Eds.), *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 703–710). Columbus: The Ohio State University.

Burkhardt, H. & Swan, M. (2013). Task design for systemic improvement: principles and frameworks (pp. 431–440). *Proceedings of the International Commission on Mathematical Instruction*, Oxford.

Clements, D., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning, 6*(2), 81–89.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307–334). Mahwah: Erlbaum.

Cobb, P. (2003). Investigating students' reasoning about linear measurement as a paradigm case of design research. In N. Pateman (Series Ed.), M. Stephan, J. Bowers, & P. Cobb (with K. Gravemeijer) (Vol. Eds.), *Journal for Research in Mathematics Education Monograph Series: Vol. 12. Supporting students' development of measuring conceptions: analyzing students' learning in social context* (pp. 1–16). Reston: National Council of Teachers of Mathematics.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13. doi:10.3102/0013189X032001009.

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symbolizing: the emergence of chains of signification in one first-grade classroom. In D. Kirshner & J. A. Whitson (Eds.), *Situated cognition: social, semiotic, and psychological perspectives* (pp. 151–233). Mahwah: Erlbaum.

Cobb, P., & Jackson, K. (2011). Towards an empirically grounded theory of action for improving the quality of mathematics teaching at scale. *Mathematics Teacher Education and Development, 13*(1), 6–33.

Cobb, P., & Jackson, K. (2015). Supporting teachers' use of research-based instructional sequences. *ZDM*—The International Journal on Mathematics Education, *47*(6) (this issue).

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*, 175–190.

Confrey, J., & Lachance, A. (2000). Transformative reading experiments through conjecture-driven research design. In A. E. Kelly & A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 231–266). Mahwah: Erlbaum.

Confrey, J., Maloney, A., Nguyen, K., Mojica, G., & Myers, M. (2009). Equipartitioning/splitting as a foundation of rational number reasoning. In M. Tzekaki, M. Kaldrimidou, & C. Sakonidis (Eds.), *Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 345–352). Thessaloniki: PME.

Corcoran, T., Mosher, F., & Rogat, A. (2009). *Learning progressions in science: an evidence-based approach to reform*. Philadelphia: Consortium for Policy Research in Education.

Daro, P., Mosher, F., & Cocoran, T. (2011). *Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction*. Philadelphia: Consortium for Policy Research in Education.

De Beer, H., Gravemeijer, K., & van Eijck, M. (2015). Discrete and continuous reasoning about change in primary school classrooms. *ZDM—The International Journal on Mathematics Education, 47*(6) (this issue).

Doorman, M., & Gravemeijer, K. (2008). Emergent modeling: discrete graphs to support the understanding of change and velocity. *ZDM—The International Journal on Mathematics Education, 41*, 199–211. doi:10.1007/s11858-008-0130-z.

Fosnot, C. T., & Dolk, M. (2005). "Mathematics" or "mathematizing"? In C. T. Fosnot (Ed.), *Constructivism: theory, perspectives and practice* (2nd ed., pp. 175–191). New York: Teachers College Press.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: D. Reidel.

Gallardo, A. (2002). The extension of the natural-number domain to the integers in the transition from arithmetic to algebra. *Educational Studies in Mathematics, 49*, 171–192. doi:10.1023/A:1016210906658.

Garcia, J. & Ruiz-Higueras, L (2013). Task design within the anthropological theory of the didactics: study and research courses for pre-school (pp. 421–430). In *Proceedings of the International Commission on Mathematical Instruction*, Oxford.

Glaeser, G. (1981). Epistémologie des nombres relatifs. *Recherches en Didactique des Mathématiques, 2*, 303–346.

Glaser, B., & Strauss, A. (1967). *The discovery of grounded research: strategies for qualitative research*. New York: Aldine De Gruyter.

Gravemeijer, K. (1994). *Developing realistic mathematics education*. Utrecht: CD Press.

Gravemeijer, K. (2004). Local instruction theories as a means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105–128.

Gravemeijer, K., & Stephan, M. (2002). Emergent models as an instructional design heuristic. In K. Gravemeijer, R. Lehrer, B. van Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and* *tool use in mathematics education* (pp. 145–169). Dordrecht: Kluwer.

Gravemeijer, K., & van Eerde, D. (2009). Design research as a means for building a knowledge vase for teachers and teaching in mathematics education. *The Elementary School Journal, 109*(5), 510–524.

Koichu, B., Zaslavsky, O., & Dolev, L. (2013). Effects of variations in task design using different representations of mathematical objects on learning: a case of a sorting task. In C. Margolinas (Ed.), *Task design in mathematics education* (pp. 461–470). Oxford: Proceedings of the International Commission on Mathematical Instruction.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M.K. Stein, & L. Kucan (Eds.), *Instructional explanations in the disciplines.* doi:10.1007/978-1-4419-0594-9_9. New York: Springer, LLC.

Lampert, M., Franke, M., Kazemi, E., Ghousseini, H., Turrou, A., Beasley, H., et al. (2013). Keeping it complex: using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education, 64*(3), 226–243. doi:10.1177/0022487112473837.

Lappan, G., Fey, J., Fitzgerald, W., Friel, S., & Phillips, E. (2002). *Connected mathematics*. Upper Saddle River: Prentice Hall.

Linchevski, L., & Williams, J. (1999). Using intuition from everyday life in "filling" the gap in children's extension of their number concept to include the negative numbers. *Educational Studies in Mathematics, 39*, 131–147. doi:10.1023/A:1003726317920.

Lytle, P. A. (1994). Investigation of a model based on the neutralization of opposites to teach integer addition and subtraction. In J. P. da Ponte & J. F. Matos (Eds.), *Proceedings of the 18th international conference for the psychology of mathematics education* (Vol. III, pp. 192–199). Lisbon: University of Lisbon.

Peled, I., Mukhopadhyay, S., & Resnick, L. B. (1989). Formal and informal sources of mental models for negative numbers. In G. Vergnaud, J. Rogalski, & M. Artigue (Eds.), *Proceedings of the 13th international conference for the psychology of mathematics education* (Vol. III, pp. 106–110). Paris: PME.

Prediger, S. & Krägeloh, K. (2015). Low achievers learning to crack algebraic word problems—a design research project for aligning a strategic scaffolding tool to students' mental processes. *ZDM—The International Journal on Mathematics Education*, *47*(6) (this issue).

Rasmussen, C., Stephan, M., & Whitehead, K. (2004). Classroom mathematical practices and gesturing. *Journal of Mathematical Behavior, 23*, 301–323.

Schoenfeld, A. (2000). Models of the teaching process. *Journal of Mathematical Behavior, 18*(3), 243–261.

Schwarz, B. B., Kohn, A. S., & Resnick, L. B. (1993/1994). Positives about negatives: a case study of an intermediate model for signed numbers. *The Journal of the Learning Sciences*, *3*, 37–92. doi:10.1207/s15327809jls0301_2.

Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher, 75*(2), 4–14.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114–145. doi:10.2307/749205.

Smith, J. P. (1995). *The effects of a computer microworld on middle school students' use and understanding of integers* (unpublished doctoral dissertation). The Ohio State University, Columbus. http://rave.ohiolink.edu/etdc/view?acc_num=osu1248798217.

Steffe, L., & Olive, J. (2010). *Children's fractional knowledge*. New York: Springer.

Steffe, L. P., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: philosophy, theory, and application*. New York: Praeger.

Stephan, M., & Akyuz, D. (2012). A proposed instructional theory for integer addition and subtraction. *Journal for Research in Mathematics Education, 43*(4), 428–464.

Stephan, M., Akyuz, D., McManus, G., & Smith, J. (2012). Conditions that support the creation of a middle-school mathematics communities of learners. *NCSM Journal of Mathematics Education Leadership, 14*(1), 19–27.

Stephan, M., Bowers, J., Cobb, P., & Gravemeijer K. (Eds.). (2003). *Journal for Research in Mathematics Education Monograph Series: Vol. 12. Supporting students' development of measuring conceptions: analyzing students' learning in social context* (N. Pateman, Series Ed.). Reston: National Council of Teachers of Mathematics.

Stephan, M. & Cobb, P. (2013). Teachers engaging in integer design research. In T. Plomp & N. Nieveen (Eds.), *Educational design research: introduction and illustrative cases* (pp. 277–298). Enschede: SLO (Netherlands institute for curriculum development).

Streefland, L. (1996). Negative numbers: reflections of a learning researcher. *Journal of Mathematical Behavior, 15*, 57–79. doi:10.1016/S0732-3123(96)90040-1.

Thompson, P. W., & Dreyfus, T. (1988). Integers as transformations. *Journal for Research in Mathematics Education, 19*, 115–133. doi:10.2307/749406.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (in press). Realistic mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Dordrecht: Springer.

Vlassis, J. (2008). The role of mathematical symbols in the development of number conceptualization: the case of the minus sign. *Philosophical Psychology, 21*, 555–570. doi:10.1080/09515080802285552.