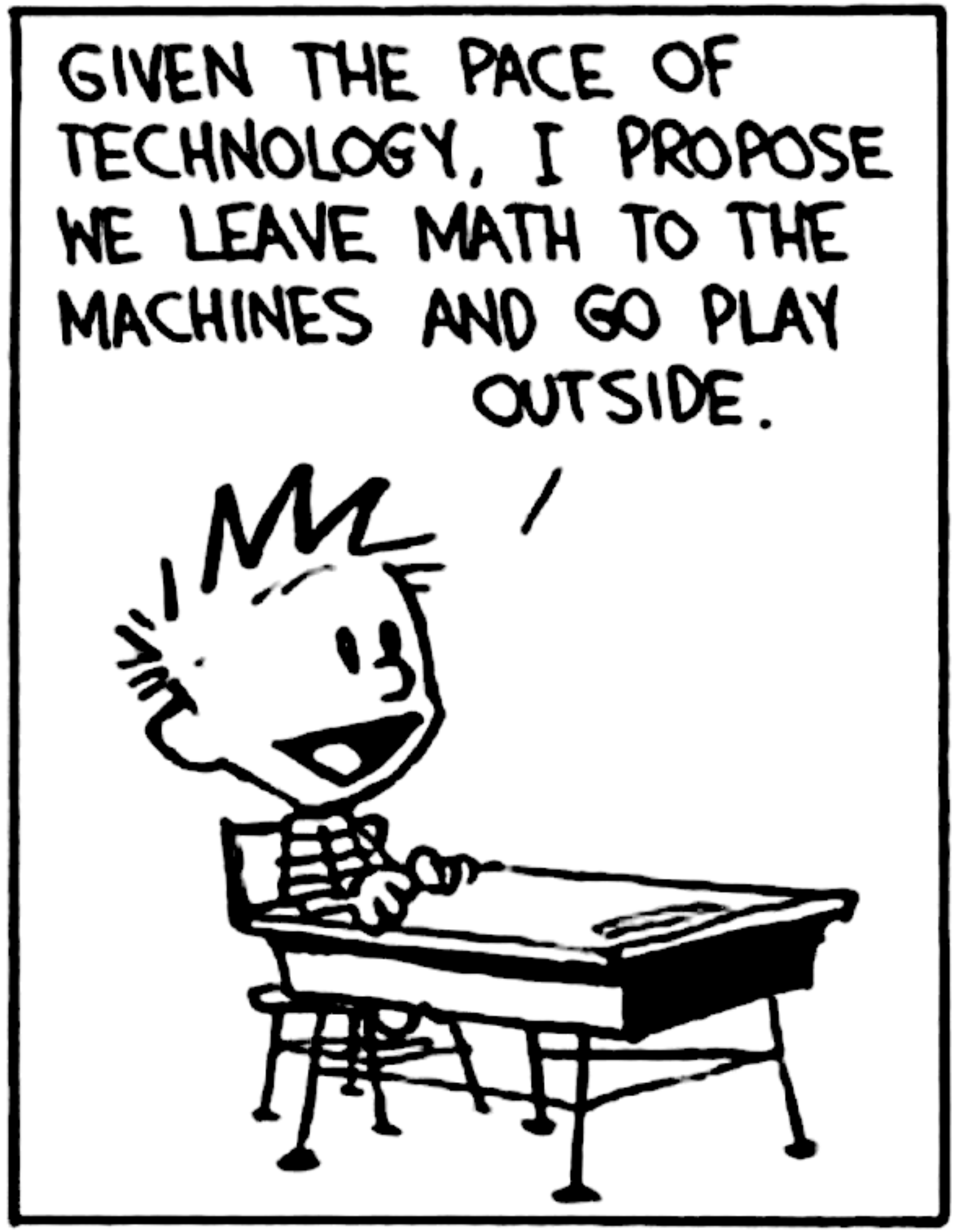
Mediation and Moderation



02/10/2020

Logistics

Things that came up

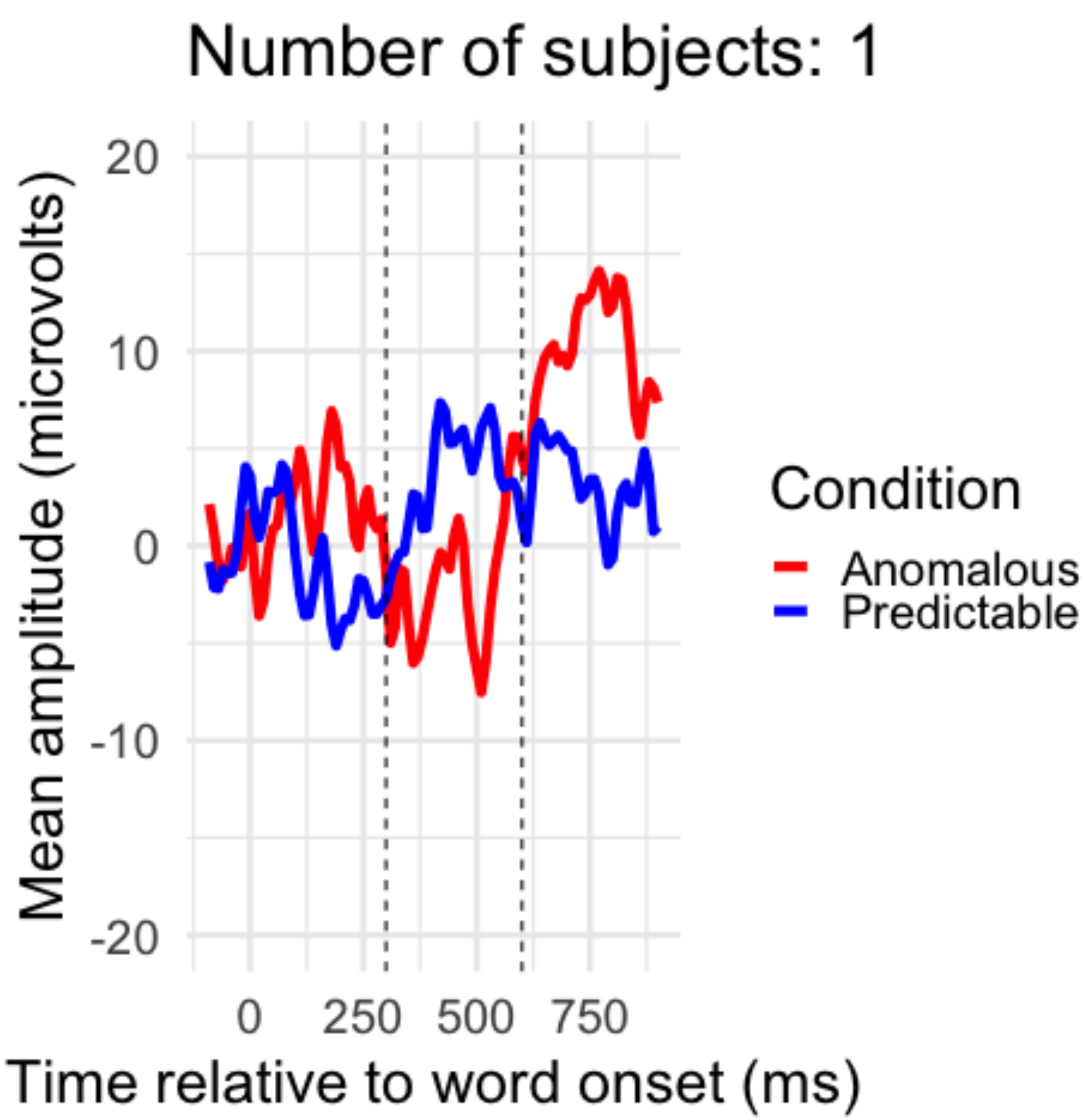
Great use of `gganimate`

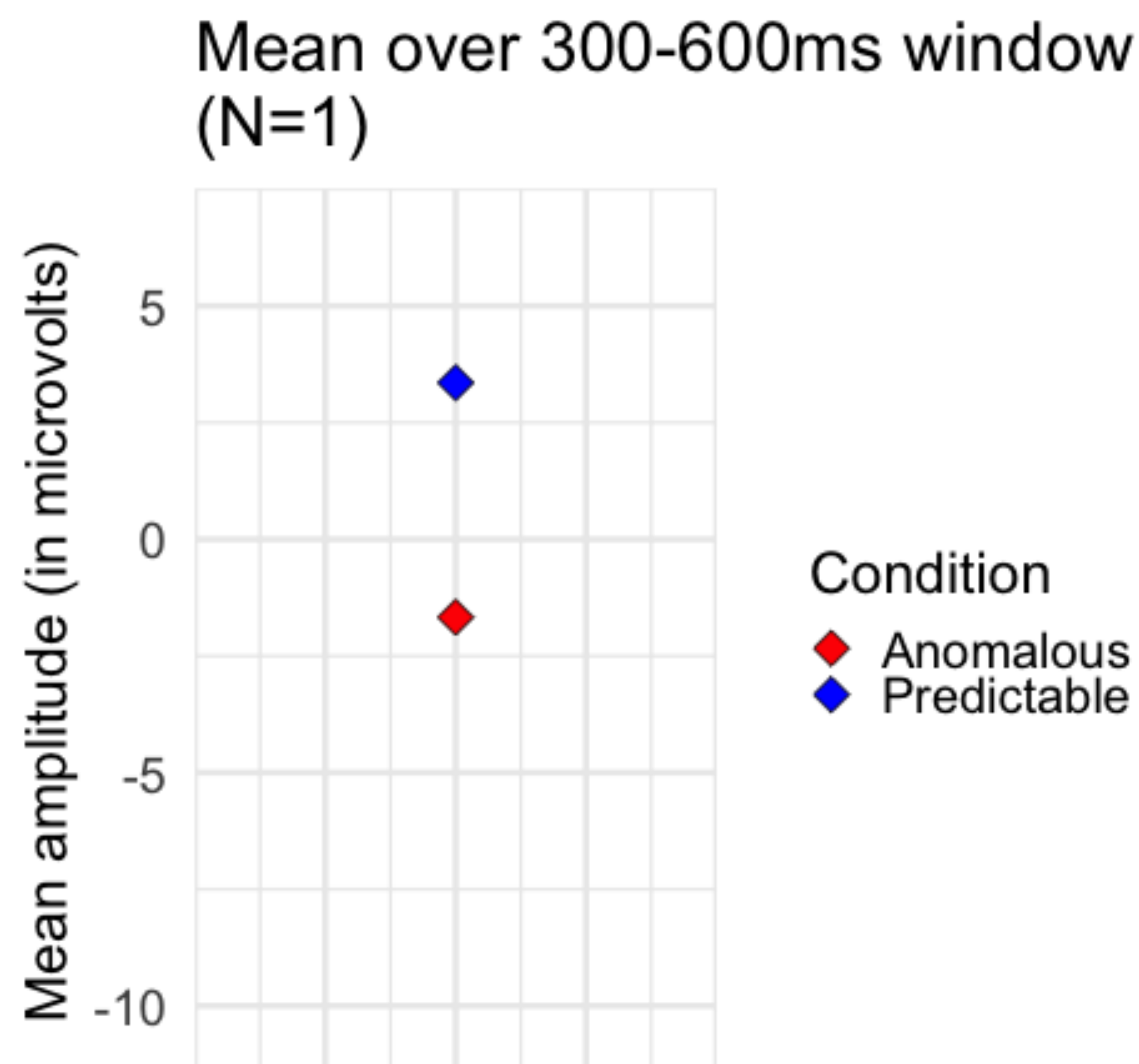
Rachel A. Ryskin

`ryskin@mit.edu`

Curriculum Vitae

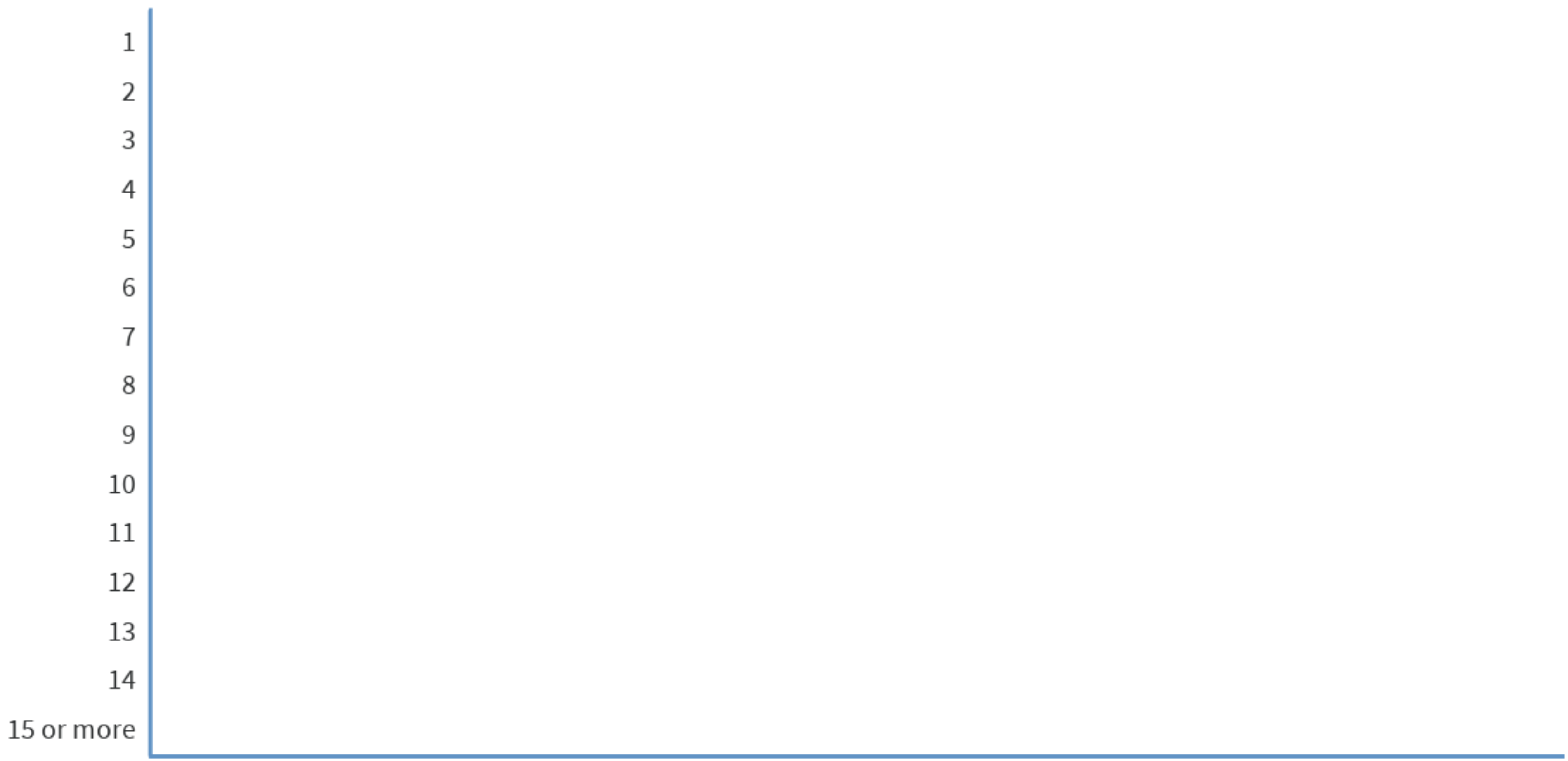
Google Scholar | GitHub | Twitter





Homework 4

How many hours did it take you to complete Homework 4?



Midterm

2 small errors in earliest release

if you downloaded the midterm before 3pm

Question 12

Question 14: Make two residual plots (for the model fitted in Question 5). One that shows model predictions on the x-axis and residuals on the y-axis, and another that shows a density plot of the residuals. Do these residual plots look ok? Describe briefly what to look for when checking the model assumptions.

```
# YOUR CODE HERE
```

Your answer:

Question 12

Question 15: Compare the augmented model (which includes whether a person has any kids as a predictor) with the model you've fit in (Question X). Report the results with PRE as your effect size.

```
# YOUR CODE HERE
```

Your answer:

Feel free to add packages

Load libraries

```
library("knitr")       # for knitting
library("pwr")         # for power analysis
library("broom")       # for tidying model fits
library("effectsize") # for effect size measures
library("tidyverse")  # for everything else
```

code chunk for question 19

Question 19 (4 points): What could be a reason that age is not a significant predictor of life satisfaction when controlling for the number of kids? Give a brief explanation.

```
# YOUR CODE HERE
```

Your answer:

not necessary to use this code chunk

Plan for today

- Questions? Answers!
  - degrees of freedom
  - t-test vs. permutation test
  - overlapping confidence intervals
- Power analysis (continued)
- Controlling for variables
- Mediation
- Moderation

Questions? Answers!

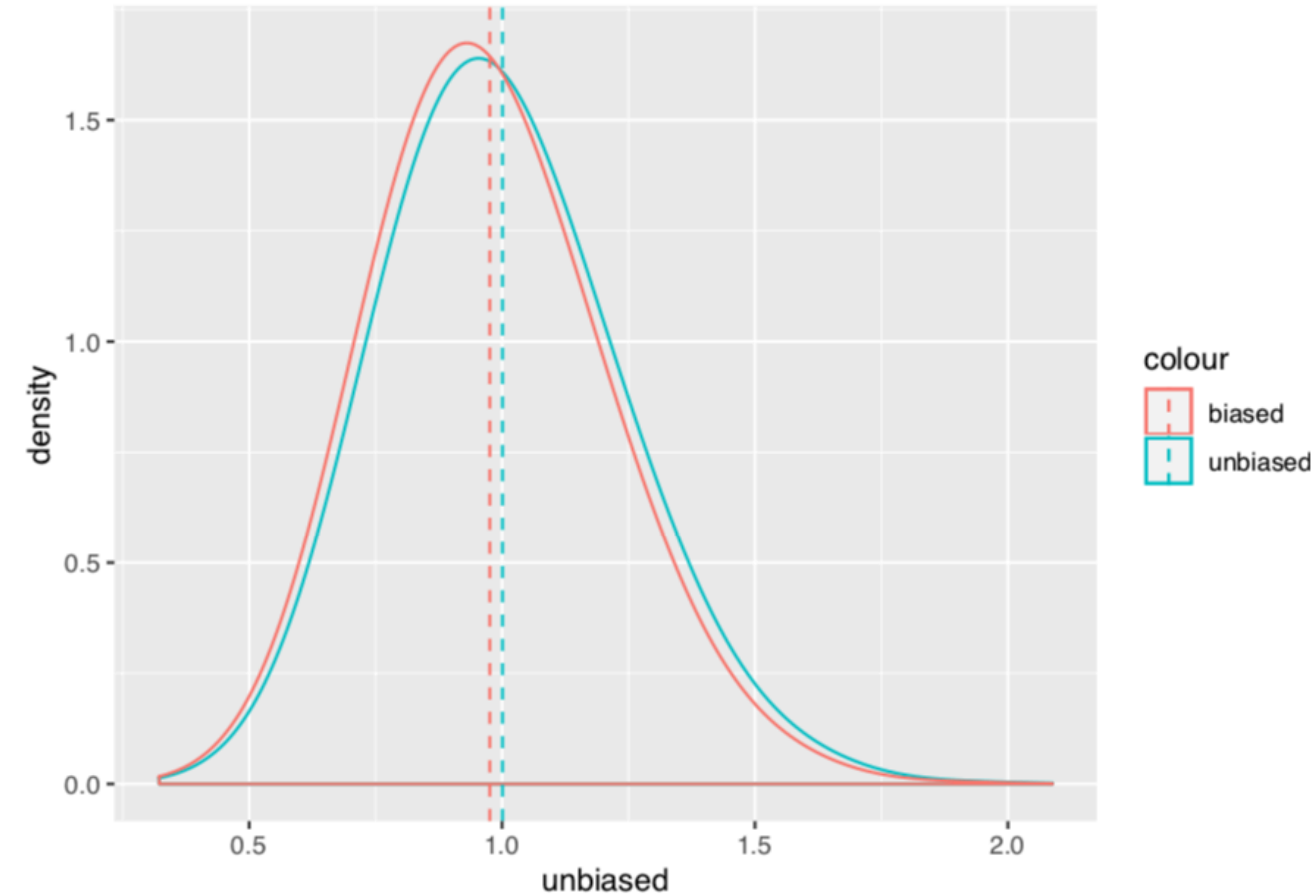
degrees of freedom

degrees of freedom

$$sd(Y) = \sqrt{\sum_{i=1}^{n} \frac{(Y_i - \overline{Y})^2}{\boxed{n-1}}}$$

Why?

makes sure that the sample standard deviation is an unbiased estimator of the population standard deviation




```r
# 7. Calculate which of the two measures s^2 or s'^2 is a better
#    estimator of the true population variance
sample_variances %>%
  summarize(error.unbiased = population_variance - mean(unbiased),
            error.biased = population_variance - mean(biased))
```

```
## # A tibble: 1 x 2
##   error.unbiased error.biased
##            <dbl>        <dbl>
## 1      -0.000786       0.0242
```

degrees of freedom

$$sd(Y) = \sqrt{\sum_{i=1}^{n} \frac{(Y_i - \boxed{\bar{Y}})^2}{n-1}}$$

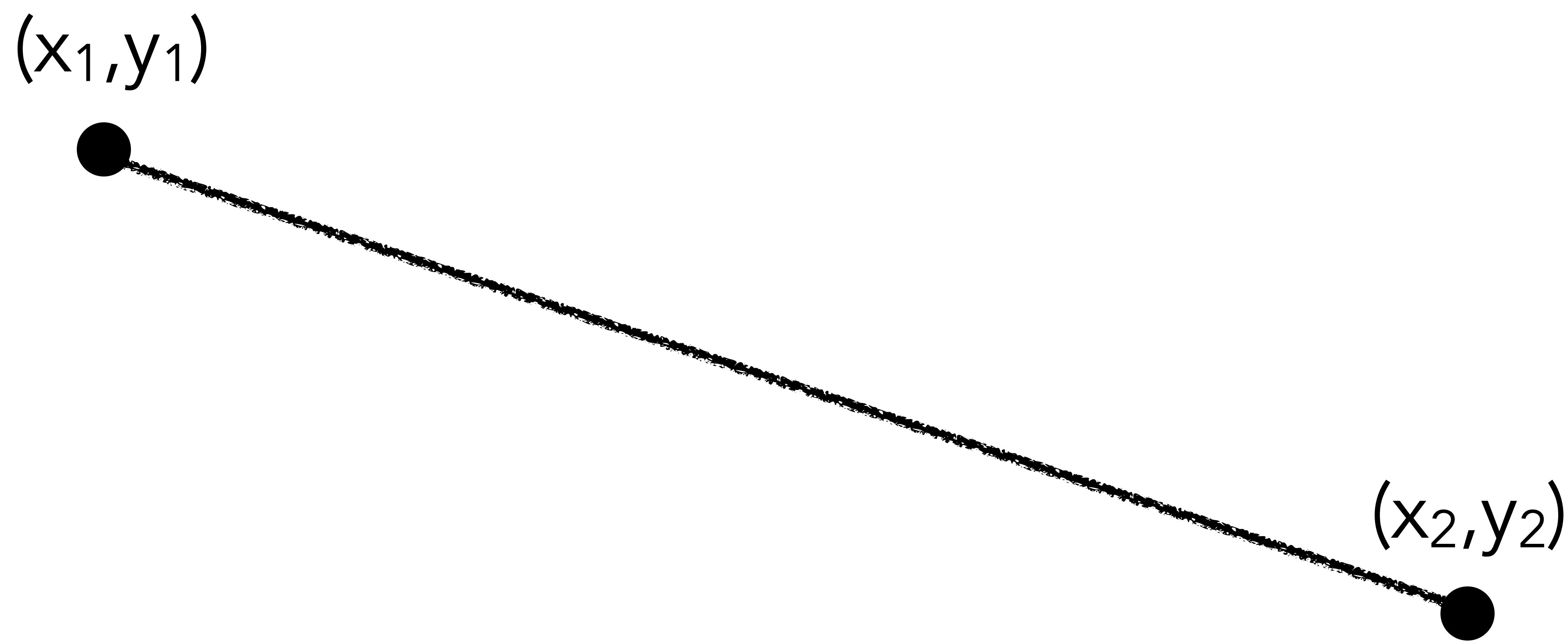
estimating the population mean

In statistics, the number of degrees of freedom is the **number of values** in the final calculation of a statistic **that are free to vary**.

https://en.wikipedia.org/wiki/Degrees_of_freedom_(statistics)

degrees of freedom

How many degrees of freedom does a **line** have?



degrees of freedom

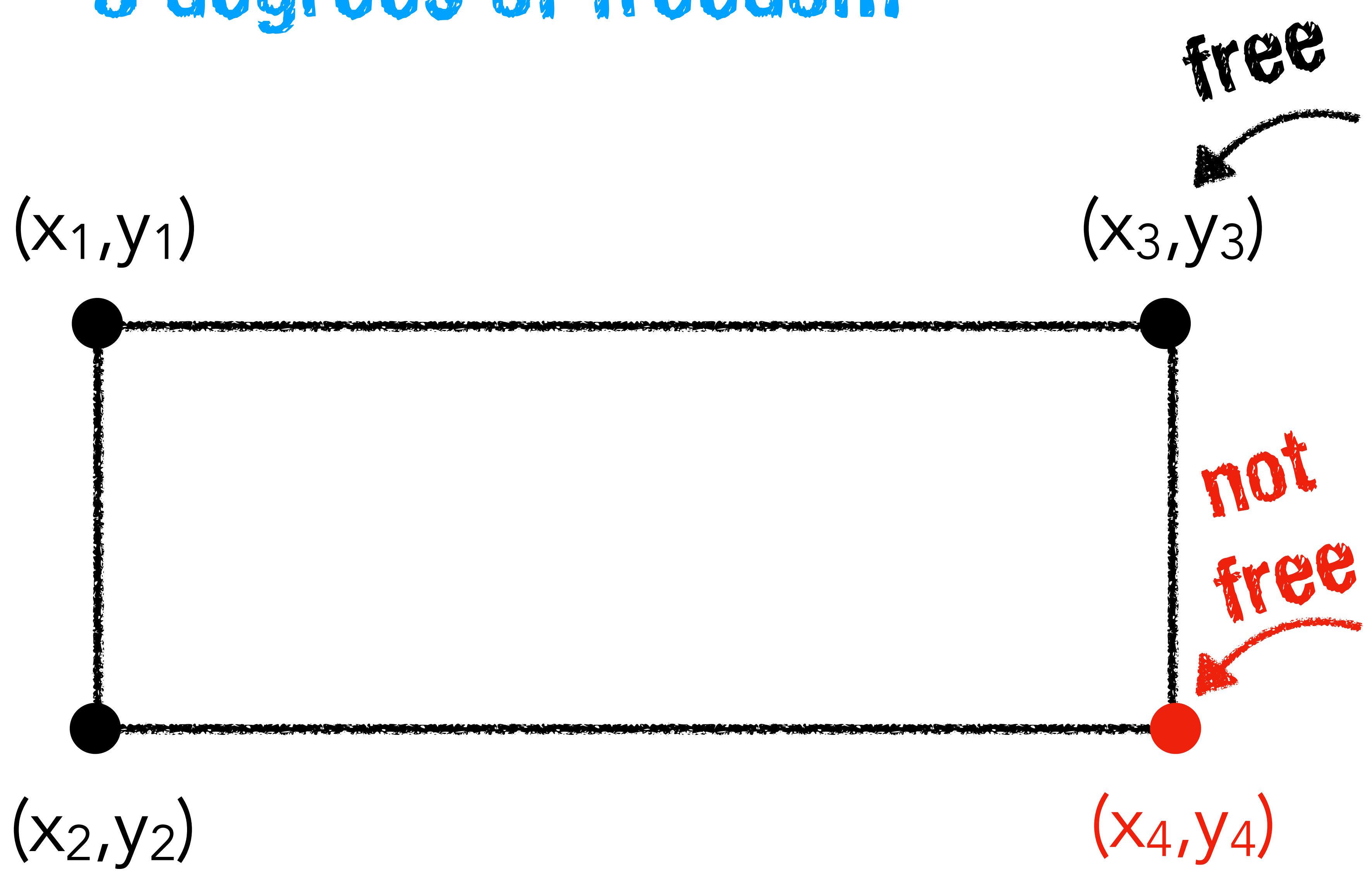
How many degrees of freedom does a **line** have?

2 degrees of freedom

degrees of freedom

How many degrees of freedom does a **rectangle** have?

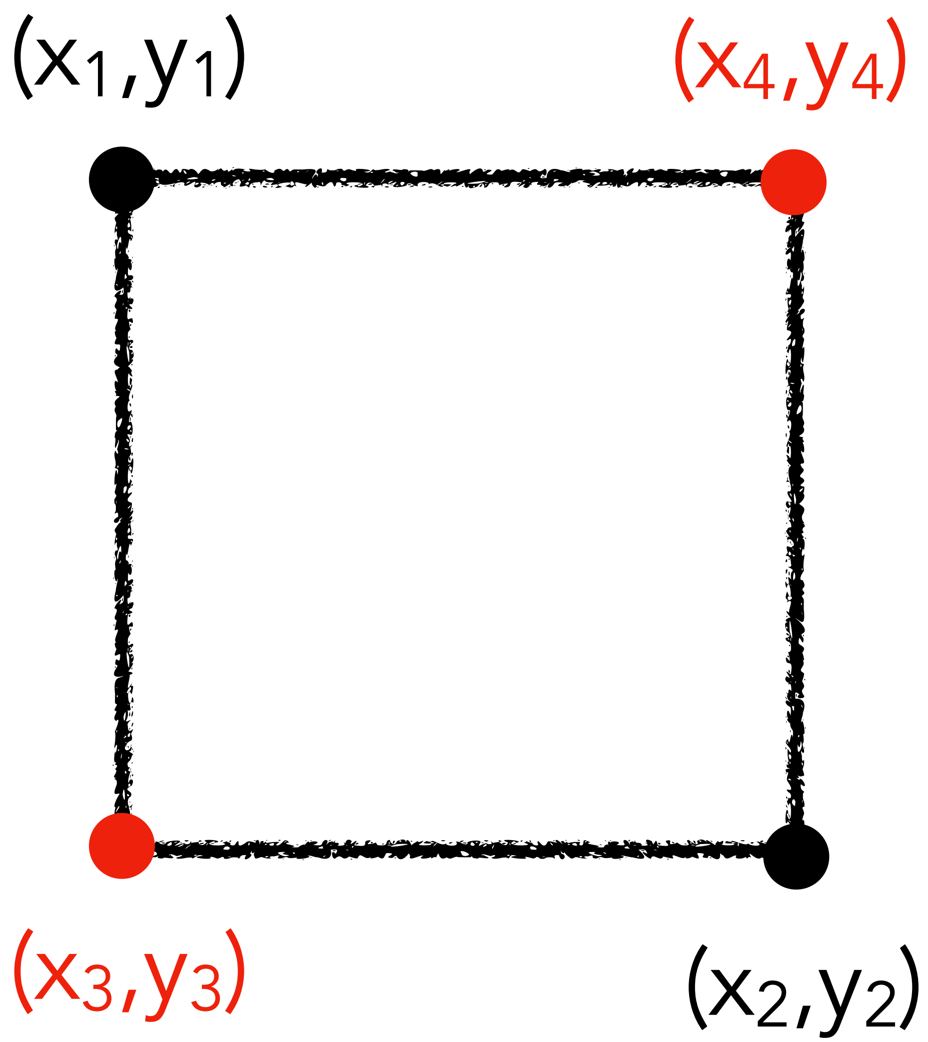
3 degrees of freedom



degrees of freedom

How many degrees of freedom does a **square** have?

2 degrees of freedom



degrees of freedom

How many degrees of freedom does a **mean** have?

the mean as an estimator has n - 1 degrees of freedom

mean

$\bar{X} = 10$

free free not free

8,13,9

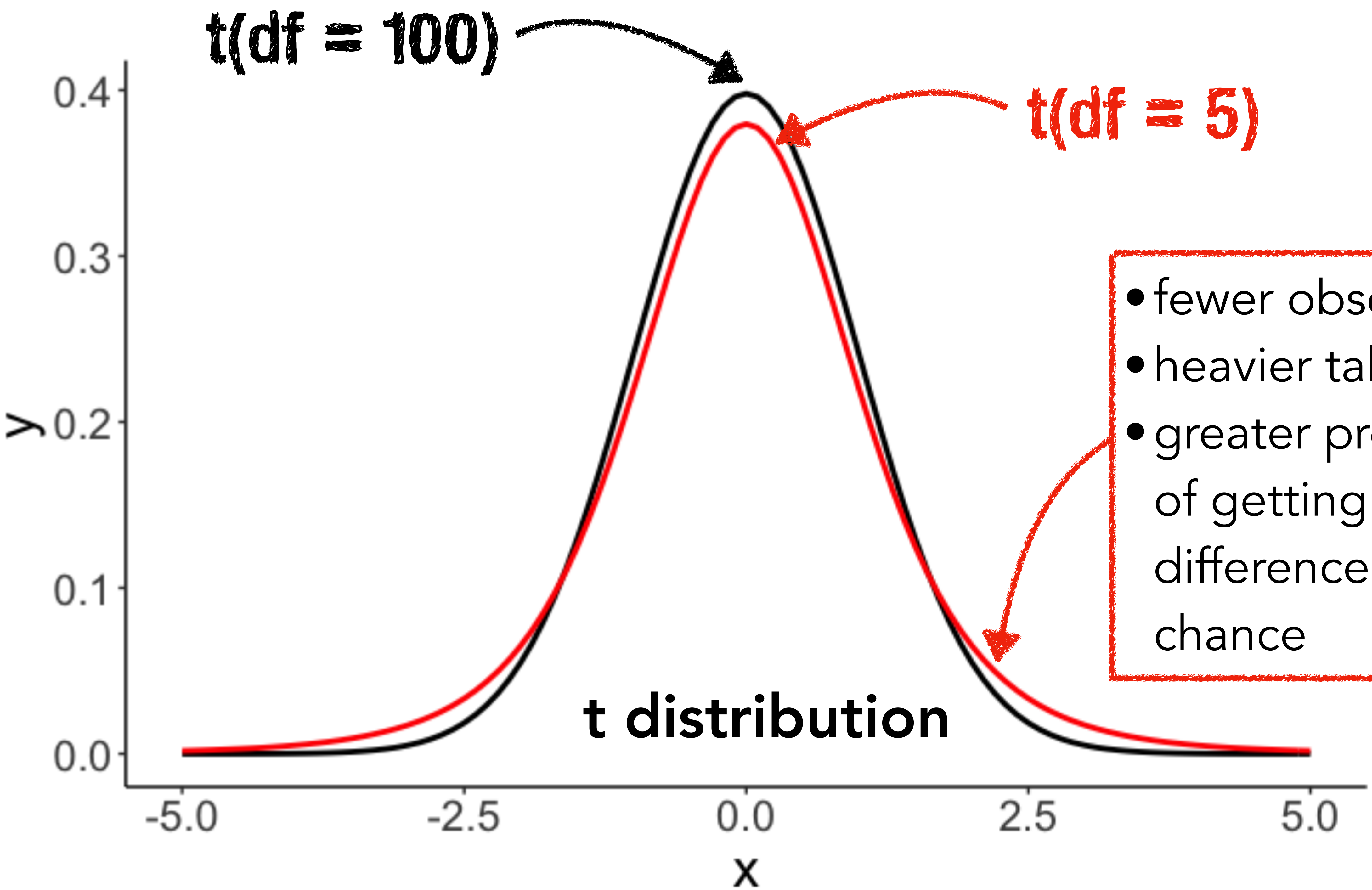
$N = 3$

number of observations

degrees of freedom

the degrees of freedom generally equals the number of observations (or pieces of information) minus the number of parameters estimated

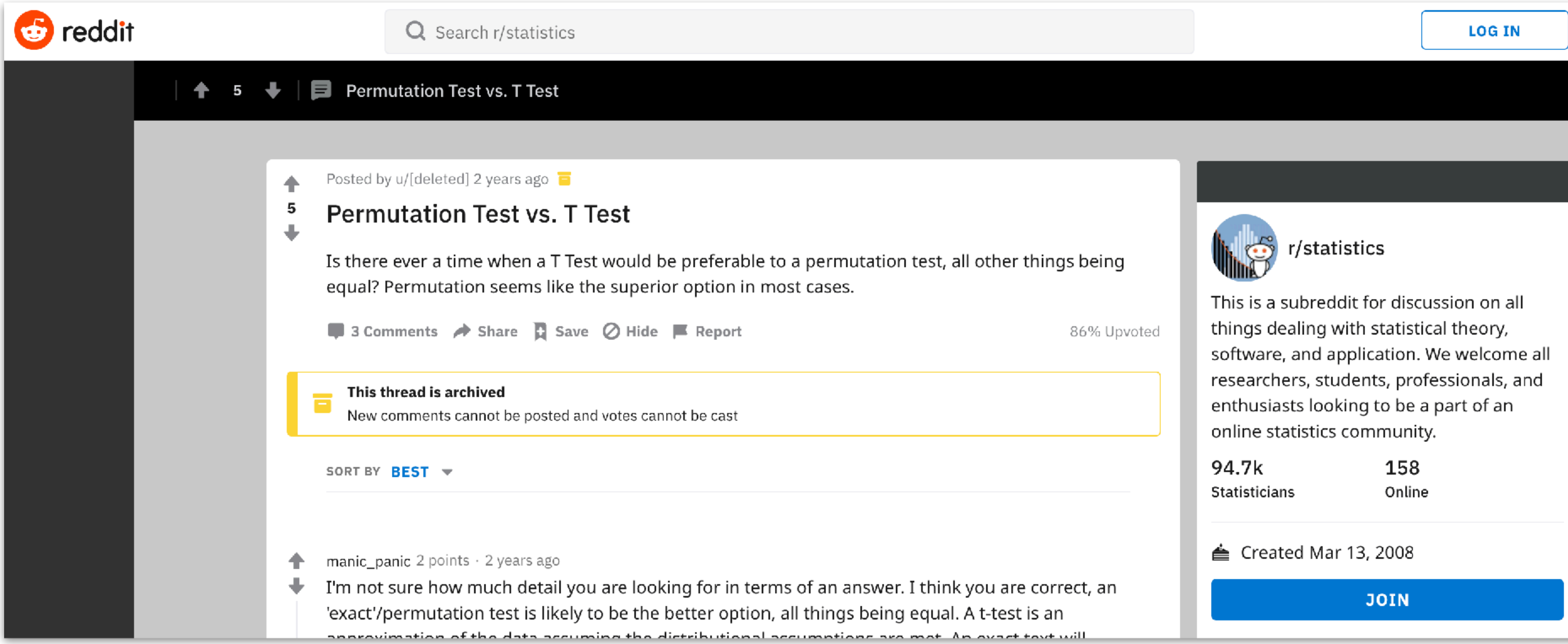
determine the shape of the sampling distributions for our test statistic



- fewer observations
- heavier tales
- greater probability of getting larger differences by chance

t-test vs. permutation

t-test vs. permutation test



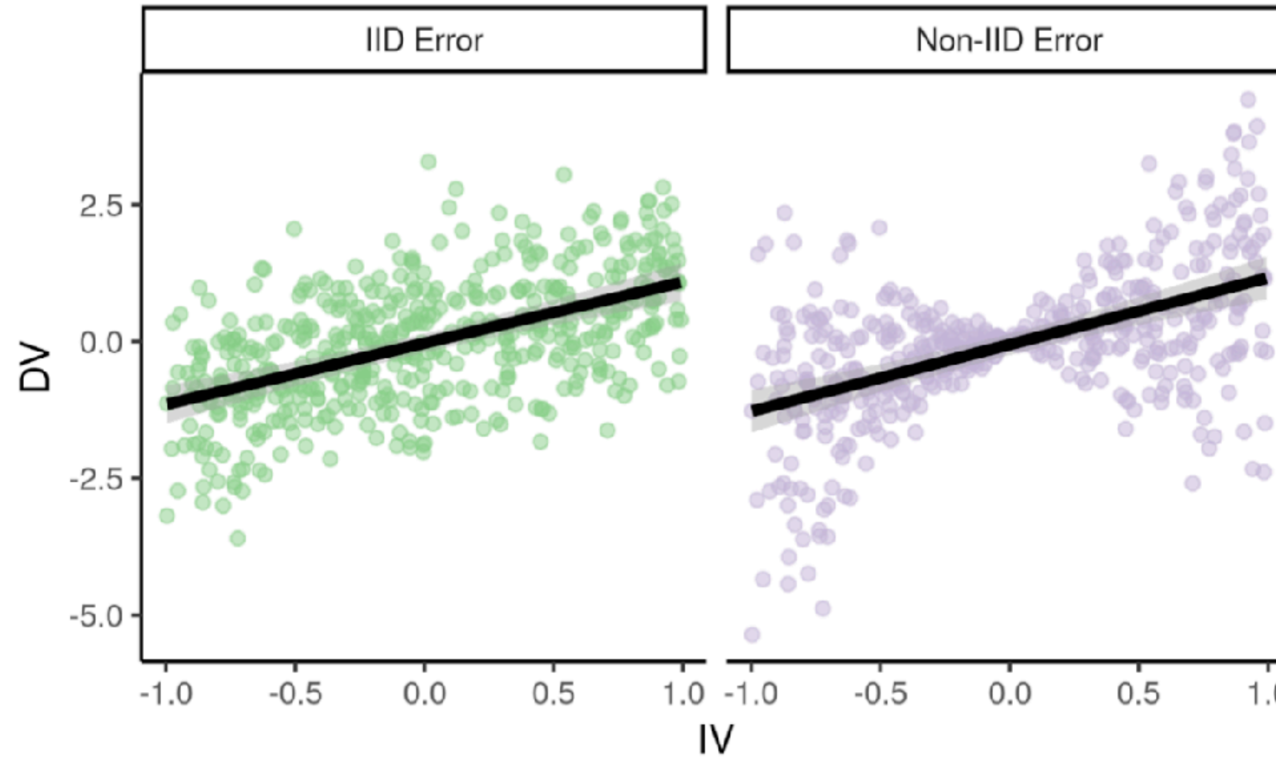
Case_Control 2 points · 2 years ago

In practice, its primarily computing time. When appropriate, I can run a t-test which gets me 99.99% of the way there almost instantly or I can wait forever for a permutation test to run across a million records.

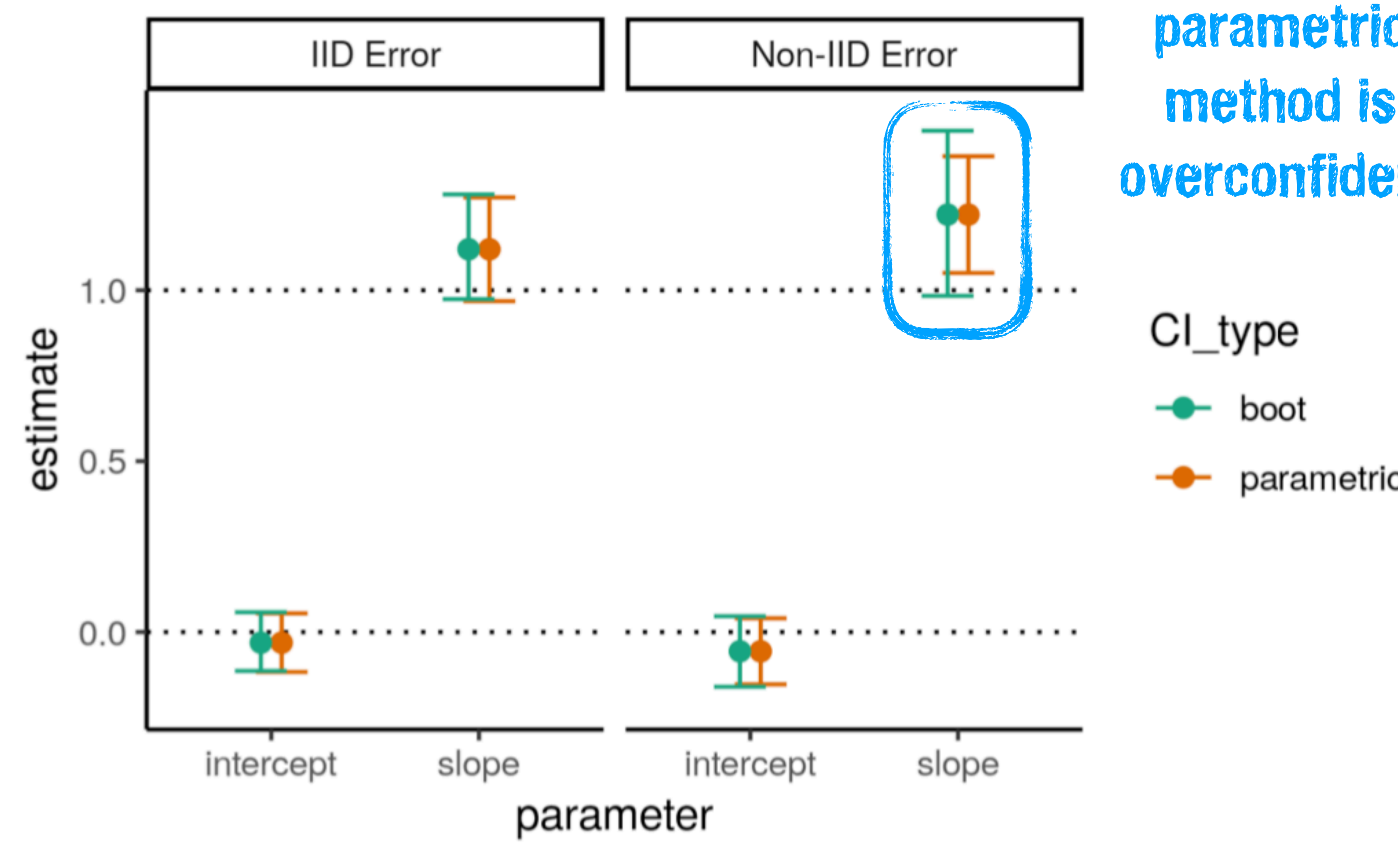
t-test vs. bootstrap

t-test assumptions:
normally distributed residuals
equality of variance

heteroscedasticity



parametric method is overconfident



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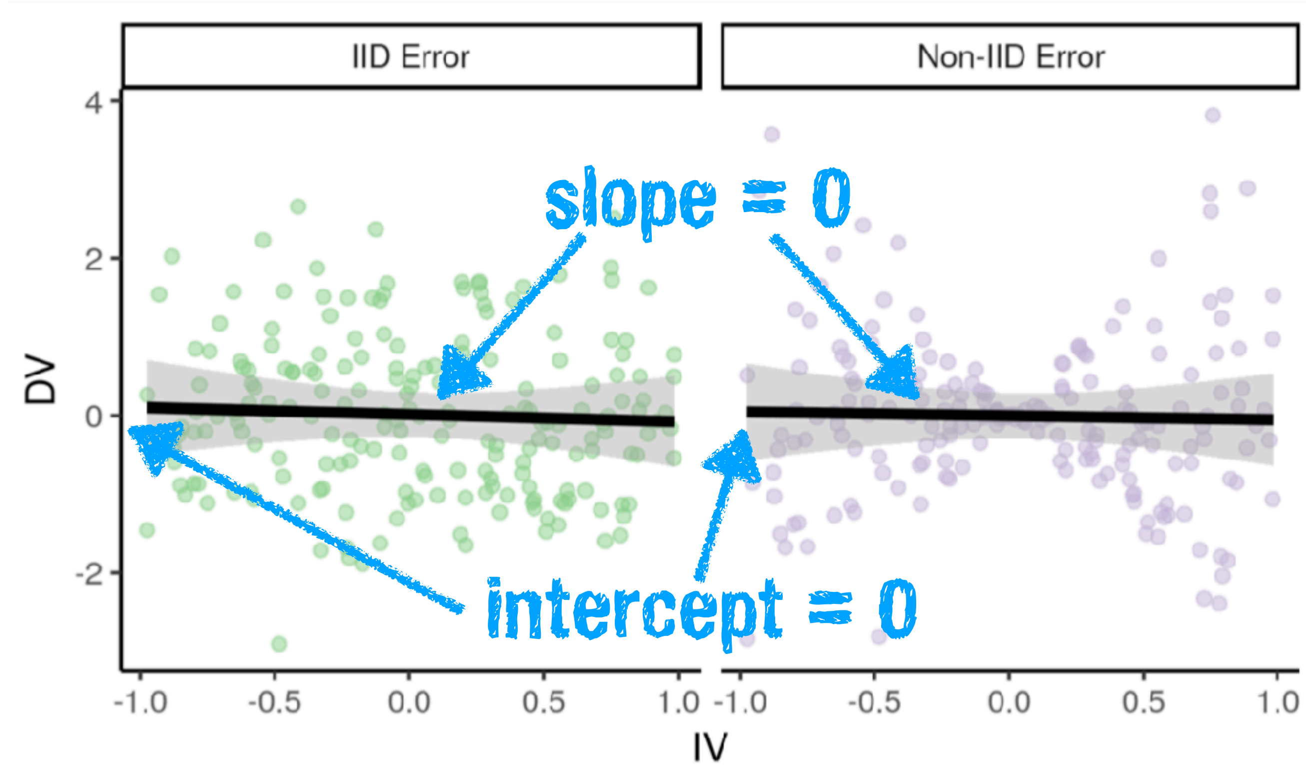
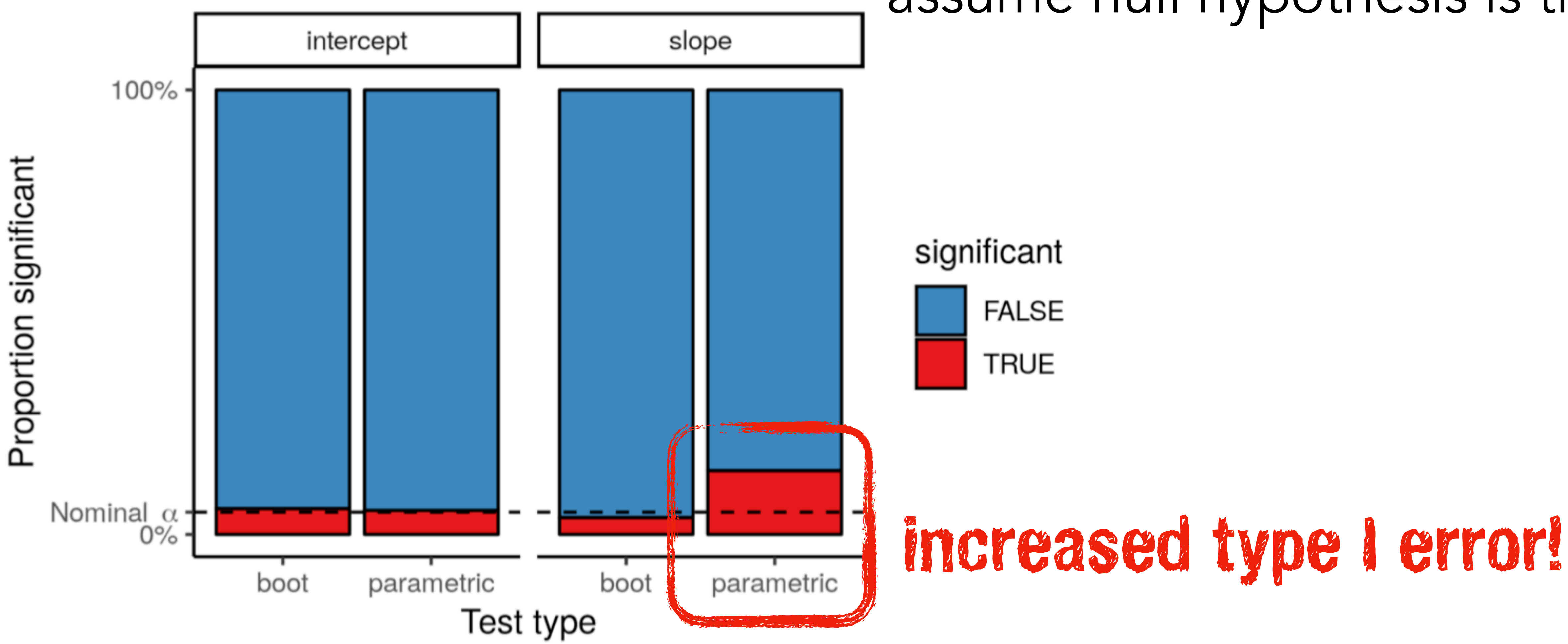


assume null hypothesis is true



t-test vs. bootstrap

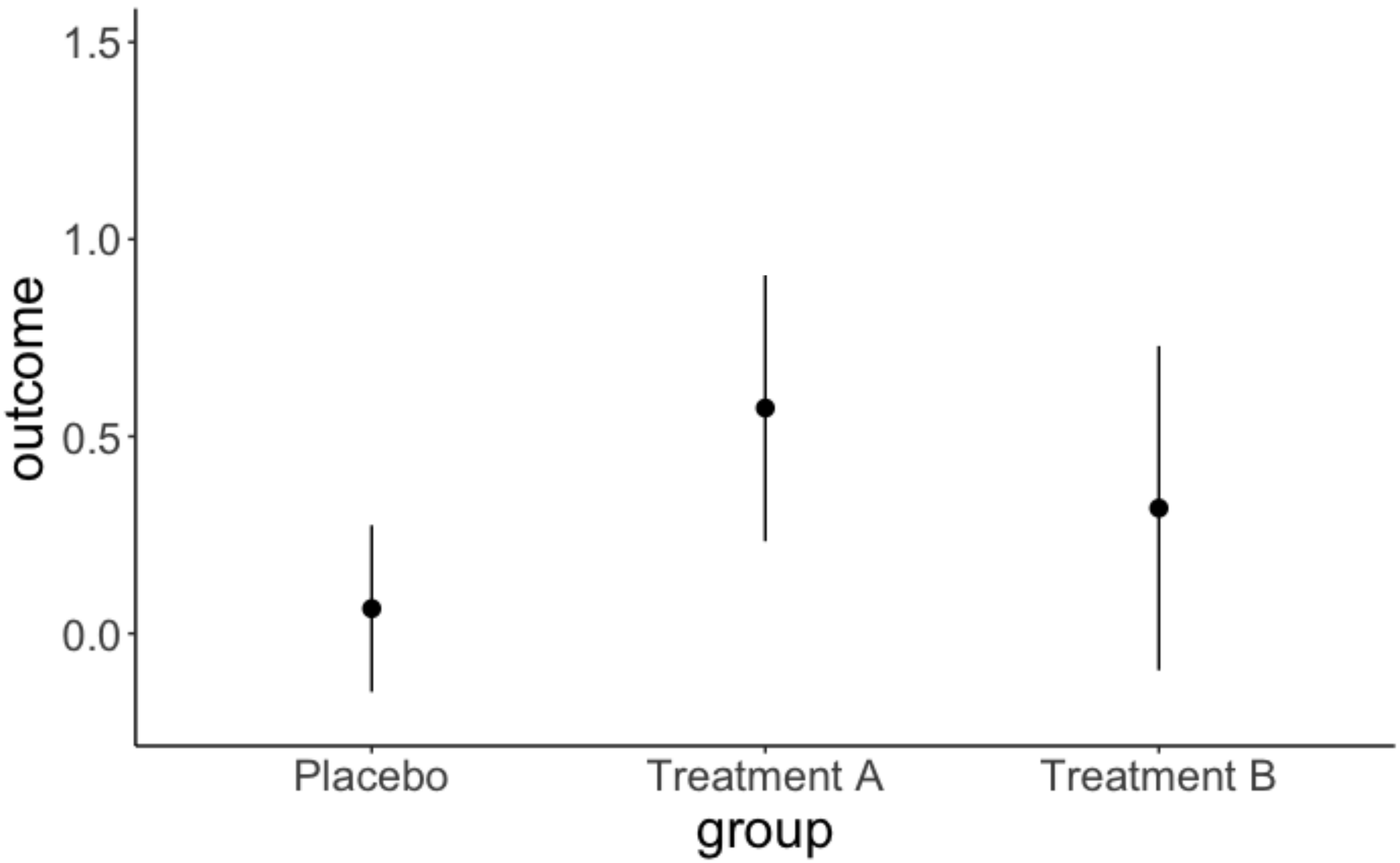
t-test assumptions:

normally distributed residuals

equality of variance

difference in significance vs. significant differences

significant differences

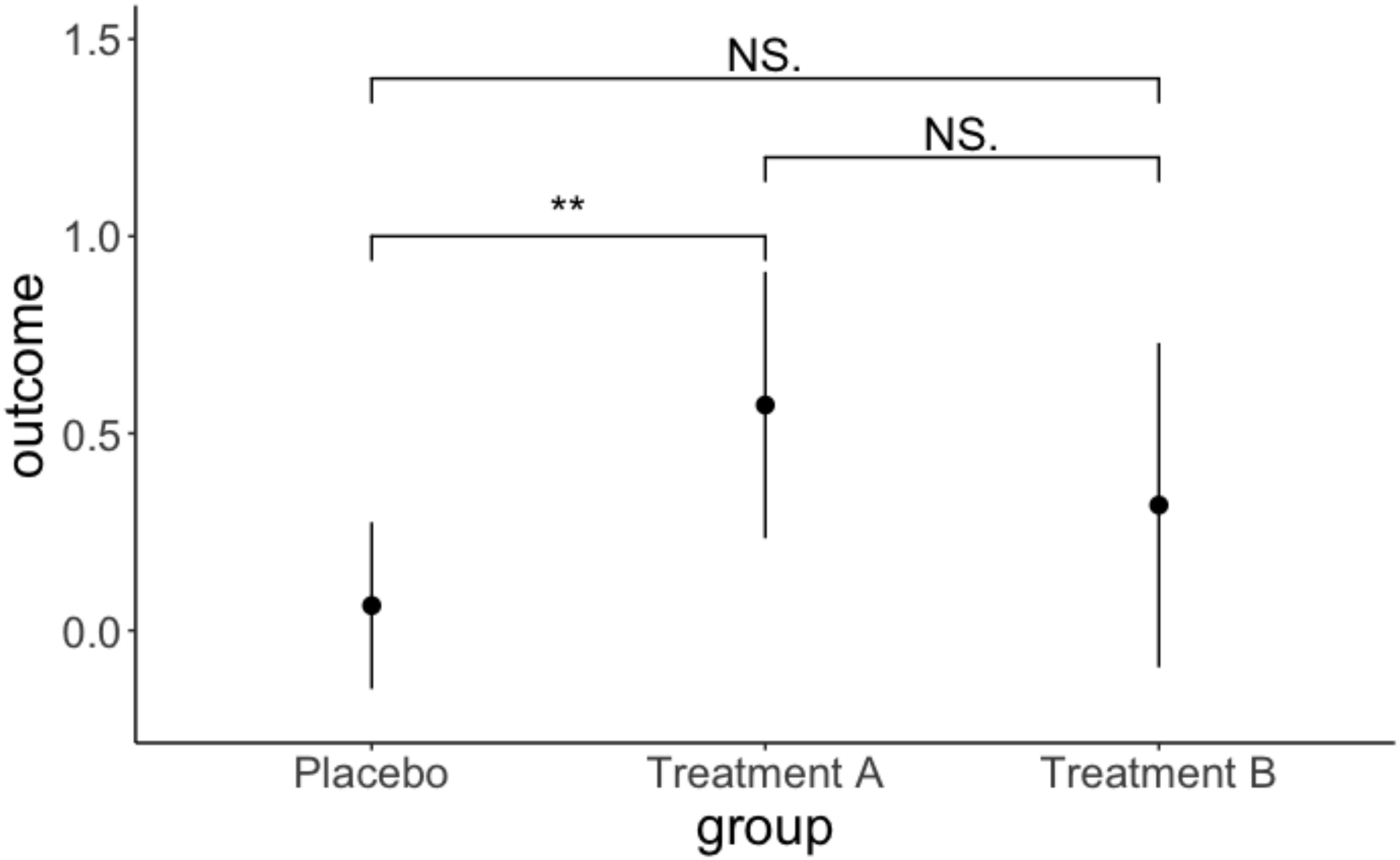


"We compared treatments A and B with a placebo. Treatment A showed a significant benefit over placebo, while treatment B had no statistically significant benefit. Therefore, treatment A is better than treatment B."

https://www.statisticsdonewrong.com/significant-differences.html

significant differences

"Our tendency to look for a difference in significance should be replaced by a check for the significance of the difference."



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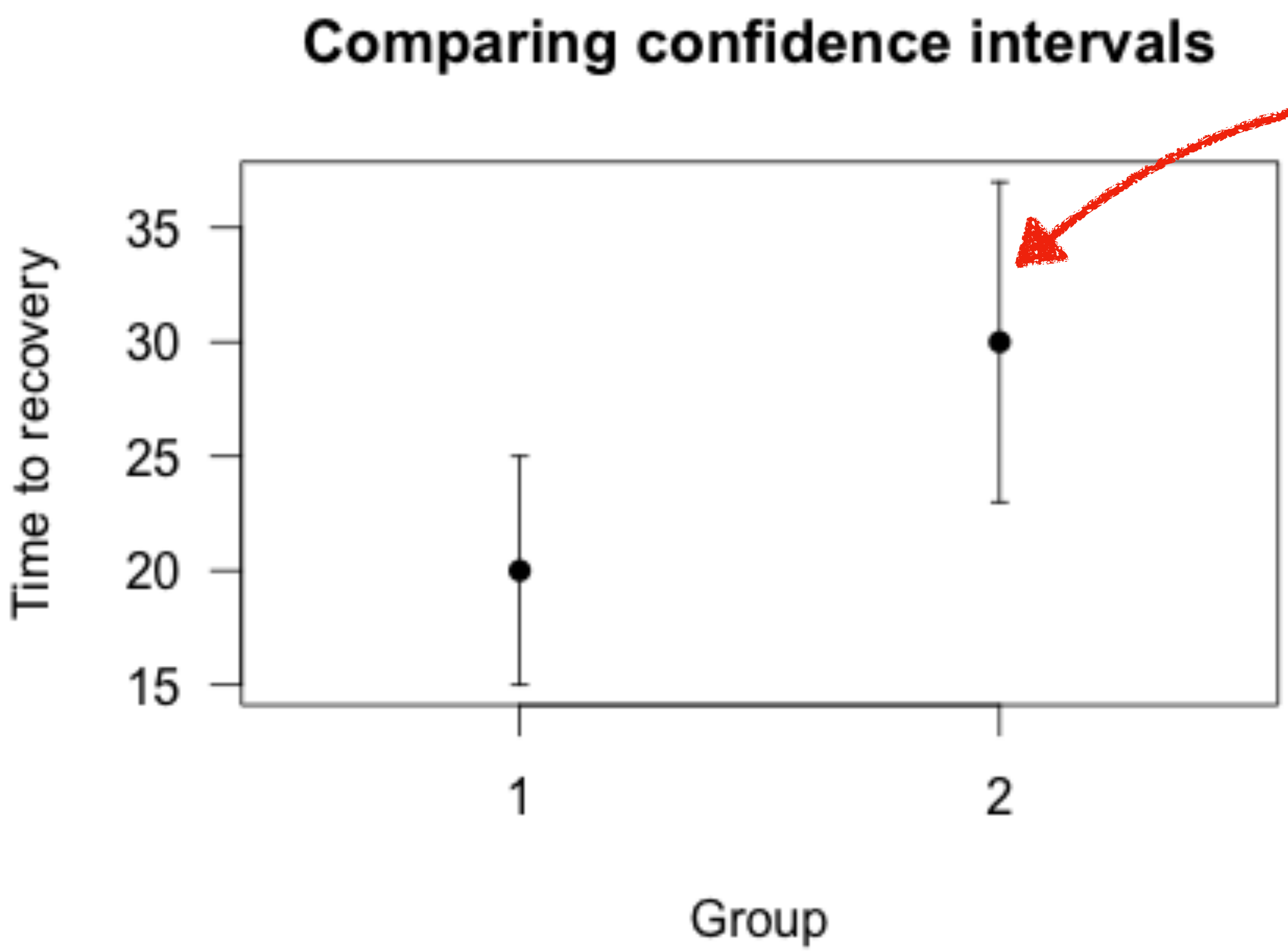
significant differences

Significant difference between Group 1 and 2?



what do the
error bars mean?

standard deviation?

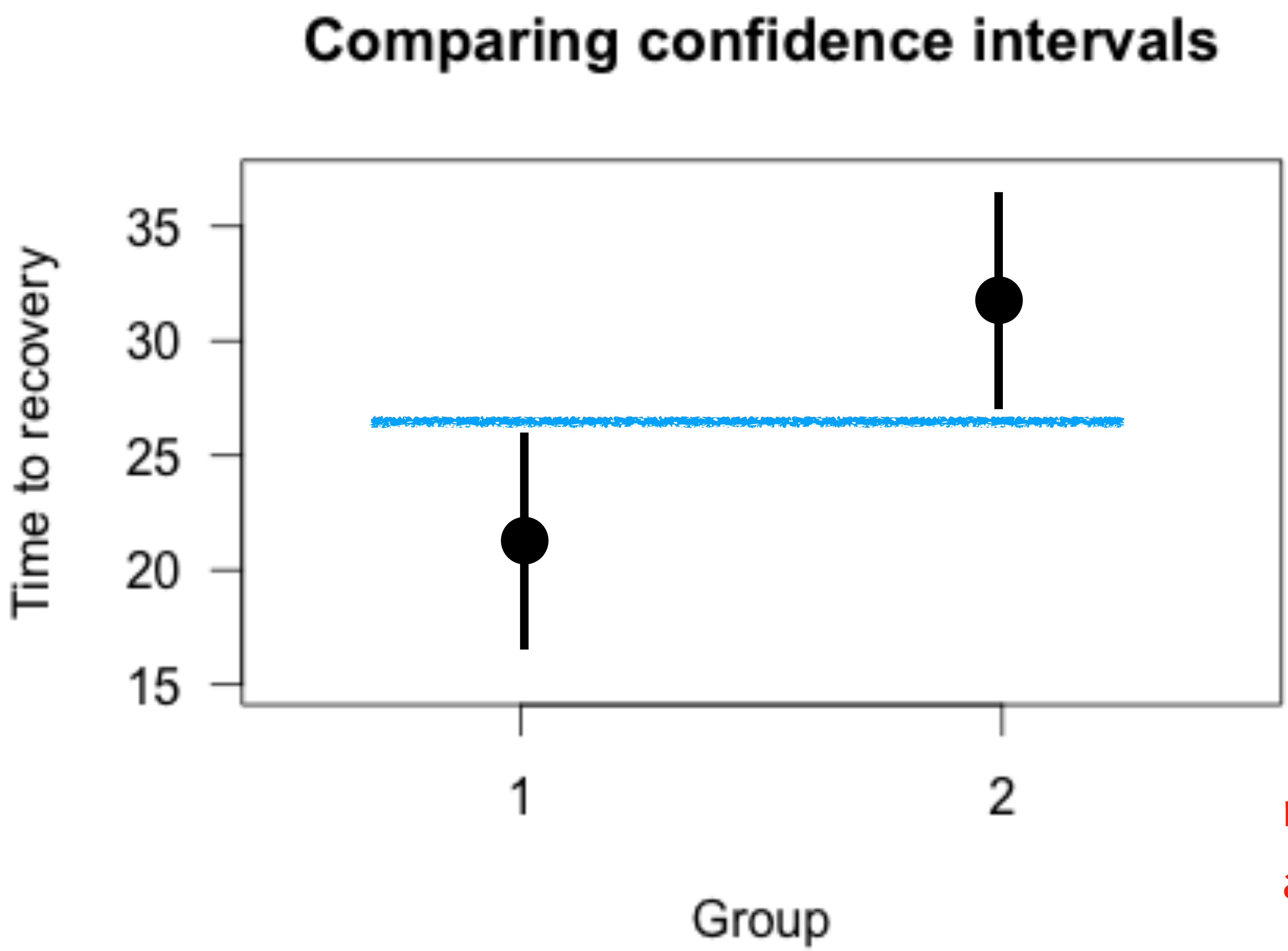
standard error of the mean?

confidence intervals?

What we would like to know, does the confidence interval of the difference between groups exclude 0?

significant differences

Significant difference between Group 1 and 2?



95% confidence intervals don't overlap

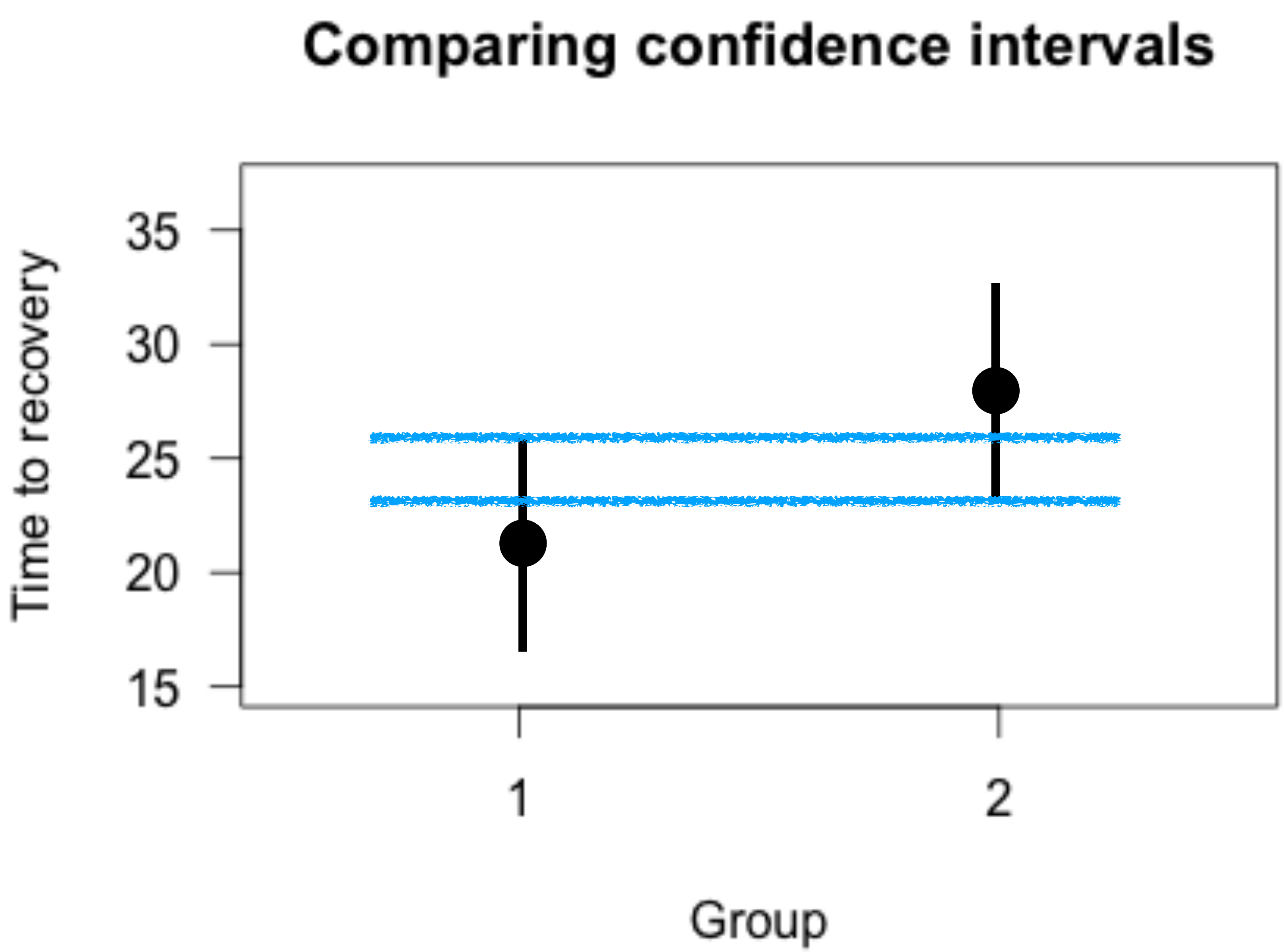
conservative test of significance

rejects H_0 **less often** than the appropriate statistical procedure

Schenker & Gentleman (2001) On judging the significance of differences by examining the overlap between confidence intervals. *The American Statistician*

significant differences

Significant difference between Group 1 and 2?



95% confidence intervals don't overlap with mean

anti-conservative test of significance

rejects H_0 **more often** than the appropriate statistical procedure

Schenker & Gentleman (2001) On judging the significance of differences by examining the overlap between confidence intervals. *The American Statistician*

Power analysis (continued)

RStudio®

time

Controlling for variables

Controlling for variables

1. What does "controlling" for variables mean?

2. When should I control for variables?

check this out

http://causality.cs.ucla.edu/blog/index.php/2019/08/14/a-crash-course-in-good-and-bad-control/

What does controlling for variables mean?

we are not actually "**controlling**" the variable

instead, we are taking the variable into consideration when making predictions

the hope is that we get a better estimate of the parameter that we are interested in by taking into account other factors

When should I control for variables?

recent advances in graphical models have produced a simple criterion to distinguish good from bad controls

***d*-separation**

directional

decide from a causal graph whether a set of variables `X` is independent of another set `Y`, given a third set `Z`

Goal: we want a precise (and unbiased) estimate of the predictive relationship between `X` and `Y`

we want to block all other paths from X to Y

http://bayes.cs.ucla.edu/BOOK-09/ch11-1-2-final.pdf

When should I control for variables?

How can I tell whether two variables are independent?

Recipe for independence

1. Draw the ancestral graph
2. "Moralize" the graph by "marrying" the parents
3. "Disorient" the graph by replacing arrows with edges
4. Delete the givens and their edges
5. Read the answer off the graph

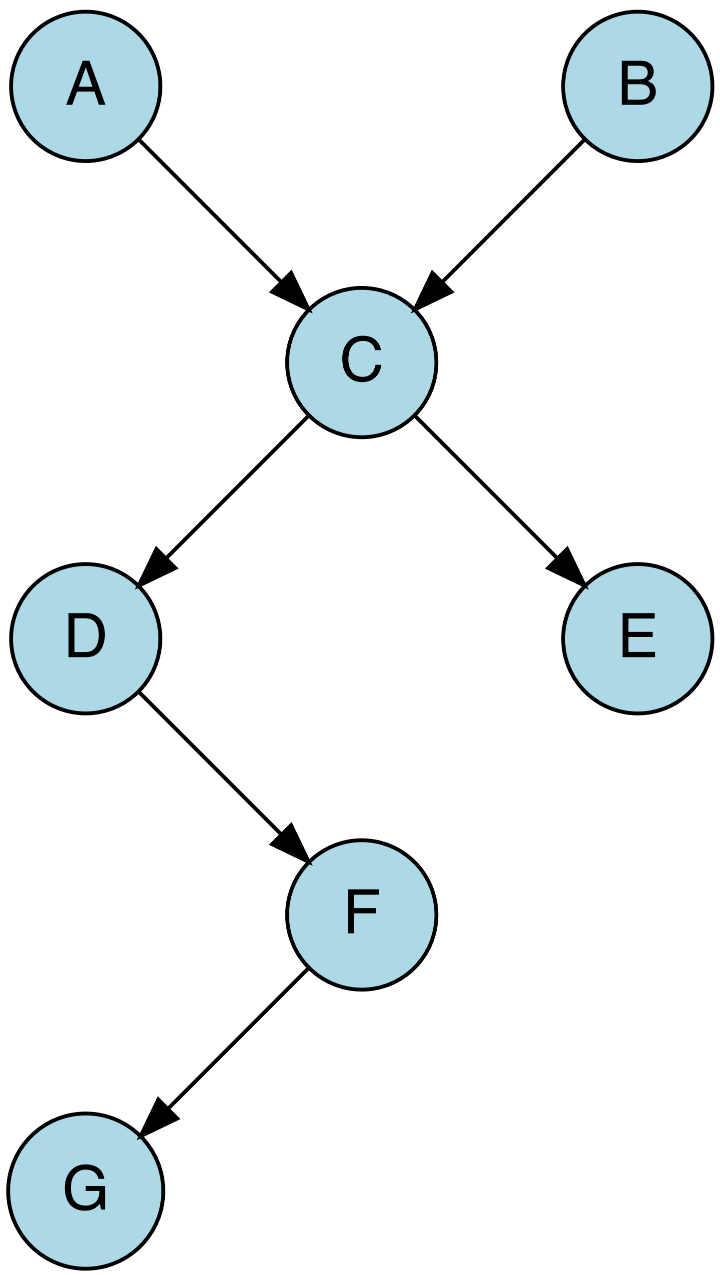
   - if variables are **disconnected** they are independent
   - if variables are connected (have a path between them) they are not guaranteed to be independent

http://web.mit.edu/jmn/www/6.034/d-separation.pdf

When should I control for variables?

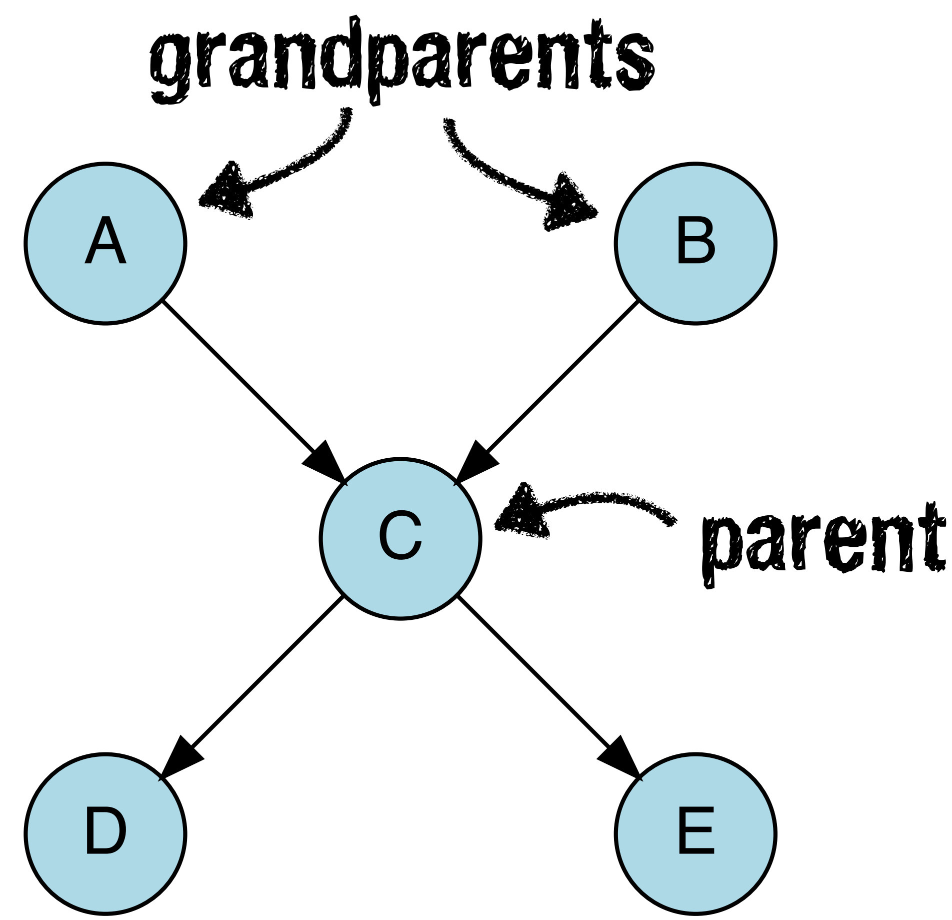
Are D and E independent?

$$p(D \mid E) = p(D) \ ?$$



1. Draw the ancestral graph

Construct the "ancestral graph" of all variables mentioned in the probability expression. This is a reduced version of the original net, consisting only of the variables mentioned and all of their ancestors (parents, parents' parents, etc.)



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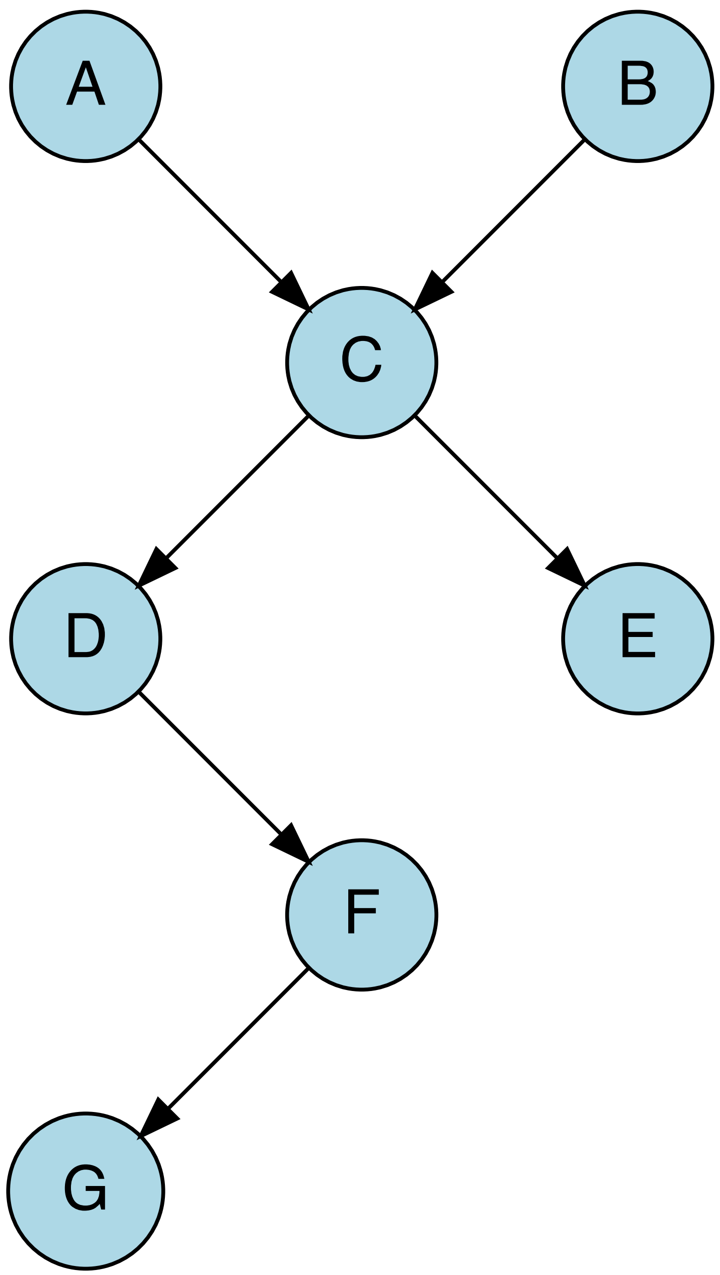
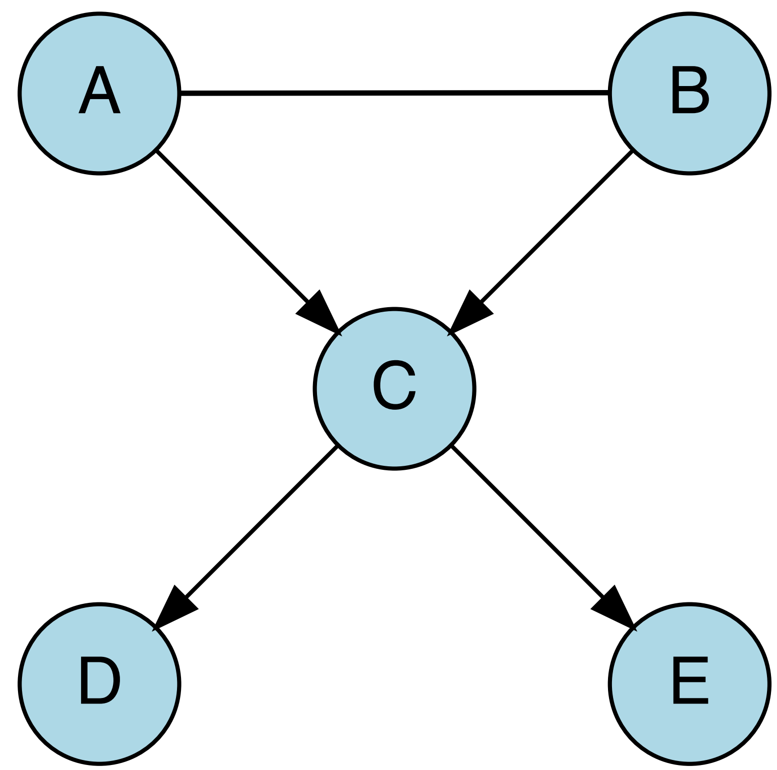


2. "Moralize" the graph

let's get married!

For each pair of variables with a common child, draw an undirected edge (line) between them. (If a variable has more than two parents, draw lines between every pair of parents.)



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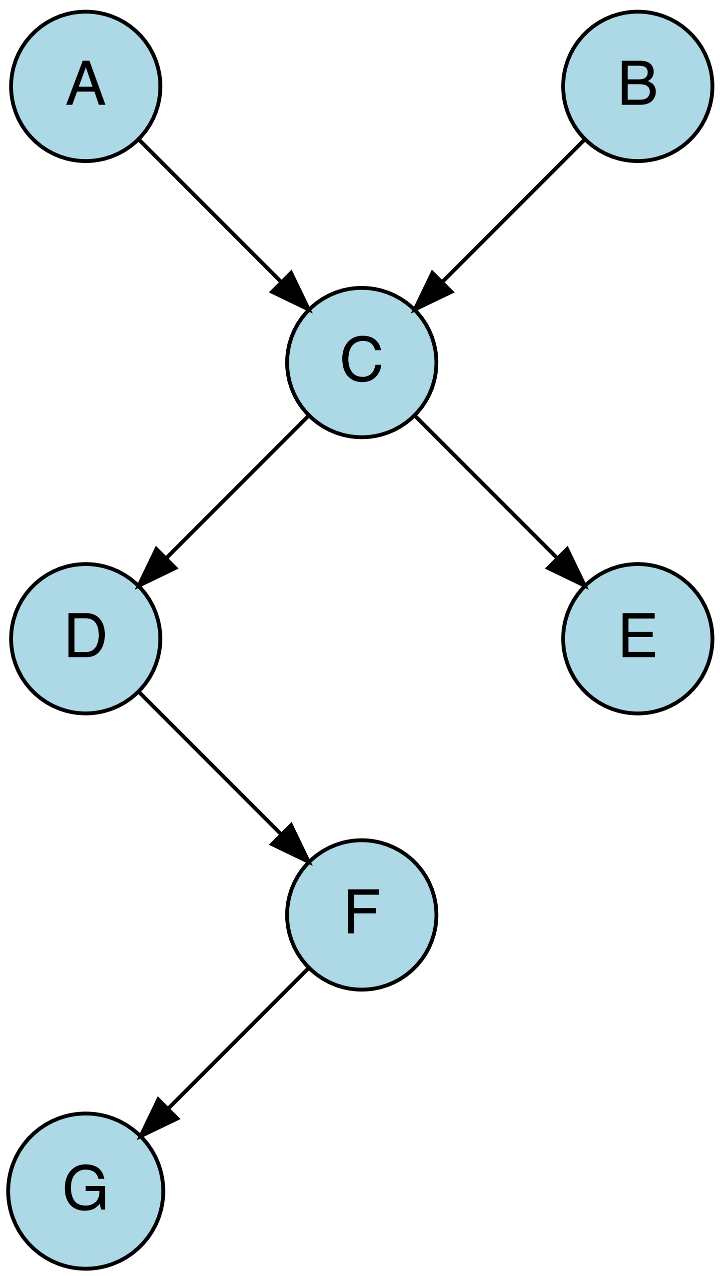
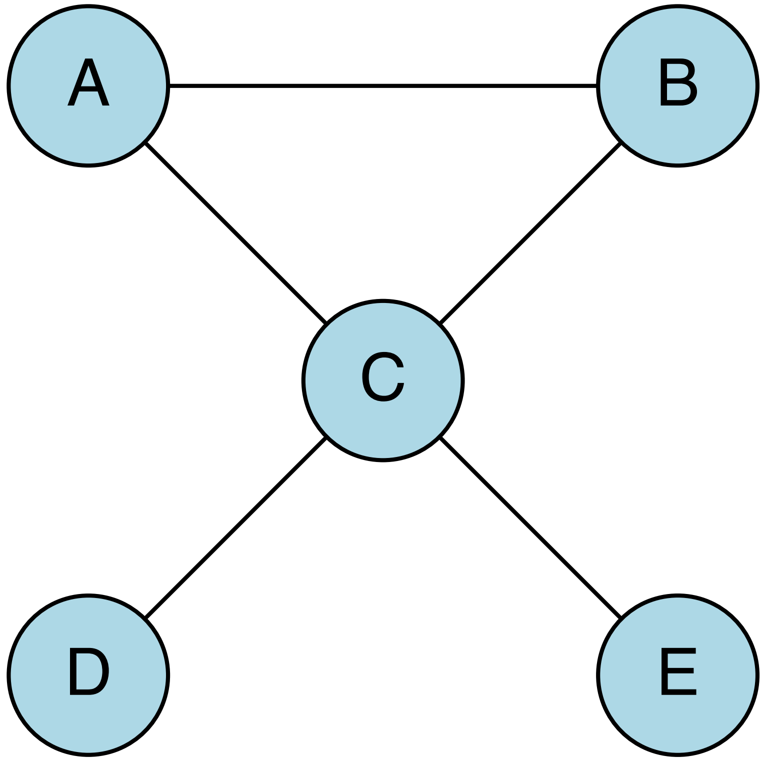


3. "Disorient" the graph

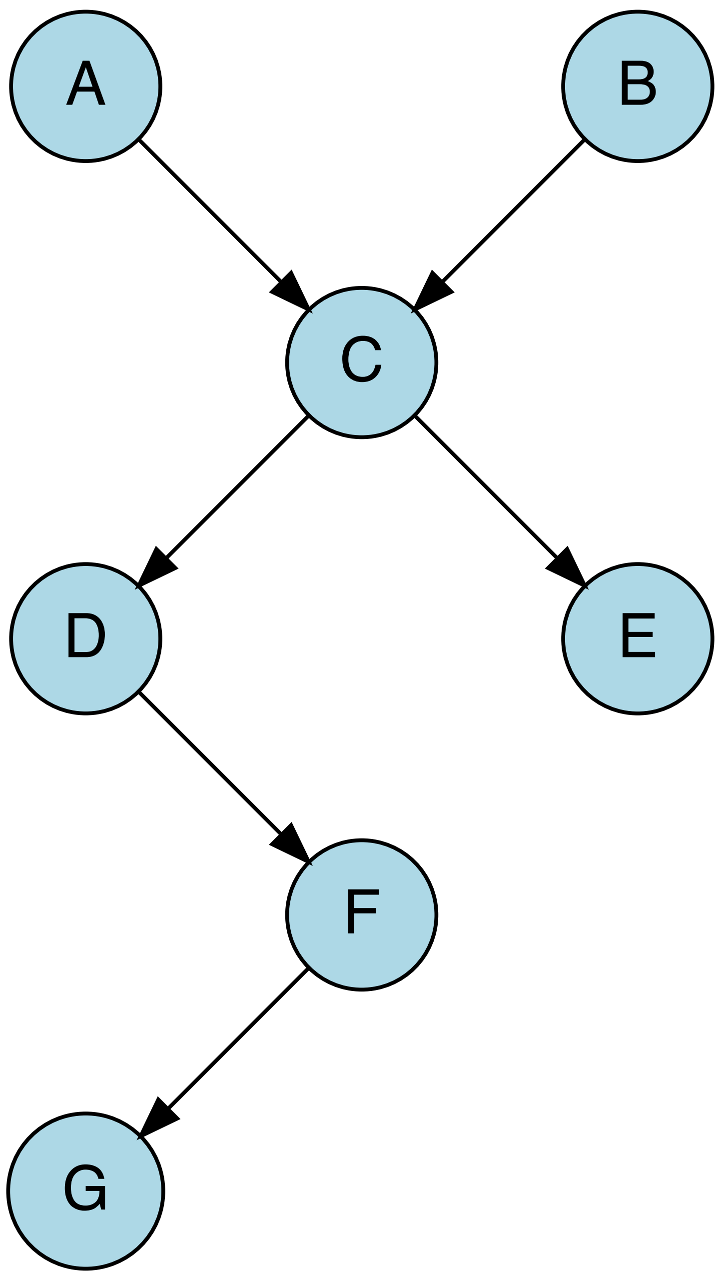
Replace arrows arrows with lines



When should I control for variables?

Are D and E independent?

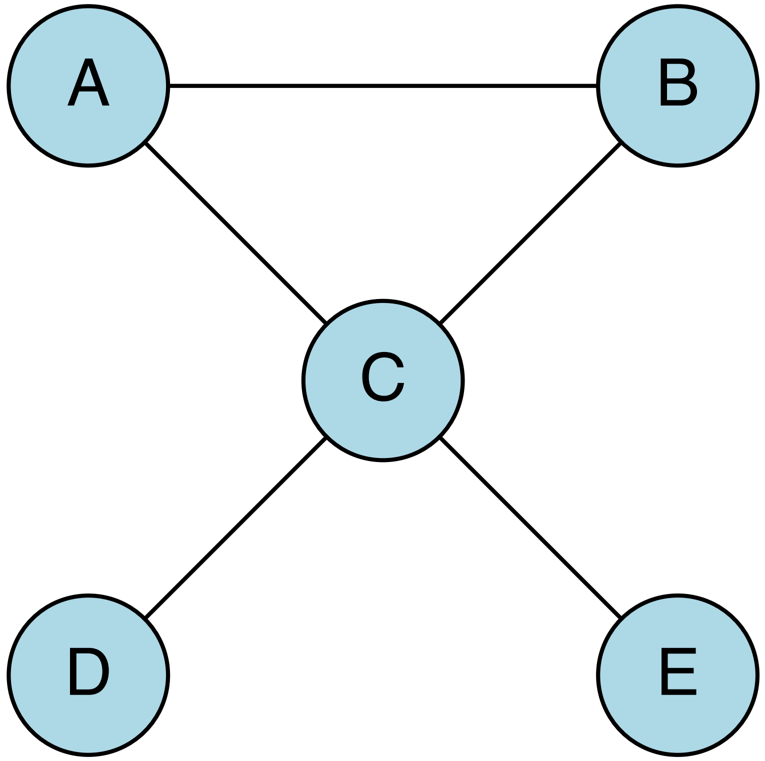
$$p(D \mid E) = p(D) \text{ ?}$$



4. Delete the givens

Remove the variables that we condition on, as well as their edges

we didn't condition on anything, so there is nothing to delete



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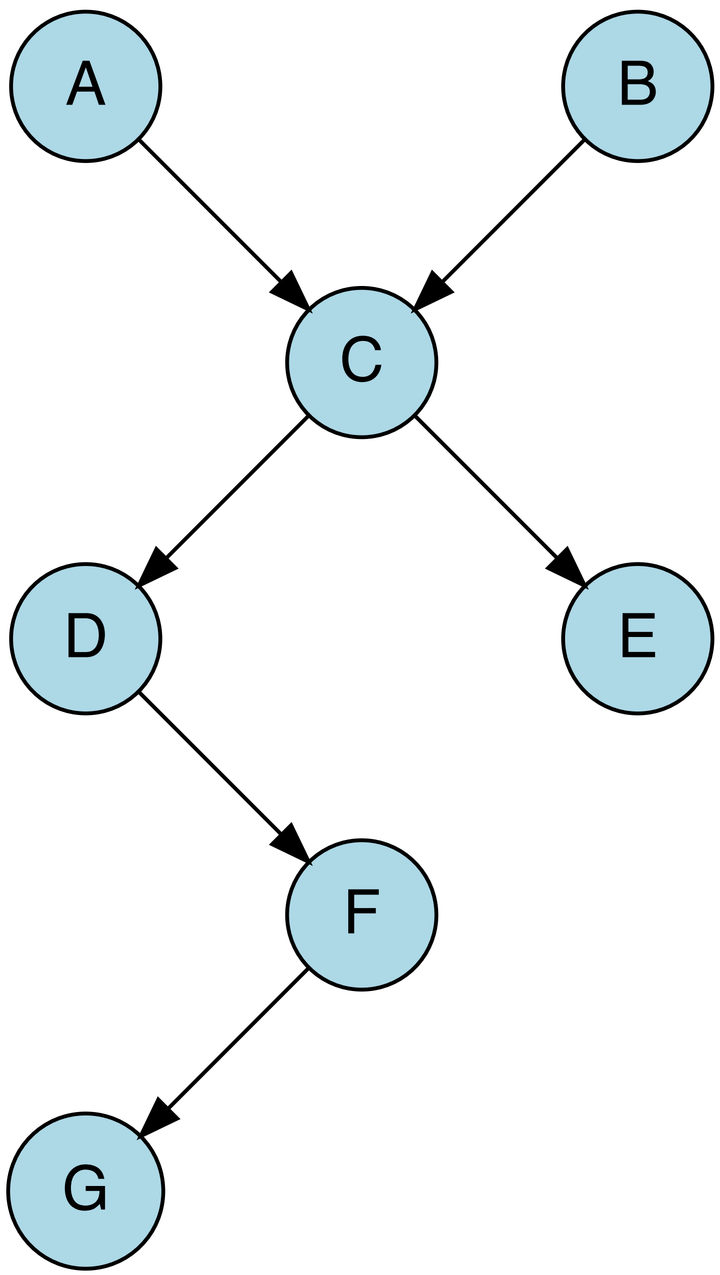

5. Read answer off the graph

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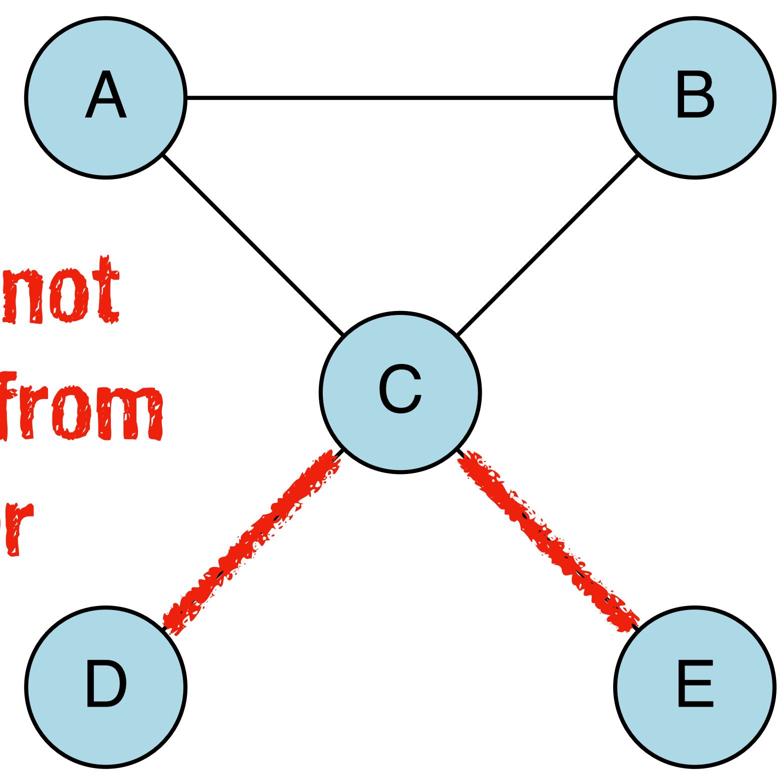


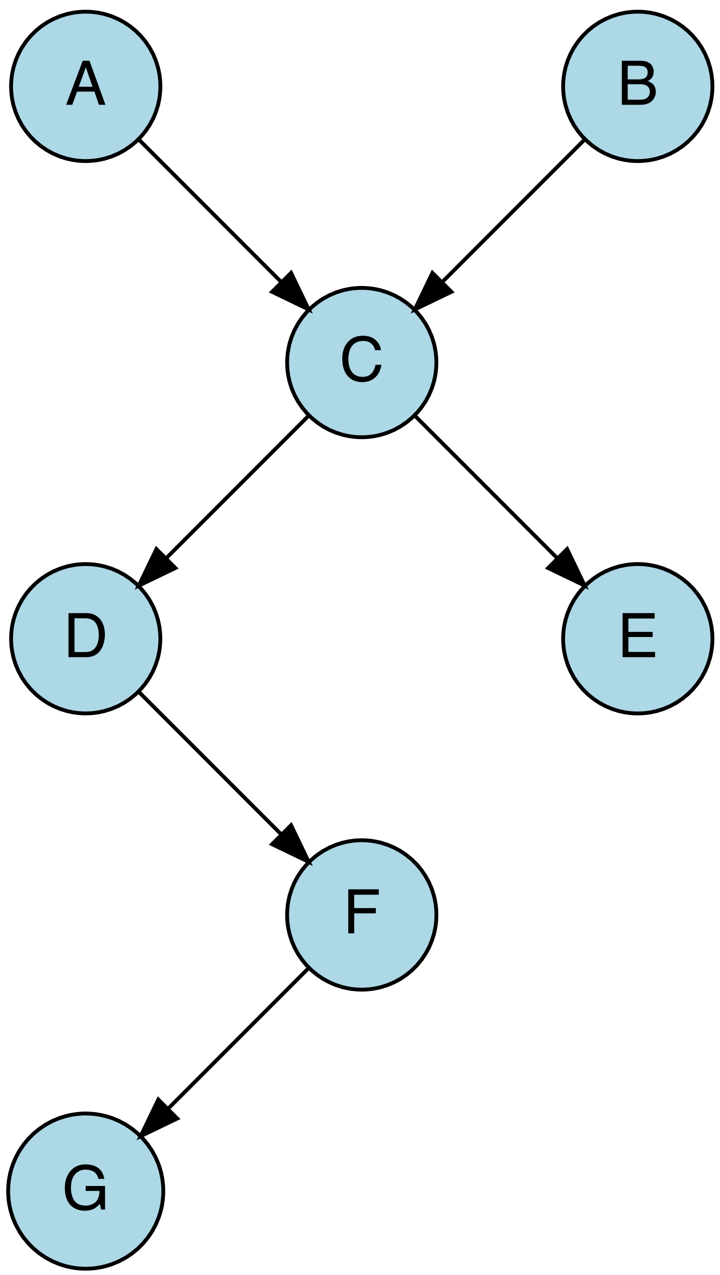
D and E are not independent from each other

they are connected via at least one path

When should I control for variables?

Are D and E independent, given C?

$$p(D \mid E, C) = p(D \mid C) \ ?$$



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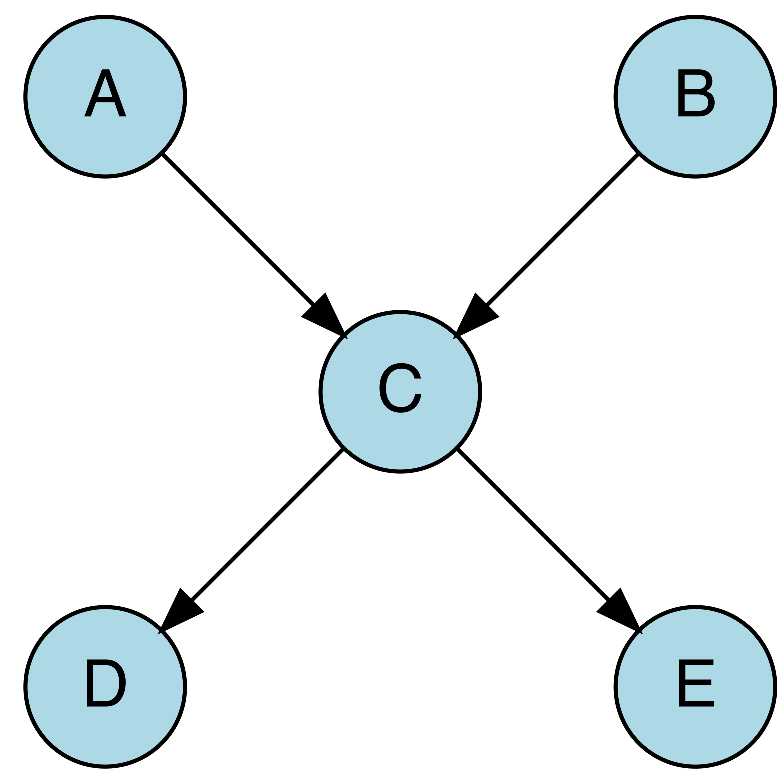
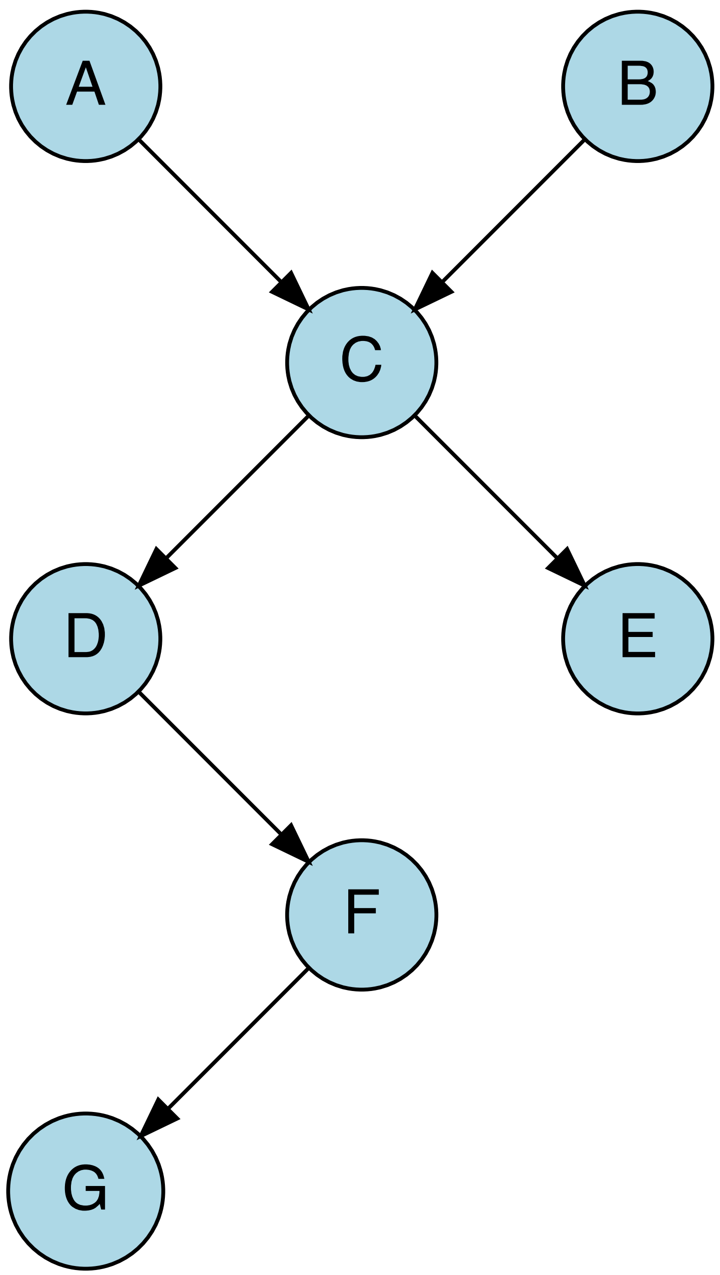


When should I control for variables?

Are D and E independent, given C?

$$p(D \mid E, C) = p(D \mid C) \; ?$$



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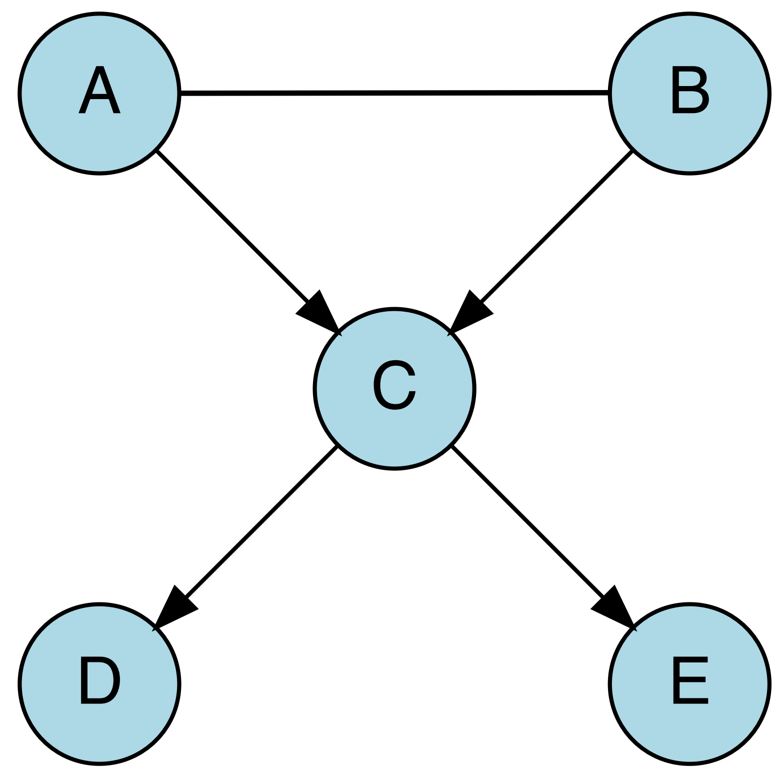

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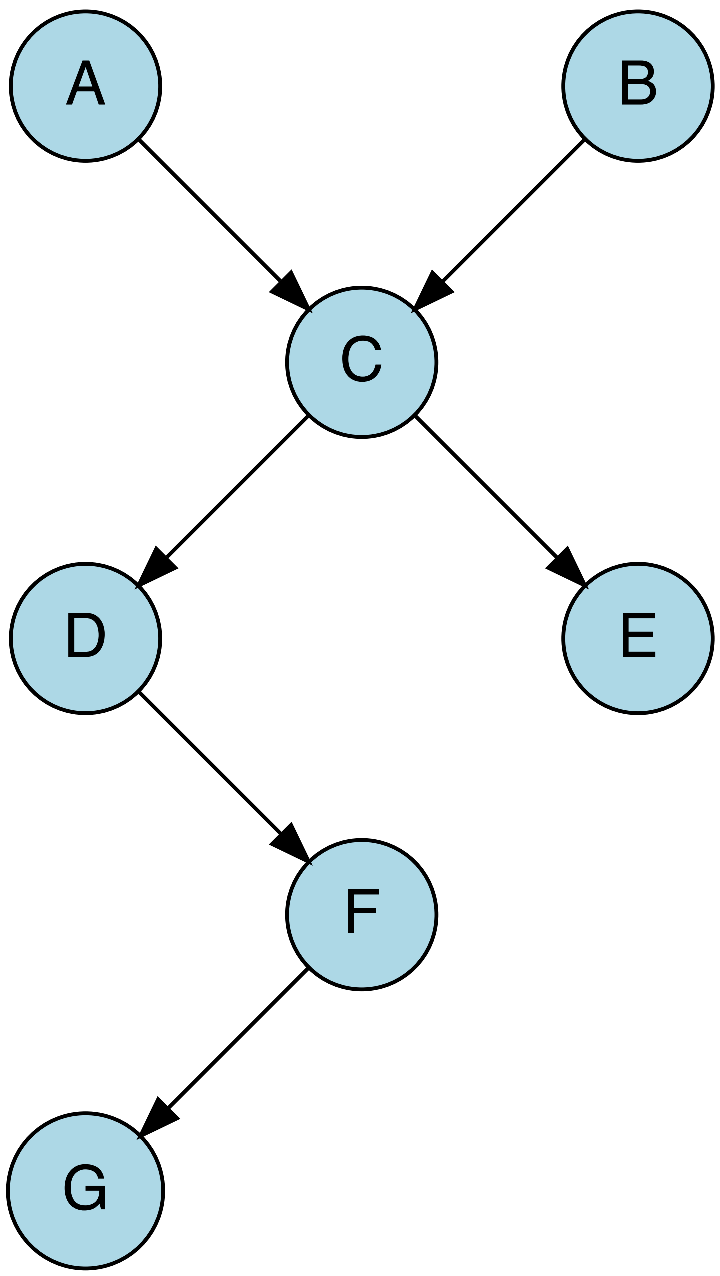

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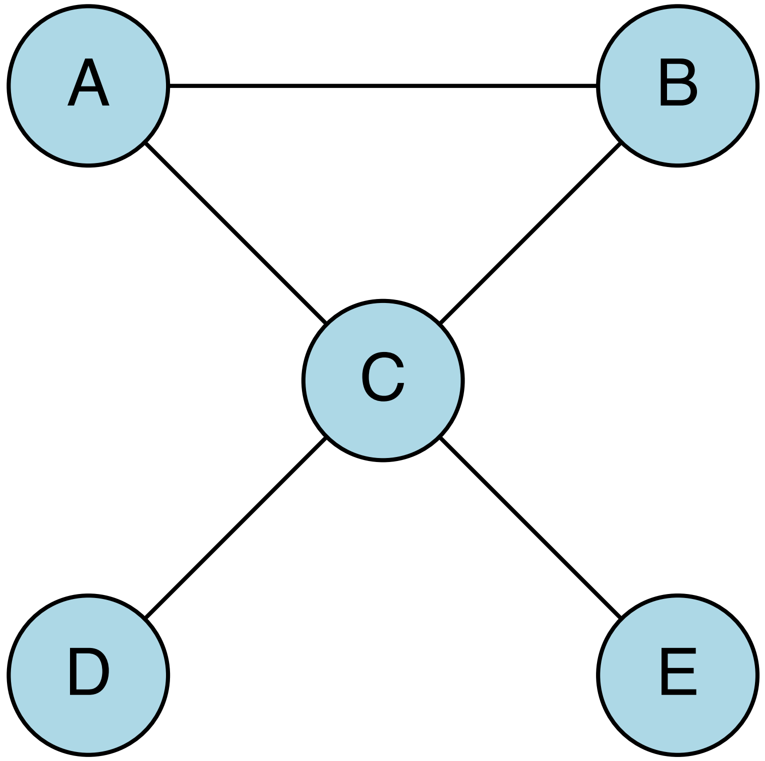

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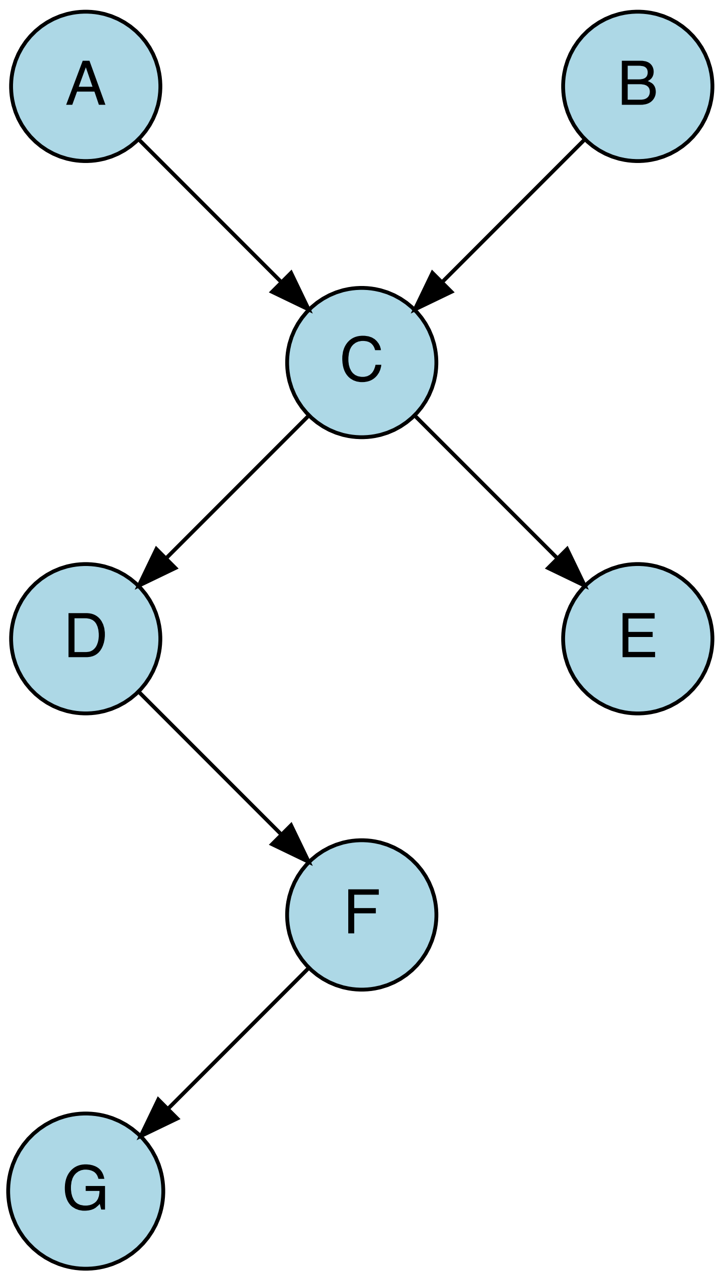
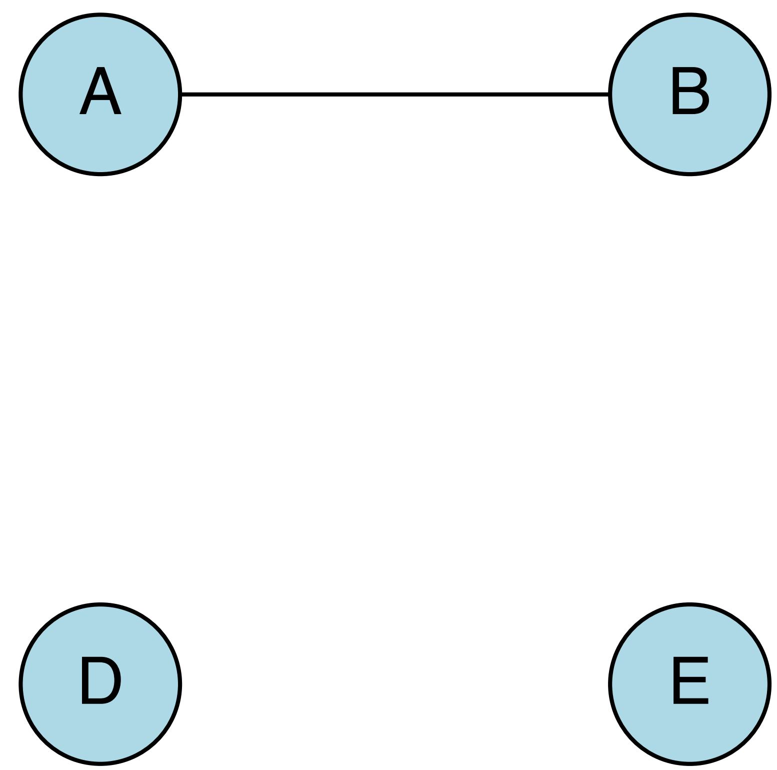


4. Delete the givens

Remove the variables that we condition on, as well as their edges

we conditioned on C!



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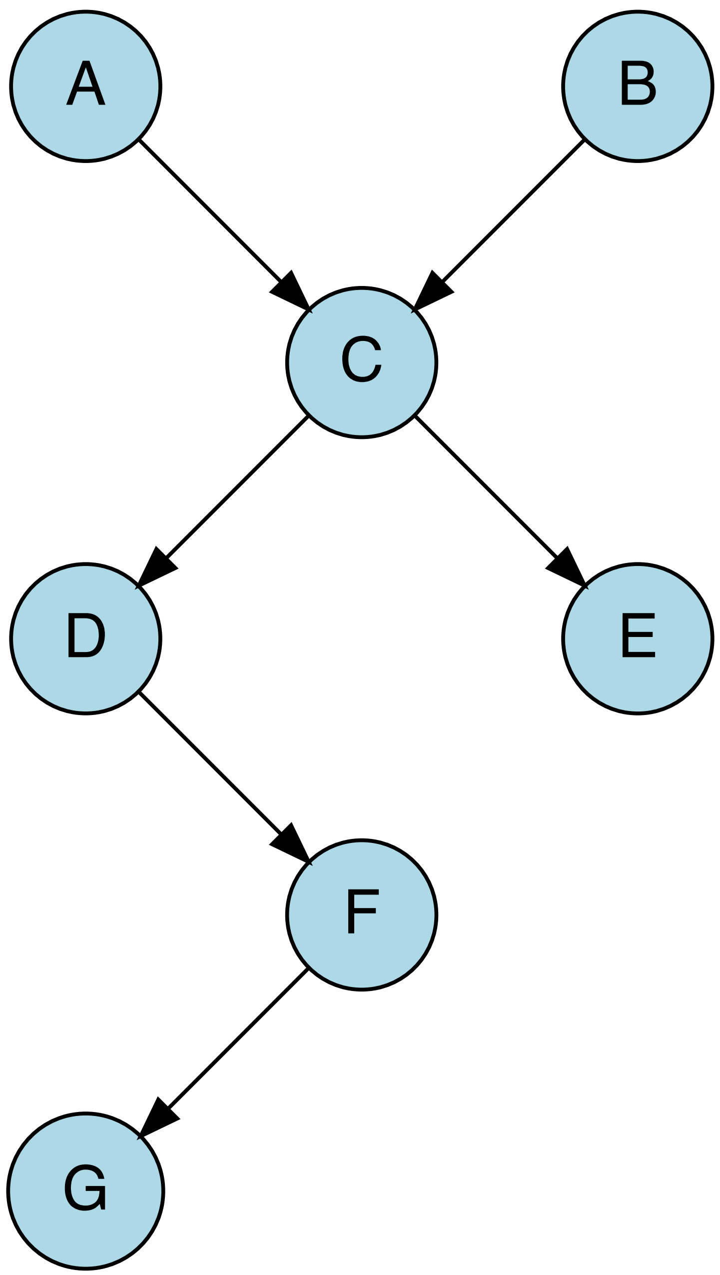

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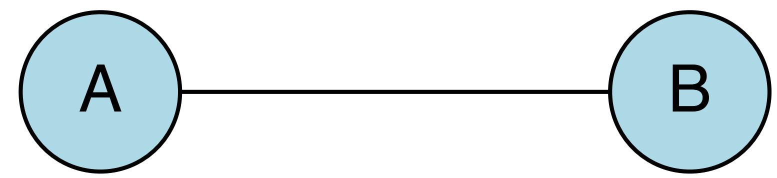


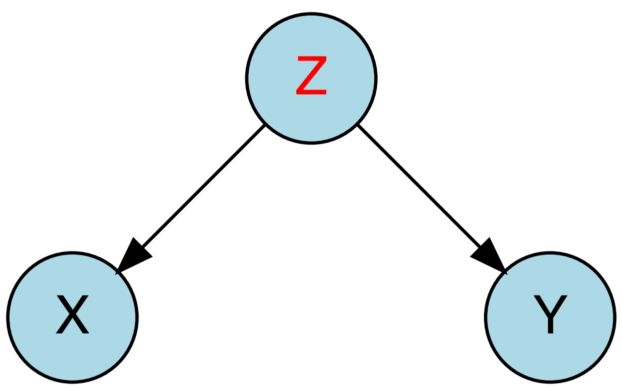
D and E are independent from each other conditioned on C

they aren't connected via a path

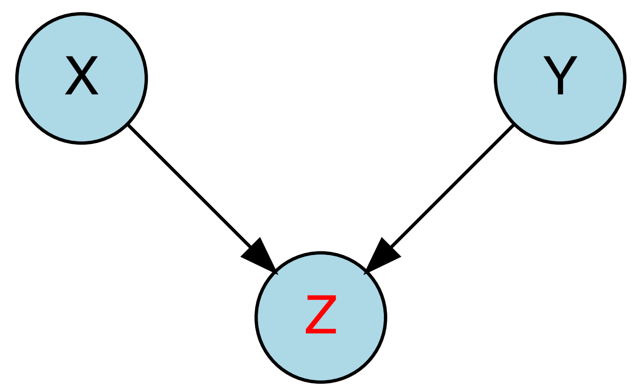
So what?

Patterns of inference

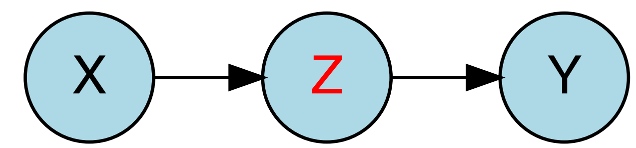
We want to estimate the (causal) relationship between X and Y



common cause



common effect



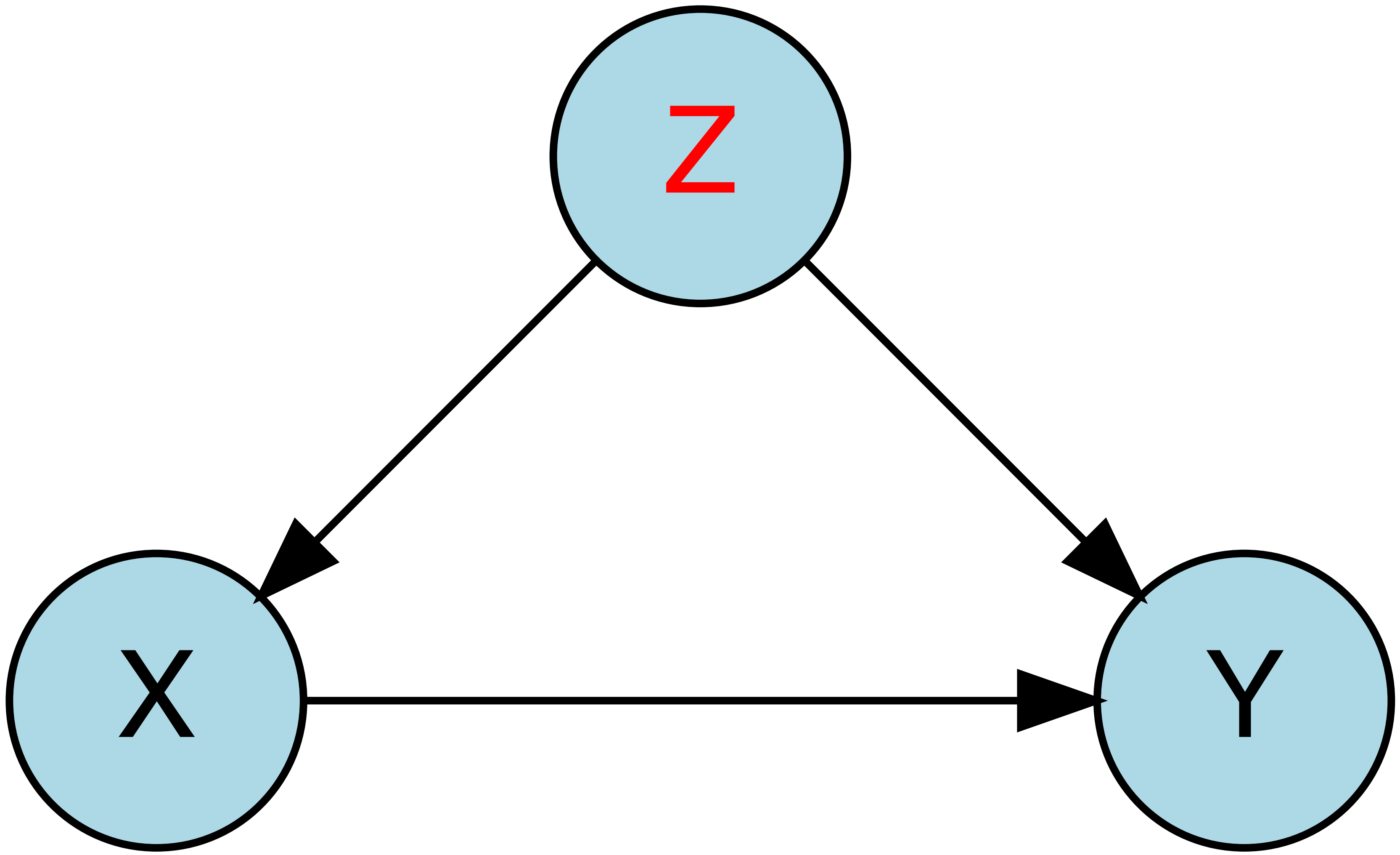
causal chain

by controlling for Z we hope to get a better estimate of the relationship between X and Y

d-separation helps us tell apart **good controls** from **bad controls**

When should I control for variables?

I want to estimate the effect that X has on Y



Is Z a **good** or a **bad** control here?

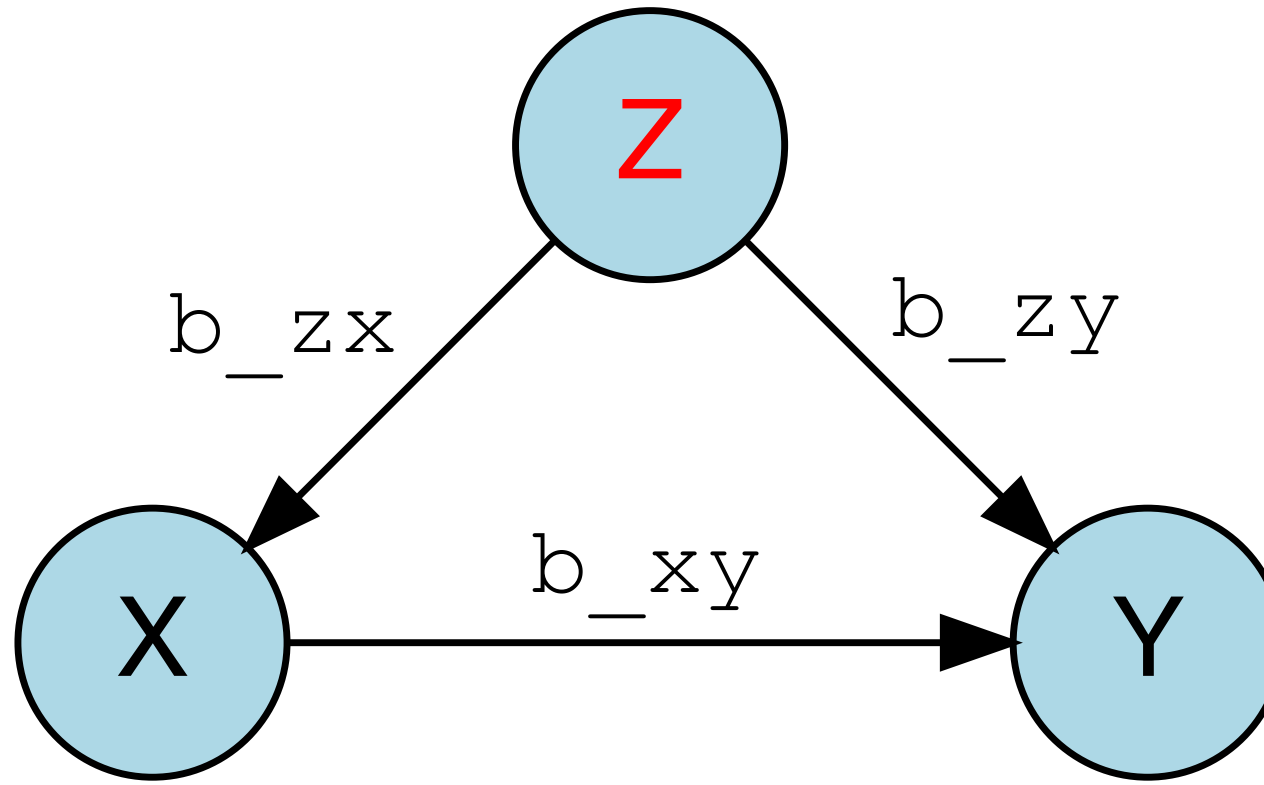
When should I control for variables?

```r
set.seed(1)

n = 1000
b_zx = 2
b_xy = 2
b_zy = 2
sd = 1

fun_error = function(n, sd){
  rnorm(n = n,
        mean = 0,
        sd = sd)
}

df = tibble(z = fun_error(n, sd),
            x = b_zx * z + fun_error(n, sd),
            y = b_zy * z + b_xy * x + fun_error(n, sd))
```



$$Y = b_0 + b_1 \cdot X + e$$

```r
# without control
lm(formula = y ~ x,
   data = df) %>%
  summary()
```

overestimating X's effect on Y

```
Call:
lm(formula = y ~ x, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.6011 -0.9270 -0.0506  0.9711  4.0454 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.02449    0.04389   0.558    0.577    
x            2.82092    0.01890 149.225   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.388 on 998 degrees of freedom
Multiple R-squared:  0.9571,	Adjusted R-squared:  0.9571 
F-statistic: 2.227e+04 on 1 and 998 DF,  p-value: < 2.2e-16
```

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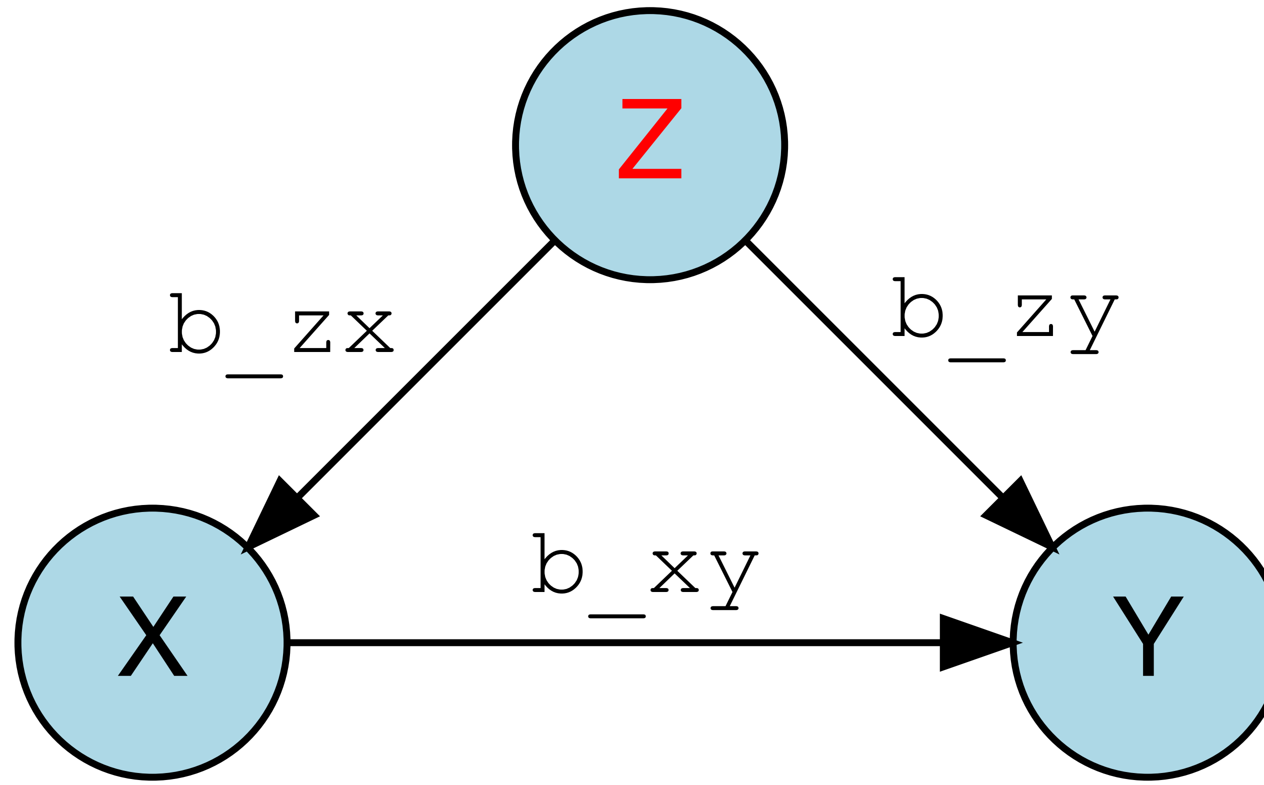
        sd = sd)
}

df = tibble(z = fun_error(n, sd),
            x = b_zx * z + fun_error(n, sd),
            y = b_zy * z + b_xy * x + fun_error(n, sd))
```



$$Y = b_0 + b_1 \cdot X + b_2 \cdot Z + e$$

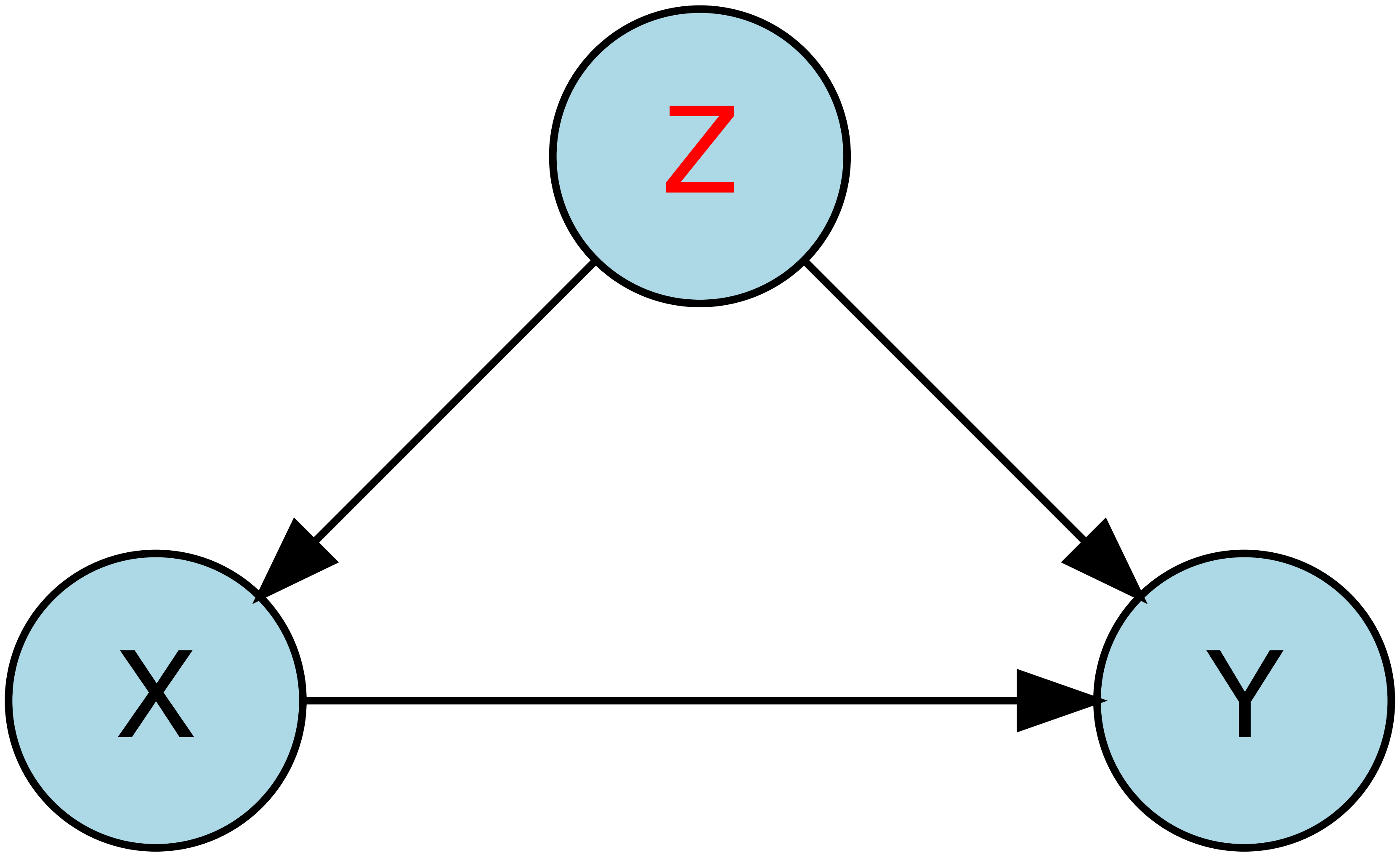
```r
# with control
lm(formula = y ~ x + z,
   data = df) %>%
  summary()
```

accurate estimate of X's effect on Y

```
Call:
lm(formula = y ~ x + z, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.6151 -0.6564 -0.0223  0.6815  2.8132 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.01624    0.03260   0.498    0.618    
x            2.02202    0.03135  64.489   <2e-16 ***
z            2.00501    0.07036  28.497   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.031 on 997 degrees of freedom
Multiple R-squared:  0.9764,	Adjusted R-squared:  0.9763 
F-statistic: 2.059e+04 on 2 and 997 DF,  p-value: < 2.2e-16
```

When should I control for variables?

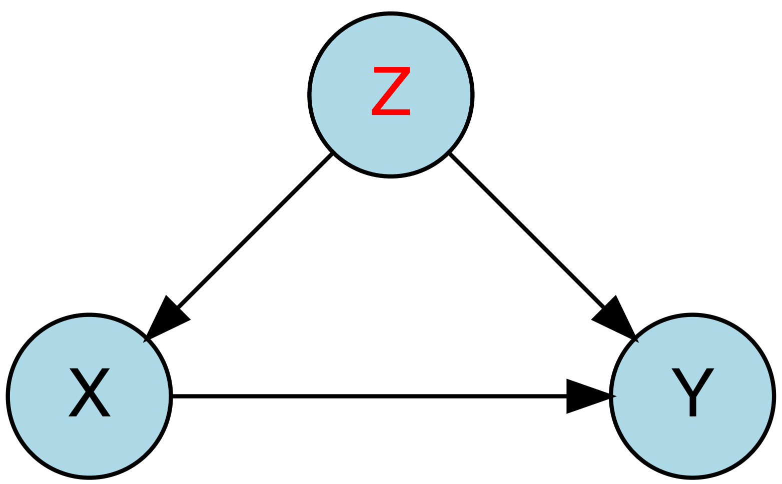
I want to estimate the effect that X has on Y



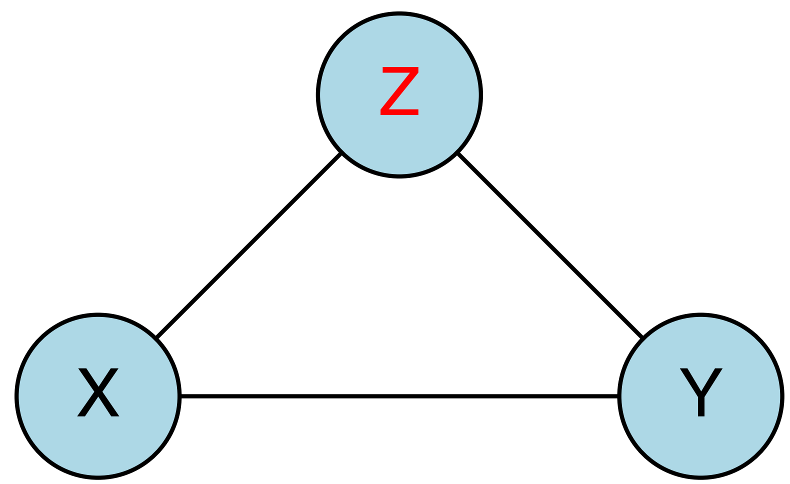
Z is a **good** control here!

When should I control for variables?

I want to estimate the effect that X has on Y

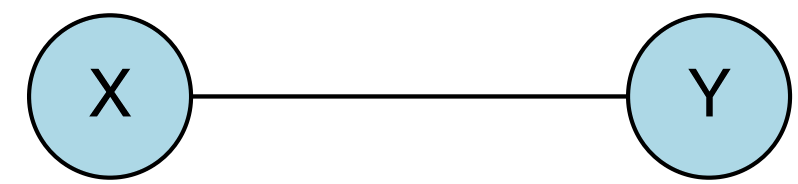


not controlling for Z



multiple paths

controlling for Z

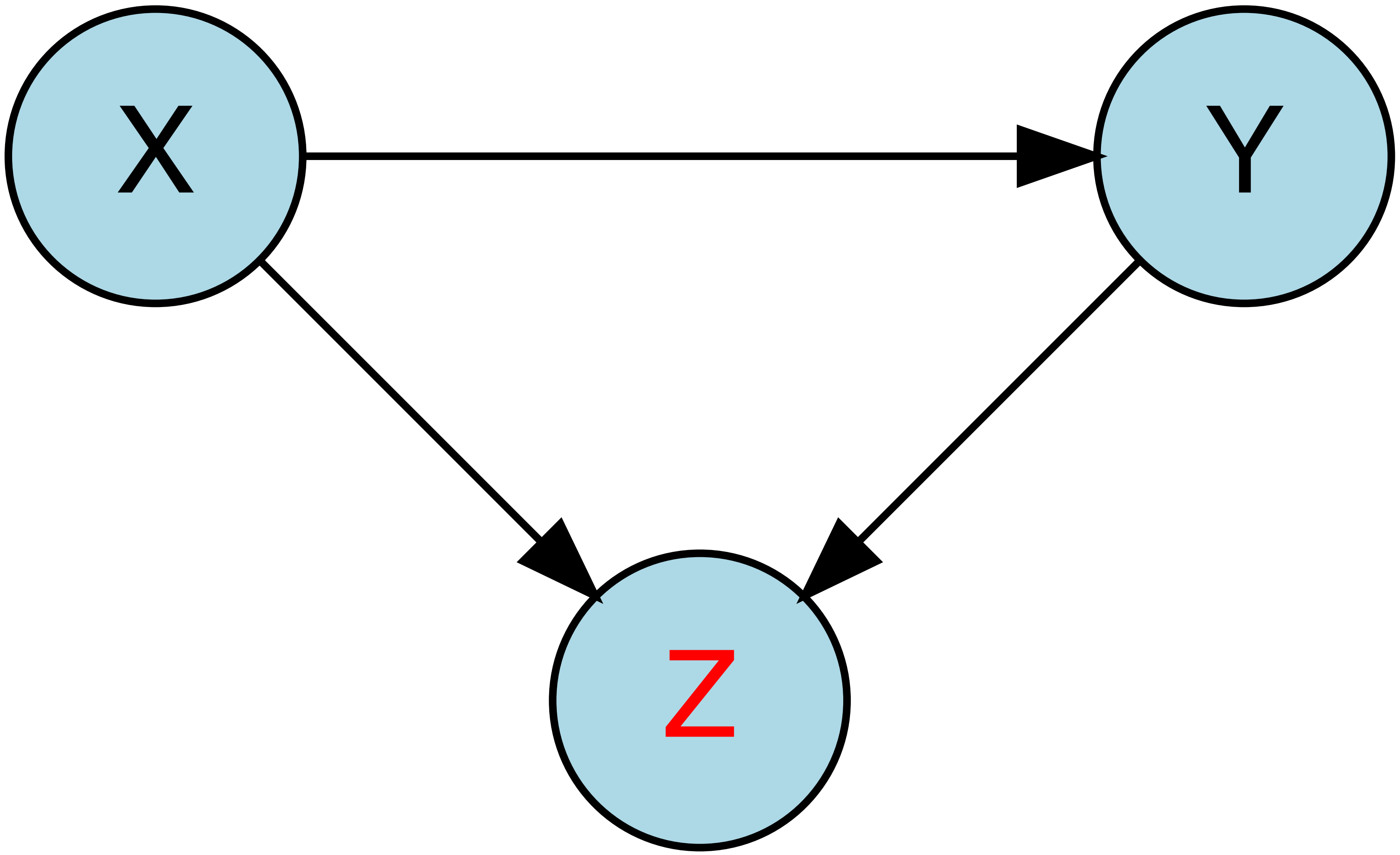


one path

Z is a **good** control here!

When should I control for variables?

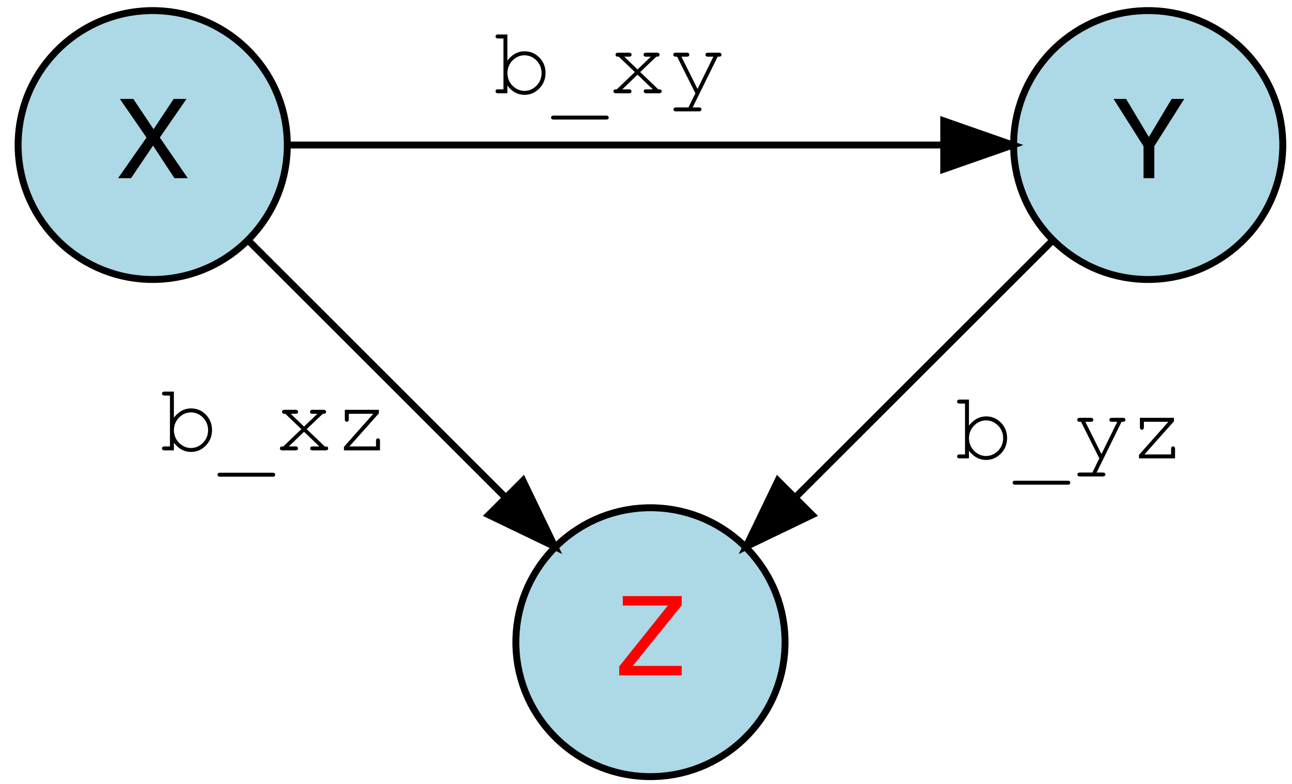
I want to estimate the effect that X has on Y



Is Z a **good** or a **bad** control here?

When should I control for variables?

```r
set.seed(1)
n = 1000
b_xy = 2
b_xz = 2
b_yz = 2
sd = 1

fun_error = function(n, sd){
  rnorm(n = n,
        mean = 0,
        sd = sd)
}

df = tibble(x = fun_error(n, sd),
            y = x * b_xy + fun_error(n, sd),
            z = x * b_xz + y * b_yz + fun_error(n, sd))
```



$$Y = b_0 + b_1 \cdot X + e$$

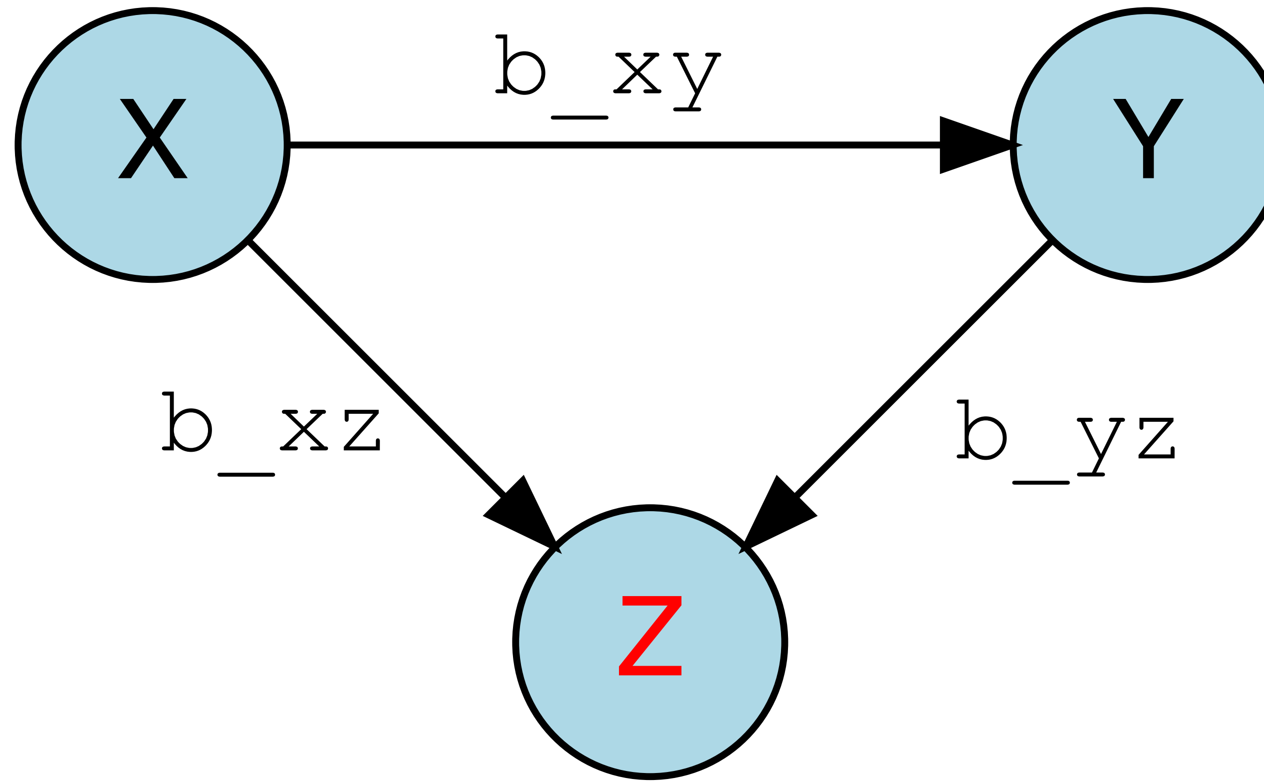
```r
# without control
lm(formula = y ~ x,
   data = df) %>%
  summary()
```

accurate estimate of X's effect on Y

```
Call:
lm(formula = y ~ x, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.2484 -0.6720 -0.0138  0.7554  3.6443 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.01619    0.03290  -0.492    0.623    
x            2.00643    0.03181  63.078   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.04 on 998 degrees of freedom
Multiple R-squared:  0.7995,	Adjusted R-squared:  0.7993 
F-statistic:  3979 on 1 and 998 DF,  p-value: < 2.2e-16
```

When should I control for variables?

```r
set.seed(1)
n = 1000
b_xy = 2
b_xz = 2
b_yz = 2
sd = 1

fun_error = function(n, sd){
  rnorm(n = n,
        mean = 0,
        sd = sd)
}

df = tibble(x = fun_error(n, sd),
            y = x * b_xy + fun_error(n, sd),
            z = x * b_xz + y * b_yz + fun_error(n, sd))
```



$$Y = b_0 + b_1 \cdot X + b_2 \cdot Z + e$$

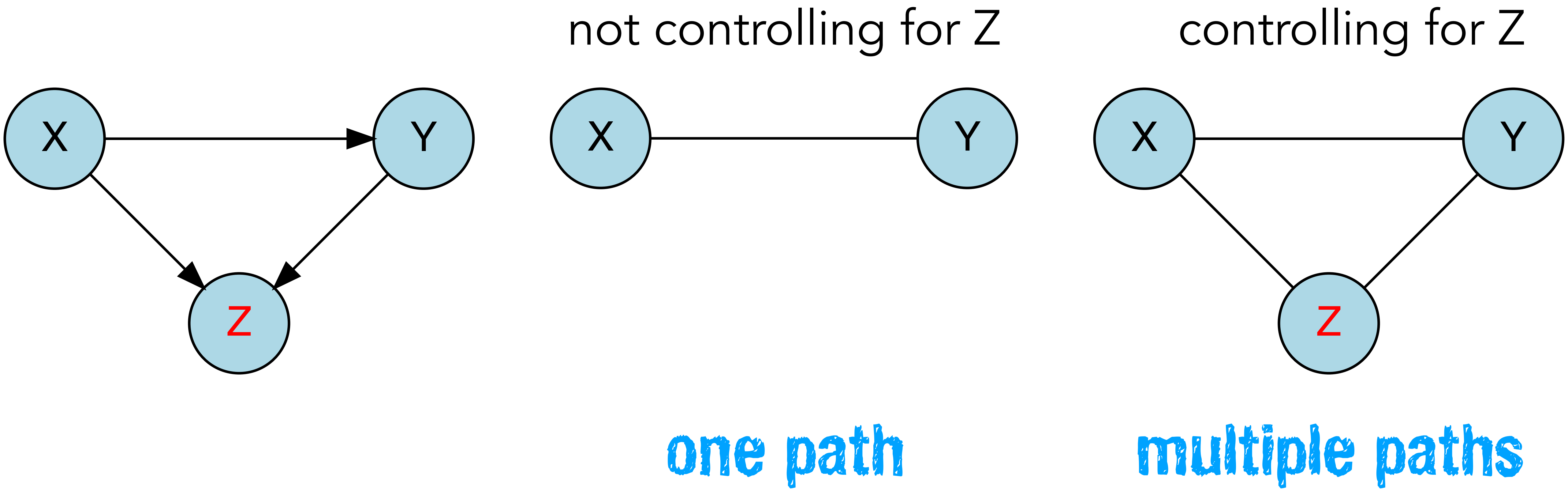
```r
# with control
lm(formula = y ~ x + z,
   data = df) %>%
  summary()
```

in accurate estimate of X's effect on Y

```
Call:
lm(formula = y ~ x + z, data = df)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.35547 -0.30016  0.00298  0.31119  1.73408 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.009608   0.014477  -0.664    0.507    
x           -0.411847   0.040026 -10.290   <2e-16 ***
z            0.398921   0.006186  64.489   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4578 on 997 degrees of freedom
Multiple R-squared:  0.9612,	Adjusted R-squared:  0.9611 
F-statistic: 1.236e+04 on 2 and 997 DF,  p-value: < 2.2e-16
```

When should I control for variables?

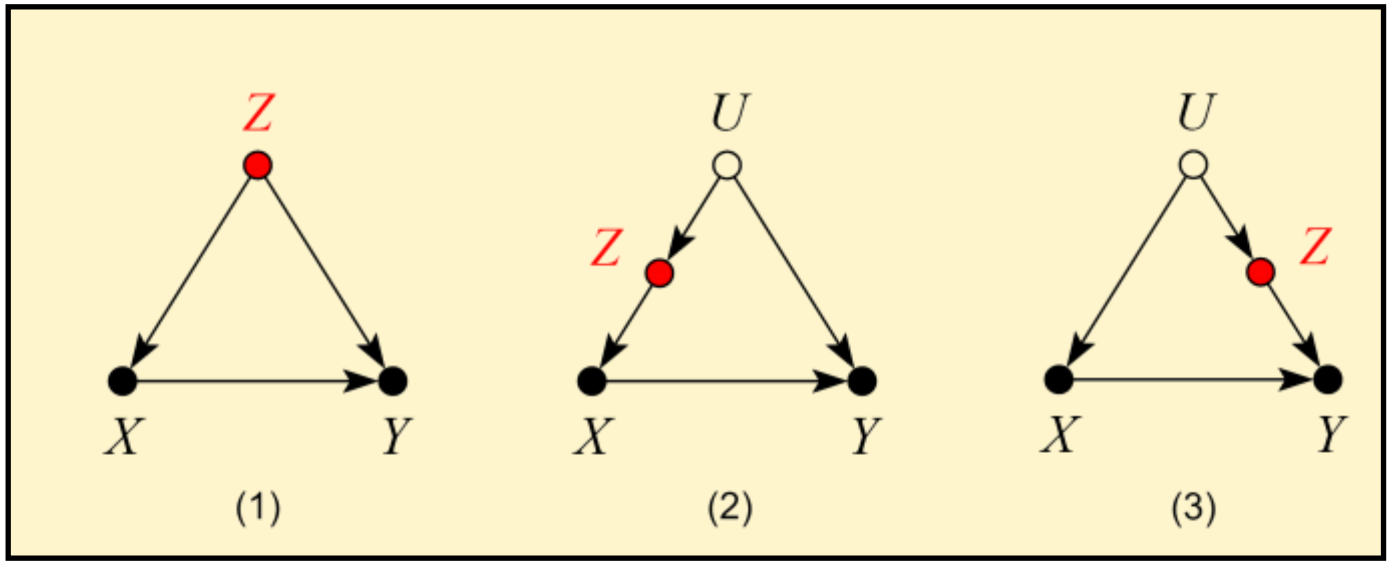
I want to estimate the effect that X has on Y



Z is a **bad** control here!

When should I control for variables?

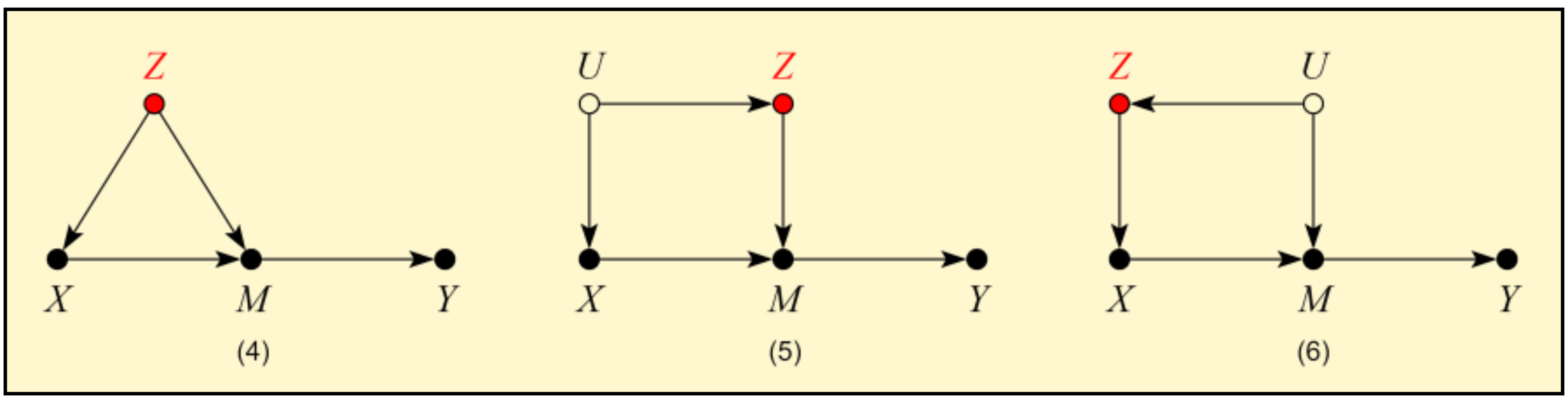
I want to estimate the effect that X has on Y



Good controls

U = unobserved variable

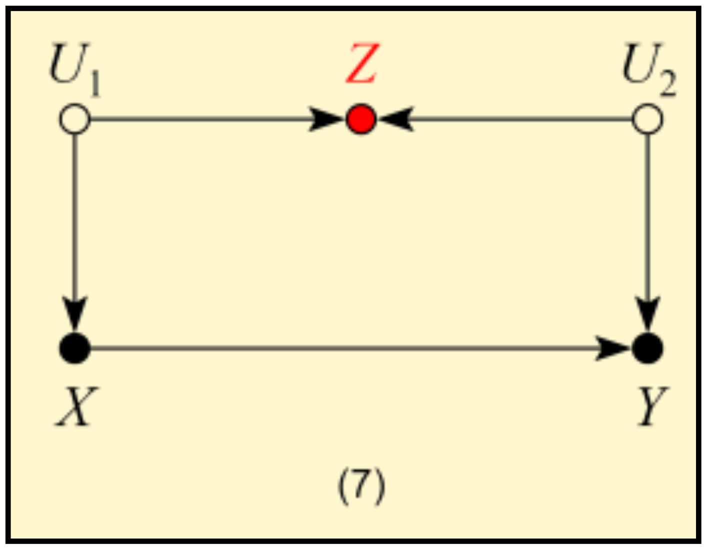
M = mediating variable



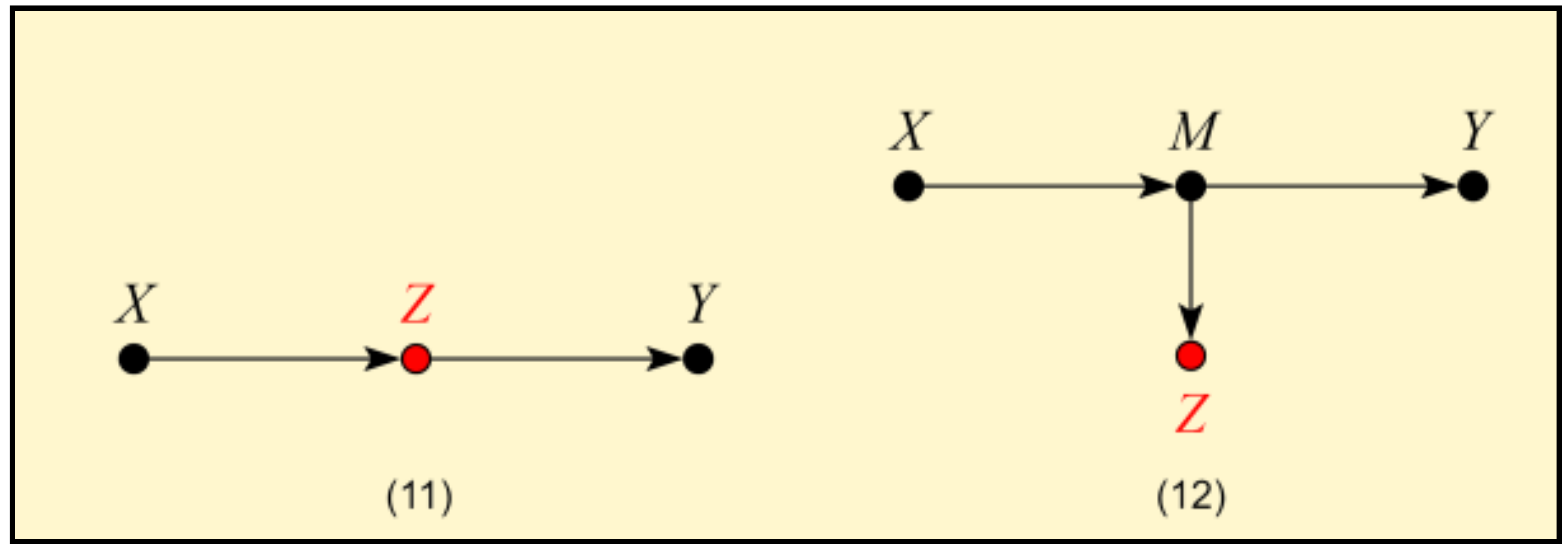
http://causality.cs.ucla.edu/blog/index.php/2019/08/14/a-crash-course-in-good-and-bad-control/

When should I control for variables?

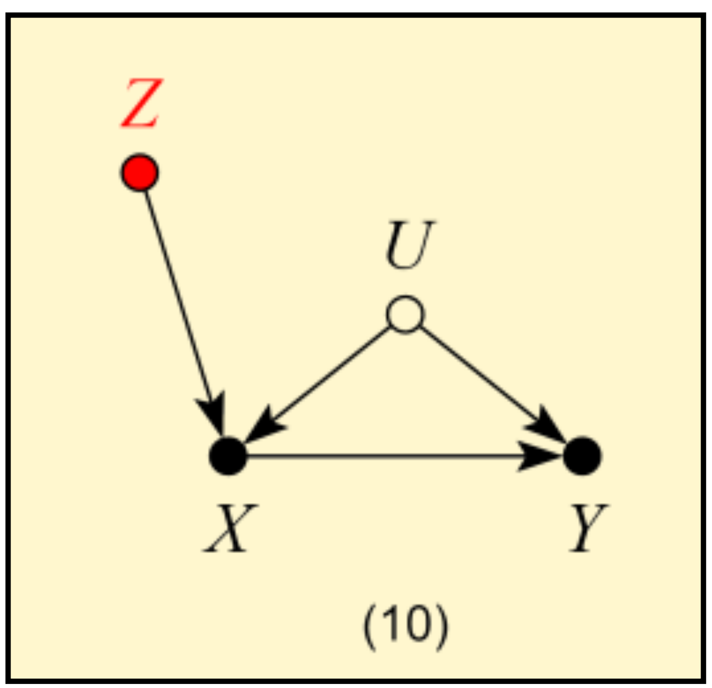
I want to estimate the effect that X has on Y



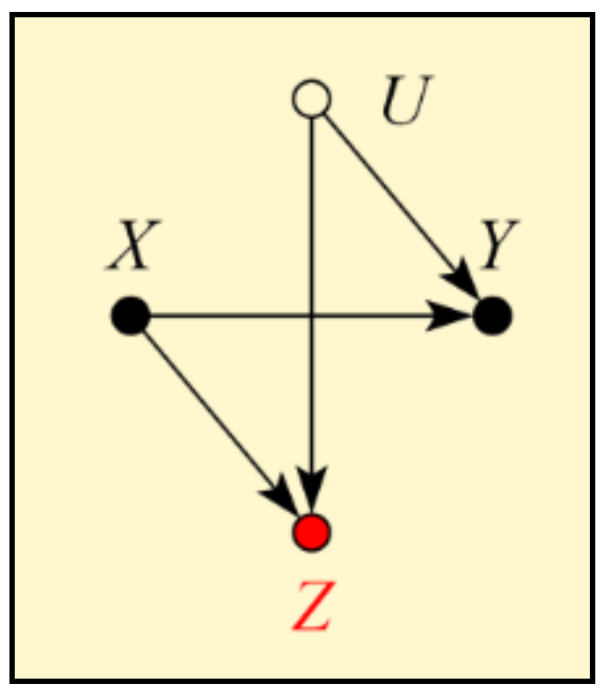
(7)

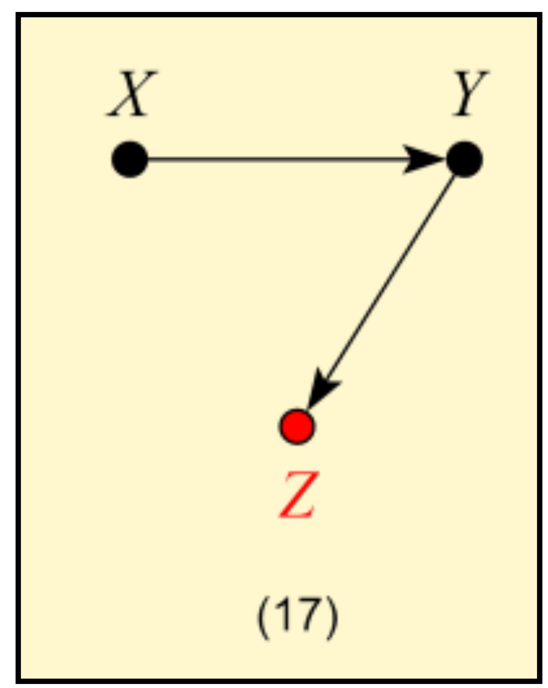


(11) (12)



(10)





(17)

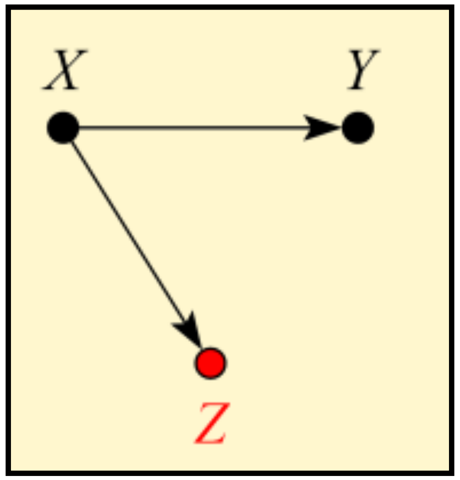
Bad controls

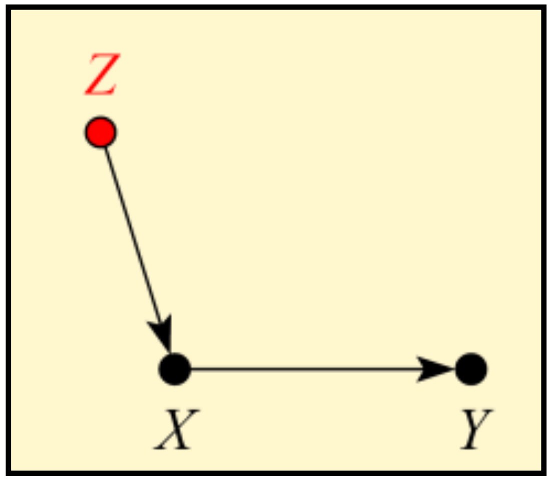
http://causality.cs.ucla.edu/blog/index.php/2019/08/14/a-crash-course-in-good-and-bad-control/

When should I control for variables?

I want to estimate the effect that X has on Y

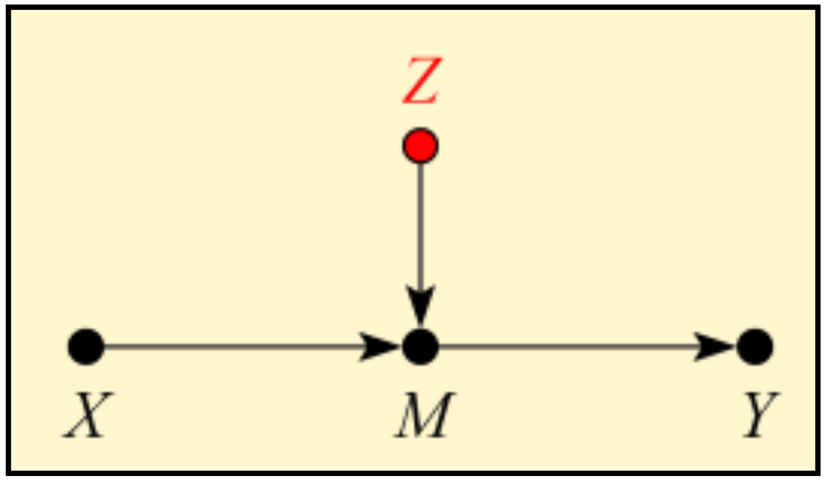
possibly bad for precision

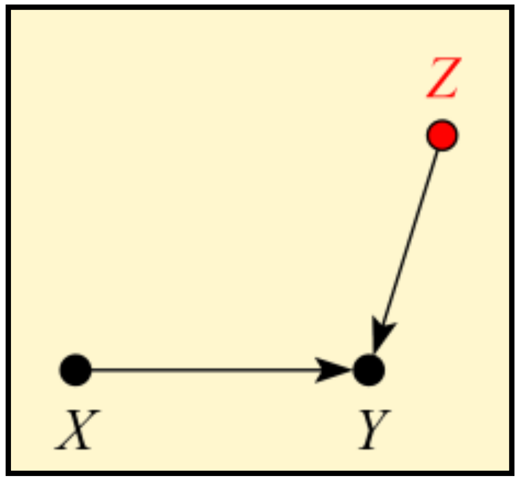




conditioning on Z reduces X's variance

possibly good for precision





conditioning on Z reduces variance of mediator or outcome variable

Neutral controls

don't introduce any bias

http://causality.cs.ucla.edu/blog/index.php/2019/08/14/a-crash-course-in-good-and-bad-control/

When should I control for variables?

Babies Learning Language

Thoughts on language learning, child development, and fatherhood; experimental methods, reproducibility, and open science; theoretical musings on cognitive science more broadly.

Tuesday, October 8, 2019

Confounds and covariates

(tl;dr: explanation of confounding and covariate adjustment)

Every year, one of the trickiest concepts for me to teach in my experimental methods course is the difference between experimental confounds and covariates. Although this distinction seems simple, it's pretty deeply related to the definition of what an experiment is and why experiments lead to good causal inferences. It's also caught up in a number of methodological problems that come up again and again in my class. This post is my attempt to explain the distinction and how it relates to different problems and cultural practices in psychology.

About

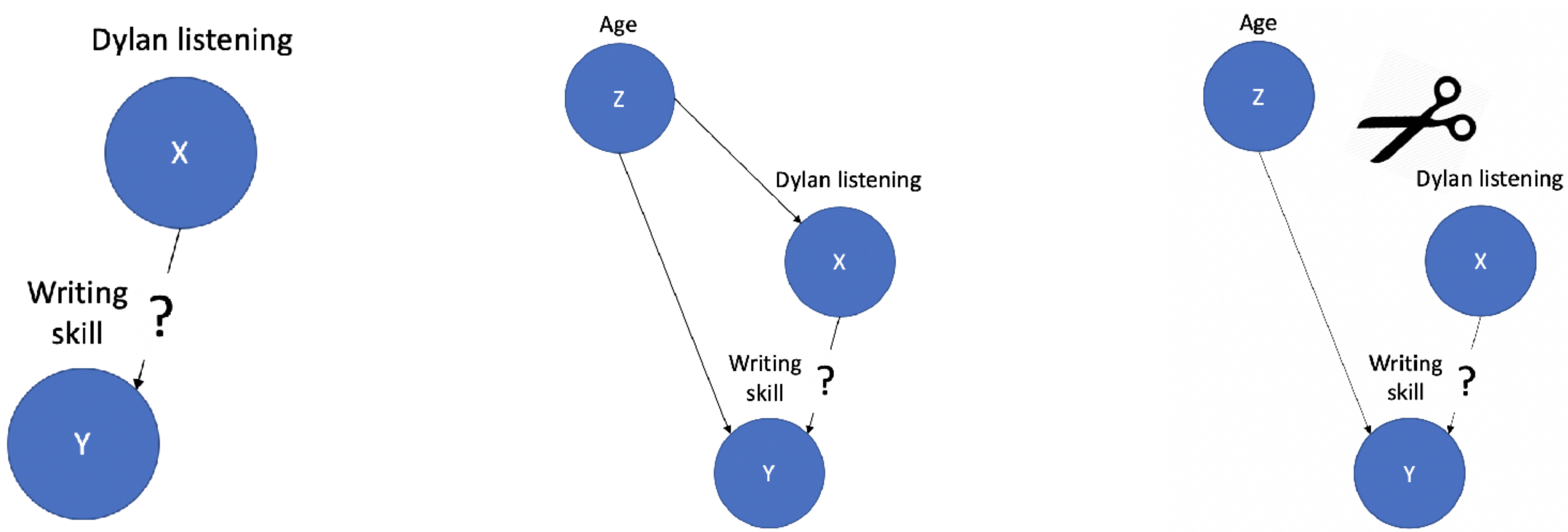
Michael Frank

Developmental Psychologist at Stanford University. Interested in language acquisition, cognitive development, thoughts, numbers, learning, and teaching. Visit my lab's website.

View my complete profile

Labels

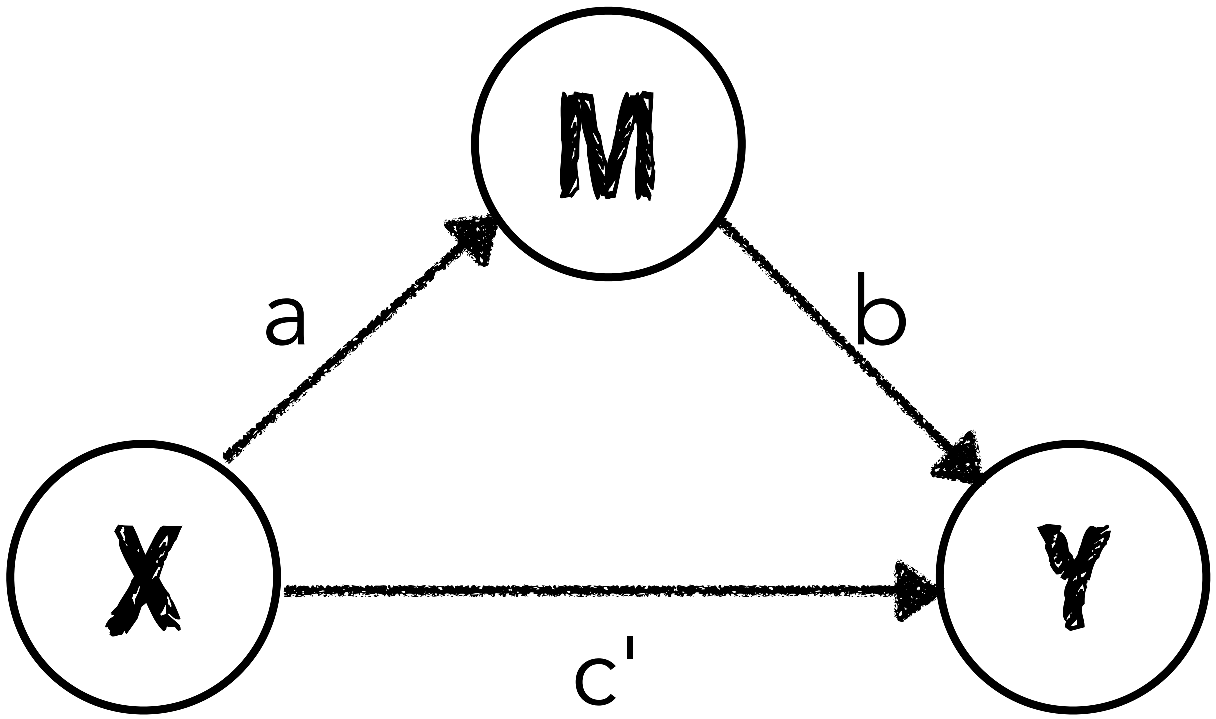
- Development (40)



http://babieslearninglanguage.blogspot.com/2019/10/confounds-and-covariates.html

Mediation

Definition

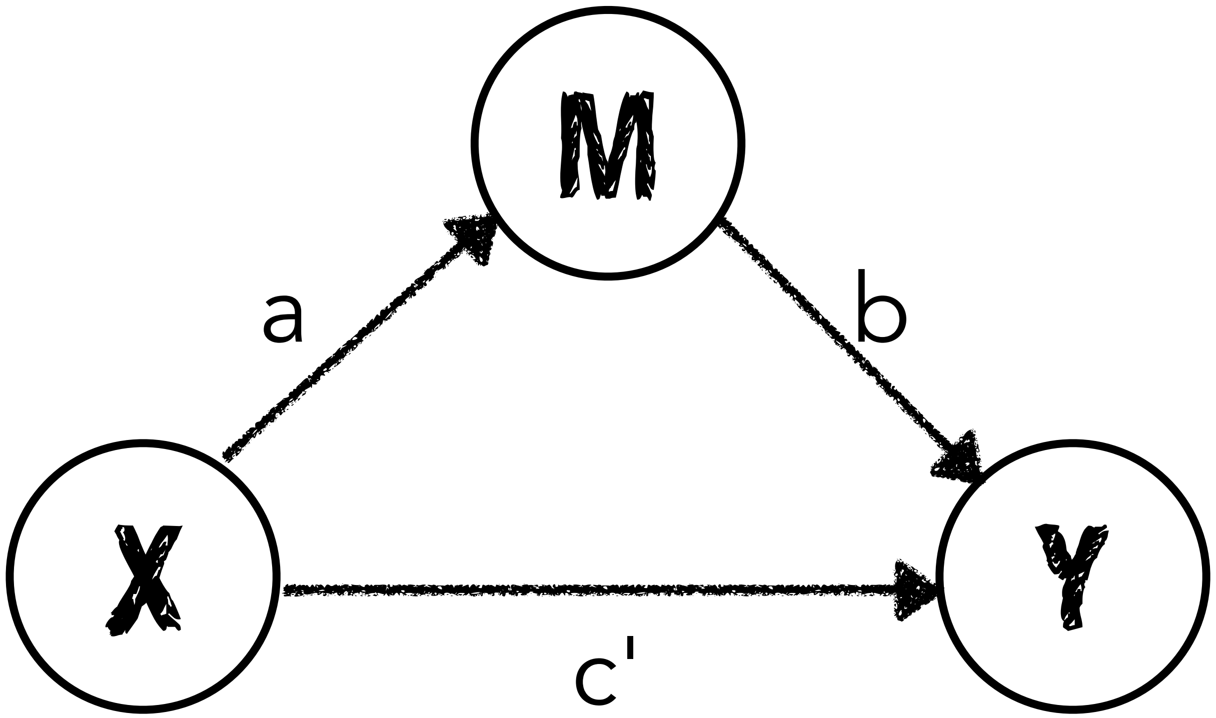


Rather than a direct causal relationship between **X** and **Y**, a mediation model proposes that **X** influences the mediator variable **M**, which in turn influences **Y**. Thus, the mediator variable serves to clarify the nature of the relationship between **X** and **Y**.

Adapted from Wikipedia

https://en.wikipedia.org/wiki/Mediation_(statistics)

Example



X = grades in Psych 252

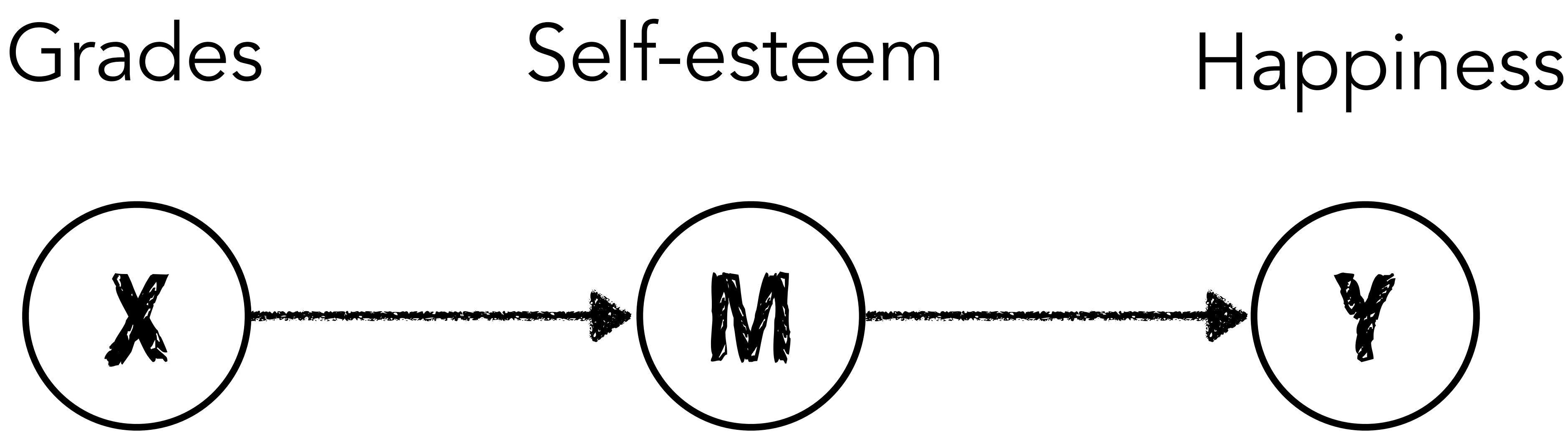
M = feelings of self-esteem

Y = happiness

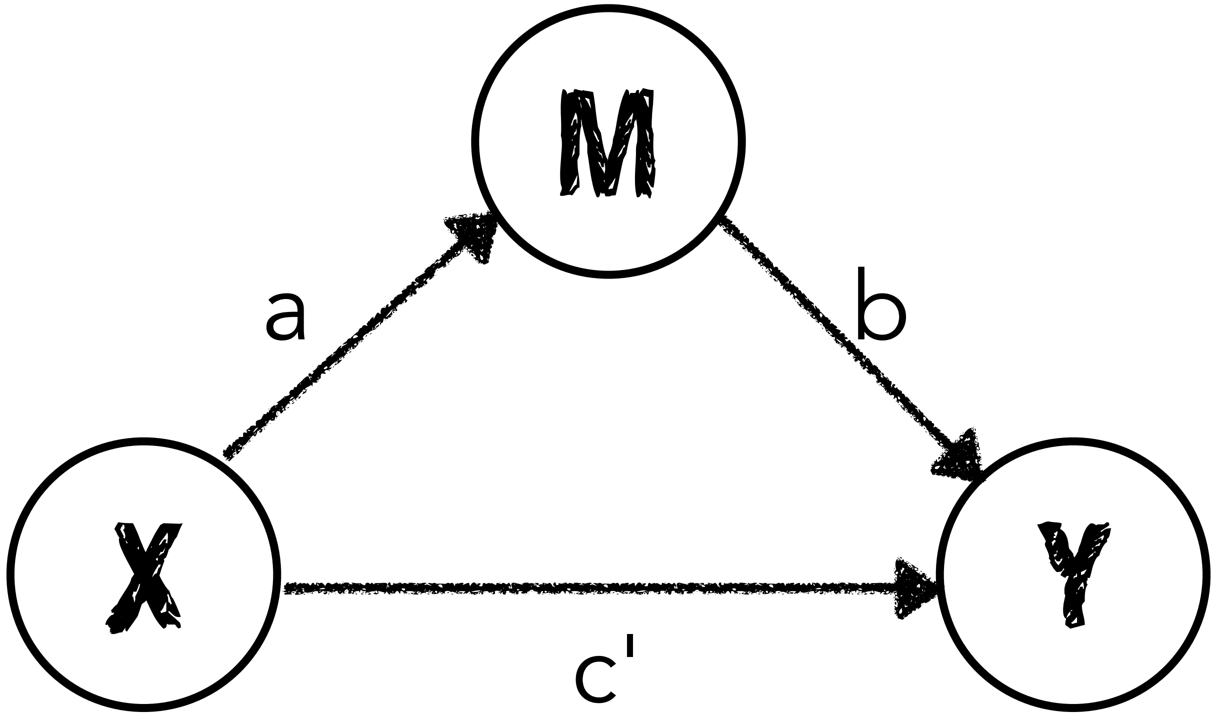
Is the relationship between grades in Psych 252 and happiness mediated by feelings of self-esteem?

Simulate a mediation analysis

```r
# number of participants
n = 100

# generate data
df.mediation = tibble(
  x = rnorm(n, 75, 7),                  # grades
  m = 0.7 * x + rnorm(n, 0, 5), # self-esteem
  y = 0.4 * m + rnorm(n, 0, 5)   # happiness
)
```



Baron and Kenny's (1986) steps for mediation



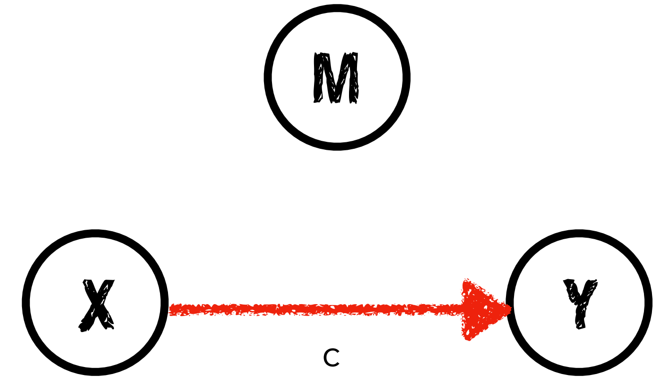
Sequence of regression models

1. Is there a relationship between **X** and **Y**?
2. Is there a relationship between **X** and **M**?
3. Does the relationship between **X** and **Y** change, once we control for **M**?

Baron, R. M. & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology*, 51(6), 1173-1182.

Is there a relationship between **X** and **Y**?

$$\hat{y} = b_0 + b_1 \cdot x$$



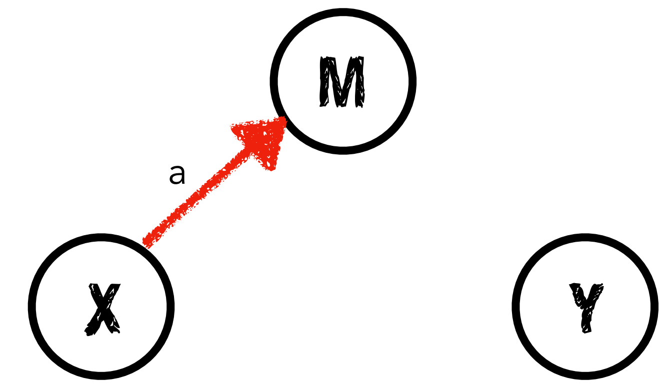

```r
# fit the model
fit.y_x = lm(formula = y ~ 1 + x,
             data = df.mediation)

# summarize the results
fit.y_x %>% summary()
```

```
Call:
lm(formula = y ~ 1 + x, data = df.mediation)

Residuals:
    Min      1Q  Median      3Q     Max
-10.917  -3.738  -0.259   2.910  12.540

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  8.78300    6.16002   1.426   0.1571
x            0.16899    0.08116   2.082   0.0399 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.16 on 98 degrees of freedom
Multiple R-squared:  0.04237,	Adjusted R-squared:  0.0326
F-statistic: 4.336 on 1 and 98 DF,  p-value: 0.03993
```

significant relationship

Is there a relationship between **X** and **M**?

$$\hat{m} = b_0 + b_1 \cdot x$$



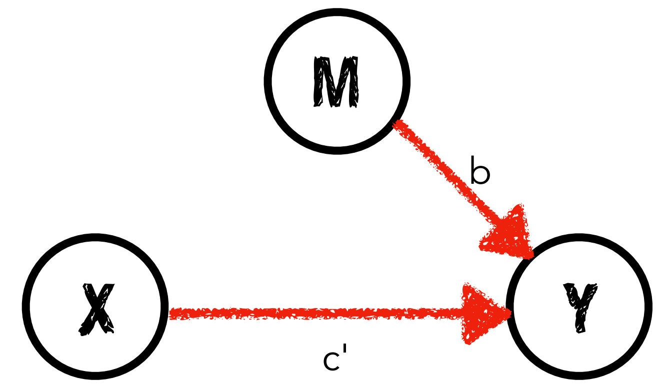

```r
# fit the model
fit.m_x = lm(formula = m ~ 1 + x,
             data = df.mediation)

# summarize the results
fit.m_x %>% summary()
```

```
Call:
lm(formula = m ~ 1 + x, data = df.mediation)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.5367 -3.4175 -0.4375  2.9032 16.4520 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  6.04494   13.41692   0.451    0.653    
x            0.66252    0.07634   8.678 8.87e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.854 on 98 degrees of freedom
Multiple R-squared:  0.4346,	Adjusted R-squared:  0.4288 
F-statistic: 75.31 on 1 and 98 DF,  p-value: 8.872e-14
```

significant relationship

Is there a relationship between **X** and **Y,** controlling for **M**?

$$\hat{y} = b_0 + b_1 \cdot m + b_2 \cdot x$$

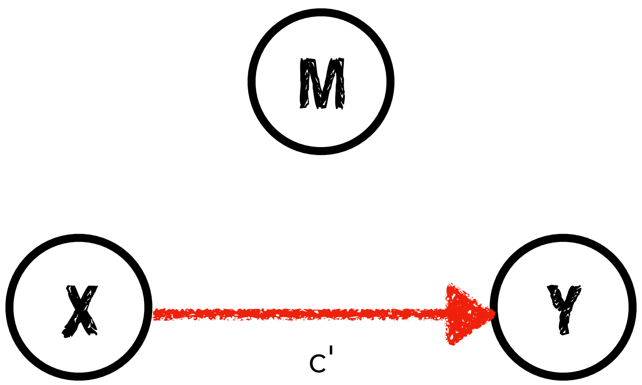



```
# fit the model
fit.y_mx = lm(formula = m ~ 1 + m + x,
              data = df.mediation)

# summarize the results
fit.y_mx %>% summary()
```

```
Call:
lm(formula = y ~ 1 + m + x, data = df.mediation)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.3651 -3.3037 -0.6222  3.1068 10.3991 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  7.80952    5.68297   1.374    0.173    
m            0.42381    0.09899   4.281 4.37e-05 ***
x           -0.11179    0.09949  -1.124    0.264    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.756 on 97 degrees of freedom
Multiple R-squared:  0.1946,	Adjusted R-squared:  0.1779 
F-statistic: 11.72 on 2 and 97 DF,  p-value: 2.771e-05
```

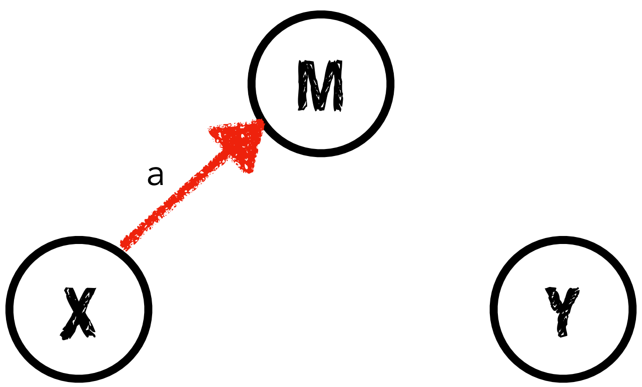
not significant

3 Step procedure



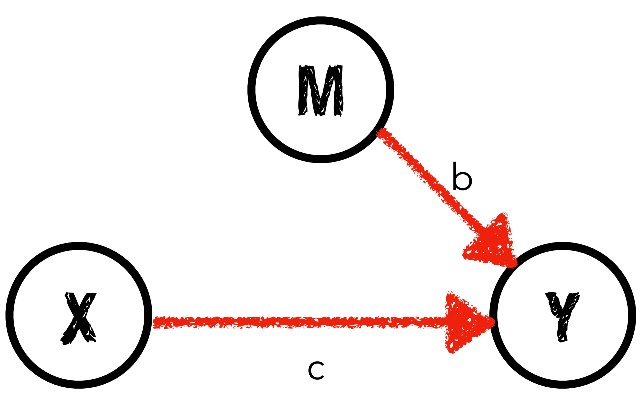
Relationship between **X** and **Y**?

$$\hat{y} = b_0 + \boxed{b_1 \cdot x}$$



Relationship between **X** and **M**?

$$\hat{m} = b_0 + b_1 \cdot x$$



Relationship between **X** and **Y**, controlling for **M**?

$$\hat{y} = b_0 + b_1 \cdot m + \boxed{b_2 \cdot x}$$

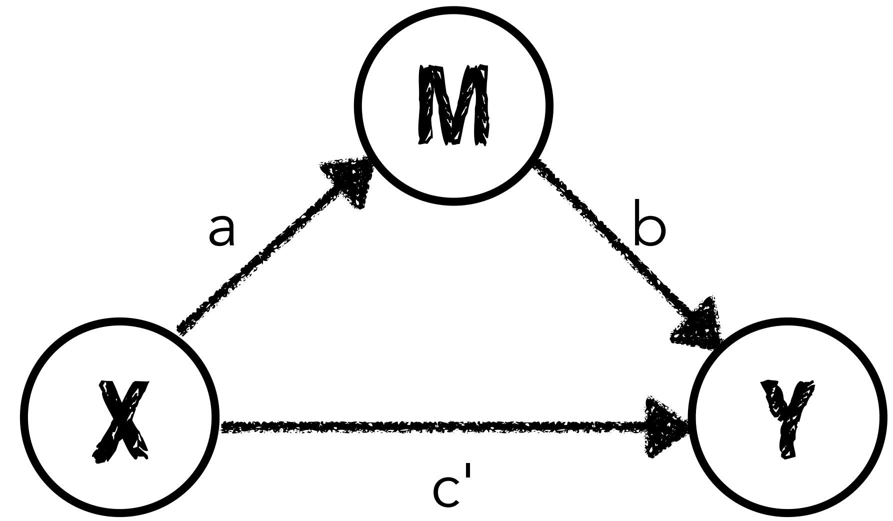
significant change?

just because it changes from significant to not significant, does not mean the change was significant!

Is the mediation significant?

1. **Sobel test**
   - assumes normally distributed data
   - has low power
2. **Bootstrapping**
   - no assumption about how the data is distributed
   - has higher power
3. **Bayesian mediation analysis**

1. Sobel test




```r
library("multilevel")

# run the sobel test
fit.sobel = sobel(pred = df.mediation$x,
                  med = df.mediation$m,
                  out = df.mediation$y)

# calculate the p-value
(1 - pnorm(fit.sobel$z.value))*2
```

product of the coefficients

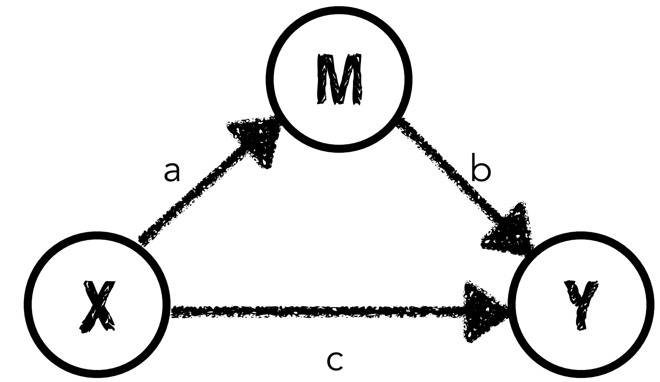
$$Z = \frac{ab}{a^2\sigma_b^2 + b^2\sigma_b^2 + \sigma_a^2\sigma_b^2}$$

standard error of a x b

"It is becoming increasingly more difficult to publish tests of mediation based purely on the Baron and Kenny method or tests that make distributional assumptions such as the Sobel test."

A Wikipedia person

2. Bootstrapping




```r
library("mediation")

# bootstrapped mediation
fit.mediation = mediate(model.m = fit.m_x,
                        model.y = fit.y_mx,
                        treat = "x",
                        mediator = "m",
                        boot = T)

# summarize results
fit.mediation %>% summary()
```

$$\hat{m} = b_0 + b_1 \cdot x$$

$$\hat{y} = b_0 + b_1 \cdot m + b_2 \cdot x$$

```
Causal Mediation Analysis

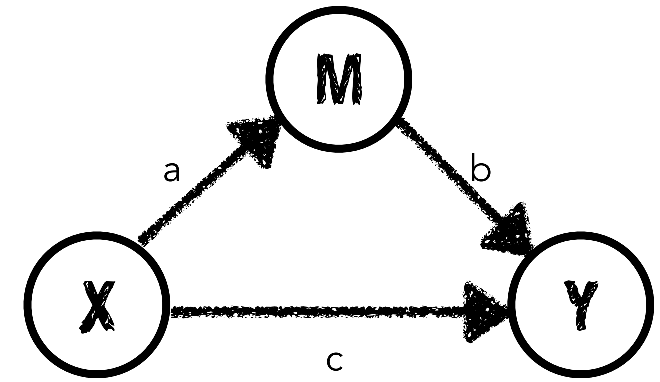
Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value    
ACME            0.28078      0.14059         0.42  <2e-16 ***
ADE            -0.11179     -0.29276         0.10   0.272    
Total Effect    0.16899     -0.00415         0.34   0.064 .  
Prop. Mediated  1.66151     -3.22476        11.46   0.064 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100


Simulations: 1000
```

2. Bootstrapping



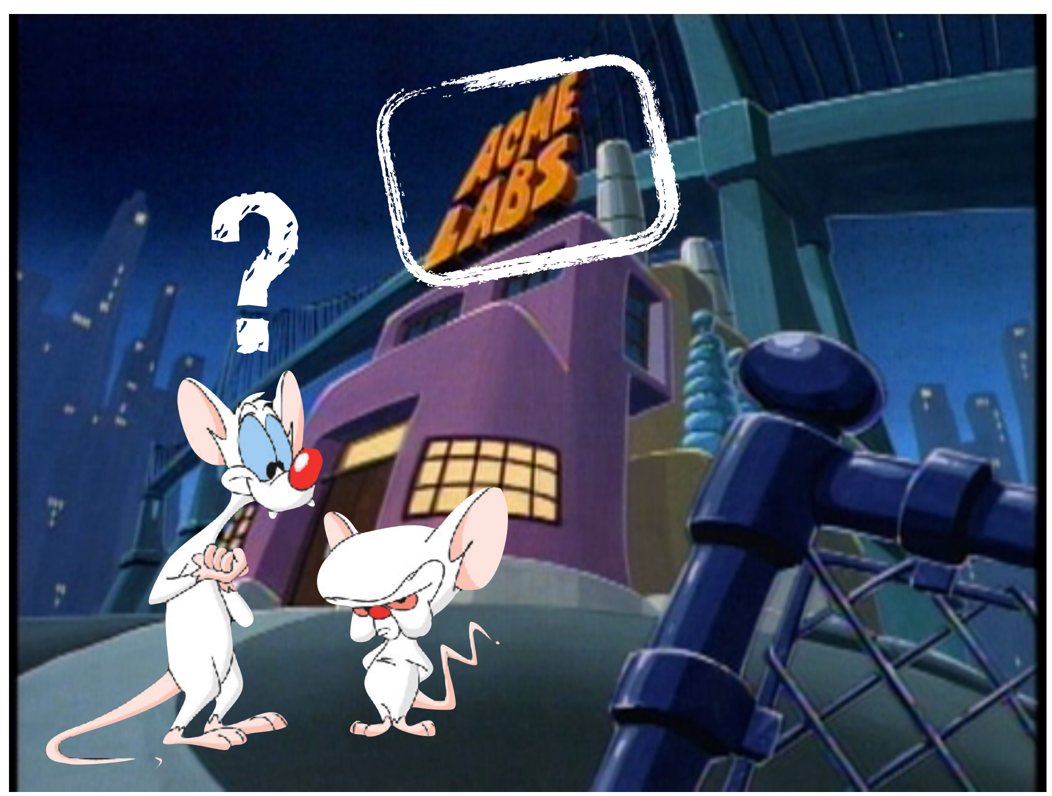

```
Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value    
ACME            0.28078      0.14059         0.42  <2e-16 ***
ADE            -0.11179     -0.29276         0.10   0.272    
Total Effect    0.16899     -0.00415         0.34   0.064 .  
Prop. Mediated  1.66151     -3.22476        11.46   0.064 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100


Simulations: 1000
```



2. Bootstrapping

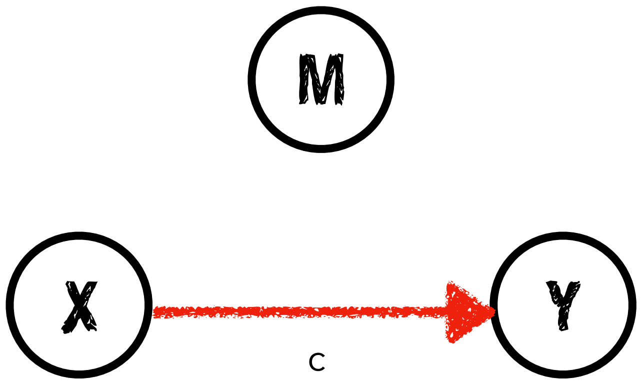
```
Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value    
ACME            0.28078      0.14059         0.42  <2e-16 ***
ADE            -0.11179     -0.29276         0.10   0.272    
Total Effect    0.16899     -0.00415         0.34   0.064 .  
Prop. Mediated  1.66151     -3.22476        11.46   0.064 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100


Simulations: 1000
```



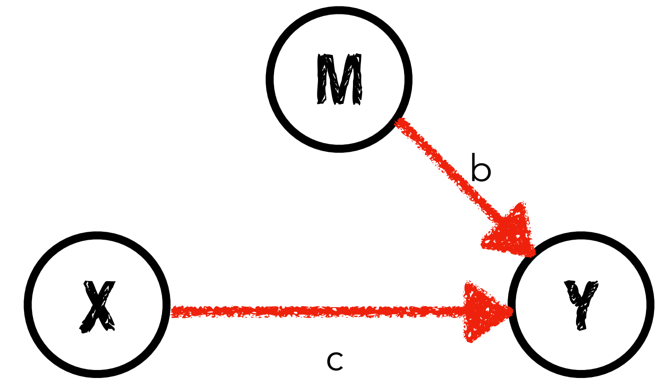
$$\hat{y} = b_0 + b_1 \cdot x$$

```
Call:
lm(formula = y ~ 1 + x, data = df.mediation)

Residuals:
    Min      1Q  Median      3Q     Max 
-10.917  -3.738  -0.259   2.910  12.540 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)  
(Intercept)  8.78300    6.16002   1.426   0.1571  
x            0.16899    0.08116   2.082   0.0399 *
```

2. Bootstrapping

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Causal Mediation Analysis

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---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100


Simulations: 1000
```

$$\hat{y} = b_0 + b_1 \cdot m + b_2 \cdot x$$ **ADE:** Average direct effect

```
Call:
lm(formula = y ~ 1 + m + x, data = df.mediation)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.3651 -3.3037 -0.6222  3.1068 10.3991 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  7.80952    5.68297   1.374    0.173    
m            0.42381    0.09899   4.281 4.37e-05 ***
x           -0.11179    0.09949  -1.124    0.264    
```

2. Bootstrapping

```
Causal Mediation Analysis

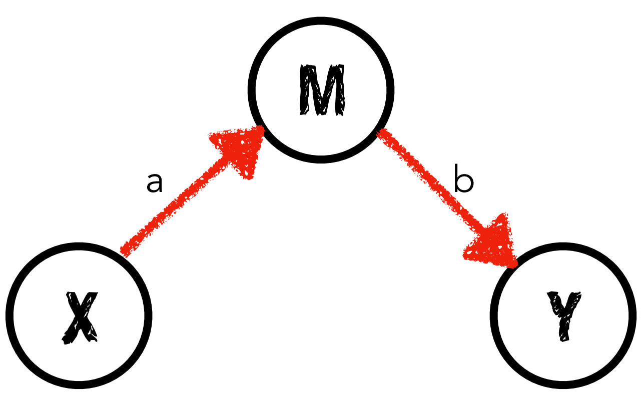
Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value    
ACME            0.28078      0.14059         0.42  <2e-16 ***
ADE            -0.11179     -0.29276         0.10   0.272    
Total Effect    0.16899     -0.00415         0.34   0.064 .  
Prop. Mediated  1.66151     -3.22476        11.46   0.064 .  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Sample Size Used: 100


Simulations: 1000
```

ACME



indirect effect: a * b

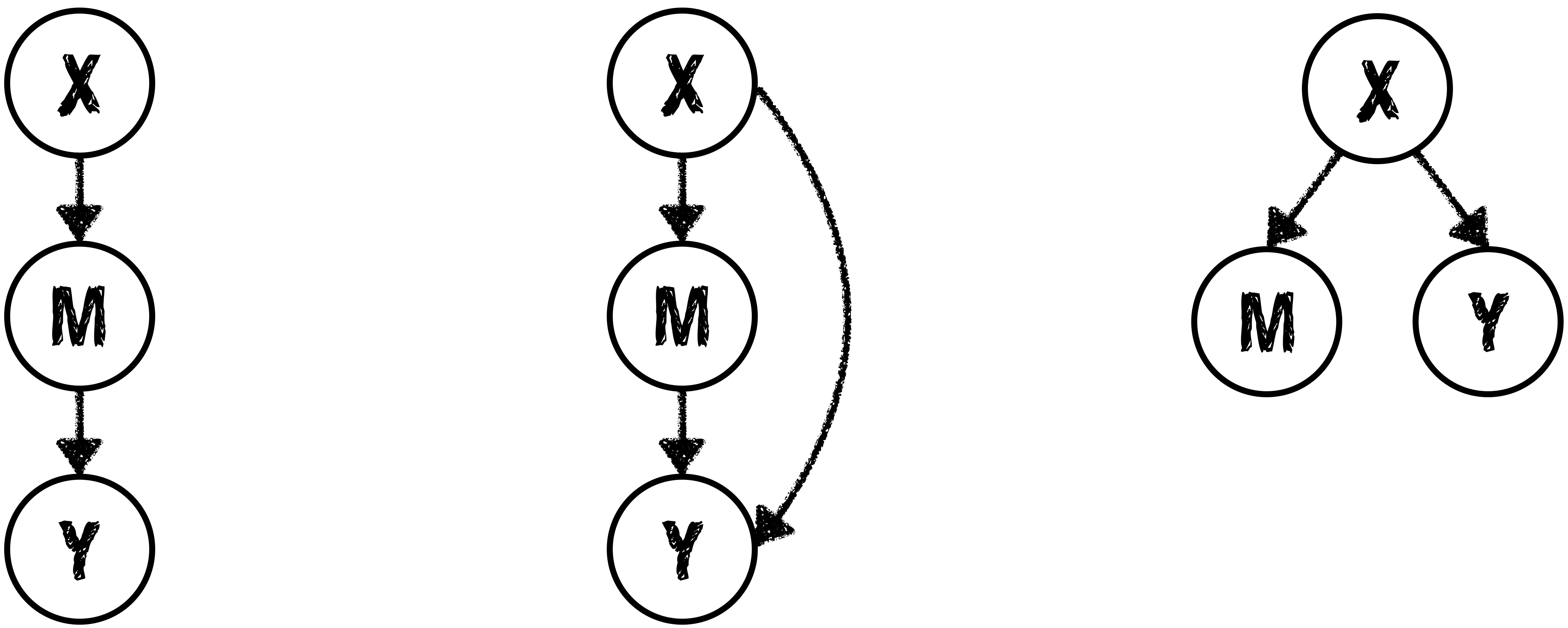
ADE: Average direct effect

ACME: Average causal mediation effect

ACME = Total effect - ADE

Underlying causal model

Full mediation **Partial mediation** **No mediation**



Full mediation: When the effect of **X** on **Y** completely disappears, **M** fully mediates between **X** and **Y**.

Partial mediation: When the effect of **X** on **Y** still exists, but in a smaller magnitude, **M** partially mediates between **X** and **Y**.

Limitations

- correlational analysis
  - we need theories / experiments to tease apart causes and effects to properly map our variables onto the diagram

Contents lists available at ScienceDirect

Journal of Experimental Social Psychology

journal homepage: www.elsevier.com/locate/jesp

Reports

What mediation analysis can (not) do

Klaus Fiedler [a,*], Malte Schott [a], Thorsten Meiser [b]

[a] University of Heidelberg, Germany
[b] University of Mannheim, Germany

Fiedler, K., Schott, M., & Meiser, T. (2011). What mediation analysis can (not) do. *Journal of Experimental Social Psychology*, 47(6), 1231-1236.

Limitations

many-to-one mapping

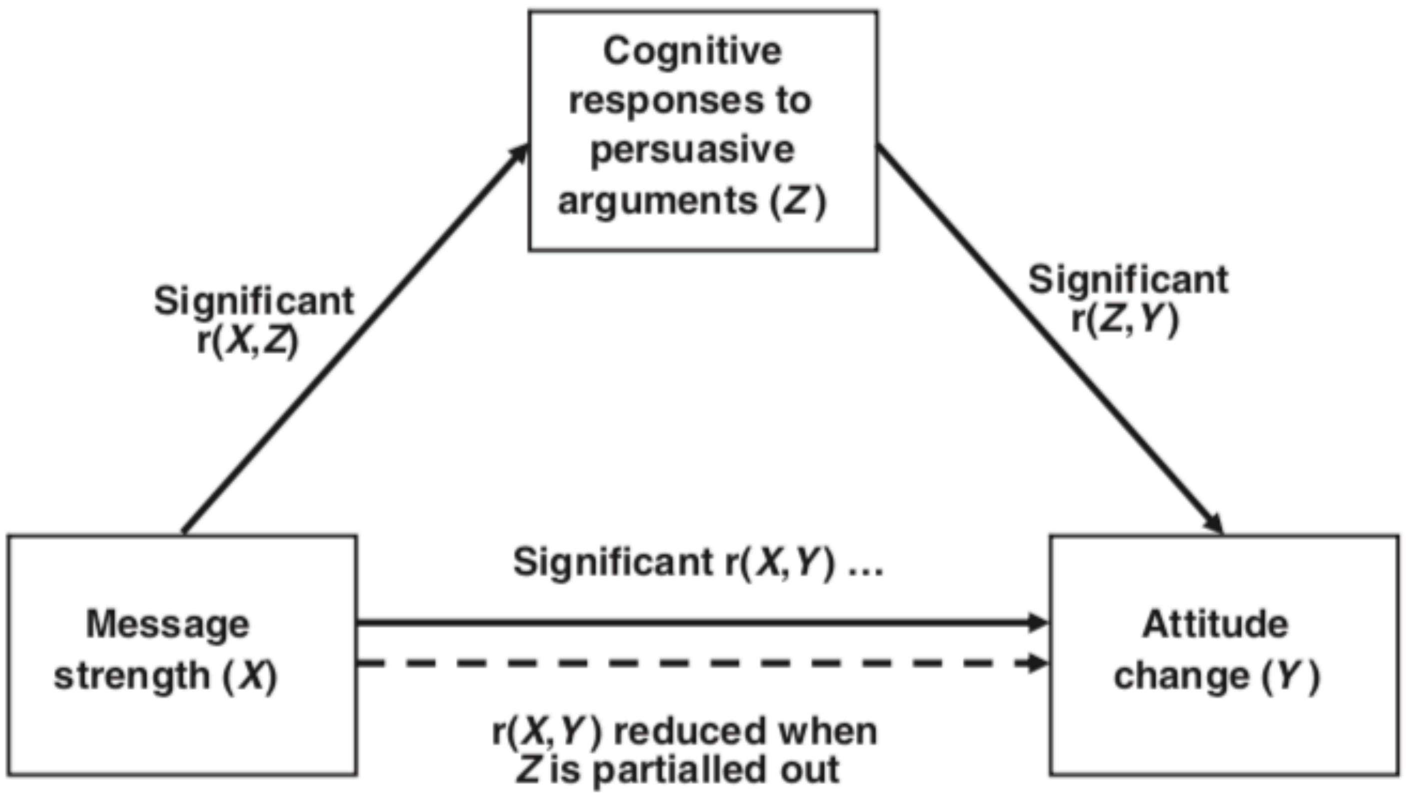


Fig. 1. Illustration of a mediation analysis in the context of persuasion research: The impact of message strength (X) on attitude change (Y) is supposed to be mediated by the recipients' cognitive responses (Z).

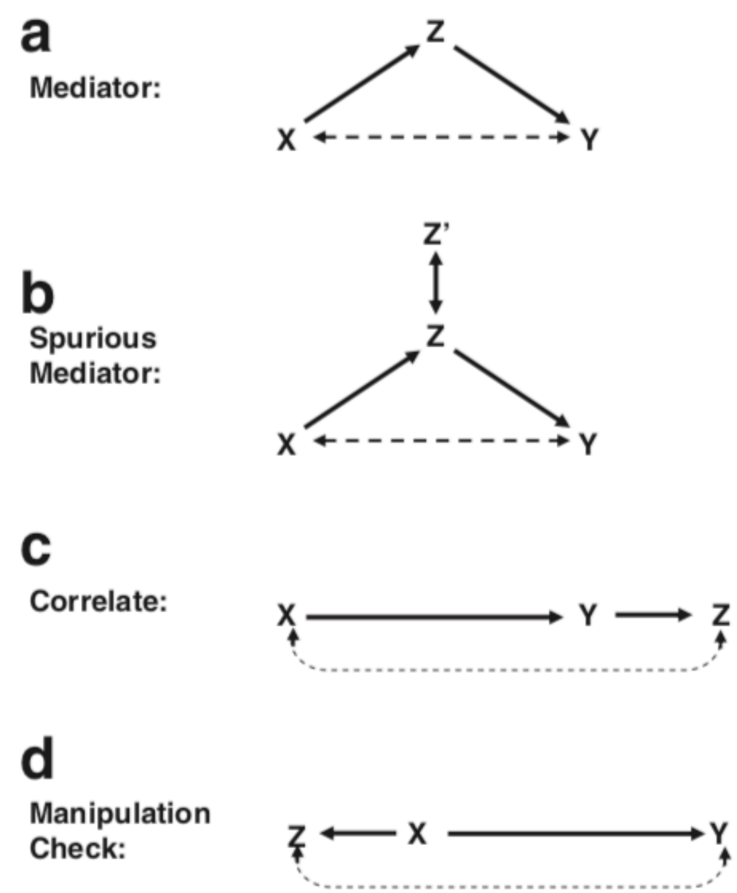
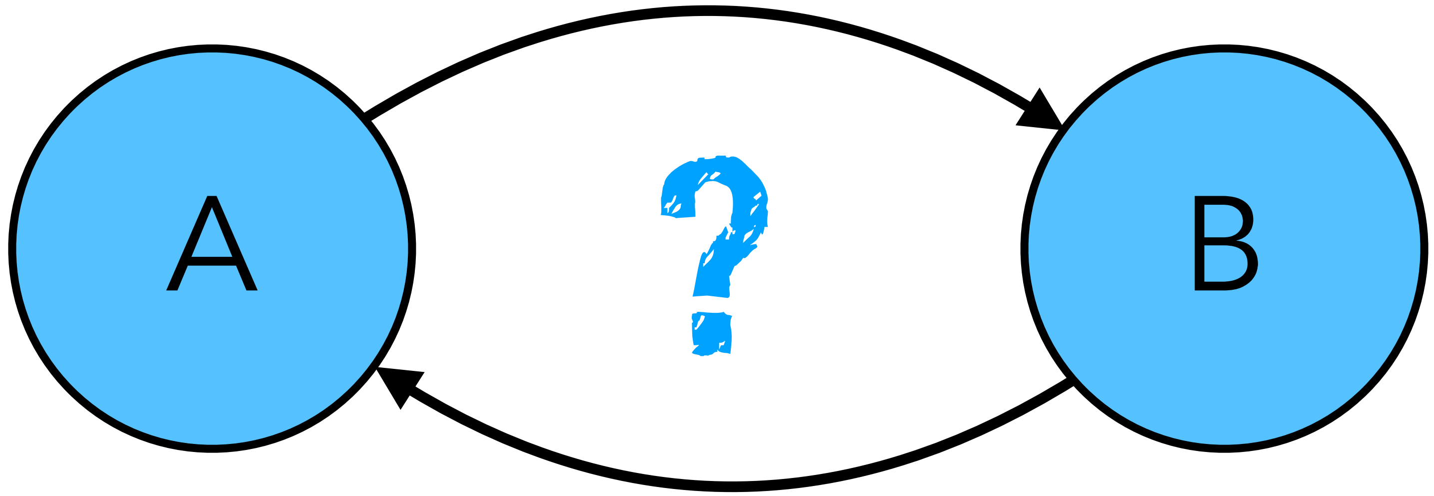


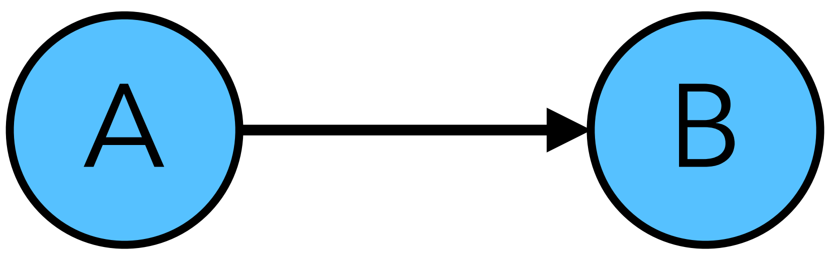
Fig. 2. Different causal models for the interpretation of the same intercorrelations between three variables, X, Y, and Z.

only experiments allow us to tell apart possible causal structures

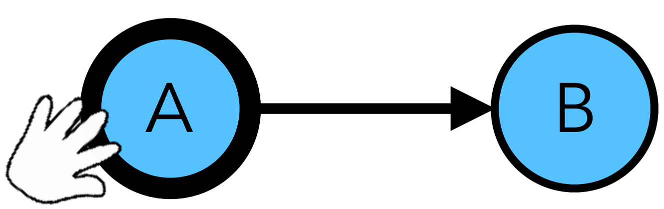
Fiedler, K., Schott, M., & Meiser, T. (2011). What mediation analysis can (not) do. *Journal of Experimental Social Psychology*, 47(6), 1231-1236.

Inferring causal structure through intervention

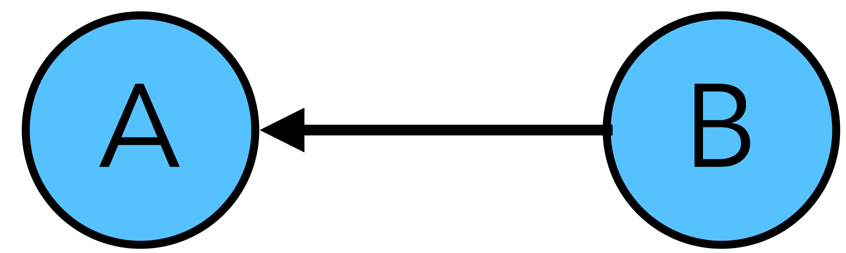




$$p(B \mid \mathrm{do}(A)) = p(B \mid A)$$

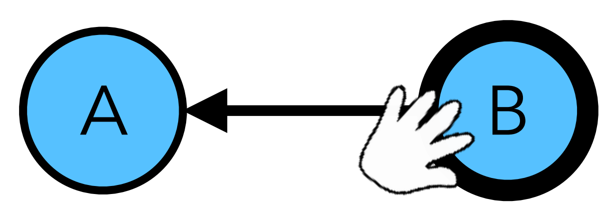


$$p(A \mid \mathrm{do}(B)) = p(A)$$



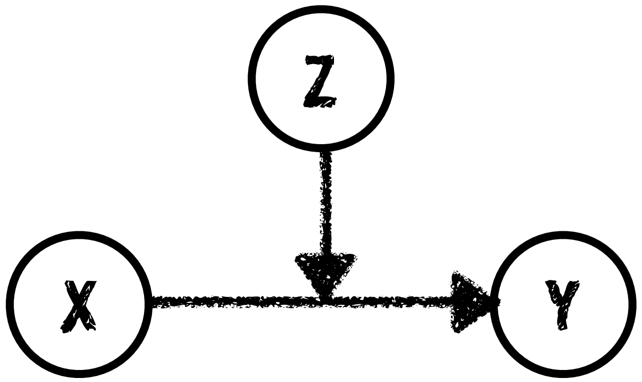
$$p(B \mid \mathrm{do}(A)) = p(B)$$

$$p(A \mid \mathrm{do}(B)) = p(A \mid B)$$



Moderation

Definition

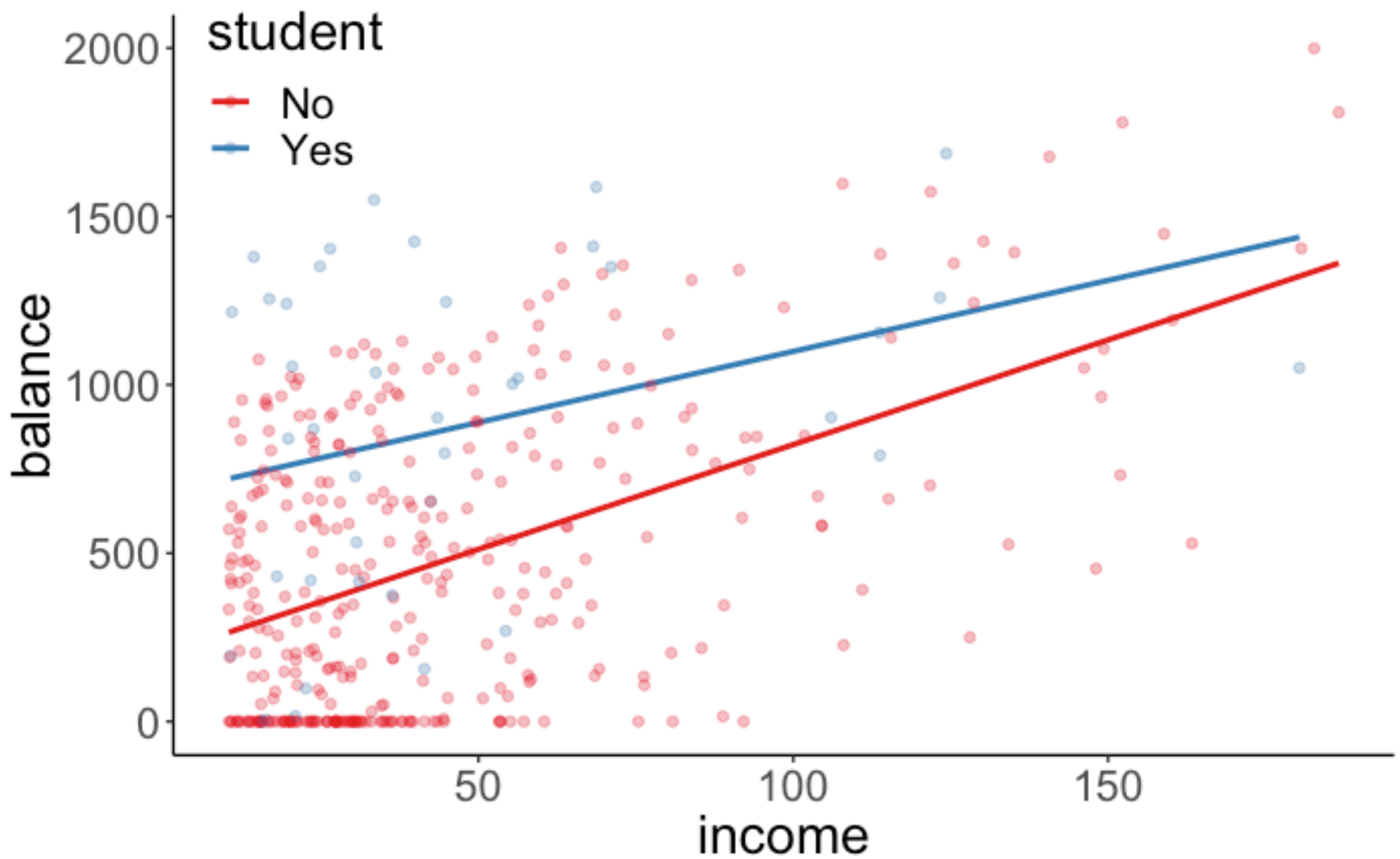


Moderation means that the effect of a predictor depends on the value of another.

Here, the nature of the relationship between **X** and **Y** depends on **Z**.

Have we come across moderation already?

Relationship between credit card balance, income, and whether the person is a student.



$$\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot \text{student}_i - 2.00 \cdot (\text{income}_i \times \text{student}_i)$$

if student = "No" $\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i$

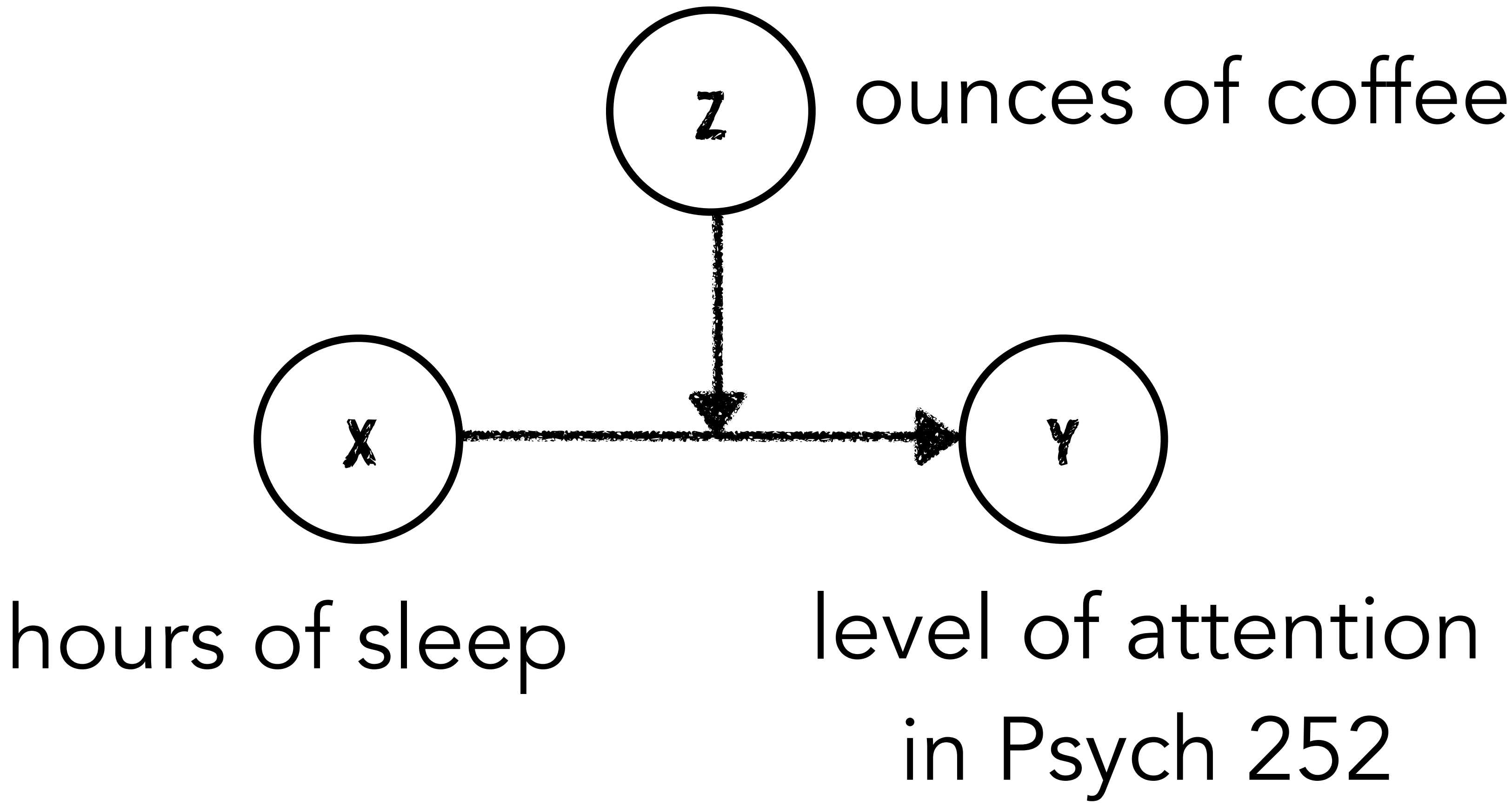
if student = "Yes"

$$\begin{aligned}\widehat{\text{balance}}_i &= 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot 1 - 2.00 \cdot (\text{income}_i \times 1) \\ &= 677.3 + 6.22 \cdot \text{income}_i - 2.00 \cdot \text{income}_i \\ &= 677.3 + 4.22 \cdot \text{income}_i\end{aligned}$$

Simulating a moderation

```r
# number of participants
n  = 100

df.moderation = tibble(
  x  = abs(rnorm(n, 6, 4)), # hours of sleep
  x1 = abs(rnorm(n, 60, 30)), # adding some systematic variance to our DV
  z  = rnorm(n, 30, 8), # ounces of coffee consumed
  y  = abs((-0.8 * x) * (0.2 * z) - 0.5 * x - 0.4 * x1 + 10 + rnorm(n, 0, 3)) # attention Paid
)
```



Simulating a moderation

```r
# scale the predictors
df.moderation = df.moderation %>%
  mutate_at(vars(x, z), "scale")

# run regression model with interaction
fit.moderation = lm(formula = y ~ 1 + x * z,
                    data = df.moderation)

# summarize result
fit.moderation %>%
  summary()
```

```
Call:
lm(formula = y ~ 1 + x * z, data = df.moderation)

Residuals:
    Min      1Q  Median      3Q     Max 
-21.466  -8.972  -0.233   6.180  38.051 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   48.544      1.173  41.390  < 2e-16 ***
x             17.863      1.196  14.936  < 2e-16 ***
z              8.393      1.181   7.108 2.08e-10 ***
x:z            6.094      1.077   5.656 1.59e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.65 on 96 degrees of freedom
Multiple R-squared:  0.7661,	Adjusted R-squared:  0.7587 
F-statistic: 104.8 on 3 and 96 DF,  p-value: < 2.2e-16
```

significant interaction

Simulating a moderation

```r
# generate data grid with three levels of the moderator
df.newdata = df.moderation %>%
  expand(x = c(min(x),
               max(x)),
         z = c(mean(z) - sd(z),
               mean(z),
               mean(z) + sd(z))) %>%
  mutate(moderator = rep(c("low", "average", "high"), nrow(.)/3))

# predictions for the three levels of the moderator
df.prediction = fit.moderation %>%
  augment(newdata = df.newdata) %>%
  mutate(moderator = factor(moderator, levels = c("high", "average", "low")))
```

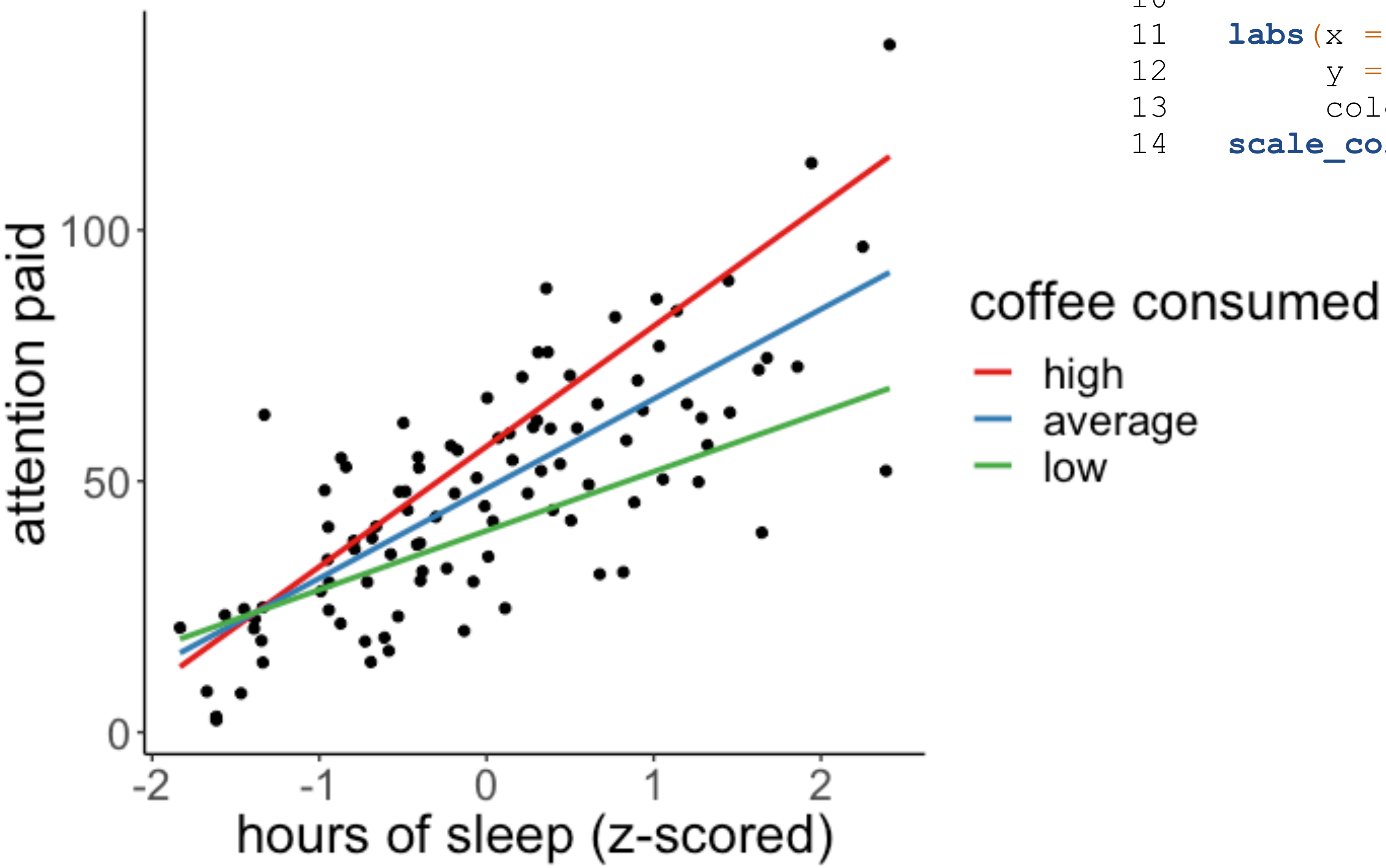
minimum and maximum

average ± 1SD

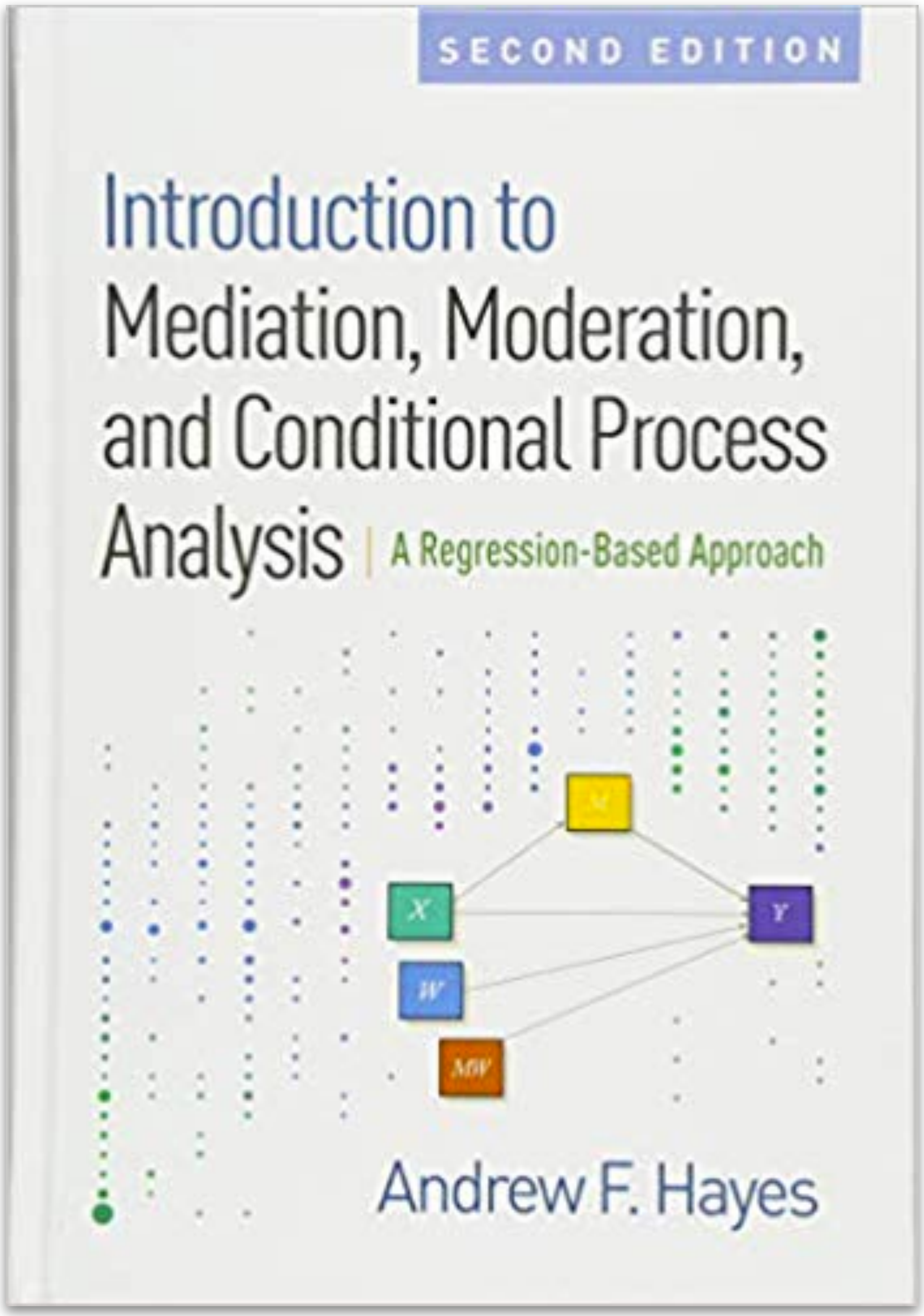
x	z	moderator	.fitted	.se.fit
-1.83	-1	low	18.58	3.75
-1.83	0	average	15.80	2.51
-1.83	1	high	13.02	2.99
2.41	-1	low	68.52	4.32
2.41	0	average	91.60	3.09
2.41	1	high	114.68	4.12

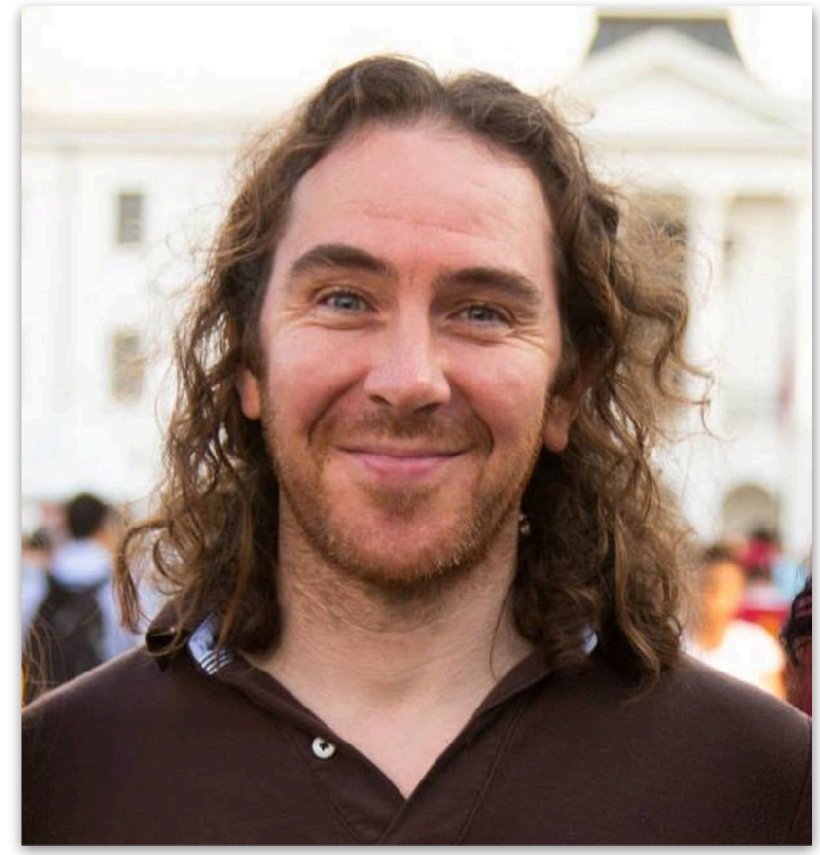
Simulating a moderation

```r
# visualize the result
df.moderation %>%
  ggplot(aes(x = x,
             y = y)) +
  geom_point() +
  geom_line(aes(y = .fitted,
                group = moderator,
                color = moderator),
            data = df.prediction,
            size = 1) +
  labs(x = "hours of sleep (z-scored)",
       y = "attention paid",
       color = "coffee consumed") +
  scale_color_brewer(palette = "Set1")
```



Learn more about mediation and moderation





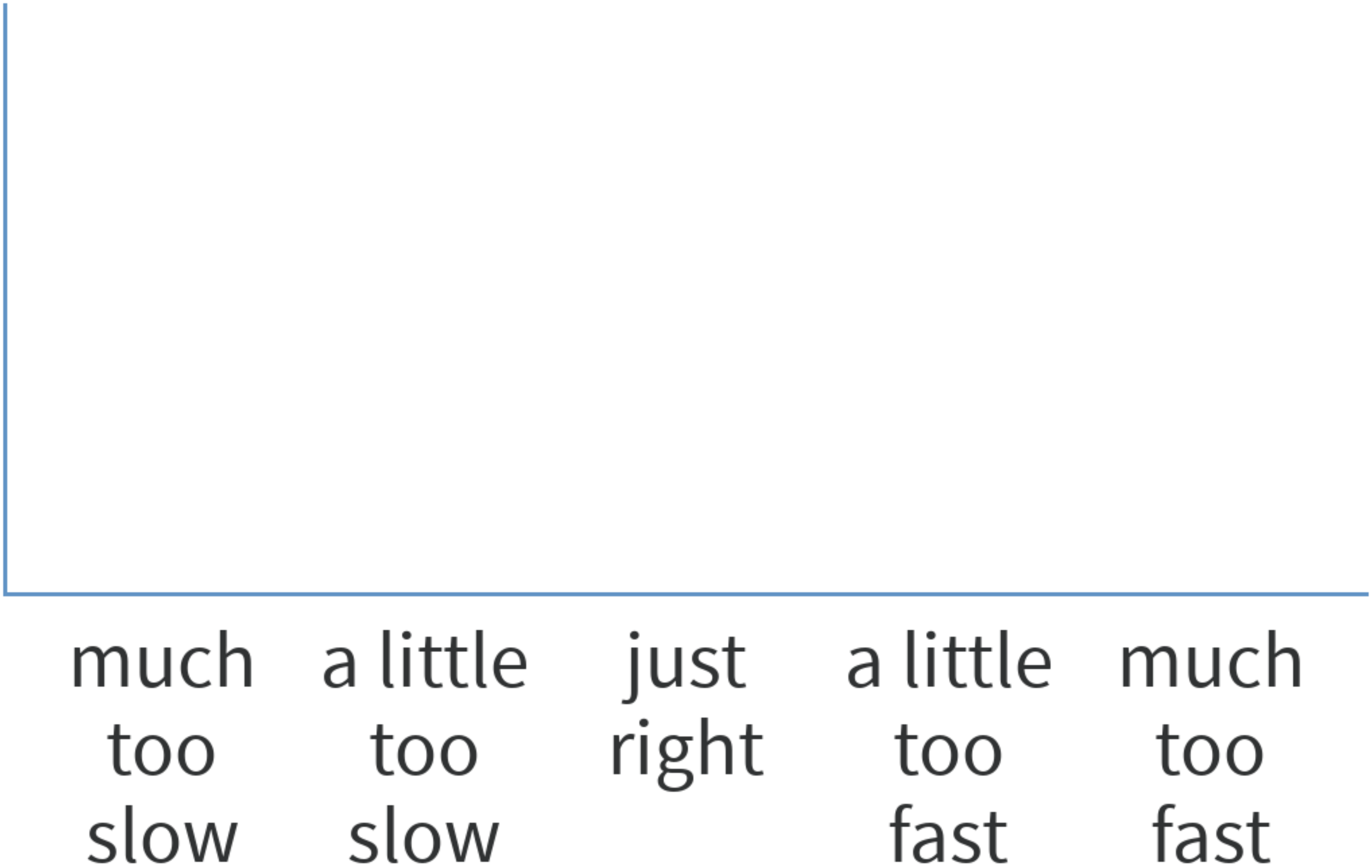
Recoded with `brms` by Solomon Kurz here: https://bookdown.org/connect/#/apps/1523/access

Summary

- Questions? Answers!
  - degrees of freedom
  - t-test vs. permutation test
  - overlapping confidence intervals
- Power analysis (continued)
- Controlling for variables
- Mediation
- Moderation

Feedback

How was the pace of today's class?



How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?

Answers to this poll are anonymous

Thank you!