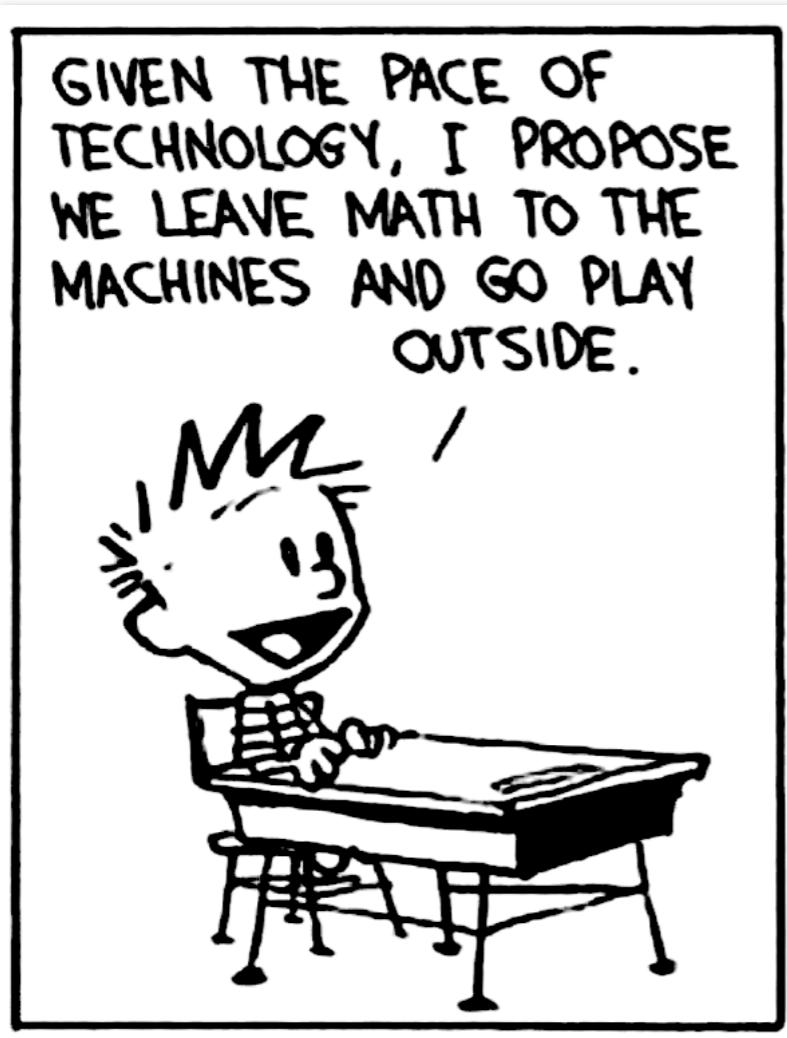


# Mediation and Moderation



Chat

Would you rather travel back in time to meet your ancestors or would you rather go to the future to meet your descendants?

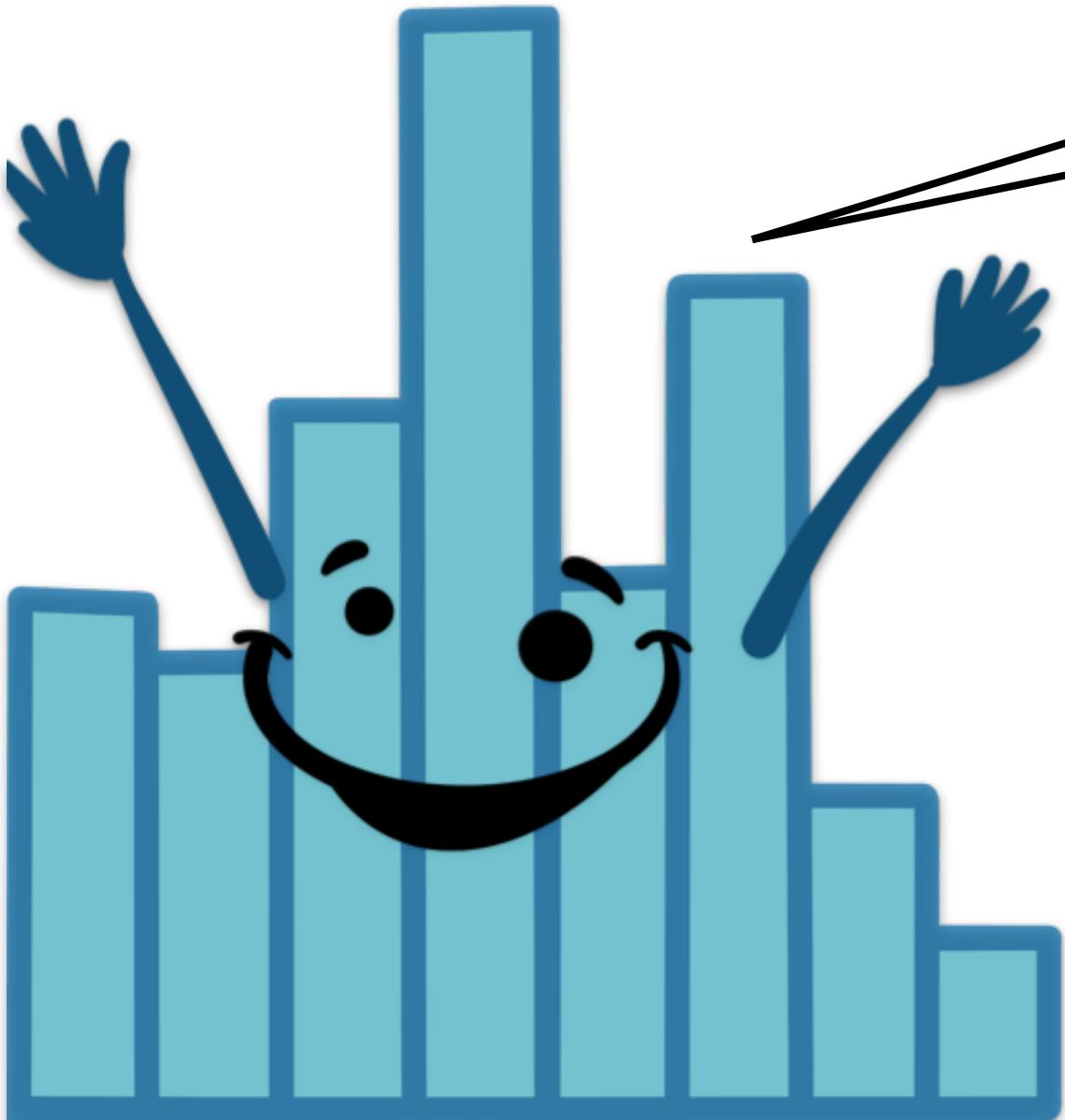
To: Everyone ▾ More ▾

Type message here...

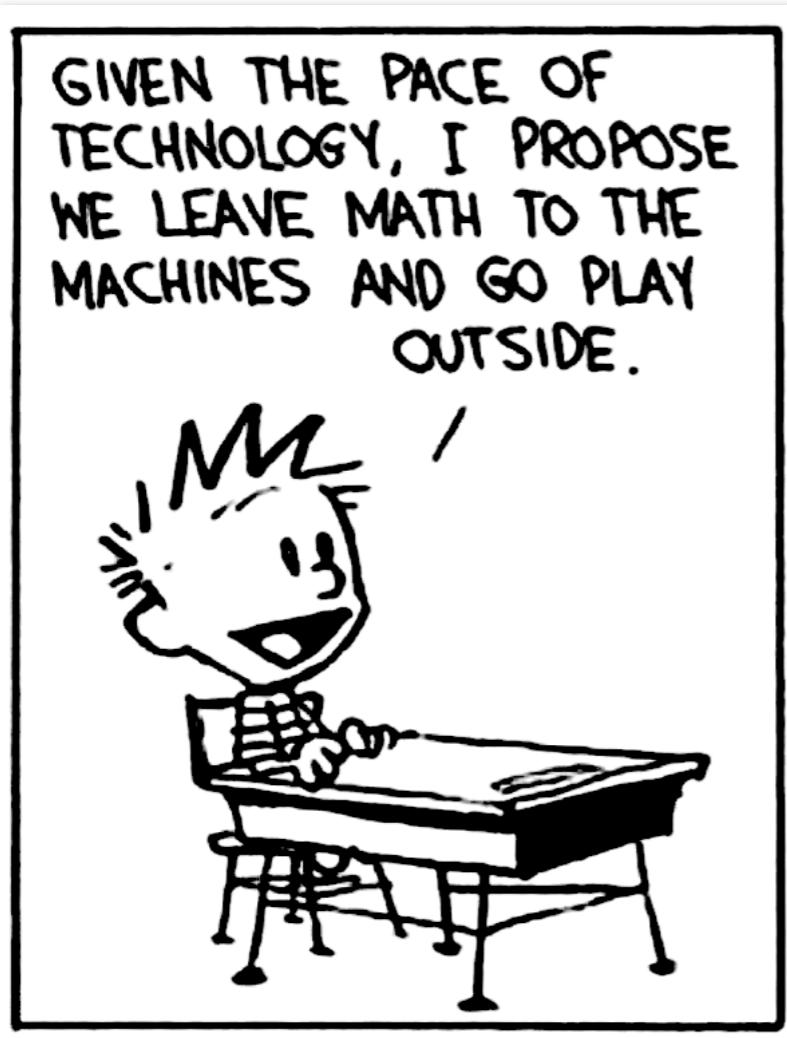


02/19/2021

Remember to  
record the  
lecture!



# Mediation and Moderation



Chat

Would you rather travel back in time to meet your ancestors or would you rather go to the future to meet your descendants?

To: Everyone ▾ More ▾

Type message here...



02/19/2021

# **Things that came up**

# gganimate

# Great use of gganimate



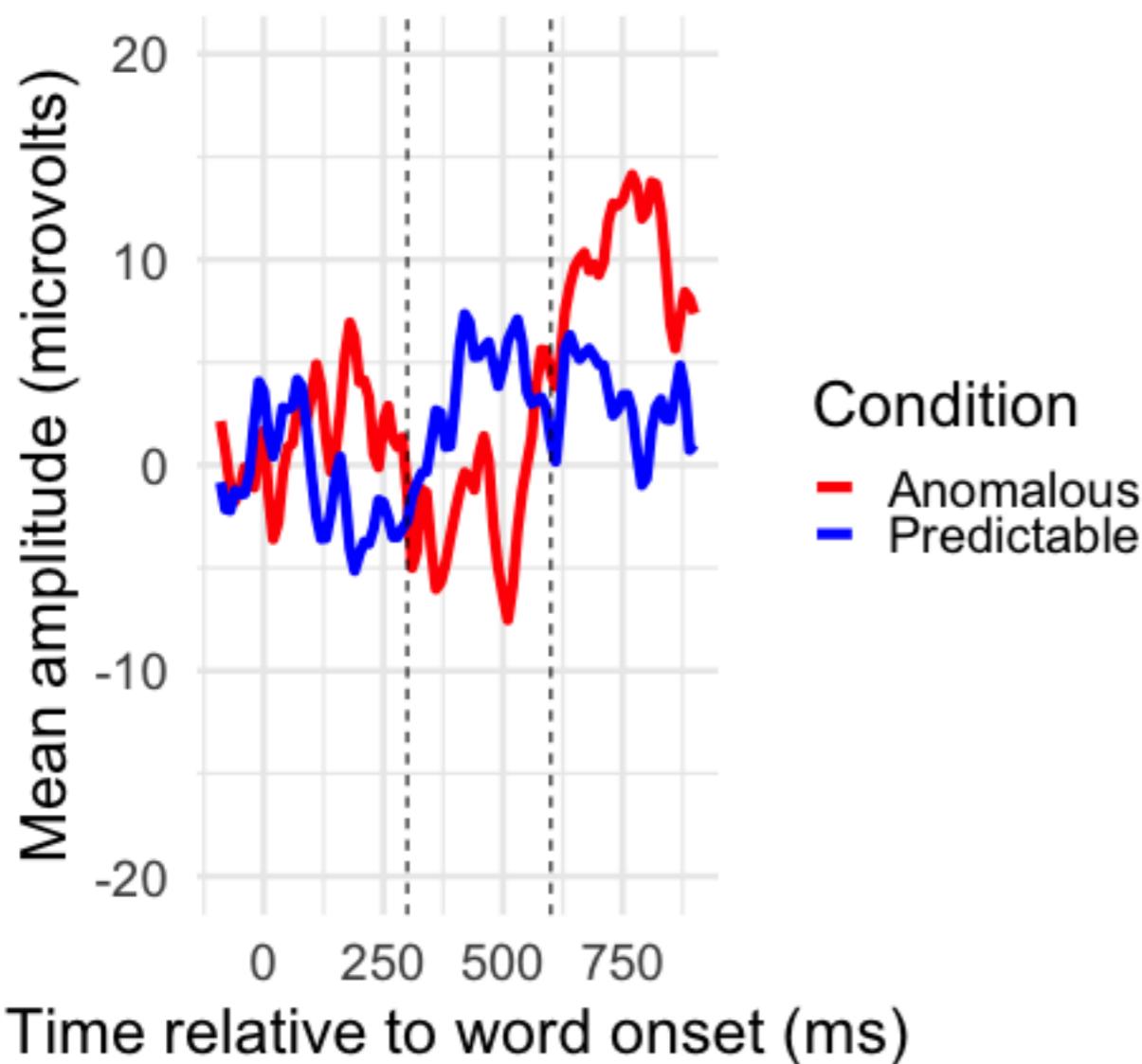
Rachel A. Ryskin

[ryskin@mit.edu](mailto:ryskin@mit.edu)

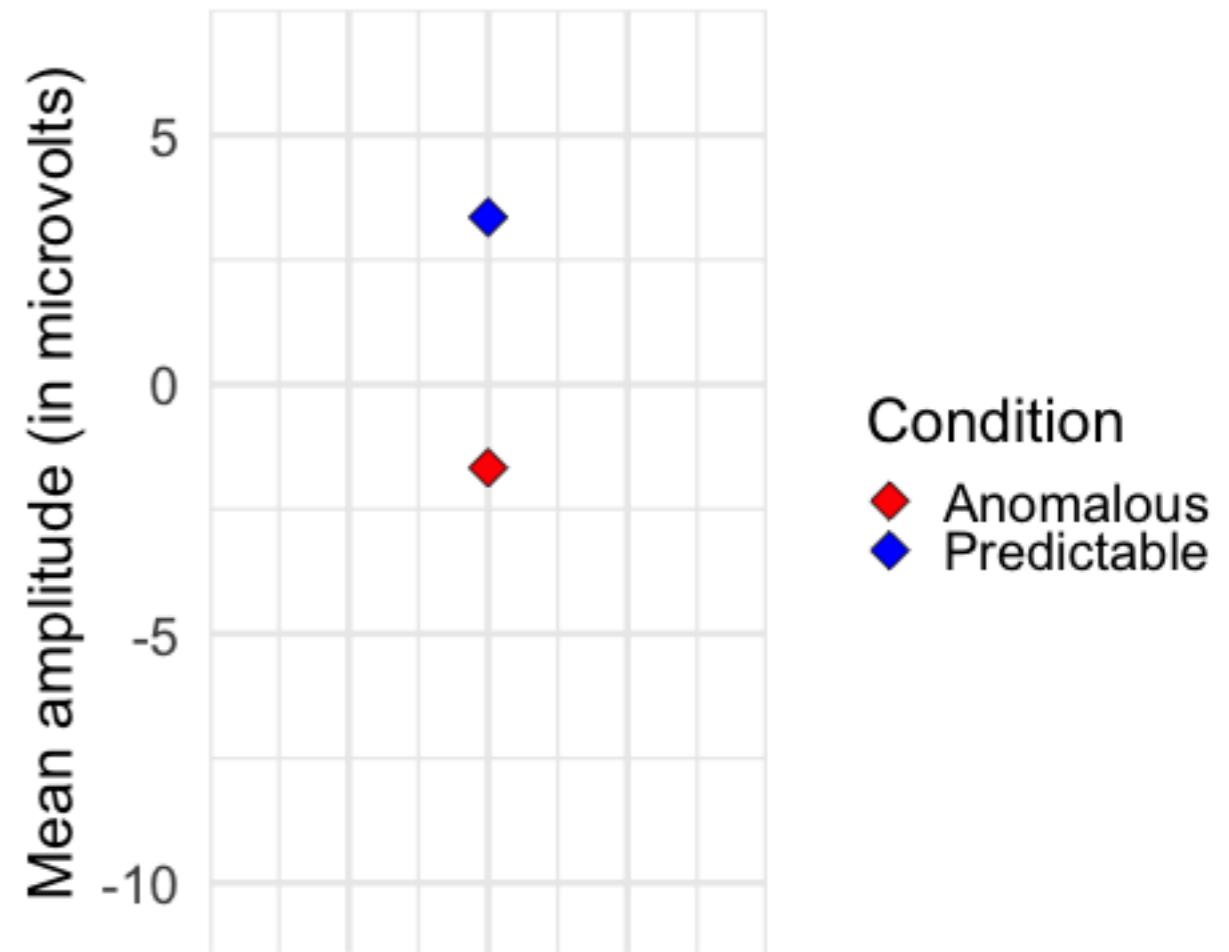
[Curriculum Vitae](#)

[Google Scholar](#) | [GitHub](#) | [Twitter](#)

Number of subjects: 1



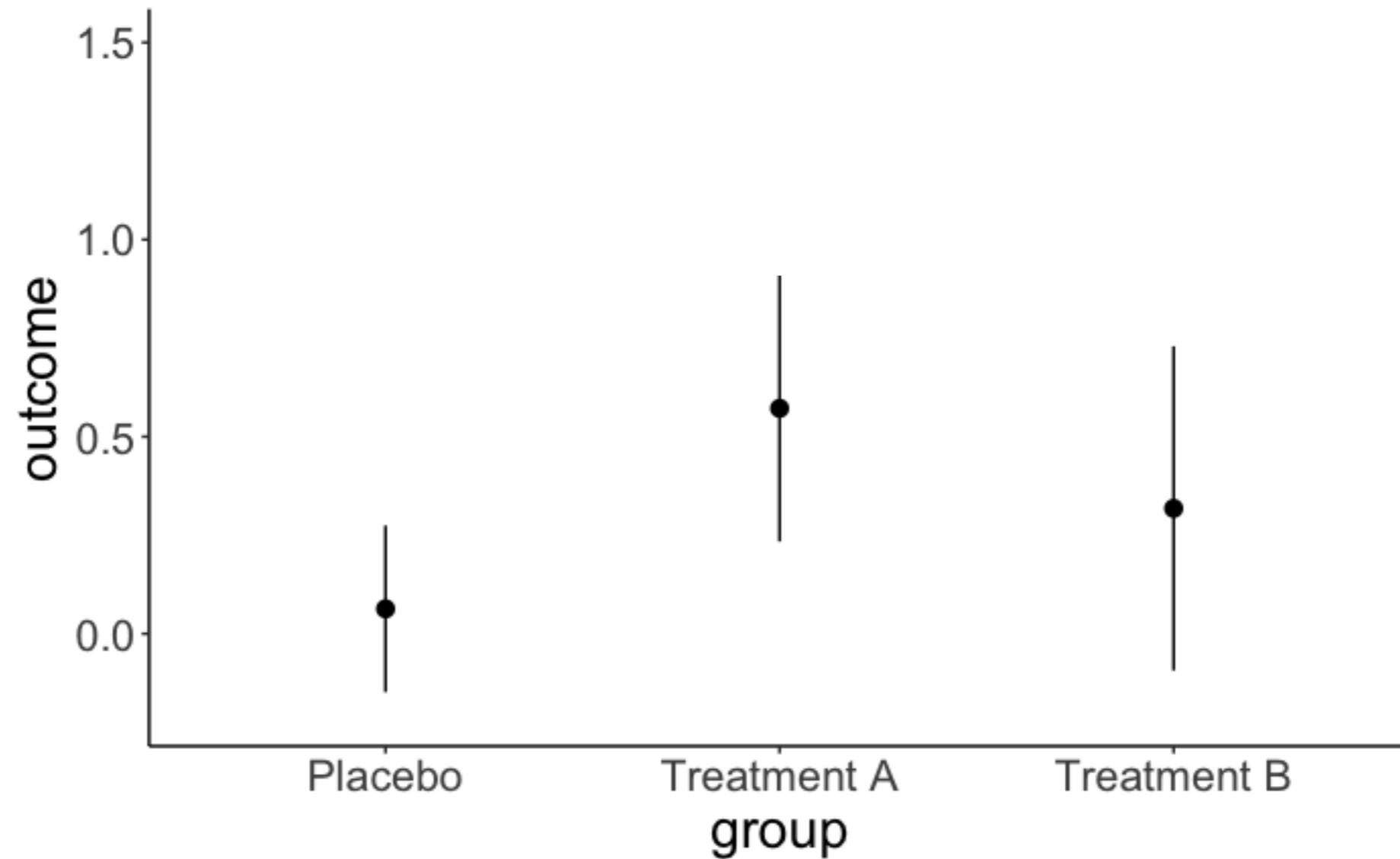
Mean over 300-600ms window  
(N=1)



**difference in significance  
vs. significant differences**

# significant differences

"Our tendency to look for a difference in significance should be replaced by a check for the significance of the difference."

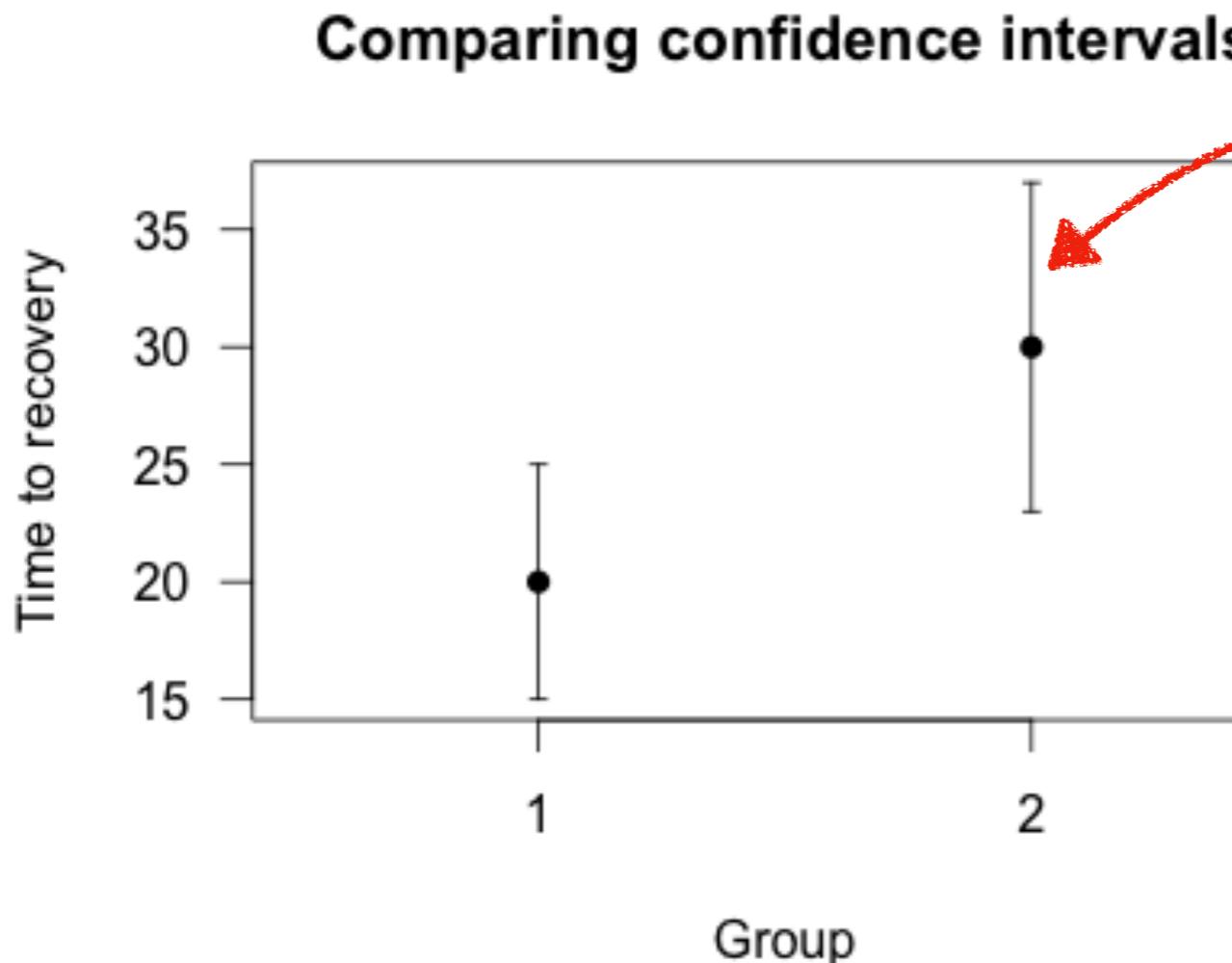


"We compared treatments A and B with a placebo. Treatment A showed a significant benefit over placebo, while treatment B had no statistically significant benefit. Therefore, treatment A is better than treatment B."

<https://www.statisticsonewrong.com/significant-differences.html>

# significant differences

## Significant difference between Group 1 and 2?



what do the  
error bars mean?

standard deviation?

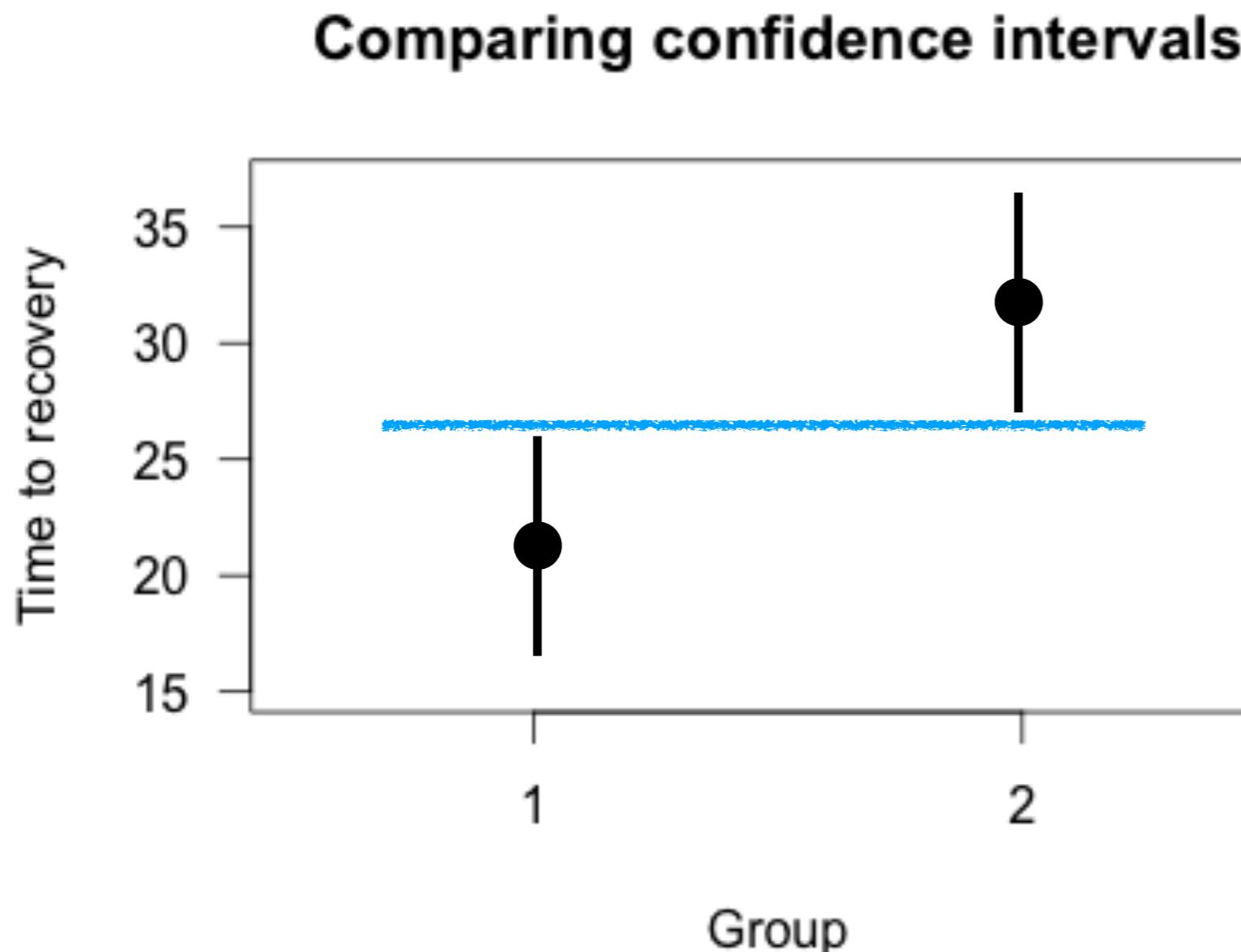
standard error of the mean?

confidence intervals?

What we would like to know, does the confidence interval of the difference between groups exclude 0?

# significant differences

## Significant difference between Group 1 and 2?



95% confidence  
intervals don't overlap

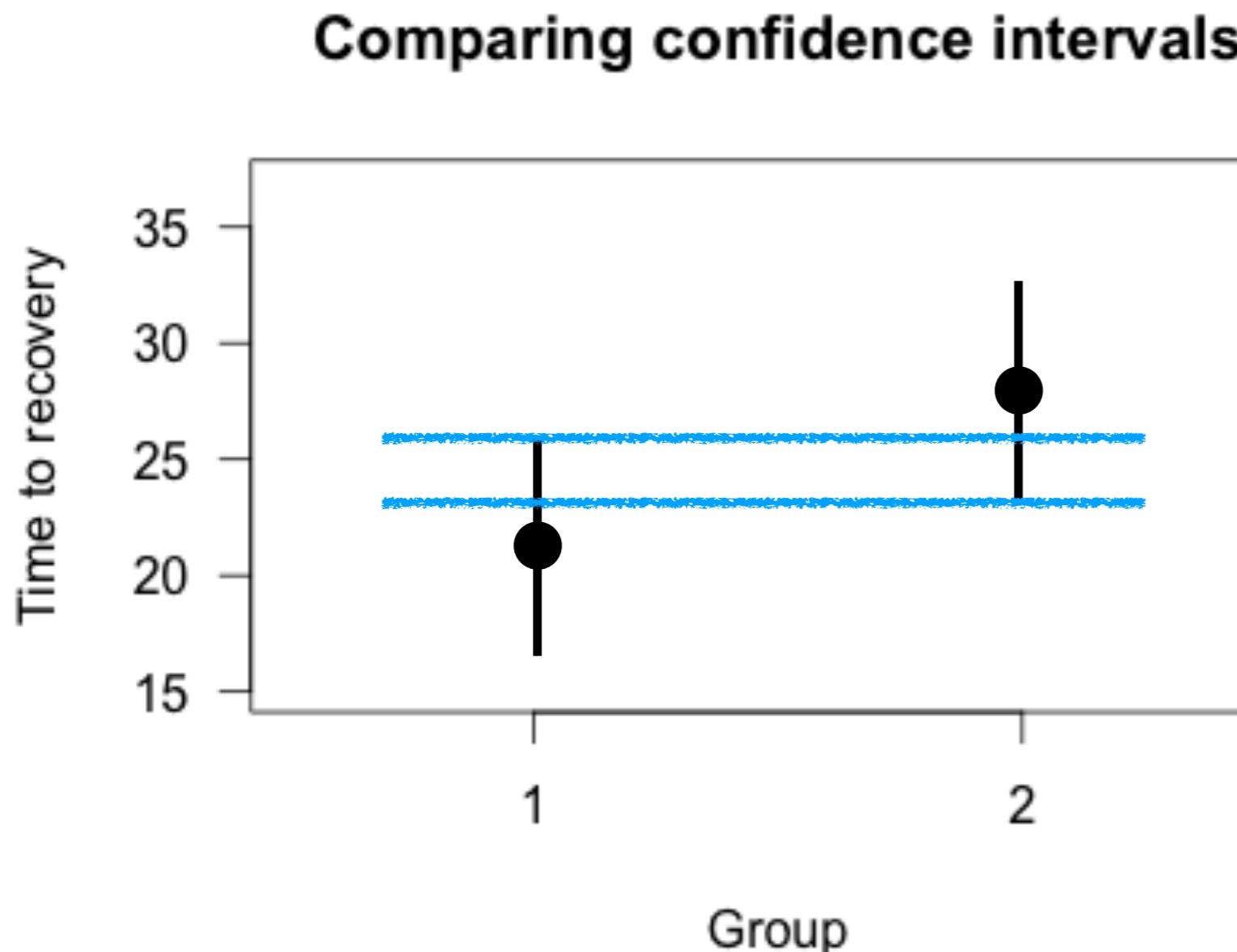
conservative test  
of significance

rejects  $H_0$  less often than the appropriate statistical procedure

Schenker & Gentleman (2001) On judging the significance of differences by examining the overlap between confidence intervals. *The American Statistician*

# significant differences

## Significant difference between Group 1 and 2?



95% confidence  
intervals don't overlap  
with mean

anti-conservative  
test of significance

rejects  $H_0$  more often than the appropriate statistical procedure

Schenker & Gentleman (2001) On judging the significance of differences by examining the overlap between confidence intervals. *The American Statistician*

# **Logistics**

# **Homework 4**

# Homework 4

Graded and solutions are on Canvas.

When poll is active, respond at **PollEv.com/psych252**

# How many hours did it take you to complete Homework 4?

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15 or more

Total Results: 0

# **Midterm**

**How many hours did it take you to complete  
the midterm? (just write one number: e.g. 9  
if it took you 9 hours)**

# Project proposal

# Project proposal

IT'S TIME FOR A...

## GROUP ASSIGNMENT!!

**Members of teams  
will all the get  
same grade!**

**maximum 3 team  
members**



Didn't attend  
any group  
meetings



Doesn't  
understand  
the material



Gave the  
presentation  
but obviously  
didn't know  
what he was  
even saying



Who is  
this guy

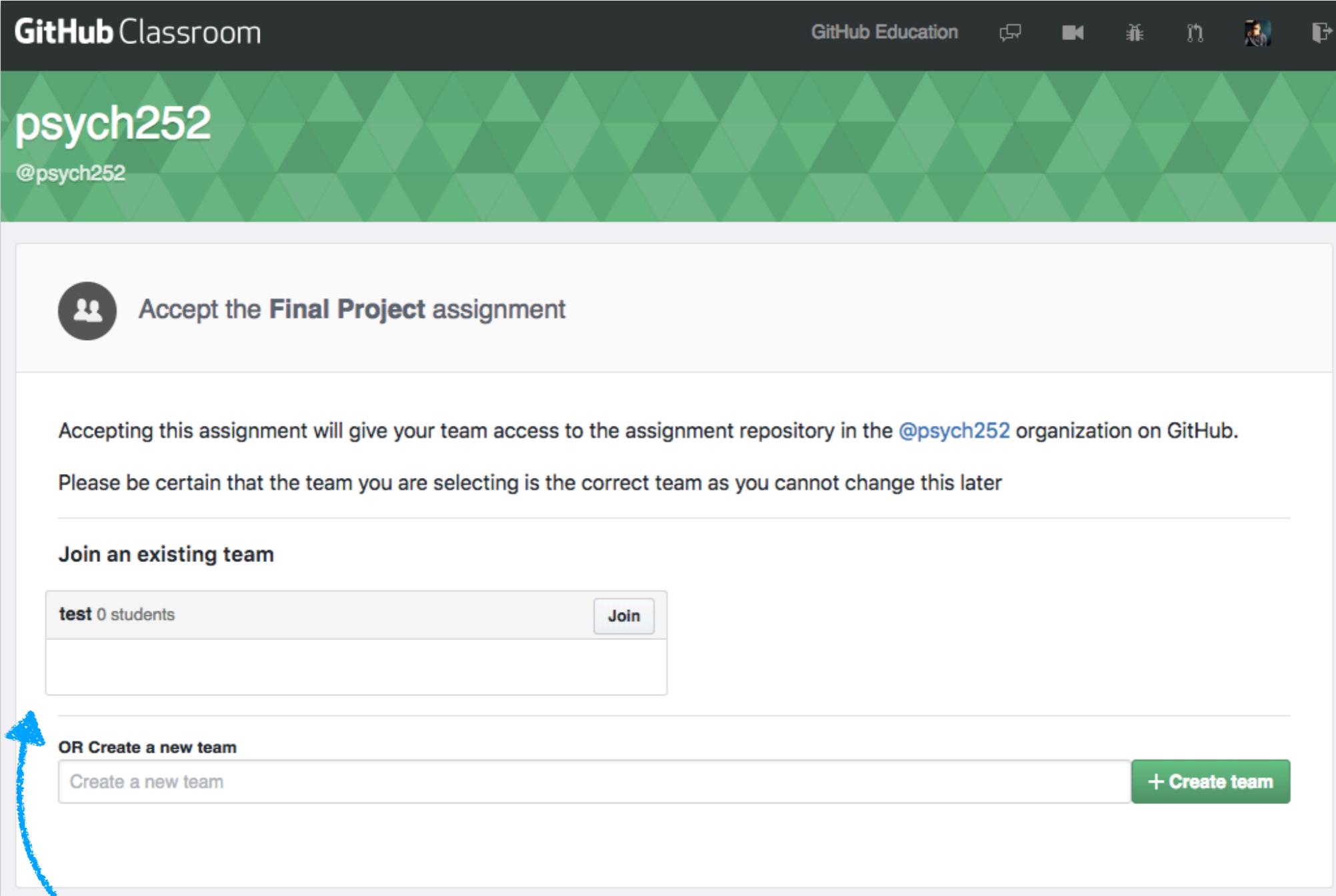


"You can  
use my  
printer"



Did all the  
research, wrote  
paper, composed  
presentation

# Project proposal



The screenshot shows the GitHub Classroom interface for the organization `psych252`. At the top, there's a navigation bar with `GitHub Classroom`, `GitHub Education`, and various icons. Below the header is a green decorative bar with the organization name `psych252` and handle `@psych252`. The main content area has a title `Accept the Final Project assignment` with a user icon. It explains that accepting the assignment will give the team access to the repository in the `@psych252` organization on GitHub. It also cautions that the team selected cannot be changed later. Below this, there's a section titled `Join an existing team` which lists a single team named `test` with 0 students, followed by a `Join` button. Underneath this, there's an option to `OR Create a new team`, with a `Create a new team` button and a `+ Create team` button. A blue arrow points from the text "join a team or make a new one" at the bottom to the `OR Create a new team` section.

join a team or make a new one

<https://classroom.github.com/g/A9487wbl>

# Project proposal

The screenshot shows a GitHub repository page for the user 'psych252'. The repository name is 'final-projects' and it is marked as 'Private'. The page includes navigation links for Code, Issues (0), Pull requests (0), Projects (0), Wiki, Insights, and Settings. Below the header, there's a section titled 'Starter code for the final project' with a 'Manage topics' link and an 'Edit' button. A summary bar shows 5 commits, 1 branch, 0 releases, and 1 contributor. The 'Branch: master' dropdown is set to 'master', and there are buttons for 'New pull request', 'Create new file', 'Upload files', 'Find file', and 'Clone or download'. The commit history lists several initial commits from a user named 'tobiasgerstenberg' made 20 minutes ago, including commits for 'code/R', 'data', 'figures', 'papers', 'presentation', 'writeup', '.gitignore', and 'README.md'. The 'Final project' section contains a note about starter code and a 'General points' section with a bulleted list of guidelines for folder and file naming, relative paths, organization, and a note about empty folders. The 'Repository structure' section is also present.

## Final project

Starter code for your final project.

### General points

- for folder and file names:
  - don't use white space in either folder or filenames, use an underscore "\_" instead
  - (almost always) use lower case only
- always use relative paths in your code
  - for example, to save a figure from an R script inside the `code/R/` folder the path should be `"../../figures/figure_name.pdf"`
- keep your folder structure organized
  - we recommend adhering to the folder structure in this repository
  - more complex projects may have additional folders such as `videos/`, `papers/`, ...
- note: some of the folders are empty except for a `.keep` file
  - the `.keep` file is just there to make sure that github includes the otherwise empty folder
  - feel free to delete the `.keep` file once you've added another file to that folder

### Repository structure

- each team will have their own private github repository
- all work on your final project should happen within this repository
- you can get **github** help in section
- post on Piazza in case you experience any problems getting set up

# Project proposal

# RMarkdown template



```
project_proposal.Rmd x
[File] [New] [Open] [Save] [ABC] [Search] Knit [Settings]
[Run] [Up] [Down] [Copy] [Cut] [Paste] [Find] [Replace] [Help] [About]

1 ---  
2 title: "A catchy project title goes here"  
3 subtitle: "My team's name goes here"  
4 author: "The team members' names go here"  
5 date: ``r Sys.time()``  
6 urlcolor: blue # to show hyperlinks in blue when printed as pdf  
7  
8 # edit the output format below  
9 # output: html_document # use this to render to html  
10 output: pdf_document # use this to render to pdf  
11  
12 ---  
13  
14 *Instructions*  
15  
16 The project proposal is due on Thursday, February 25th at 8pm. It should not be  
longer than 700 words. It may contain code (code doesn't count toward the word  
limit).  
17  
18 # Research question (3 pt)  
19  
20 ## What's your main research question? (1 pt)  
21  
22 > YOUR ANSWER HERE  
23  
24 ## Which hypotheses are you trying to test? (2 pt)  
25  
26 _Enumerate 1-3 hypotheses in terms of directional relationships. For example, you might  
write something like "X is predicted to increase as Y increases", or "We predict that  
performance in Group 2 will be significantly better than performance in Group 1"._  
27  
28 > YOUR ANSWER HERE  
29  
30 # Methods (4 pt)  
31
```

Upload the  
pdf to canvas

# Project proposal

The project proposal is due on  
**Thursday, February 25<sup>th</sup> at 8pm**

**instructions of how to submit  
will be released later today**

# Plan for today

- Controlling for variables
- Mediation
- Moderation

# **Controlling for variables**

# Breakout rooms

**Tasks:**



1. What does "controlling" for variables mean?
2. When should I control for variables?

**Size:** ~3 people

**Time:** 5 minutes

**Report:** Share your group's thinking in the main room.

# Controlling for variables

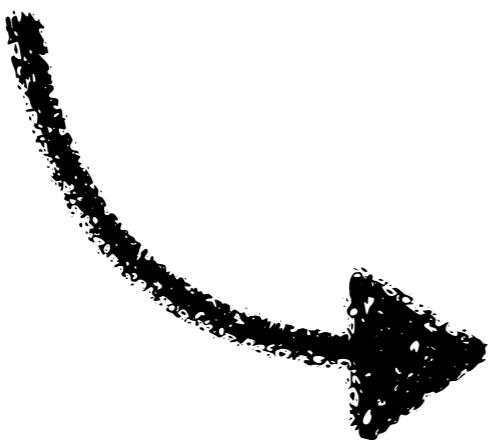
1. What does "controlling" for variables mean?
2. When should I control for variables?

 check this out

# What does controlling for variables mean?

we are not actually "**controlling**" the variable

instead, we are taking the variable into consideration when making predictions



**the hope is that we get a better estimate of the parameter that we are interested in by taking into account other factors**

# When should I control for variables?

recent advances in graphical models have produced a way to help distinguish good from bad controls

 **d-separation**  
**directional**

decide from a causal graph whether a set of variables  $X$  is independent of another set  $Y$ , given a third set  $Z$

**Goal:** we want a precise (and unbiased) estimate of the predictive relationship between  $X$  and  $Y$

 **we want to block all other paths from  $X$  to  $Y$**

# When should I control for variables?

## How can I tell whether two variables are independent?

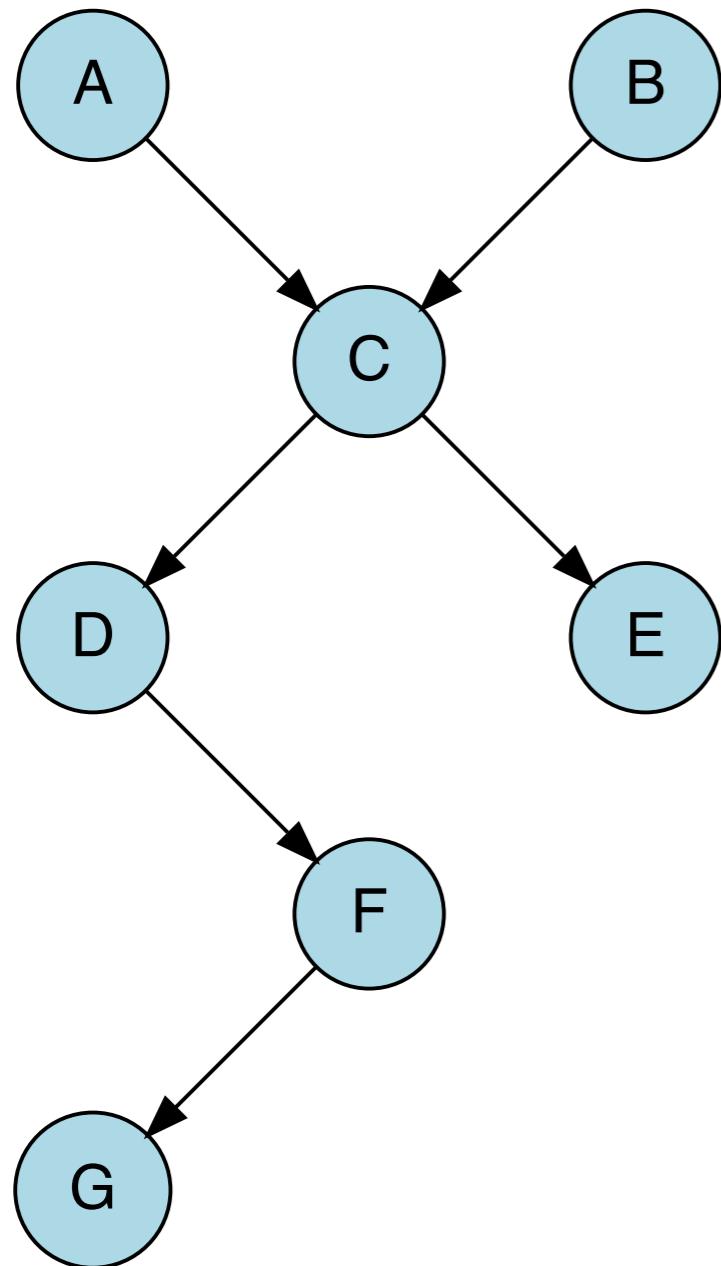
### Recipe for independence

1. Draw the ancestral graph
  2. "Moralize" the graph by "marrying" the parents
  3. "Disorient" the graph by replacing arrows with edges
  4. Delete the givens and their edges
  5. Read the answer off the graph
- if variables are **disconnected** they are independent  
- if variables are connected (have a path between them)  
they are not guaranteed to be independent

# When should I control for variables?

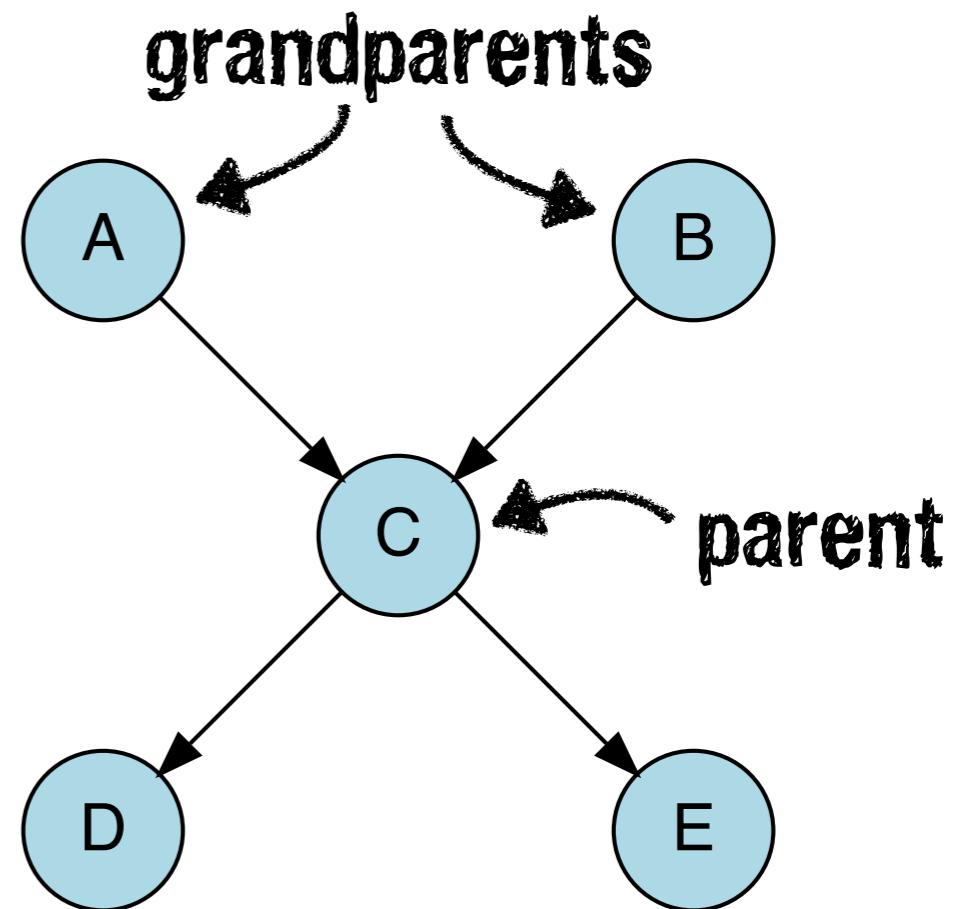
**Are D and E independent?**

$$p(D | E) = p(D) ?$$



## 1. Draw the ancestral graph

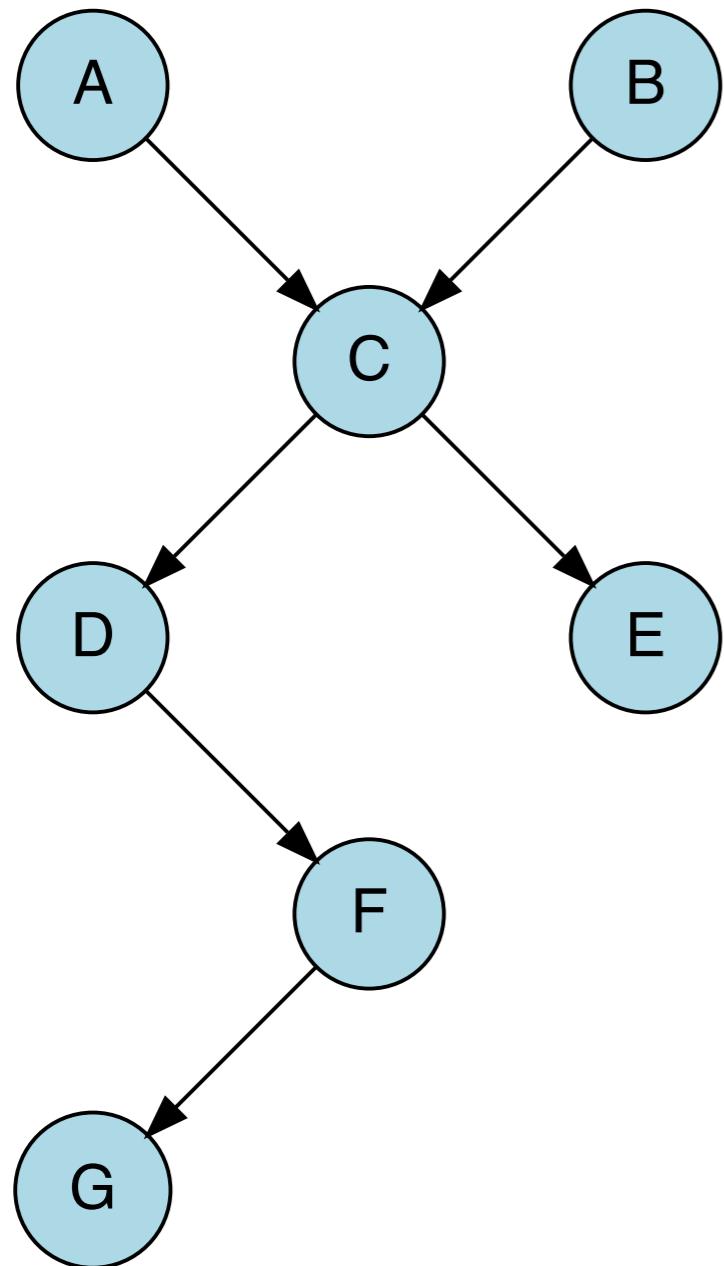
Construct the "ancestral graph" of all variables mentioned in the probability expression. This is a reduced version of the original net, consisting only of the variables mentioned and all of their ancestors (parents, parents' parents, etc.)



# When should I control for variables?

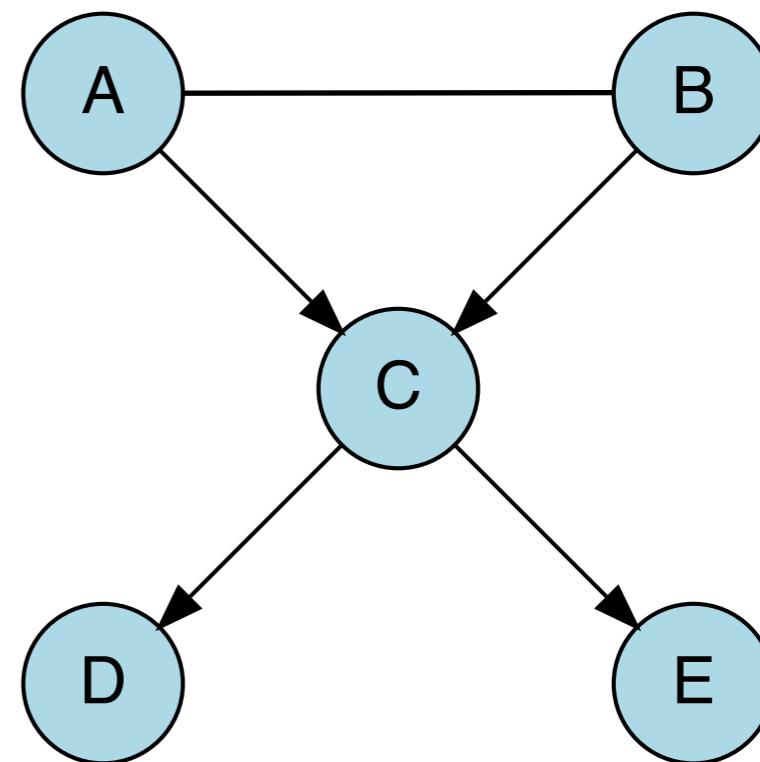
**Are D and E independent?**

$$p(D | E) = p(D) ?$$



**2. "Moralize" the graph**  
**let's get married!**

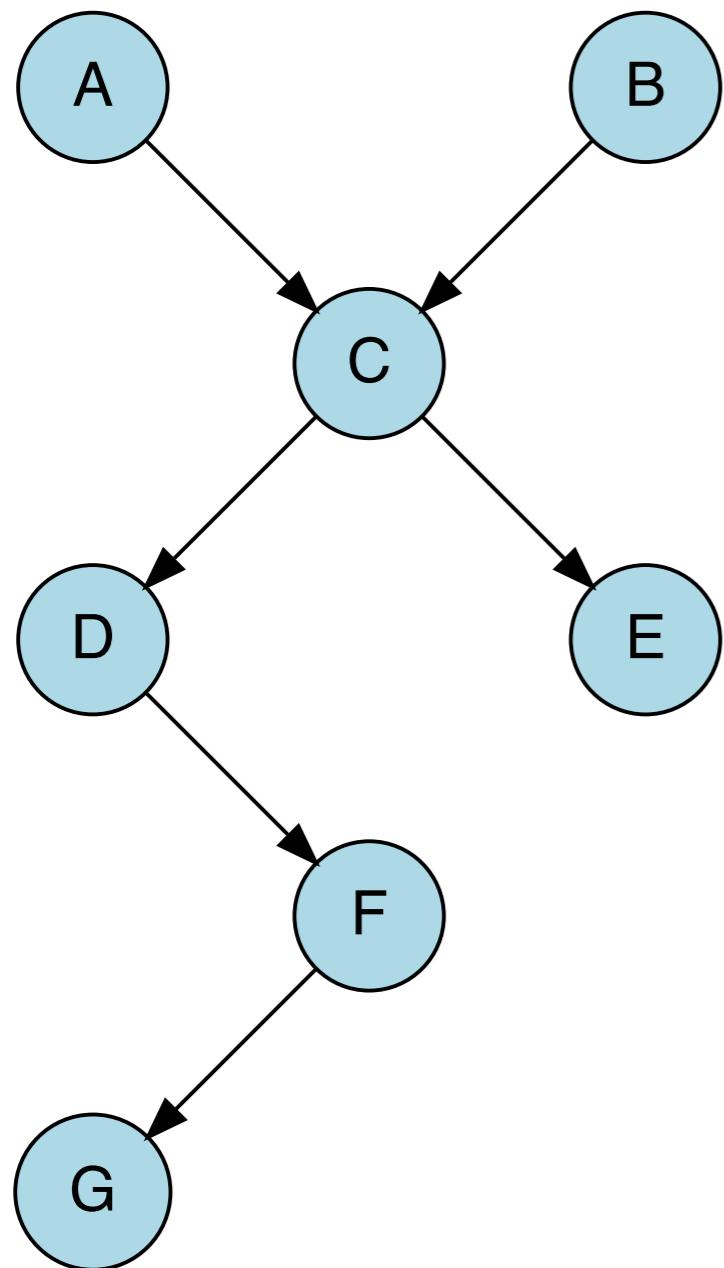
For each pair of variables with a common child, draw an undirected edge (line) between them. (If a variable has more than two parents, draw lines between every pair of parents.)



# When should I control for variables?

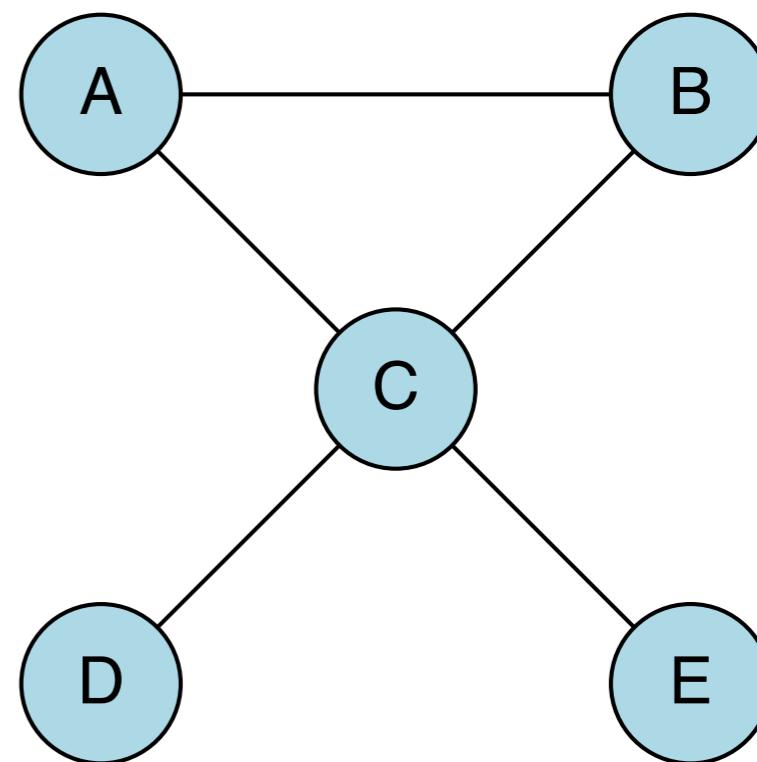
**Are D and E independent?**

$$p(D | E) = p(D) ?$$



## 3. "Disorient" the graph

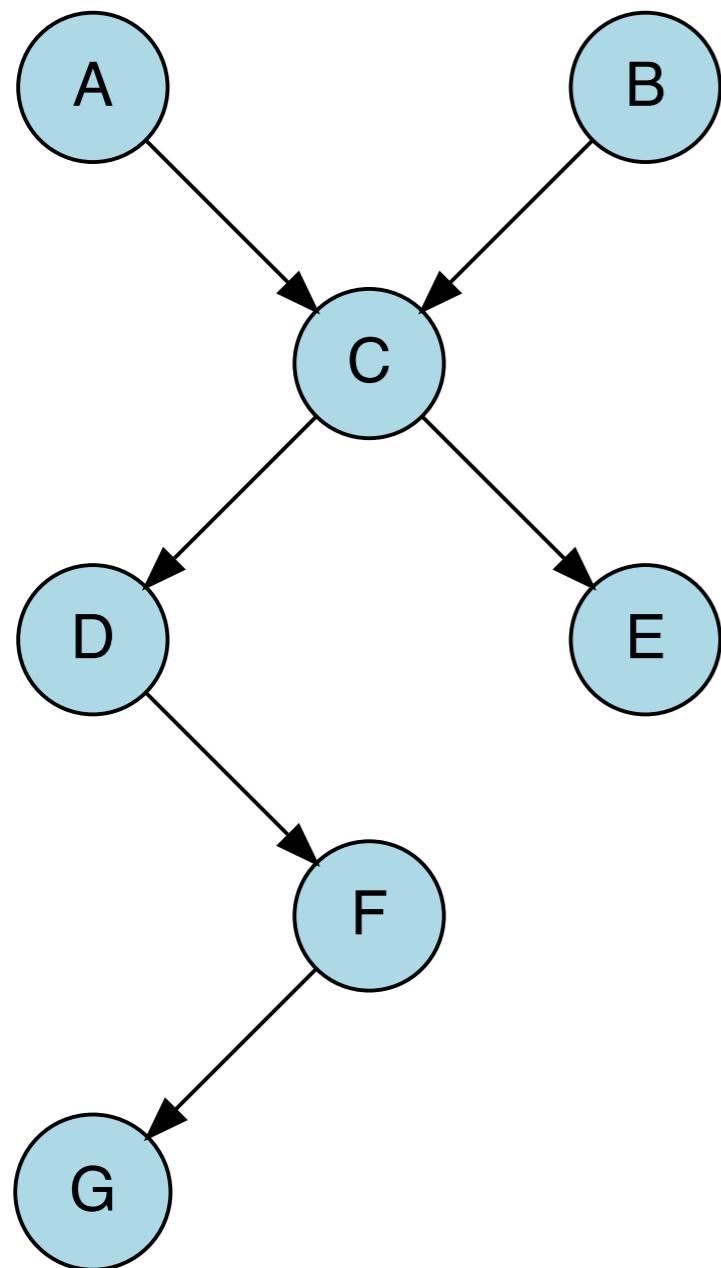
Replace arrows with lines



# When should I control for variables?

**Are D and E independent?**

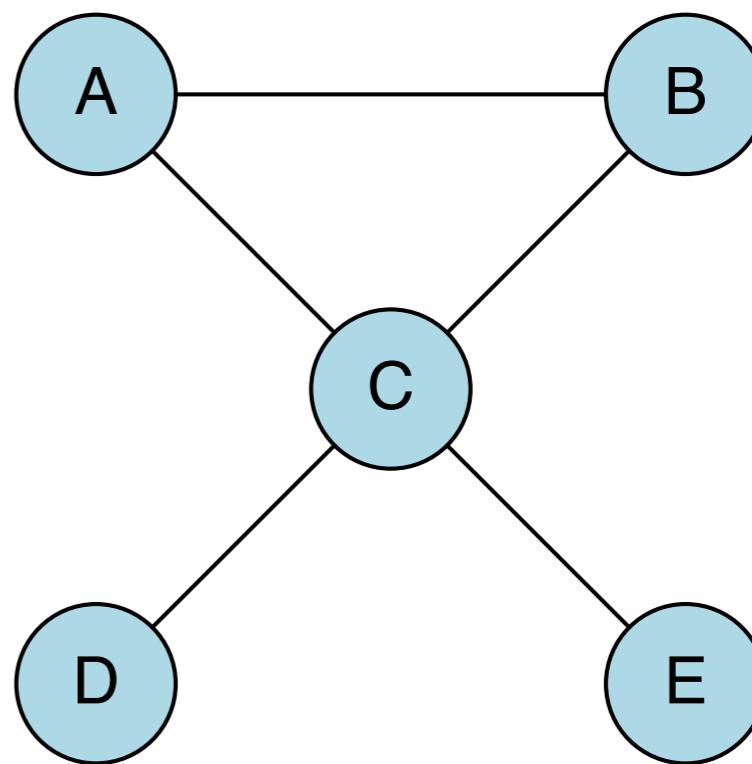
$$p(D | E) = p(D) ?$$



## 4. Delete the givens

Remove the variables that we condition on, as well as their edges

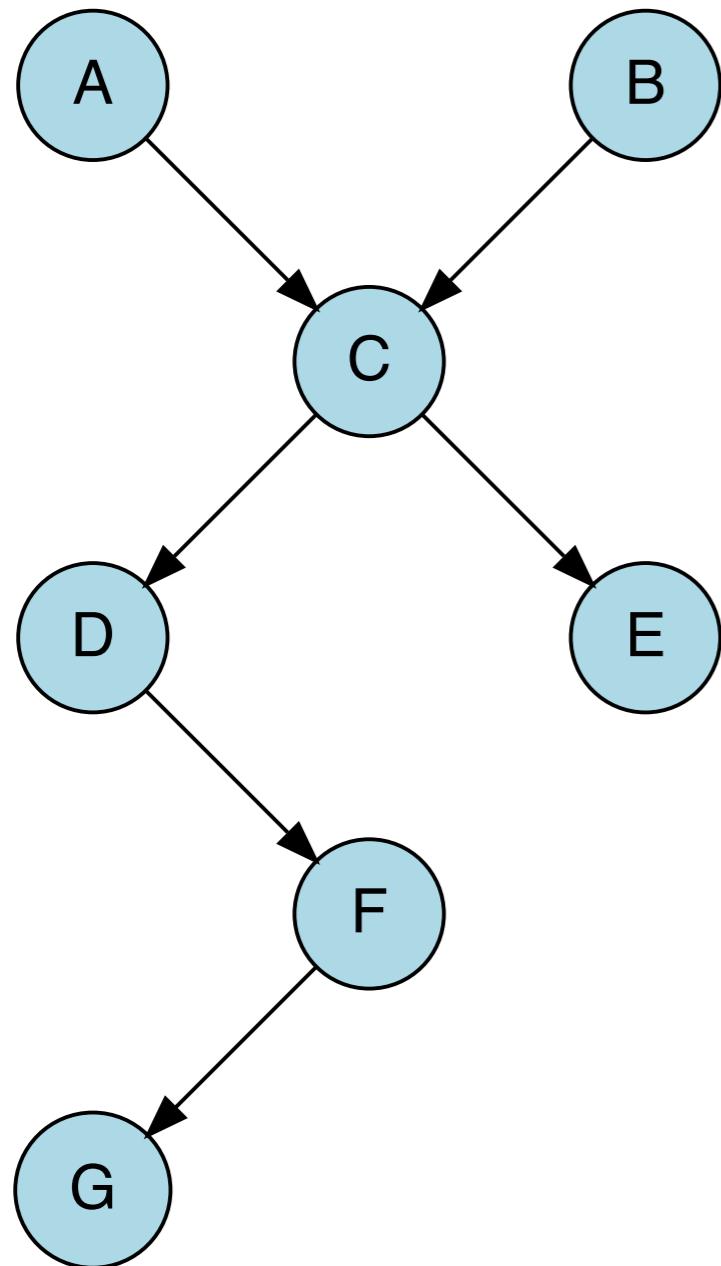
**we didn't condition on anything,  
so there is nothing to delete**



# When should I control for variables?

**Are D and E independent?**

$$p(D | E) = p(D) ?$$



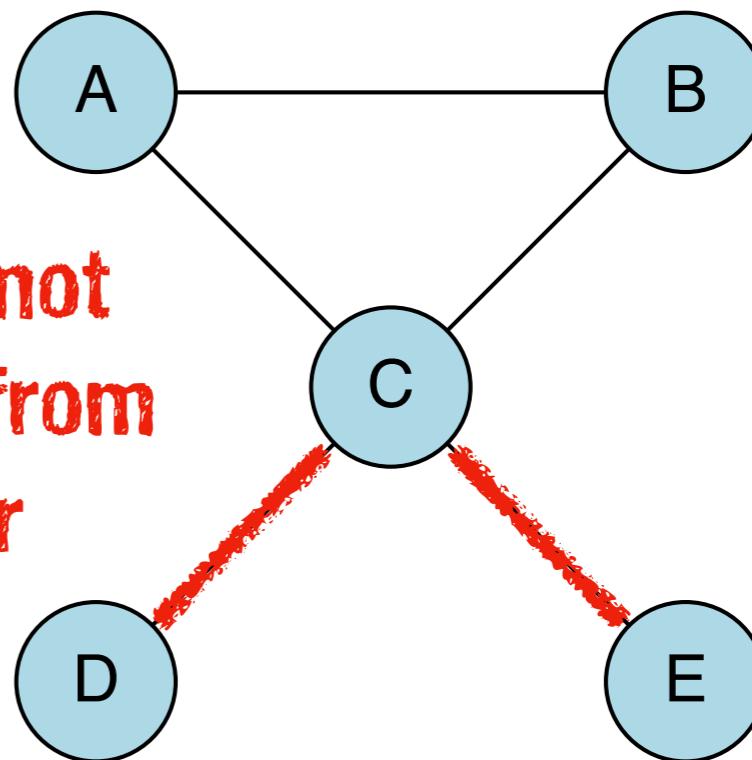
**5. Read answer off the graph**

- if variables are **disconnected** they are independent
- if variables are connected (have a path between them) they are not guaranteed to be independent

**D and E are not independent from each other**



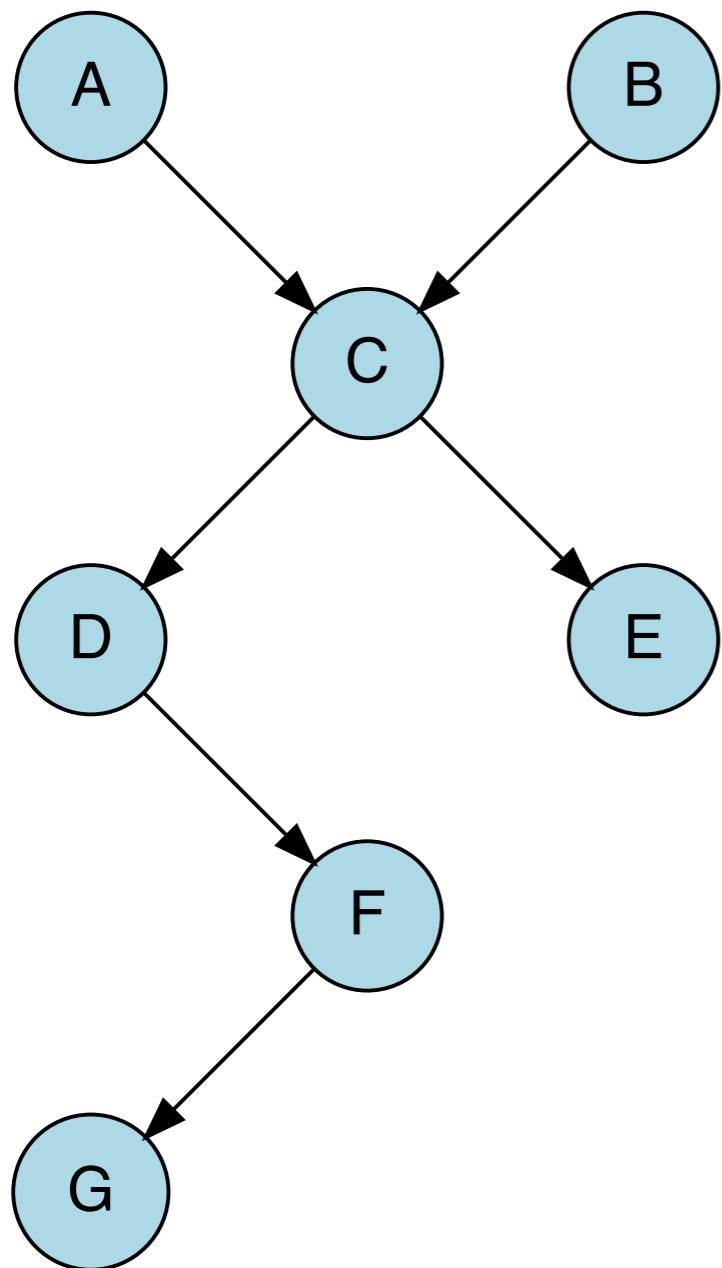
**they are connected via at least one path**



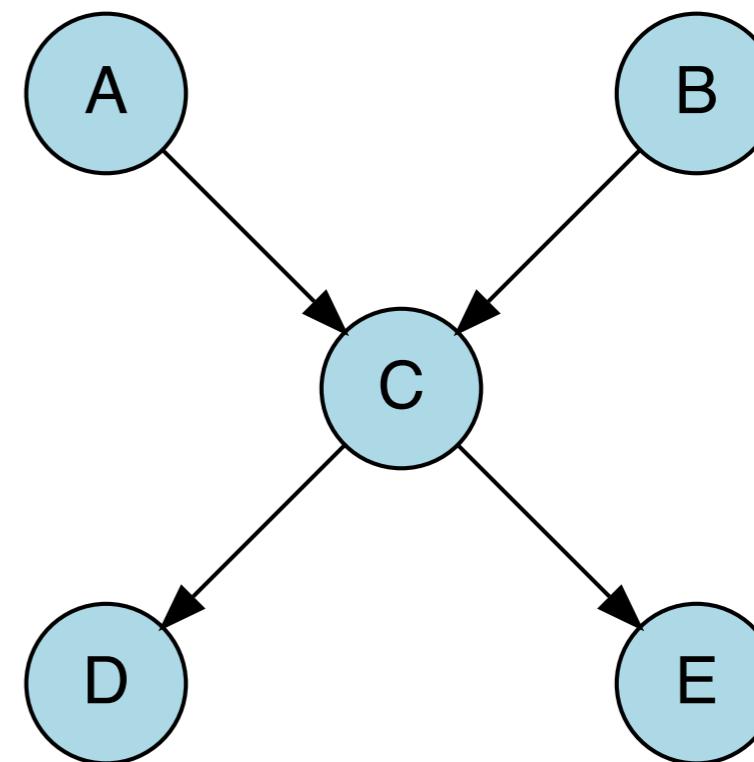
# When should I control for variables?

**Are D and E independent, given C?** 1. Draw the ancestral graph

$$p(D | E, C) = p(D | C) ?$$



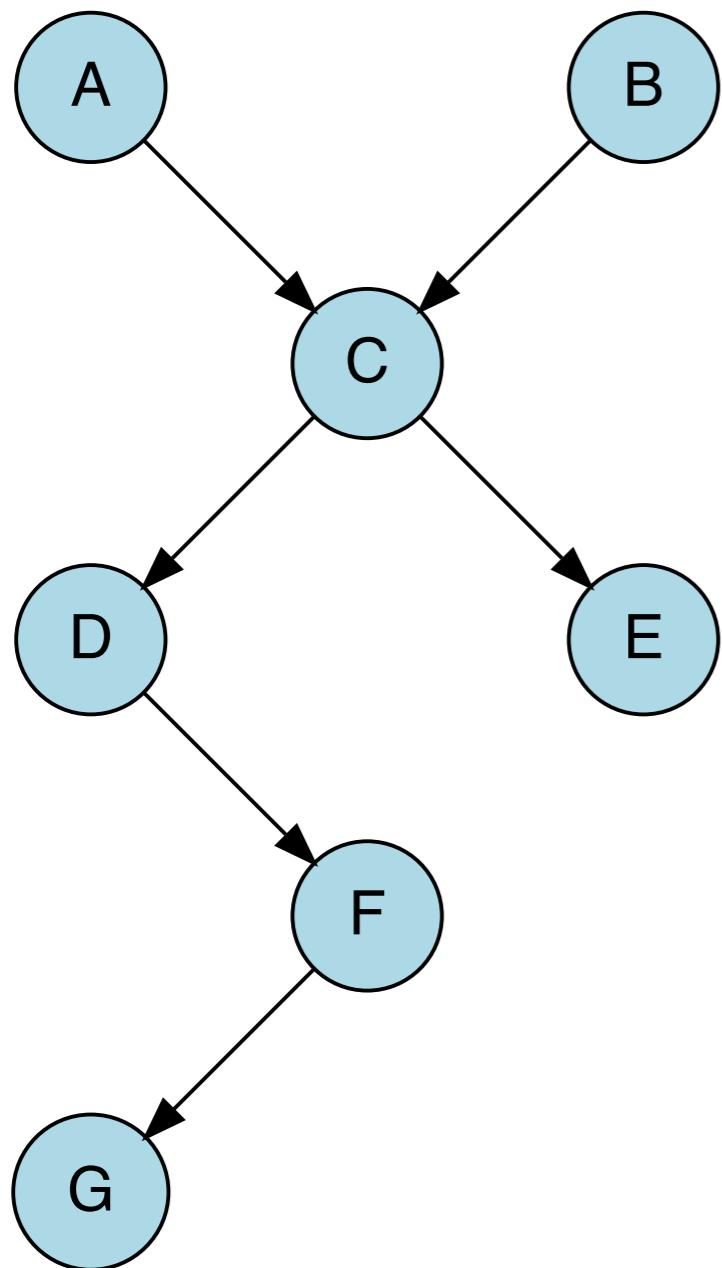
Construct the "ancestral graph" of all variables mentioned in the probability expression. This is a reduced version of the original net, consisting only of the variables mentioned and all of their ancestors (parents, parents' parents, etc.)



# When should I control for variables?

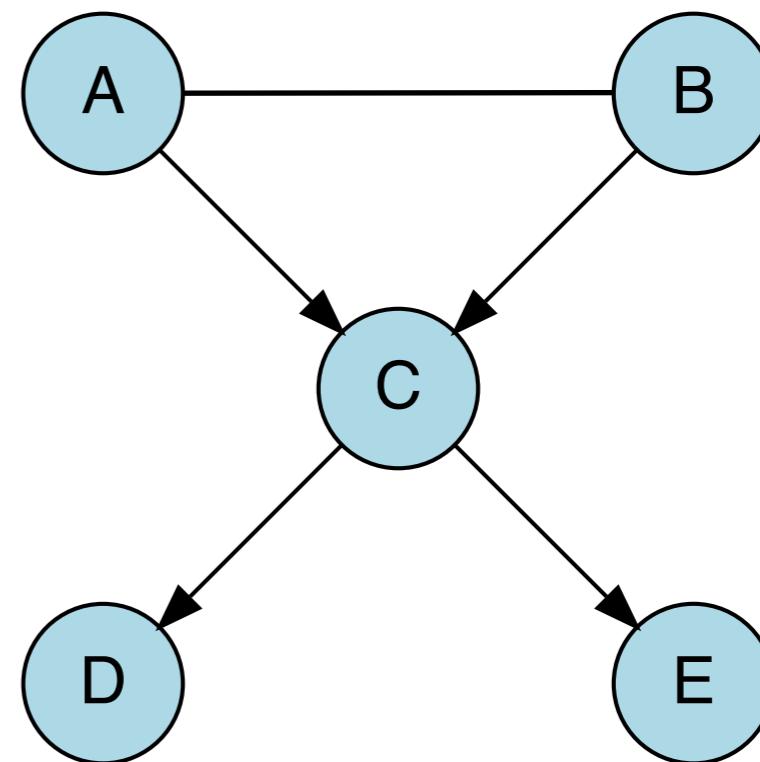
**Are D and E independent, given C? 2. "Moralize" the graph**

$$p(D | E, C) = p(D | C) ?$$



**let's get married!**

For each pair of variables with a common child, draw an undirected edge (line) between them. (If a variable has more than two parents, draw lines between every pair of parents.)

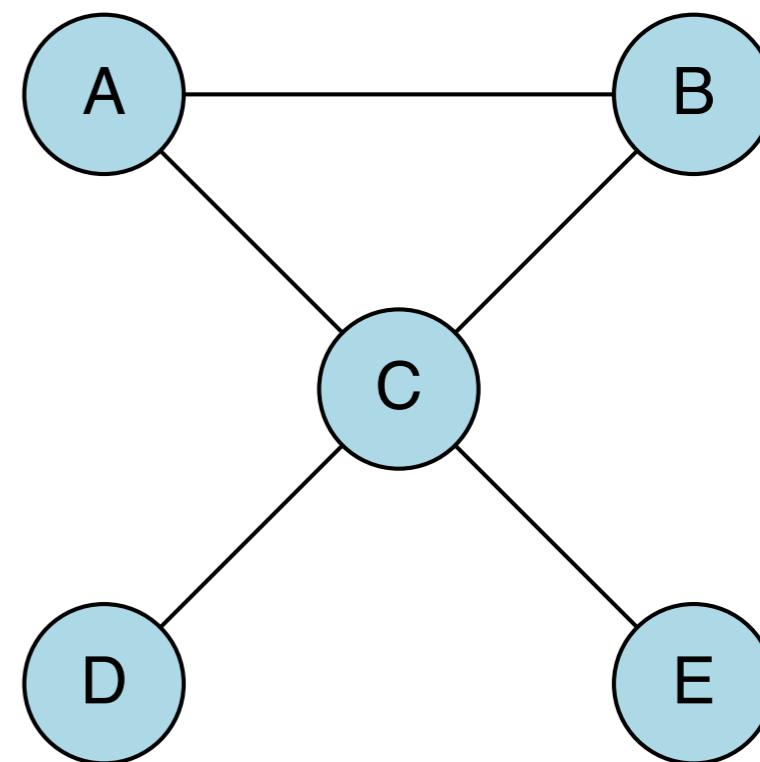
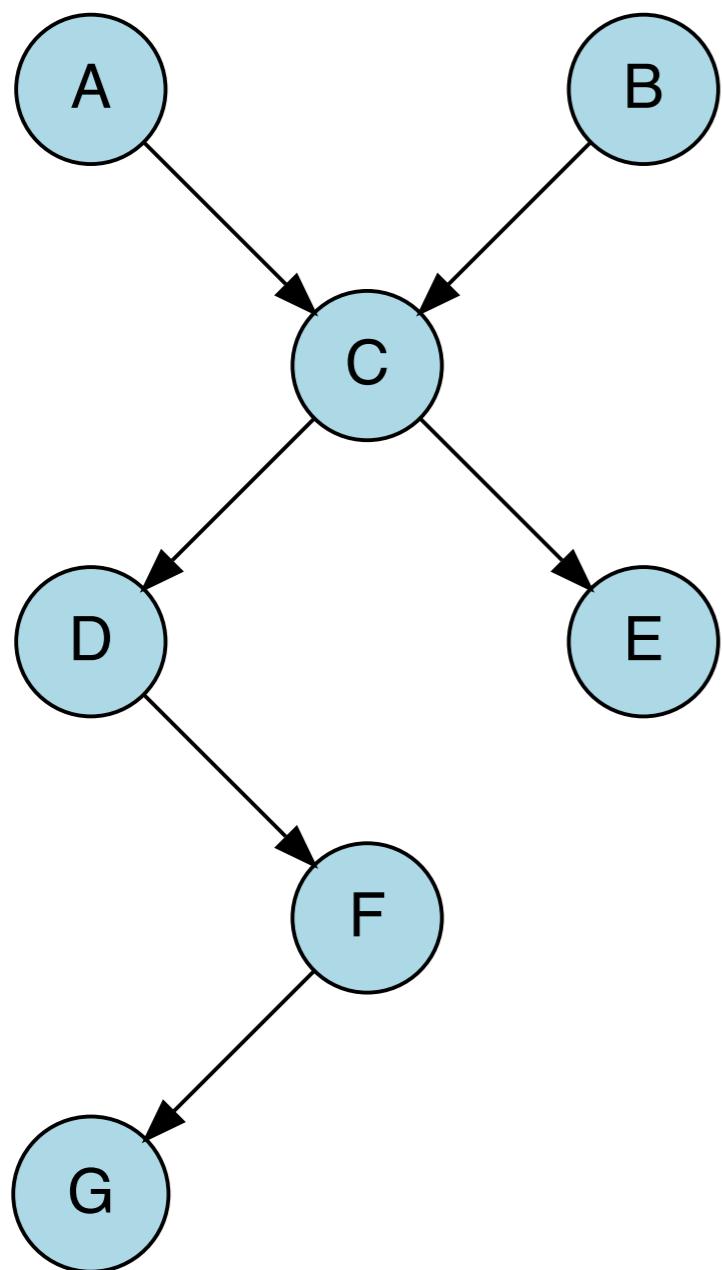


# When should I control for variables?

**Are D and E independent, given C? 3. "Disorient" the graph**

$$p(D | E, C) = p(D | C) ?$$

Replace arrows with lines



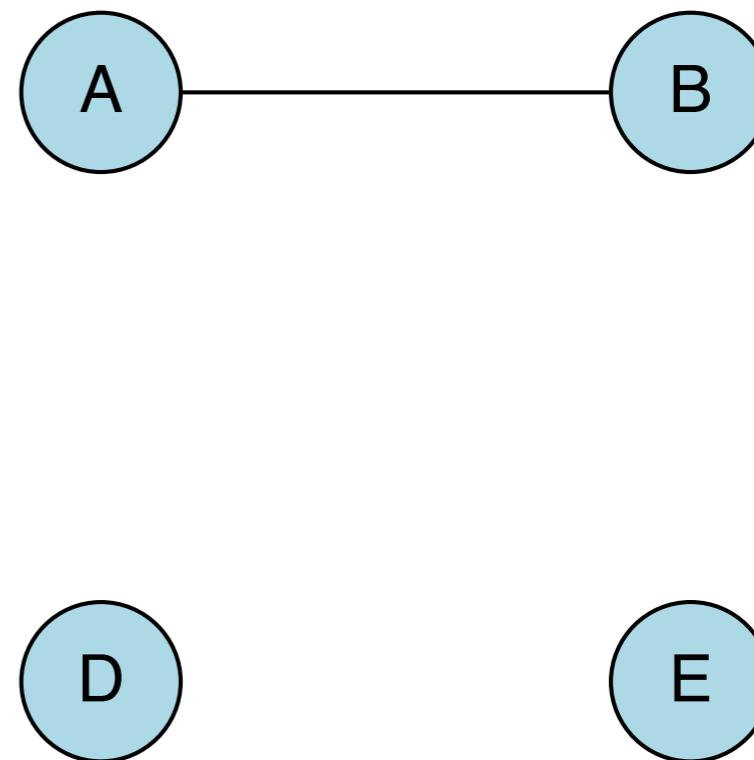
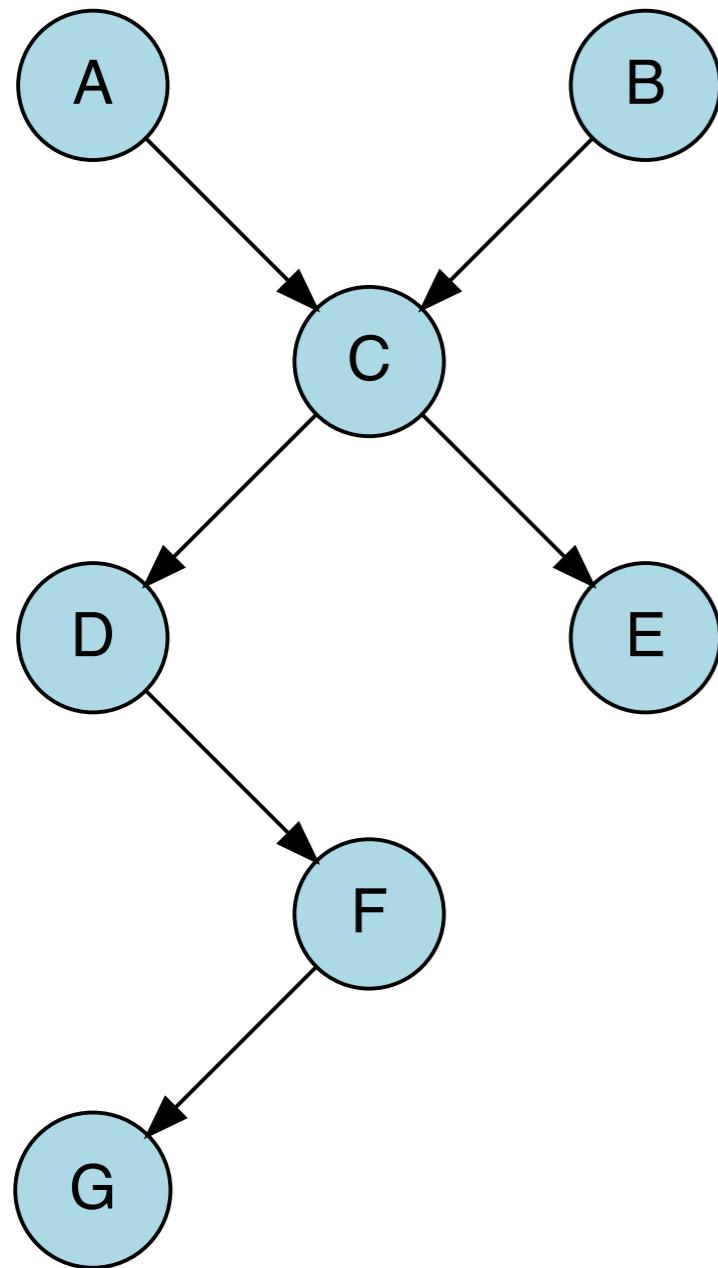
# When should I control for variables?

**Are D and E independent, given C? 4. Delete the givens**

$$p(D | E, C) = p(D | C) ?$$

Remove the variables that we condition on, as well as their edges

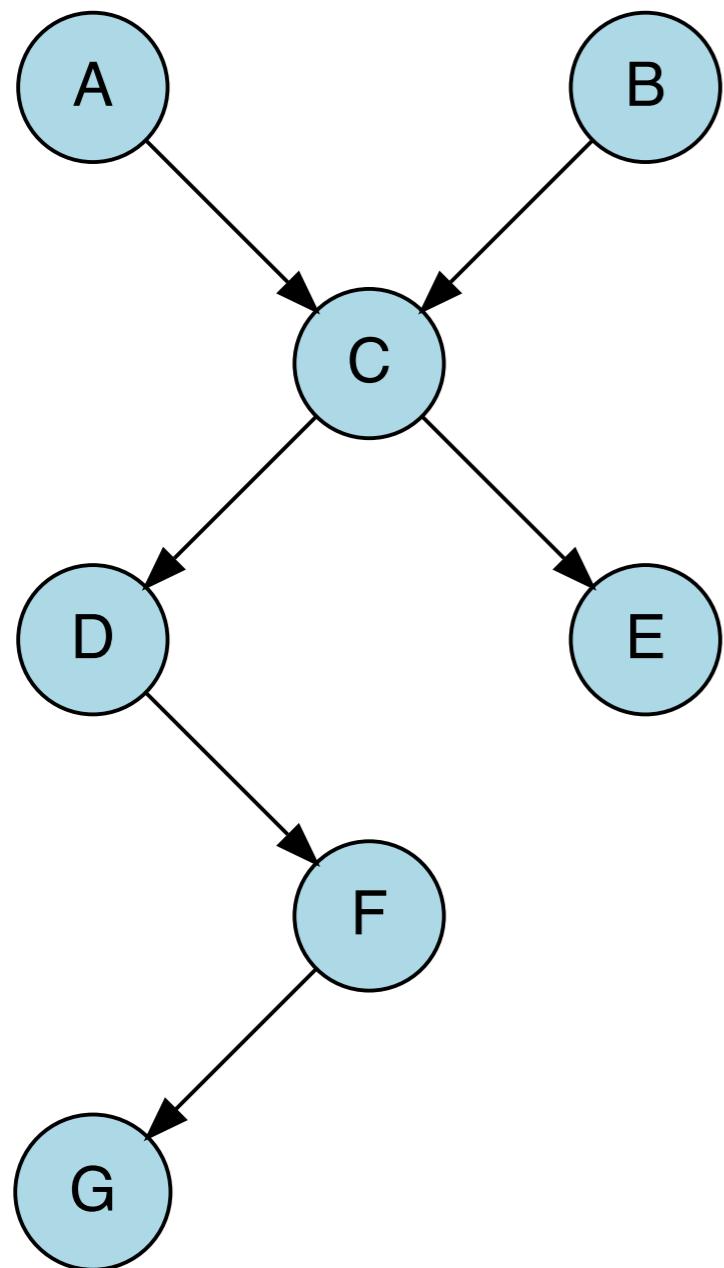
**we conditioned on C!**



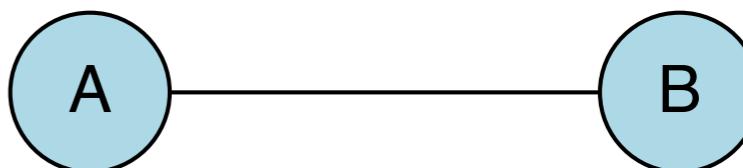
# When should I control for variables?

**Are D and E independent, given C? 5. Read answer off the graph**

$$p(D | E, C) = p(D | C) ?$$



- if variables are **disconnected** they are independent
- if variables are connected (have a path between them) they are not guaranteed to be independent



**D and E are independent from each other conditioned on C**



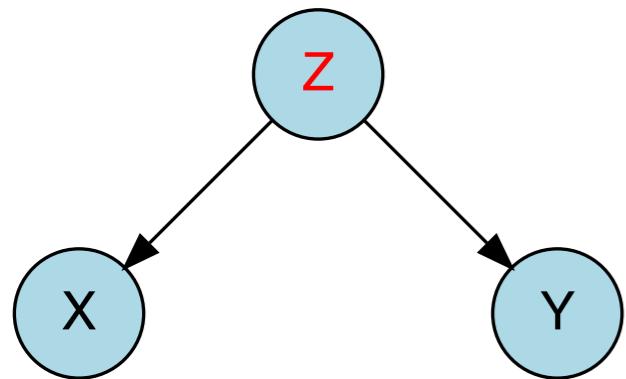
**they aren't connected via a path**

# So what?

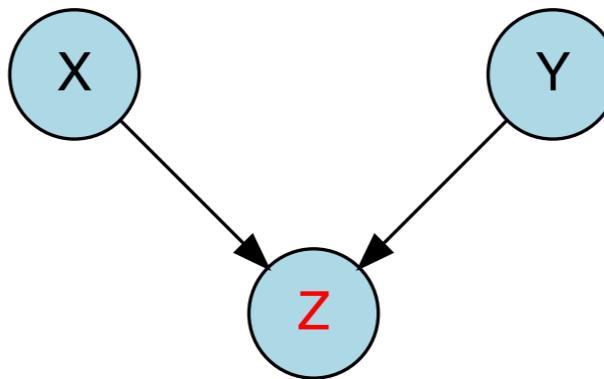


# Patterns of inference

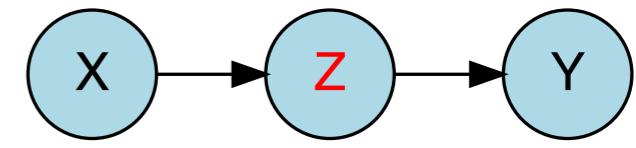
We want to estimate the (causal) relationship between X and Y



common cause



common effect



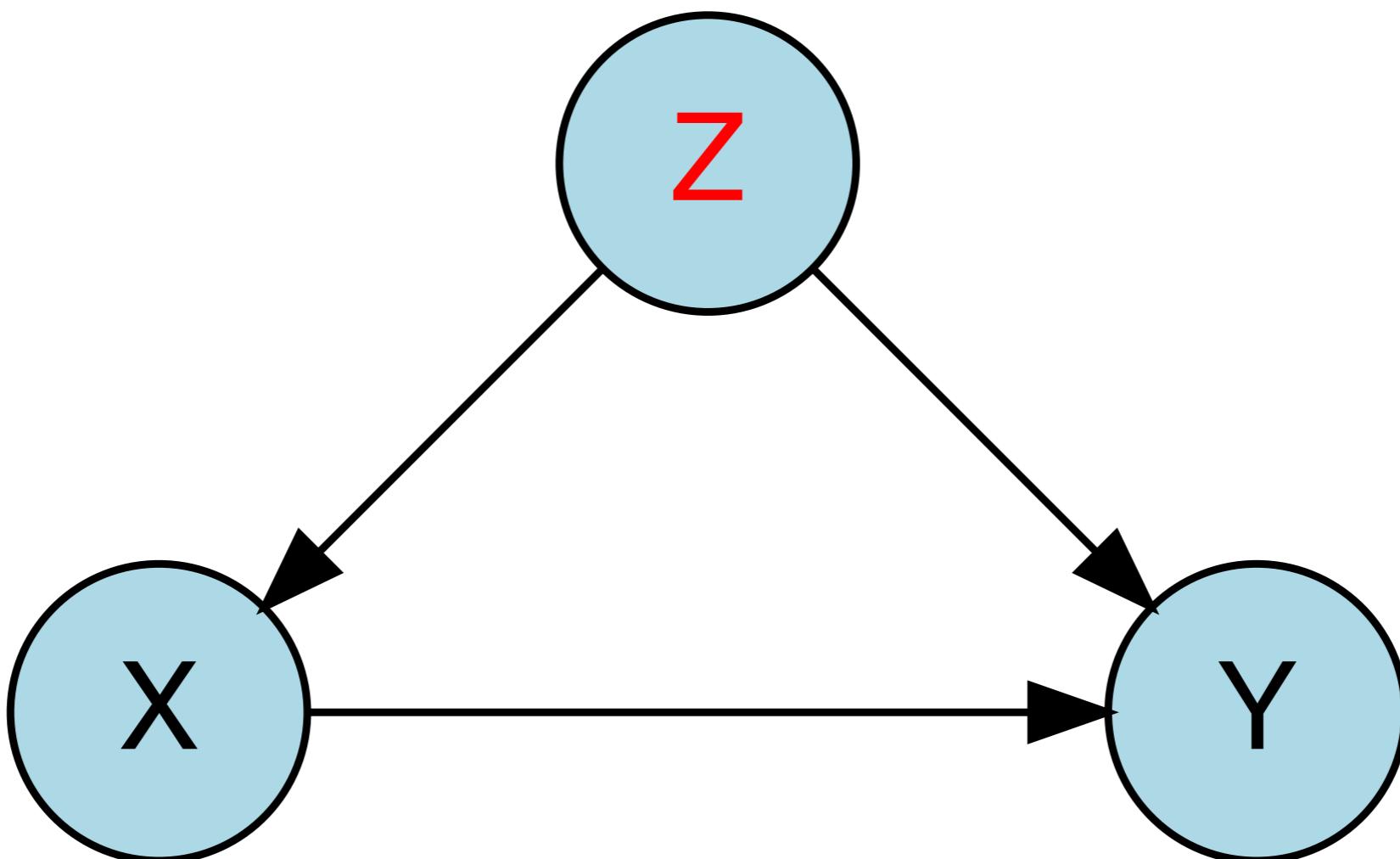
causal chain

by controlling for Z we hope to get a better estimate of the relationship between X and Y

**d-separation** helps us tell apart **good controls** from **bad controls**

# When should I control for variables?

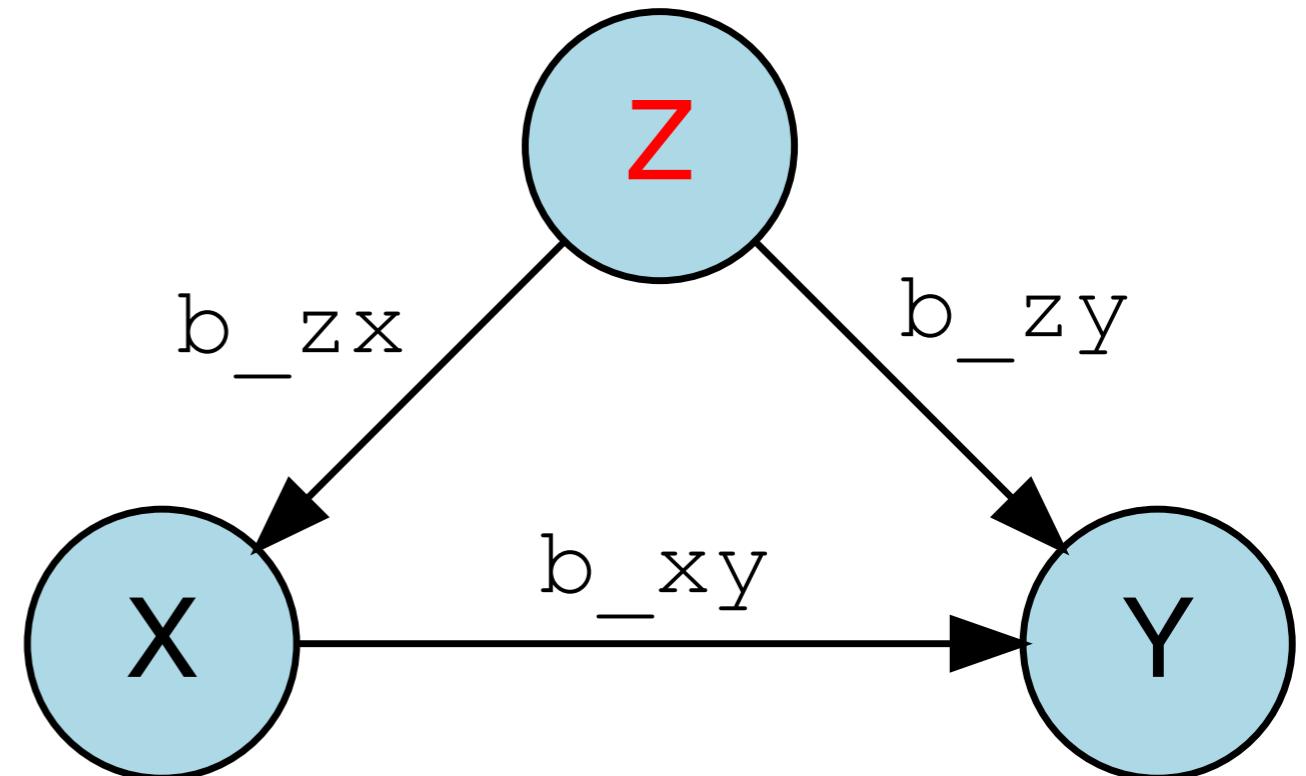
I want to estimate the effect that X has on Y



Is Z a **good** or a **bad** control here?

# When should I control for variables?

```
1 set.seed(1)
2
3 n = 1000
4 b_zx = 2
5 b_xy = 2
6 b_zy = 2
7 sd = 1
8
9 fun_error = function(n, sd) {
10   rnorm(n = n,
11         mean = 0,
12         sd = sd)
13 }
14
15 df = tibble(z = fun_error(n, sd),
16               x = b_zx * z + fun_error(n, sd),
17               y = b_zy * z + b_xy * x + fun_error(n, sd))
```



overestimating  
X's effect on Y

$$Y = b_0 + b_1 \cdot X + e$$

```
1 # without control
2 lm(formula = y ~ x,
3     data = df) %>%
4     summary()
```

```
Call:
lm(formula = y ~ x, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-4.6011 -0.9270 -0.0506  0.9711  4.0454

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.02449   0.04389   0.558   0.577
x           2.82092   0.01890 149.225 <2e-16 ***
---
Signif. codes:  0 '****' 0.001 '***' 0.01 '**' 0.05 '*' 0.1 '.' 1

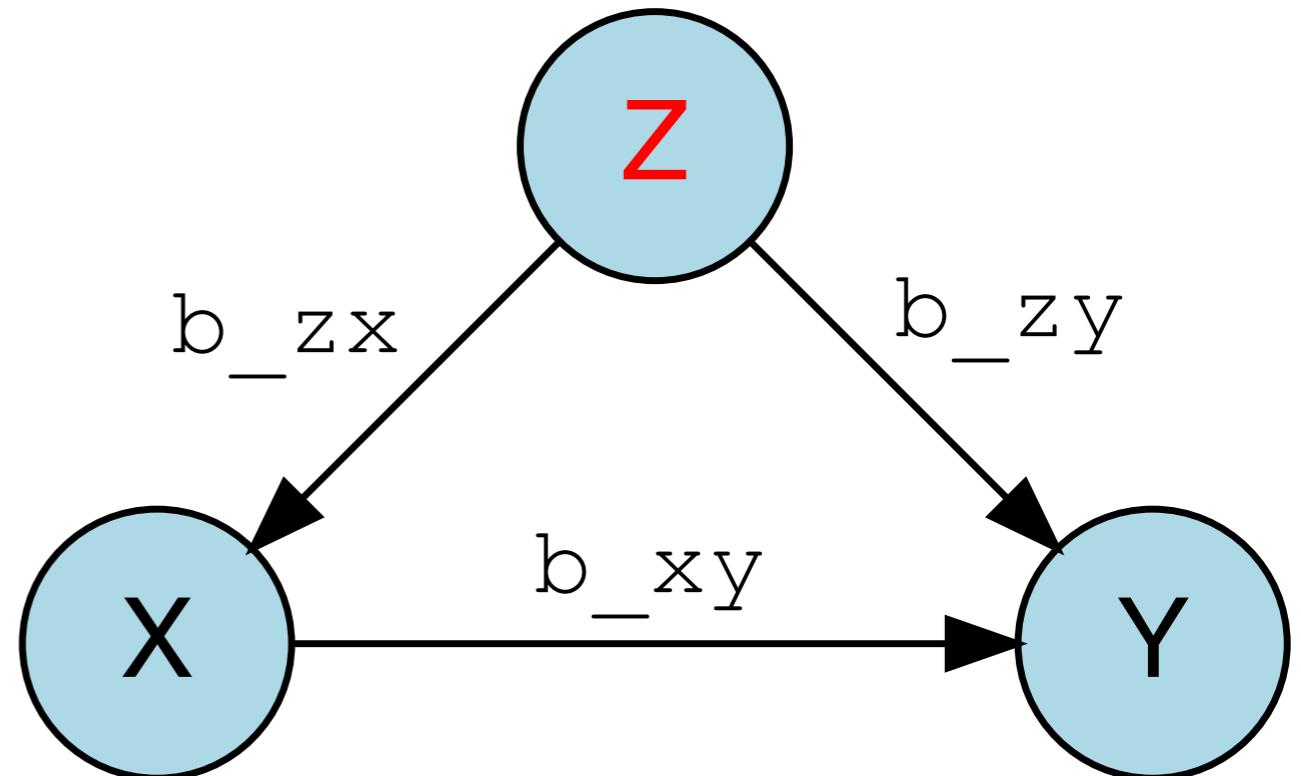
Residual standard error: 1.388 on 998 degrees of freedom
Multiple R-squared:  0.9571,    Adjusted R-squared:  0.9571
F-statistic: 2.227e+04 on 1 and 998 DF,  p-value: < 2.2e-16
```

# When should I control for variables?

```
1 set.seed(1)
2
3 n = 1000
4 b_zx = 2
5 b_xy = 2
6 b_zy = 2
7 sd = 1
8
9 fun_error = function(n, sd) {
10   rnorm(n = n,
11         mean = 0,
12         sd = sd)
13 }
14
15 df = tibble(z = fun_error(n, sd),
16               x = b_zx * z + fun_error(n, sd),
17               y = b_zy * z + b_xy * x + fun_error(n, sd))
```

$$Y = b_0 + b_1 \cdot X + b_2 \cdot Z + e$$

```
1 # with control
2 lm(formula = y ~ x + z,
3     data = df) %>%
4   summary()
```



Call:  
lm(formula = y ~ x + z, data = df)

Residuals:

Min	1Q	Median	3Q	Max
-3.6151	-0.6564	-0.0223	0.6815	2.8132

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.01624	0.03260	0.498	0.618
x	2.02202	0.03135	64.489	<2e-16 ***
z	2.00501	0.07036	28.497	<2e-16 ***

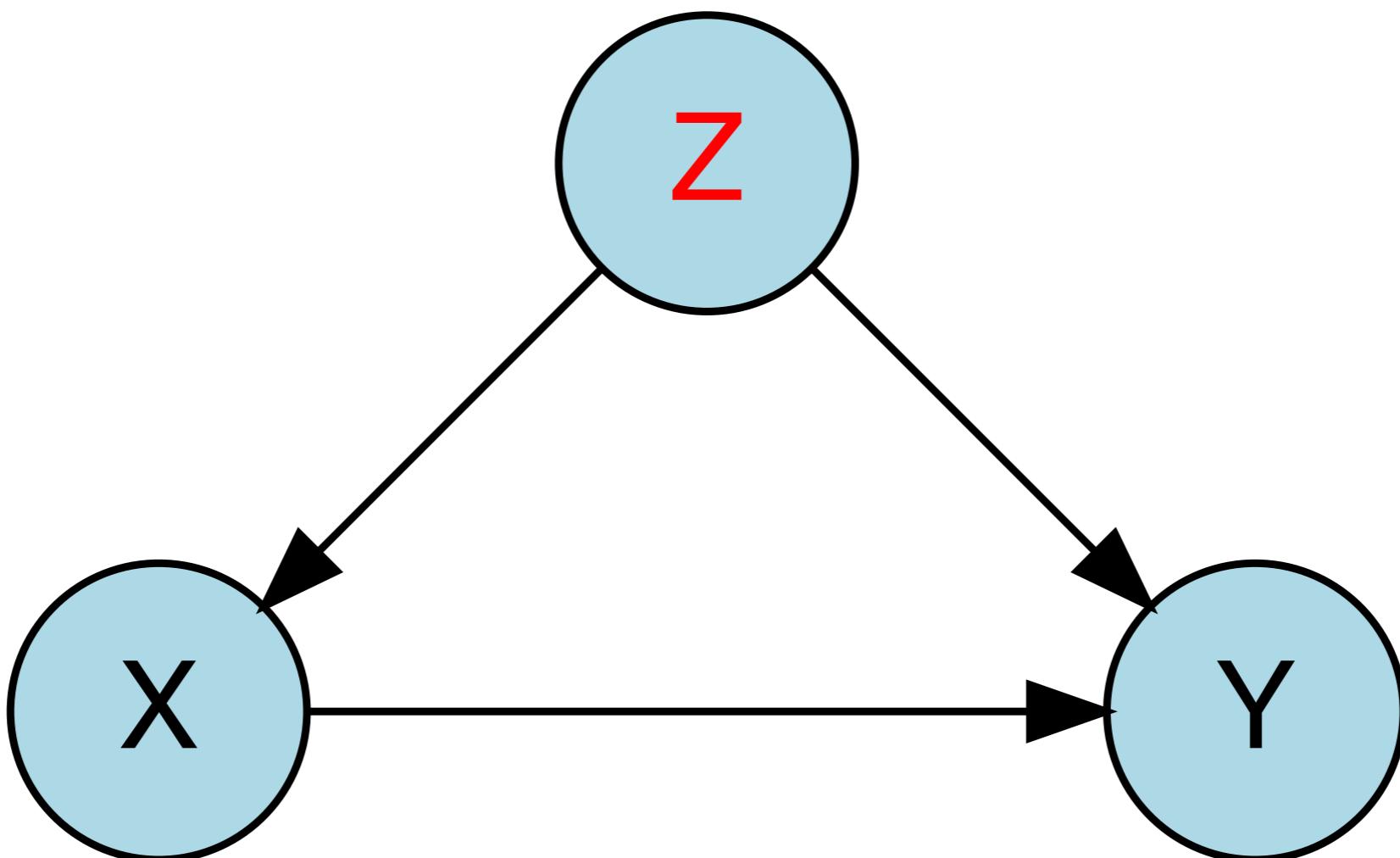
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.031 on 997 degrees of freedom  
Multiple R-squared: 0.9764, Adjusted R-squared: 0.9763  
F-statistic: 2.059e+04 on 2 and 997 DF, p-value: < 2.2e-16

**accurate estimate  
of X's effect on Y**

# When should I control for variables?

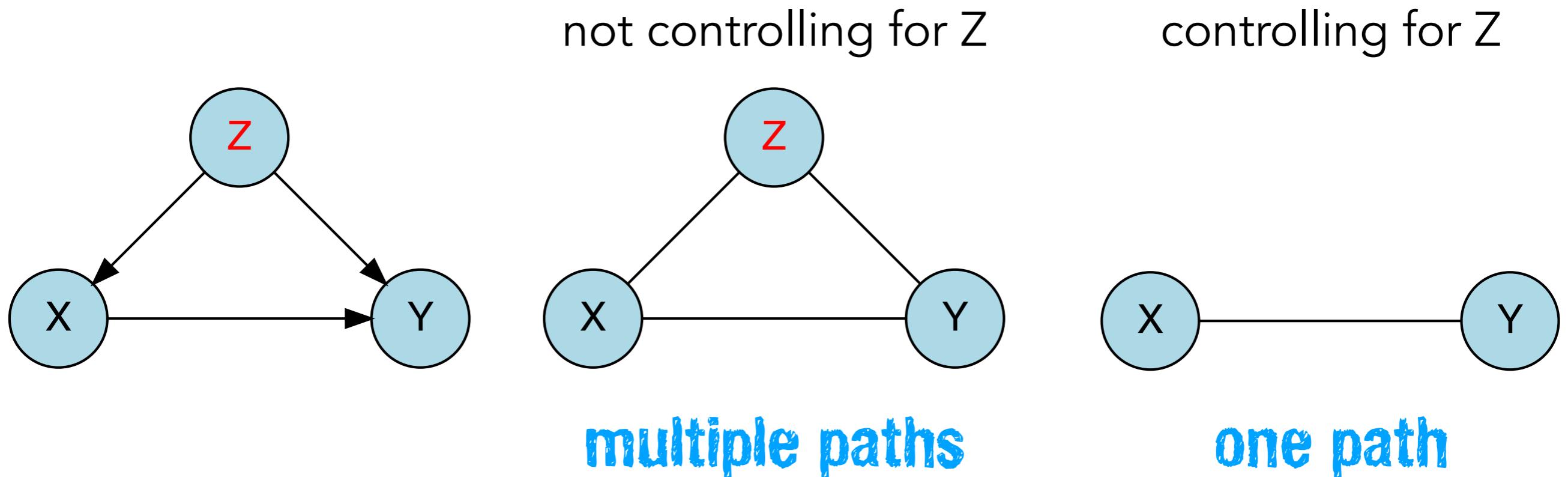
I want to estimate the effect that X has on Y



Z is a **good** control here!

# When should I control for variables?

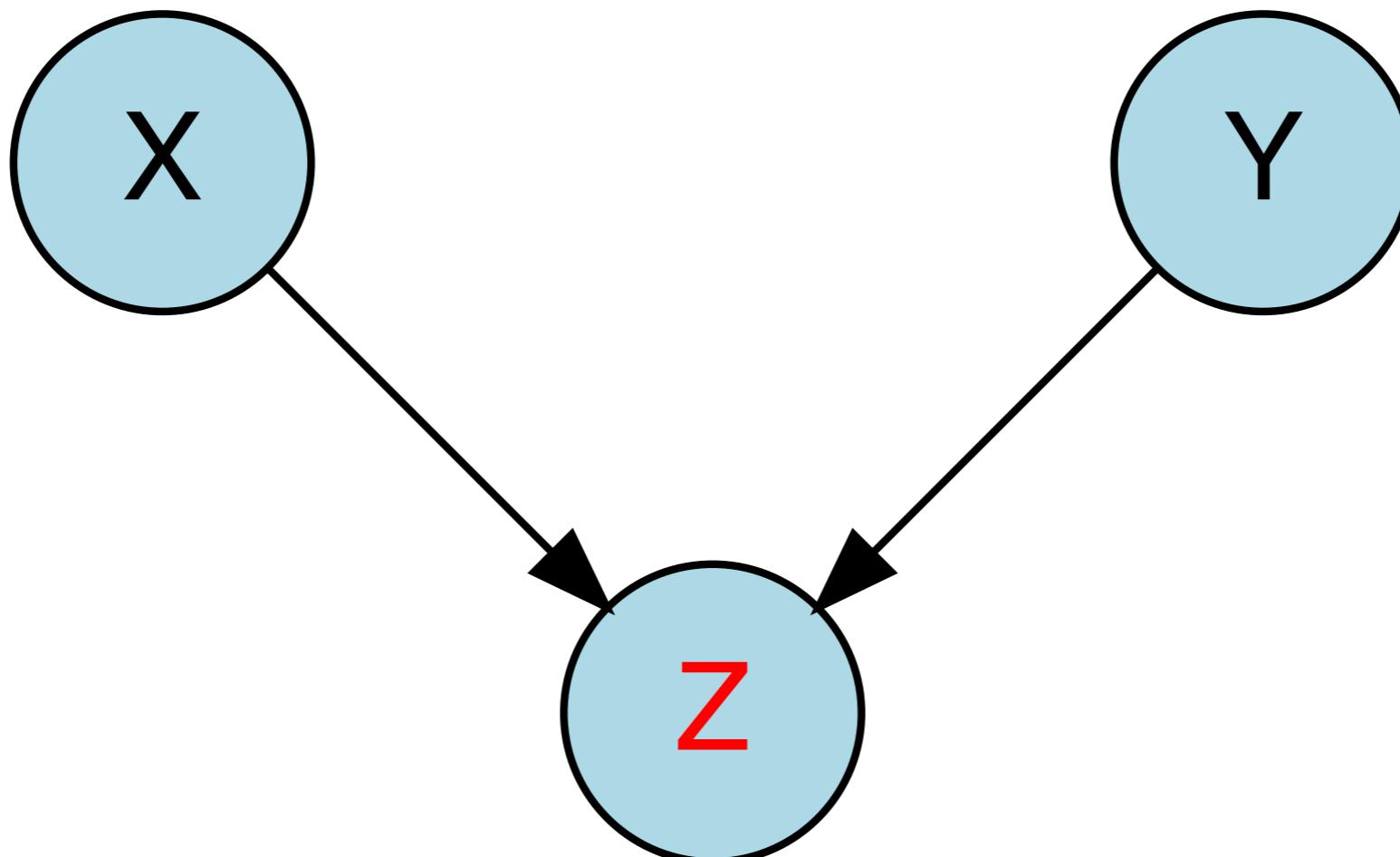
I want to estimate the effect that X has on Y



Z is a **good** control here!

# When should I control for variables?

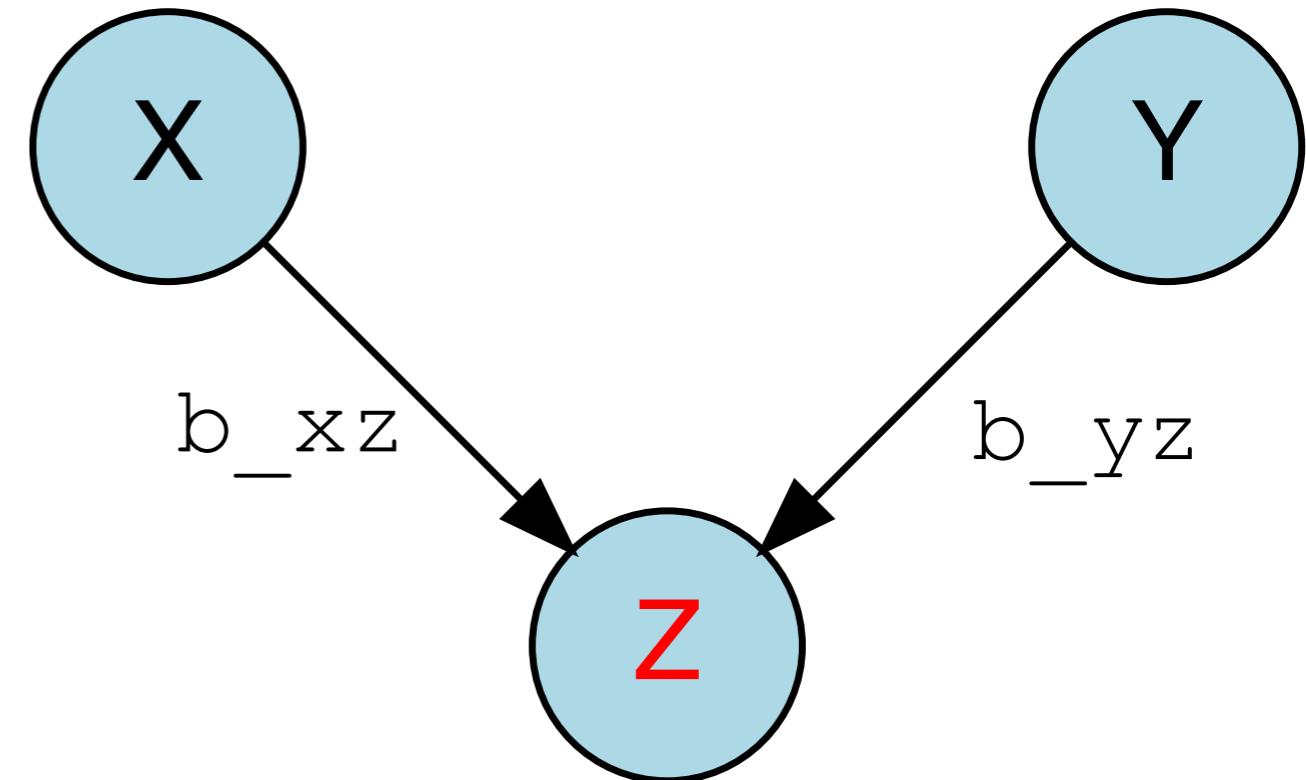
I want to estimate the effect that X has on Y



Is Z a **good** or a **bad** control here?

# When should I control for variables?

```
1 set.seed(1)
2 n = 1000
3 b_xz = 2
4 b_yz = 2
5 sd = 1
6
7 fun_error = function(n, sd){
8   rnorm(n = n,
9         mean = 0,
10        sd = sd)
11 }
12
13 df = tibble(x = fun_error(n, sd),
14               y = fun_error(n, sd),
15               z = x * b_xz + y * b_yz + fun_error(n, sd))
```



accurate estimate  
of X's effect on Y

$$Y = b_0 + b_1 \cdot X + e$$

```
1 # without control
2 lm(formula = y ~ x,
3     data = df) %>%
4   summary()
```

```
Call:
lm(formula = y ~ x, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.2484 -0.6720 -0.0138  0.7554  3.6443 

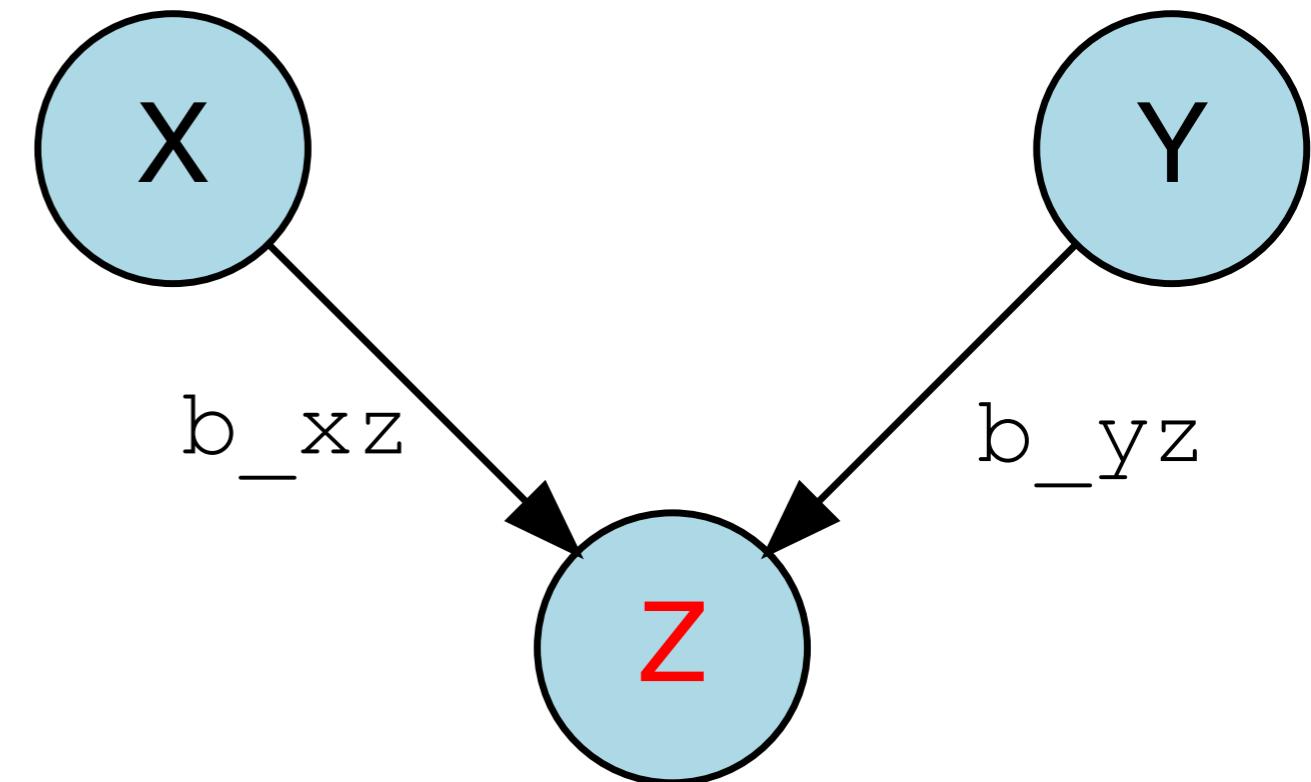
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.016187  0.032905 -0.492   0.623    
x            0.006433  0.031809  0.202   0.840    
                                                        
Residual standard error: 1.04 on 998 degrees of freedom
Multiple R-squared:  4.098e-05, Adjusted R-squared: -0.000961 
F-statistic: 0.0409 on 1 and 998 DF, p-value: 0.8398
```

# When should I control for variables?

```
1 set.seed(1)
2 n = 1000
3 b_xz = 2
4 b_yz = 2
5 sd = 1
6
7 fun_error = function(n, sd){
8   rnorm(n = n,
9         mean = 0,
10        sd = sd)
11 }
12
13 df = tibble(x = fun_error(n, sd),
14               y = fun_error(n, sd),
15               z = x * b_xz + y * b_yz + fun_error(n, sd))
```

$$Y = b_0 + b_1 \cdot X + b_2 \cdot Z + e$$

```
1 # with control
2 lm(formula = y ~ x + z,
3     data = df) %>%
4   summary()
```



inaccurate  
estimate of X's  
effect on Y

```
Call:
lm(formula = y ~ x + z, data = df)

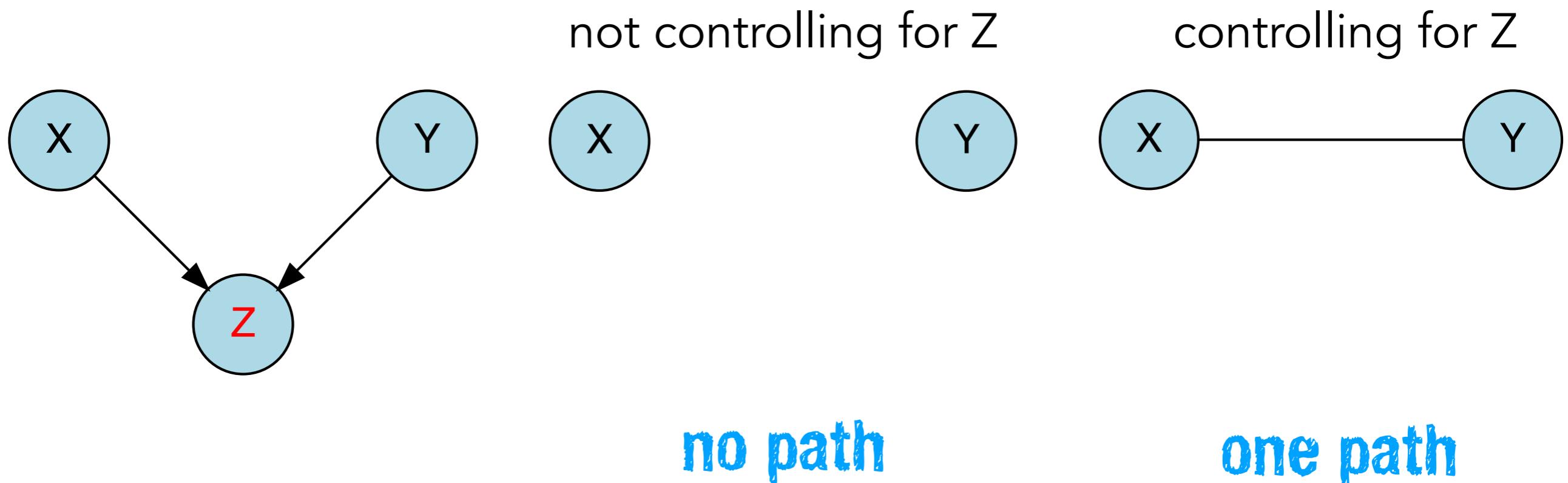
Residuals:
    Min      1Q  Median      3Q     Max 
-1.35547 -0.30016  0.00298  0.31119  1.73408 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.009608  0.014477  -0.664   0.507    
x            -0.816164  0.018936 -43.102 <2e-16 ***  
z             0.398921  0.006186  64.489 <2e-16 ***  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4578 on 997 degrees of freedom
Multiple R-squared:  0.8066,    Adjusted R-squared:  0.8062 
F-statistic: 2079 on 2 and 997 DF,  p-value: < 2.2e-16
```

# When should I control for variables?

I want to estimate the effect that X has on Y



Z is a **bad** control here!

# When should I control for variables?

## Babies Learning Language

Thoughts on language learning, child development, and fatherhood; experimental methods, reproducibility, and open science; theoretical musings on cognitive science more broadly.

Tuesday, October 8, 2019

### Confounds and covariates

(tl;dr: explanation of confounding and covariate adjustment)

Every year, one of the trickiest concepts for me to teach in my experimental methods course is the difference between experimental confounds and covariates. Although this distinction seems simple, it's pretty deeply related to the definition of what an experiment is and why experiments lead to good causal inferences. It's also caught up in a number of methodological problems that come up again and again in my class. This post is my attempt to explain the distinction and how it relates to different problems and cultural practices in psychology.

#### About



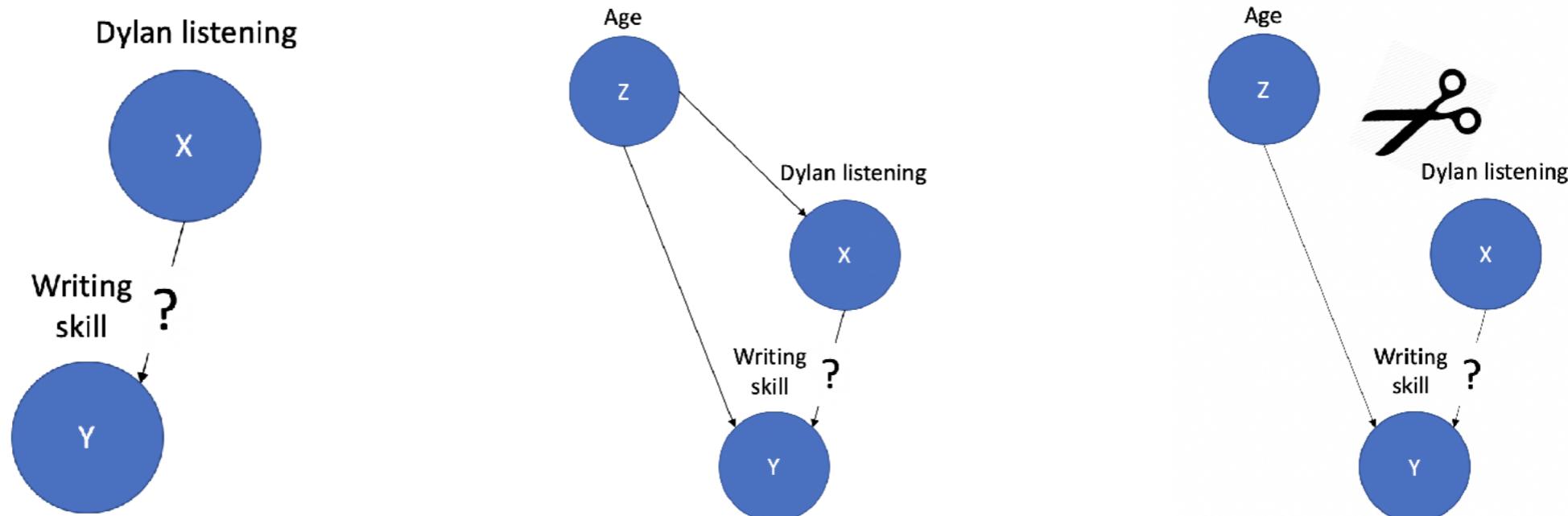
[Michael Frank](#)

Developmental Psychologist at Stanford University. Interested in language acquisition, cognitive development, thoughts, numbers, learning, and teaching. [Visit my lab's website](#).

[View my complete profile](#)

#### Labels

- [Development](#) (40)

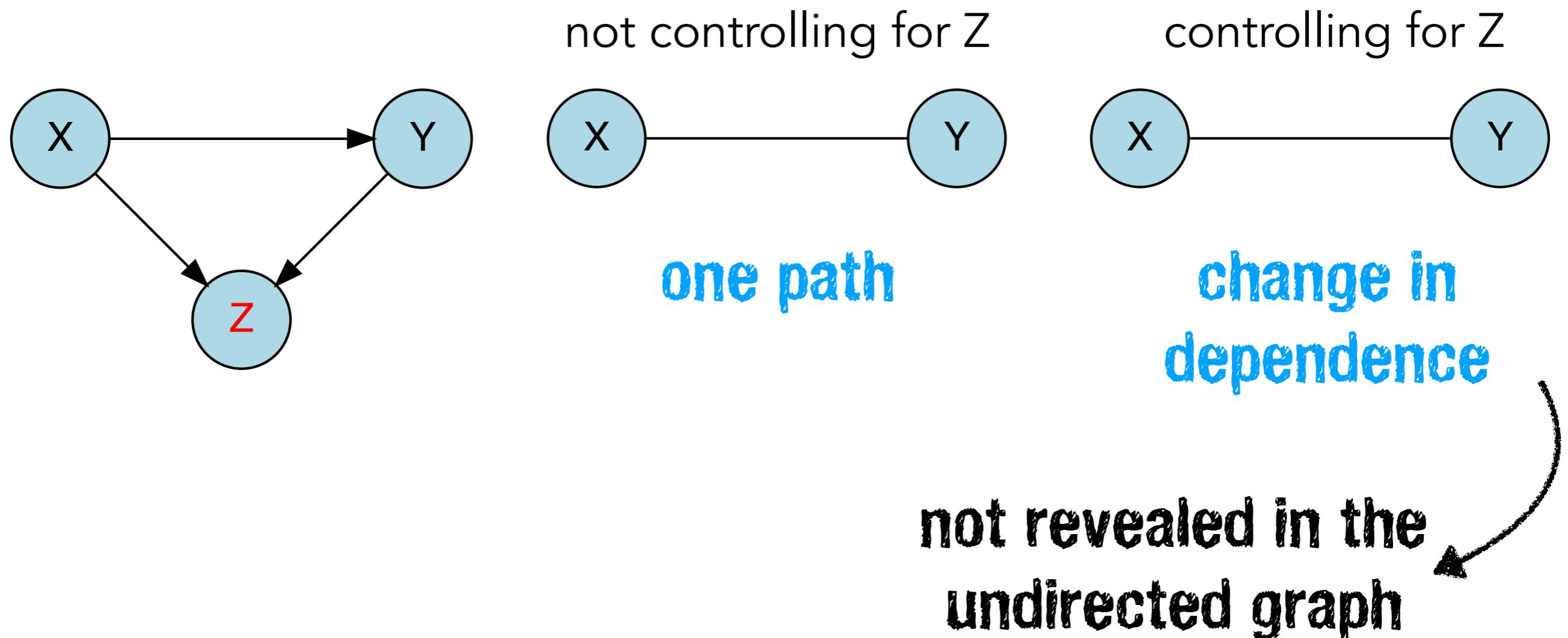


# When should I control for variables?

- checking for **d-separation** tells us whether or not variables are (conditionally) independent
- it also tells us whether paths of dependence "open up", or get "closed down"
- the graphical procedure doesn't necessarily reveal whether the dependence between variables changes: it reveals the **structure** of dependence but not the **strength**
- you can always double check via running simulations in R

# When should I control for variables?

I want to estimate the effect that X has on Y



Z is a **bad** control here!

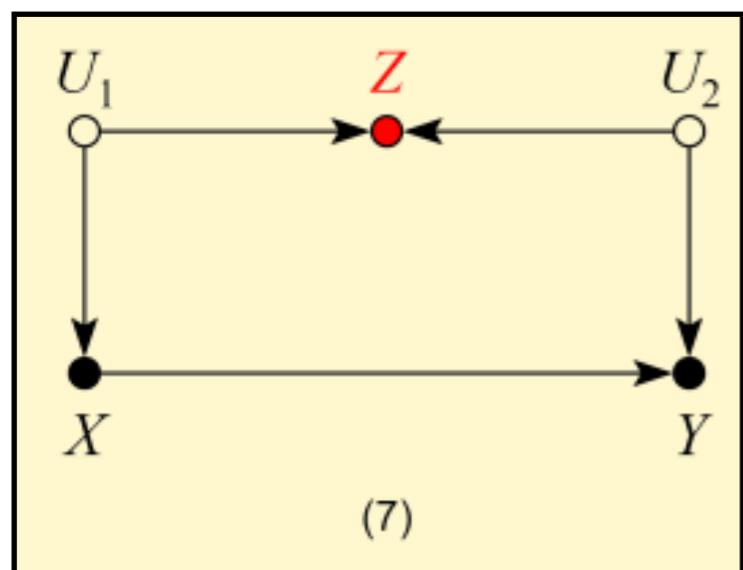
# When should I control for variables?

- **good controls** reduce additional paths from X to Y apart from the direct path we are interested in estimating
- **bad controls** introduce additional paths (or change existing ones) that lead to a biased estimate of the direct path between X and Y

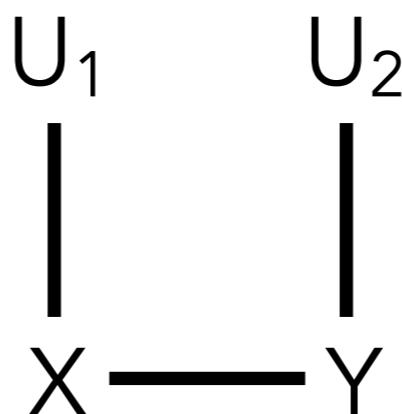
# When should I control for variables?

"Important topic. I feel like this procedure for determining good and bad controls is a bit convoluted. I feel like there's a simpler shorthand. Does another factor influence both x and y? Does another factor influence x? What's wrong with that simpler algorithm?"

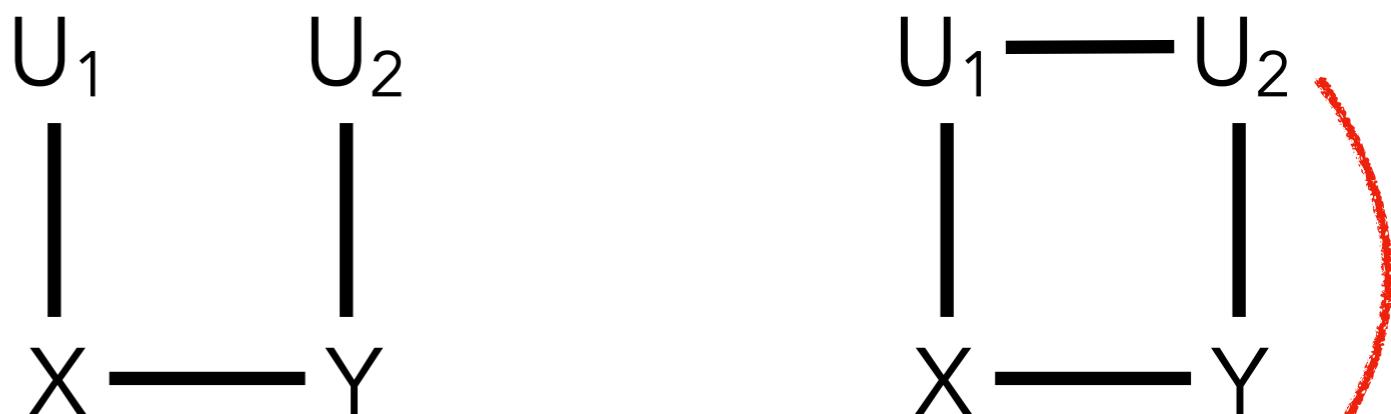
**not enough unfortunately**



not controlling for Z



controlling for Z

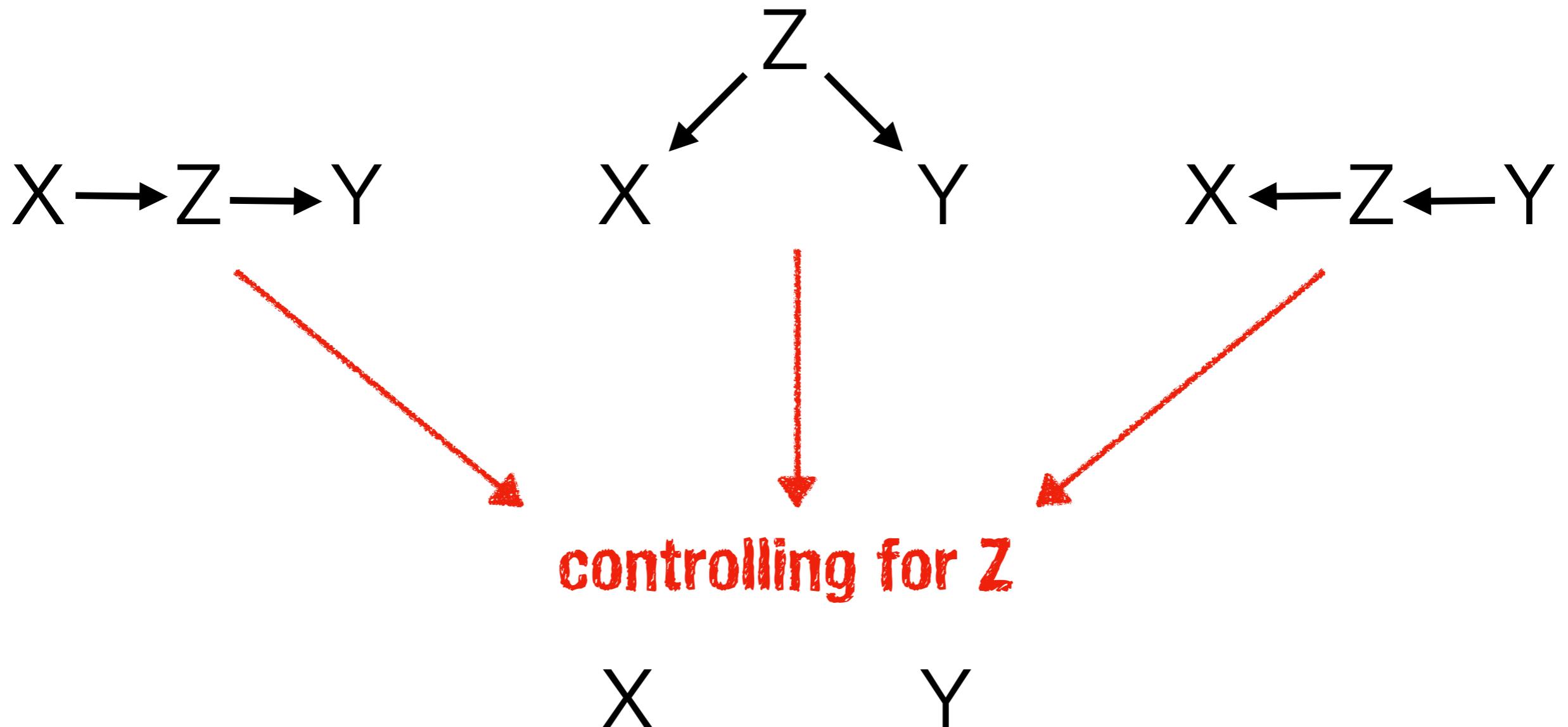


Z is a **bad** control here!

**additional path** ↗

# When should I control for variables?

**Problem: We don't know the ground truth ...**

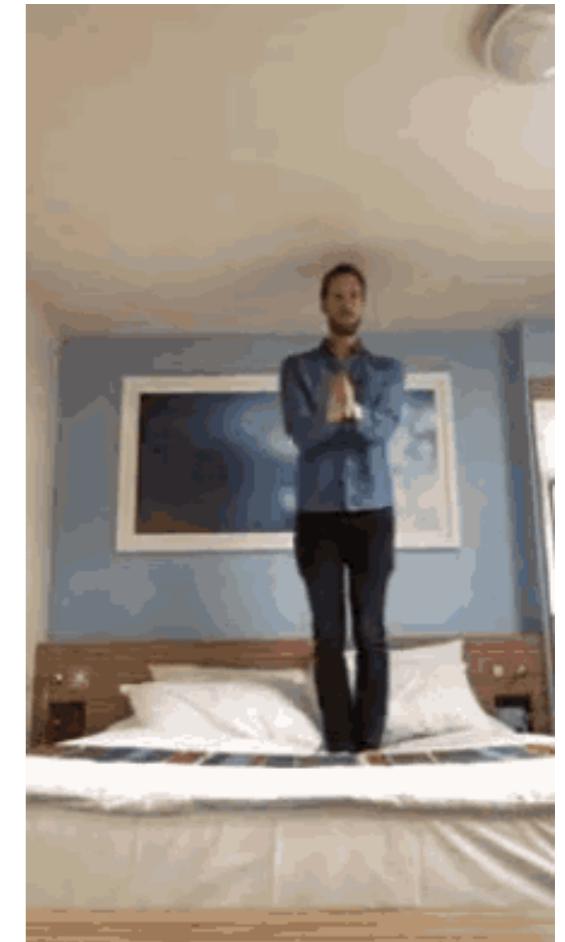
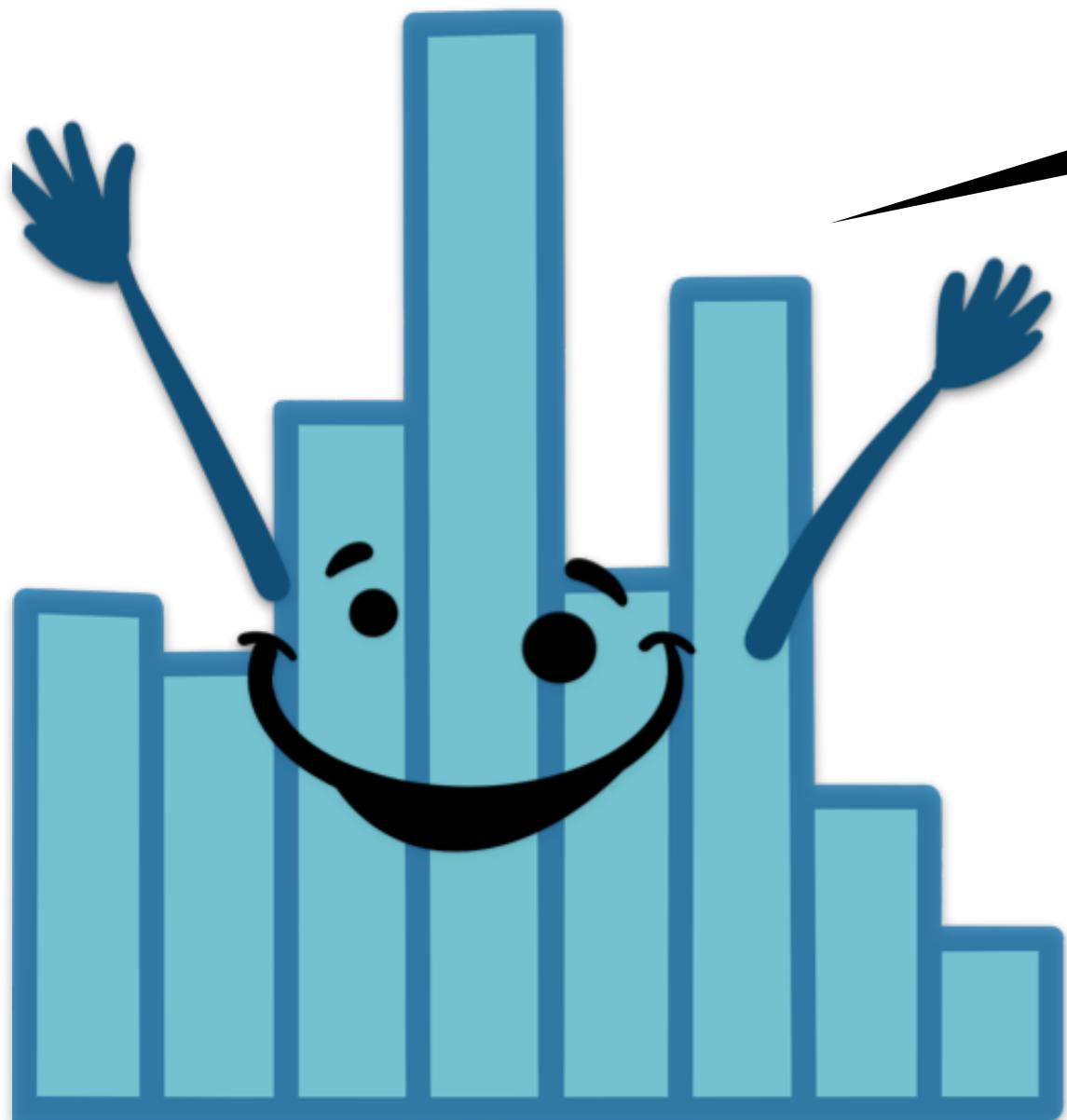


we need to manipulate  $X$  experimentally to tell these apart\*

\* sort of (see next slide)

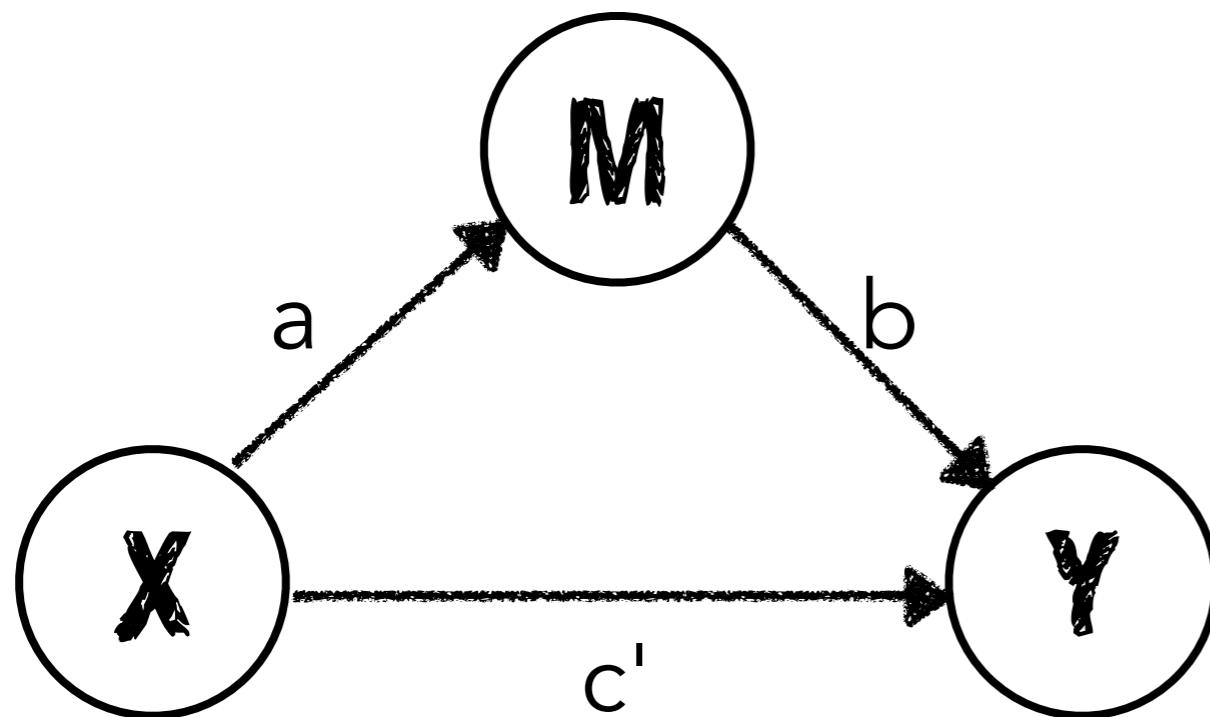
01:00

stretch break!



# **Mediation**

# Definition

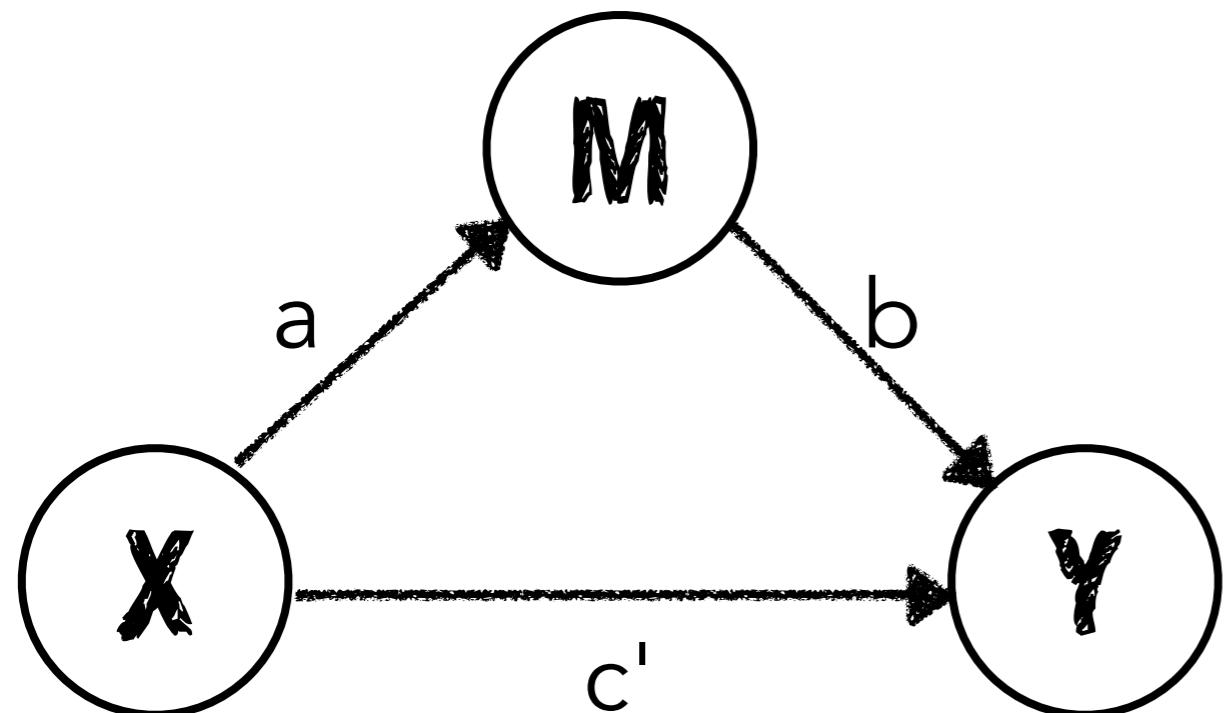


Rather than a direct causal relationship between **X** and **Y**, a mediation model proposes that **X** influences the mediator variable **M**, which in turn influences **Y**. Thus, the mediator variable serves to clarify the nature of the relationship between **X** and **Y**.

**Adapted from Wikipedia**

[https://en.wikipedia.org/wiki/Mediation\\_\(statistics\)](https://en.wikipedia.org/wiki/Mediation_(statistics))

# Example



**X** = grades in Psych 252

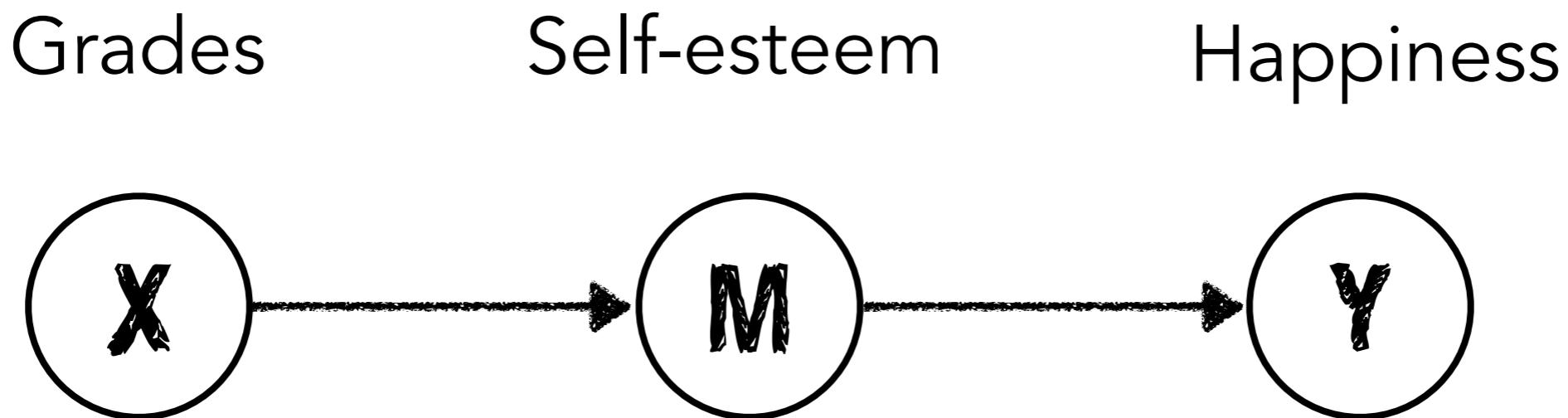
**M** = feelings of self-esteem

**Y** = happiness

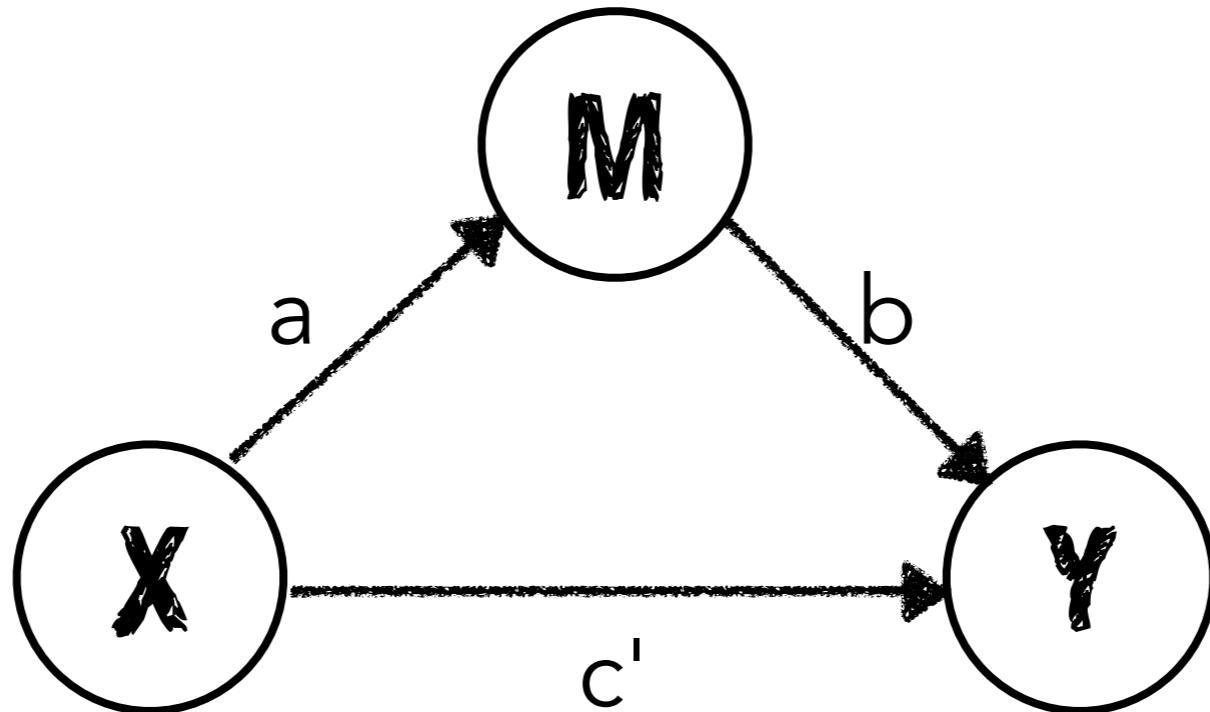
Is the relationship between grades in Psych 252 and happiness mediated by feelings of self-esteem?

# Simulate a mediation analysis

```
1 # number of participants
2 n = 100
3
4 # generate data
5 df.mediation = tibble(
6   x = rnorm(n, 75, 7),           # grades
7   m = 0.7 * x + rnorm(n, 0, 5), # self-esteem
8   y = 0.4 * m + rnorm(n, 0, 5) # happiness
9 )
```



# Baron and Kenny's (1986) steps for mediation



## Sequence of regression models

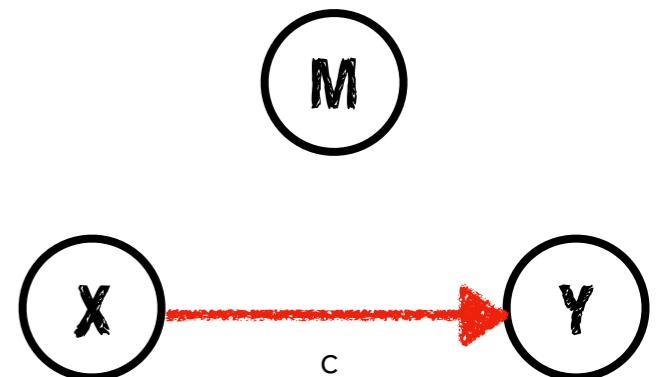
1. Is there a relationship between **X** and **Y**?
2. Is there a relationship between **X** and **M**?
3. Does the relationship between **X** and **Y** change, once we control for **M**?

Baron, R. M. & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology*, 51(6), 1173-1182.

# Is there a relationship between X and Y?

$$\hat{y} = b_0 + b_1 \cdot x$$

```
1 # fit the model
2 fit.y_x = lm(formula = y ~ 1 + x,
3               data = df.mediation)
4
5 # summarize the results
6 fit.y_x %>% summary()
```



```
Call:
lm(formula = y ~ 1 + x, data = df.mediation)

Residuals:
    Min      1Q  Median      3Q     Max 
-10.917 -3.738 -0.259  2.910 12.540 

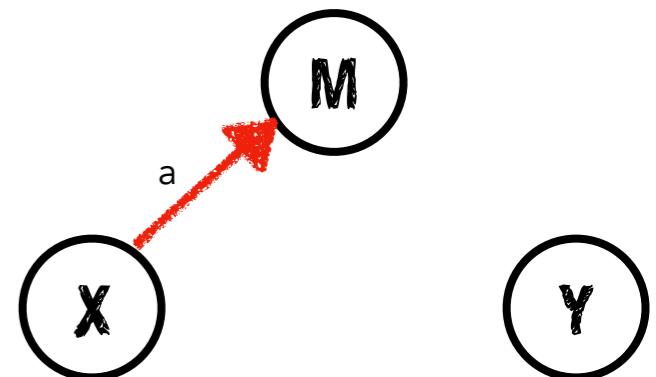
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 8.78300   6.16002   1.426   0.1571    
x            0.16899   0.08116   2.082   0.0399 *  
                                 ***
Signif. codes:  0 '****' 0.001 '***' 0.01 '**' 0.05 '*' 0.1 ' ' 1

Residual standard error: 5.16 on 98 degrees of freedom
Multiple R-squared:  0.04237, Adjusted R-squared:  0.0326 
F-statistic: 4.336 on 1 and 98 DF,  p-value: 0.03993
```

**significant  
relationship**

# Is there a relationship between X and M?

$$\hat{m} = b_0 + b_1 \cdot x$$



```
1 # fit the model
2 fit.m_x = lm(formula = m ~ 1 + x,
3               data = df.mediation)
4
5 # summarize the results
6 fit.m_x %>% summary()
```

```
Call:
lm(formula = m ~ 1 + x, data = df.mediation)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.5367 -3.4175 -0.4375  2.9032 16.4520 

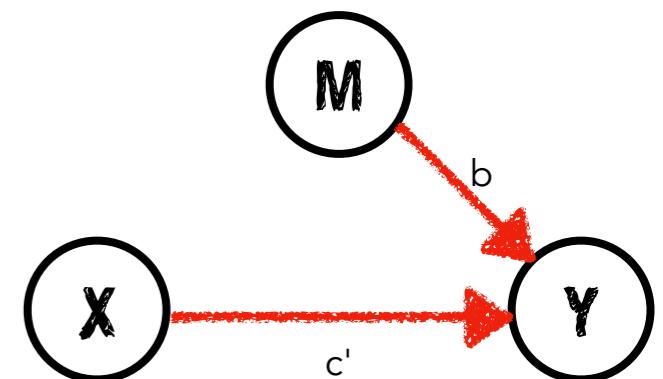
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 6.04494   13.41692   0.451   0.653    
x            0.66252   0.07634   8.678 8.87e-14 ***  
---
Signif. codes:  0 '****' 0.001 '***' 0.01 '**' 0.05 '*' 0.1 '.' 1

Residual standard error: 4.854 on 98 degrees of freedom
Multiple R-squared:  0.4346,    Adjusted R-squared:  0.4288 
F-statistic: 75.31 on 1 and 98 DF,  p-value: 8.872e-14
```

significant  
relationship

# Is there a relationship between X and Y, controlling for M?

$$\hat{y} = b_0 + b_1 \cdot m + b_2 \cdot x$$



```
1 # fit the model
2 fit.y_mx = lm(formula = y ~ 1 + m + x,
3                 data = df.mediation)
4
5 # summarize the results
6 fit.y_mx %>% summary()
```

```
Call:
lm(formula = y ~ 1 + m + x, data = df.mediation)

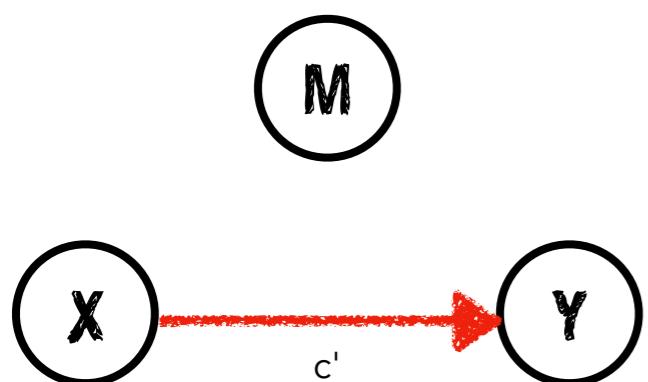
Residuals:
    Min      1Q  Median      3Q     Max 
-9.3651 -3.3037 -0.6222  3.1068 10.3991 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 7.80952   5.68297   1.374   0.173    
m           0.42381   0.09899   4.281 4.37e-05 *** 
x          -0.11179   0.09949  -1.124   0.264    
---
Signif. codes:  0 '****' 0.001 '***' 0.01 '**' 0.05 '*' 0.1 '.' 1 

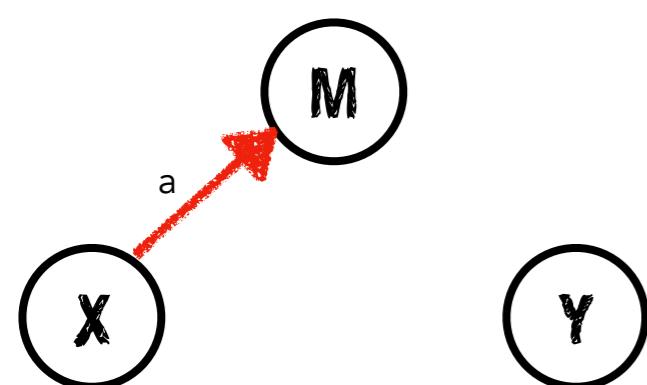
Residual standard error: 4.756 on 97 degrees of freedom
Multiple R-squared:  0.1946, Adjusted R-squared:  0.1779 
F-statistic: 11.72 on 2 and 97 DF,  p-value: 2.771e-05
```

not significant

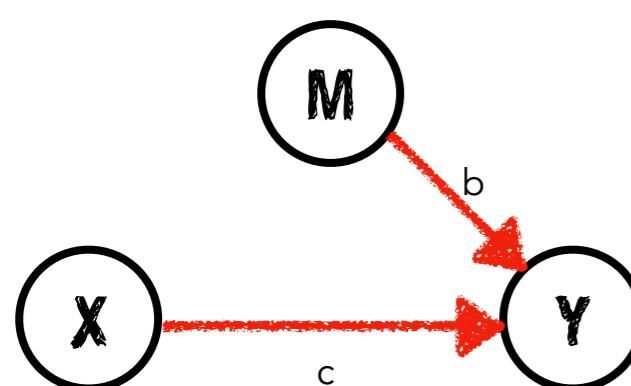
# 3 Step procedure



Relationship between **X** and **Y**?  
 $\hat{y} = b_0 + b_1 \cdot x$



Relationship between **X** and **M**?  
 $\hat{m} = b_0 + b_1 \cdot x$



Relationship between **X** and **Y**,  
controlling for **M**?  
 $\hat{y} = b_0 + b_1 \cdot m + b_2 \cdot x$

**significant  
change?**

**Mantra:**  
**a change in significance is not  
always a significant change**

# Is the mediation significant?

## 1. Sobel test

- assumes normally distributed data
- has low power

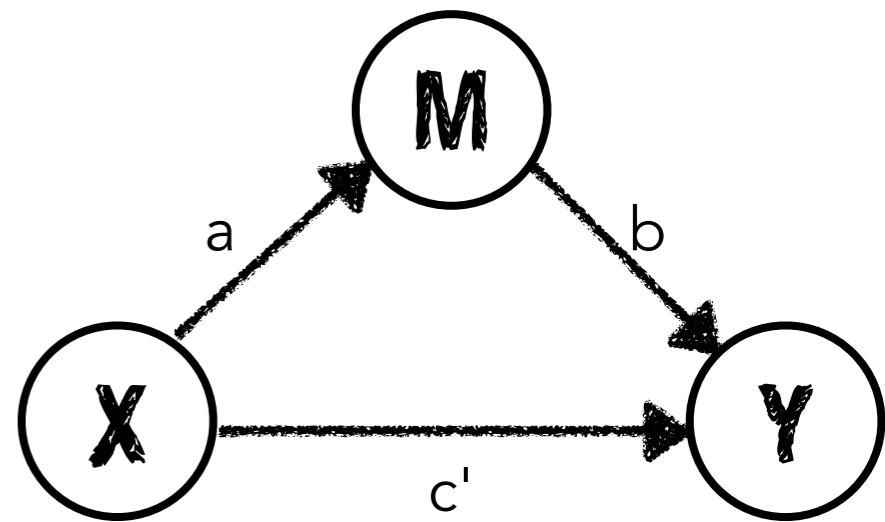
## 2. Bootstrapping

- no assumption about how the data is distributed
- has higher power

## 3. Bayesian mediation analysis

# 1. Sobel test

```
1 library("multilevel")
2
3 # run the sobel test
4 fit.sobel = sobel(pred = df.mediation$x,
5                     med = df.mediation$m,
6                     out = df.mediation$y)
7
8 # calculate the p-value
9 (1 - pnorm(fit.sobel$z.value)) ^ 2
```



"It is becoming increasingly more difficult to publish tests of mediation based purely on the Baron and Kenny method or tests that make distributional assumptions such as the Sobel test."

A Wikipedia person

## 2. Bootstrapping

```
1 library("mediation")
```

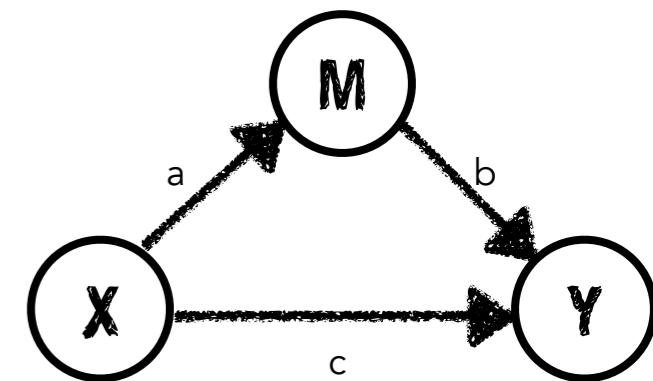
```
2  
3 # bootstrapped mediation
```

```
4 fit.mediation = mediate(model.m = fit.m_x, ←  $\hat{m} = b_0 + b_1 \cdot x$   
5 model.y = fit.y_mx, ←  $\hat{y} = b_0 + b_1 \cdot m + b_2 \cdot x$   
6 treat = "x",  
7 mediator = "m",  
8 boot = T)
```

```
9
```

```
10 # summarize results
```

```
11 fit.mediation %>% summary()
```



Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

	Estimate	95% CI Lower	95% CI Upper	p-value	
ACME	0.28078	0.14059	0.42	<2e-16	***
ADE	-0.11179	-0.29276	0.10	0.272	
Total Effect	0.16899	-0.00415	0.34	0.064	.
Prop. Mediated	1.66151	-3.22476	11.46	0.064	.

---

Signif. codes: 0 '\*\*\*\*' 0.001 '\*\*\*' 0.01 '\*\*' 0.05 '\*' 0.1 '.' 1

Sample Size Used: 100

Simulations: 1000

# 2. Bootstrapping

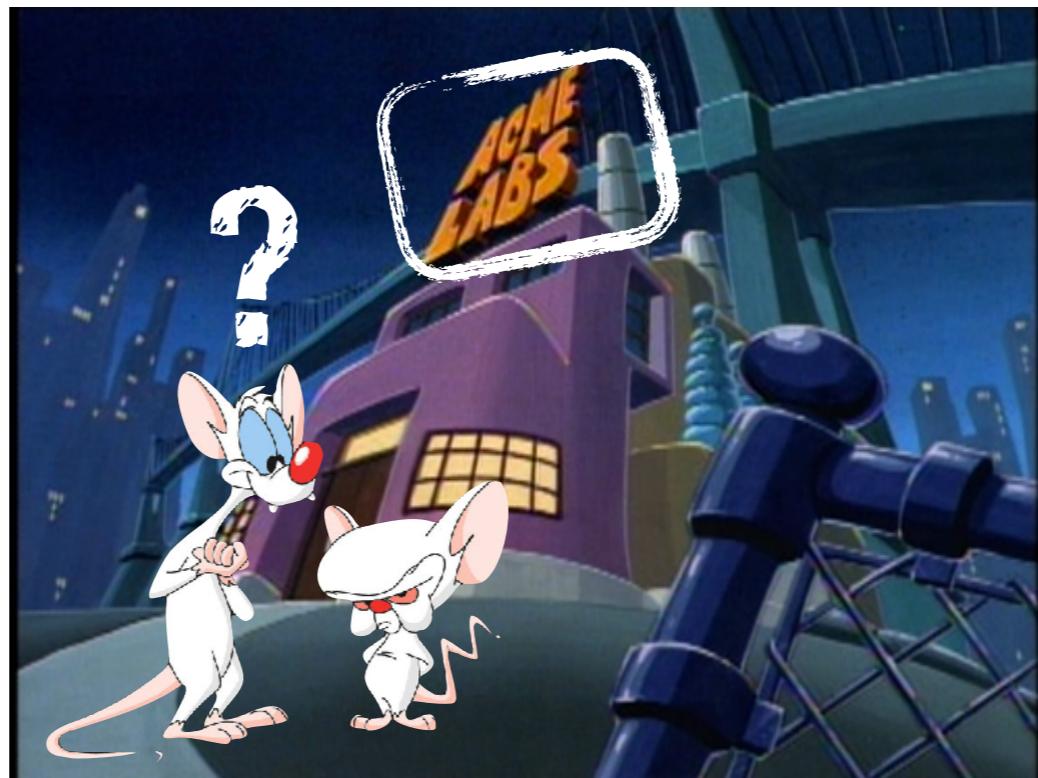
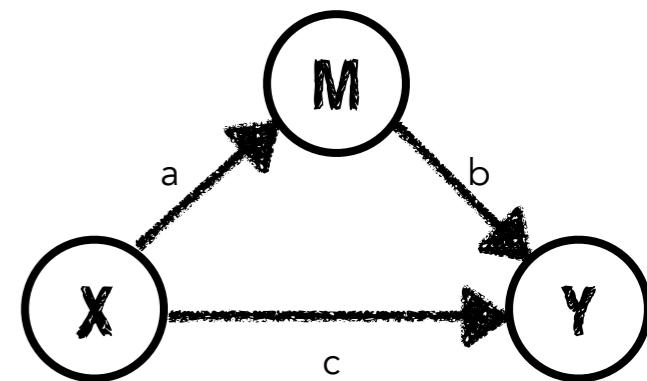
Causal Mediation Analysis

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Prop. Mediated	1.66151	-3.22476	11.46	0.064	.
---					
Signif. codes:	0 '***'	0.001 '**'	0.01 '*'	0.05 '.'	0.1 ' '
					1

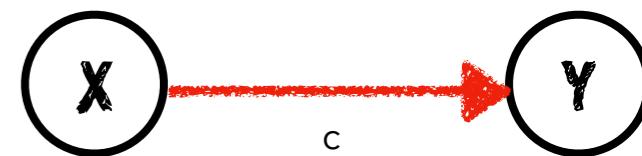
Sample Size Used: 100

Simulations: 1000



# 2. Bootstrapping

M



Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

	Estimate	95% CI Lower	95% CI Upper	p-value	
ACME	0.28078	0.14059	0.42	<2e-16	***
ADE	-0.11179	-0.29276	0.10	0.272	
Total Effect	0.16899	-0.00415	0.34	0.064	.
Prop. Mediated	1.66151	-3.22476	11.46	0.064	.

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100

Simulations: 1000

$$\hat{y} = b_0 + b_1 \cdot x$$

Call:

```
lm(formula = y ~ 1 + x, data = df.mediation)
```

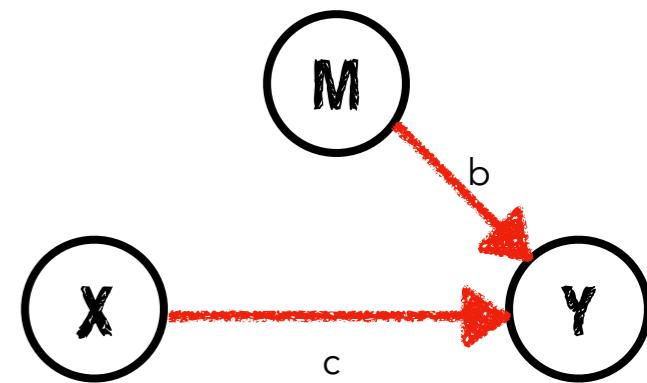
Residuals:

Min	1Q	Median	3Q	Max
-10.917	-3.738	-0.259	2.910	12.540

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	8.78300	6.16002	1.426	0.1571
x	0.16899	0.08116	2.082	0.0399 *

## 2. Bootstrapping



Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

	Estimate	95% CI Lower	95% CI Upper	p-value	
ACME	0.28078	0.14059	0.42	<2e-16	***
ADE	-0.11179	-0.29276	0.10	0.272	
Total Effect	0.16899	-0.00415	0.34	0.064	.
Prop. Mediated	1.66151	-3.22476	11.46	0.064	.
---					
Signif. codes:	0 '***'	0.001 '**'	0.01 '*'	0.05 '.'	0.1 ' '
Sample Size Used:	100				

Simulations: 1000

$$\hat{y} = b_0 + b_1 \cdot m + b_2 \cdot x \quad \text{ADE: Average direct effect}$$

Call:  
lm(formula = y ~ 1 + m + x, data = df.mediation)

Residuals:  
Min 1Q Median 3Q Max  
-9.3651 -3.3037 -0.6222 3.1068 10.3991

Coefficients:  
Estimate Std. Error t value Pr(>|t|)  
(Intercept) 7.80952 5.68297 1.374 0.173  
m 0.42381 0.09899 4.281 4.37e-05 \*\*\*  
x -0.11179 0.09949 -1.124 0.264

## 2. Bootstrapping

Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

	Estimate	95% CI Lower	95% CI Upper	p-value	
ACME	0.28078	0.14059	0.42	<2e-16	***
ADE	-0.11179	-0.29276	0.10	0.272	
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Prop. Mediated	1.66151	-3.22476	11.46	0.064	.
---					
Signif. codes:	0 '***'	0.001 '**'	0.01 '*'	0.05 '.'	0.1 ' '
Sample Size Used:	100				

Simulations: 1000

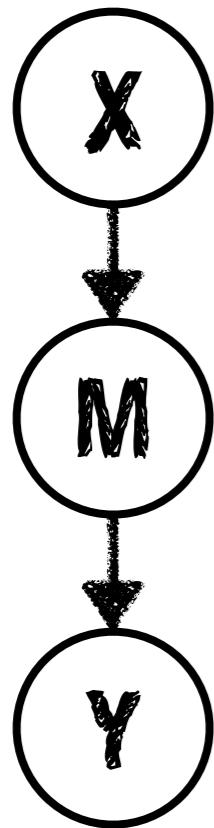
**ACME:** Average causal mediation effect

**ACME** = Total effect - ADE

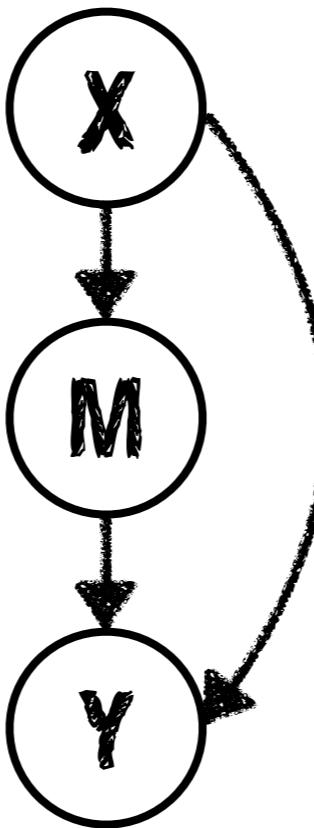
**ADE:** Average direct effect

# Underlying causal model

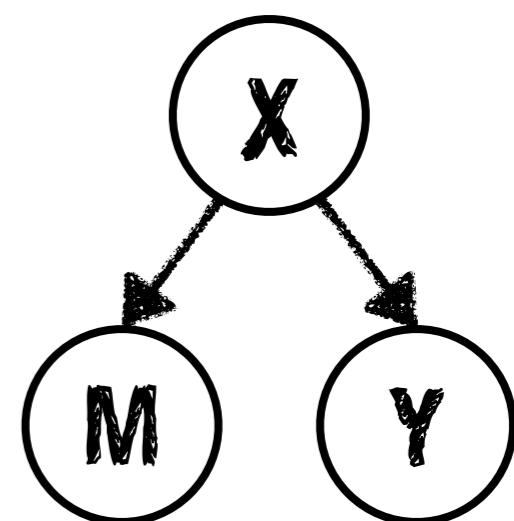
**Full mediation**



**Partial mediation**



**No mediation**

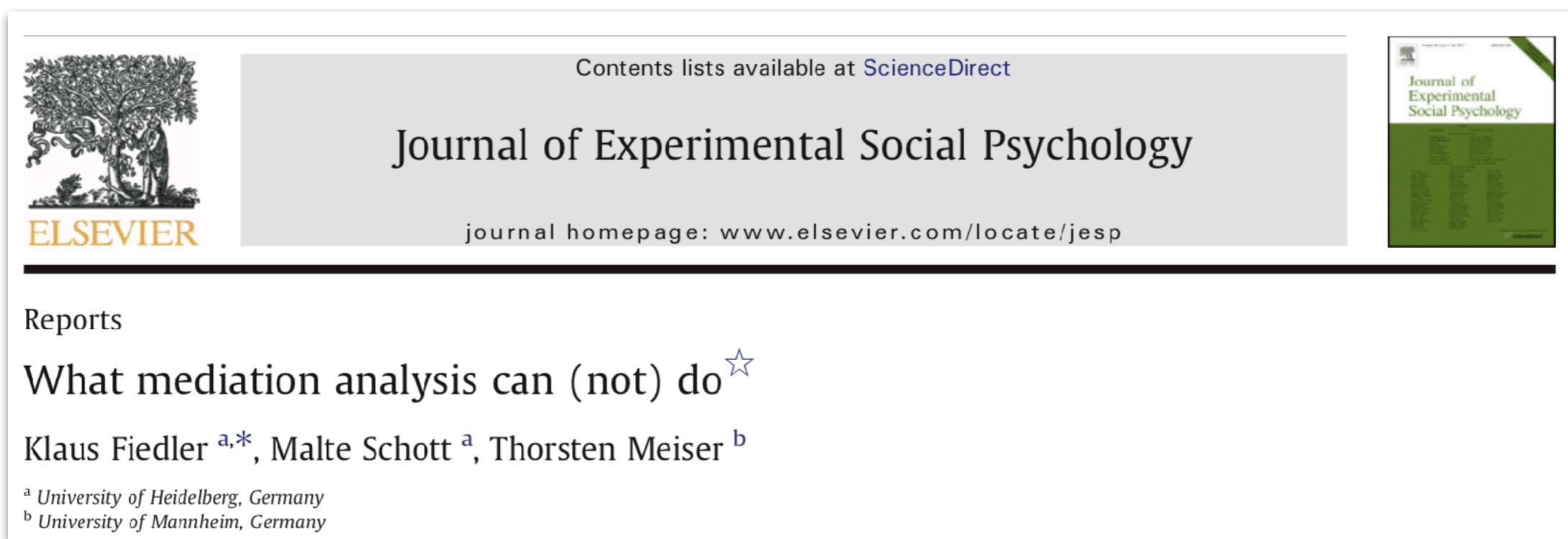


**Full mediation:** When the effect of **X** on **Y** completely disappears, **M** fully mediates between **X** and **Y**.

**Partial mediation:** When the effect of **X** on **Y** still exists, but in a smaller magnitude, **M** partially mediates between **X** and **Y**.

# Limitations

- correlational analysis
  - we need theories / experiments to tease apart causes and effects to properly map our variables onto the diagram

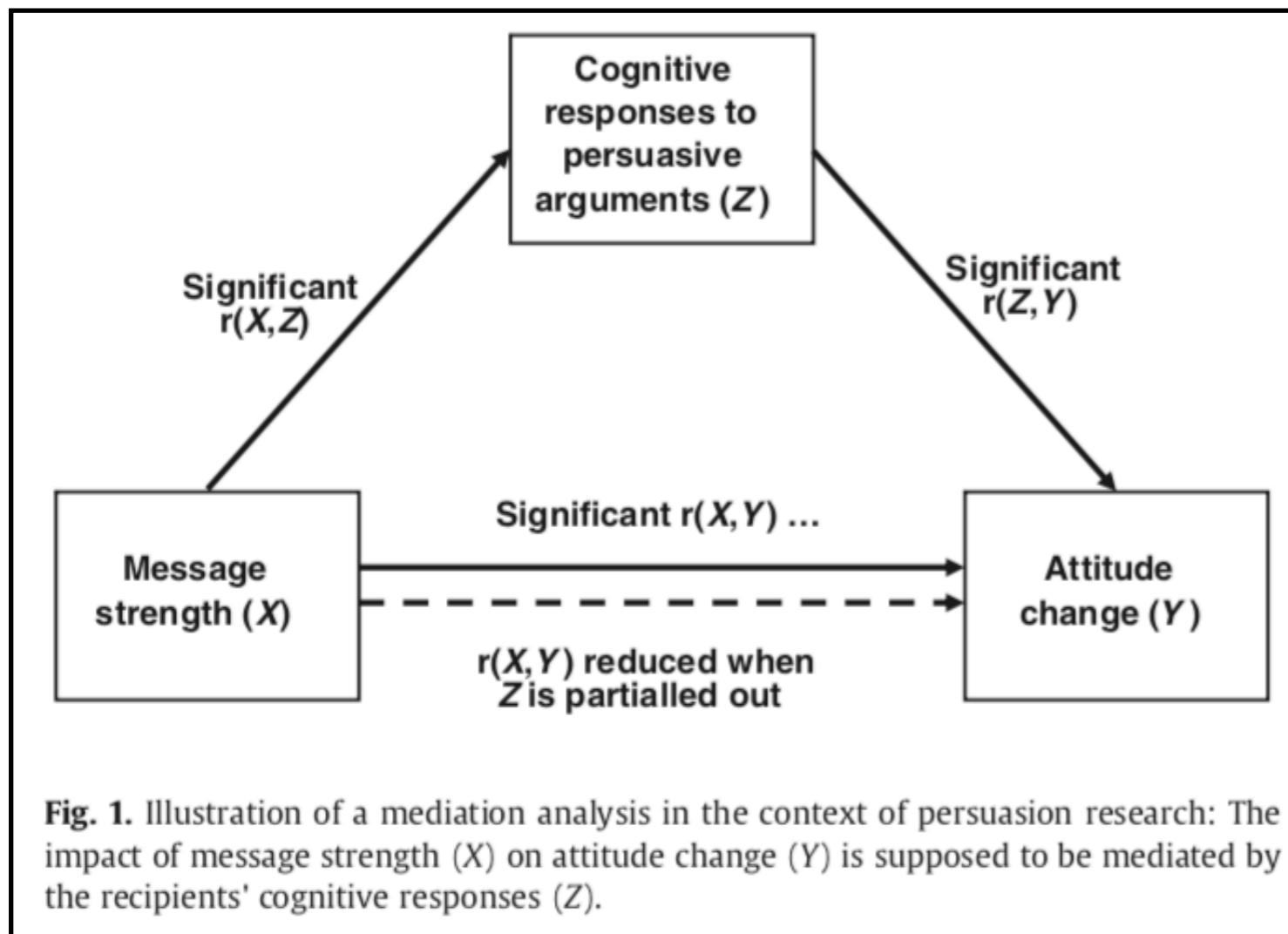


The image shows a journal article page from the Journal of Experimental Social Psychology. At the top left is the Elsevier logo, which includes a tree and the word 'ELSEVIER'. To the right of the logo is the journal title 'Journal of Experimental Social Psychology' and its website 'journal homepage: www.elsevier.com/locate/jesp'. Above the journal title is a link 'Contents lists available at ScienceDirect'. To the right of the journal title is a small thumbnail image of the journal cover. Below the header, the word 'Reports' is followed by the article title 'What mediation analysis can (not) do' with a blue star icon. The authors listed are Klaus Fiedler <sup>a,\*</sup>, Malte Schott <sup>a</sup>, and Thorsten Meiser <sup>b</sup>. Below the authors are two footnotes: <sup>a</sup> University of Heidelberg, Germany and <sup>b</sup> University of Mannheim, Germany.

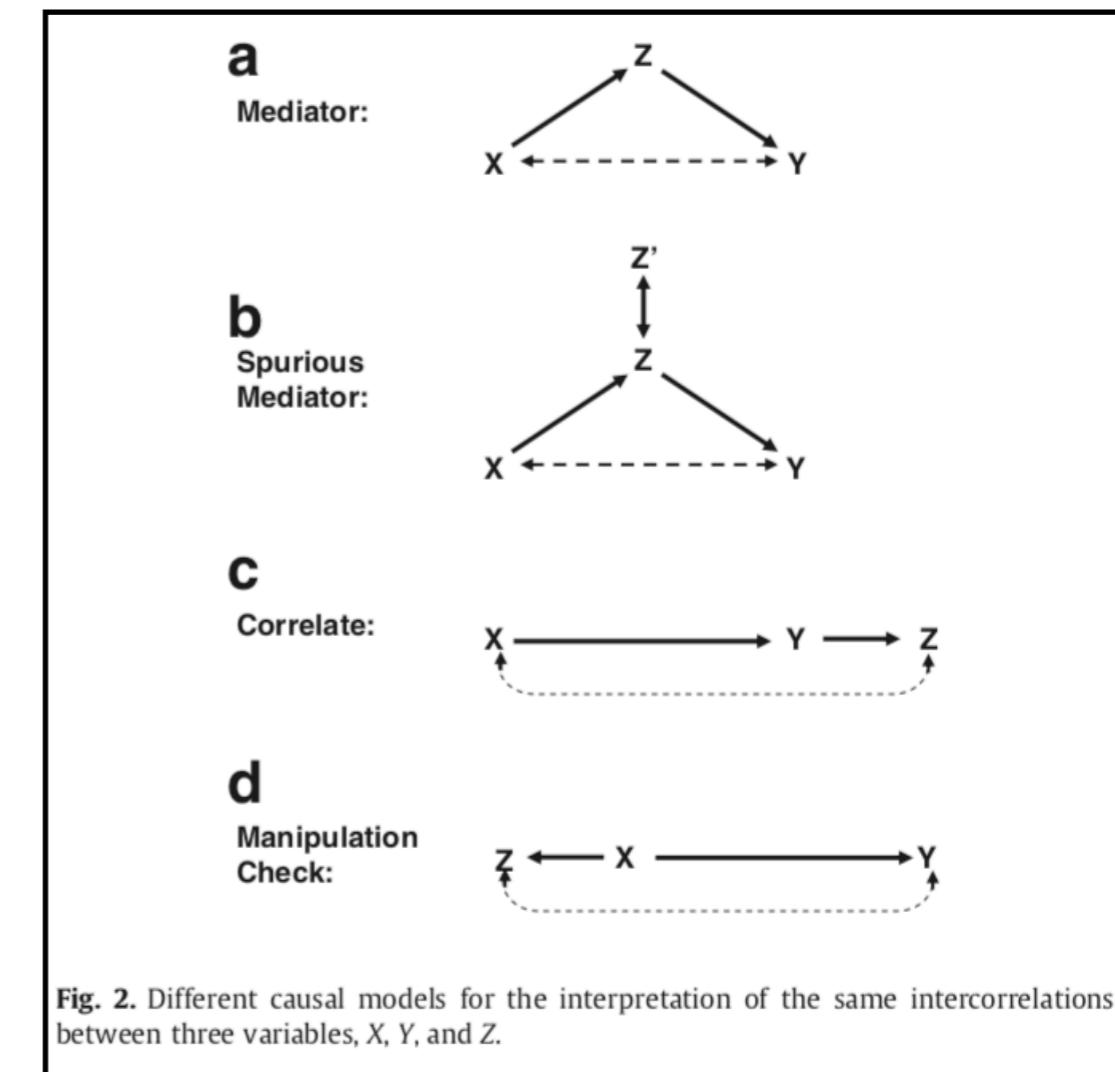
Fiedler, K., Schott, M., & Meiser, T. (2011). What mediation analysis can (not) do. *Journal of Experimental Social Psychology*, 47(6), 1231-1236. 76

# Limitations

## many-to-one mapping



**Fig. 1.** Illustration of a mediation analysis in the context of persuasion research: The impact of message strength ( $X$ ) on attitude change ( $Y$ ) is supposed to be mediated by the recipients' cognitive responses ( $Z$ ).



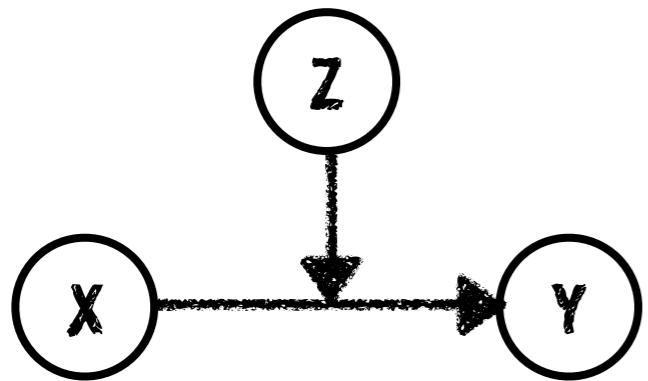
**Fig. 2.** Different causal models for the interpretation of the same intercorrelations between three variables,  $X$ ,  $Y$ , and  $Z$ .

only experiments allow us to tell apart possible causal structures

Fiedler, K., Schott, M., & Meiser, T. (2011). What mediation analysis can (not) do. *Journal of Experimental Social Psychology*, 47(6), 1231-1236.

# Moderation

# Definition

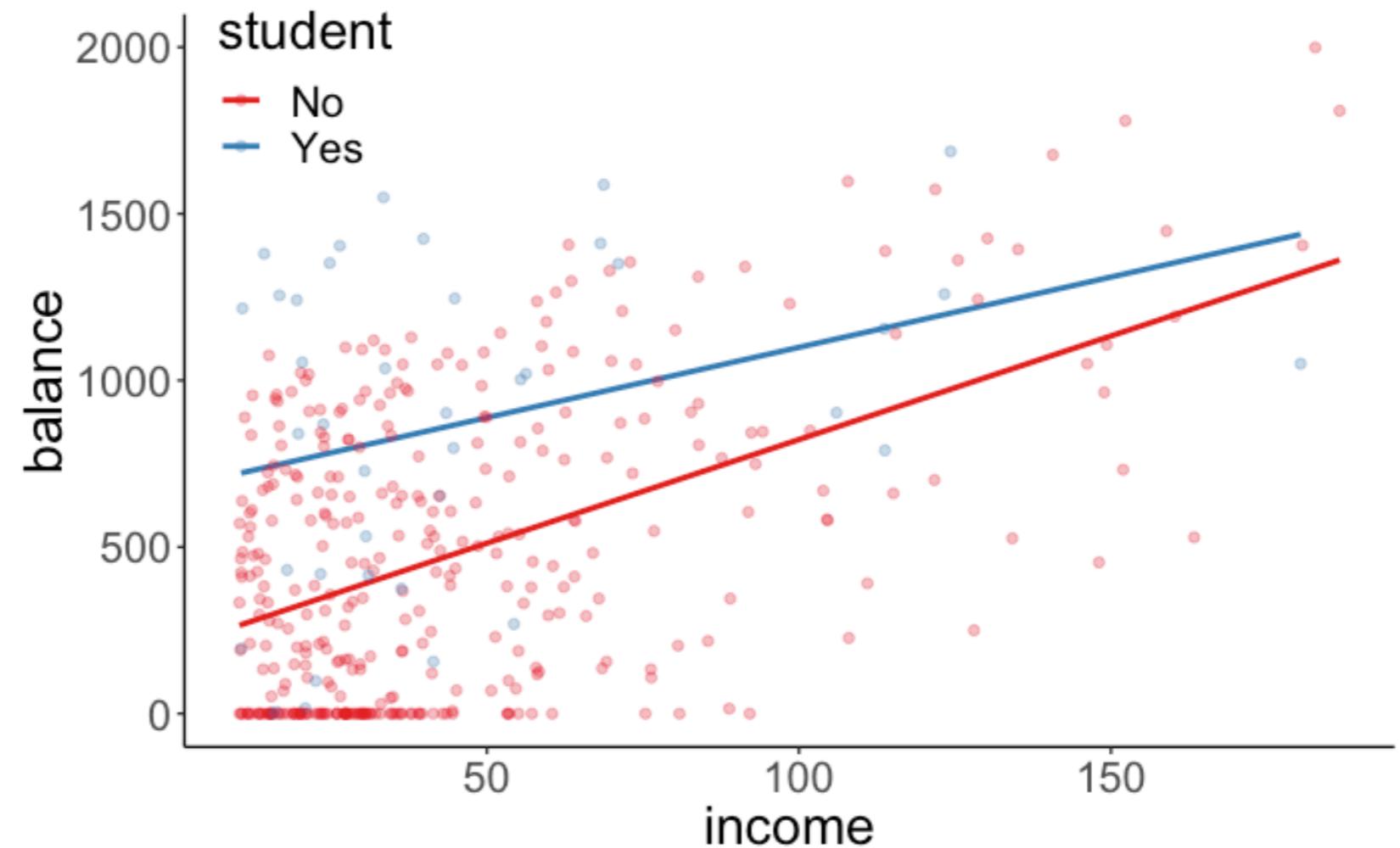


**Moderation** means that the effect of a predictor depends on the value of another.

Here, the nature of the relationship between **X** and **Y** depends on **Z**.

**Have we come across moderation already?**

Relationship  
between credit card  
balance, income,  
and whether the  
person is a student.



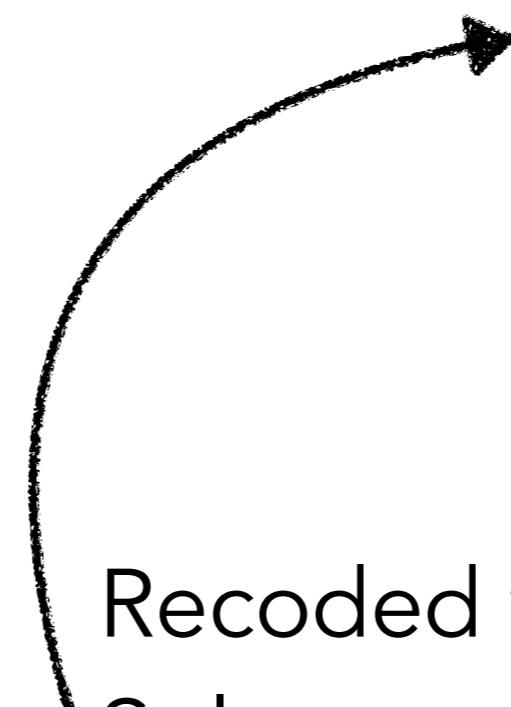
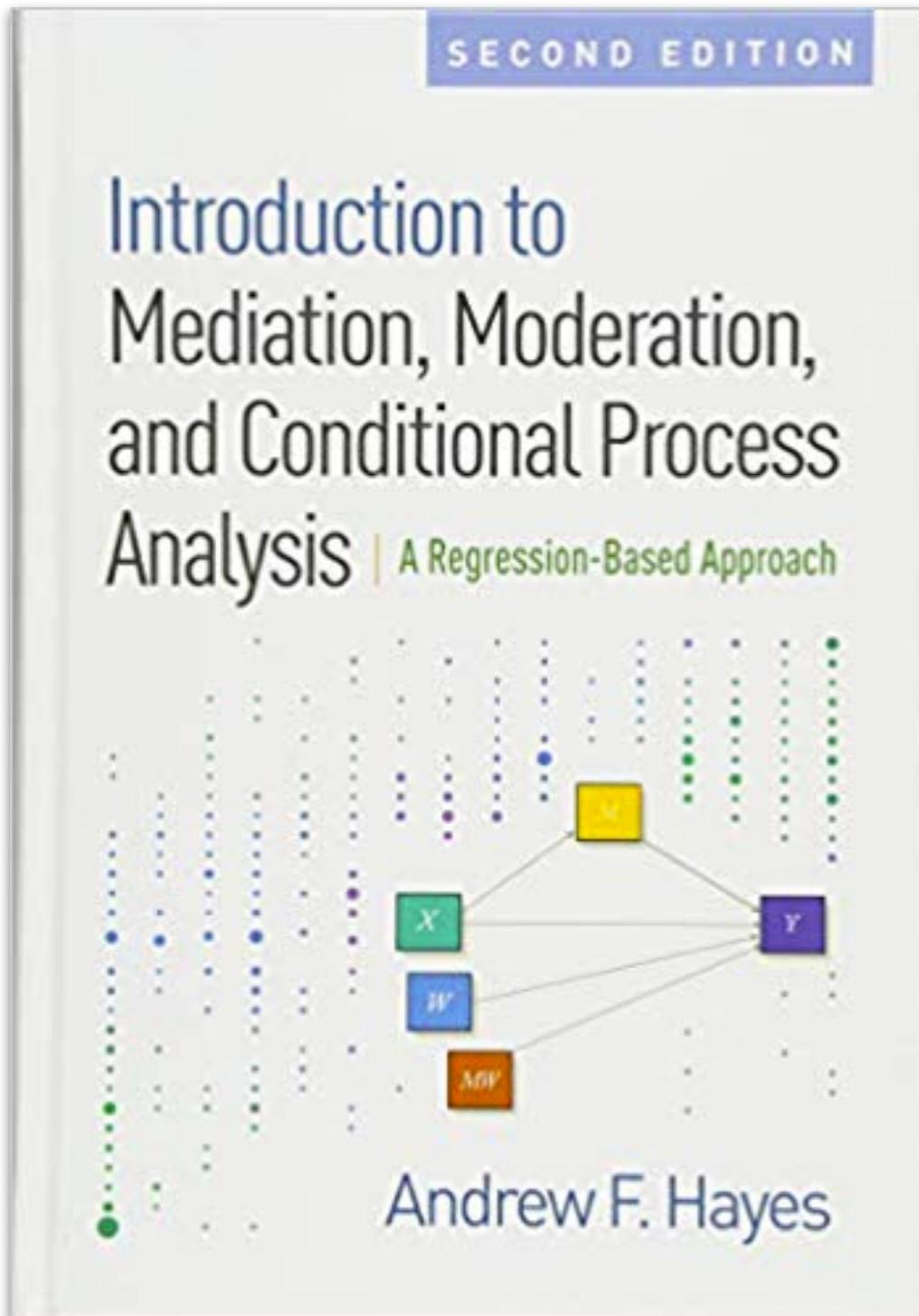
$$\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot \text{student}_i - 2.00 \cdot (\text{income}_i \times \text{student}_i)$$

**if student = "No"**  $\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i$

**if student = "Yes"**

$$\begin{aligned}
 \widehat{\text{balance}}_i &= 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot 1 - 2.00 \cdot (\text{income}_i \times 1) \\
 &= 677.3 + 6.22 \cdot \text{income}_i - 2.00 \cdot \text{income}_i \\
 &= 677.3 + 4.22 \cdot \text{income}_i
 \end{aligned}$$

# Learn more about mediation and moderation



Recoded with `brms` by  
Solomon Kurz here:  
[https://bookdown.org/  
connect/#/apps/1523/access](https://bookdown.org/connect/#/apps/1523/access)

# Plan for today

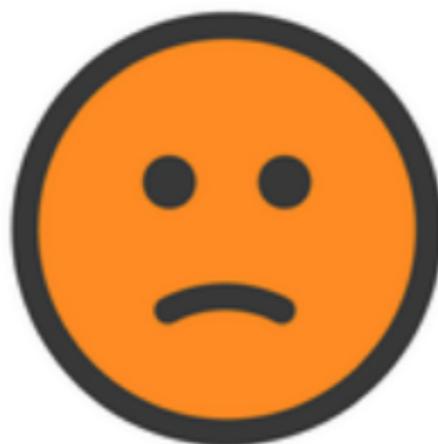
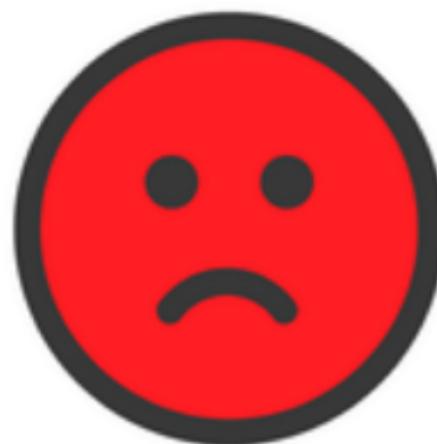
- Controlling for variables
- Mediation
- Moderation

# **Feedback**

# How was the pace of today's class?

much    a little    just    a little    much  
too        too        right      too        too  
slow      slow                                    fast      fast

# How happy were you with today's class overall?



**What did you like about today's class? What could be improved next time?**

Thank you!