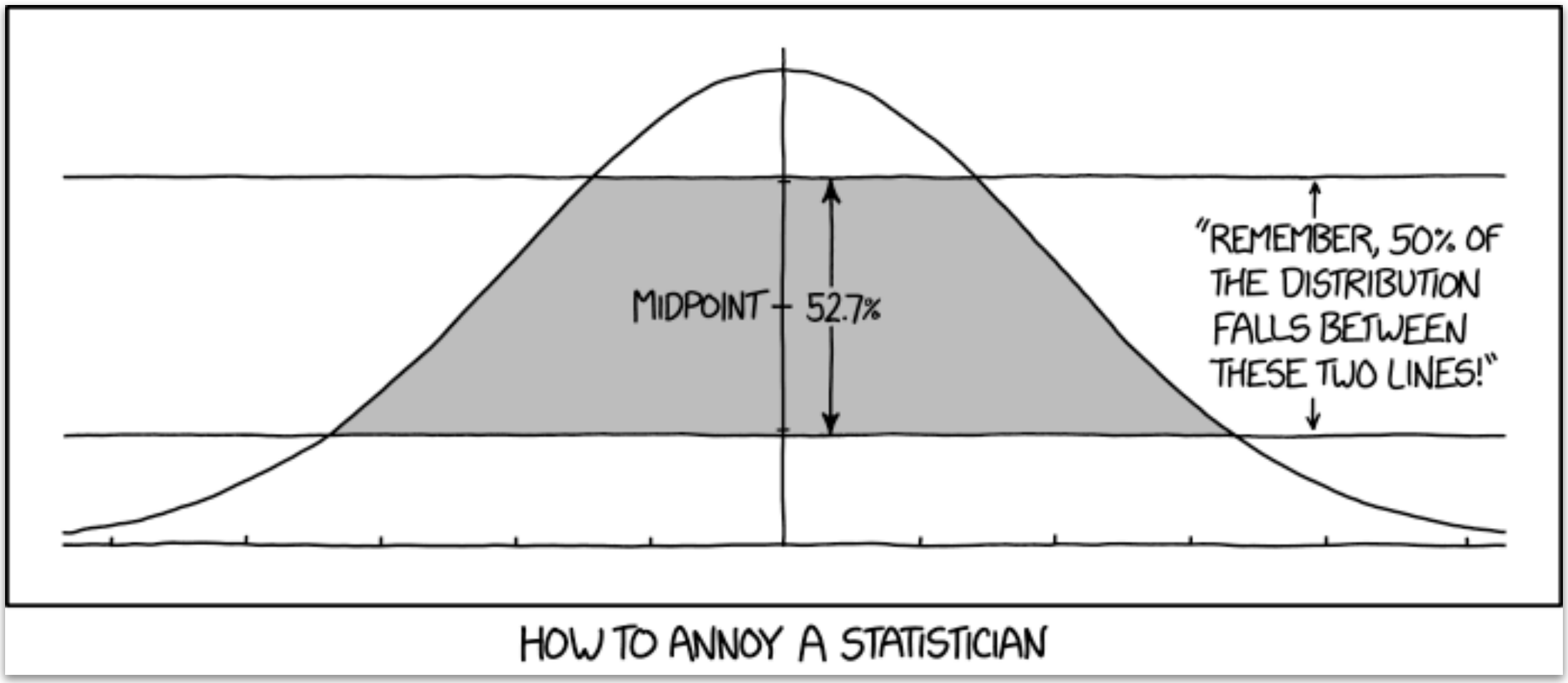
PSYCH 252: Statistical Methods for Behavioral and Social Sciences

Bayesian data analysis 2

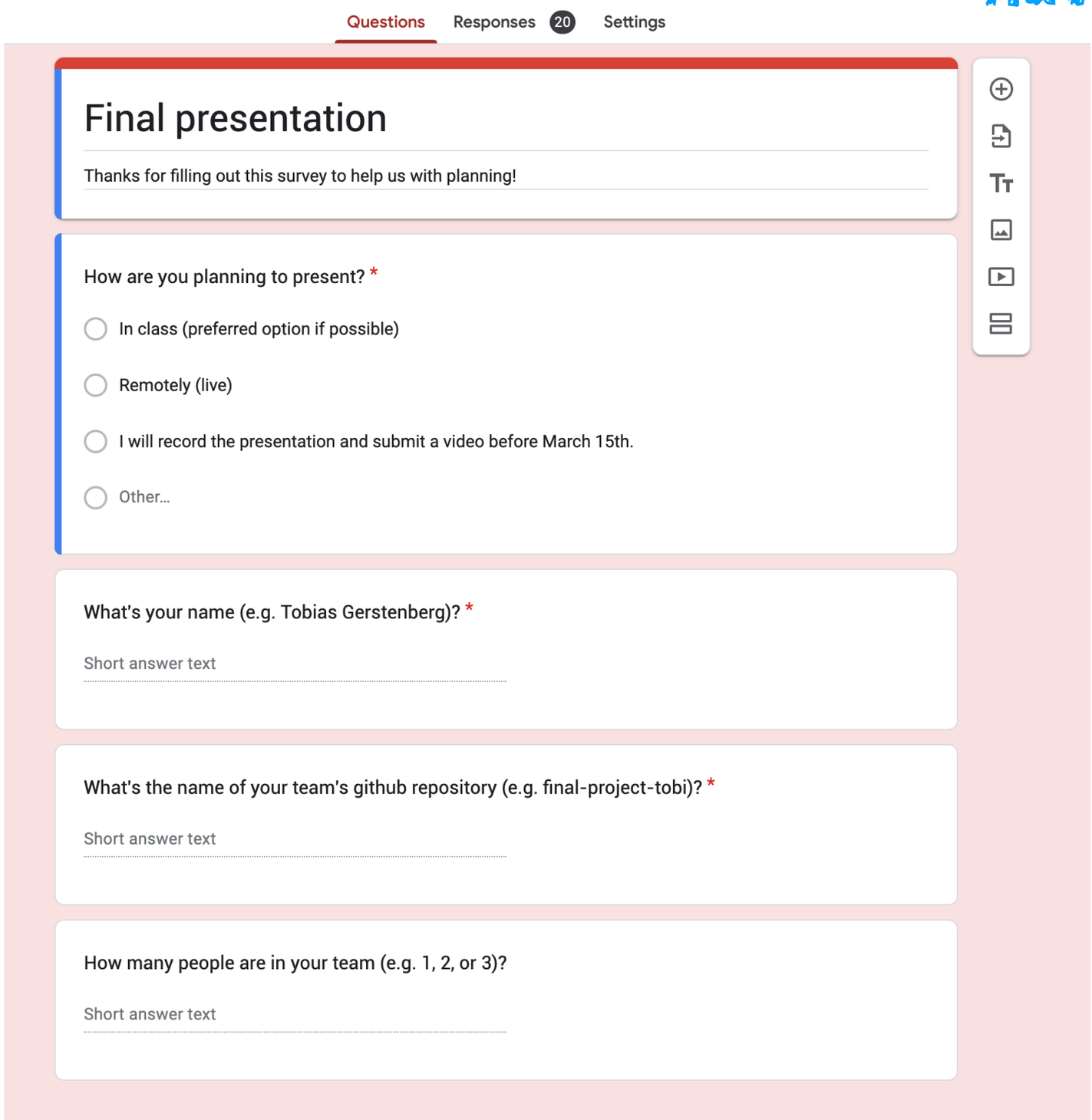


03/06/2024

Logistics

Final presentation

please fill out if you haven't already



https://tinyurl.com/psych252presentation24

Homework 7 (the last one, yay)

submission is optional

My name goes here

The names of the people I have worked w

2022-03-03 22:20:30

1 Instructions

This homework is due by **Thursday, March 10th, 8:00pm**.

As per usual, please upload your rendered pdf on Canvas.

Note:

- Some code chunks contain some skeleton code. The code chunk opt so that knitting the RMarkdown document doesn't throw any erro
- Make sure to show the results of your calculations in the knitted pd `eval=T` when you knit your homework, so that your calculations a
- Some questions ask for a short written response as indicated by t function at the end of a code chunk.

Good luck with the homework! If you have any questions, make sure t Thursday and/or post your questions on EdStem.

1.1 Load data

For the logistic regression question, we will use a data set which has inf transplant patients.

```
data(heart_transplant, package = "openintro")
df.heart = heart_transplant %>%
  mutate(survived = ifelse(survived == "dead", 0, 1))
```

Here is a description of the data set:

> The Stanford University Heart Transplant Study was conducted perimental heart transplant program increased lifespan. Each pat designated officially a heart transplant candidate, meaning that would most likely benefit from a new heart. Then the actual hear few weeks to several months depending on the availability of a don this waiting period show improvement and get deselected as a hear the purposes of this experiment those patients were kept in the da

1.2 Part 1: Logistic regression (3 points)

Question 1.1: (1 point)

Fit a logistic regression where you predict whether or not a person `sur` the model summary.


```
### YOUR CODE HERE ###

######################
```

Question 1.2: (1 point)

Use inverse logit to transform the log odds of the model coefficients into a probability of s interpretable, and describe these results from the model.

```
### YOUR CODE HERE ###

######################
```

Your answer:

Question 1.3: (1 point)

Visualize the results of the logistic regression model that you've fitted in Question 2. Th describe the relationship between age and the probability of survival.

```
### YOUR CODE HERE ###

######################
```

Your answer:

1.3 Part 2: Bayesian inference "by hand" (8 points)

This homework is from "PSYCH 10: Introduction to Statistical Methods" which is taugh You may find taking a look at this online chapter useful: Doing Bayesian Estimation

Question 2.1: (2 points)

Let's say that we are interested in the probability that a new drug called Bayesium w disease of frequentitis. For our purposes we are happy to estimate this variable (which we the nearest of 0.1.

Create a data frame called bayesium that includes the following variables:

- `theta`: a vector containing values of theta ranging from 0.0 to 1.0 in steps of 0.1
- `flat_prior`: a vector representing a flat, uniform prior across all possible values o the same prior value for each level of theta). This should be a probability distributio sum to one.
- `bayes_prior`: a vector containing a prior based on a binomial distribution with a reflecting the prior of a Bayesian who strongly believes that the treatment will have be obtained using the command: `dbinom(seq(0,10), 10, p = 0.9)`
- `freq_prior`: a vector containing a prior based on a binomial distribution with a reflecting the prior of a frequentist who strongly believes that the treatment will h
- `dogmatic_prior`: a vector containing a prior with all of its density on theta = 0 other values of theta, reflecting the prior of an very dogmatic Bayesian with very s

```
### YOUR CODE HERE ###

######################
```

Question 2.2: (2 points)

Let's say that we perform a clinical trial of the new drug in 20 people and we find that 10 are saved by the drug. Create a variable within the bayesium data frame called `likeli` the binomial likelihood for these data given each value of theta. (Hint: use an R function

function of the binomial distribution to calculate the number of 'successes' or saved lives in a certain number of trials or participants).

Then compute the posterior probabilities for each different prior and add them to the bayesium data frame within the following variables:

- `posterior_flat`: posterior given `flat_prior`
- `posterior_bayes`: posterior given `bayes_prior`
- `posterior_freq`: posterior given `freq_prior`
- `posterior_dogmatic`: posterior given `dogmatic_prior`

Each of these should be normalized so that they are probabilities (i.e. they sum to one).

```
df.bayesium = df.bayesium %>%
  ### YOUR CODE HERE ###
  mutate(likelihood =   ) %>%

  # compute posterior probabilities for each type of prior, and add to df

  ######################
```

Question 2.3: (4 points)

For each of the four priors, make a separate figure in which you plot the prior across all values of theta, and include the following: the prior with a black dotted line, the likelihood as a red dashed line, and the posterior with a solid line in blue. Add a title to the plot that includes the name of the prior. You can combine the figures into one using the "patchwork" or "cowplot" library (take a look at notes from the class `03_visualization2`). Alternatively, you can also just create one figure and make figure panels using the `facet_wrap()` or `facet_grid()` function.

```
### YOUR CODE HERE ###

# plot priors, likelihood, and posteriors

######################
```

Then, in your own words, describe how the different priors affect the respective maximum posterior estimates.

Your answer:

1.4 Part 3: Bayesian data analysis (4 points)

Question 3.1: (1 point)

Build a bayesian model from the `df.heart` data similar to the frequentist model in Question 1.1. Call this model `fit.brm`

```
### YOUR CODE HERE ###
fit.brm = brm(formula =
              family =
              data =
              file = "cache/brm",
              seed = 1)
######################
```

Question 3.2: (1 point)

Print the summary of this model and interpret its coefficients. Compare these to the coefficients from the frequentist model you built in Question 1.1.

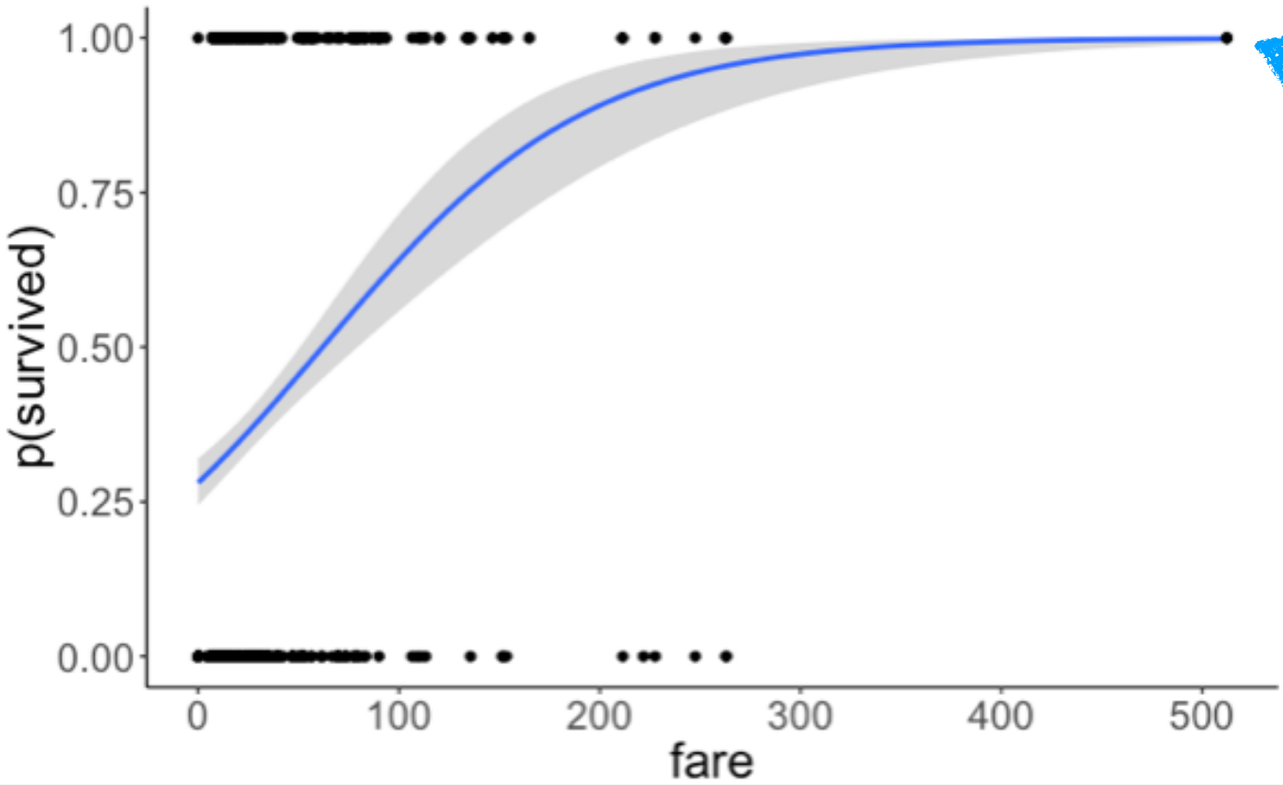
the best 6 of your homeworks count

Things that came up

Outlier in logistic regression

Visualize the model's predictions

```
1 ggplot(data = df.titanic,
2        mapping = aes(x = fare,
3                      y = survived)) +
4   geom_smooth(method = "glm",
5               method.args = list(family = "binomial")) +
6   geom_point() +
7   labs(y = "p(survived)")
```



logistic regression is robust to **consistent** outliers

Outlier in logistic regression

```
1 fit.glm = glm(formula = survived ~ 1 + fare,
2               family = "binomial",
3               data = df.titanic)
```

```
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.941330   0.095129  -9.895  < 2e-16 ***
fare         0.015197   0.002232   6.810 9.79e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
1 fit.glm = glm(formula = survived ~ 1 + fare,
2               family = "binomial",
3               data = df.titanic %>%
4                 filter(fare < 500))
```

```
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.941130   0.095192  -9.887  < 2e-16 ***
fare         0.015189   0.002236   6.794 1.09e-11 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

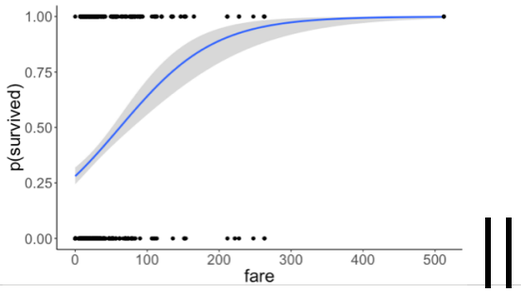
removing the consistent outlier makes almost no difference

Outlier in logistic regression

```
1 fit.glm = glm(formula = survived ~ 1 + fare,
2                family = "binomial",
3                data = df.titanic)
```

Visualize the model's predictions

```
1 ggplot(data = df.titanic,
2        mapping = aes(x = fare,
3                      y = survived)) +
4   geom_smooth(method = "glm",
5               method.args = list(family = "binomial")) +
6   geom_point() +
7   labs(y = "p(survived)")
```



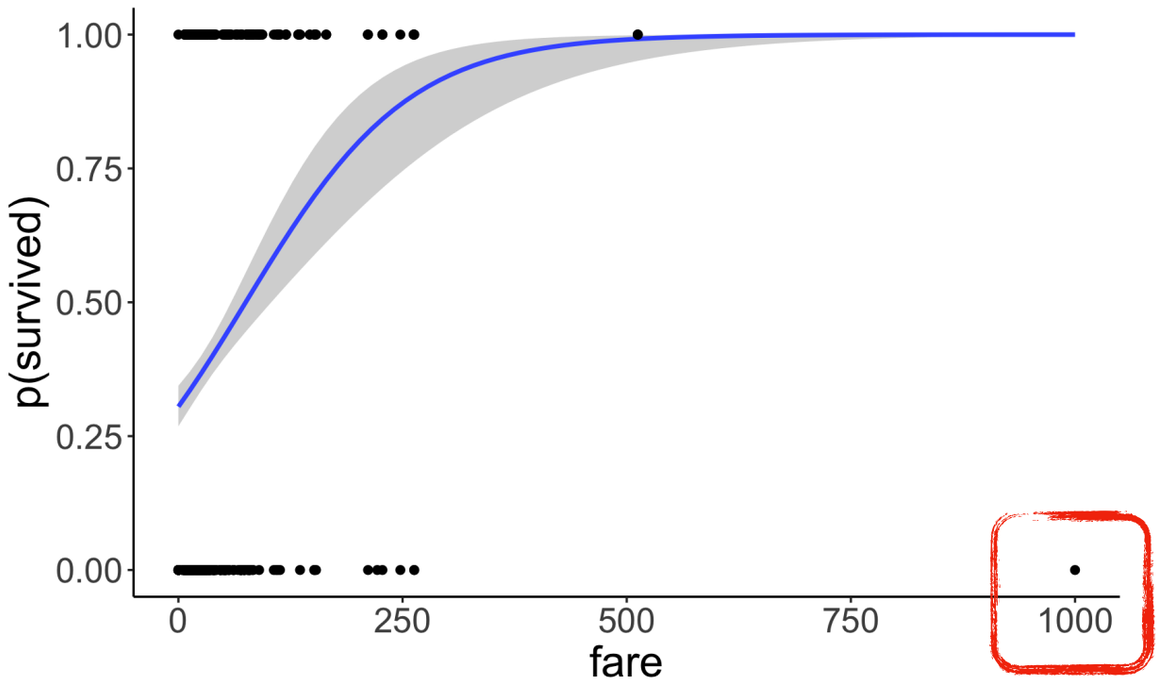

```
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.941330   0.095129  -9.895  < 2e-16 ***
fare         0.015197   0.002232   6.810 9.79e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

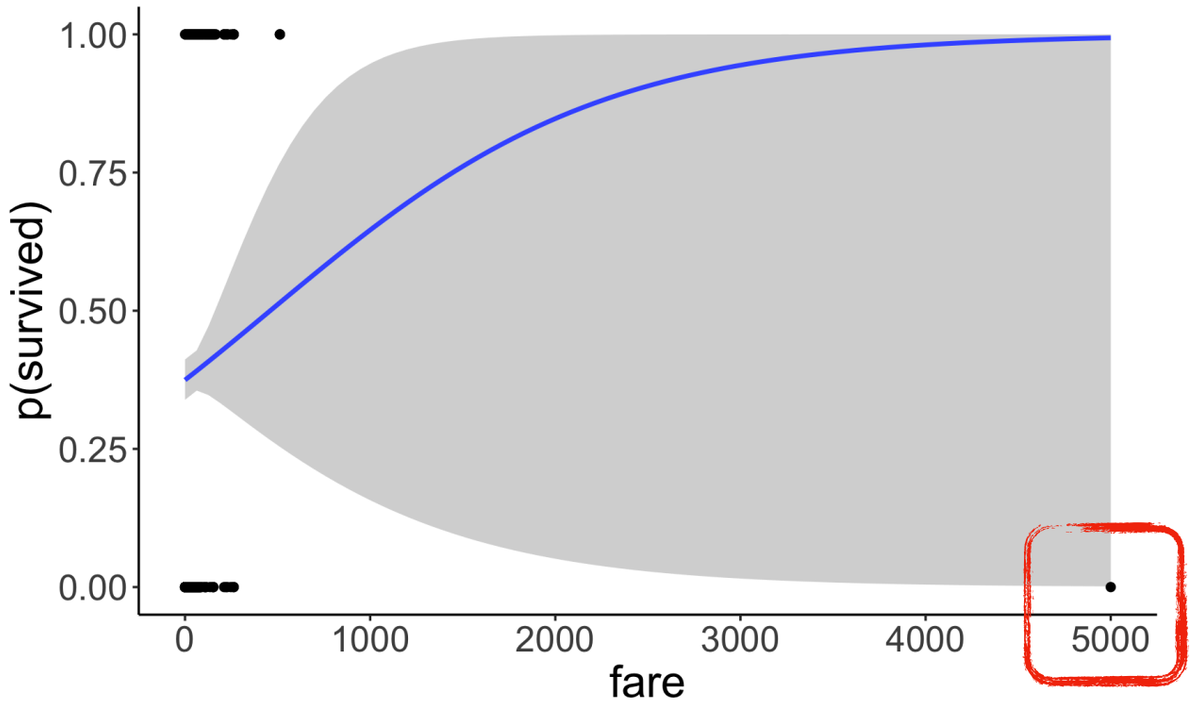
```
1 fit.glm = glm(formula = survived ~ 1 + fare,
2                family = "binomial",
3                data = df.titanic %>%
4                  mutate(fare = ifelse(passenger_id == 1, 10000, fare)))
```

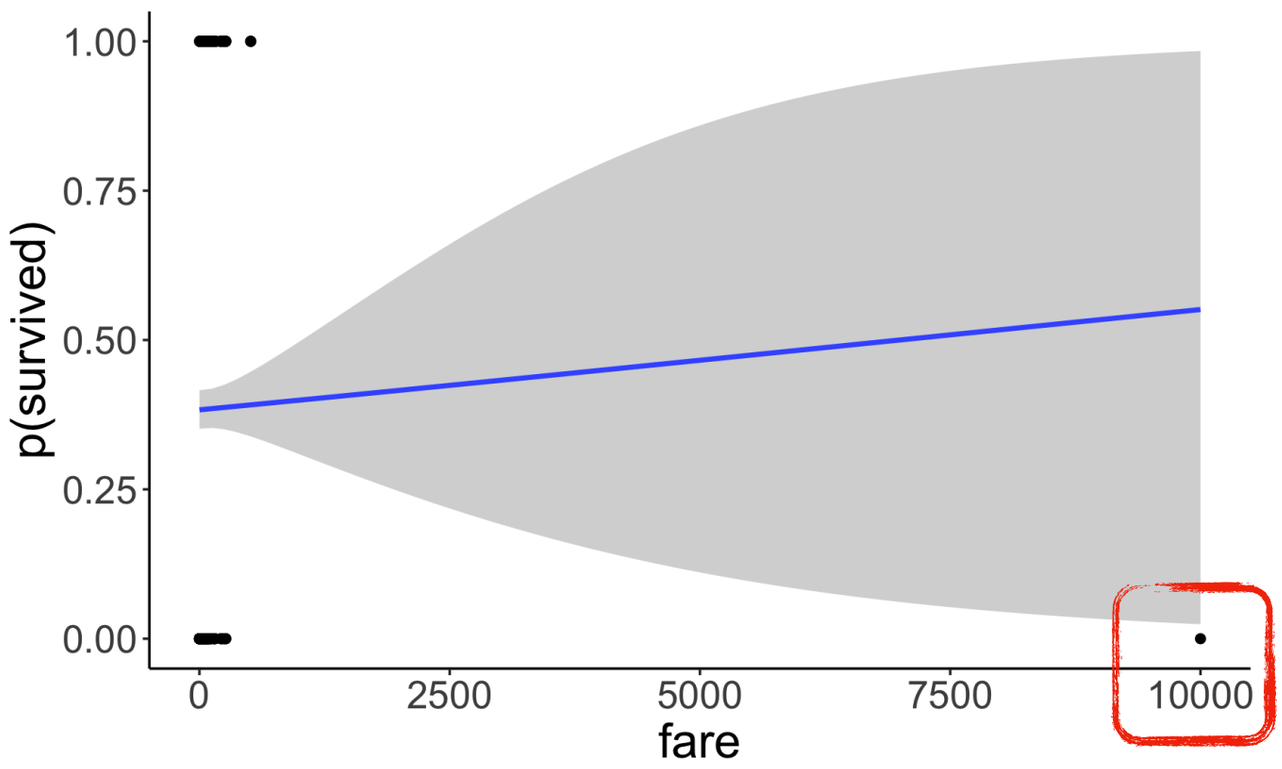
```
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.4762784  0.0694495  -6.858 6.99e-12 ***
fare         0.0000681  0.0001997   0.341    0.733
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

adding an inconsistent outlier makes a big difference!

Outlier in logistic regression







In linear regression, it is very easy to visualize outliers using a scatter plot. ... However, whereas a Y value in linear regression may be arbitrarily large, the maximum fitted distance between a fitted and observed logistic value is bounded. Does that mean that a logistic regression is robust to outliers? **Absolutely not.**

https://stats.stackexchange.com/questions/279010/how-does-outlier-impact-logistic-regression

Plan for today

- Quick recap
- Doing Bayesian data analysis with greta
- Doing Bayesian data analysis **with BRMS**
  - Testing hypotheses
  - Model evaluation

Quick recap

Quick recap: Bayesian data analysis philosophy

Goal of data analysis: Inference about the world

Frequentist statistics

- generate a sampling distribution of the test statistic assuming H_0
- compare observed value of the test statistic with the sampling distribution
- reject the H_0 if probability of observed value (or more extreme values) is less than α

Bayesian statistics

- directly test hypotheses of interest
- define prior over hypotheses **p(H)**
- compute likelihood of the data for each hypothesis **p(D|H)**
- use Bayes' rule to infer the posterior over hypotheses given the data **p(H|D)**

Clue guide to probability

$$p(B \mid A) = \frac{p(A \mid B) \cdot p(B)}{p(A)}$$

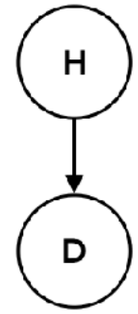
we derived this using the definition of conditional probability

$$\text{posterior}\ p(H \mid D) = \frac{\text{likelihood}\ p(D \mid H) \cdot \text{prior}\ p(H)}{p(D)}$$

subjective probability interpretation

H = Hypothesis

D = Data



formal framework for learning from data

updating our prior belief $p(H)$, to a posterior belief $p(H \mid D)$ given some data

Bayesian Recipe

- Hypotheses
- Prior over hypotheses
- Data
- Likelihood of the data given each hypothesis
- Posterior over hypotheses given the data

+ a healthy dose of Bayes' rule

$$p(H \mid D) = \frac{p(D \mid H) \cdot p(H)}{p(D)}$$

Coin flip example

```
1  # data
2  data = rep(0:1, c(8, 2))
3
4  # parameters
5  theta = c(0.1, 0.5, 0.9)
6
7  # prior
8  prior = c(0.25, 0.5, 0.25)
9
10 # likelihood
11 likelihood = dbinom(sum(data == 1), size = length(data), prob = theta)
12
13 # posterior
14 posterior = likelihood * prior / sum(likelihood * prior)
```

8 tails and 2 heads

hypotheses

prior over the hypotheses

binomial distribution

multiply

re-normalize

theta	prior	likelihood	prior_x_like	posterior
0.1	0.25	0.19	0.0475	0.69
0.5	0.50	0.04	0.02	0.31
0.9	0.25	0.00	0.00	0.00

Quick recap: What affects the posterior?

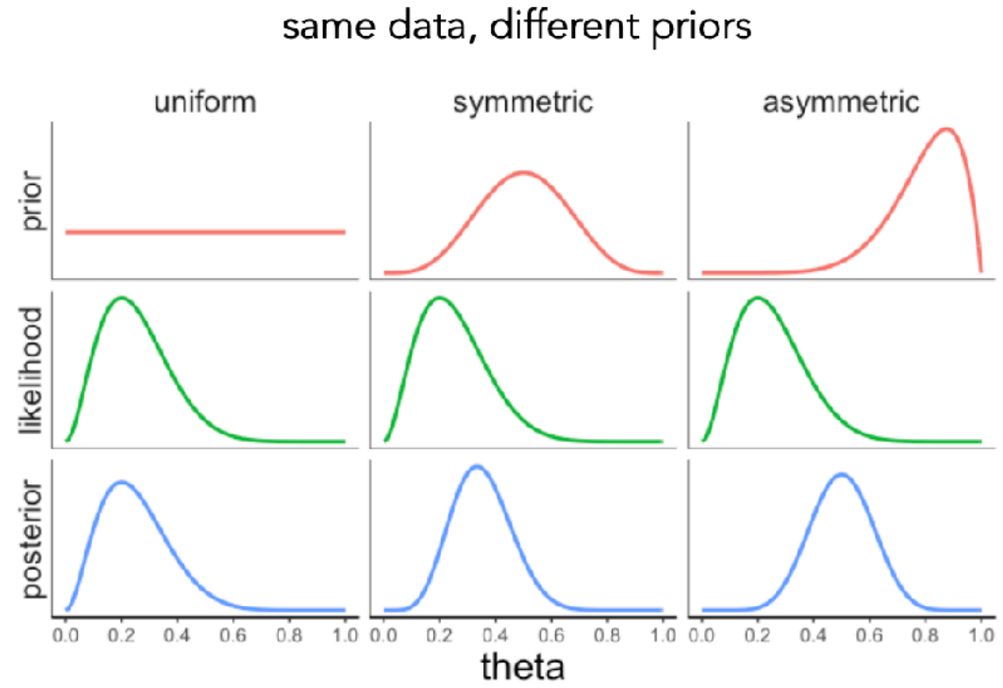
What affects the posterior?

1. the prior over hypotheses
2. the likelihood of the data given each hypothesis

$$\underbrace{p(H \mid D)}_{\text{Posterior}} = \frac{\overbrace{p(D \mid H)}^{\text{Likelihood}} \cdot \overbrace{p(H)}^{\text{Prior}}}{\underbrace{p(D)}_{\text{Normalizing constant}}}$$

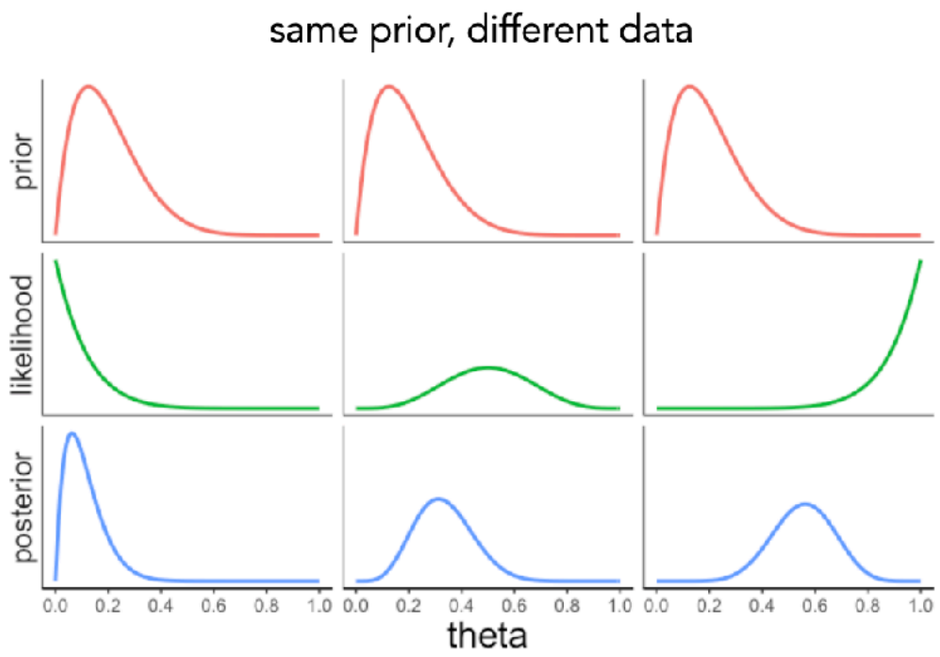
45

The effect of the **prior**



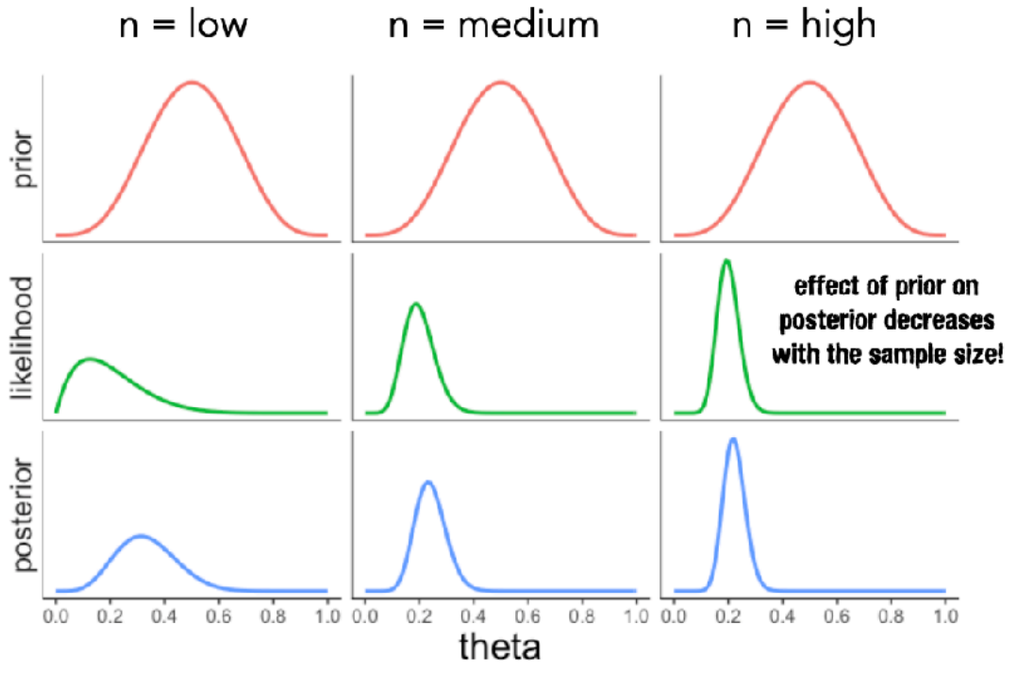
46

The effect of the **likelihood**



47

The effect of **sample size**



48

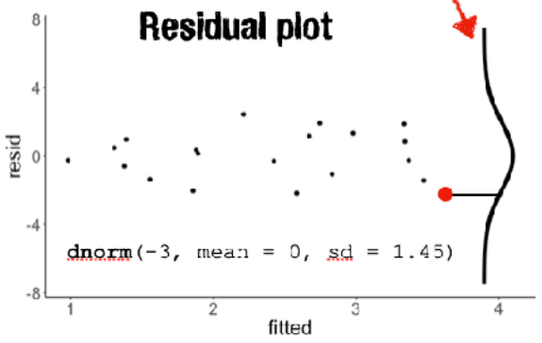
Quick recap: **likelihood**, prior, inference

Likelihood

Gaussian distribution

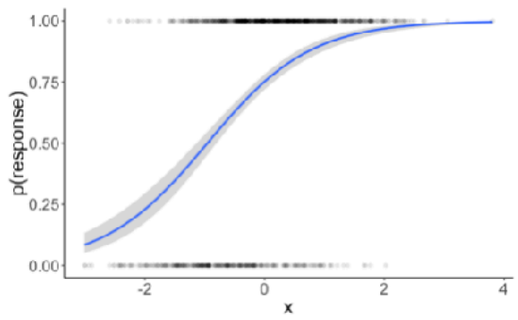
$$Y_i = b_0 + b_1 \cdot x_i + e_i$$

$$e_i \sim \mathcal{N}(0, \sigma)$$

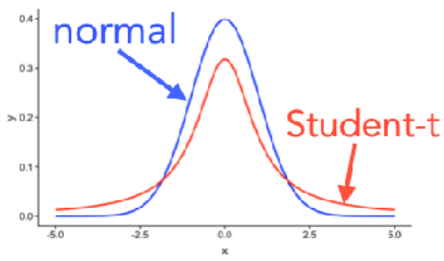


Binomial distribution

```
fit.glm = glm(formula = survived ~ 1 + fare,
              family = "binomial",
              data = df.titanic)
```

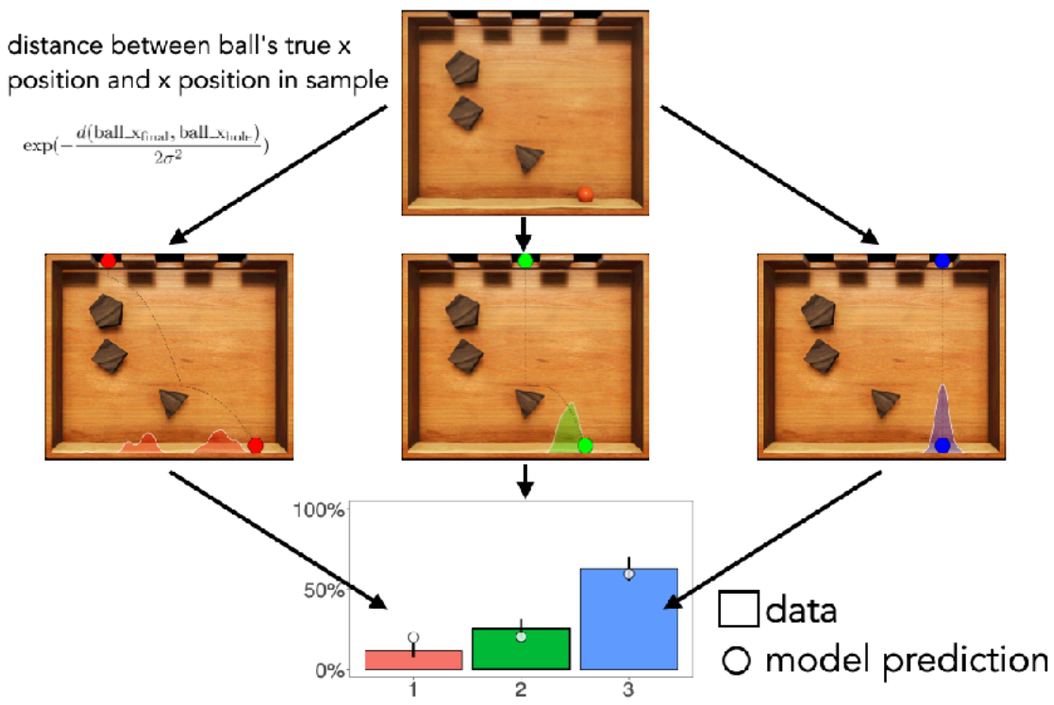


Likelihood



- **Bernoulli:**
  - binary data
  - a single trial
- **Poisson**: count of discrete events
- **Beta-binomial**: like binomial but probability of success may change across trials
- **Student-t**:
  - same as Normal
  - handles greater variability in the data (distribution has **fat tails**)

- …

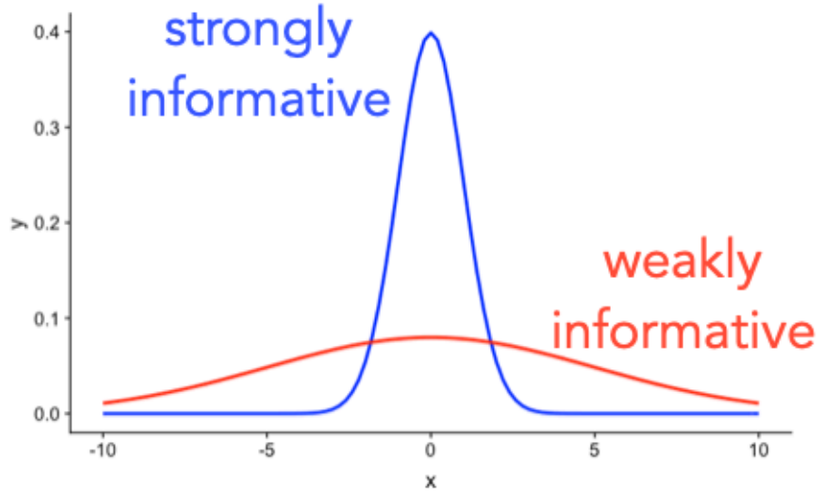


Quick recap: likelihood, **prior**, inference

Prior

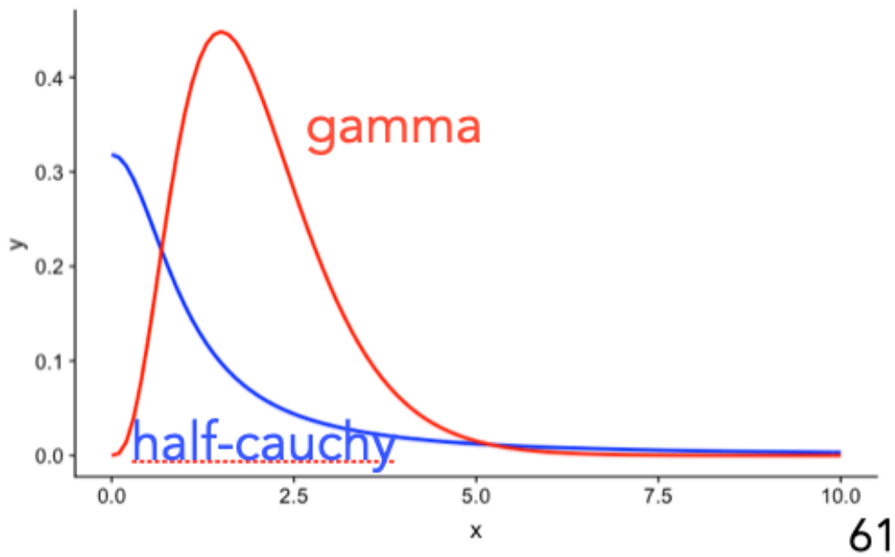
- **uniform**:
  - continuous or discrete
  - bounded between minimum and maximum

for **beta coefficients** in a regression



- **gaussian:**
  - sd determines how informative the prior is
- **gamma, half-cauchy**:
  - for parameters we know are positive

for **standard deviation** of the Gaussian



61

Quick recap: likelihood, prior, **inference**

Posterior **Likelihood** **Prior**

$$p(H \mid D) = \frac{p(D \mid H) \cdot p(H)}{p(D)}$$

Normalizing constant

the devil is in the denominator ...

Doing Bayesian inference

Discrete hypothesis space

$$p(H \mid D) = \frac{p(D \mid H) \cdot p(H)}{\sum_{i=1}^{n} p(D \mid H_i) \cdot p(H_i)}$$

sum over all possibilities

Continuous hypothesis space

$$p(H \mid D) = \frac{p(D \mid H) \cdot p(H)}{\int_{-\infty}^{\infty} p(D \mid H_i) \cdot p(H_i) dH_i}$$

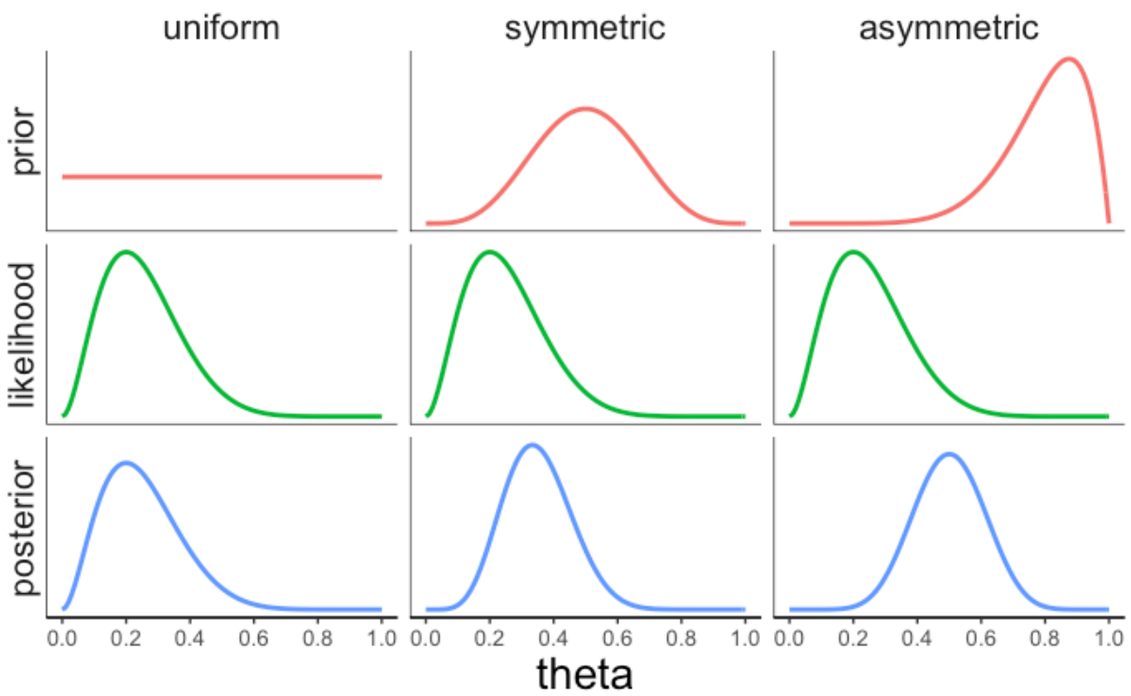
integral over all possibilities

Discretizing the parameters

```
# grid
theta = seq(0, 1, 0.01)

# data
data = rep(0:1, c(8, 2))

# calculate posterior
df.prior = tibble(theta = theta,
                  prior_uniform = dbeta(theta, shape1 = 1, shape2 = 1),
                  prior_normal = dbeta(theta, shape1 = 5, shape2 = 5),
                  prior_biased = dbeta(theta, shape1 = 8, shape2 = 2)) %>%
  pivot_longer(cols = -theta,
               names_to = "prior_index",
               values_to = "prior") %>%
  mutate(likelihood = dbinom(sum(data == 1),
                             size = length(data),
                             prob = theta)) %>%
  group_by(prior_index) %>%
  mutate(posterior = likelihood * prior / sum(likelihood * prior)) %>%
  ungroup() %>%
  pivot_longer(cols = -c(theta, prior_index),
               names_to = "index",
               values_to = "value")
```

100 discrete values

for 3 variables, we would already need 1 Mio combinations



The CURSE of dimensionality

Inference via sampling

- we cannot directly calculate the probability of the posterior (because it might have a pretty weird shape)
- **but**: we can draw random samples from the posterior
- we can then use our data wrangling and visualization skills to make inferences based on these samples

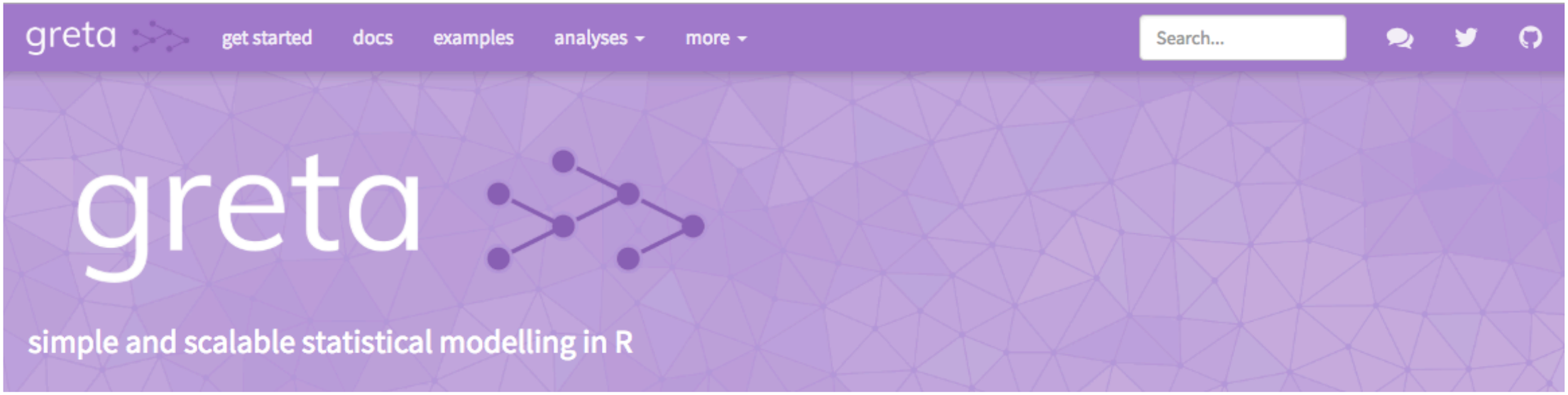
It's as if ...

we don't have **`pnorm()`** but we do have **`rnorm()`**

Inference via sampling

- Bayesian data analysis is becoming more popular because:
  - computers are getting more powerful
  - inference techniques are getting better
  - software packages become easier to use

Doing Bayesian data analysis with greta

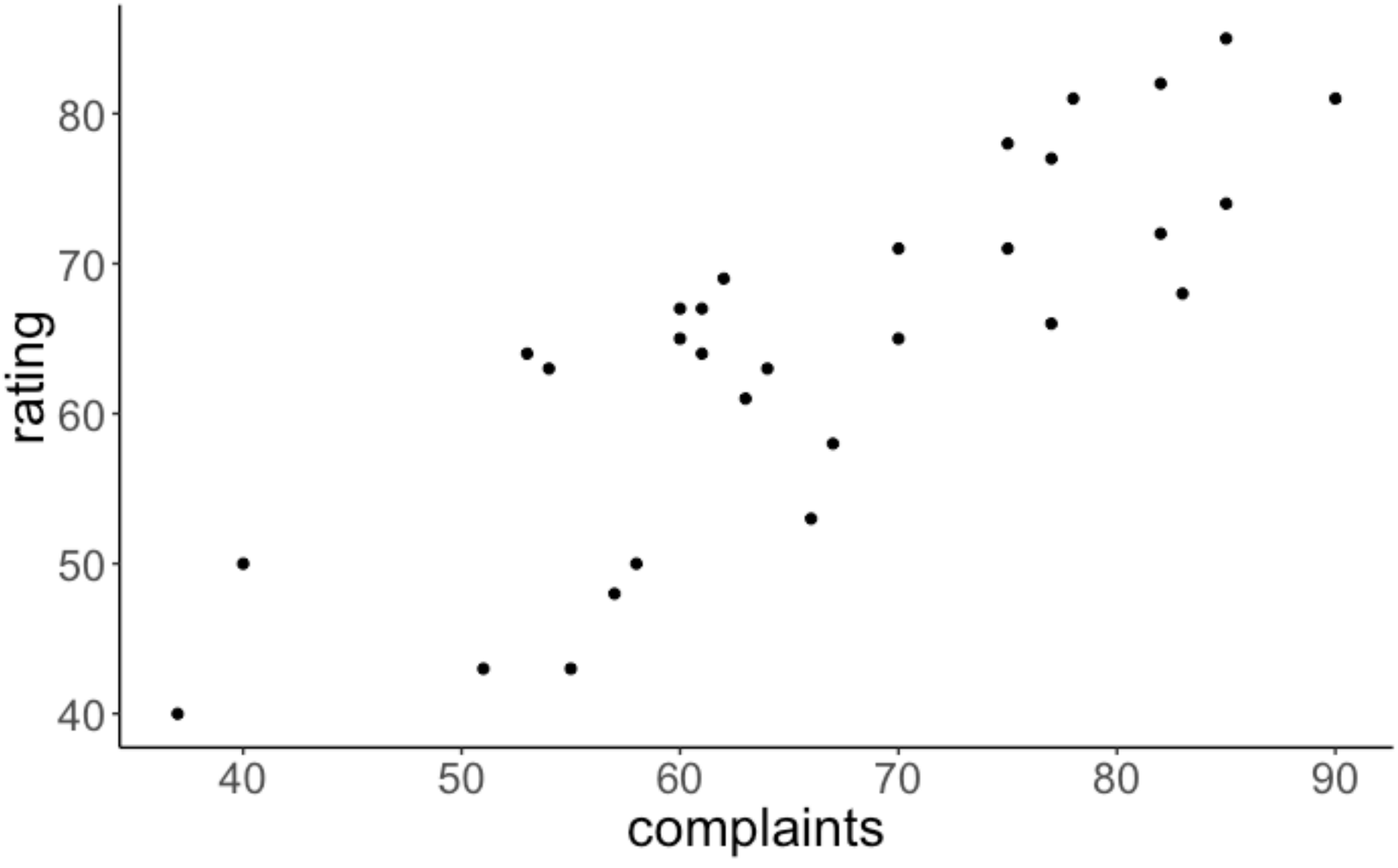


- let's us write Bayesian models directly in R with a simple syntax
- uses Tensorflow to implement Hamiltonian Monte Carlo sampling (a fast inference algorithm ...)

unfortunately doesn't work on Apple silicone (e.g. M1)

Attitude data set

What's the relationship between how well an employee handles complaints and their overall rating?



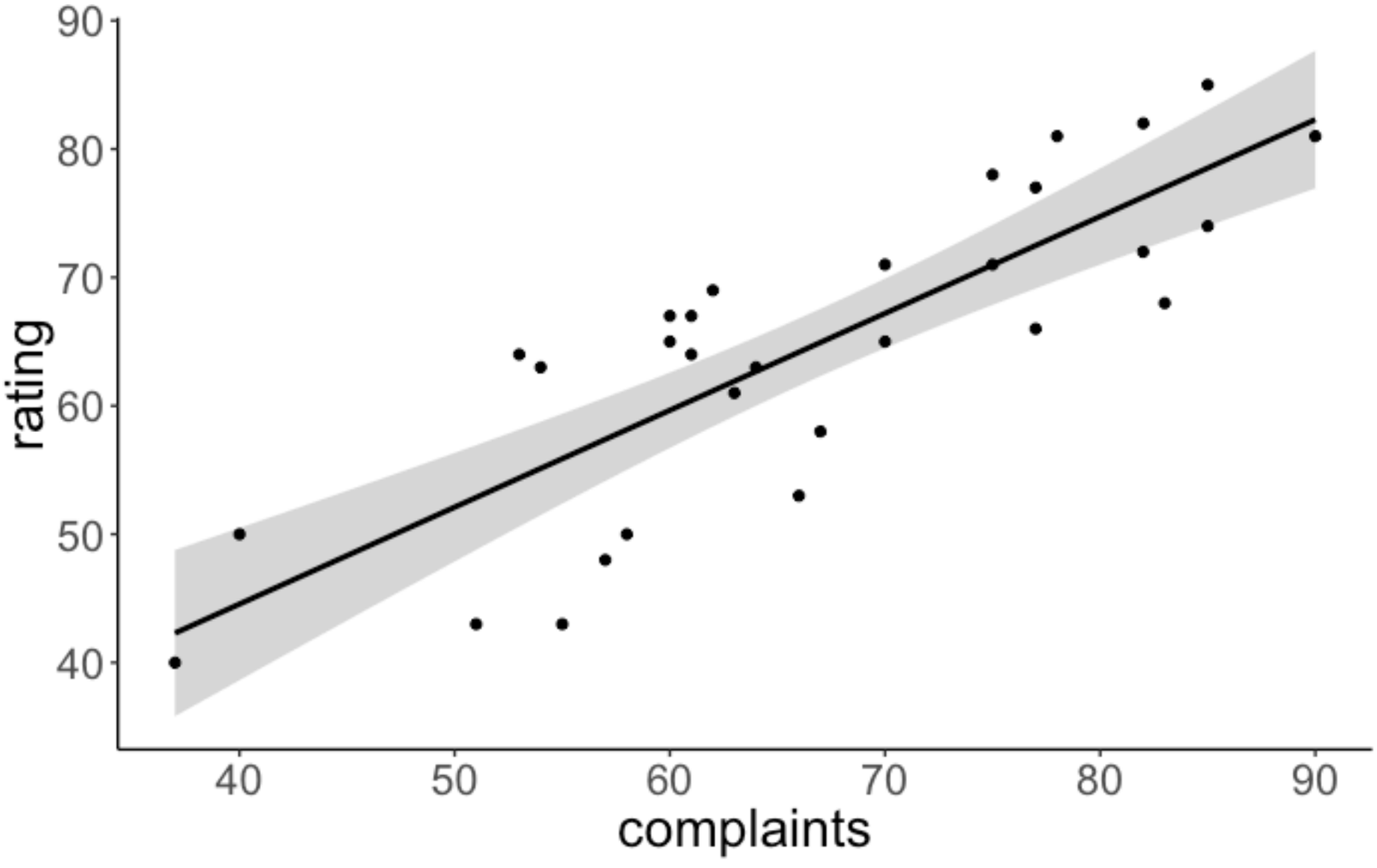
Frequentist analysis

Frequentist analysis

```r
# fit model
fit = lm(formula = rating ~ 1 + complaints,
         data = df.attitude)

# print summary
fit %>% summary()
```

```
Call:
lm(formula = rating ~ 1 + complaints, data = df.attitude)

Residuals:
     Min       1Q   Median       3Q      Max
-12.8799  -5.9905   0.1783   6.2978   9.6294

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 14.37632    6.61999   2.172   0.0385 *
complaints   0.75461    0.09753   7.737 1.99e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.993 on 28 degrees of freedom
Multiple R-squared:  0.6813,  Adjusted R-squared:  0.6699
F-statistic: 59.86 on 1 and 28 DF,  p-value: 1.988e-08
```

Visualize model predictions



Best-fitting regression line with confidence interval

Bayesian analysis

Model specification

```
 1 library("greta")
 2 library("tidybayes")
 3
 4 # variables & priors
 5 b0 = normal(0, 10)
 6 b1 = normal(0, 10)
 7 sd = cauchy(0, 3, truncation = c(0, Inf))
 8
 9 # linear predictor
10 mu = b0 + b1 * attitude$complaints
11
12 # observation model (likelihood)
13 distribution(attitude$rating) = normal(mu, sd)
14
15 # define the model
16 m = model(b0, b1, sd)
```

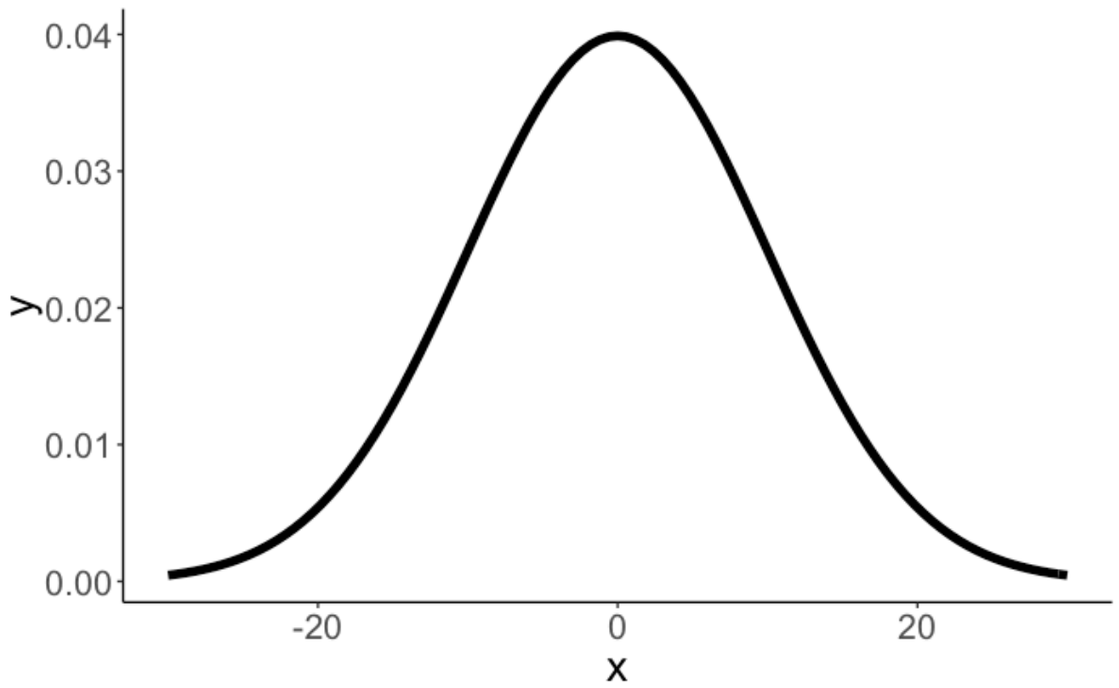
priors

linear combination

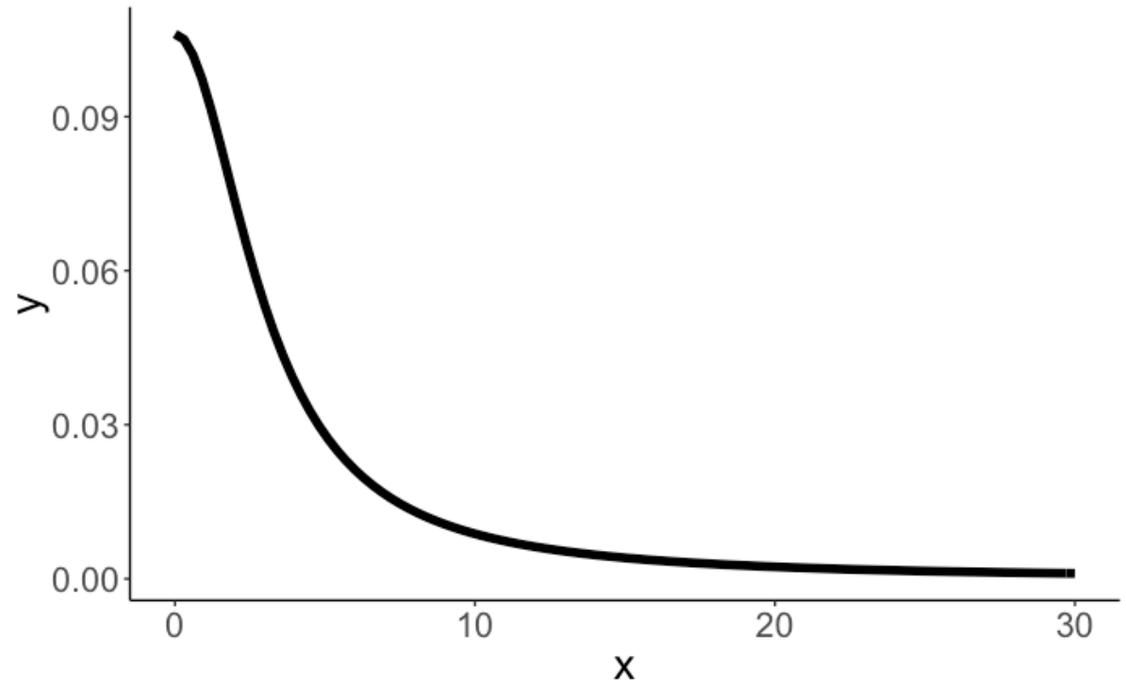
Gaussian likelihood

build the model

Priors



Gaussian prior on intercept and coefficient

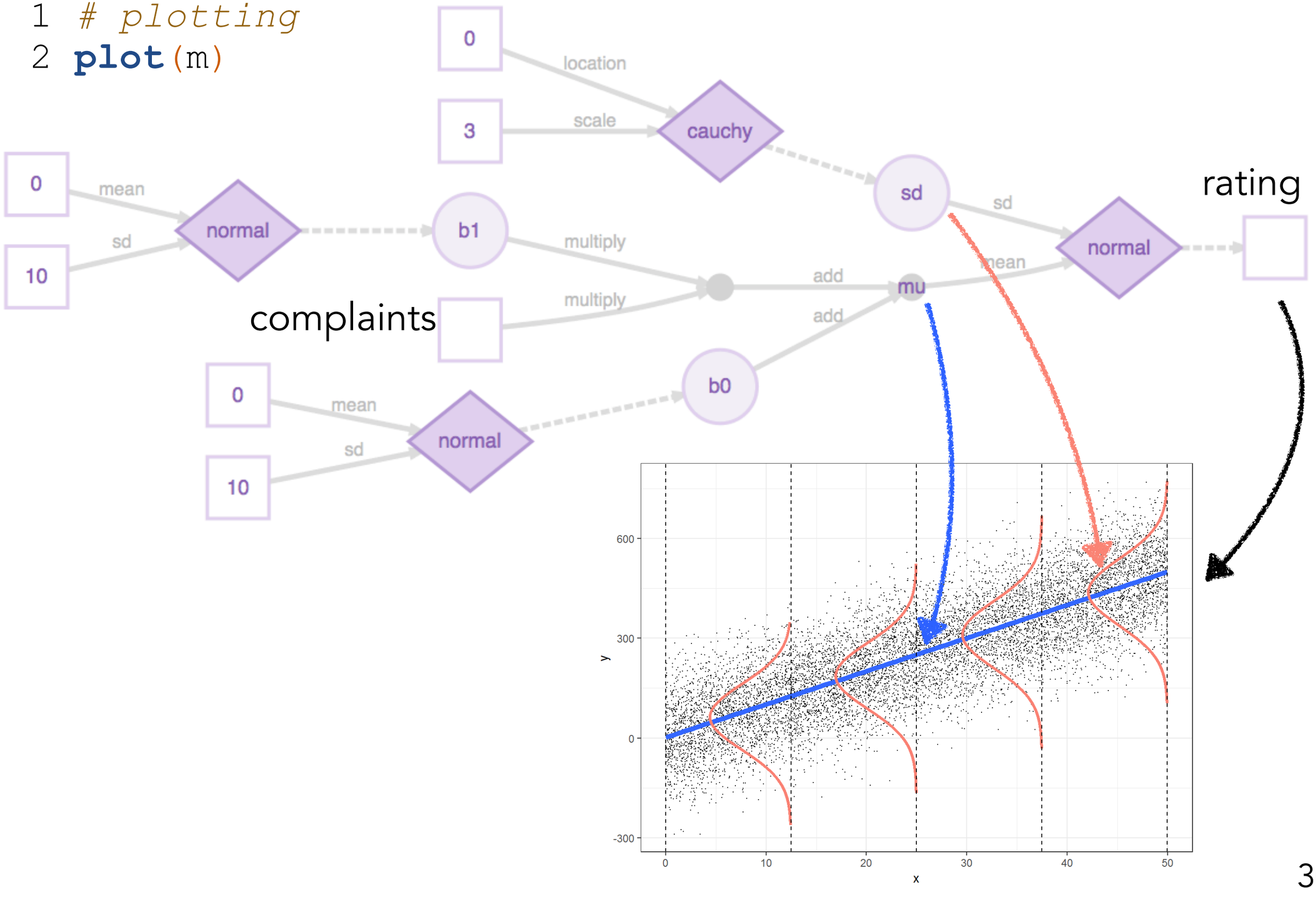


Truncated Cauchy prior on the standard deviation

weakly informative priors (allow for a wide range of possible values)

Graphical representation of the model

```
1 # plotting
2 plot(m)
```



Inference via sampling

Markov Chain Monte Carlo inference

```
1 # sampling
2 draws = mcmc(m, n_samples = 1000)
3
4 # tidy up the draws
5 df.draws = tidy_draws(draws) %>%
6   clean_names()
```

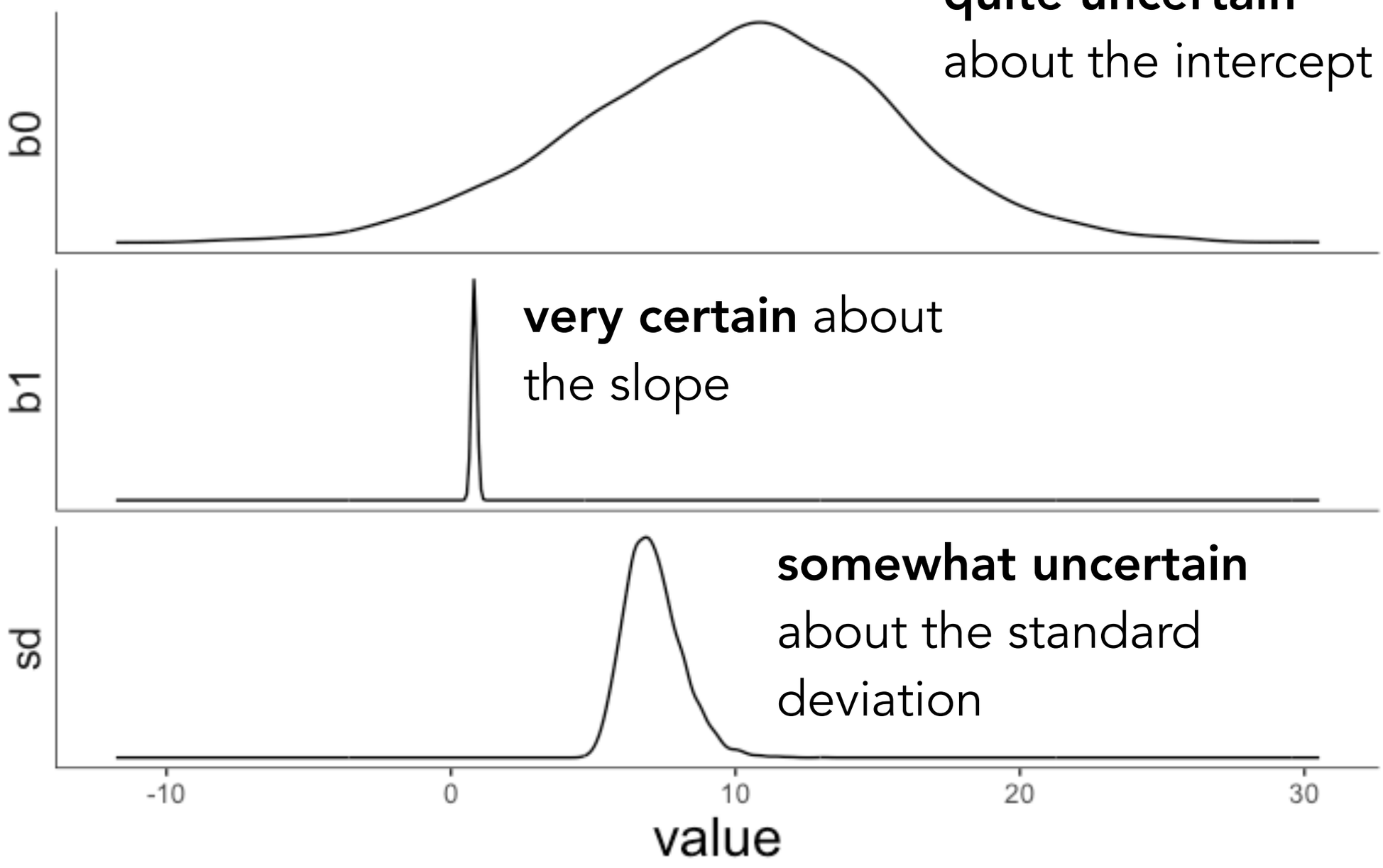
chain	iteration	draw	b0	b1	sd
1	1	1	6.08	0.87	7.60
1	2	2	1.12	0.95	7.66
1	3	3	-1.83	0.99	7.01
1	4	4	-4.23	1.02	6.64
1	5	5	3.26	0.87	7.96
1	6	6	-1.04	0.98	7.67
1	7	7	-0.83	0.97	10.12
1	8	8	-1.41	0.97	8.02
1	9	9	9.46	0.81	6.30
1	10	10	10.02	0.84	6.57

each of these is a solution for explaining the data

nice visualization of MCMC samplers

https://github.com/chi-feng/mcmc-demo

Visualize the posterior



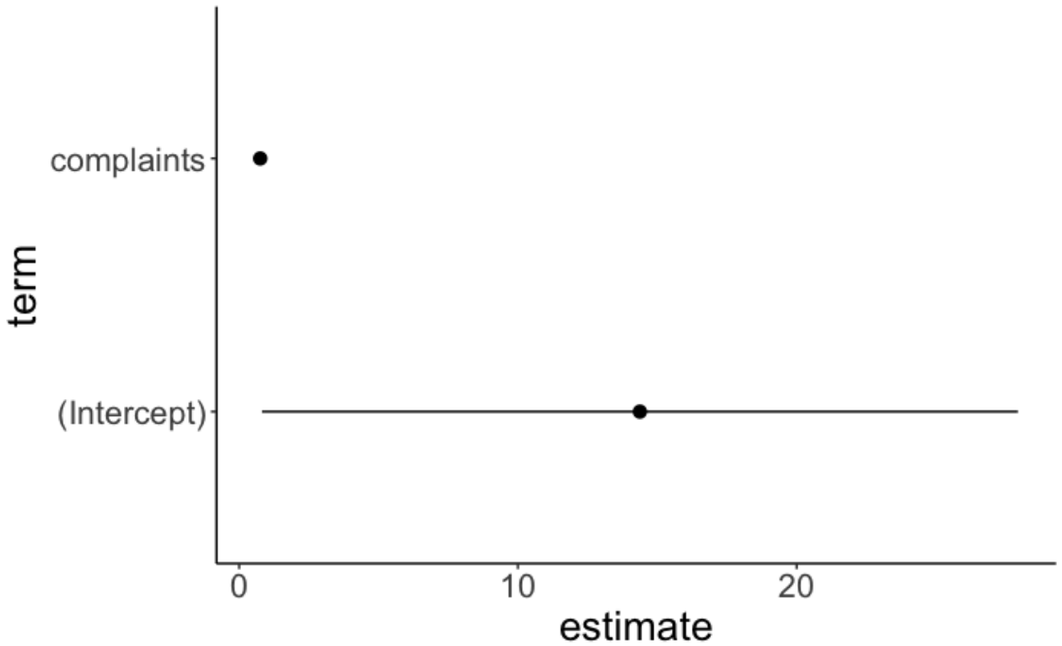
This is the solution of a Bayesian analysis.

A posterior distribution over each parameter in our model.

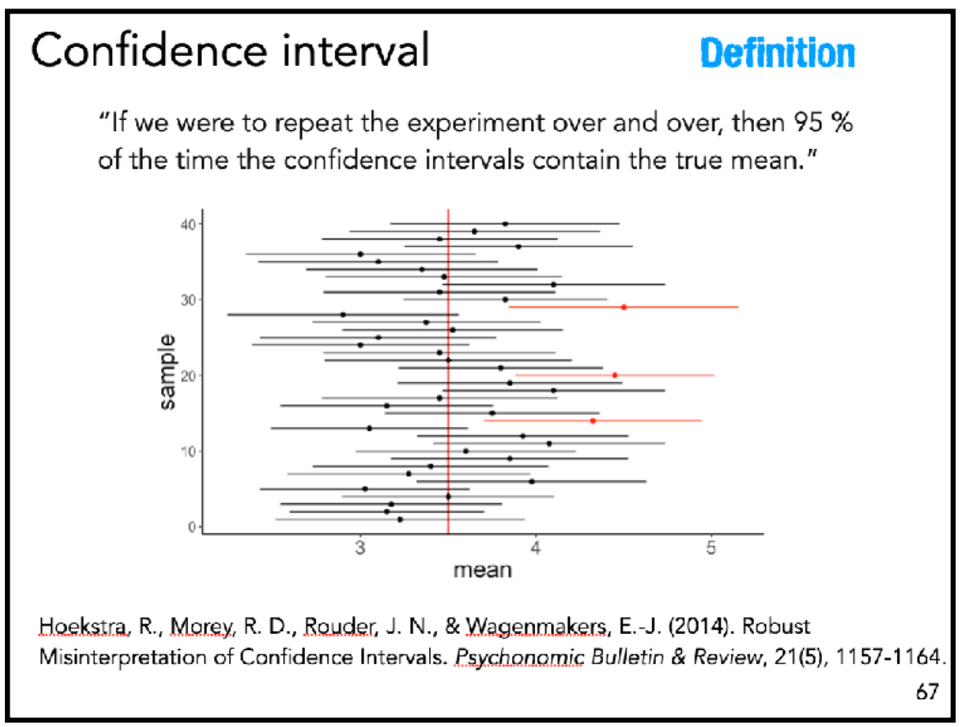
We can use this to visualize model predictions, and to test hypotheses.

Confidence interval

```
1 fit.lm = lm(formula = rating ~ 1 + complaints,
2              data = df.attitude)
```

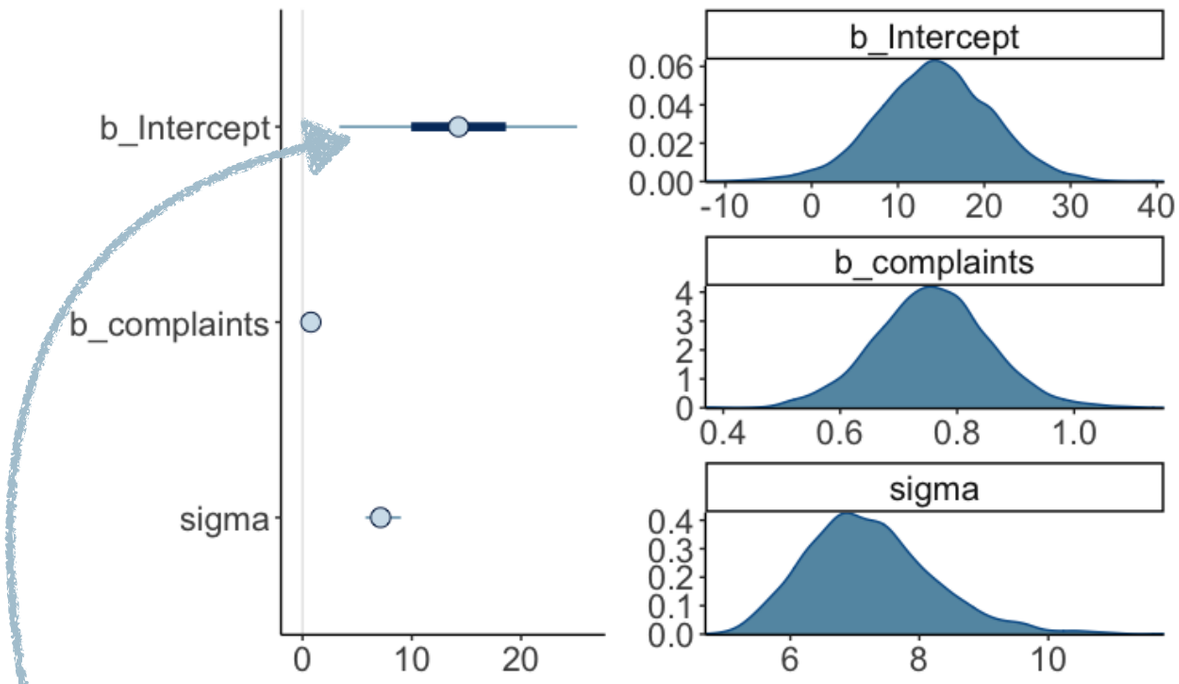


point estimate + confidence interval



Credible interval

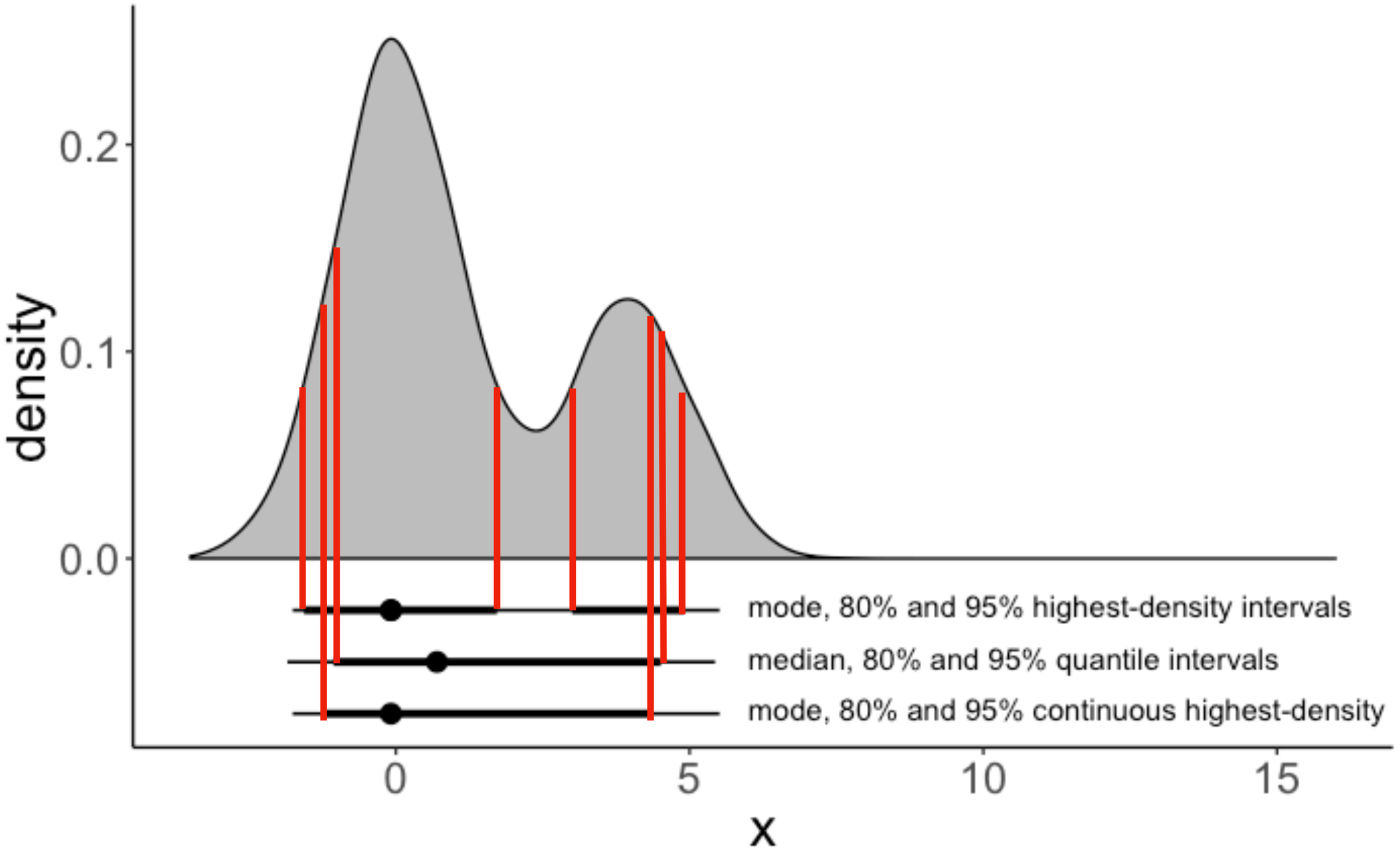
```
1 fit.brm = brm(formula = rating ~ 1 + complaints,
2                data = df.attitude)
```



full posterior distribution over each parameter

with 90% the true parameter lies within this range

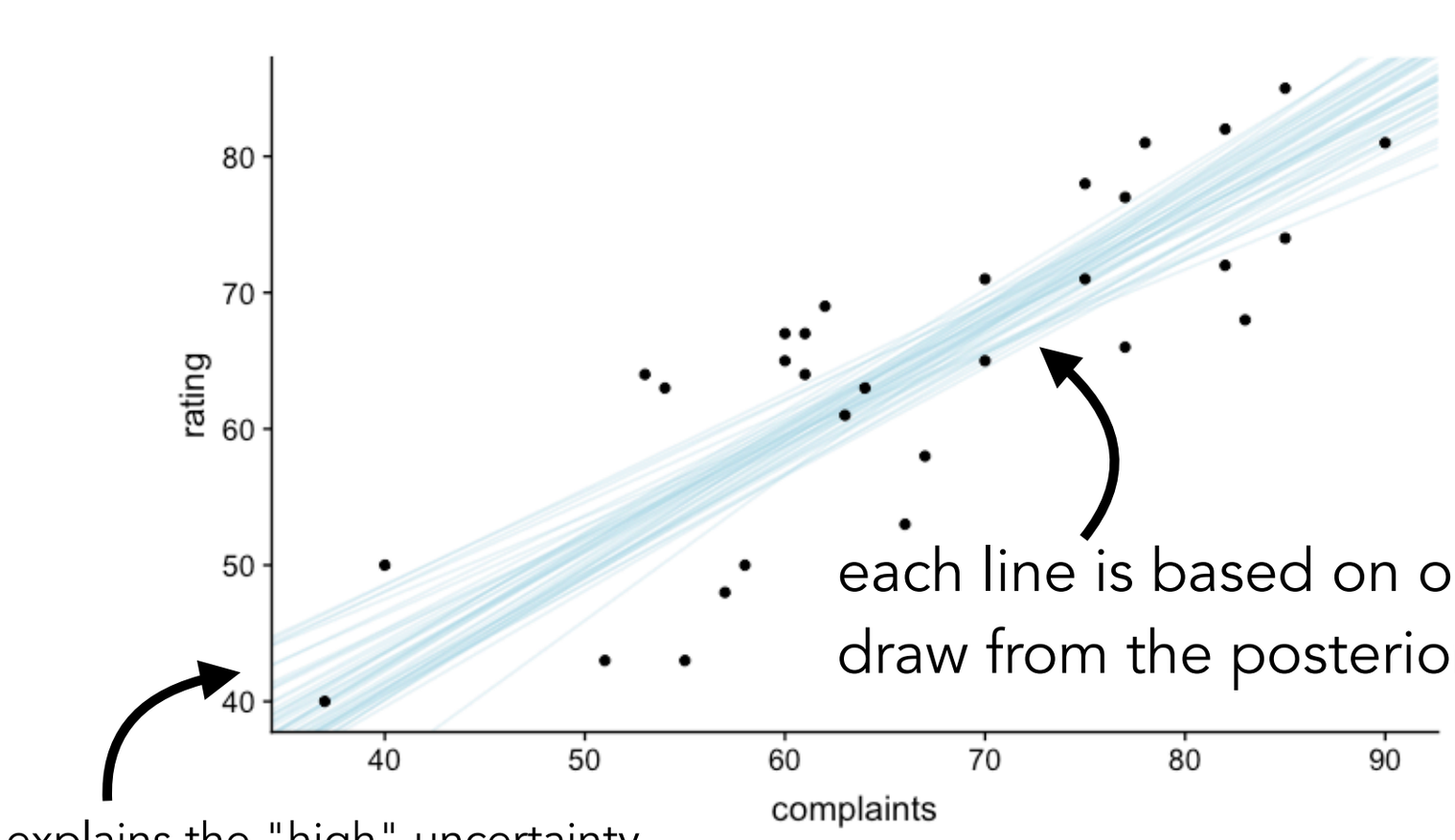
Different kinds of credible intervals



ways of summarizing the posterior distribution

Visualize the model predictions

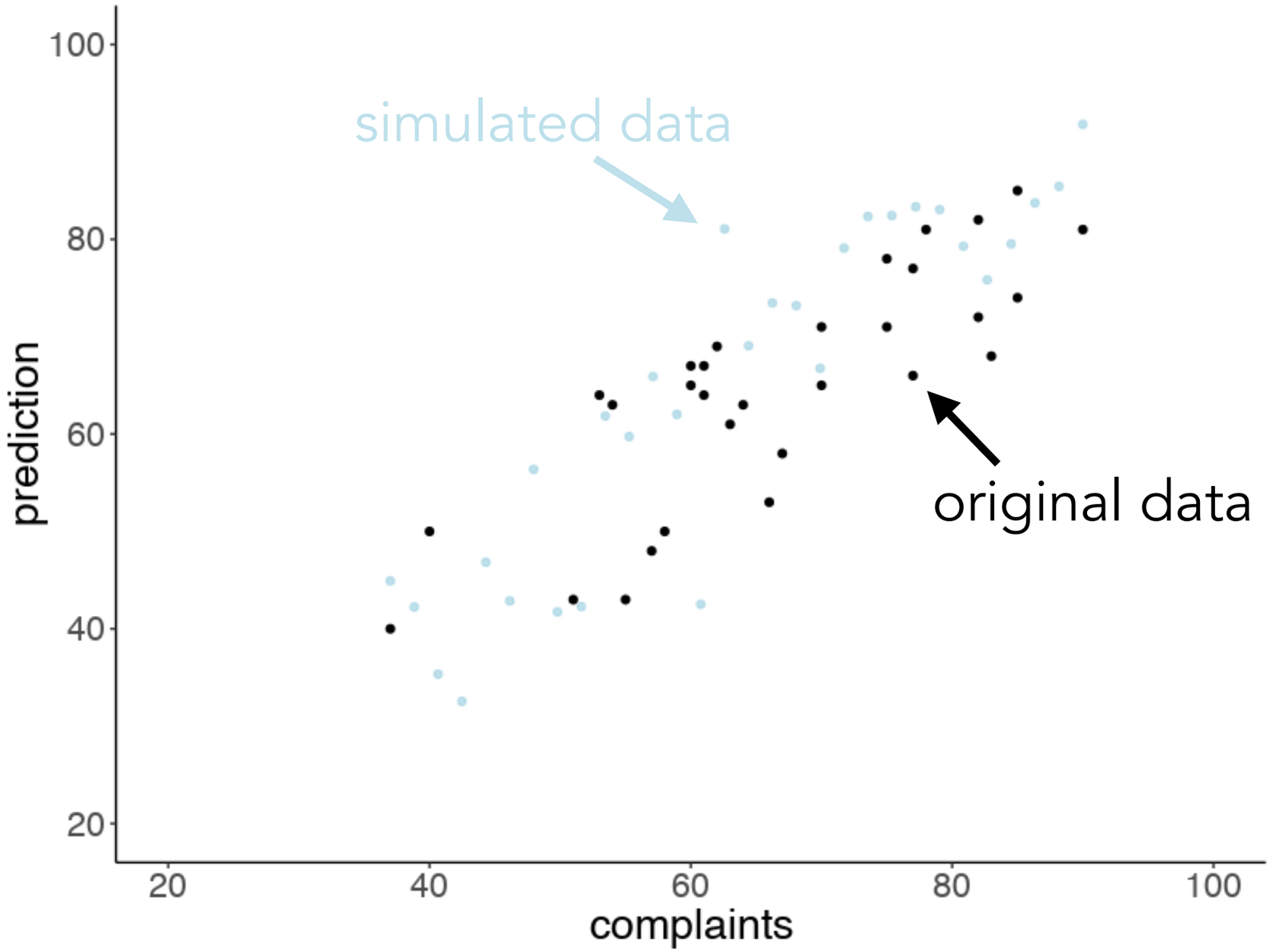
```
ggplot(data = df.attitude,
       mapping = aes(x = complaints,
                     y = rating)) +
  geom_abline(data = df.draws %>%
                sample_n(size = 50),
              aes(intercept = b0,
                  slope = b1),
              alpha = 0.3,
              color = "lightblue") +
  geom_point()
```



chain	iteration	draw	b0	b1	sd
1	1	1	6.08	0.87	7.60
1	2	2	1.12	0.95	7.66
1	3	3	-1.83	0.99	7.01
1	4	4	-4.23	1.02	6.64
1	5	5	3.26	0.87	7.96
1	6	6	-1.04	0.98	7.67
1	7	7	-0.83	0.97	10.12
1	8	8	-1.41	0.97	8.02
1	9	9	9.46	0.81	6.30
1	10	10	10.02	0.84	6.57

Posterior predictive check

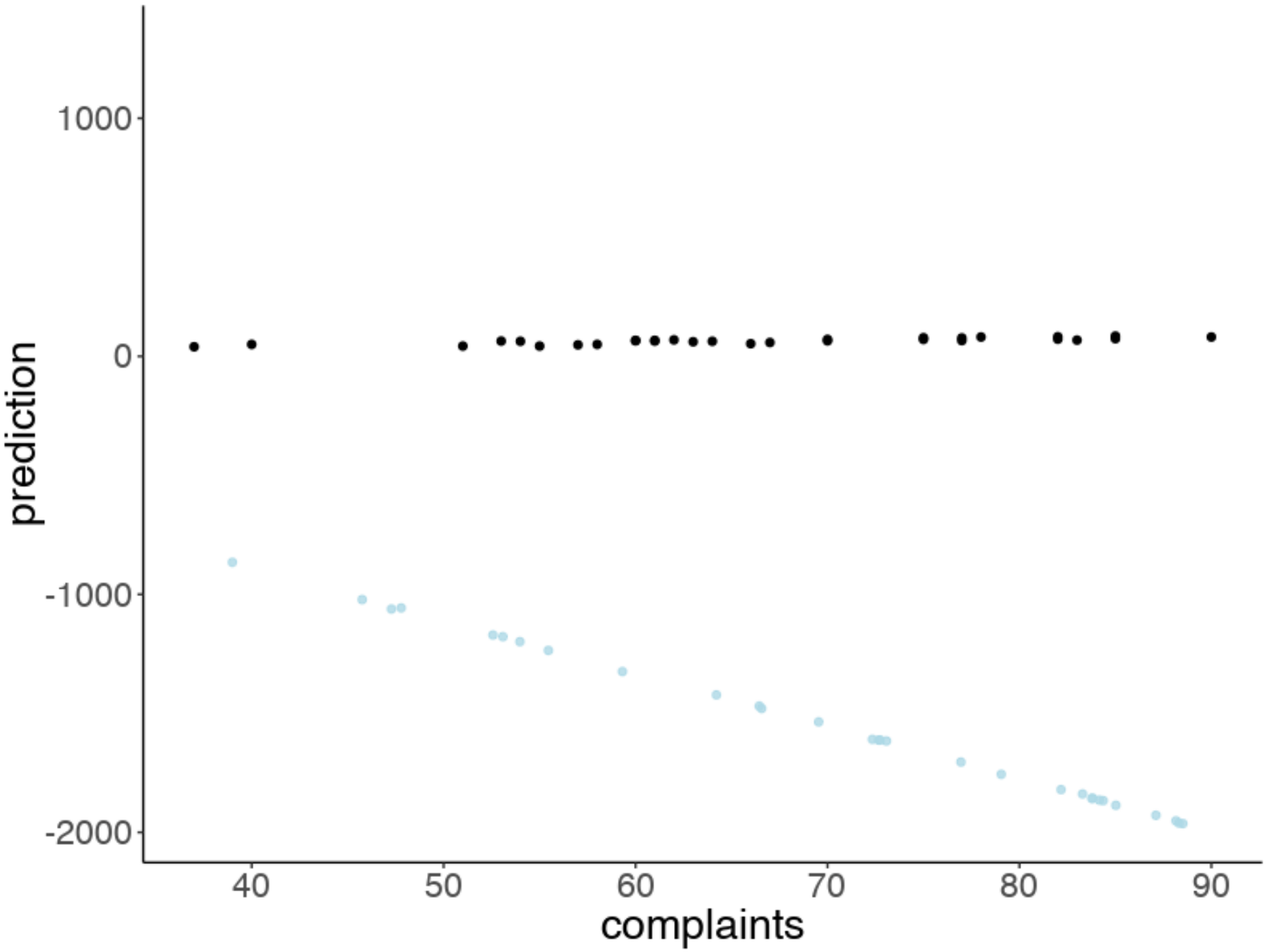
1. sample parameters from the **posterior distribution**
2. generate data using these parameters (using the likelihood function)



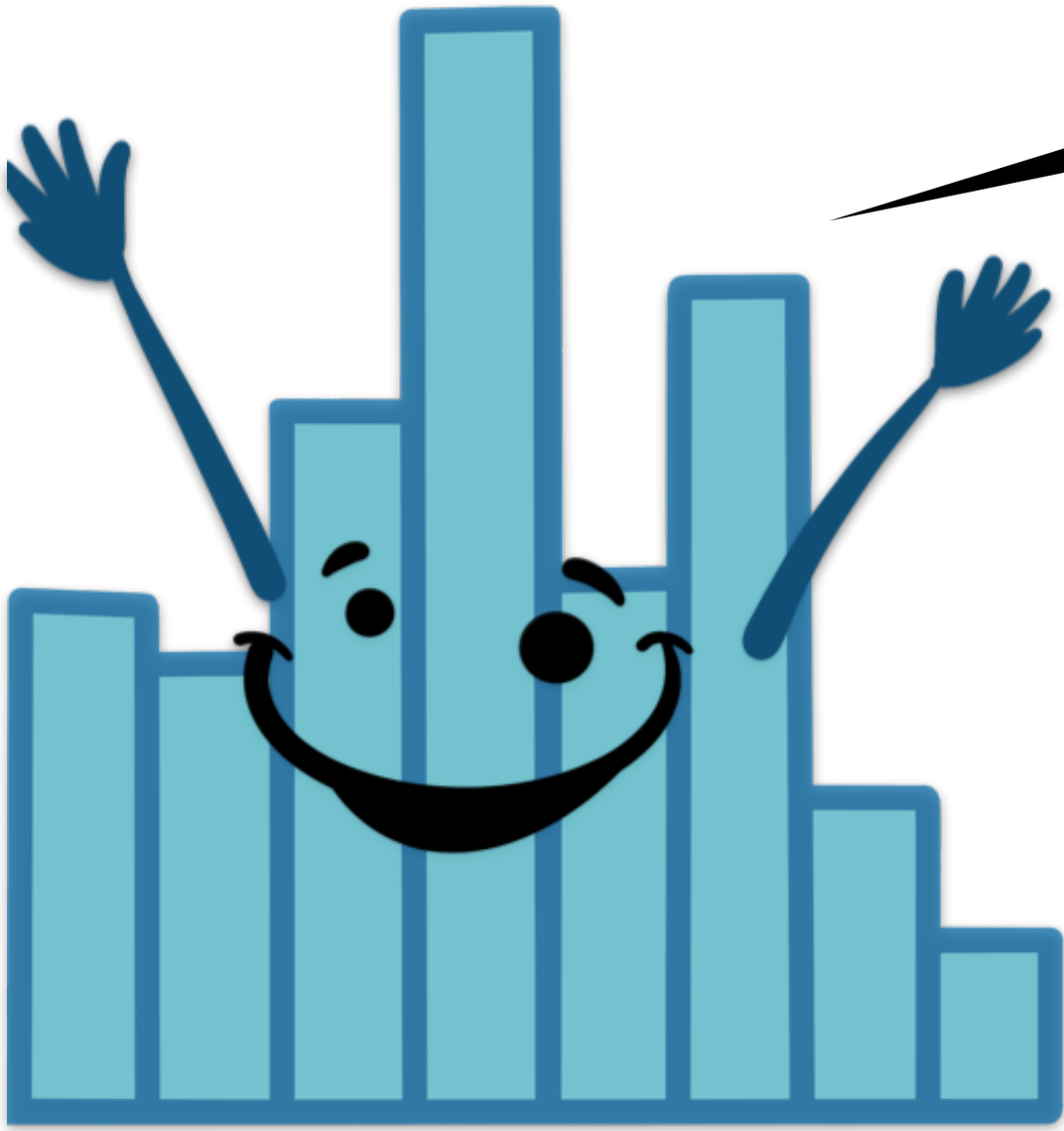
Hypothetical outcome plot

Prior predictive check

1. sample parameters from the **prior distribution**
2. generate data using these parameters (using the likelihood function)



Hypothetical outcome plot



02:00

stretch break!

Paul Bürkner

Doing Bayesian data analysis with BRMS

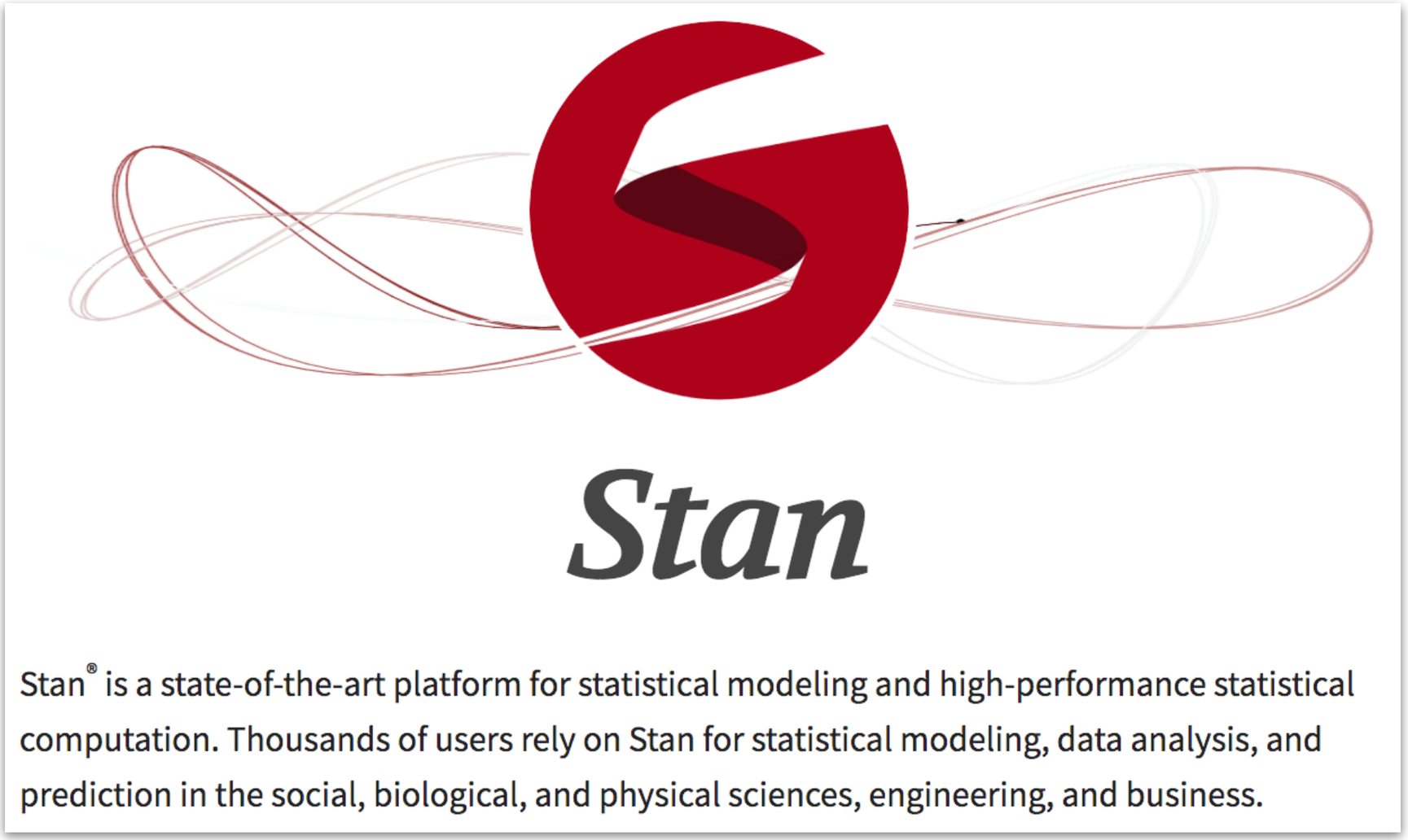
Software packages

`library("brms")`

Bayesian regression modeling with Stan

- very powerful package that makes it easy to run Bayesian regression models
- we specify models using the same syntax we've already learned based on **`lm`**`()`, **`glm`**`()`, and **`lmer`**`()`
- `brms` turns this into Stan code and fits the model
- we can then use `tidybayes` to investigate the posterior

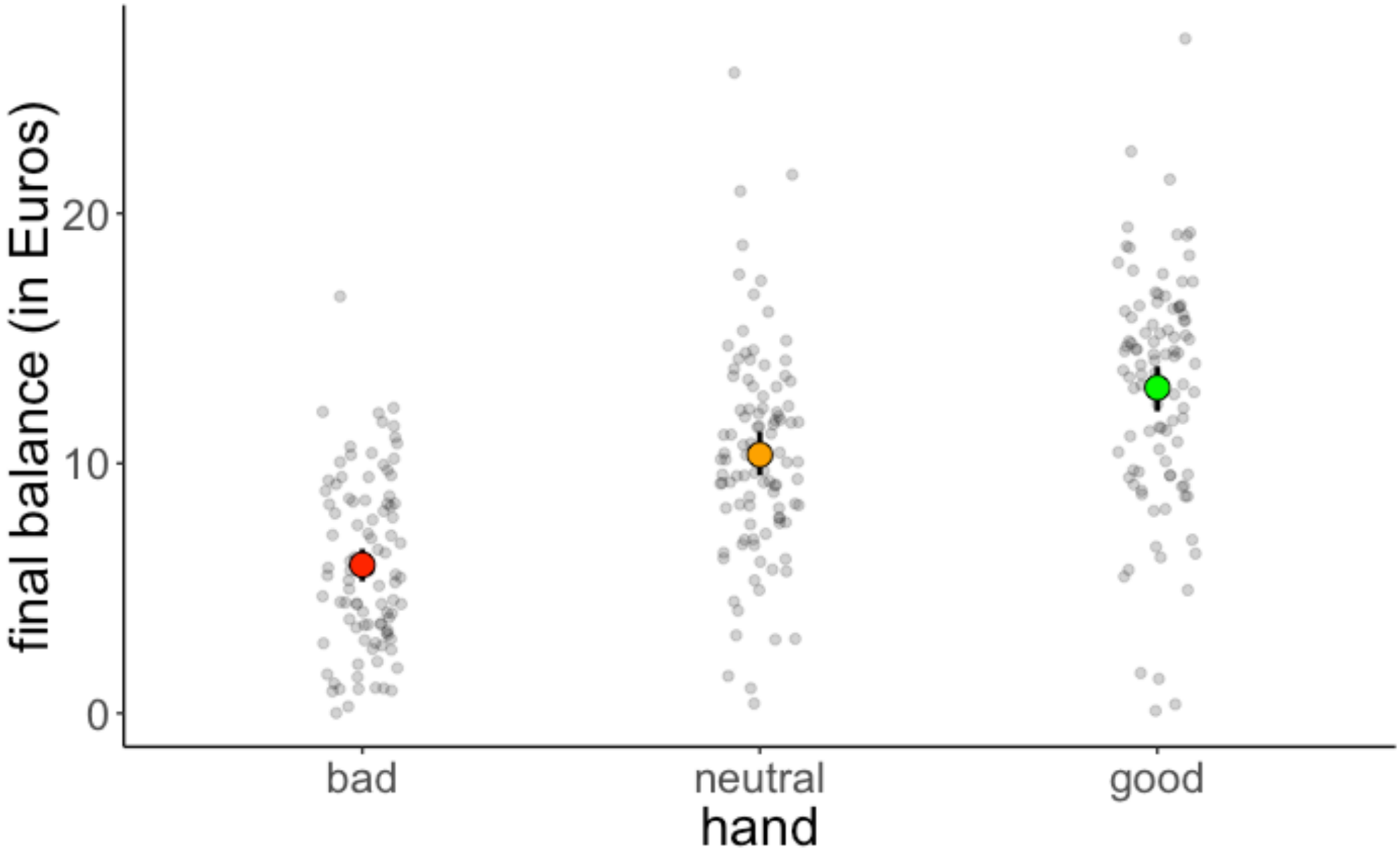
Software packages



https://mc-stan.org/

Poker data

Poker data

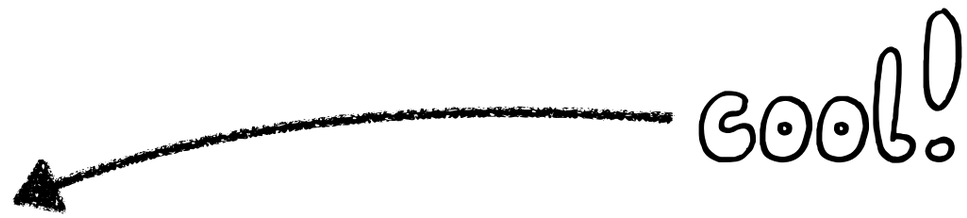


Using `lm()`

```
1 fit.lm_poker = lm(formula = balance ~ 1 + hand,
2                   data = df.poker)
3
4 fit.lm_poker %>% summary()
```

```
Call:
lm(formula = balance ~ 1 + hand, data = df.poker)

Residuals:
     Min       1Q   Median       3Q      Max
-12.9264  -2.5902  -0.0115   2.6573  15.2834

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   5.9415     0.4111  14.451  < 2e-16 ***
handneutral   4.4051     0.5815   7.576 4.55e-13 ***
handgood      7.0849     0.5815  12.185  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.111 on 297 degrees of freedom
Multiple R-squared:  0.3377, Adjusted R-squared:  0.3332
F-statistic:  75.7 on 2 and 297 DF,  p-value: < 2.2e-16
```

Using `brm()`




```
1 fit.brm_poker = brm(formula = balance ~ 1 + hand,
2                     data = df.poker)
3
4 fit.brm_poker %>% summary()
```

```
 Family: gaussian
  Links: mu = identity; sigma = identity
Formula: balance ~ 1 + hand
   Data: df.poker (Number of observations: 300)
Samples: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
         total post-warmup samples = 4000

Population-Level Effects:
            Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept       5.93      0.41     5.12     6.72 1.00     2986     2744
handneutral     4.41      0.58     3.30     5.55 1.00     3497     2903
handgood        7.10      0.58     5.99     8.29 1.00     3545     2932

Family Specific Parameters:
      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sigma     4.12      0.17     3.81     4.46 1.00     3650     2921

Samples were drawn using sampling(NUTS). For each parameter, Eff.Sample
is a crude measure of effective sample size, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
```

Comparison between `lm()` and `brm()`

`lm()`

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)    5.9415     0.4111  14.451  < 2e-16 ***
handneutral    4.4051     0.5815   7.576 4.55e-13 ***
handgood       7.0849     0.5815  12.185  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

`brm()`

```
Population-Level Effects:
             Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept        5.93      0.41     5.12     6.72 1.00     2986     2744
handneutral      4.41      0.58     3.30     5.55 1.00     3497     2903
handgood         7.10      0.58     5.99     8.29 1.00     3545     2932
```

almost identical results!

What about the priors?

```
1 fit.brm_poker = brm(formula = balance ~ 1 + hand,
2                     data = df.poker)
```

By default, `brms` uses weakly informative priors for the model parameters.

There are quite a few other defaults, let's take a look under the hood ...

"Full" specification of the model

```
fit.brm_poker_full = brm(
  formula = balance ~ 1 + hand,
  family = "gaussian",
  data = df.poker,
  prior = c(
    prior(normal(0, 10), class = "b", coef = "handgood"),
    prior(normal(0, 10), class = "b", coef = "handneutral"),
    prior(student_t(3, 3, 10), class = "Intercept"),
    prior(student_t(3, 0, 10), class = "sigma")
  ),
  inits = list(
    list(Intercept = 0, sigma = 1, handgood = 5, handneutral = 5),
    list(Intercept = -5, sigma = 3, handgood = 2, handneutral = 2),
    list(Intercept = 2, sigma = 1, handgood = -1, handneutral = 1),
    list(Intercept = 1, sigma = 2, handgood = 2, handneutral = -2)
  ),
  iter = 4000,
  warmup = 1000,
  chains = 4,
  file = "cache/brm_poker_full",
  seed = 1
)
```

likelihood

priors

initialization

how many runs in the inference chain

how long for the warmup

how many chains

save the model result

make reproducible

fitting Bayesian models takes some time, so storing results is key

Turned into Stan code

```
// generated with brms 2.7.0
functions {
}
data {
  int<lower=1> N;  // total number of observations
  vector[N] Y;  // response variable
  int<lower=1> K;  // number of population-level effects
  matrix[N, K] X;  // population-level design matrix
  int prior_only;  // should the likelihood be ignored?
}
transformed data {
  int Kc = K - 1;
  matrix[N, K - 1] Xc;  // centered version of X
  vector[K - 1] means_X;  // column means of X before centering
  for (i in 2:K) {
    means_X[i - 1] = mean(X[, i]);
    Xc[, i - 1] = X[, i] - means_X[i - 1];
  }
}
parameters {
  vector[Kc] b;  // population-level effects
  real temp_Intercept;  // temporary intercept
  real<lower=0> sigma;  // residual SD
}
transformed parameters {
}
model {
  vector[N] mu = temp_Intercept + Xc * b;
  // priors including all constants
  target += normal_lpdf(b[1] | 0, 10);
  target += normal_lpdf(b[2] | 0, 10);
  target += student_t_lpdf(temp_Intercept | 3, 3, 10);
  target += student_t_lpdf(sigma | 3, 0, 10)
    - 1 * student_t_lccdf(0 | 3, 0, 10);
  // likelihood including all constants
  if (!prior_only) {
    target += normal_lpdf(Y | mu, sigma);
  }
}
generated quantities {
  // actual population-level intercept
  real b_Intercept = temp_Intercept - dot_product(means_X, b);
}
```

- probabilistic programming language
- flexible construction of Bayesian models
- ports have been written for R, Python, Julia, ...
- implements a fast inference algorithm

Results

posterior samples

b_Intercept	b_handneutral	b_handgood	sigma
5.97	4.27	7.48	3.94
5.11	5.25	7.40	3.91
7.03	3.78	5.80	4.48
5.72	4.18	7.25	4.00
6.01	4.44	6.15	4.57
5.94	4.69	6.72	4.36
6.39	3.84	6.40	3.92
5.24	5.15	7.69	4.16
6.12	4.51	7.20	4.14
6.43	3.71	6.37	4.13
5.85	5.01	7.32	4.00
6.51	3.58	6.62	3.95
5.85	4.45	7.62	4.17
5.80	5.45	6.36	4.10
5.48	5.51	7.22	3.99

⋮

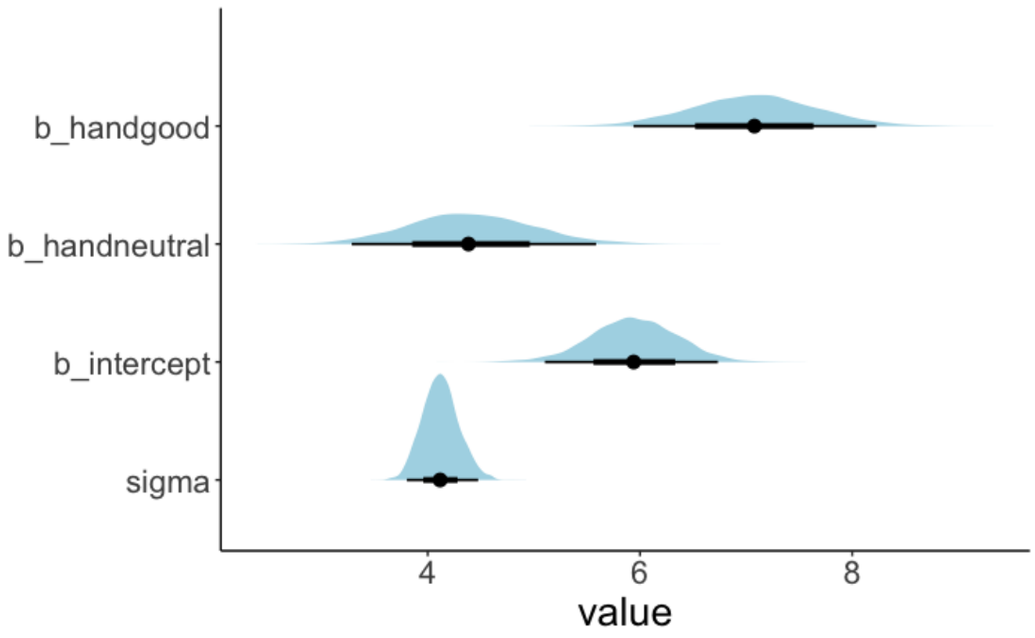
maximum a posteriori

summary of posterior

parameter	lower	mode	upper
b_handgood	5.97	7.07	8.27
b_handneutral	3.21	4.43	5.51
b_intercept	5.17	5.95	6.77
sigma	3.81	4.12	4.47

MAP estimate and 95% highest density interval

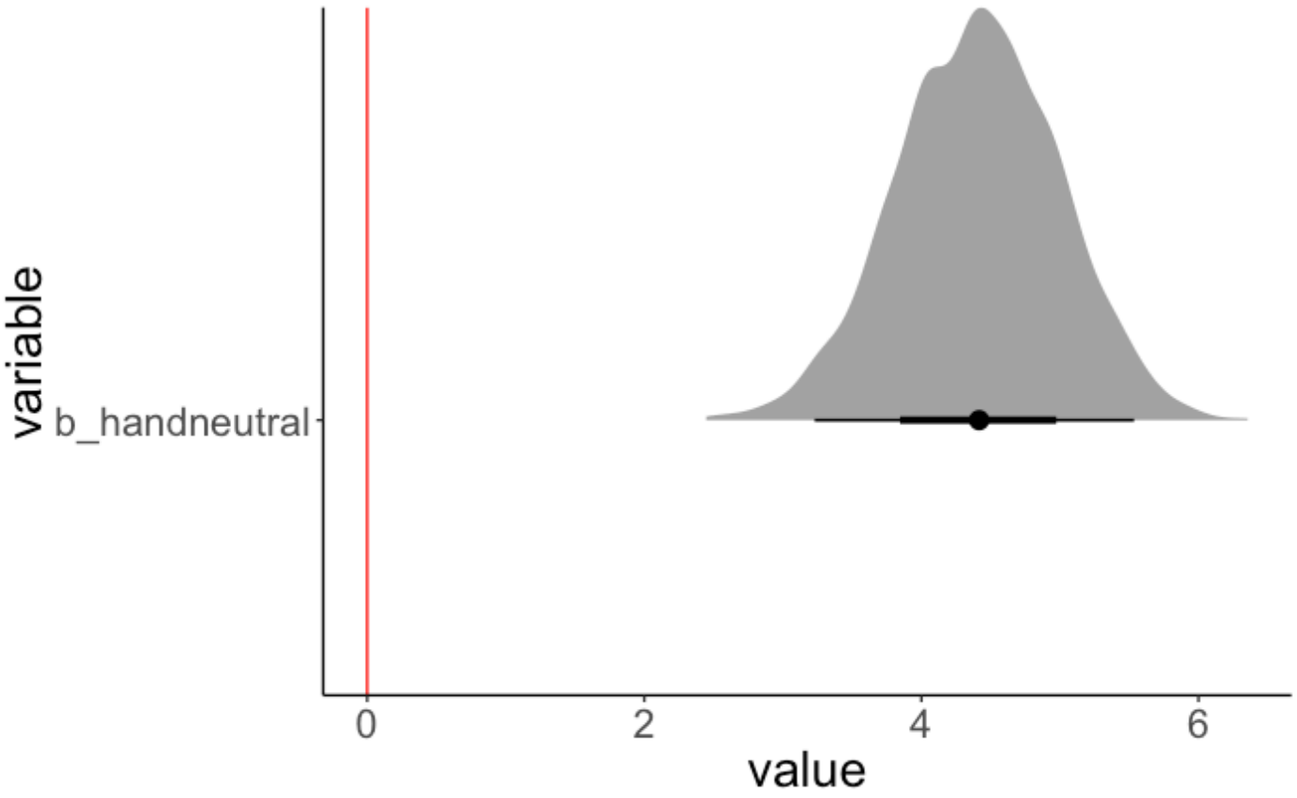
visualization



Testing hypotheses

Asking questions based on the posterior

Do neutral hands earn more money than bad hands?



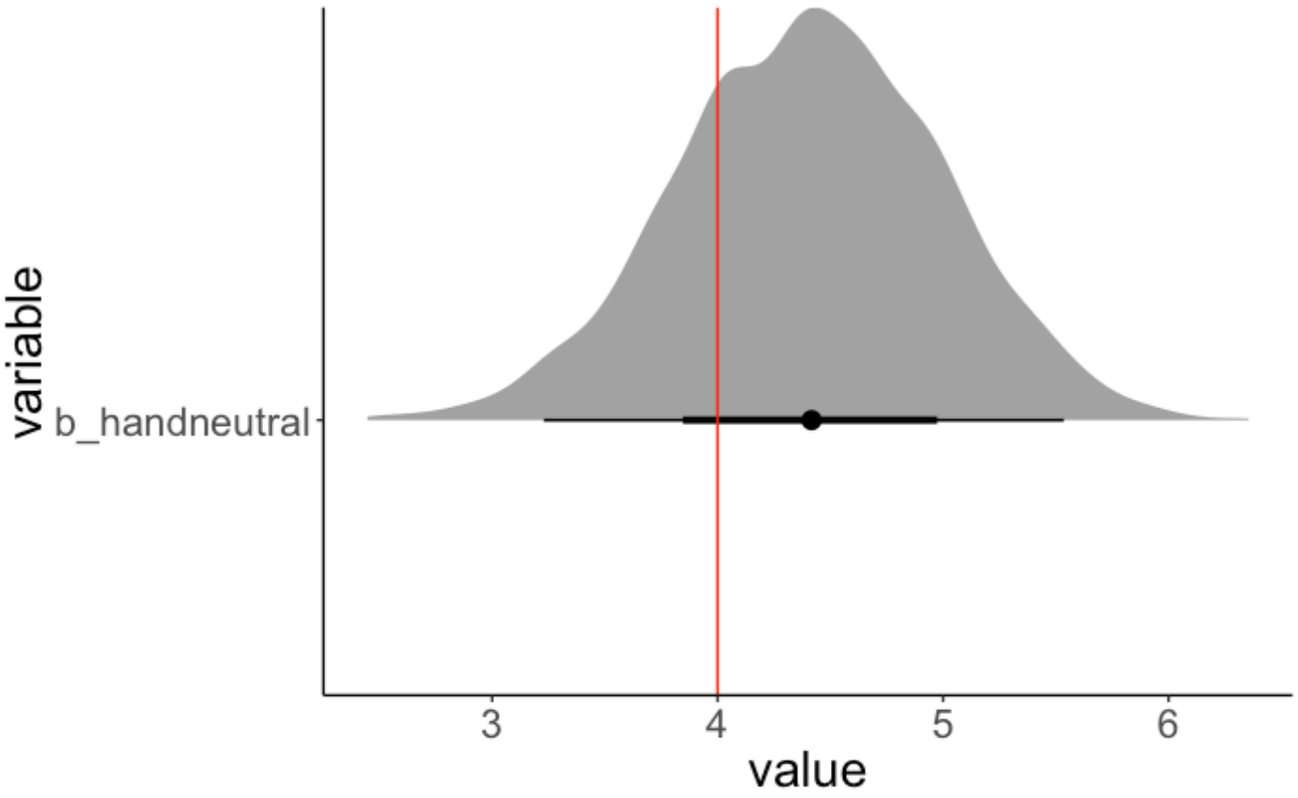
What's the probability that handneutral is less than 0?

```
1 hypothesis(fit.brm,
2              hypothesis = "handneutral < 0")
```

p = 0

Asking questions based on the posterior

Do neutral hands earn much more money than bad hands?



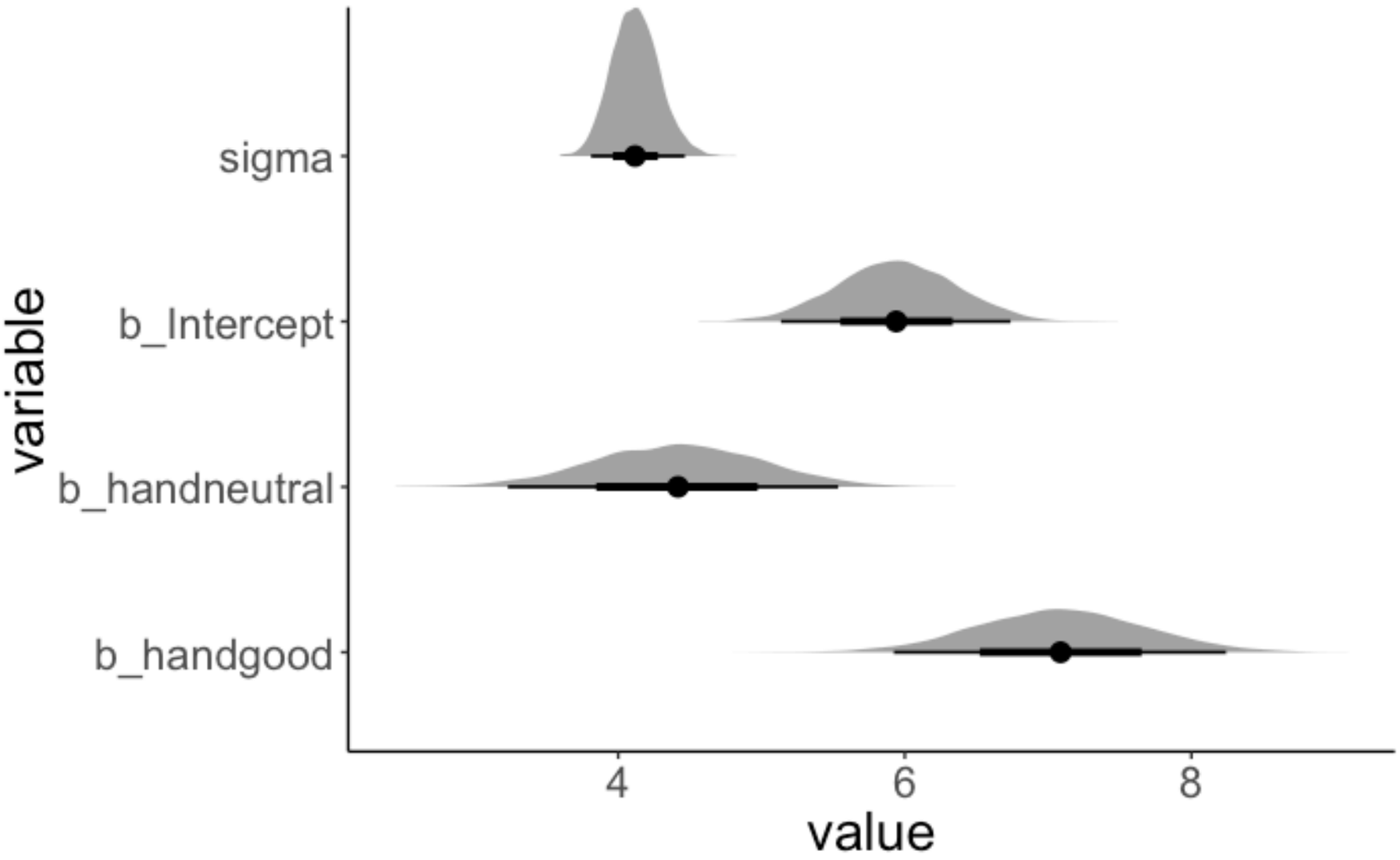
What's the probability that the difference between neutral and bad hands is **more than 4**?

```
1 hypothesis(fit.brm,
2              hypothesis = "handneutral > 4")
```

p = 0.75

Asking questions based on the posterior

Do good hands make twice as much as bad hands?

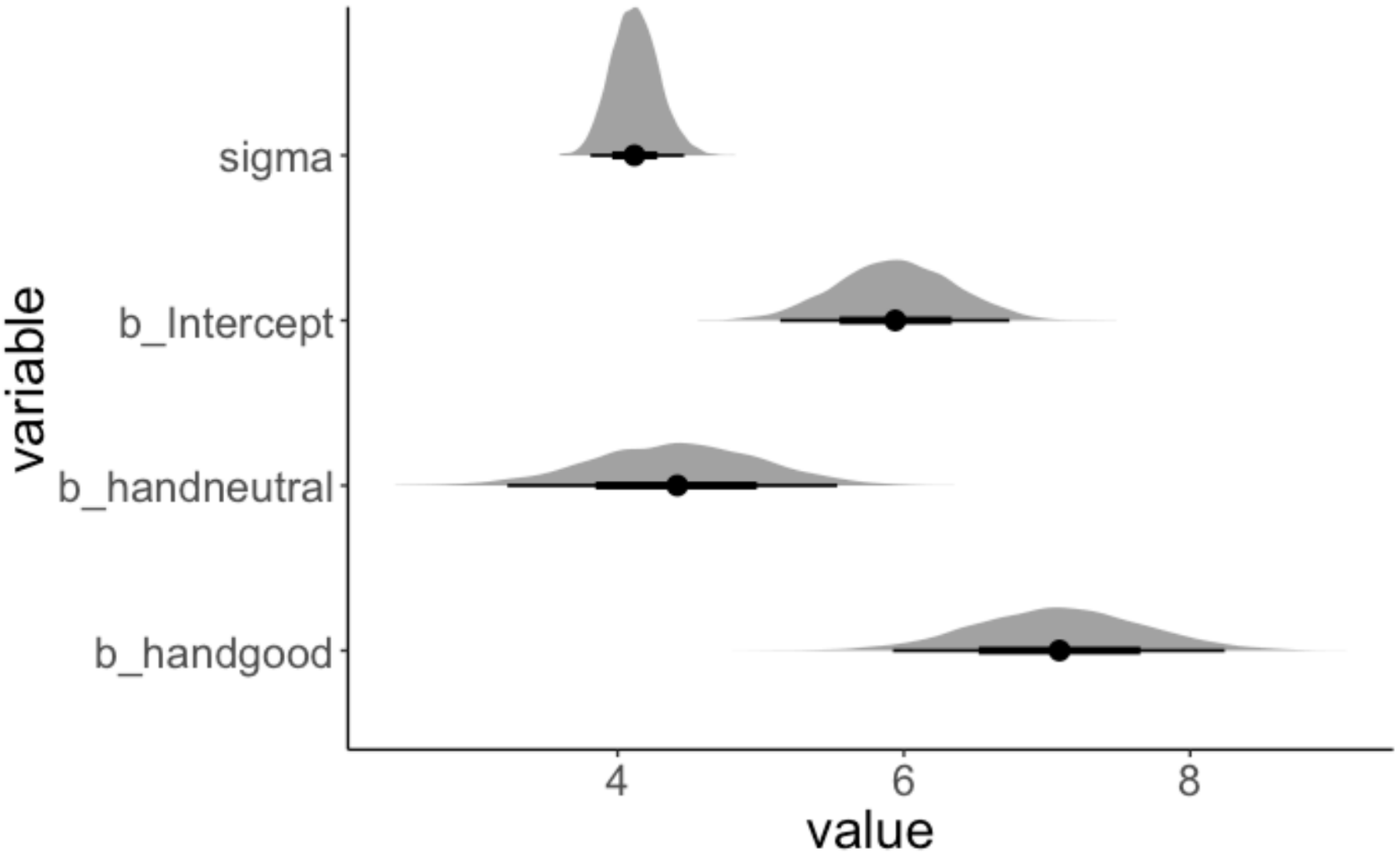



```
1 hypothesis(fit.brm,
2              hypothesis = "handgood + Intercept > 2 * Intercept")
```

p = 0.89

Asking questions based on the posterior

Are neutral hands worse than bad and good hands combined?




```
1 hypothesis(fit.brm,
2              hypothesis = "Intercept + handneutral < (Intercept + Intercept + handgood) / 2")
```

p = 0.04

Testing hypothesis

```
df.hypothesis = fit.brm %>%
  posterior_samples() %>%
  clean_names() %>%
  select(starts_with("b_")) %>%
  mutate(neutral = b_intercept + b_handneutral,
         bad_good_average = (b_intercept + b_intercept + b_handgood)/2,
         hypothesis = neutral < bad_good_average)
```

samples from the posterior

b_intercept	b_handneutral	b_handgood	neutral	bad_good_average	hypothesis
6.07	4.10	7.20	10.17	9.67	FALSE
6.06	4.44	6.95	10.49	9.53	FALSE
5.88	5.00	6.73	10.87	9.24	FALSE
5.85	4.78	6.18	10.63	8.94	FALSE
5.86	4.46	7.68	10.32	9.70	FALSE

```
df.hypothesis %>%
  summarize(p = sum(hypothesis)/n())
```

p = 0.04

Testing hypotheses

Having a posterior distribution allows us to ask questions about the data in a very flexible way!

The "emmeans" package is your friend!

```
1 fit.brm_poker %>%
2   emmeans(specs = consec ~ hand)
```

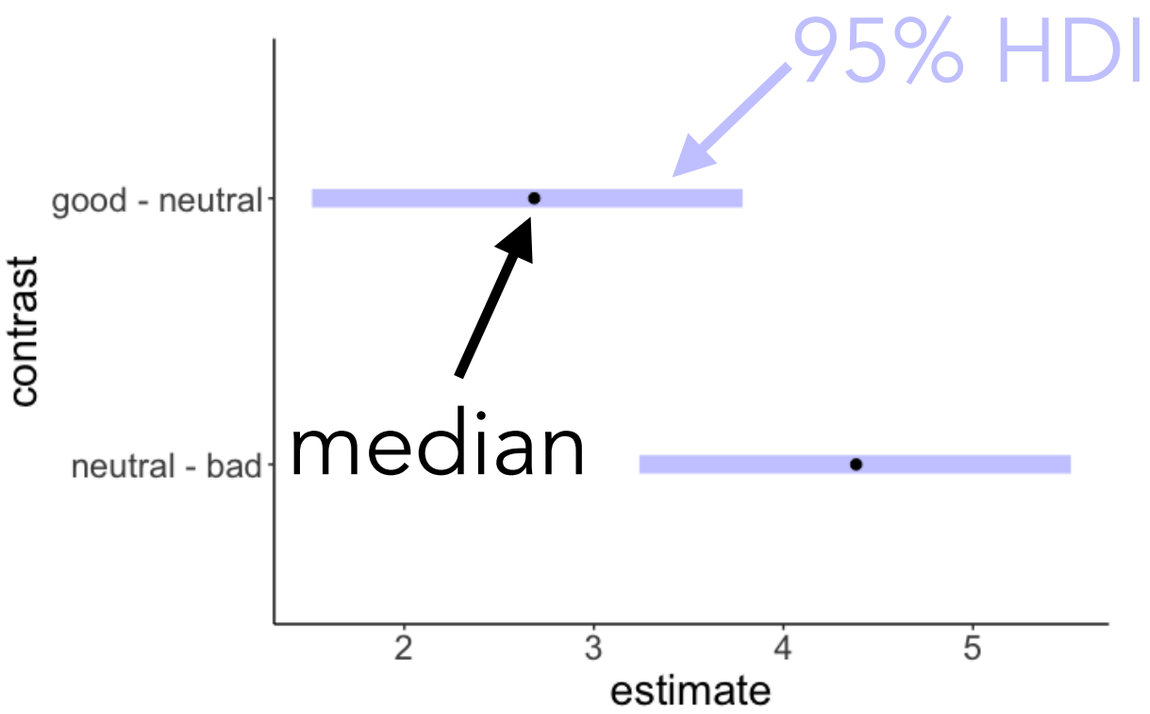
estimated mean for each group

```
$emmeans
 hand    emmean lower.HPD upper.HPD
 bad       5.94      5.16      6.78
 neutral  10.34      9.55     11.15
 good     13.02     12.22     13.82

Point estimate displayed: median
HPD interval probability: 0.95

$contrasts
 contrast       estimate lower.HPD upper.HPD
 neutral - bad      4.38      3.24      5.52
 good - neutral     2.69      1.51      3.78

Point estimate displayed: median
HPD interval probability: 0.95
```

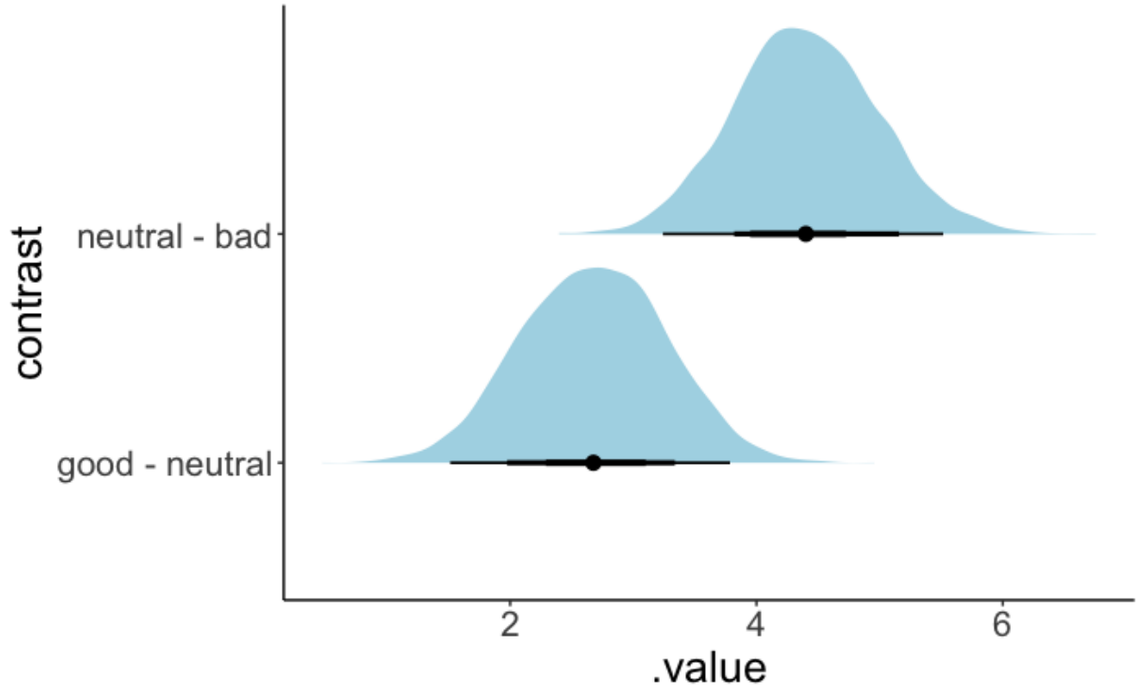
contrasts

Visualizing the contrasts

```
1 fit.brm_poker %>%
2   emmeans(specs = consec ~ hand) %>%
3   pluck("contrasts") %>%
4   plot()
```




```
1 fit.brm_poker %>%
2   emmeans(specs = consec ~ hand) %>%
3   pluck("contrasts") %>%
4   gather_emmeans_draws() %>%
5   ggplot(mapping = aes(y = contrast,
6                        x = .value)) +
7   stat_halfeye(fill = "lightblue",
8                point_interval = mean_hdi,
9                .width = c(0.5, 0.75, 0.95))
```



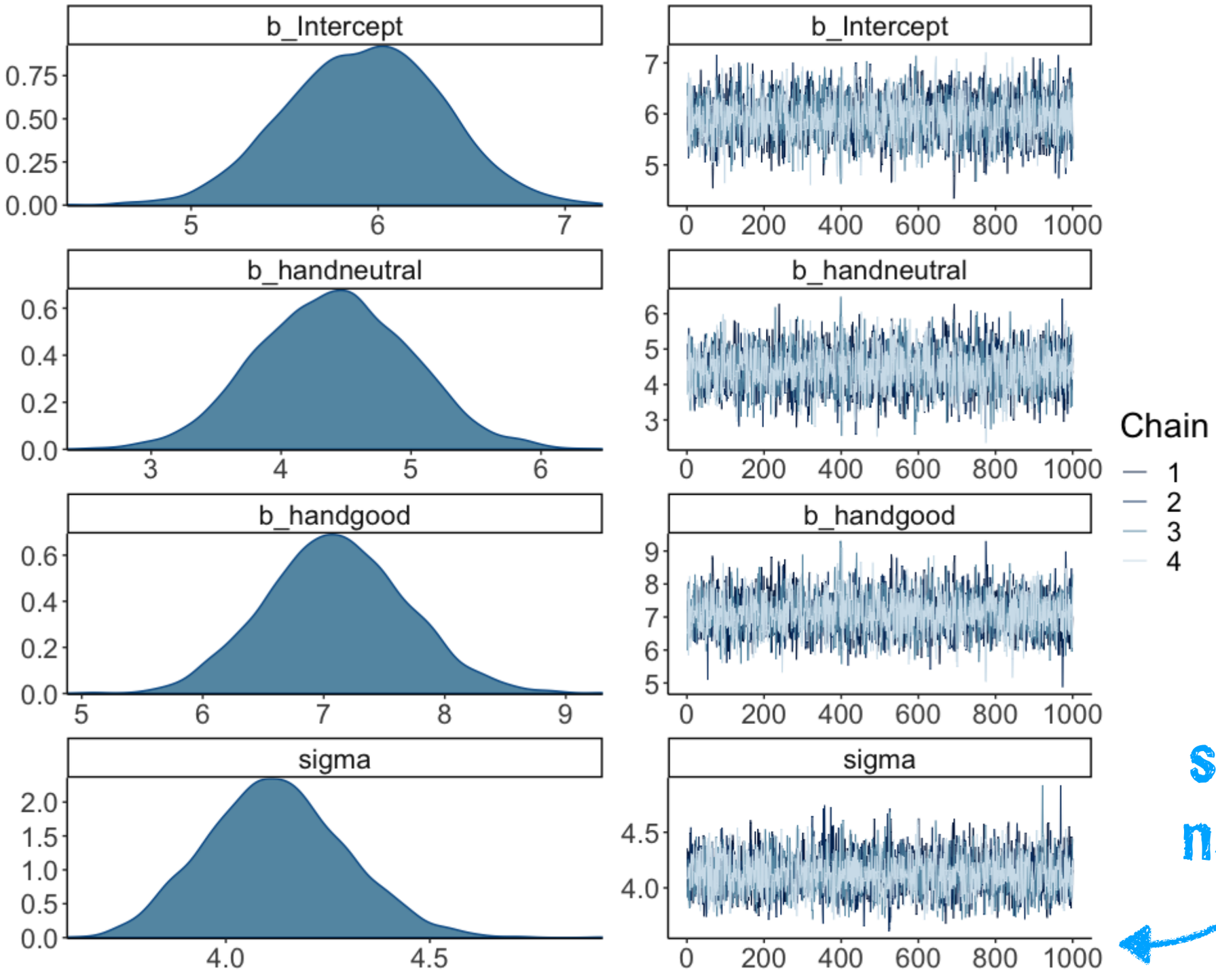
mean, 50% HDI, 75% HDI, 95% HDI

Model evaluation

1. Check whether inference worked

Can we trust the model results?

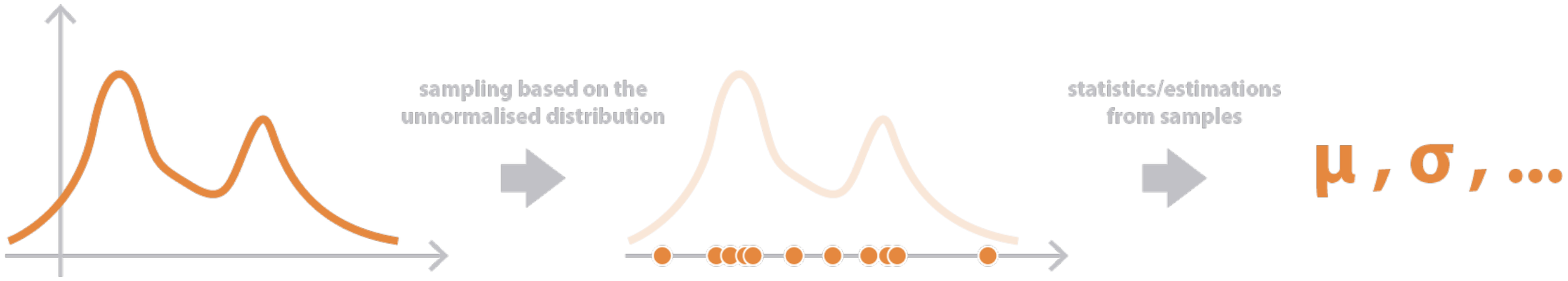
```
plot(fit.brm_poker)
```



https://chi-feng.github.io/mcmc-demo/

Can we trust the model results?

Inference via Markov Chain Monte Carlo (MCMC)

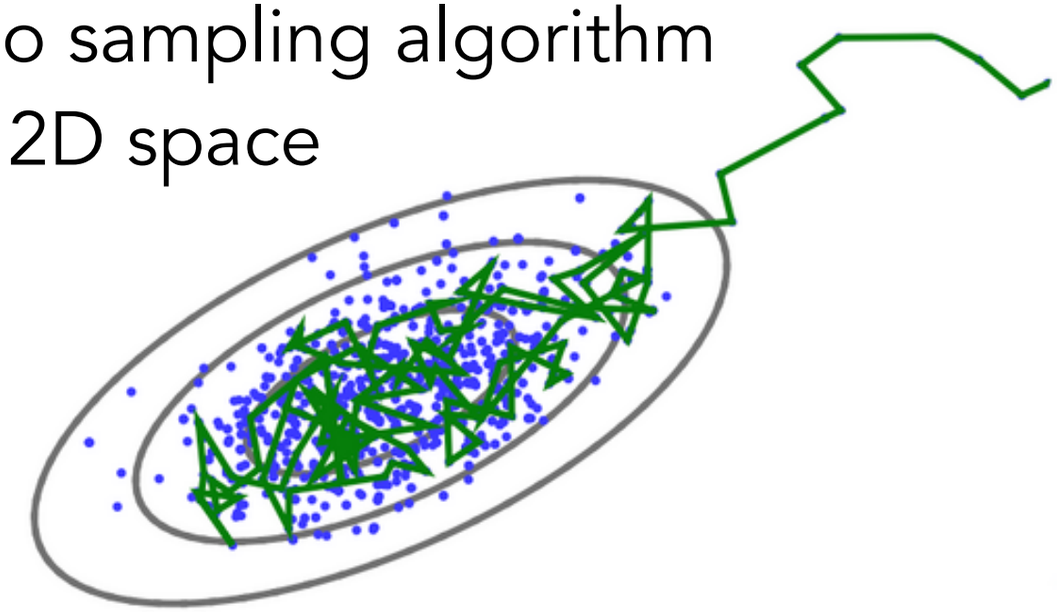


Unnormalised distribution
whose normalisation factor computation is intractable

Samples
that can be obtained with MCMC and without proceeding to the normalisation

Statistics or estimations
that can be computed based on the generated samples

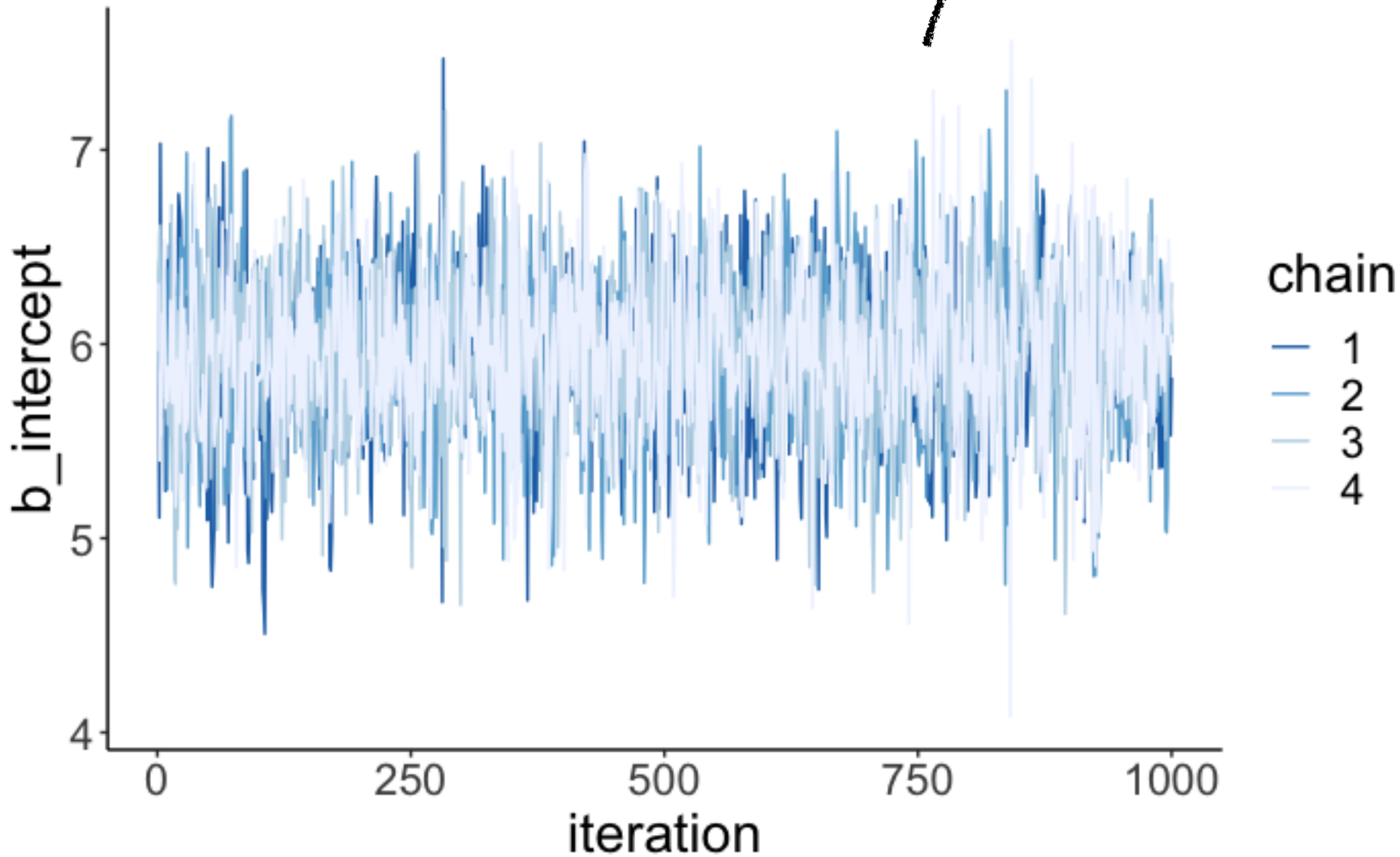
Markov Chain Monte Carlo sampling algorithm in a 2D space



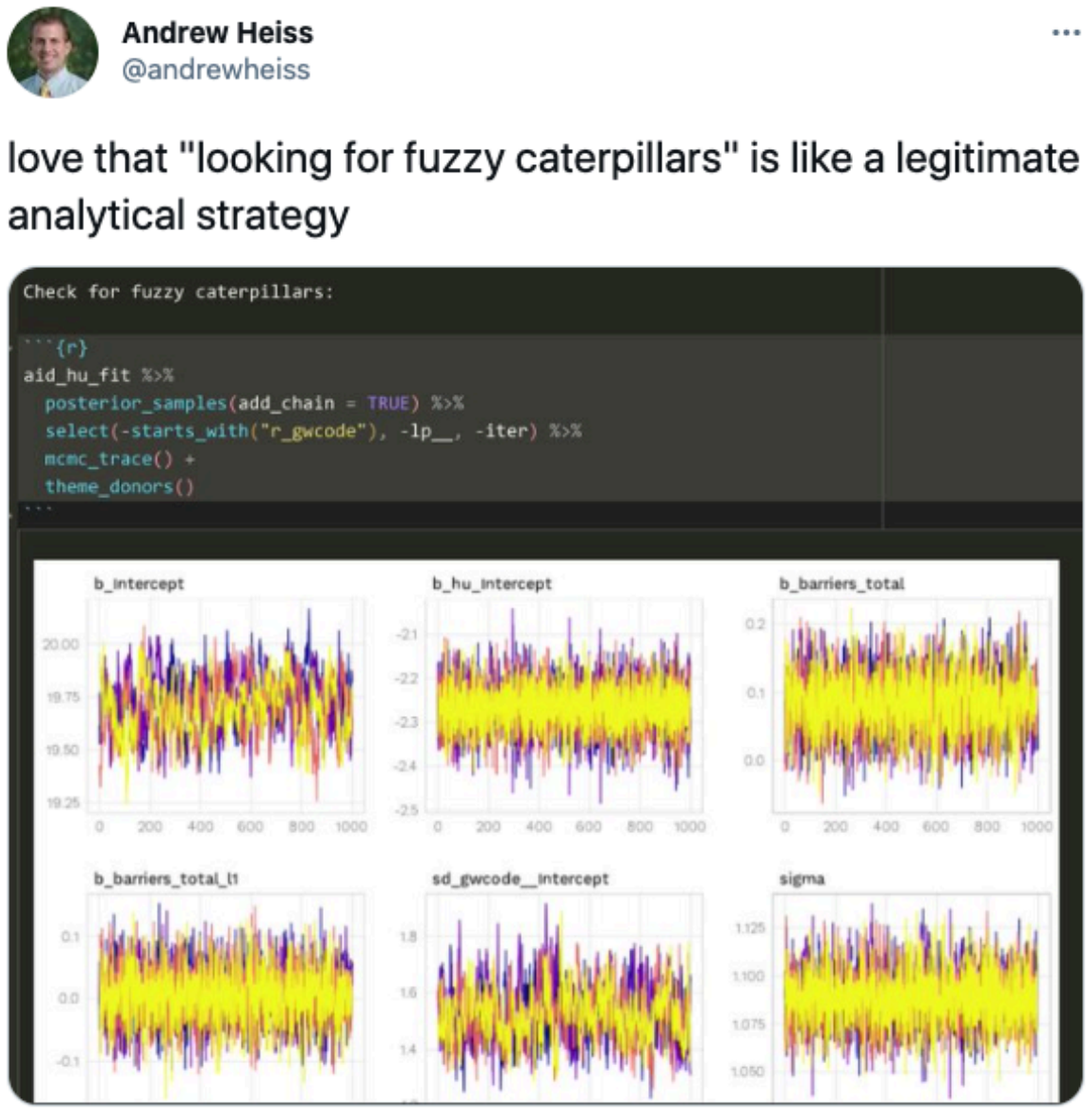
goal: draw **independent** samples from the posterior distribution

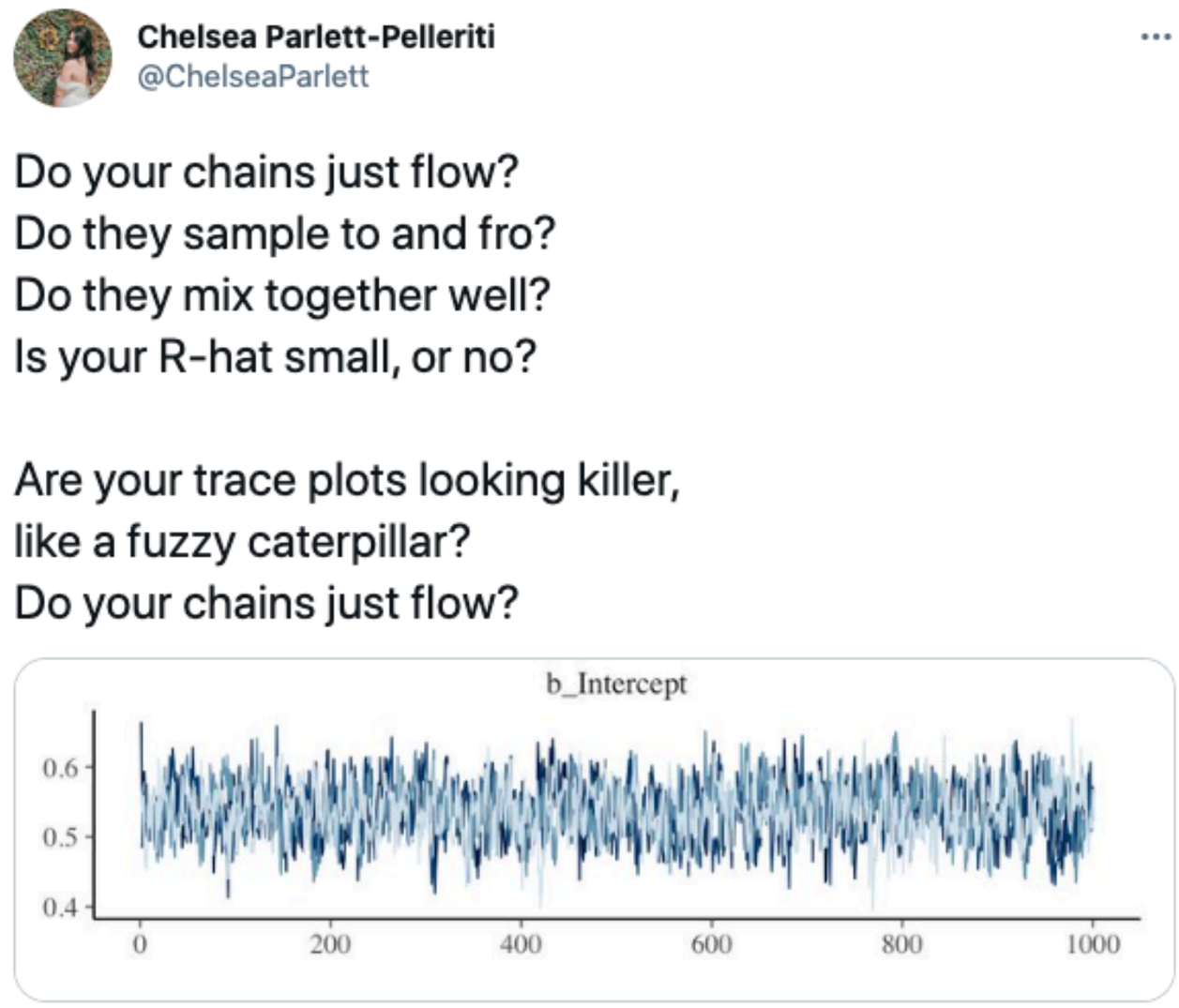
Can we trust the model results?

looks like a fuzzy caterpillar



Stats twitter chimes in ...





When things don't work out

only two data points!

incredibly wide uniform priors

10000000000

```
df.data = tibble(y = c(-1, 1))

fit.brm_wrong = brm(data = df.data,
                    family = gaussian,
                    formula = y ~ 1,
                    prior = c(prior(uniform(-1e10, 1e10), class = Intercept),
                              prior(uniform(0, 1e10), class = sigma)),
                    inits = list(list(Intercept = 0, sigma = 1),
                                 list(Intercept = 0, sigma = 1)),
                    iter = 4000,
                    warmup = 1000,
                    chains = 2,
                    file = "cache/brm_wrong")
```

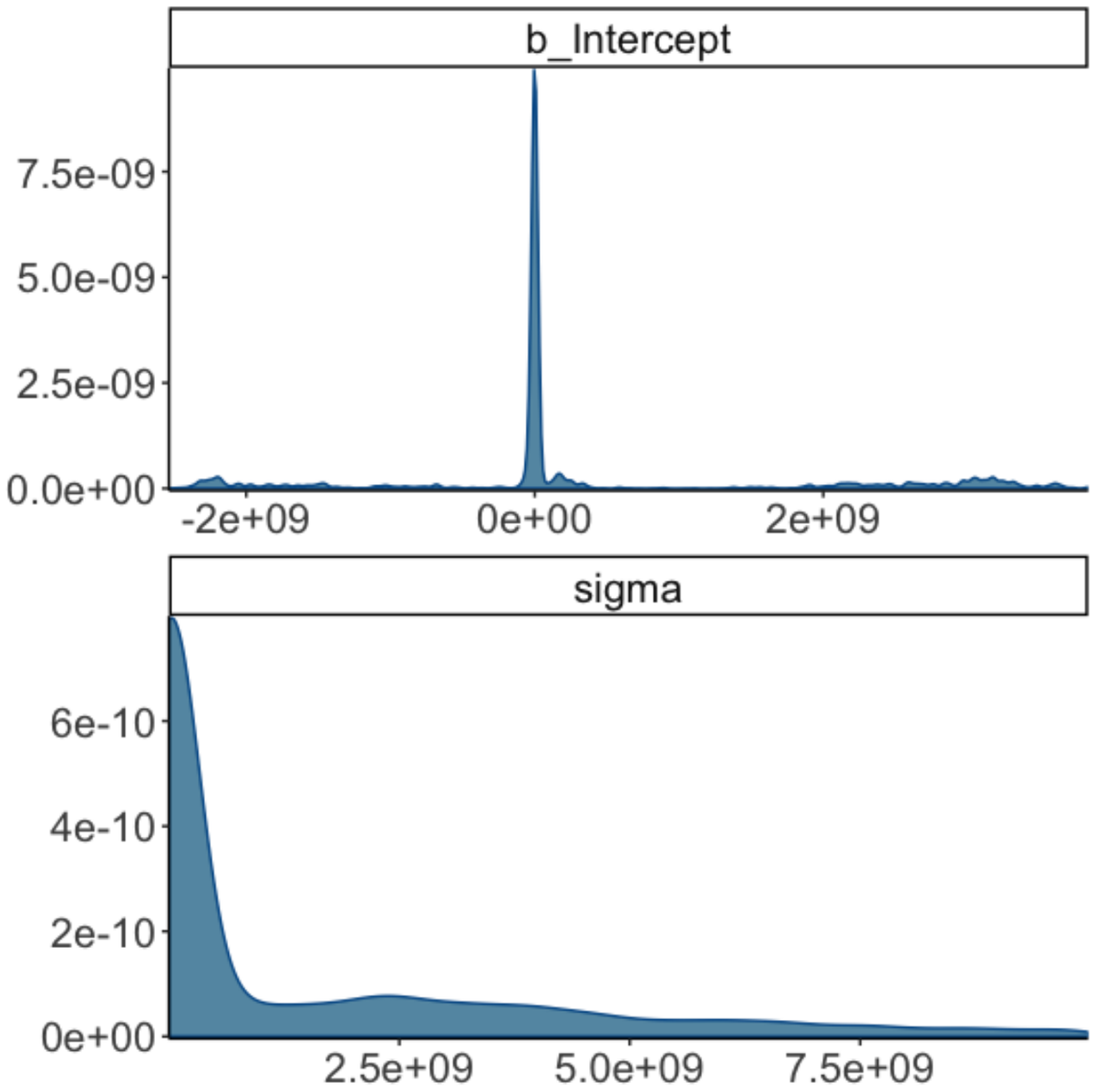
When things don't work out

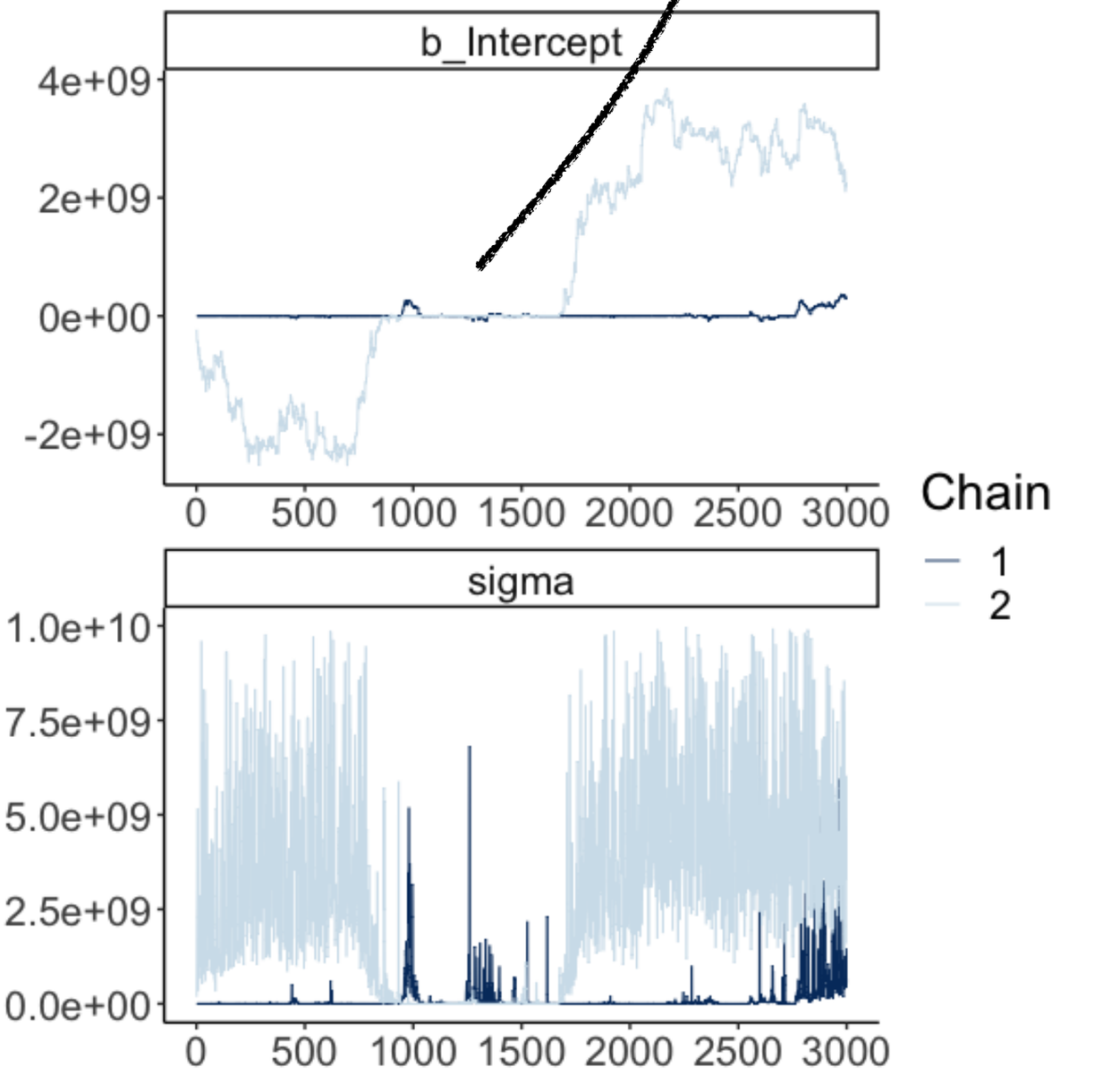
```
summary(fit.brm_wrong)
```

```
The model has not converged (some Rhats are > 1.1). Do not analyse the results!
We recommend running more iterations and/or setting stronger priors.There were 1203
divergent transitions after warmup. Increasing adapt_delta above 0.8 may help.
See http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup Family:
gaussian
  Links: mu = identity; sigma = identity
Formula: y ~ 1
   Data: df.data (Number of observations: 2)
Samples: 2 chains, each with iter = 4000; warmup = 1000; thin = 1;
         total post-warmup samples = 6000

Population-Level Effects:
              Estimate     Est.Error       l-95% CI      u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept 357550121.58 1416057299.71 -2244033111.47 3333594132.43 1.78        3       24

Family Specific Parameters:
            Estimate     Est.Error l-95% CI      u-95% CI Rhat Bulk_ESS Tail_ESS
sigma 1524412740.64 2392424321.98 21668.93 8317582240.06 1.40        4       41

Samples were drawn using sampling(NUTS). For each parameter, Eff.Sample
is a crude measure of effective sample size, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
```

When things don't work out

doesn't look like a fuzzy caterpillar





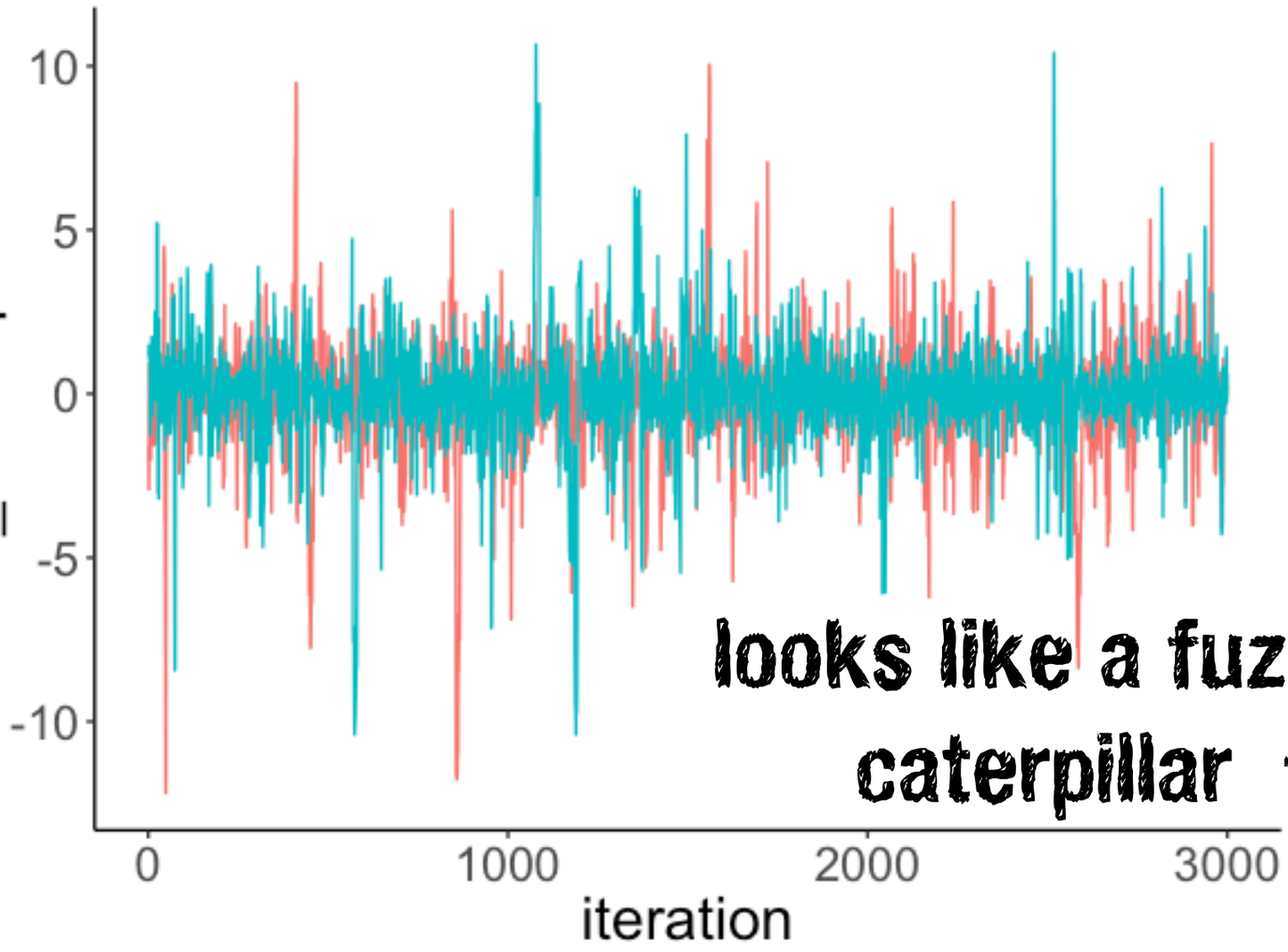
Having somewhat informative priors fixes things

```
fit.brm_right = brm(data = df.data,
                    family = gaussian,
                    formula = y ~ 1,
                    prior = c(prior(normal(0, 10), class = Intercept), # more reasonable priors
                              prior(cauchy(0, 1), class = sigma)),
                    iter = 4000,
                    warmup = 1000,
                    chains = 2,
                    seed = 1,
                    file = "cache/brm_right")
```

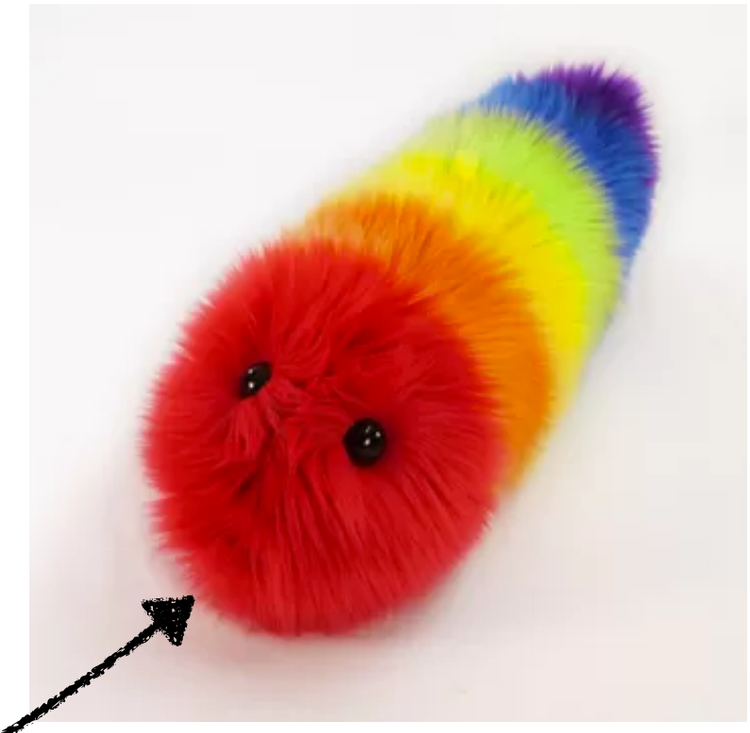
more reasonable priors

```
Family: gaussian
  Links: mu = identity; sigma = identity
Formula: y ~ 1
   Data: list(y = c(-1, 1)) (Number of observations: 2)
Samples: 2 chains, each with iter = 4000; warmup = 1000; thin = 1;
         total post-warmup samples = 6000

Population-Level Effects:
          Estimate Est.Error l-95% CI u-95% CI Eff.Sample Rhat
Intercept    -0.06      1.72    -3.78     3.27       1033 1.00

Family Specific Parameters:
      Estimate Est.Error l-95% CI u-95% CI Eff.Sample Rhat
sigma     2.21      6.99     0.61     6.92       1006 1.00

Samples were drawn using sampling(NUTS). For each parameter, Eff.Sample
is a crude measure of effective sample size, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
```

Having somewhat informative priors fixes things



looks like a fuzzy caterpillar



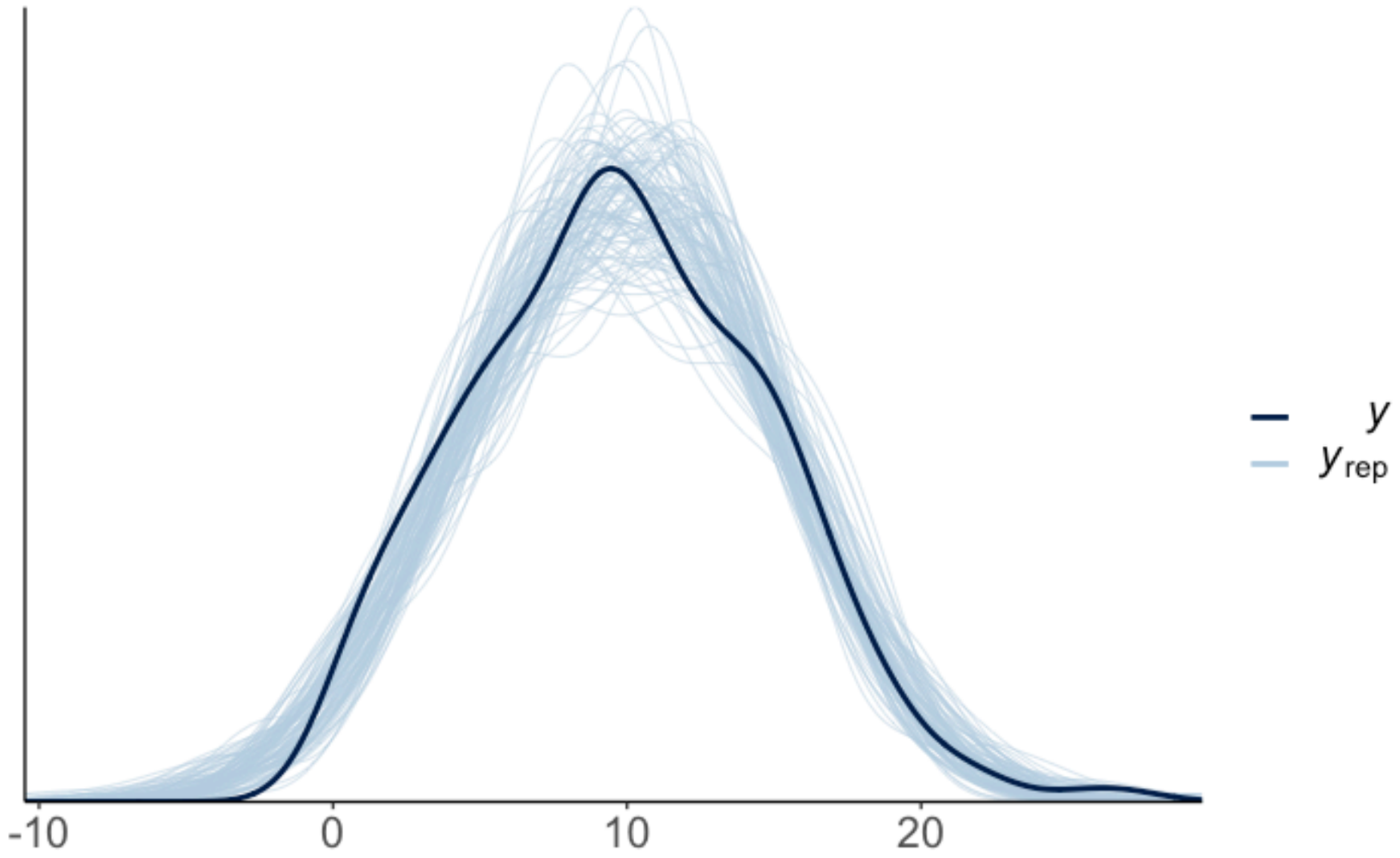
if things go wrong:

- set more informative priors
- run more warm-up samples
- adjust the sampling algorithm as suggested via the `control` argument

2. Visualize model predictions

Posterior predictive check

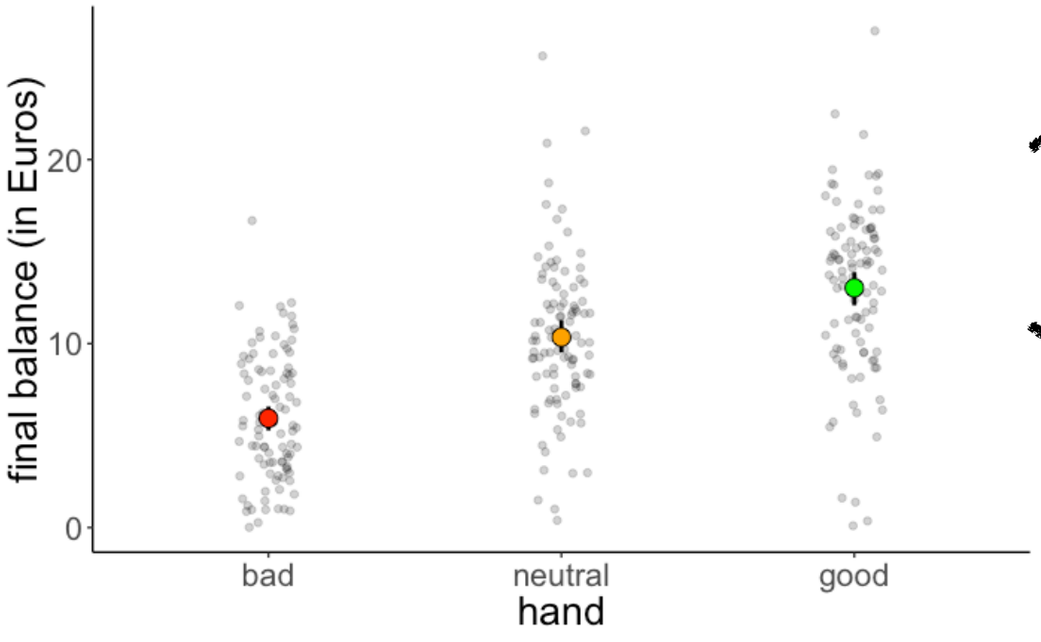
```
pp_check(fit.brm, nsamples = 100)
```



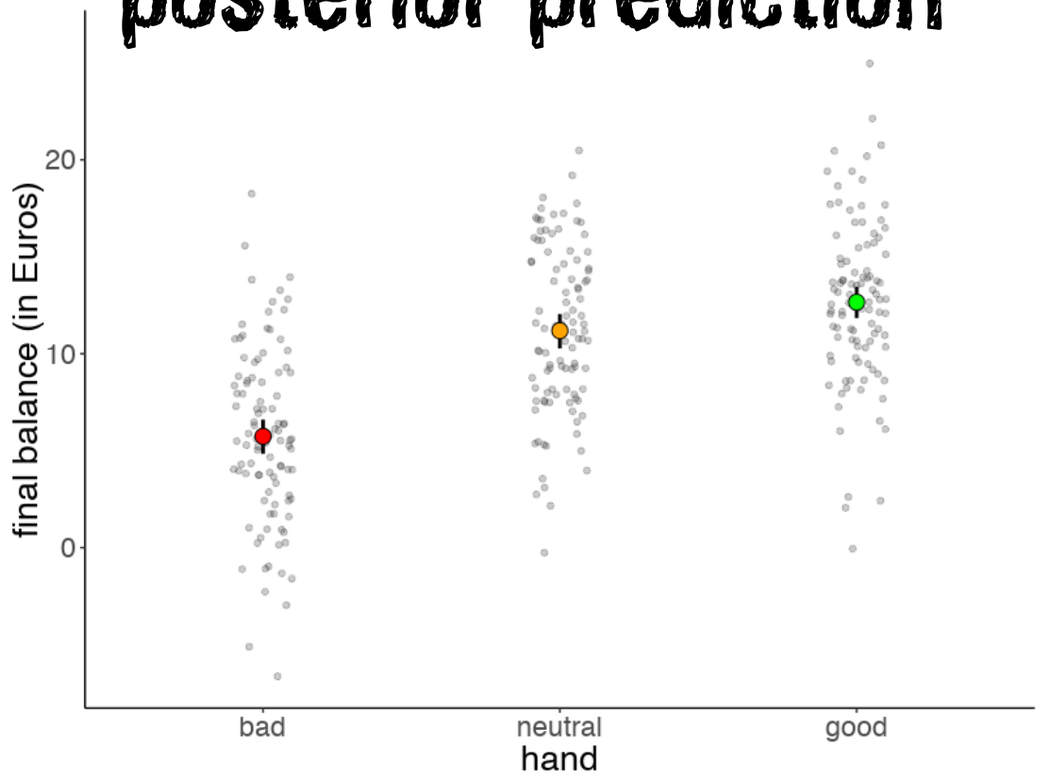
The model accurately captures the distribution of the response variable

Posterior predictive check

original data

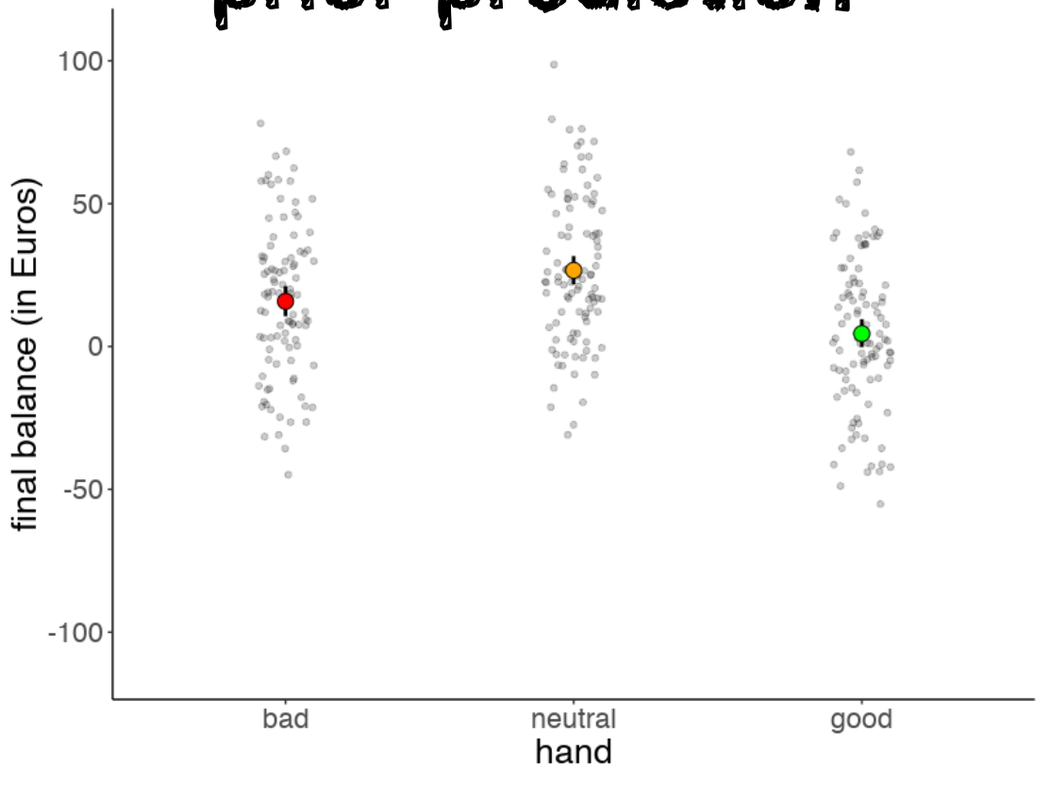


posterior prediction



take a look at course notes for how to make these

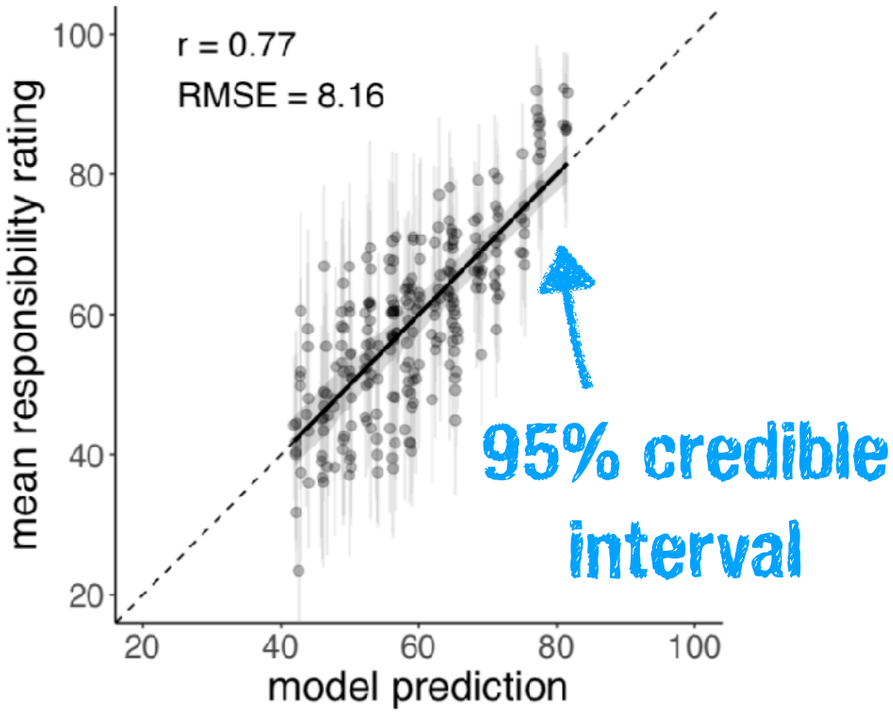
prior prediction



Reporting results

Reporting results

Plots



Tables

Table 1
Estimates of the mean, standard error, and 95% HDIs of the different predictors in the Bayesian mixed effects model. Note: `n_causes` *= number of causes.*

`responsibility ~ 1 + surprise + pivotality + n_causes + (1 + surprise + pivotality + n_causes | participant)`

term	estimate	std.error	lower 95% HDI	upper 95% HDI
intercept	59.94	3.25	54.70	65.22
surprise	21.68	4.57	14.17	29.23
pivotality	13.52	1.82	10.47	16.53
n_causes	-5.72	0.50	-6.55	-4.90

model formula

parameter estimates

Text

We computed a Bayesian mixed effects model with random intercepts and slopes to predict participants' responsibility judgments (see Table 1). Figure 6b shows a scatter plot of the model predictions and participants' responsibility judgments for the full set of 170 scenarios (with 250 judgments). Overall, the model predicts participants' responsibility judgments well with r = .77 and RMSE = 8.16. Table 1 shows the estimates of the different predictors. As can be seen, none of the predictors' 95% HDIs overlap with 0.[1]

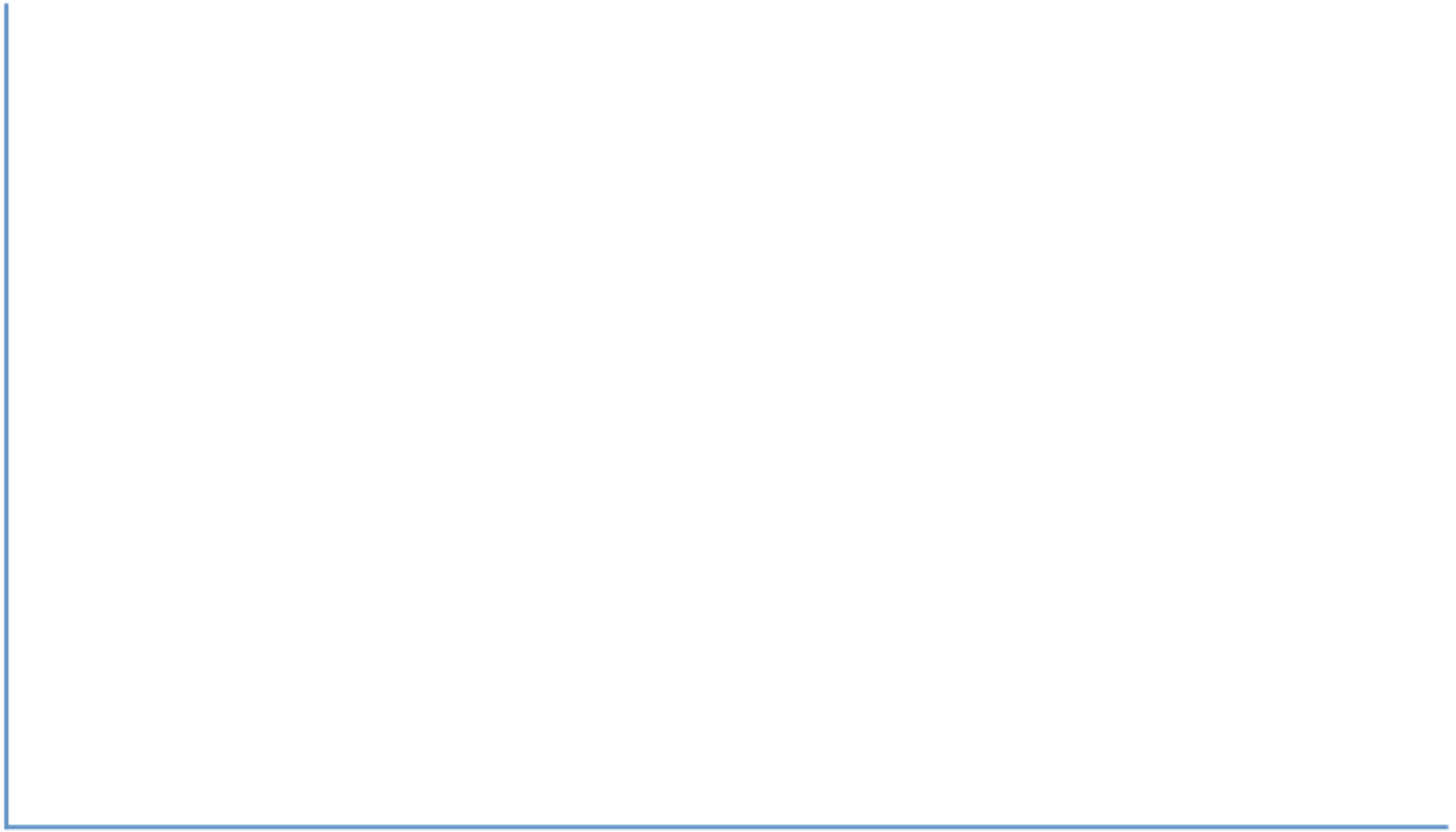
[1]For any statistical claim, we report the mean of the posterior distribution together with the 95% highest-density interval (HDI). All Bayesian models were written in Stan (Carpenter et al., 2017) and accessed with the brms package (Bürkner, 2017) in R (R Core Team, 2019).

Summary

- Quick recap
- Doing Bayesian data analysis with greta
- Doing Bayesian data analysis **with BRMS**
  - Testing hypotheses
  - Model evaluation

Feedback

How was the pace of today's class?



How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?

Thank you!