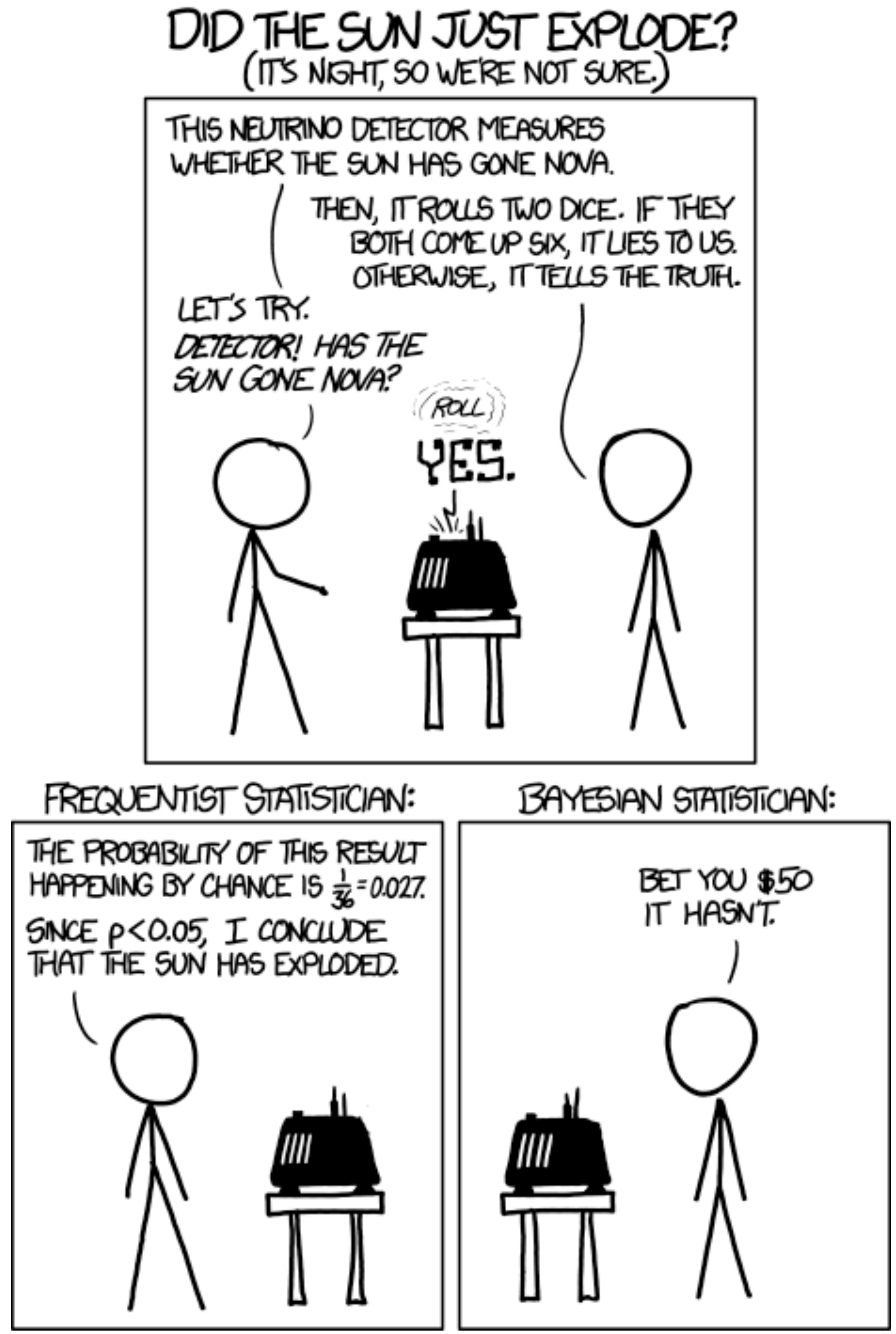
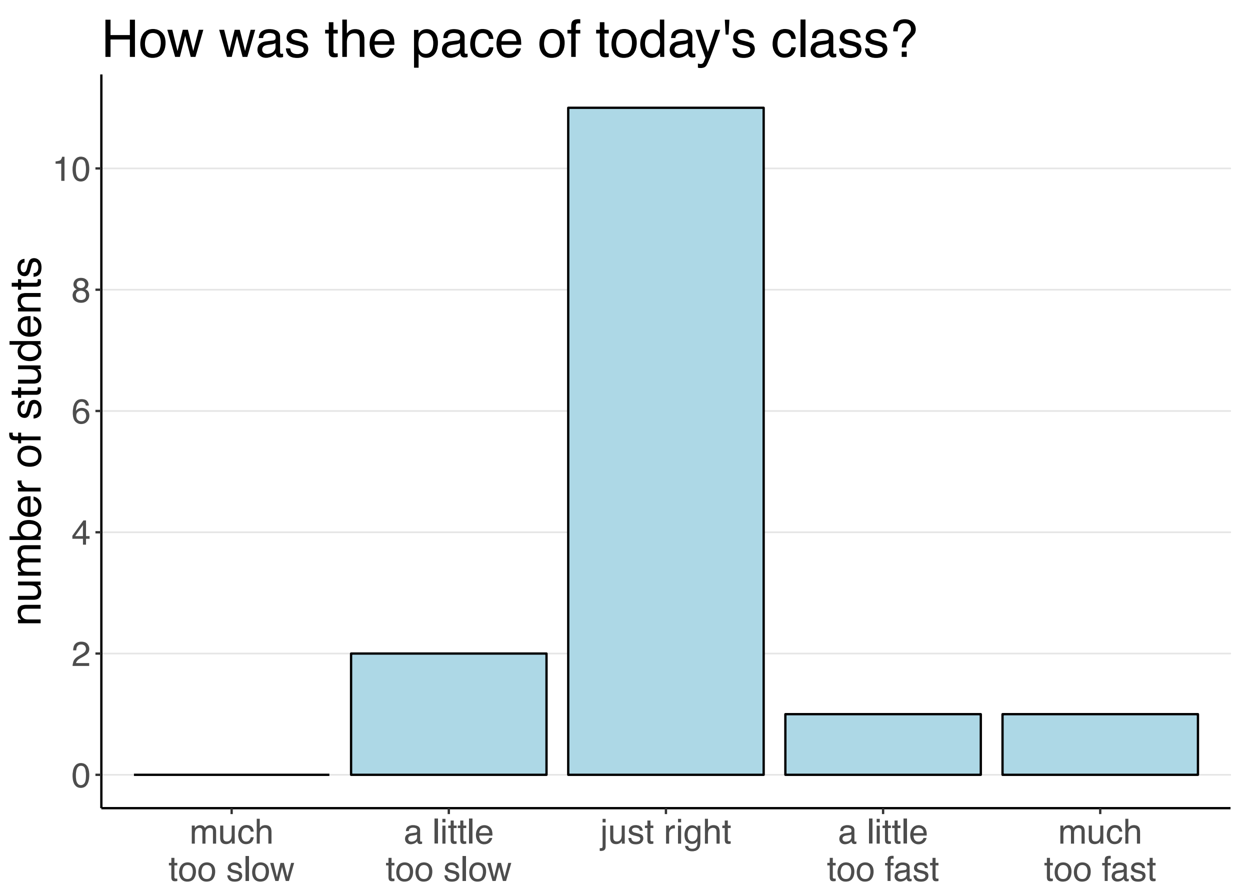
Bayesian data analysis 1

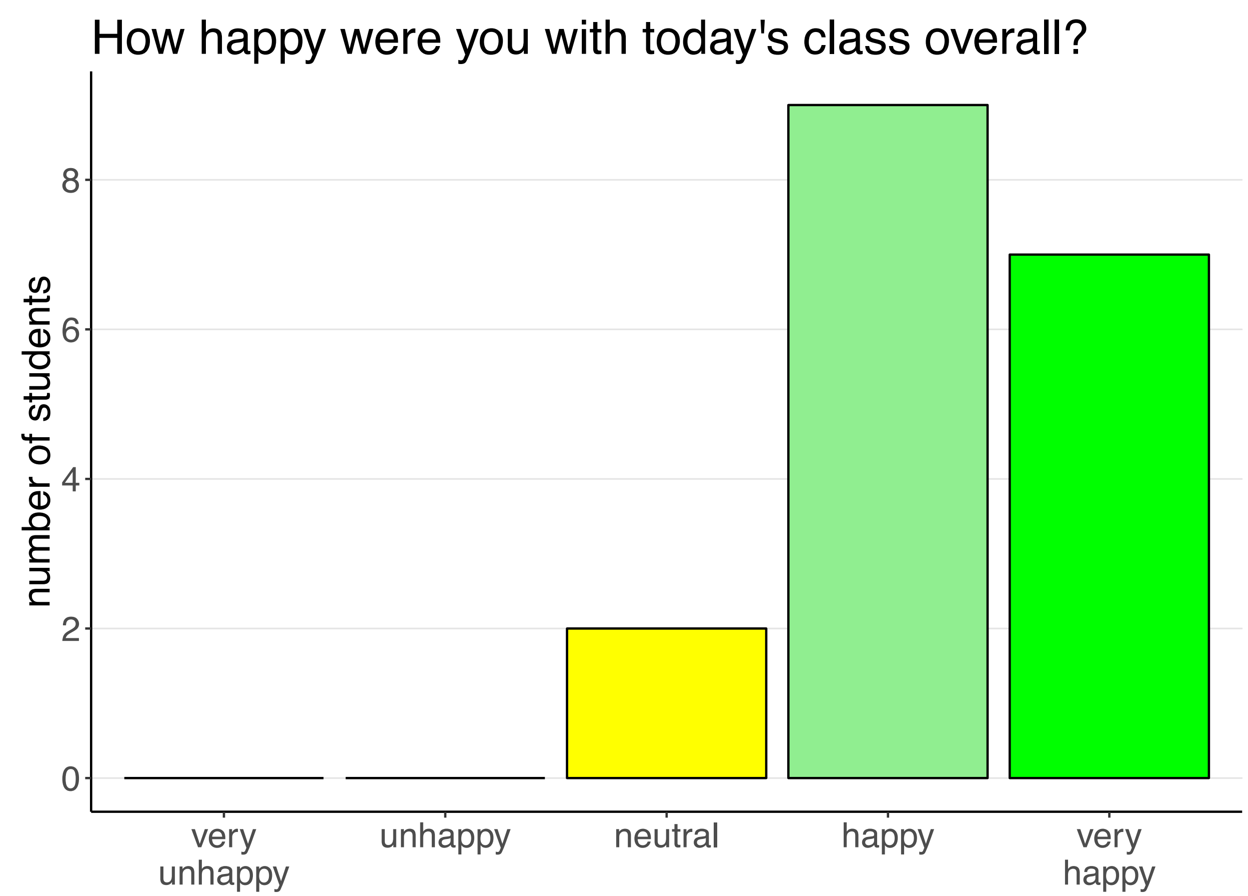


03/04/2019

Your feedback

Your feedback





Homework 5

this looks much better ...

Things that came up

Interactions in random effects?!

Mixed Model Analyses with Interactions in the Random Effects Structure

Ask Question

2

2

I'm new to mixed models, and I'm having trouble trying to figure out what the random structure should be for a specific example.

I'm using a shooter bias task (the paradigm where subjects are faster to shoot an armed black target) and I am interested in a 3-way interaction between Target Race (Black/White), Target Object(Gun/NoGun), Inhibitory Ability (continuous variable where higher numbers are better inhibitory ability).

DV = Reaction time

Fixed Factor 1 = Target Race (dichotomous variable; repeated measure) Fixed Factor 2 = Target Object (dichotomous variable; repeated measure) Fixed Factor 3= Inhibitory Ability (continuous variable)

Random Factor = SubjectID

I am interested in the 3-way interaction, and while I understand the fixed factor model, specifying the random factor model has never made sense to me.

Assuming, I'll be using LME for R...

Reaction Time ~ Target Race + Target Object + Inhibitory ability+ all two-way interaction terms + three-way interaction term + ...random model....?

would the random model be something like (Target Race:Target Object:Inhibitory Ability | SubjectID)? I really don't get how to specify the random model. So, any help would be appreciated. Including readings on how to specify the random effects structure would be appreciated.

random-effects-model mixed-model lme4-nlme

asked 3 years, 4 months ago

viewed 1,641 times

active 3 years, 4 months ago

Linked

2 Random effects structure lme

Related

9 Questions about specifying linear mixed models in R for repeated measures data with additional nesting structure

1 A mixture of mixed-model regression and fixed-effects-only regression: what is it exactly?

4 How would you model this random effects structure?

0 Mixed effects model: model fitting vs conceptual sense

1 Interactions between random effects in mixed models

0 Mixed model with clustered and repeated-measures data

```
reaction_time ~ 1 + target_race * target_object *
                inhibitory_ability + (1 + target_race *
                target_object | subject)
```

"effects" package

```r
library("effects")

# fit
fit.glm = glm(formula = survived ~ 1 + fare + sex,
              data = df.titanic,
              family = "binomial")

# report effects
allEffects(mod = fit.glm)
```

calculate all effects

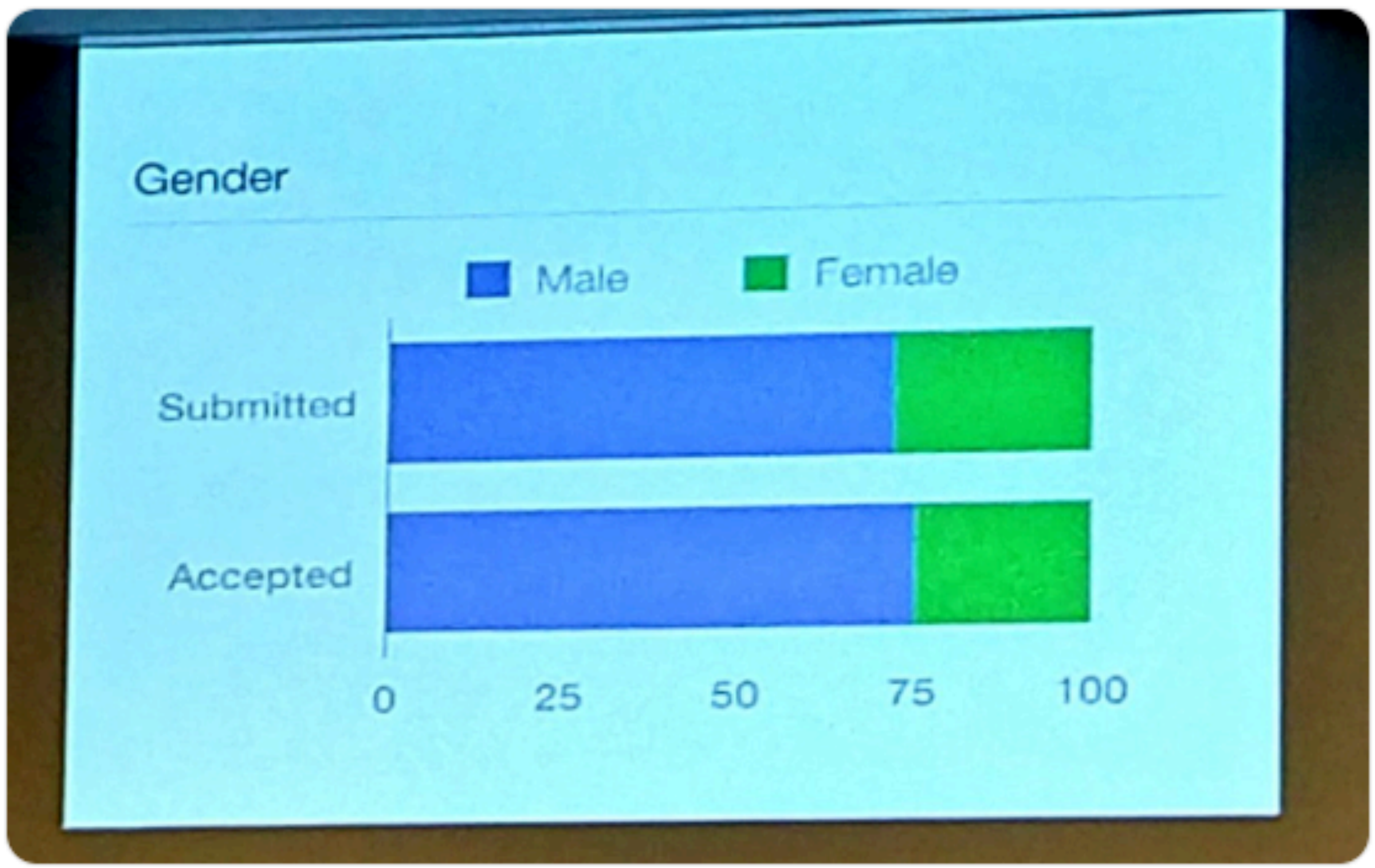
```
 model: survived ~ 1 + fare + sex

 fare effect
fare
        0       100       300       400       500 
0.2845816 0.5497251 0.9200021 0.9724491 0.9908535 

 sex effect
sex
   female      male 
0.7326757 0.1955222 
```

Bias in conference admission?

Adam J Calhoun
@neuroecology

Follow

Accepted and submitted abstracts by gender roughly the same at #cosyne19 cc @neimarkgeffen @TrackingActions



10:29 AM - 28 Feb 2019

Bias in conference admission?

Adam J Calhoun @neuroecology · Feb 28

Accepted and submitted abstracts by gender roughly the same at #cosyne19 cc @neimarkgeffen @TrackingActions

4 10

Yael Niv
@yael_niv

Follow

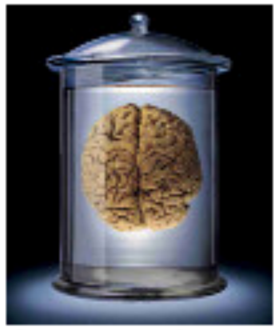
Replying to @neuroecology @neimarkgeffen @TrackingActions

Doesn't look the same to me...

10:55 AM - 2 Mar 2019

Bias in conference admission?

COSYNE



Cosyne 19
Meeting program

Computational and Systems Neuroscience (Cosyne) 2019

Main meeting: 28 February-03 March 2019 in Lisbon, Portugal

Download Cosyne 2019 Main Meeting Program

Workshops: 04 March-05 March 2019 in Cascais, Portugal

Download Cosyne 2019 Workshops Program

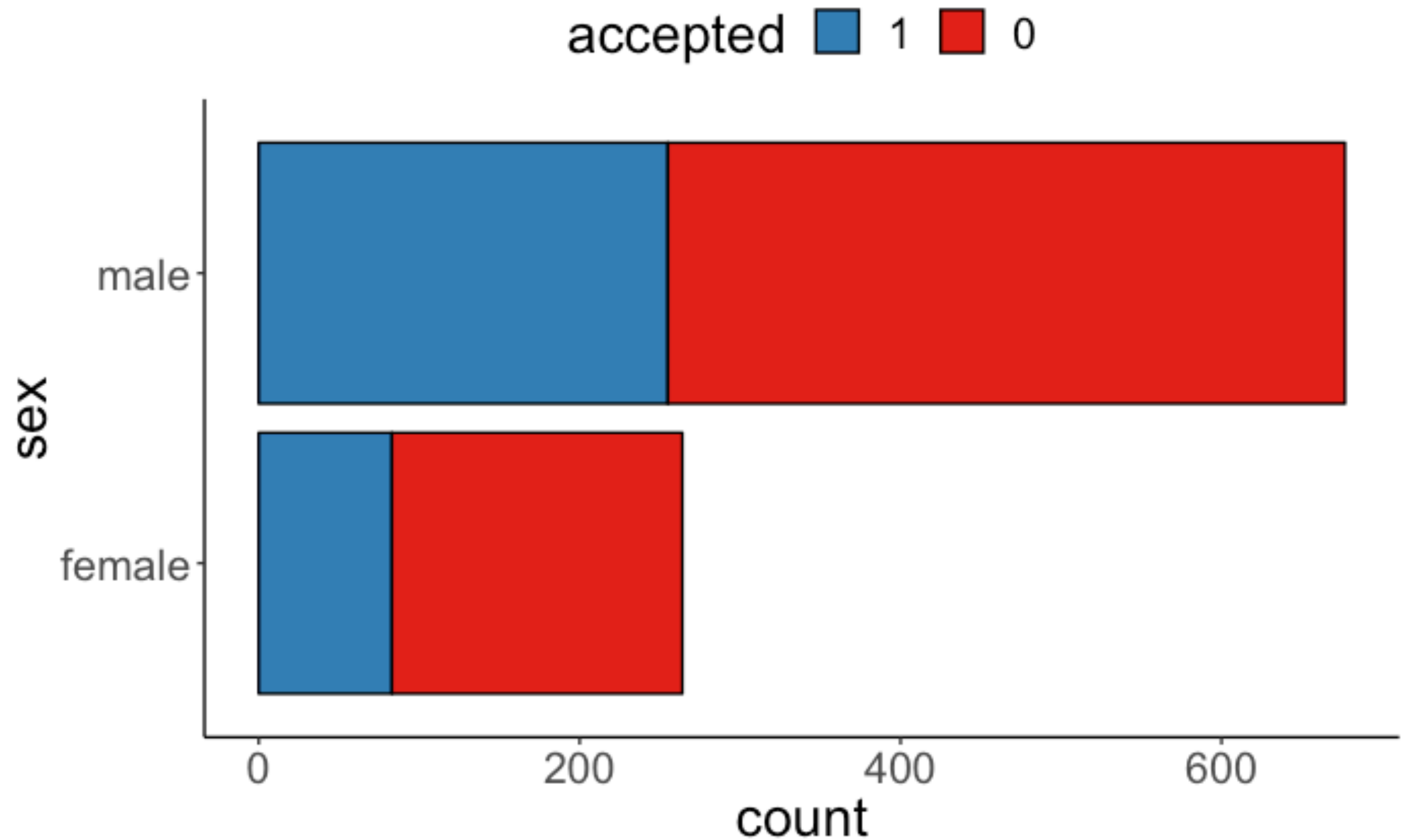
About Cosyne

The annual Cosyne meeting provides an inclusive forum for the exchange of experimental and theoretical/computational approaches to problems in systems neuroscience.

To encourage interdisciplinary interactions, the main meeting is arranged in a single track. A set of invited talks are selected by the Executive Committee and Organizing Committee, and additional talks and posters are selected by the Program Committee, based on submitted abstracts.

Cosyne topics include (but are not limited to): neural coding, natural scene statistics, dendritic computation, neural basis of persistent activity, nonlinear receptive field mapping, representations of time and sequence, reward systems, decision-making, synaptic plasticity, map formation and plasticity, population coding, attention, and computation with spiking networks. Participants include pure experimentalists, pure theorists, and everything in between.

Bias in conference admission?



different representation of the data

Bias in conference admission?

```r
# logistic regression
fit.glm = glm(formula = accepted ~ 1 + sex,
              family = "binomial",
              data = df.conference)

# model summary
fit.glm %>%
  summary()
```

```
Call:
glm(formula = accepted ~ 1 + sex, family = "binomial", data = df.conference)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.9723  -0.9723  -0.8689   1.3974   1.5213

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  -0.7797     0.1326  -5.881 4.07e-09 ***
sexmale       0.2759     0.1545   1.786   0.0741 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1228.9  on 940  degrees of freedom
Residual deviance: 1225.6  on 939  degrees of freedom
AIC: 1229.6

Number of Fisher Scoring iterations: 4
```

Bias in conference admission?

Megan Carey @meganinlisbon · Mar 2

I presented the math for this at the #cosyne19 diversity lunch today.

Success rates for first authors with known gender:
Female: 83/264 accepted = 31.4%
Male: 255/677 accepted = 37.7%

37.7/31.4 = a 20% higher success rate for men

Adam J Calhoun @neuroecology

Accepted and submitted abstracts by gender roughly the same at #cosyne19 cc @neimarkgeffen @TrackingActions

Show this thread

9 37 83

Mehrdad Jazayeri @mjaztwit Following

Replying to @meganinlisbon

That's a really large difference. It's seems like this year we really messed up as a community. What's the distribution of difference under the null (if you do the same analysis but shuffle the gender labels)?

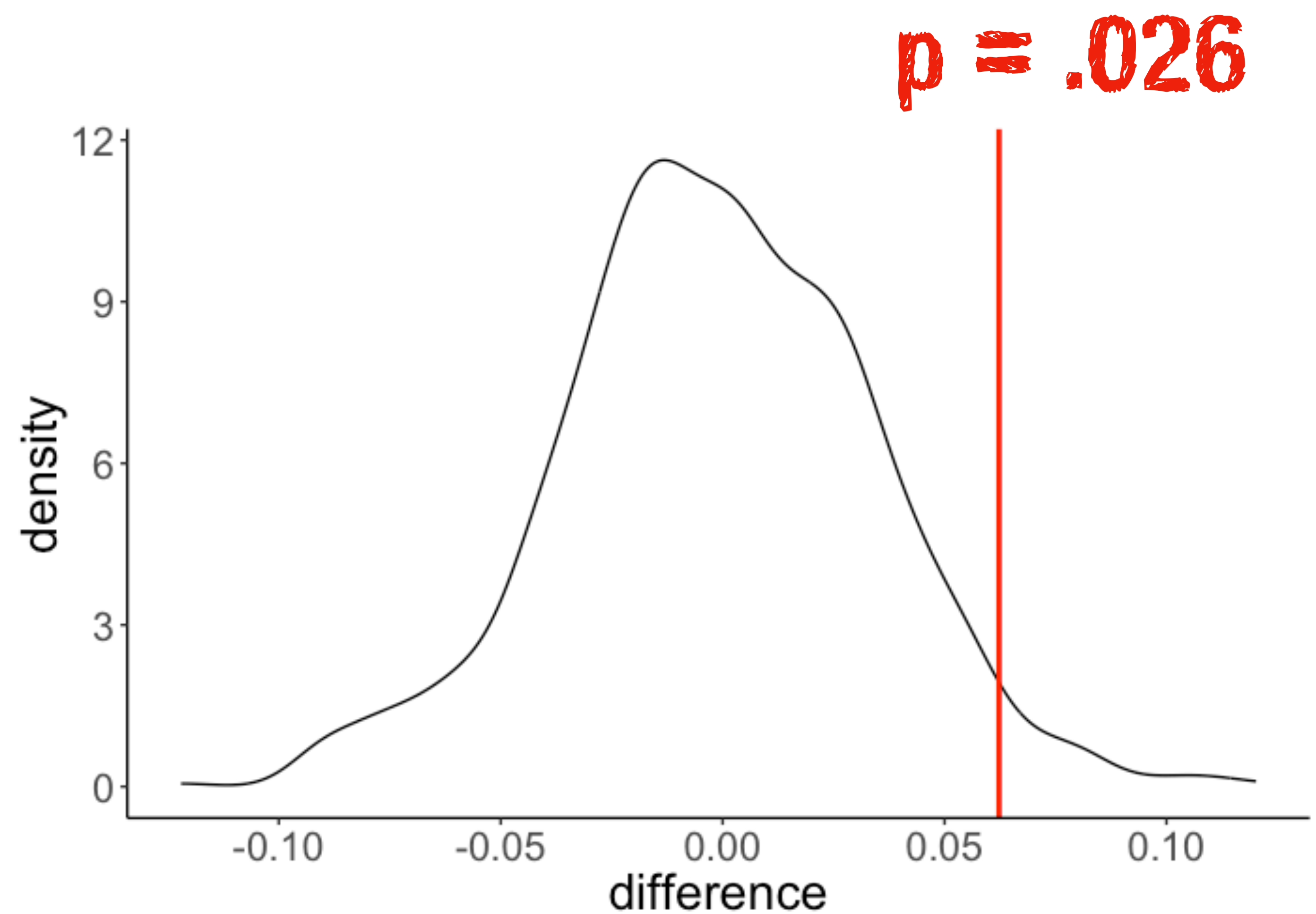
8:06 AM - 2 Mar 2019

8 Likes

permutation test!

Bias in conference admission?

```r
# difference in proportion
fun.difference = function(df){
  df %>%
    as_tibble() %>%
    count(sex, accepted) %>%
    group_by(sex) %>%
    mutate(proportion = n / sum(n)) %>%
    filter(accepted == 1) %>%
    select(sex, proportion) %>%
    spread(sex, proportion) %>%
    mutate(difference = male - female) %>%
    pull(difference)
}

# actual difference
difference = df.conference %>%
  fun.difference()

# permutation test
df.permutation = df.conference %>%
  permute(n = 1000, sex) %>%
  mutate(difference = map_dbl(perm, ~ fun.difference(.)))
```



significant association between sex and acceptance rate

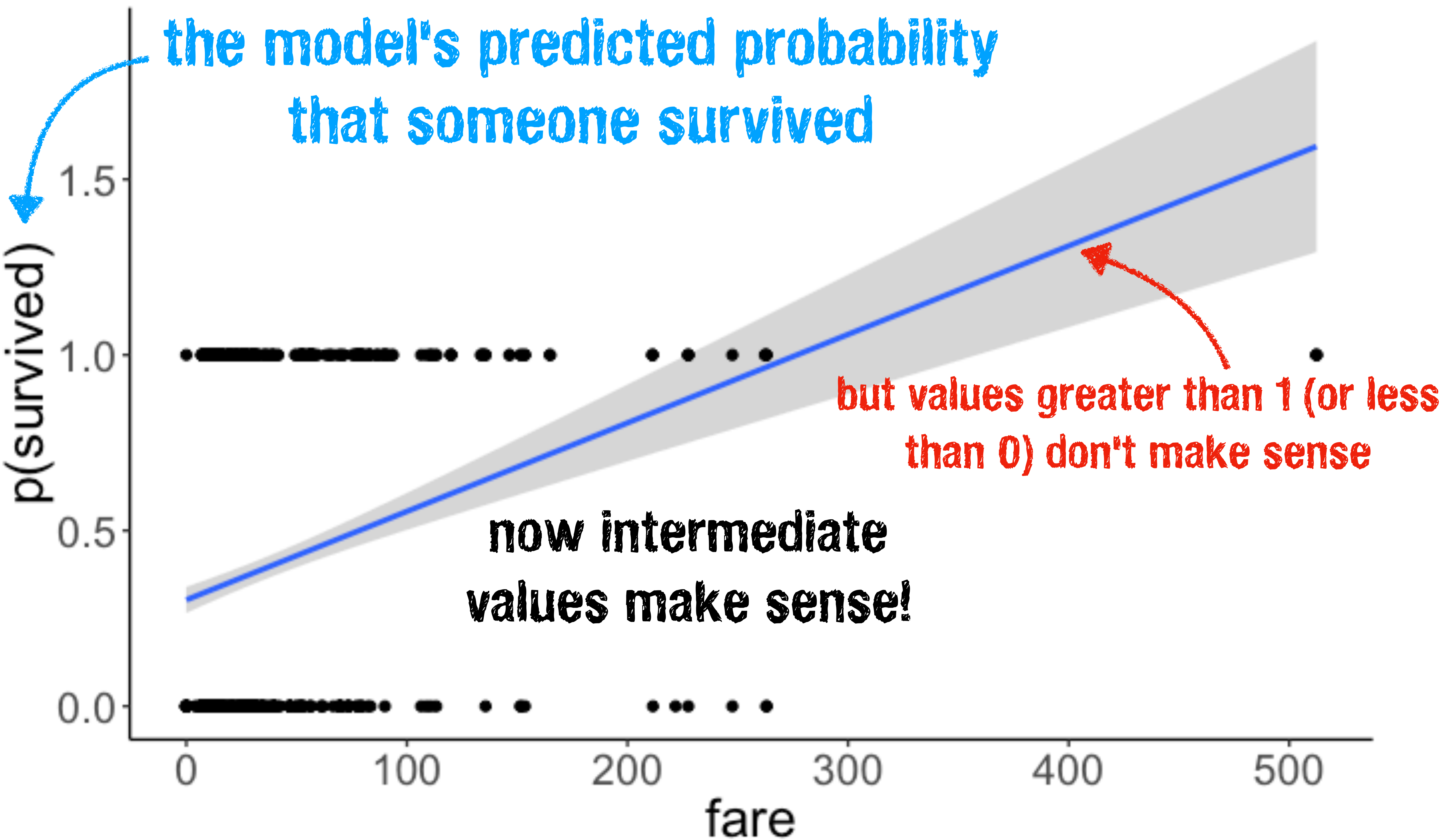
Plan for today

- Generalized linear model
  - Logistic regression
    - Simulating a logistic regression
    - Reporting results
  - Mixed effects logistic regression
- Bayesian data analysis
  - Comparison between frequentist and Bayesian data analysis
  - Quick flash from the past
  - A Bayesian model of multi-modal inference

Generalized linear model

Simulating, fitting, and reporting logistic regression

Is there a relationship between `fare` and `survived`?



Logit transform

$$\pi_i = b_0 + b_1 \cdot X_i + e_i$$

predict the probability of Y

$$\pi_i = P(Y_i = 1)$$

we need to transform the dependent variable so that it can take any value between -∞ and +∞ (we can then transform it back into a probability later)

Step 1: Calculate the "odds"

$$\frac{P(Y_i = 1)}{P(Y_i = 0)} = \frac{\pi_i}{1 - \pi_i}$$

ranges between 0 and +∞

Step 2: Take the (natural) log

ranges between -∞ and +∞

$$\ln\left(\frac{\pi_i}{1 - \pi_i}\right) = b_0 + b_1 \cdot X_i + e_i$$

Simulating a logistic regression

```
# make example reproducible
set.seed(1)

# set parameters
sample_size = 1000
b0 = 0
b1 = 1

# generate data
df.data = tibble(
  x = rnorm(n = sample_size),
  y = b0 + b1 * x,
  p = inv.logit(y)) %>%
  mutate(response = rbinom(n(), size = 1, p = p))

# fit model
fit = glm(formula = response ~ 1 + x,
          family = "binomial",
          data = df.data)

# model summary
fit %>% summary()
```

set some parameters

linear model (y is in log odds)

transform into probability

randomly draw response

fit a logistic regression

summarize the result

Simulating a logistic regression

```r
# make example reproducible
set.seed(1)

# set parameters
sample_size = 1000
b0 = 0
b1 = 1

# generate data
df.data = tibble(
  x = rnorm(n = sample_size),
  y = b0 + b1 * x,
  p = inv.logit(y)) %>%
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# fit model
fit = glm(formula = response ~ 1 + x,
          family = "binomial",
          data = df.data)

# model summary
fit %>% summary()
```

```
Call:
glm(formula = response ~ 1 + x, family = "binomial", data = df.data)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.1137  -1.0118  -0.4591   1.0287   2.2591

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.06214    0.06918  -0.898    0.369
x            0.92905    0.07937  11.705   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1385.4  on 999  degrees of freedom
Residual deviance: 1209.6  on 998  degrees of freedom
AIC: 1213.6

Number of Fisher Scoring iterations: 3
```

Assessing the model fit

actual value

predicted value

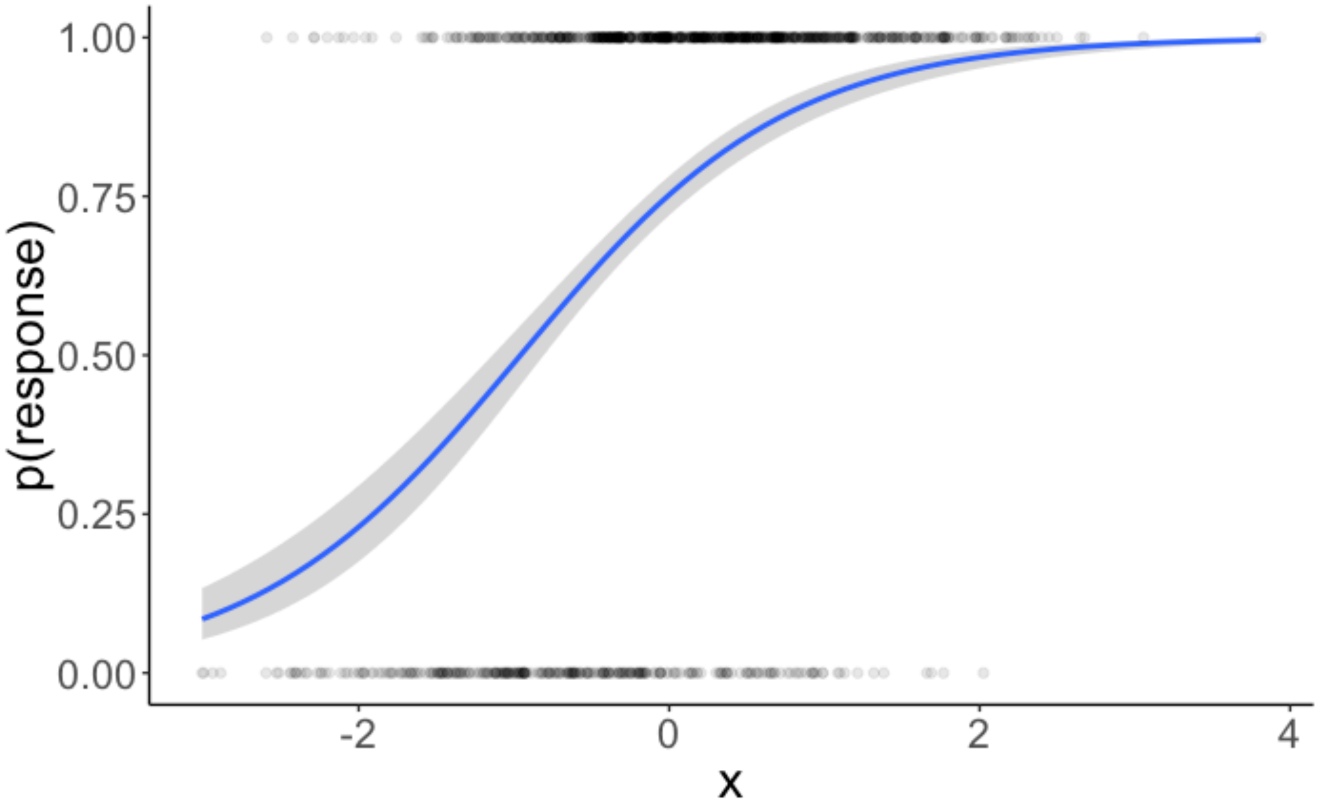
$$\text{log-likelihood} = \sum_{i=1}^{n} [Y_i \cdot \ln(P(Y_i)) + (1 - Y_i) \cdot \ln(1 - P(Y_i))]$$

- calculate the probability of the observed response
- take the log of these probabilities
- sum them up to get the log-likelihood of the data (given the model)

response	p(Y = 1)	p(Y = response)	log(p(Y = response))
1	0.34	0.34	-1.07
0	0.53	0.47	-0.75
1	0.30	0.30	-1.20
1	0.81	0.81	-0.22
1	0.56	0.56	-0.58
0	0.30	0.70	-0.36
1	0.60	0.60	-0.52
1	0.65	0.65	-0.43
1	0.62	0.62	-0.48
0	0.41	0.59	-0.54

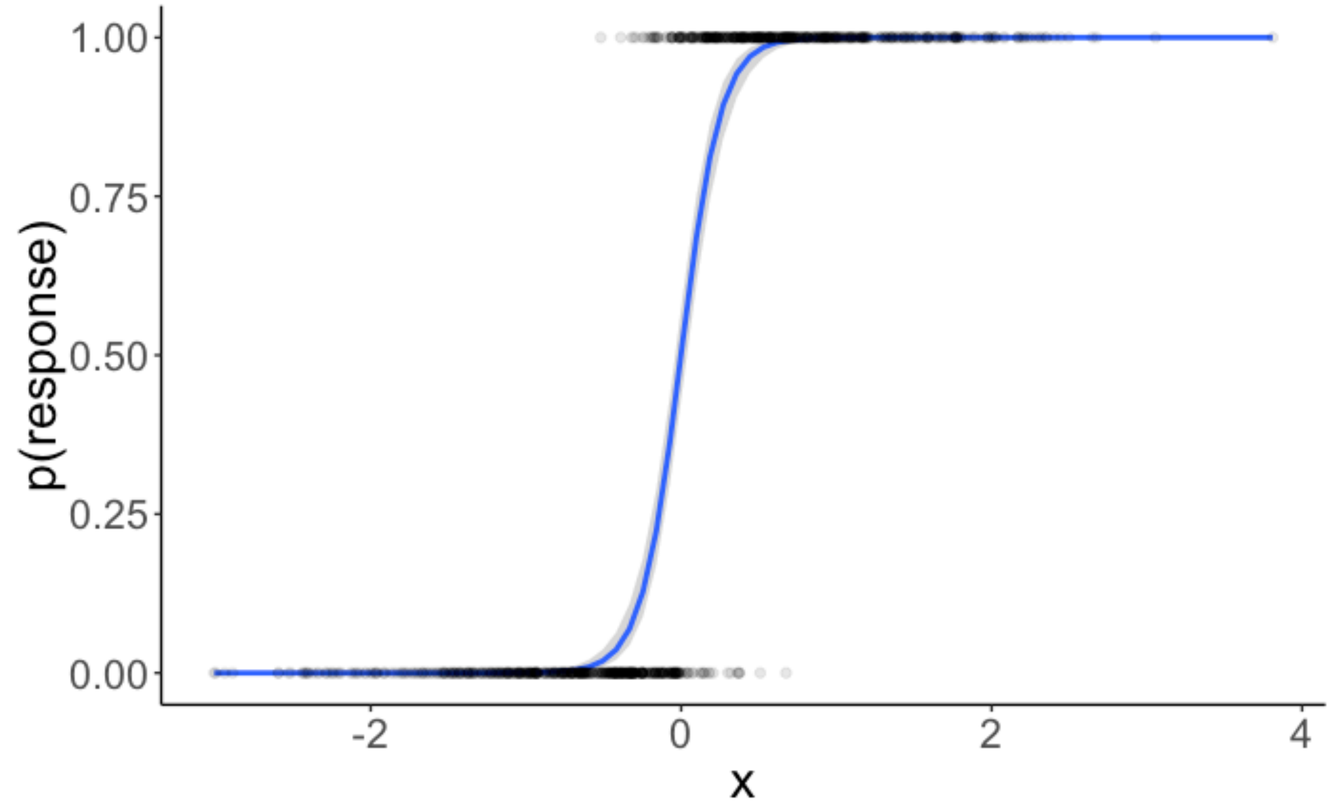
Assessing the model fit

doesn't predict the response very well



logLik	AIC	BIC
-501.65	1007.3	1017.12

predicts the response much better



logLik	AIC	BIC
-156.37	316.74	326.55

Testing hypotheses

aka checking whether it's **worth it**

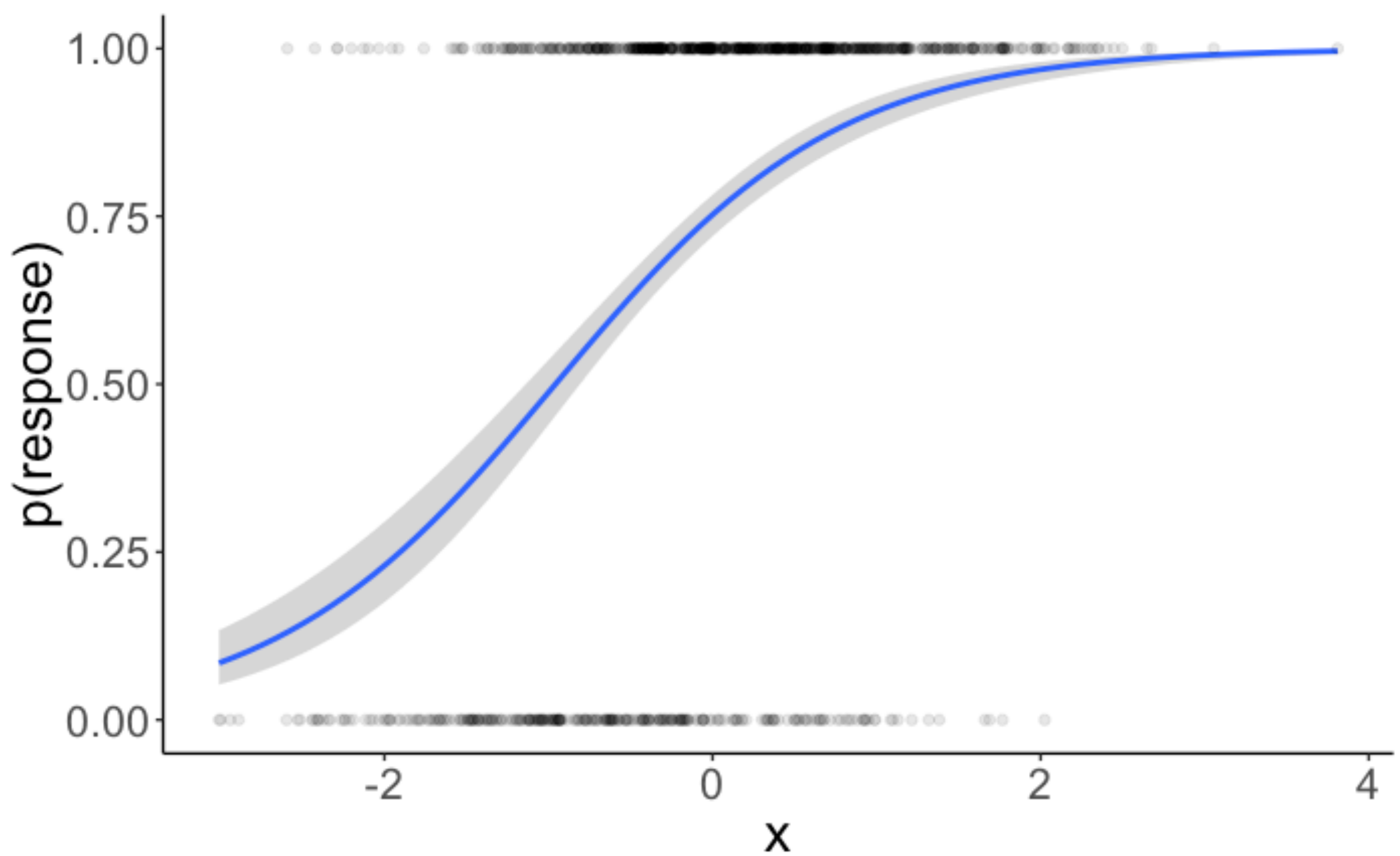
```r
# fit compact model
fit.compact = glm(formula = survived ~ 1 + fare,
                  family = "binomial",
                  data = df.titanic)

# fit augmented model
fit.augmented = glm(formula = survived ~ 1 + sex + fare,
                    family = "binomial",
                    data = df.titanic)

# likelihood ratio test
anova(fit.compact, fit.augmented, test = "LRT")
```

we need to specify that we want a likelihood ratio test

```
Analysis of Deviance Table

Model 1: survived ~ 1 + fare
Model 2: survived ~ 1 + sex + fare
  Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
1       889    1117.57
2       888     884.31  1   233.26 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Reporting results

- Visualize the data
- Show a table with the regression results
- Report significance of different factors
- Interpreting parameter estimates is tricky -- probably best to report probabilities for a few example cases (the "effects" package will make your life a little easier)



Assumptions

- linearity (between predictors and log odds)
- independence
- no multi-collinearity

- model fails to converge when there is **complete separation**:
  - if outcome variable can be perfectly predicted by a (combination of) predictor(s)

Different kinds of generalized models

Different linking functions

```
binomial(link = "logit")

gaussian(link = "identity")

Gamma(link = "inverse")

inverse.gaussian(link = "1/mu^2")

poisson(link = "log")

quasi(link = "identity", variance = "constant")

quasibinomial(link = "logit")

quasipoisson(link = "log")
```

apply different transformations to the dependent variable

Mixed effects logistic regression

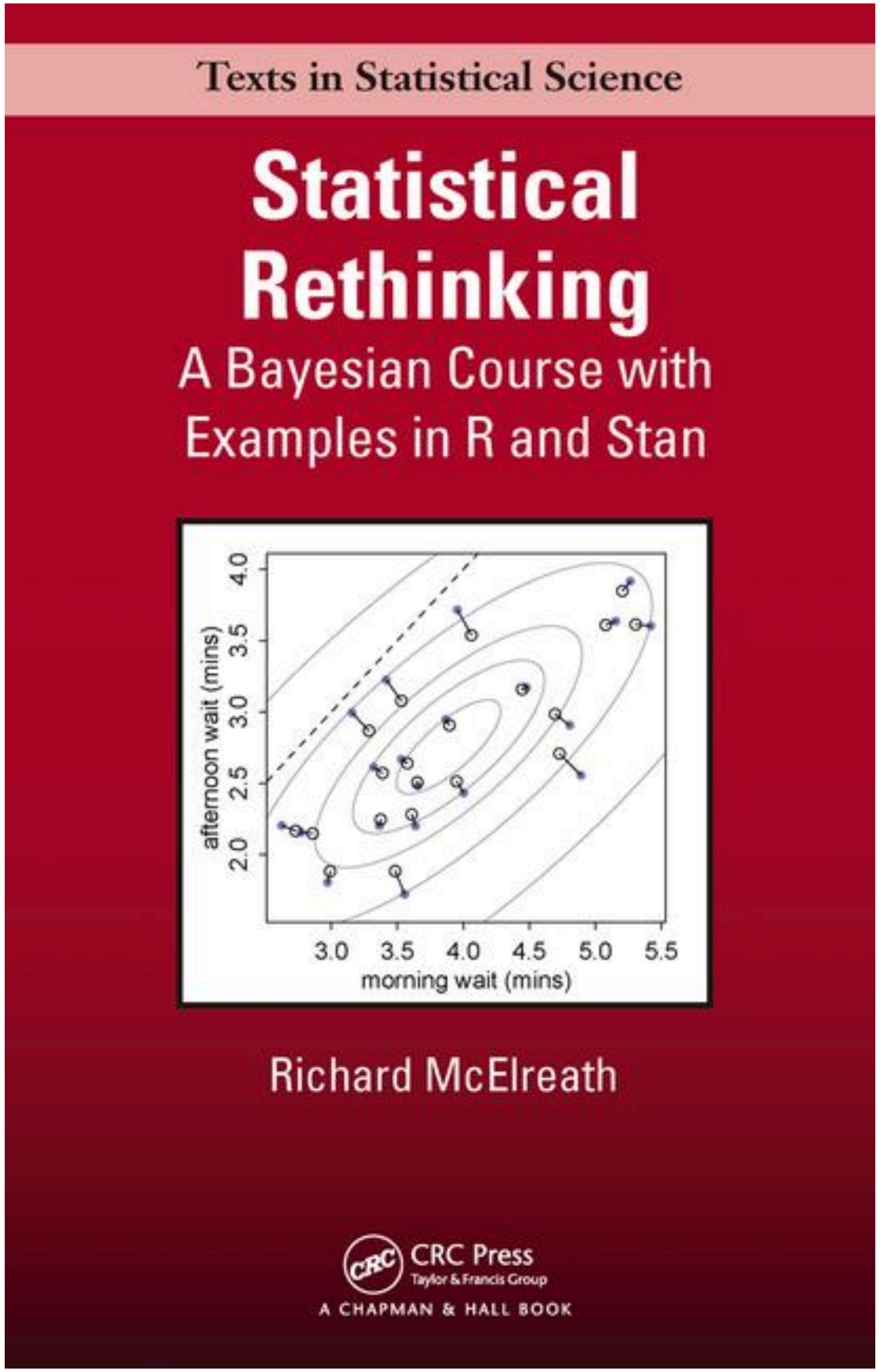
Mixed effects logistic regression

```r
fit =  glmer(repeatgr ~ 1 + ses * Minority + (1 | schoolNR),
             data = df.language,
             family = "binomial")

fit %>% summary()
```

```
Generalized linear mixed model fit by maximum likelihood (Laplace Approximation) ['glmerMod']
 Family: binomial  ( logit )
Formula: repeatgr ~ 1 + ses * Minority + (1 | schoolNR)
   Data: bdf

     AIC      BIC   logLik deviance df.resid
  1672.8   1701.5   -831.4   1662.8     2282

Scaled residuals:
    Min      1Q  Median      3Q     Max
-0.9602 -0.4071 -0.3155 -0.2219  5.9500

Random effects:
 Groups   Name        Variance Std.Dev.
 schoolNR (Intercept) 0.2583   0.5083
Number of obs: 2287, groups:  schoolNR, 131

Fixed effects:
               Estimate Std. Error z value Pr(>|z|)
(Intercept)   -0.454556   0.206103  -2.205   0.0274 *
ses           -0.061913   0.007908  -7.829 4.93e-15 ***
MinorityY      0.480047   0.471208   1.019   0.3083
ses:MinorityY  0.011938   0.022737   0.525   0.5996
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) ses    MnrtyY
ses         -0.906
MinorityY   -0.400  0.369
ses:MinrtyY  0.299 -0.321 -0.866
```

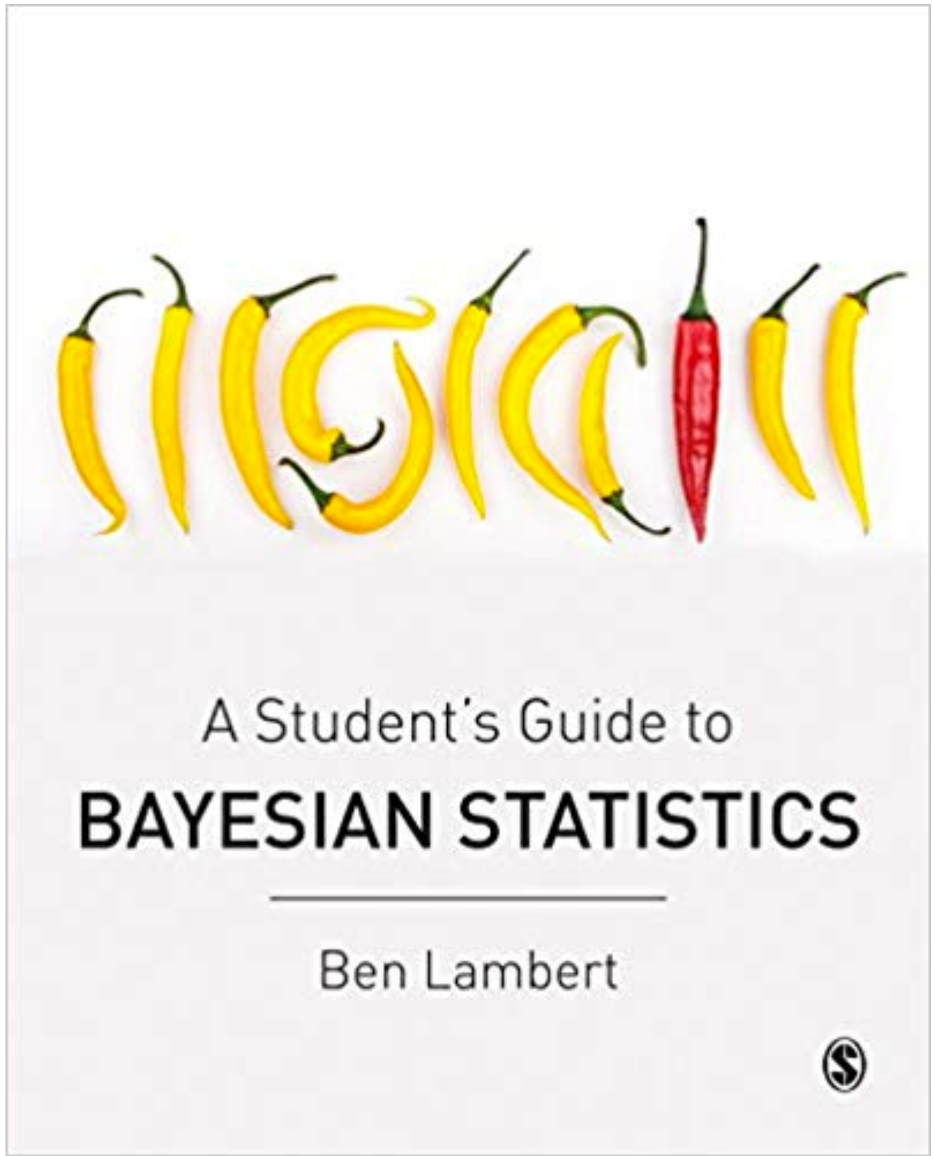
Bayesian data analysis

Good books on Bayesian data analysis



- nice hands-on book (which uses R throughout)
- unfortunately, mostly in base R
- however, rewrite of all the code with "`tidyverse`" and "`BRMS`" is here: https://bookdown.org/connect/#/apps/1850/access
- video lectures are available here: https://goo.gl/4zZWTv
- Version 2 is in the making (so worth waiting if you are thinking about buying the book)

Good books on Bayesian data analysis



- less hands-on (no R code)
- very nice visualizations of key concepts
- goes a little more into theoretical details (but in a mostly non-mathy way)
- also comes with video lectures

Goal of data analysis: Inference about the world

Frequentist statistics

- generate a sampling distribution of the test statistic assuming H_0
- compare observed value of the test statistic with the sampling distribution
- reject the H_0 if probability of observed value (or more extreme values) is less than α

Bayesian statistics

- directly test hypotheses of interest
- define prior over hypotheses
- compute likelihood of the data for each hypothesis
- use Bayes' rule to infer the posterior over hypotheses given the data **p(H|D)**

Objections to frequentist NHST

null hypothesis significance testing

- p-value is not a measure of evidential support
  - becomes smaller as *N* increases
- p-value depends on researcher intentions
  - the same data can have a different p-value depending on whether the sample size was fixed a priori vs. decided after looking at the data
- results are often misinterpreted (both p-values and confidence intervals are not particularly intuitive)
- what we want to know: $p(\text{Hypothesis} \mid \text{Data})$
- what we calculate: $p(\text{Data} \mid \text{Null Hypothesis})$

NHST can lead to paradoxical inferences

"When you have eliminated all which is impossible, then whatever remains, however improbable, must be the truth."

In NHST we can come up with situations where we would reject all (null) hypotheses…

Why don't more people use Bayesian Statistics?

- supposedly more difficult
  - relies on the logic of probability theory
- reliance on a *prior*
- reliance on computing and simulation
  - we can't just use SPSS
  - but we can use JASP (Just Another Statistics Program)

and we've already learned how to simulate and visualize data in this class!

What are (some of) the benefits of Bayesian data analysis?

- intuitive model testing and comparison
  - compare simulated data with the real data
- straightforward interpretation of results
  - Bayesian credible intervals vs. Confidence intervals
- full model flexibility
  - easy extension to capture data-generating processes of high complexity
- less opportunity for misuse of test
  - we build models from the ground up, making our assumptions explicit
- better predictions!

Flash from the past

Clue guide to probability

we derived this using the definition of conditional probability

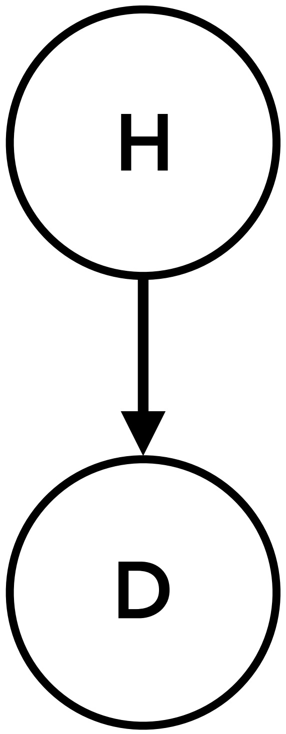
$$p(B \mid A) = \frac{p(A \mid B) \cdot p(B)}{p(A)}$$

$$\overset{\text{posterior}}{p(H \mid D)} = \frac{\overset{\text{likelihood}}{p(D \mid H)} \cdot \overset{\text{prior}}{p(H)}}{p(D)}$$

subjective probability interpretation

H = Hypothesis

D = Data



formal framework for learning from data

updating our prior belief $p(H)$, to a posterior belief $p(H \mid D)$ given some data

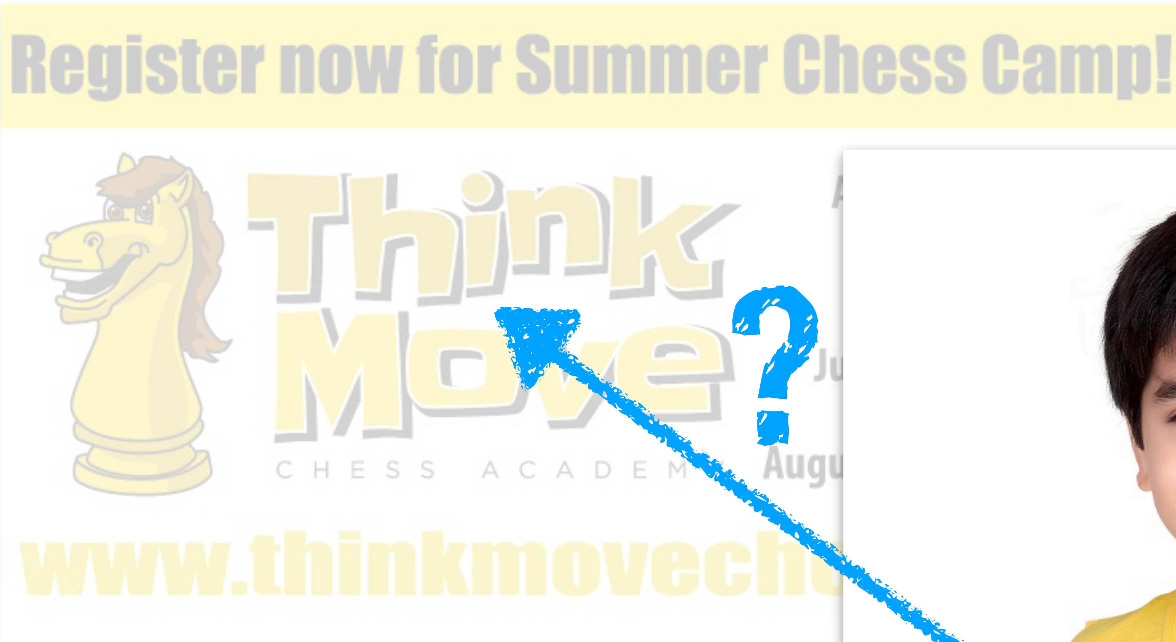
Summer camp

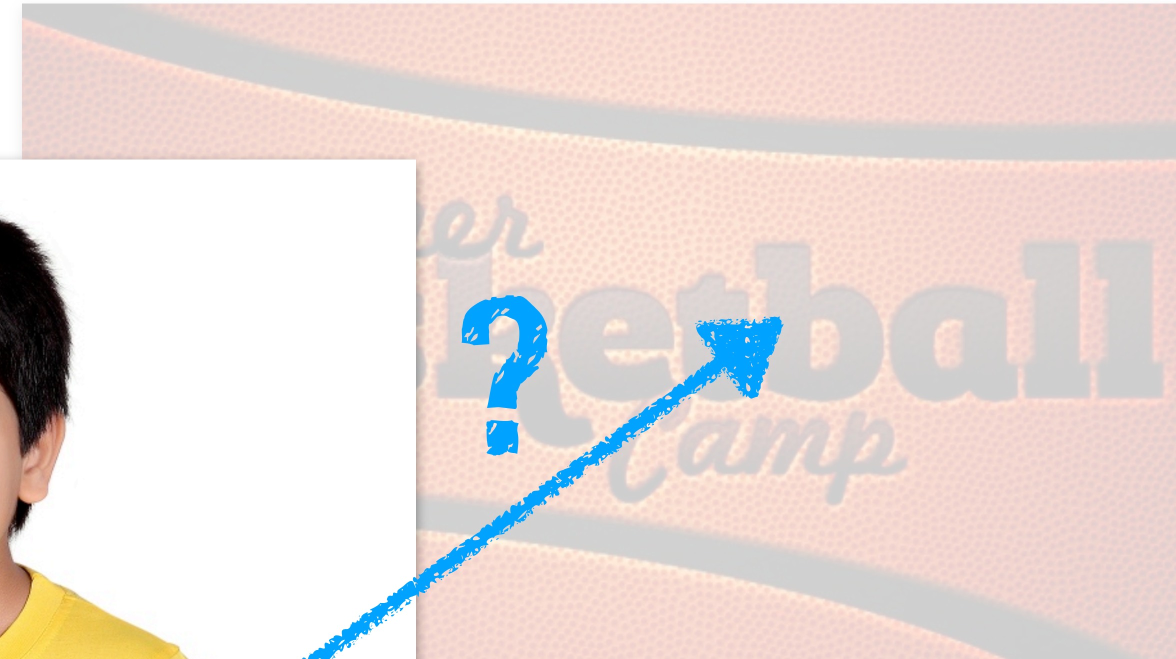
twice as many kids go to the basketball camp

$X \sim \text{Normal}(\mu = 170, \sigma = 8)$

$X \sim \text{Normal}(\mu = 180, \sigma = 10)$

Summer camp





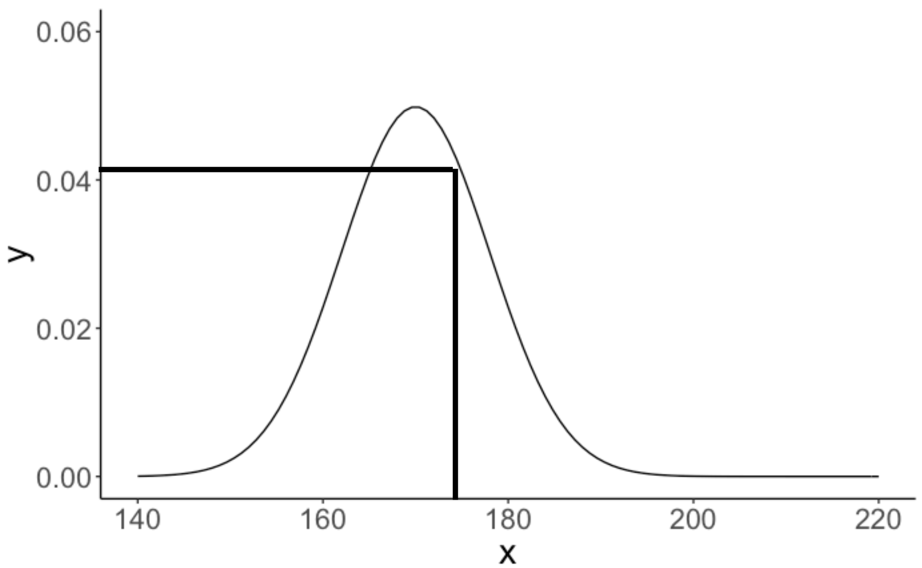
twice as ma ... etball camp

$X \sim \text{Normal}(\mu = 170$ ... $\text{al}(\mu = 180, \sigma = 10)$

Summer camp

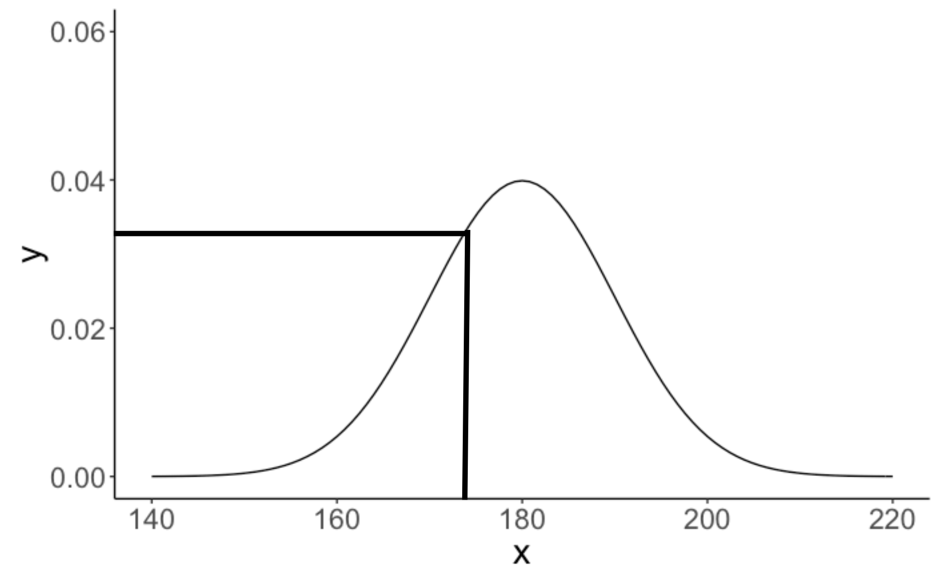
prior $p(\text{chess}) = \frac{1}{3}$ $p(\text{basketball}) = \frac{2}{3}$

likelihood



`dnorm(175, mean = 170, sd = 8)`

= 0.041



`dnorm(175, mean = 180, sd = 10)`

= 0.035

posterior

$$p(\text{sport} = \text{basketball} \mid \text{height} = 175) = \frac{p(175 \mid \text{basketball}) \cdot p(\text{basketball})}{p(175)}$$

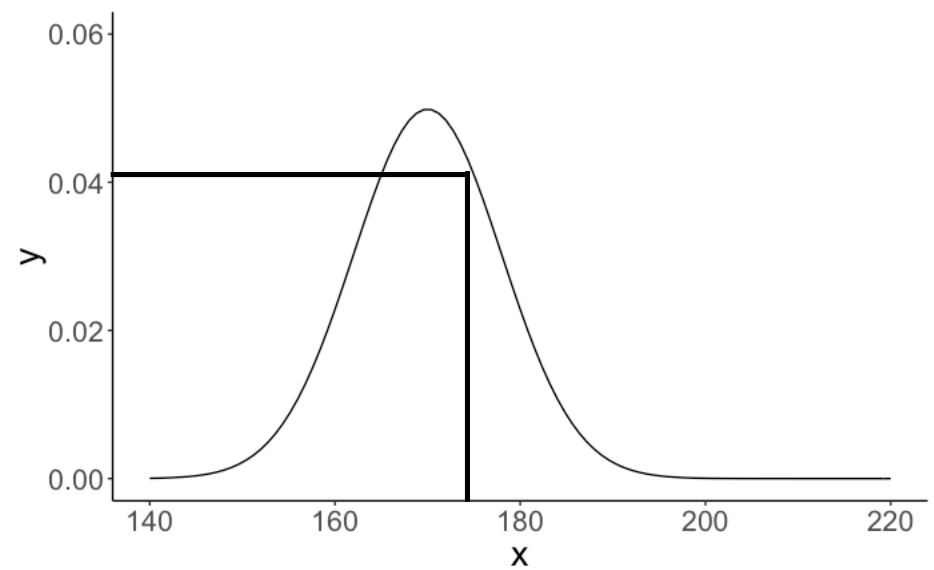
$$p(\text{basketball} \mid 175) = \frac{p(175 \mid \text{basketball}) \cdot p(\text{basketball})}{p(175 \mid \text{basketball}) \cdot p(\text{basketball}) + p(175 \mid \text{chess}) \cdot p(\text{chess})}$$

Summer camp

prior

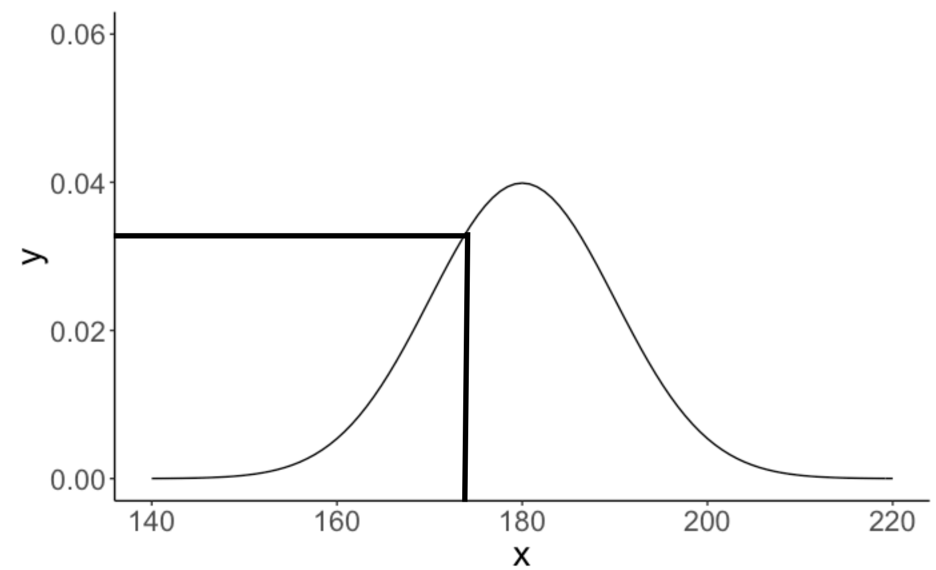
$$p(\text{chess}) = \frac{1}{3} \qquad p(\text{basketball}) = \frac{2}{3}$$

likelihood



`dnorm(175, mean = 170, sd = 8)`

= 0.041



`dnorm(175, mean = 180, sd = 10)`

= 0.035

posterior

$$p(\text{basketball} \mid 175) = \frac{p(175 \mid \text{basketball}) \cdot p(\text{basketball})}{p(175 \mid \text{basketball}) \cdot p(\text{basketball}) + p(175 \mid \text{chess}) \cdot p(\text{chess})}$$

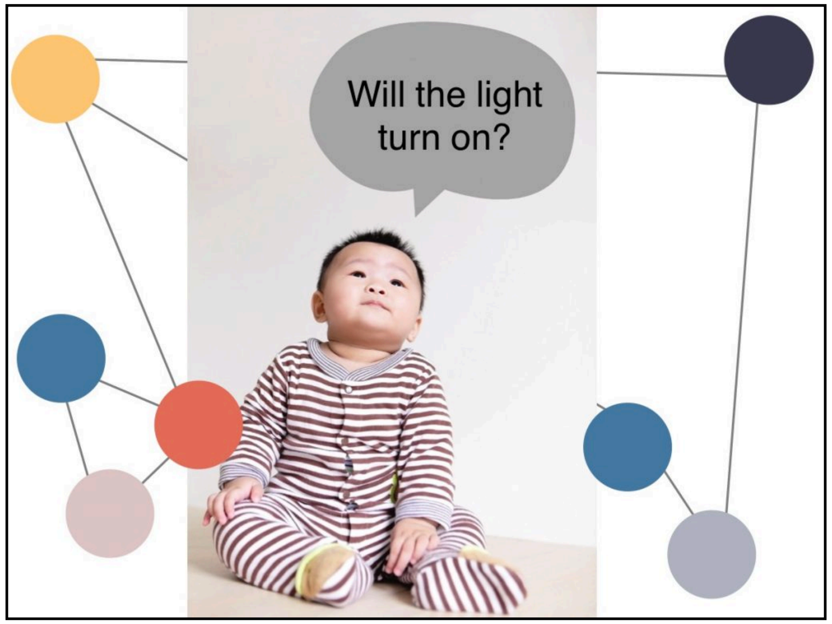
$$p(\text{basketball} \mid 175) = \frac{0.035 \cdot 2/3}{0.035 \cdot 2/3 + 0.041 \cdot 1/3} \approx 0.63$$

send the kid to the basketball gym!

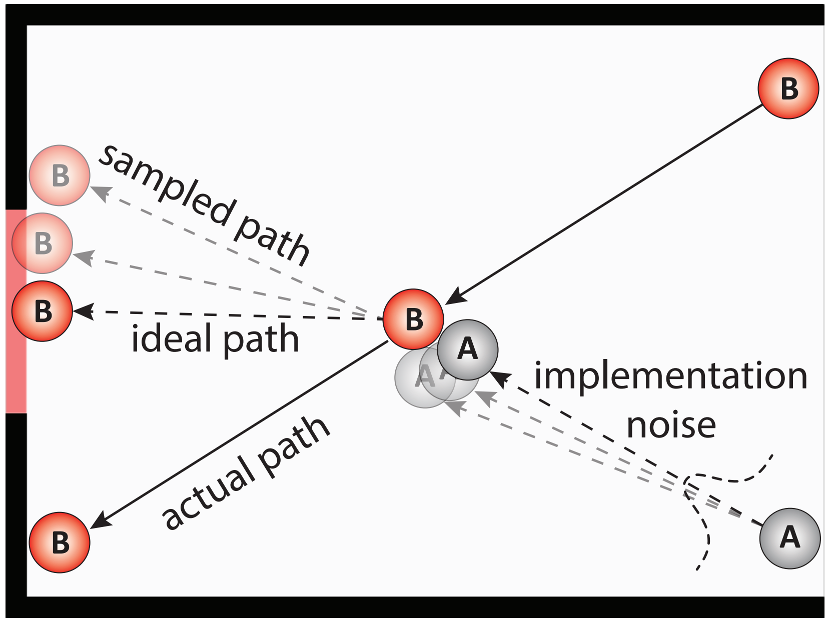
A Bayesian model of multi-modal inference

cic Causality in Cognition

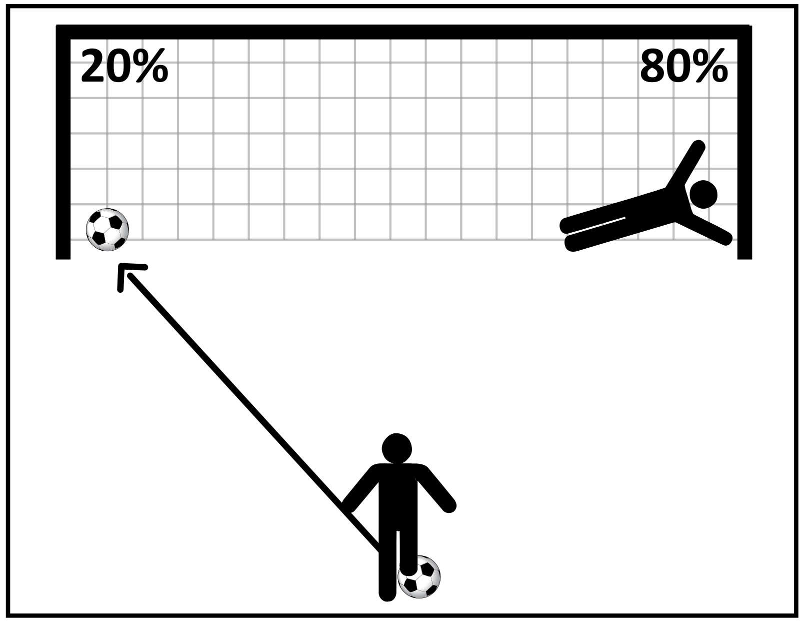
Our lab studies the role of causality in our understanding of the world, and of each other.



learning



reasoning

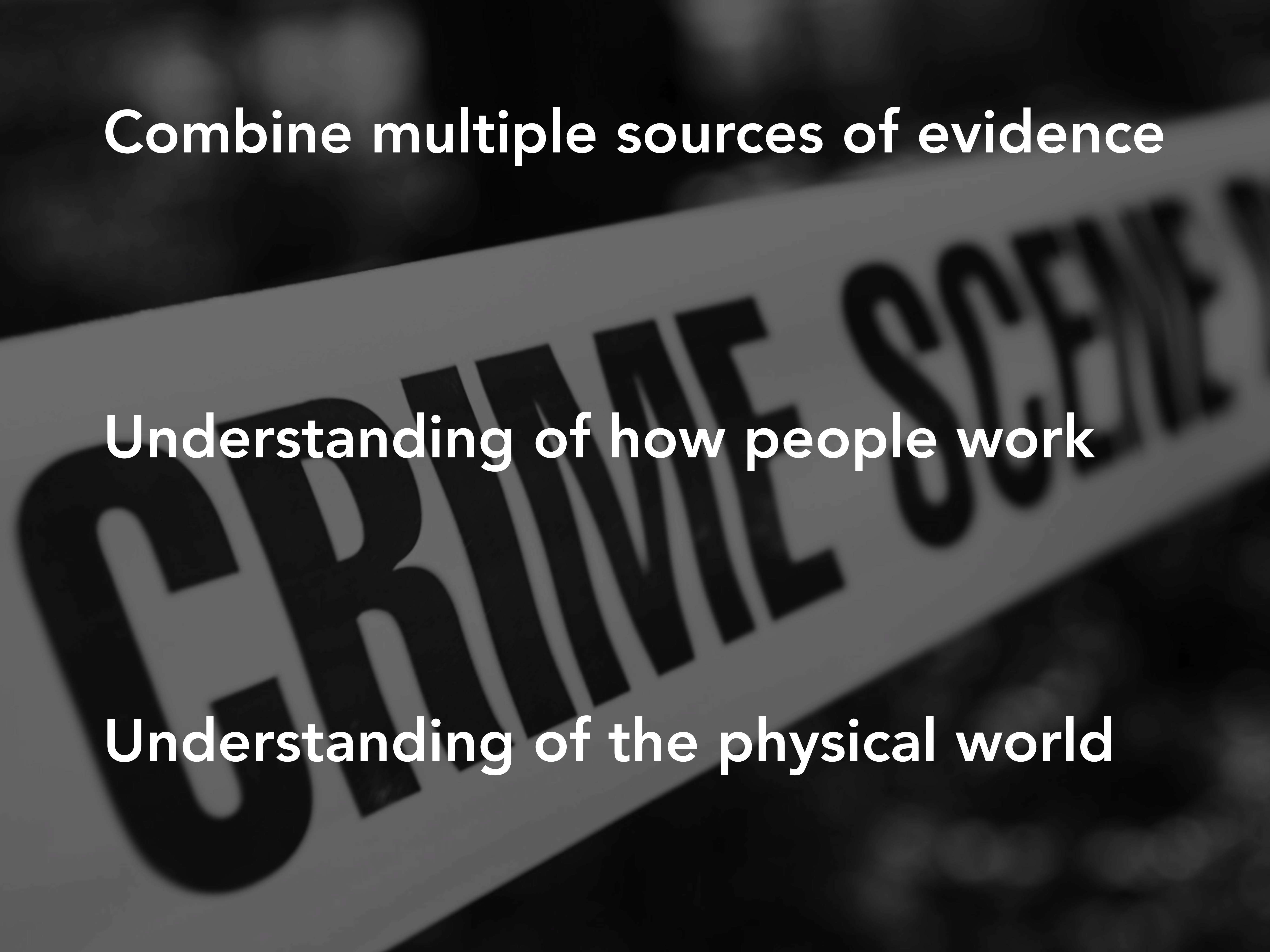


judgment

@tobigerstenberg

http://cicl.stanford.edu/

Coca-Cola
Coca-Cola
Coca-Cola
BANG
BANG



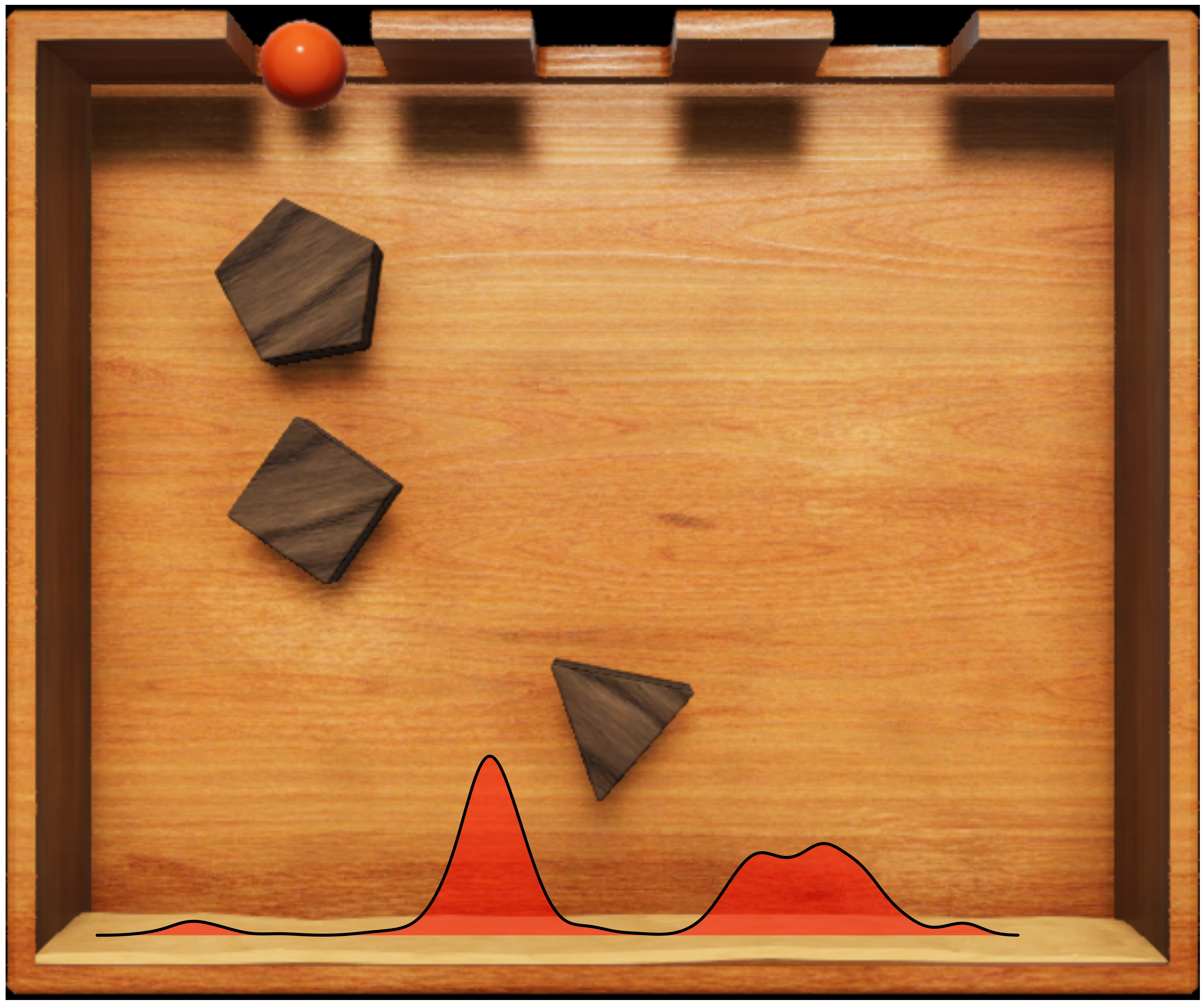
Combine multiple sources of evidence

Understanding of how people work

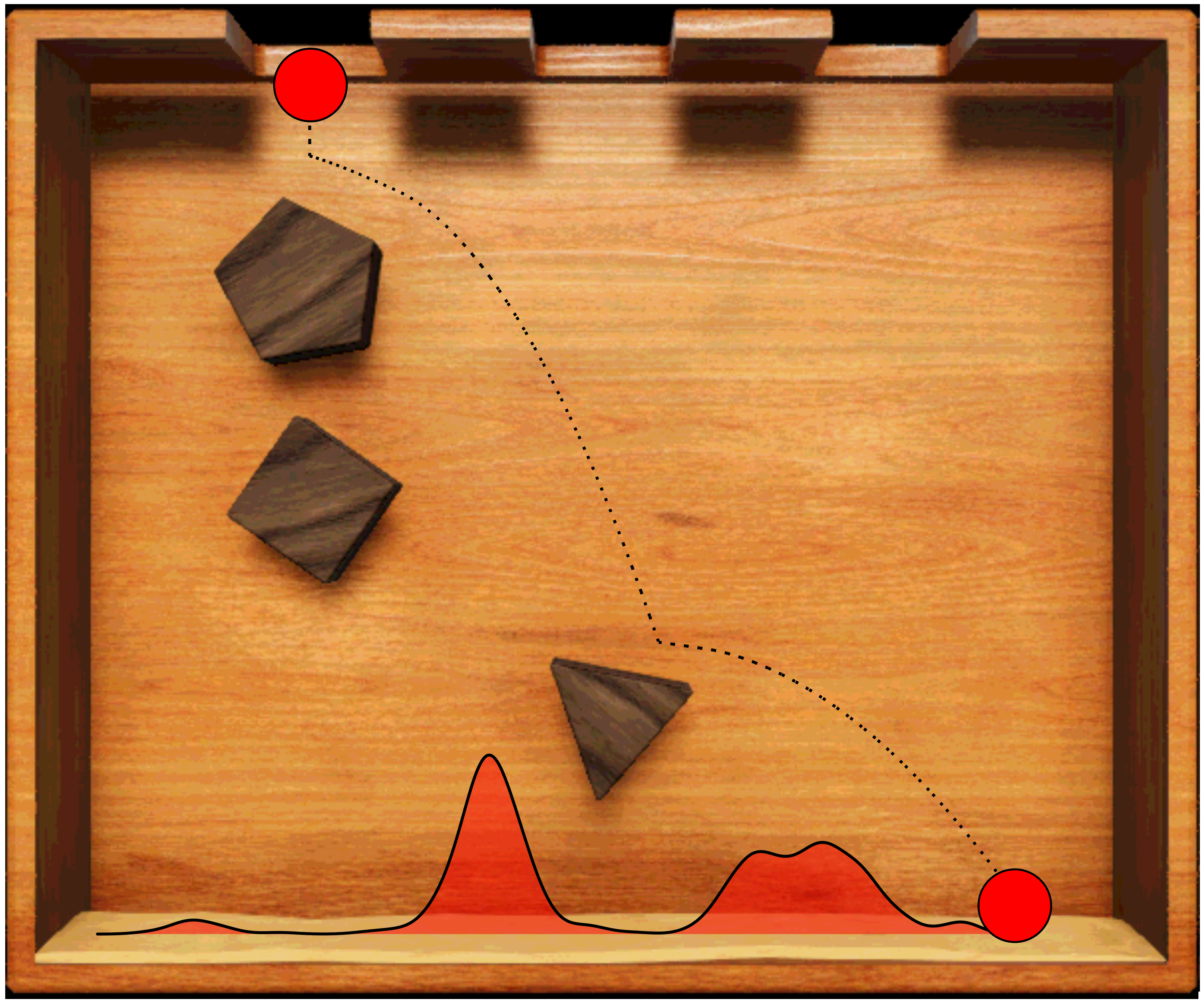
Understanding of the physical world

Prediction: Where will the ball land?

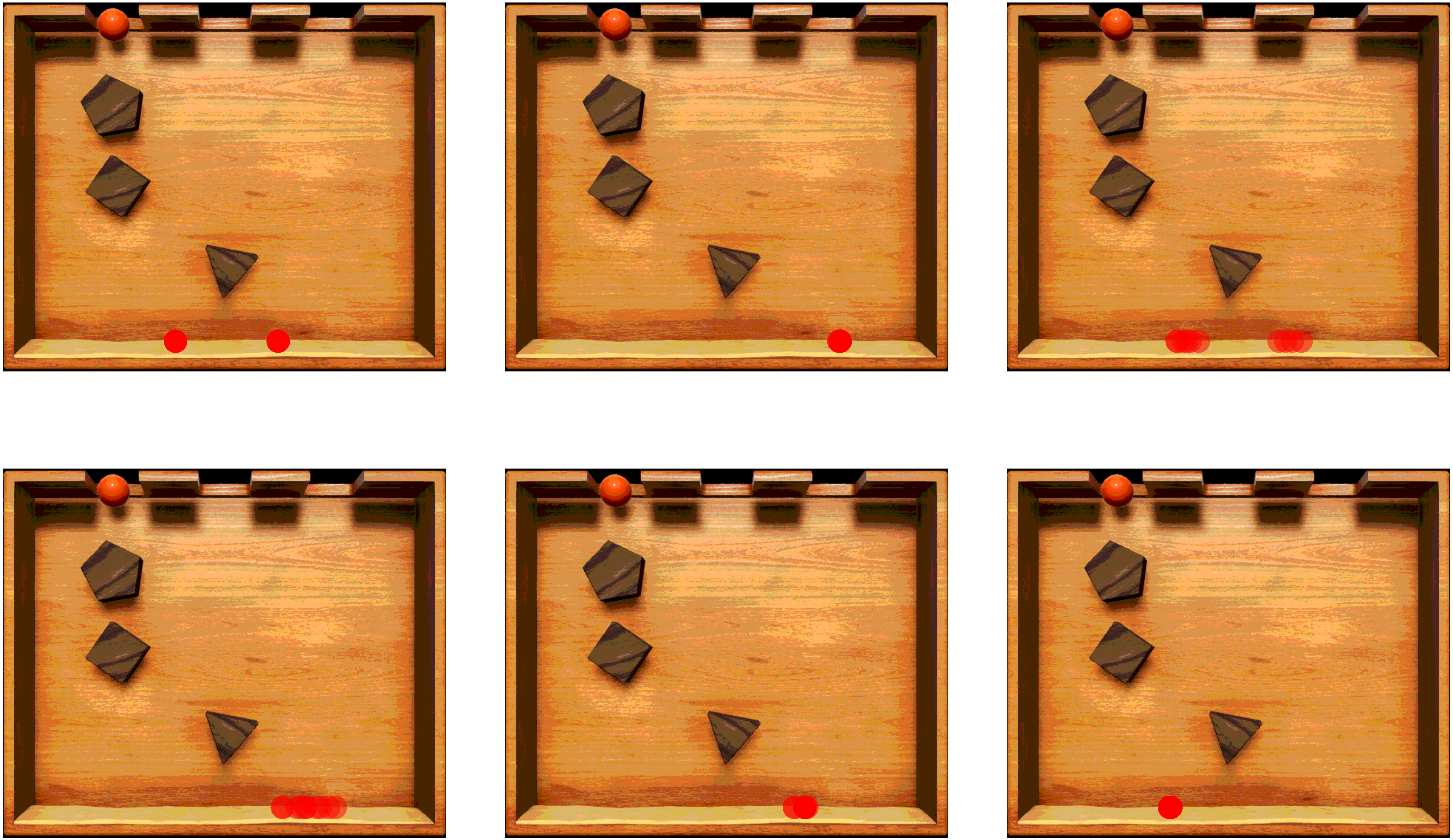
Prediction: Where will the ball land?



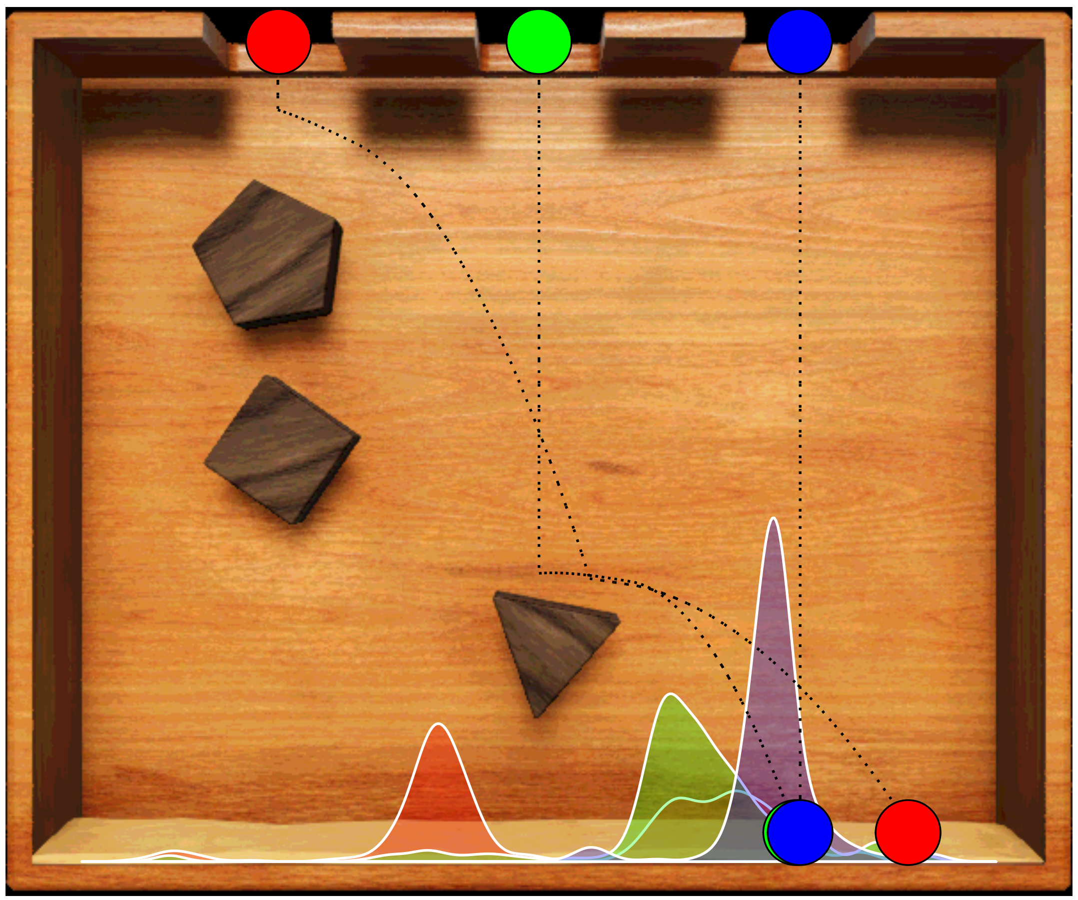
Prediction: Where will the ball land?



Prediction: Where will the ball land?



Prediction: Where will the ball land?

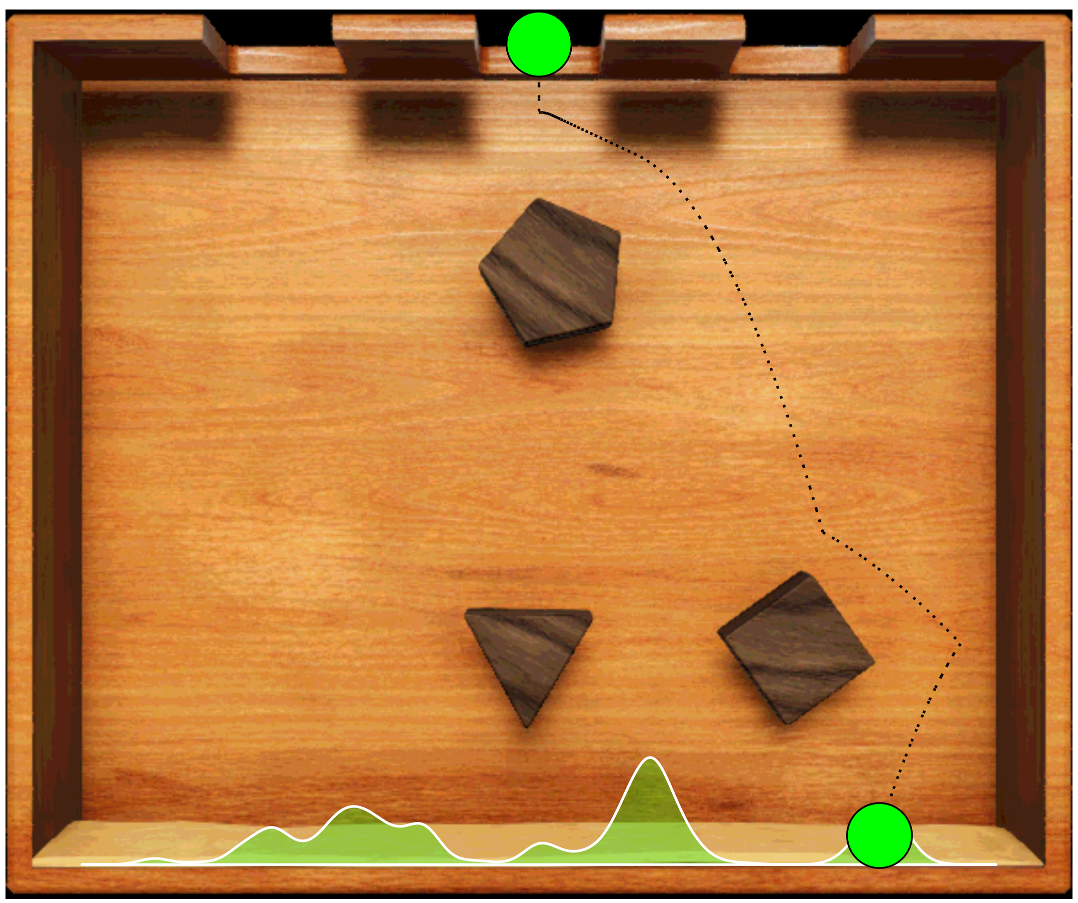


people

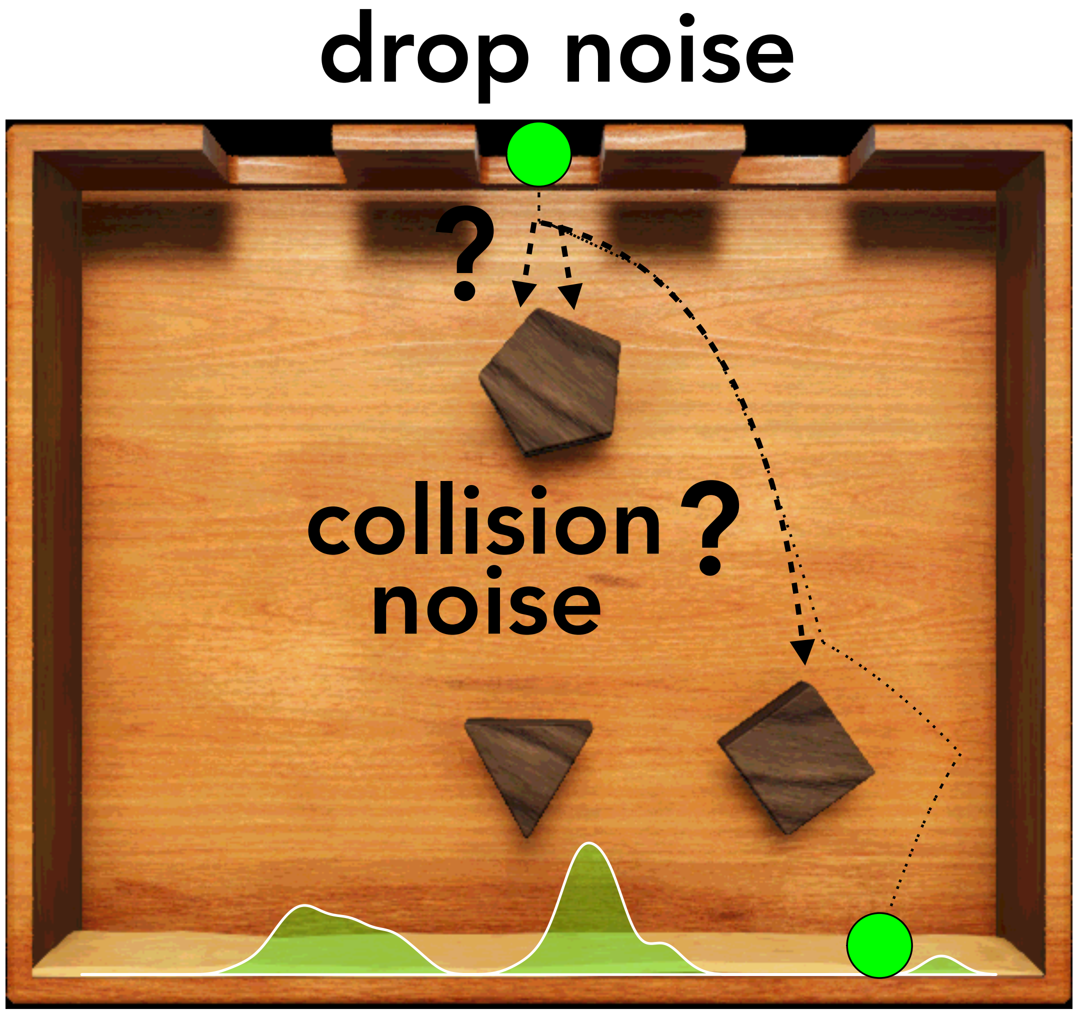
Ullman, Spelke, Battaglia, & Tenenbaum (2017) Mind Games: Game Engines as an Architecture for Intuitive Physics. *Trends in Cognitive Sciences*

Smith & Vul (2013) Sources of uncertainty in intuitive physics. *Topics in Cognitive Science*

Prediction: Where will the ball land?

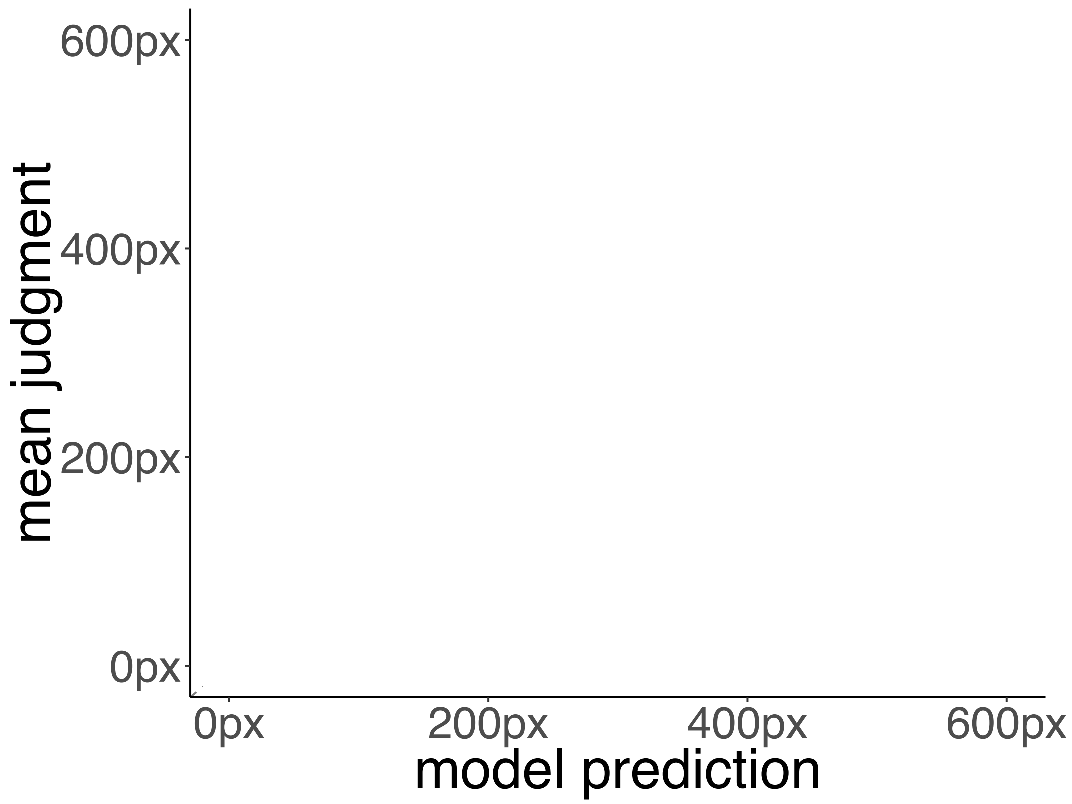


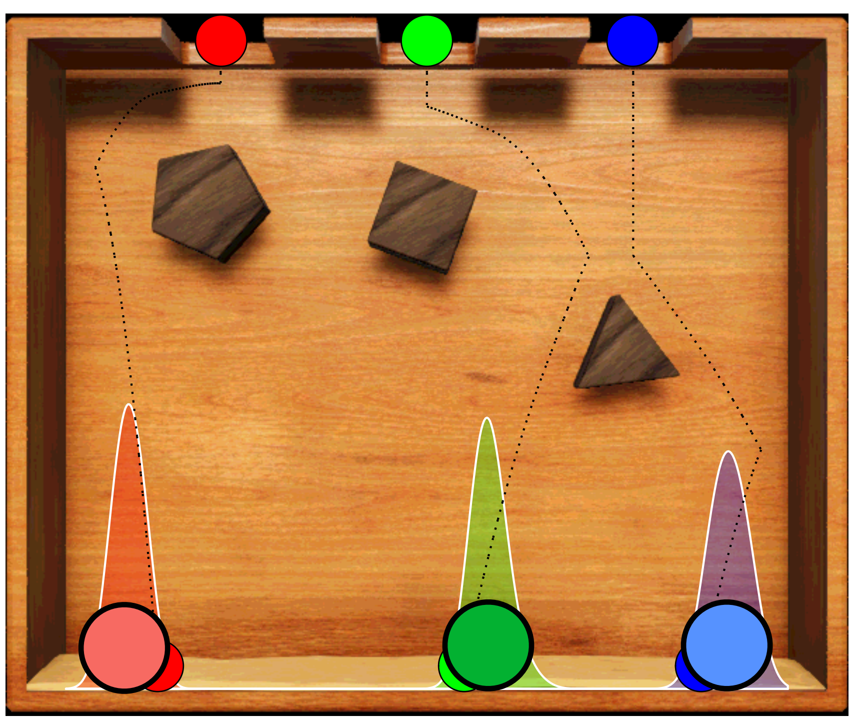
people



model

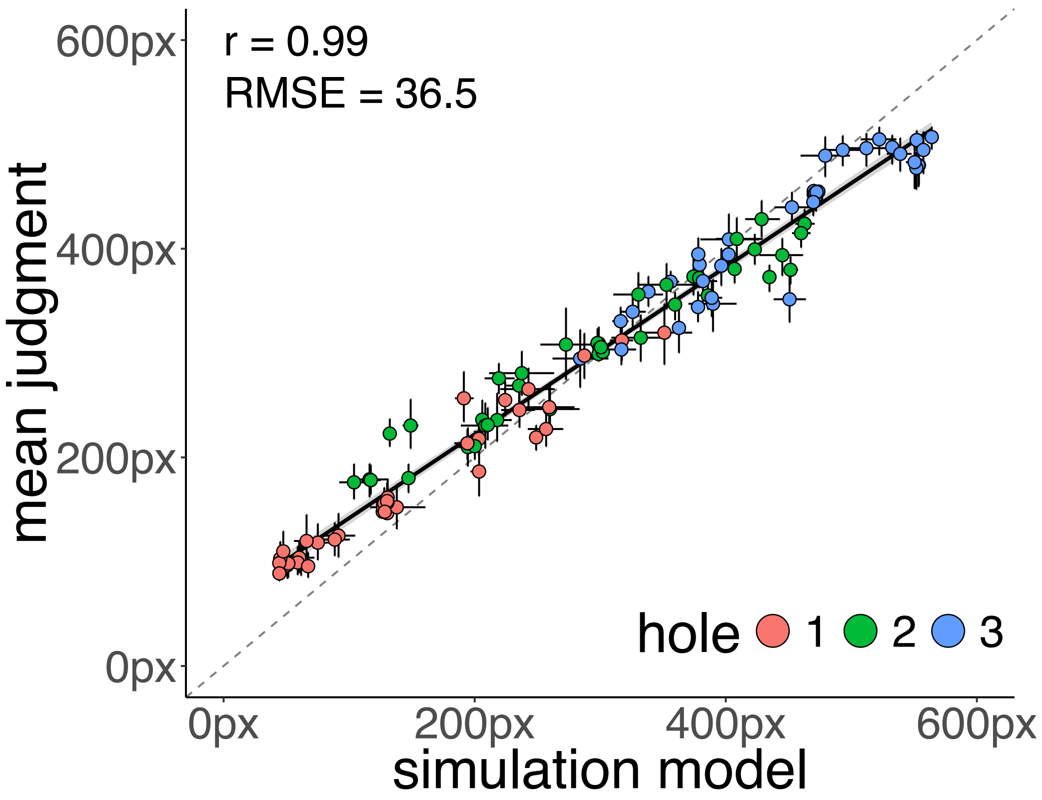
Prediction: Where will the ball land?



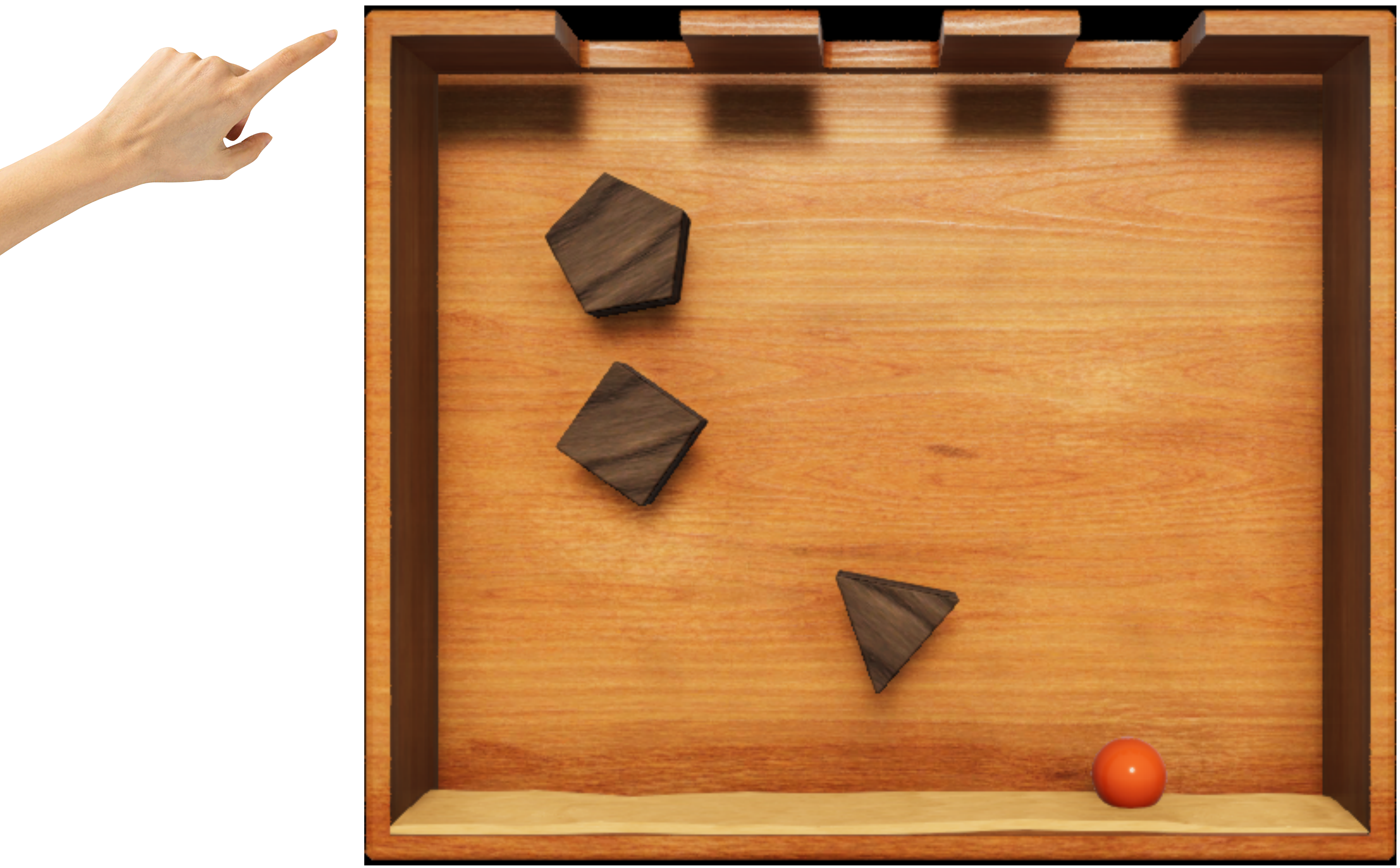


mean model prediction

Prediction: Where will the ball land?



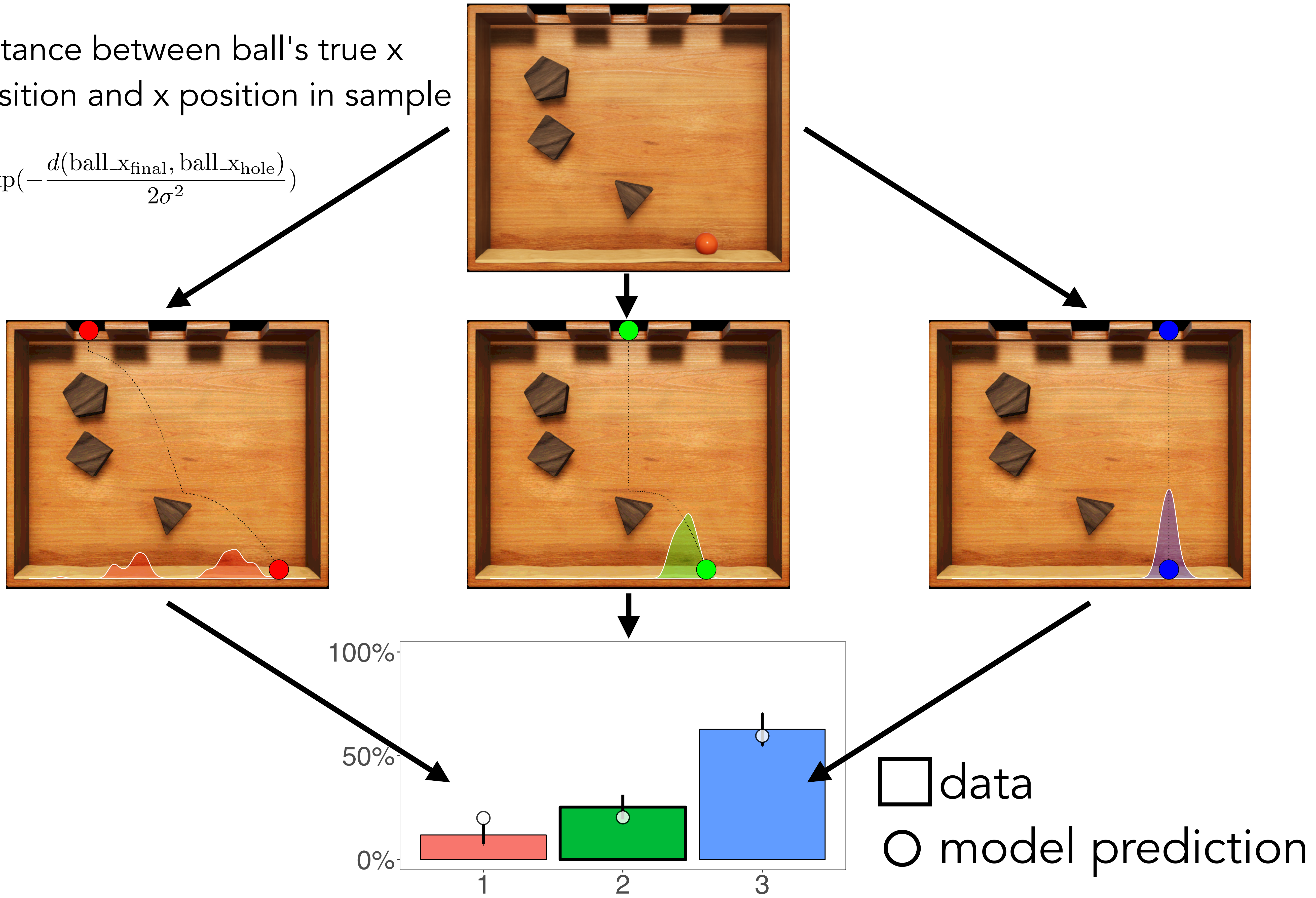
Inference: In which hole was the ball dropped?



Inference: In which hole was the ball dropped?

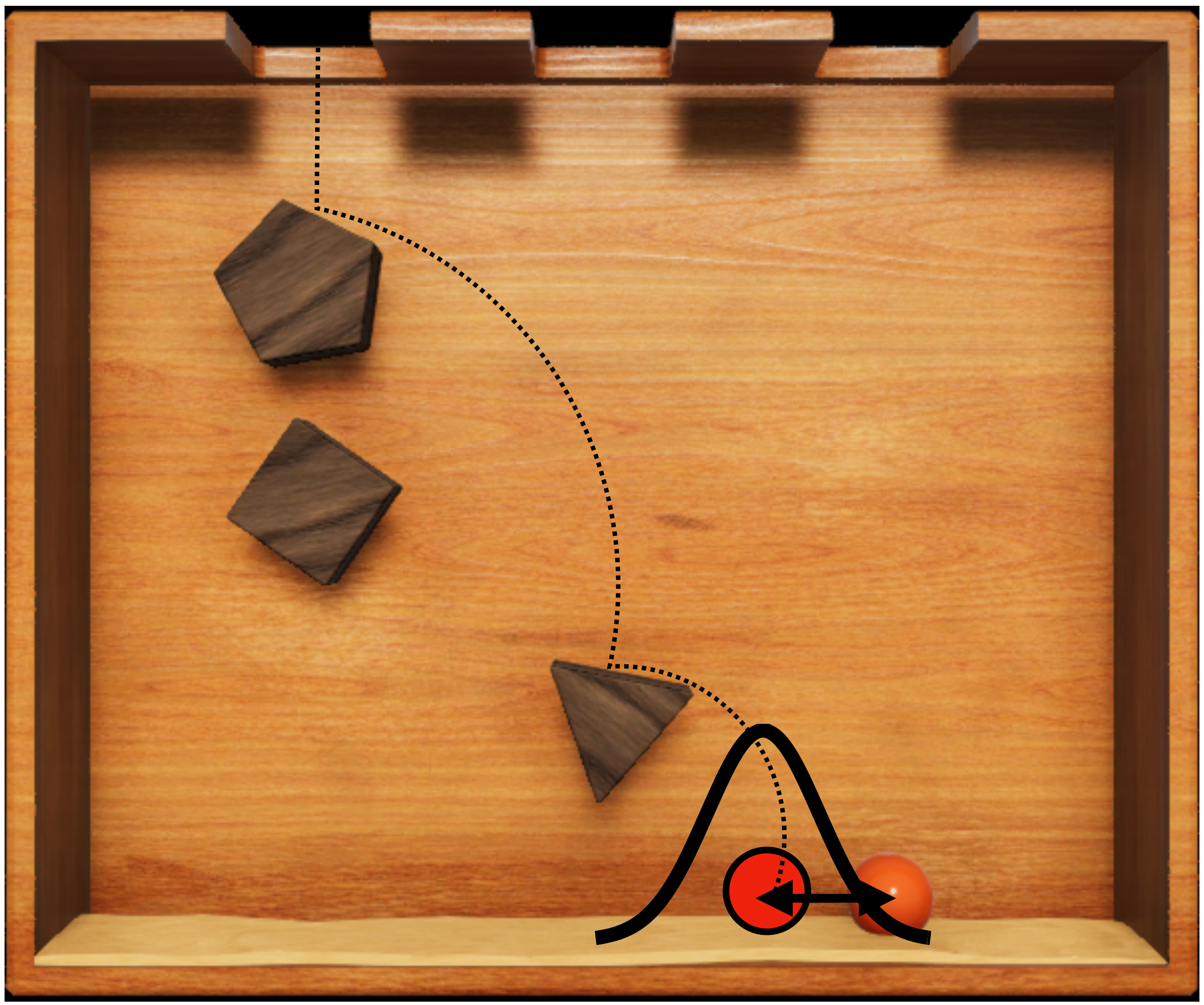
distance between ball's true x position and x position in sample

$$\exp\left(-\frac{d(\mathrm{ball\_x_{final}}, \mathrm{ball\_x_{hole}})}{2\sigma^2}\right)$$



Inference: In which hole was the ball dropped?

Uniform prior over the three different holes

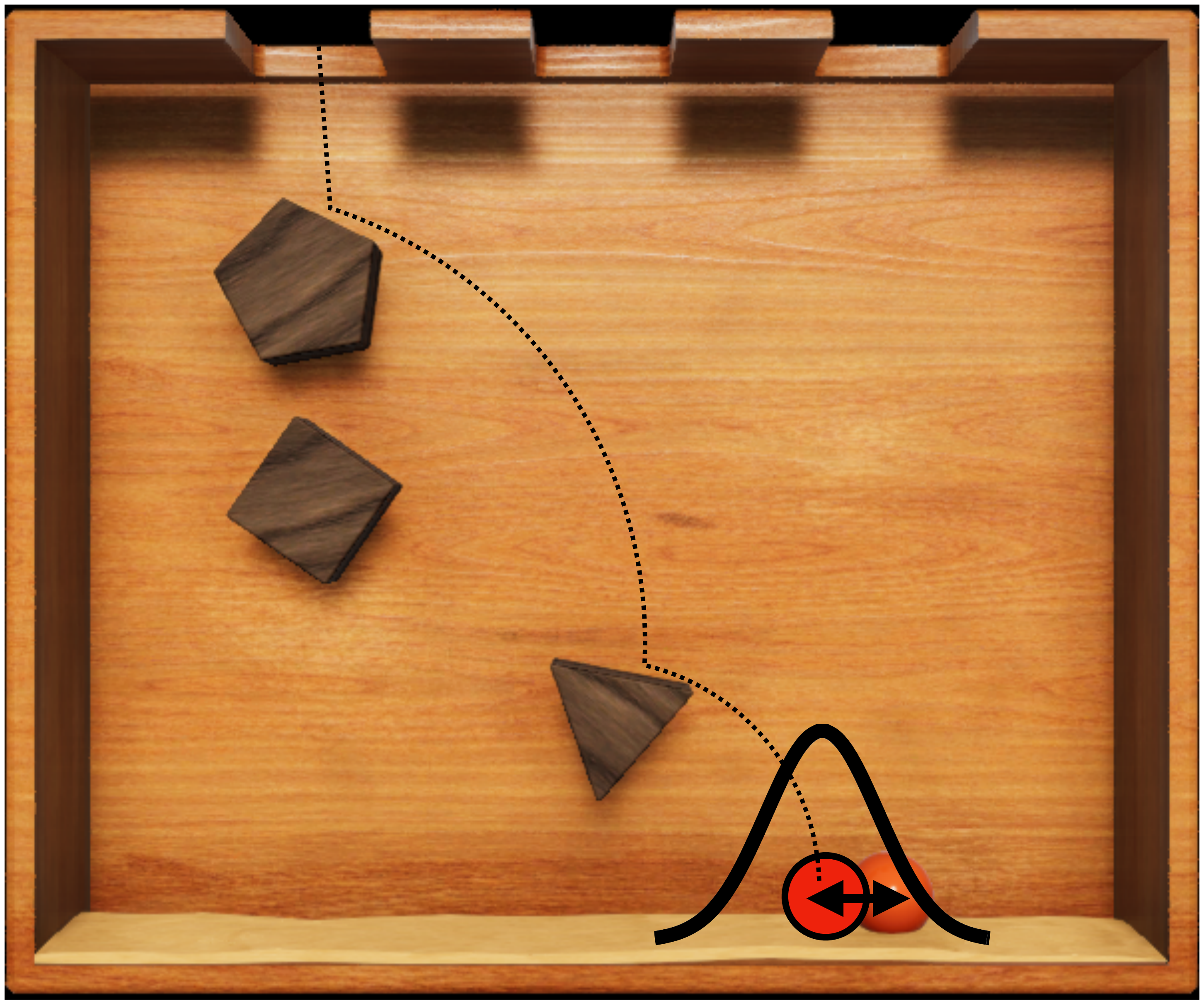


Likelihood of the ball ending up here if it was dropped in hole 1/2/3?

- drop the ball (with noise)
- look at the distance between the simulated ball and where the ball actually ended up
- convert this distance into a likelihood via a Gaussian distribution

Inference: In which hole was the ball dropped?

Uniform prior over the three different holes



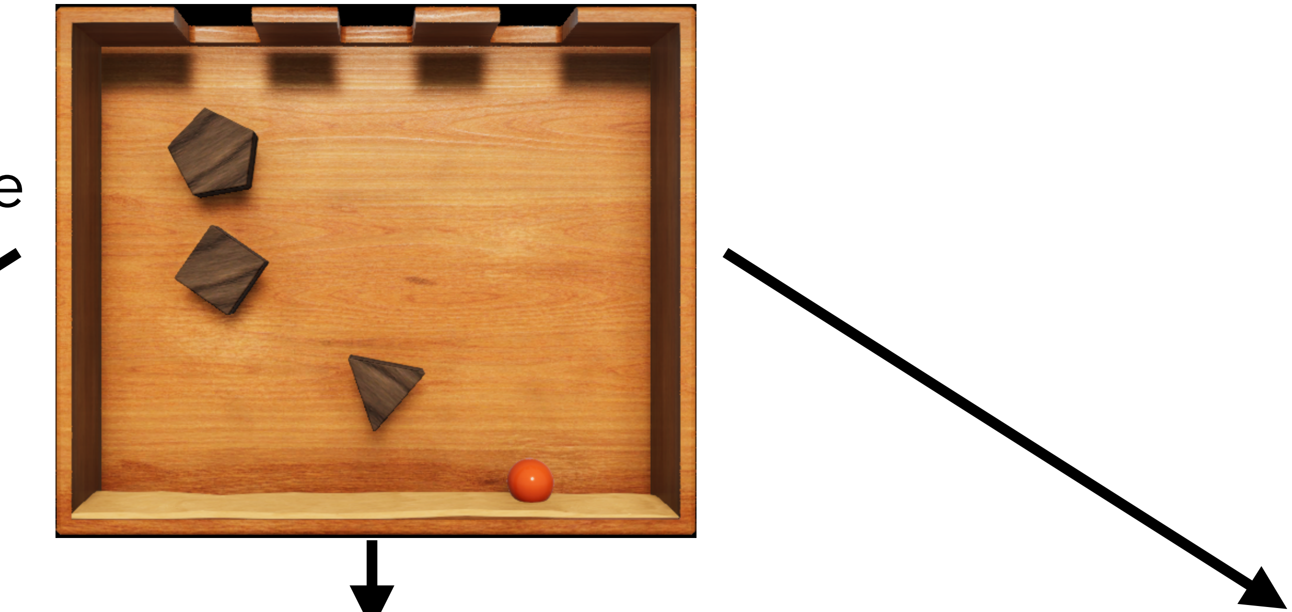
Likelihood of the ball ending up here if it was dropped in hole 1?

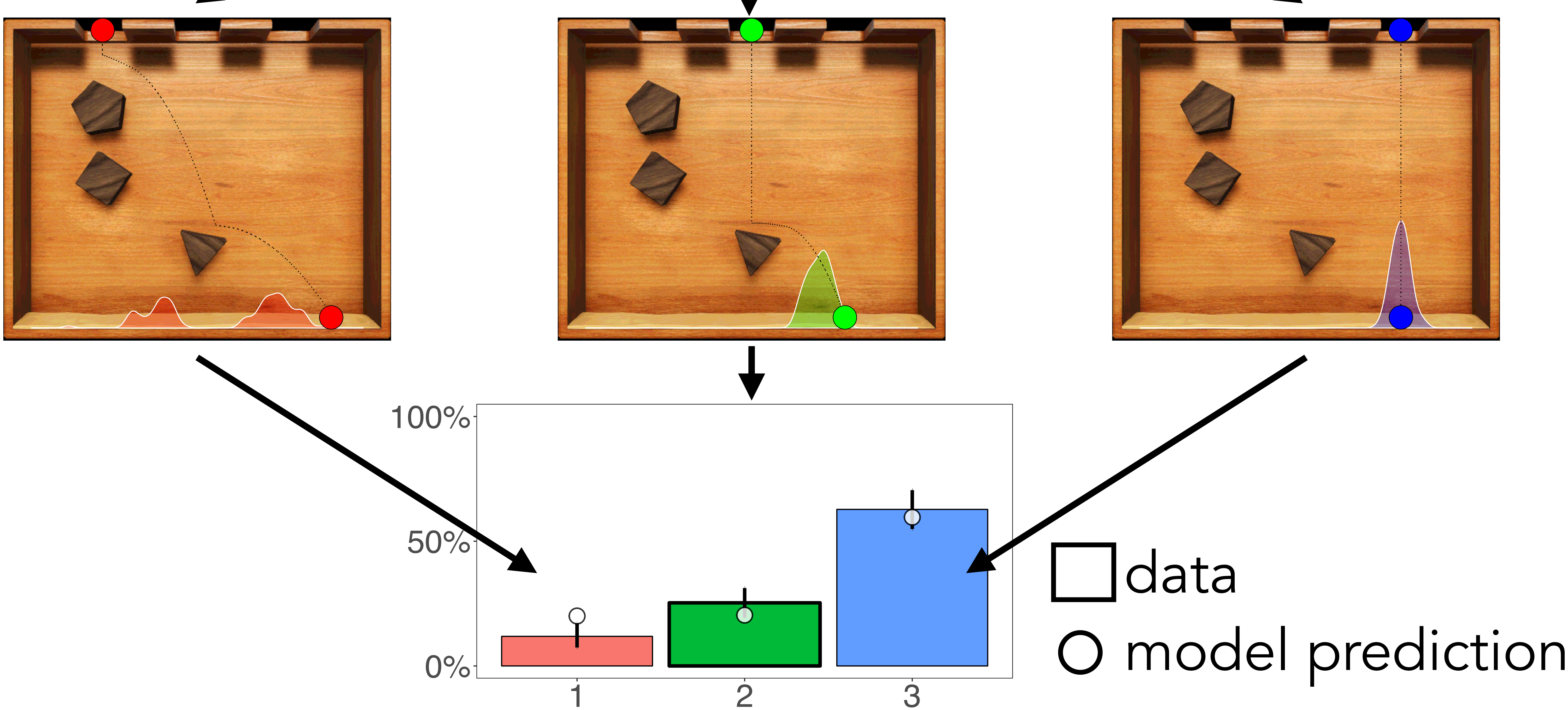
- drop the ball (with noise)
- look at the distance between the simulated ball and where the ball actually ended up
- convert this distance into a likelihood via a Gaussian distribution

Inference: In which hole was the ball dropped?

distance between ball's true x position and x position in sample

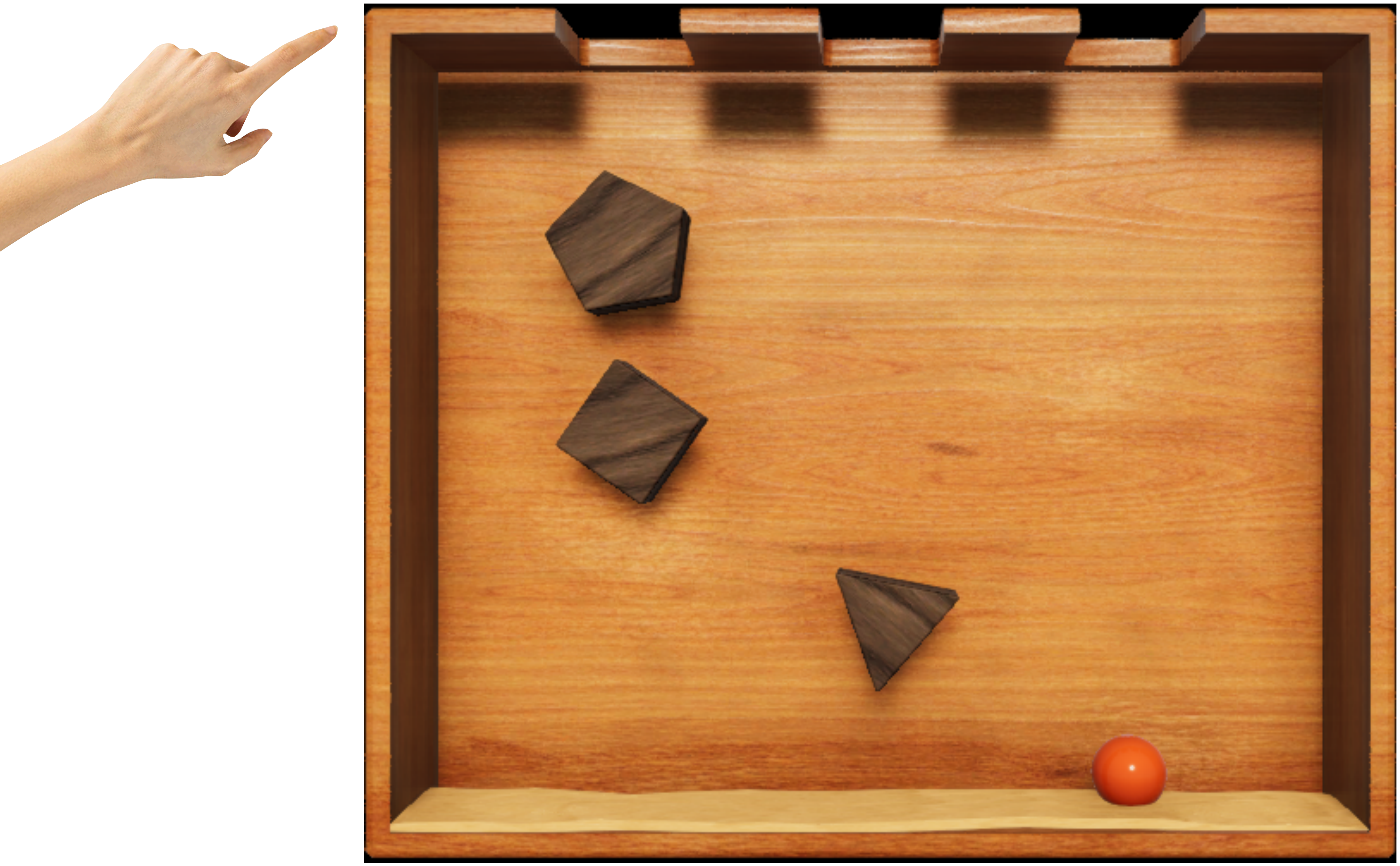
$$\exp\left(-\frac{d(\text{ball\_x}_{\text{final}}, \text{ball\_x}_{\text{hole}})}{2\sigma^2}\right)$$





Inference: In which hole was the ball dropped?

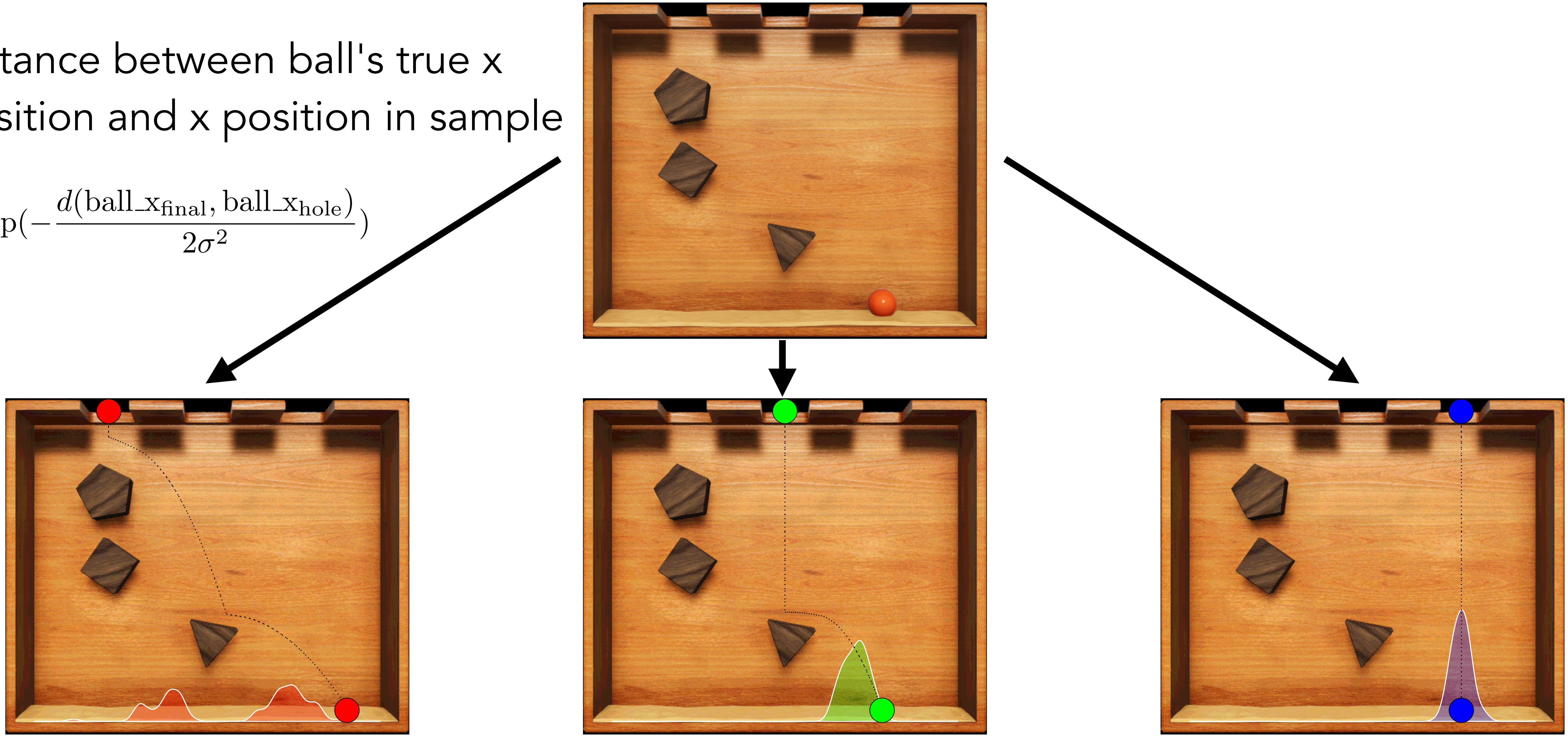
Inference: In which hole was the ball dropped?



Inference: In which hole was the ball dropped?

distance between ball's true x position and x position in sample

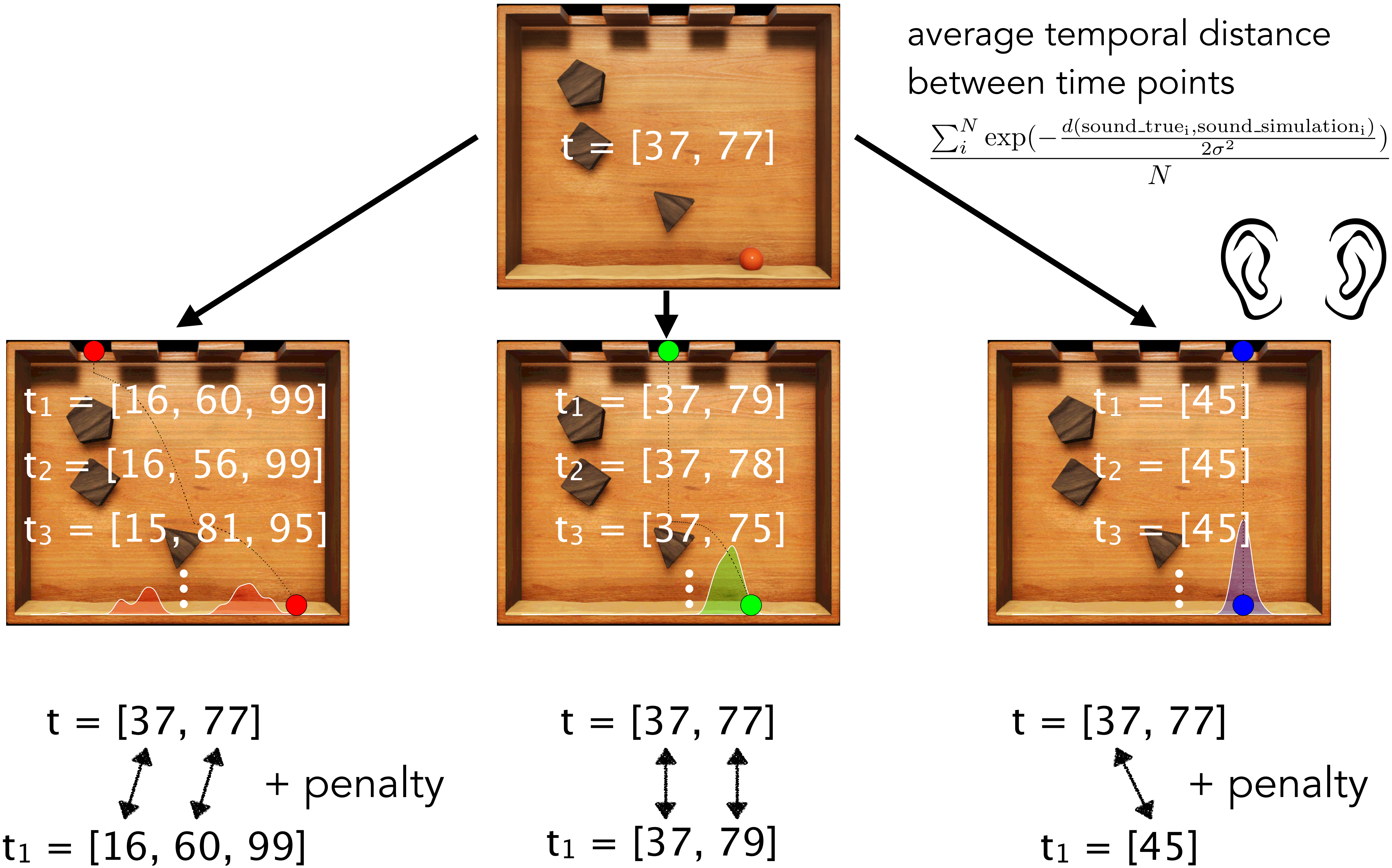
$$\exp\left(-\frac{d(\text{ball\_x}_{\text{final}}, \text{ball\_x}_{\text{hole}})}{2\sigma^2}\right)$$



Inference: In which hole was the ball dropped?

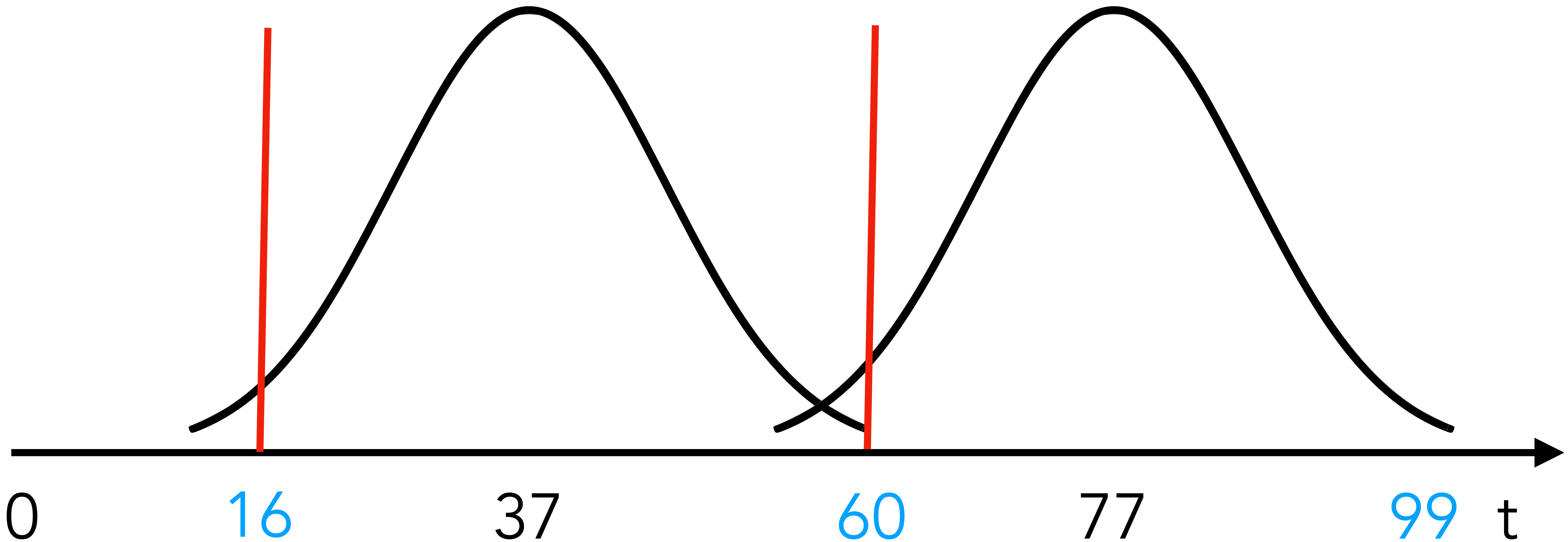
average temporal distance between time points

$$\frac{\sum_i^N \exp\left(-\frac{d(\mathrm{sound\_true_i}, \mathrm{sound\_simulation_i})}{2\sigma^2}\right)}{N}$$



Likelihood of a sequence of collision sounds

actual sounds $t = [37, 77]$



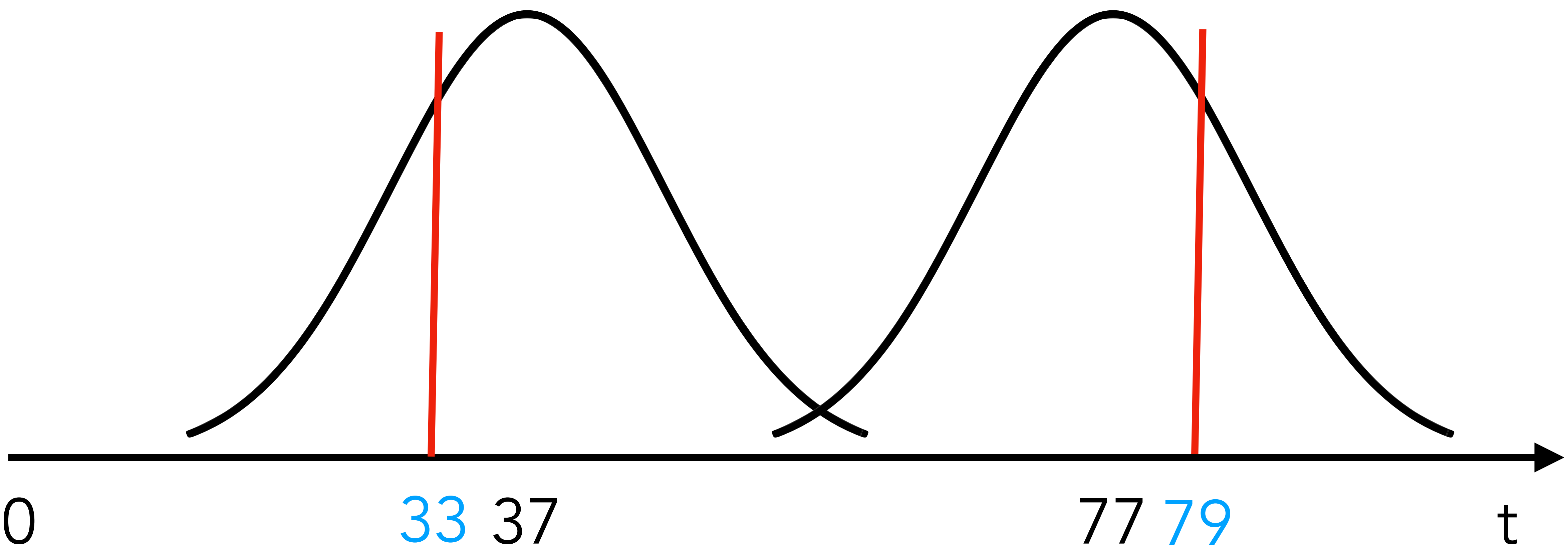
simulated sounds $t_1 = [16, 60, 99]$

fixed penalty

- determine which ones are closest to the actual sounds
- calculate their likelihood
- add a penalty for each additional (or missed) sound

Likelihood of a sequence of collision sounds

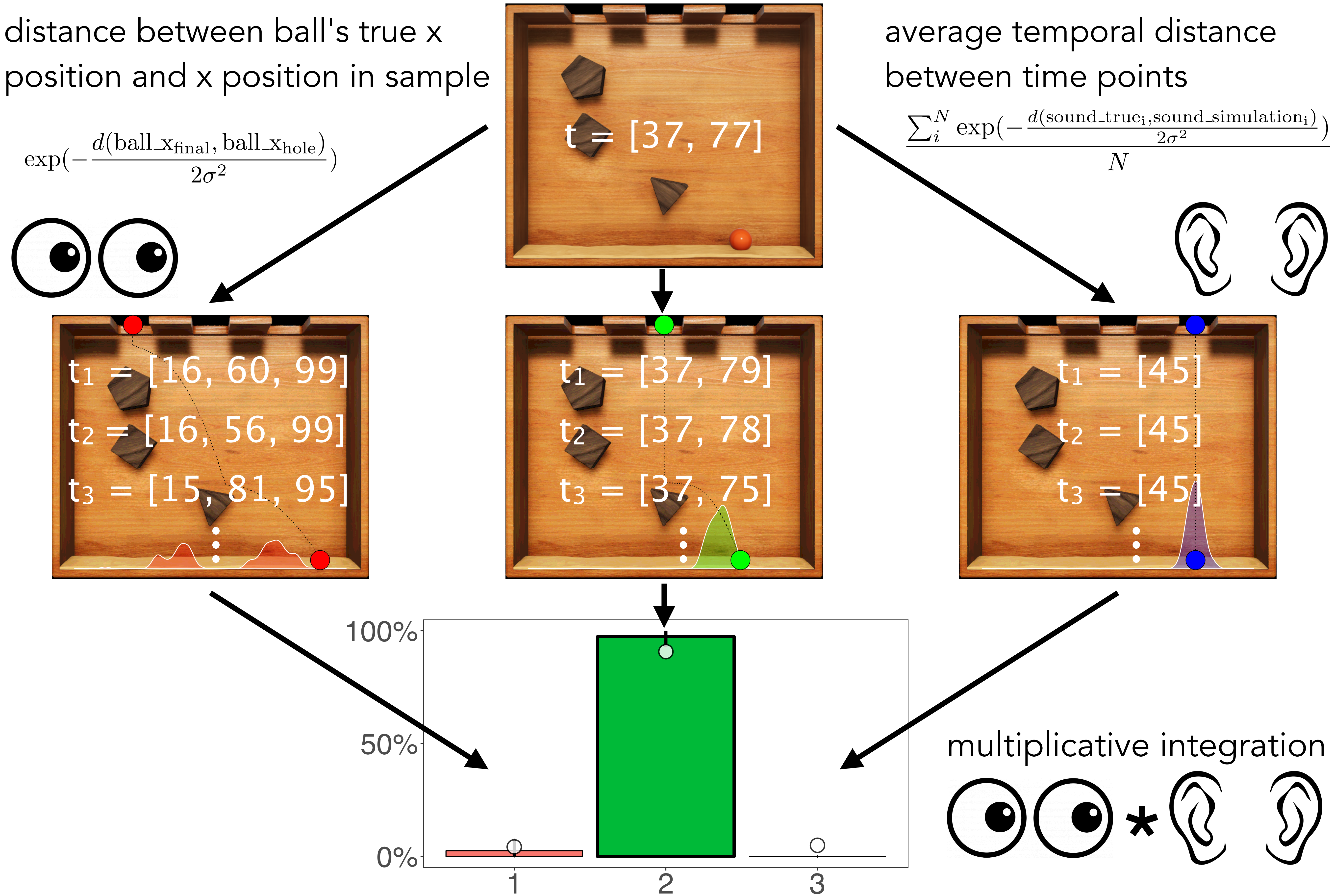
actual sounds t = [37, 77]



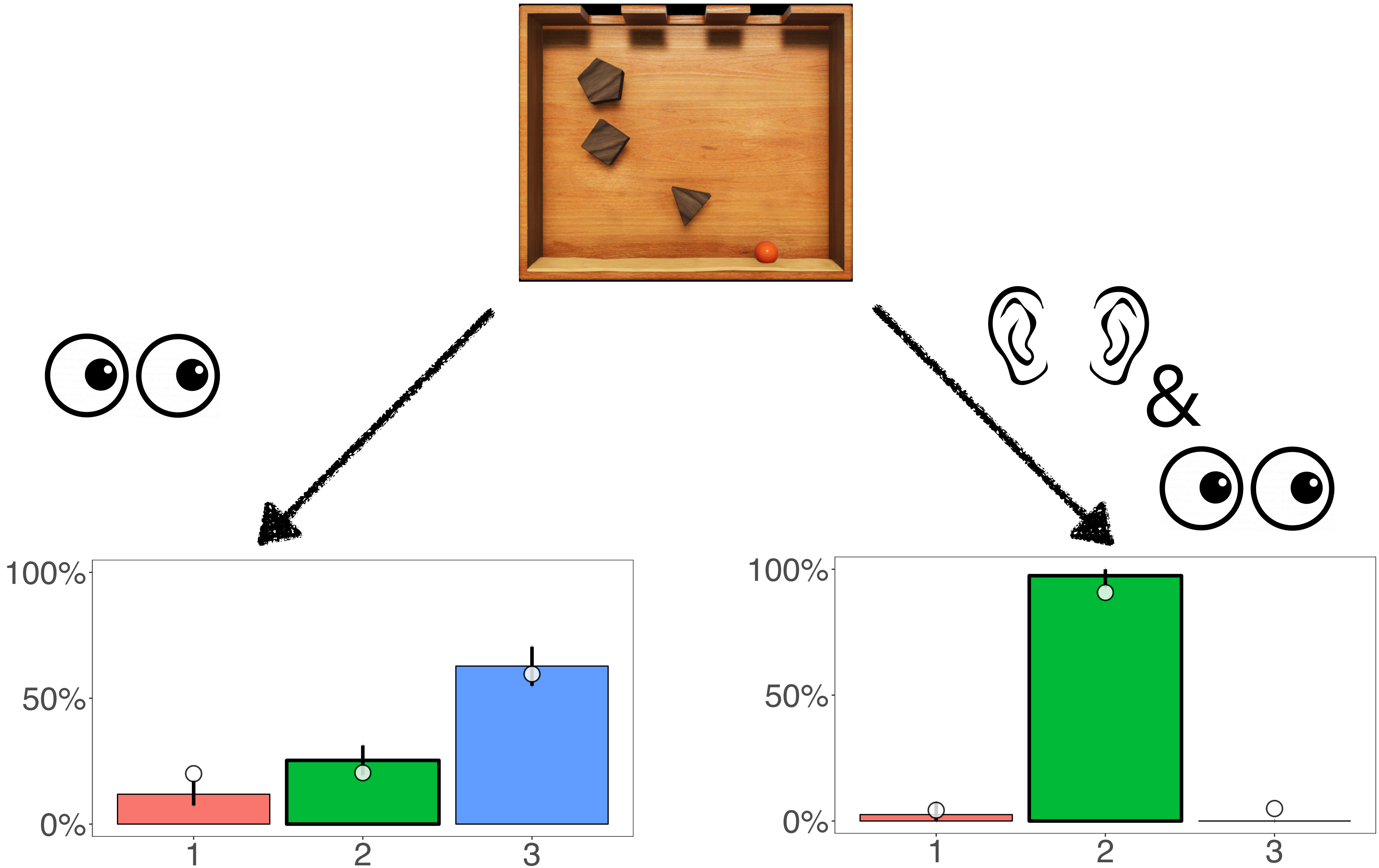
simulated sounds t_1 = [33, 79]

- determine which ones are closest to the actual sounds
- calculate their likelihood
- add a penalty for each additional (or missed) sound

Inference: In which hole was the ball dropped?



Inference: In which hole was the ball dropped?



Coin flip example

Coin flip example

Which coin did I flip?

Hypotheses

p = 0.1	p = 0.5	p = 0.9
		

Data

#8 tails, #2 heads

Bayesian Recipe

- Hypotheses
- Prior over hypotheses
- Data
- Likelihood of the data given the hypotheses
- Posterior over hypotheses given the data

+ a healthy dose of Bayes' rule

$$p(H \mid D) = \frac{p(D \mid H) \cdot p(H)}{p(D)}$$

Coin flip example

```r
# data
data = rep(0:1, c(8, 2))
```

8 tails and 2 heads

```r
# parameters
theta = c(0.1, 0.5, 0.9)
```

hypotheses

```r
# prior
prior = c(0.25, 0.5, 0.25)
```

prior over the hypotheses

```r
# likelihood
likelihood = dbinom(sum(data == 1), size = length(data), prob = theta)
```

binomial distribution

```r
# posterior
posterior = likelihood * prior / sum(likelihood * prior)
```

posterior

$$p(H \mid D) = \frac{p(D \mid H) \cdot p(H)}{p(D)}$$

law of total probability:

$$p(D) = \sum_{i=1}^{n} p(D \mid H_i) \cdot p(H_i)$$

Coin flip example

```
# data
data = rep(0:1, c(8, 2))

# parameters
theta = c(0.1, 0.5, 0.9)

# prior
prior = c(0.25, 0.5, 0.25)

# likelihood
likelihood = dbinom(sum(data == 1), size = length(data), prob = theta)

# posterior
posterior = likelihood * prior / sum(likelihood * prior)
```

8 tails and 2 heads

hypotheses

prior over the hypotheses

binomial distribution

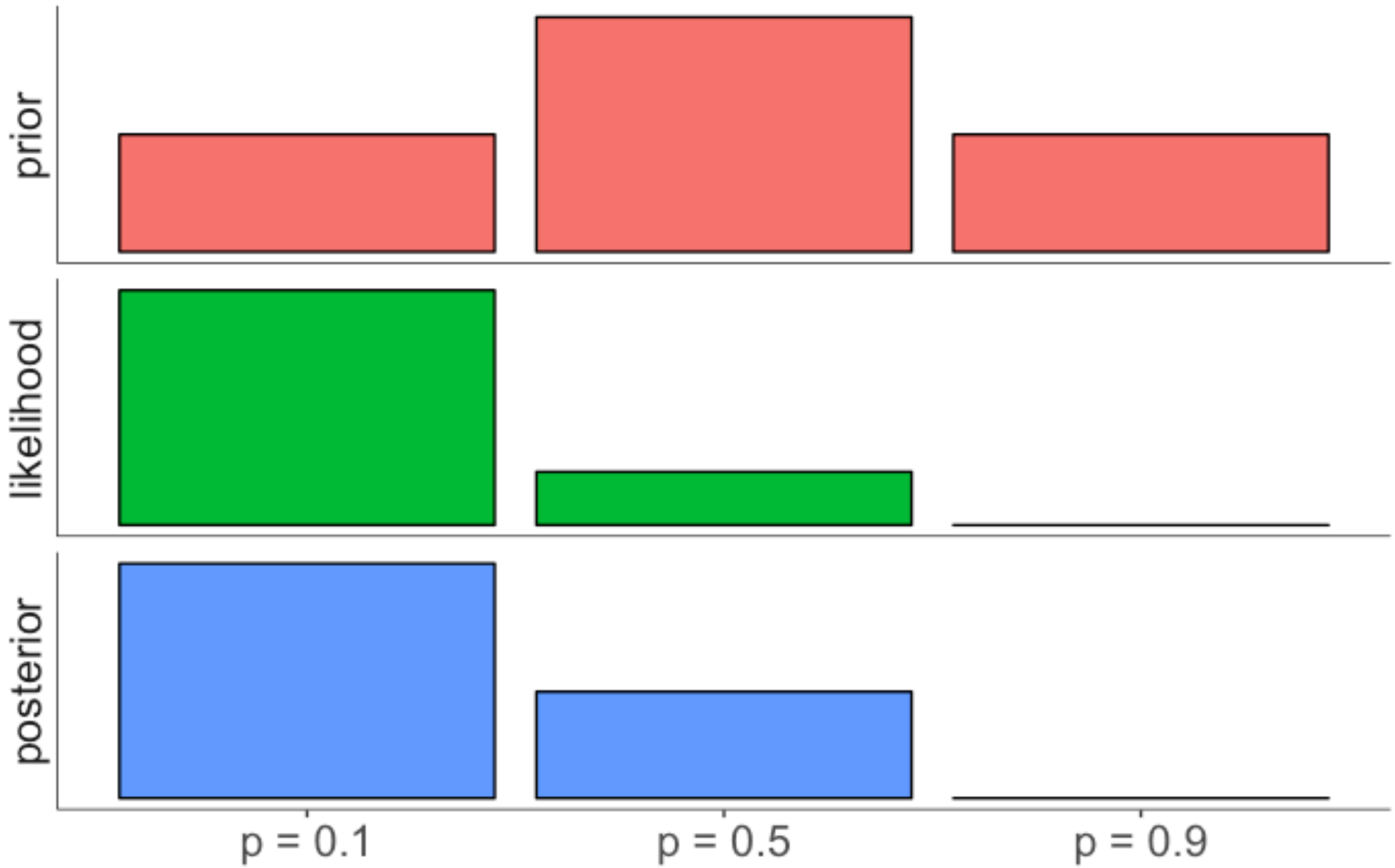
posterior

theta	prior	likelihood	posterior
0.1	0.25	0.19	0.69
0.5	0.50	0.04	0.31
0.9	0.25	0.00	0.00

Coin flip example

data: #8 tails, #2 heads

Which coin was flipped?



posterior = multiplicative weighting of prior and likelihood

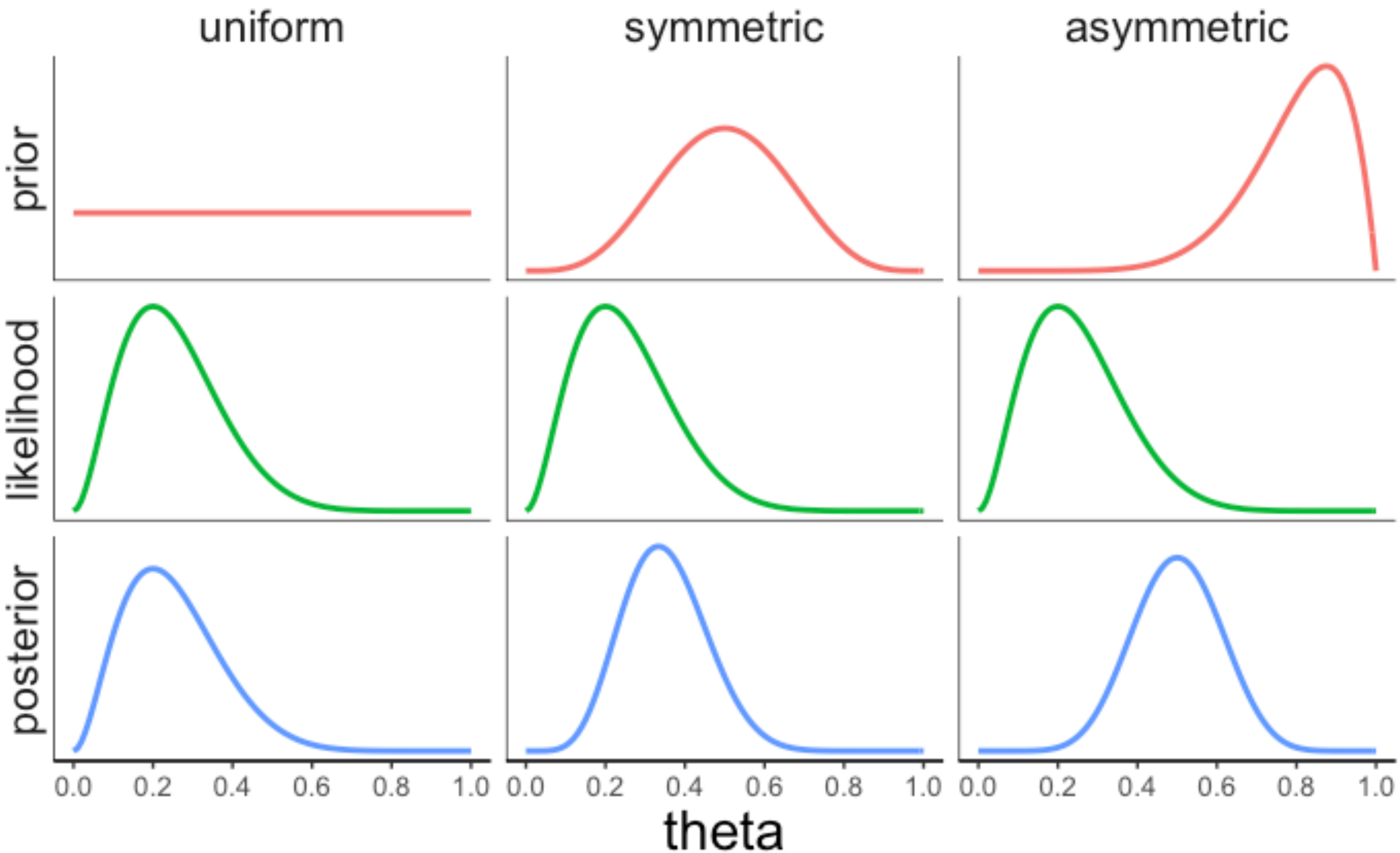
What affects the posterior?

What affects the posterior?

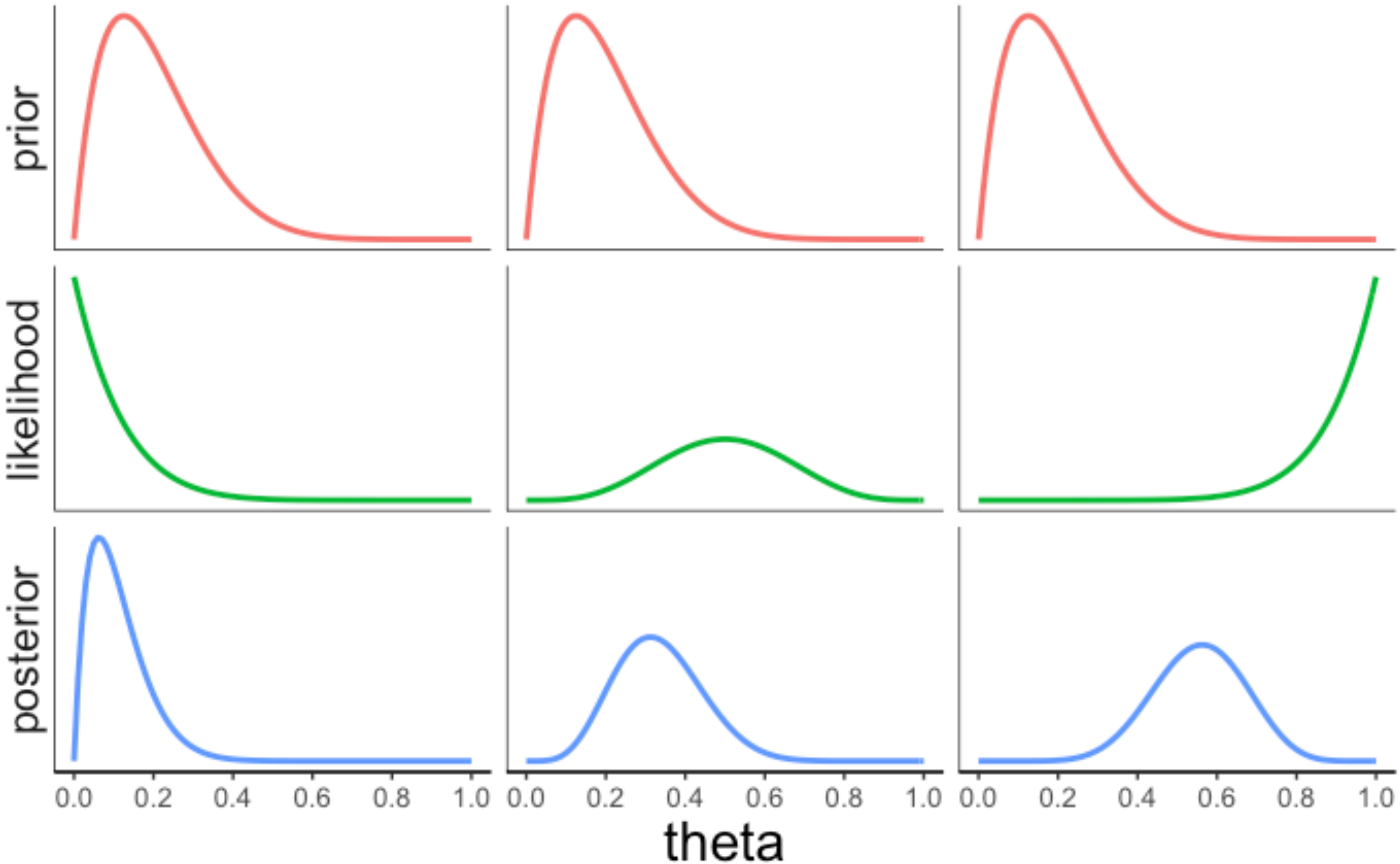
1. the prior over hypotheses
2. the likelihood of the data given each hypothesis

$$\underbrace{p(H \mid D)}_{\text{Posterior}} = \frac{\overbrace{p(D \mid H)}^{\text{Likelihood}} \cdot \overbrace{p(H)}^{\text{Prior}}}{\underbrace{p(D)}_{\text{Normalizing constant}}}$$

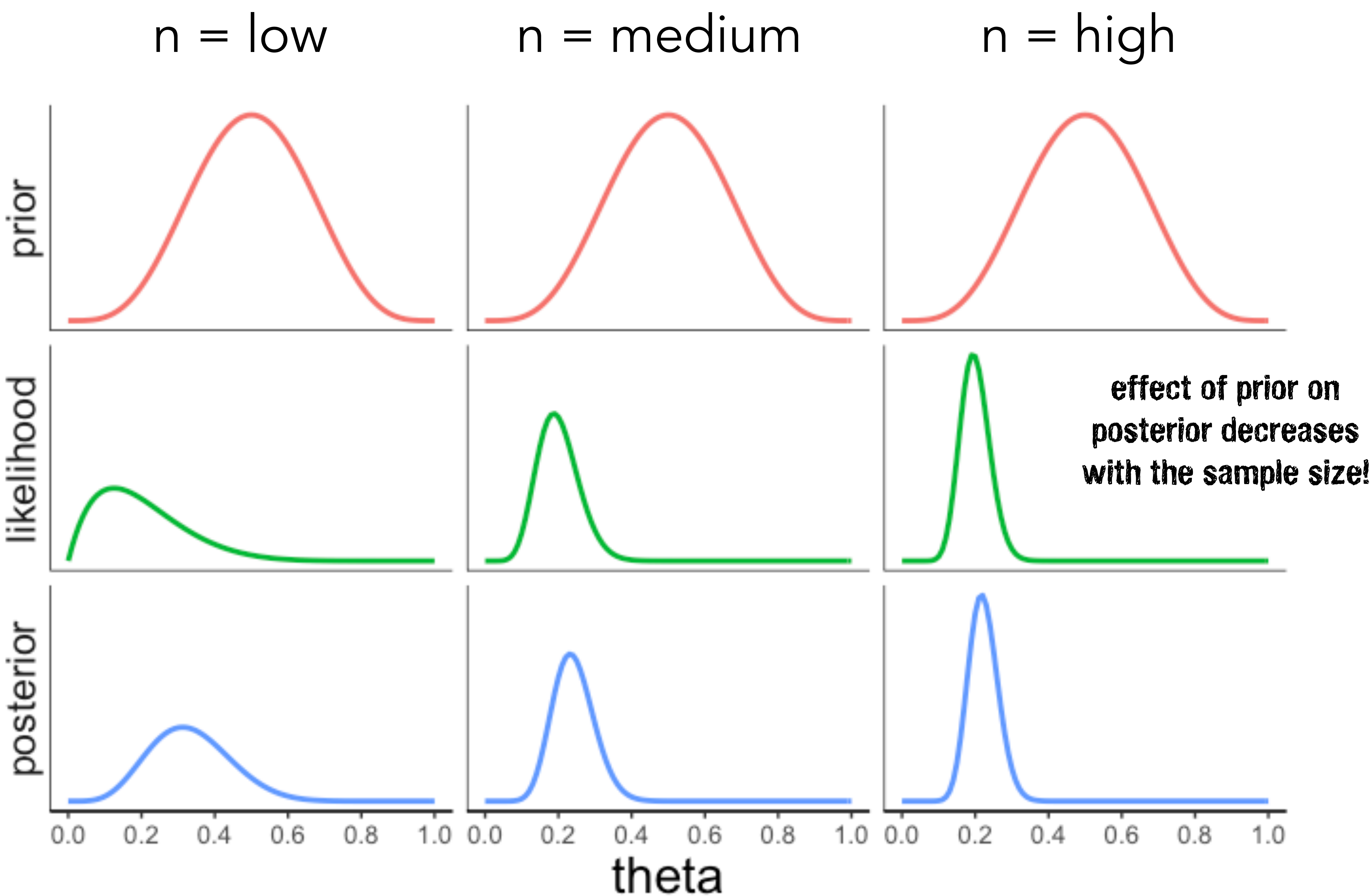
The effect of the **prior**



The effect of the **likelihood**



The effect of **sample size**



Summary

- Generalized linear model
  - Logistic regression
    - Simulating a logistic regression
    - Reporting results
  - Mixed effects logistic regression
- Bayesian data analysis
  - Comparison between frequentist and Bayesian data analysis
  - Quick flash from the past
  - A Bayesian model of multi-modal inference

Feedback

How was the pace of today's class?

How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?

Thank you!