PSYCH 252: Statistical Methods for Behavioral and Social Sciences

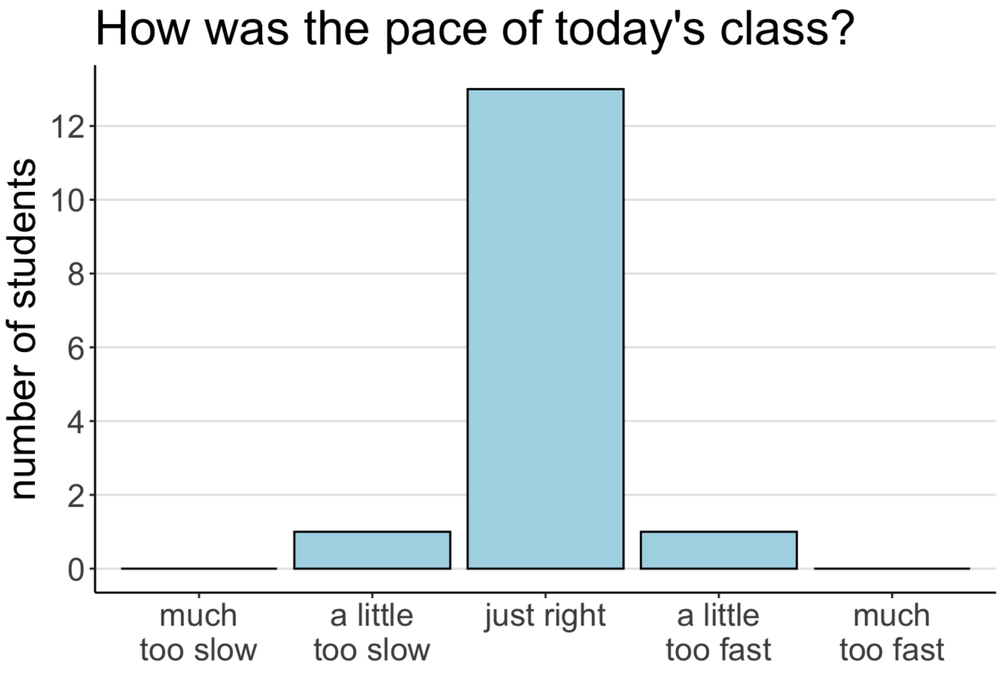
# Linear model 3

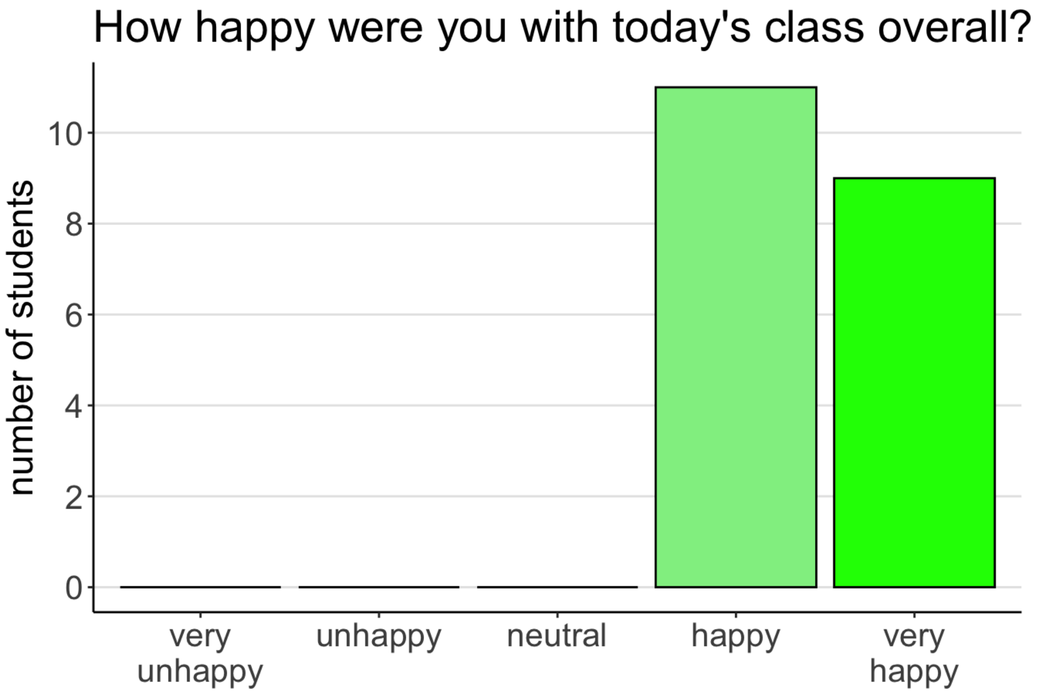


02/03/2025

# Feedback

# Your feedback




Tobi, you explain so well!! Thank you so much :)

Would be helpful to go backwards on each term. For example if you explain the use of a standard deviation also explain what would happen if you removed that or if you set it to a larger or smaller value

Might just be me but I have trouble seeing the red laser pointer on the screen. Could you try using your mouse or even your hand as well? Thanks!

I think we need more worked examples

# Things that came up

# Lecture Recordings

# Stanford Course Recording Policy

Recording and sharing your class lectures and in-class activities can make it easier for students to review class content or catch up after absences. However, there are many factors to consider. …

**Recordings should not broadly replace in-person attendance**

Communicate to your students that the recordings are intended to supplement in-class learning, provide flexibility for students with extenuating circumstances, or accommodate specific students with a disability. Recordings should not be a replacement for attending class in person. Consider strategies such as in-class learning activities and attendance policies to engage students and encourage attendance.

**Student permission to record is not required**

You do not need to obtain consent from students if the streaming and viewing of the recorded sessions are limited to in-class use, such as within the Canvas course site. However, we encourage you to provide advance notice and be transparent with students about whether the course will be recorded. The consent to use of photographic images policy provides more information.

# Stanford Course Recording Policy

**Do not share recordings publicly**

You should not post or share recordings beyond your Stanford Canvas course site. This is due to statutory and regulatory compliance issues concerning student privacy and copyright associated with publicly posting recorded lectures. Zoom and Panopto are tools within Canvas for securely posting videos for your students.

If you have a compelling reason to share the content outside of your class (i.e., make it public), you will need to obtain consent from the appropriate Dean's or Vice Provost's office.

**Stanford retains all intellectual property rights**

Course meeting recordings remain the intellectual property of Stanford University.

**Students should not record without permission or share recordings**

Students may not record audio or video of class meetings without the permission of the instructor (and guest presenters where applicable). The recording and broadcasting courses policy provides more details. ***If the instructor grants permission, students may keep recordings for personal use only and may not post, share, or otherwise distribute recordings.***

# Plan for today

- Quick recap
- Multiple regression
  - two continuous predictors
  - one categorical predictor
  - one continuous and one categorical predictor
- Interactions
- `lm()` output

# Quick recap

# Quick recap: Regression (conceptual)

Linear model: Simple regression

```
Data = Model + Error
```

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\hat{Y}_i = \beta_0 + \beta_1 X_i$$

the model is a linear combination of predictors

The general procedure

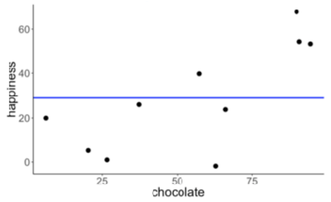
1. Define $H_0$ as Model C (compact) and $H_1$ as Model A (augmented)
2. Fit model parameters to the data
3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that $H_0$ is true)

$H_0$: Chocolate consumption and happiness are unrelated.

**Model C**

$$Y_i = \beta_0 + \epsilon_i$$

**Model prediction**




**Fitted model**

$$Y_i = 28.88 + e_i$$

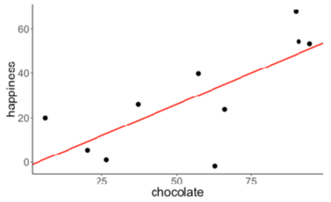
$H_1$: Chocolate consumption and happiness are related.

**Model A**

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

chocolate consumption

**Model prediction**



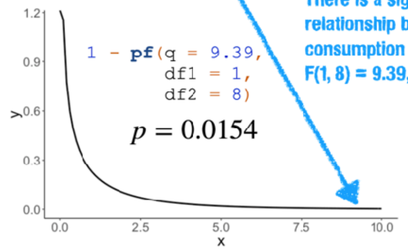

**Fitted model**

$$Y_i = -2.04 + 0.56 X_i + e_i$$

Decide whether it's **worth it**

- To compute the *F* statistic, we need:
  - PRE = 0.54
  - PC = 1
  - PA = 2
  - $n$ = 10

$$F = \frac{\text{PRE}/(\text{PA} - \text{PC})}{(1 - \text{PRE})/(n - \text{PA})} = \frac{0.54/(2-1)}{(1-0.54)/(10-2)} = 9.39$$




There is a significant relationship between chocolate consumption and happiness $F(1, 8) = 9.39$, $p = .0154$.

# Quick recap: Regression (in R)

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\text{balance}_i = b_0 + b_1 \cdot \text{income}_i + e_i$$

```
fit = lm(formula = balance ~ 1 + income, data = df.credit)
```

outcome — intercept (doesn't need to be specified explicitly) — predictor — data


```
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ 1 + income
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    399 84339912
2    398 66208745  1  18131167 108.99 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

$$\text{PRE} = 1 - \frac{66208745}{84339912} \approx 0.215$$

The augmented model reduces the error by 21.5%.

```
lm(balance ~ 1 + income, data = df.credit) %>%
  summary()
```

$R^2$

```
Residual standard error: 407.9 on 398 degrees of freedom
Multiple R-squared:  0.215,	Adjusted R-squared:  0.213
F-statistic:   109 on 1 and 398 DF,  p-value: < 2.2e-16
```

## The general procedure

1. Define $H_0$ as Model C (compact) and $H_1$ as Model A (augmented)
2. Fit model parameters to the data

```
fit_c = lm(formula = balance ~ 1,
           data = df.credit)

fit_a = lm(formula = balance ~ 1 + income,
           data = df.credit)
```

---

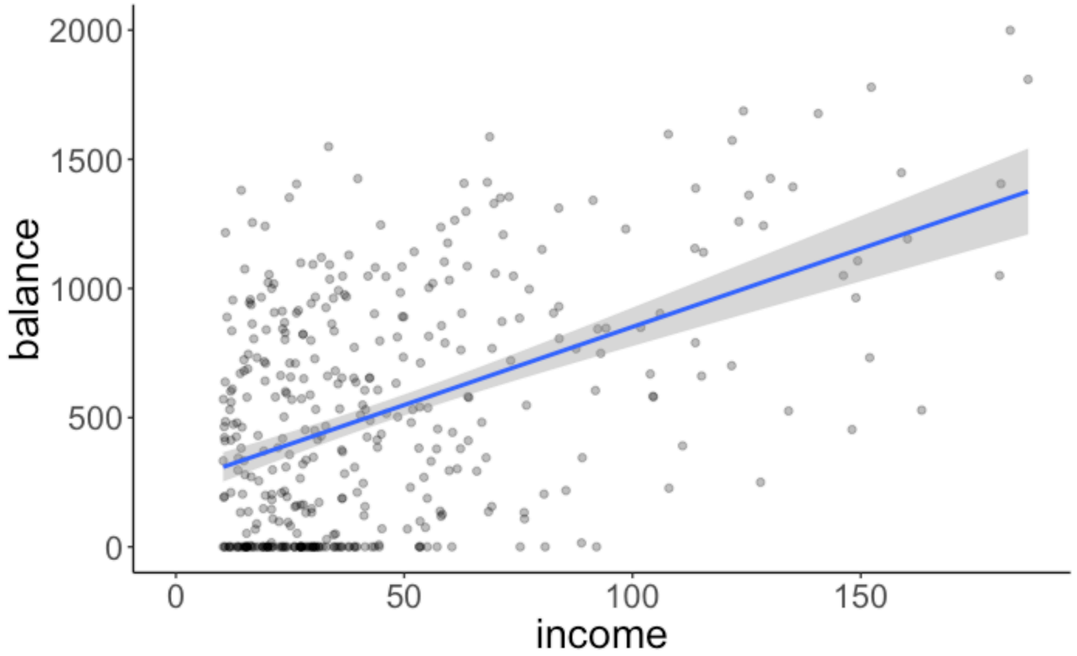
3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that $H_0$ is true)

```
anova(fit_c, fit_a)
```

70

# Quick recap: Reporting results

plot




statistical test

There is a significant relationship between a person's income and the average balance on their credit cards

$F(1, 389) = 108.99$, $p < .001$, $r = .463$.

effect size measure

With each additional $1000 of income, the average balance is predicted to increase by $6.05 [4.91, 7.19] (95% CI).

parameter estimate with confidence interval

# Multiple regression

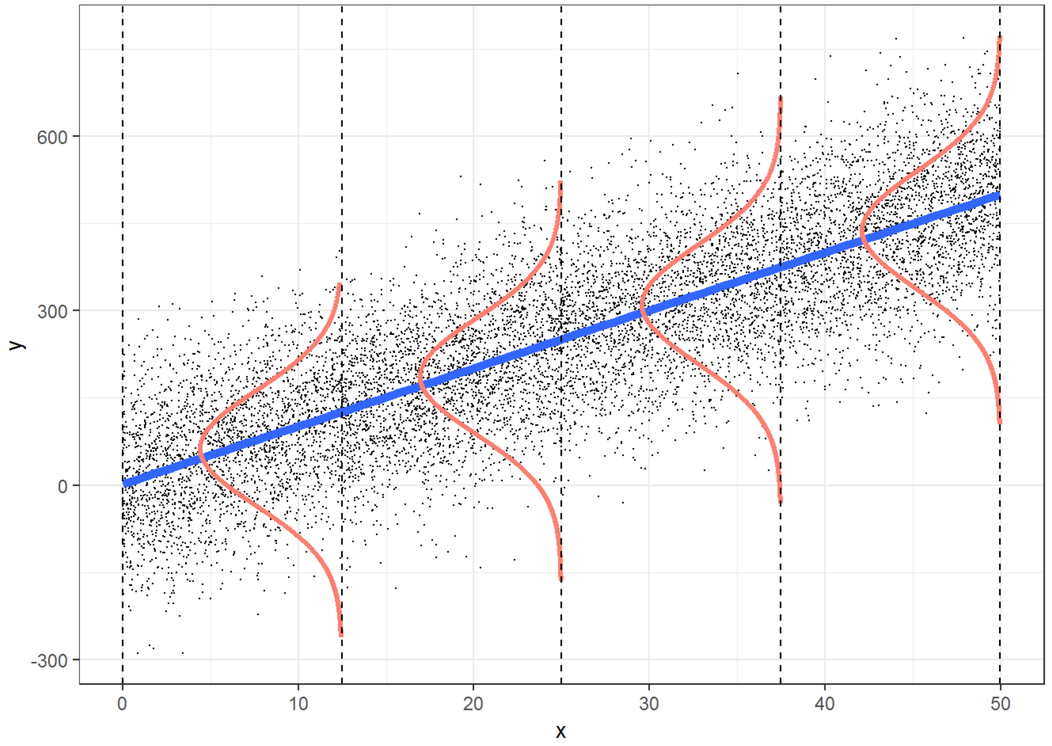
# Model assumptions of **simple** regression

- independent observations
- *Y* is continuous
- errors are normally distributed

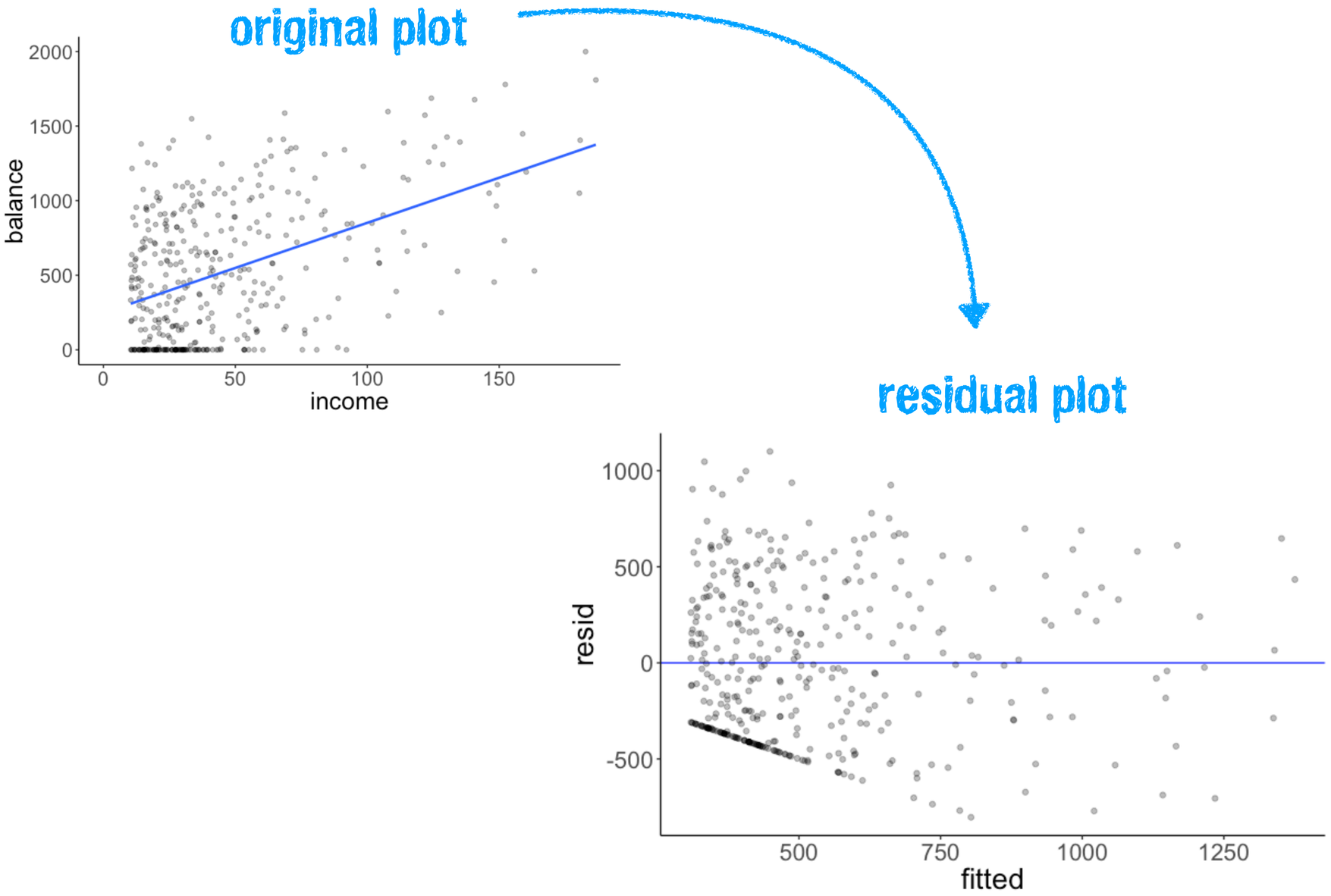
the dependent variable doesn't need to be normally distributed!

- errors have constant variance
- error terms are uncorrelated

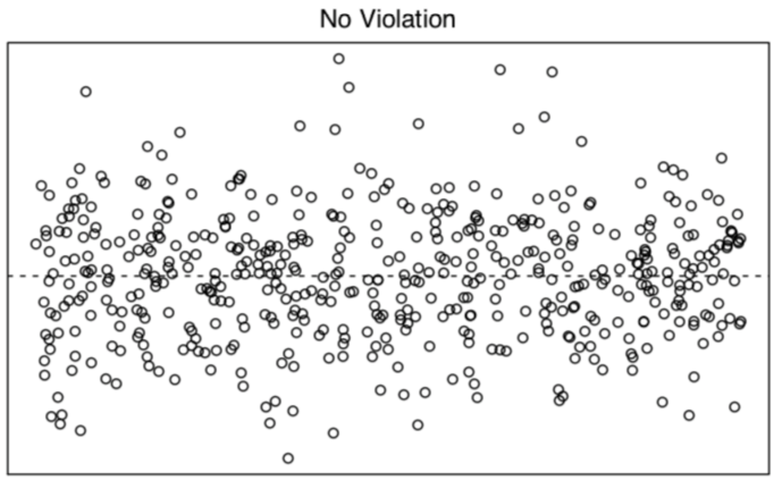
i.i.d = Independent and identically distributed (homoscedasticity of errors)

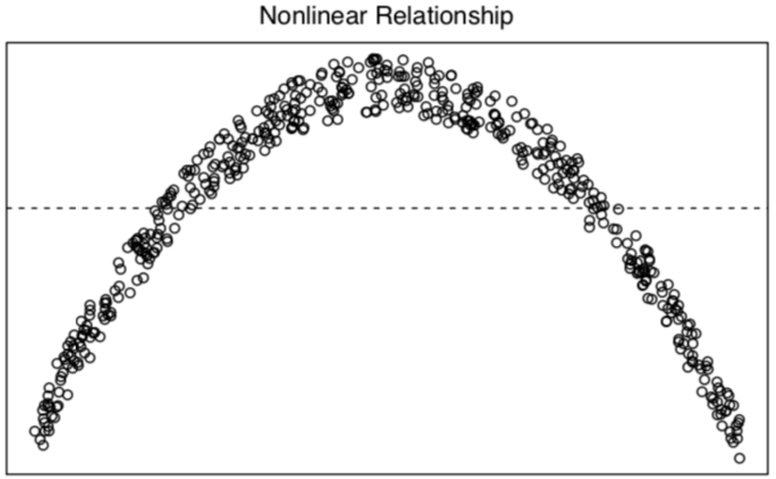


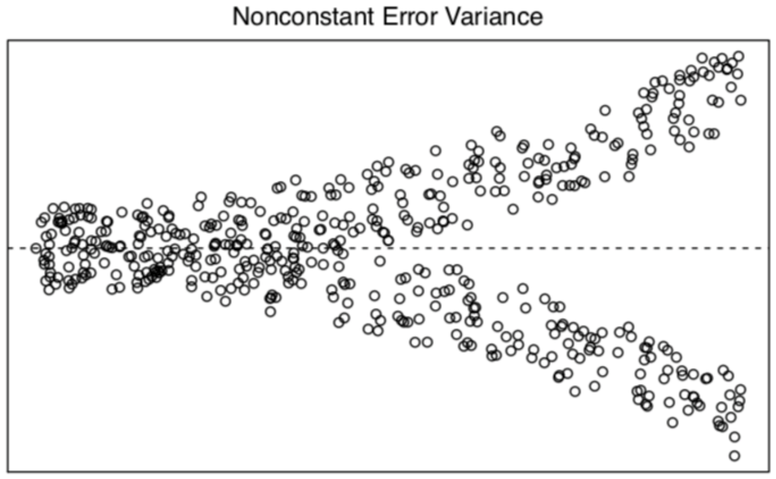
# Model assumptions of simple regression

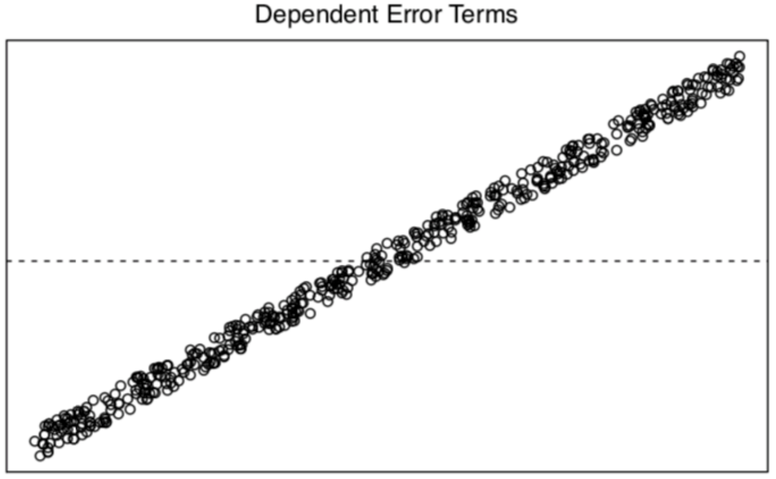


# Model assumptions of simple regression



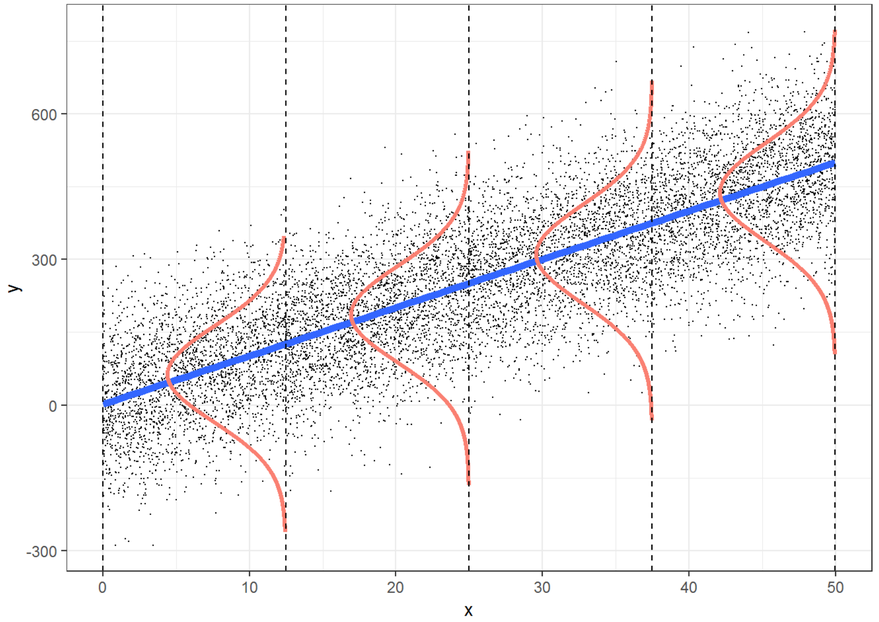



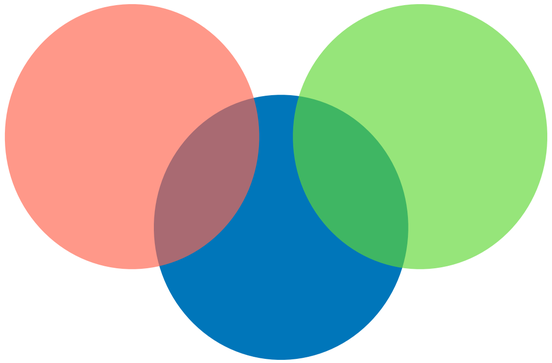
# Assumptions of **multiple** regression

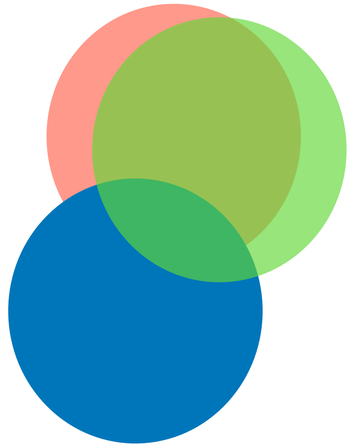
- independent observations
- *Y* is continuous
- errors are normally distributed




- errors have constant variance
- error terms are uncorrelated
- **no multicollinearity**

predictors in the model should not be highly correlated with each other





# Linear model

$$\texttt{Data} = \texttt{Model} + \texttt{Error}$$

**Simple regression**

$$Y_i = \beta_0 + \beta_1 X_{1i} + \epsilon_i$$

one predictor

**Multiple regression**

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_p X_{pi} + \epsilon_i$$

many predictors

# Advertising data set

money spent on different media (x $1000)

sales (x 1000)

- Combine several predictors to explain an outcome variable of interest

| index | tv | radio | newspaper | sales |
|---|---|---|---|---|
| 1 | 230.1 | 37.8 | 69.2 | 22.1 |
| 2 | 44.5 | 39.3 | 45.1 | 10.4 |
| 3 | 17.2 | 45.9 | 69.3 | 9.3 |
| 4 | 151.5 | 41.3 | 58.5 | 18.5 |
| 5 | 180.8 | 10.8 | 58.4 | 12.9 |
| 6 | 8.7 | 48.9 | 75.0 | 7.2 |
| 7 | 57.5 | 32.8 | 23.5 | 11.8 |
| 8 | 120.2 | 19.6 | 11.6 | 13.2 |
| 9 | 8.6 | 2.1 | 1.0 | 4.8 |
| 10 | 199.8 | 2.6 | 21.2 | 10.6 |

Model C

**Simple regression**

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + e_i$$

Model A

**Multiple regression**

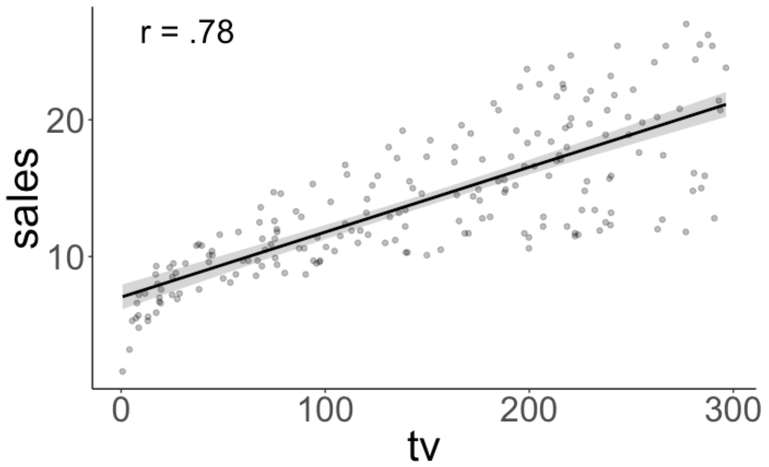
$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

Can we predict sales better when we consider radio ads in addition to TV ads?

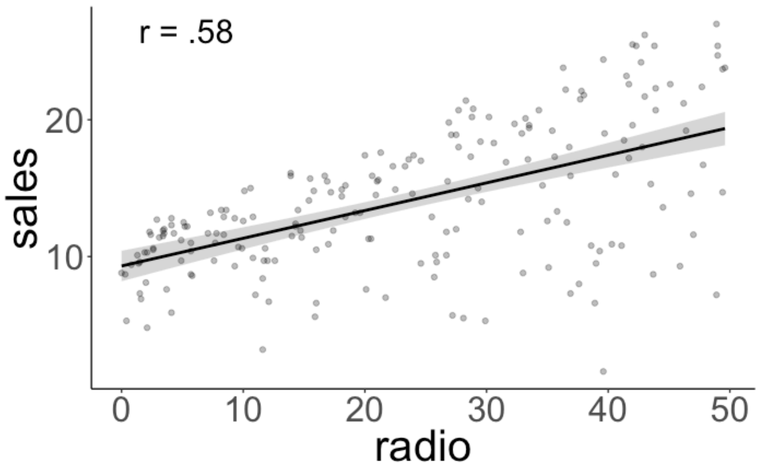
"Controlling" for TV ads, do radio ads explain any additional variance in sales?

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

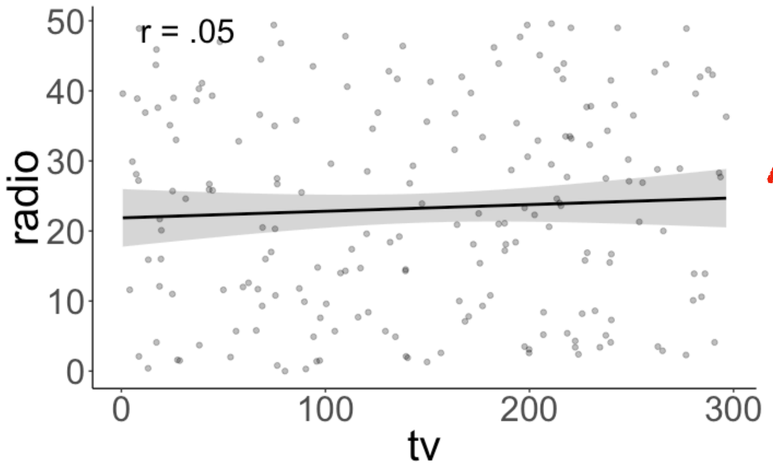
**Relationship between TV ads and sales**




**Relationship between radio ads and sales**




**Relationship between TV ads and radio ads**




**predictors are not correlated, yay!**

# Visualizing correlations

```
library("corrr")
```


```
1 df.credit %>%
2   select_if(is.numeric) %>%
3   correlate() %>%
4   rearrange() %>%
5   shave() %>%
6   fashion()
```

| rowname | index | newspaper | radio | sales | tv |
|---|---|---|---|---|---|
| index | | | | | |
| newspaper | -0.15 | | | | |
| radio | -0.11 | 0.35 | | | |
| sales | -0.05 | 0.23 | 0.58 | | |
| tv | 0.02 | 0.06 | 0.05 | 0.78 | |

# Can we predict sales better when we consider radio in addition to TV ads?

1. Define $H_0$ as Model C (compact) and $H_1$ as Model A (augmented)
2. Fit model parameters to the data
3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that $H_0$ is true)

$H_0$: Radio ads and sales are not related once we control for TV ads.

**Model C**

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + e_i$$

$H_1$: Radio ads and sales are related even when we control for TV ads.

**Model A**

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

```
1 # fit the models
2 fit_c = lm(sales ~ 1 + tv, data = df.ads)
3 fit_a = lm(sales ~ 1 + tv + radio, data = df.ads)
4
5 # do the F test
6 anova(fit_c, fit_a)
```

we reject the $H_0$

```
Analysis of Variance Table

Model 1: sales ~ 1 + tv
Model 2: sales ~ 1 + tv + radio
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1    198 2102.53
2    197  556.91  1    1545.6 546.74 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

# Evaluating the model: Model fit

```
fit_a %>%
  glance()
```

| r.squared | adj.r.squared | sigma | statistic | p.value | df | logLik | AIC | BIC | deviance | df.residual |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.897 | 0.896 | 1.681 | 859.618 | 0 | 3 | -386.197 | 780.394 | 793.587 | 556.914 | 197 |

| | |
|---|---|
| `r.squared` | The percent of variance explained by the model |
| `adj.r.squared` | r.squared adjusted based on the degrees of freedom |
| `sigma` | The square root of the estimated residual variance |
| `statistic` | F-statistic |
| `p.value` | p-value from the F test, describing whether the full regression is significant |
| `df` | Degrees of freedom used by the coefficients |
| `logLik` | the data's log-likelihood under the model |
| `AIC` | the Akaike Information Criterion |
| `BIC` | the Bayesian Information Criterion |
| `deviance` | deviance |
| `df.residual` | residual degrees of freedom |

# Evaluating the model: Model fit

```
fit_a %>%
  glance()
```

| r.squared | adj.r.squared | sigma | statistic | p.value | df | logLik | AIC | BIC | deviance | df.residual |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.897 | 0.896 | 1.681 | 859.618 | 0 | 3 | -386.197 | 780.394 | 793.587 | 556.914 | 197 |

**Compact Model**

$$\text{sales}_i = b_0 + e_i$$

The augmented model reduces 89.7% of the error compared to a compact model that just predicts the mean.

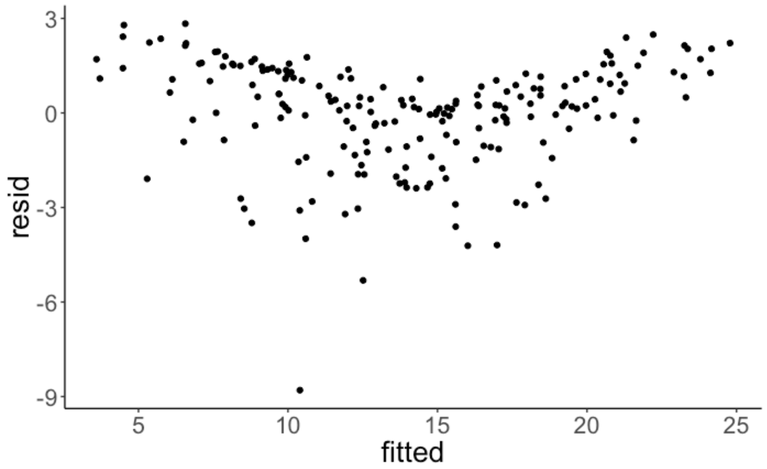
**Augmented Model**

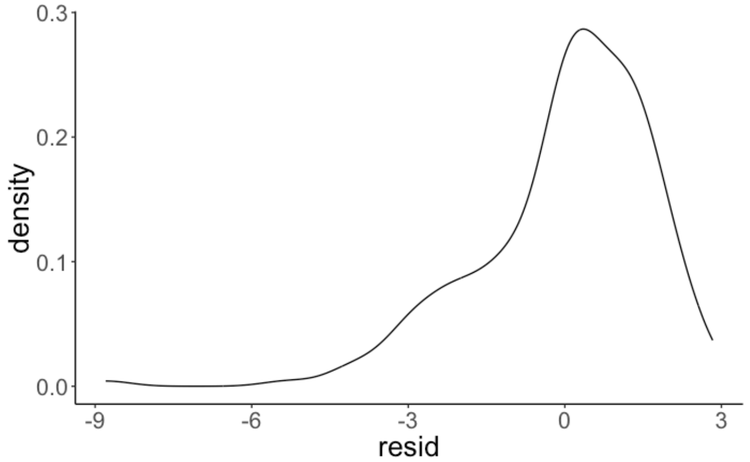
$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

$$\text{PRE} = 1 - \frac{\text{SSE}(A)}{\text{SSE}(C)} = R^2$$

# Evaluating the model: Residual plots

```
resid = sales - fitted
```




**OKish overall**

# Interpreting the results

```
fit_a %>%
  tidy(conf.int = T)
```

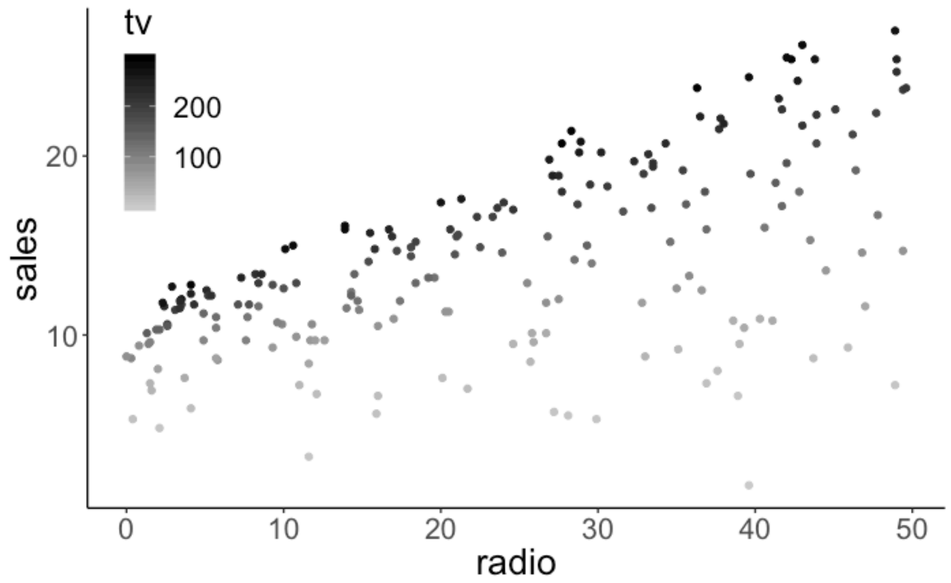
| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|---|---|---|---|---|---|---|
| (Intercept) | 2.92 | 0.29 | 9.92 | 0 | 2.34 | 3.50 |
| tv | 0.05 | 0.00 | 32.91 | 0 | 0.04 | 0.05 |
| radio | 0.19 | 0.01 | 23.38 | 0 | 0.17 | 0.20 |

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

$$\widehat{\text{sales}}_i = 2.92 + 0.05 \cdot \text{tv}_i + \boxed{0.19 \cdot \text{radio}_i}$$

For a given amount of TV advertising, an additional $1000 on radio advertising is associated with an increase in sales by 190 units.

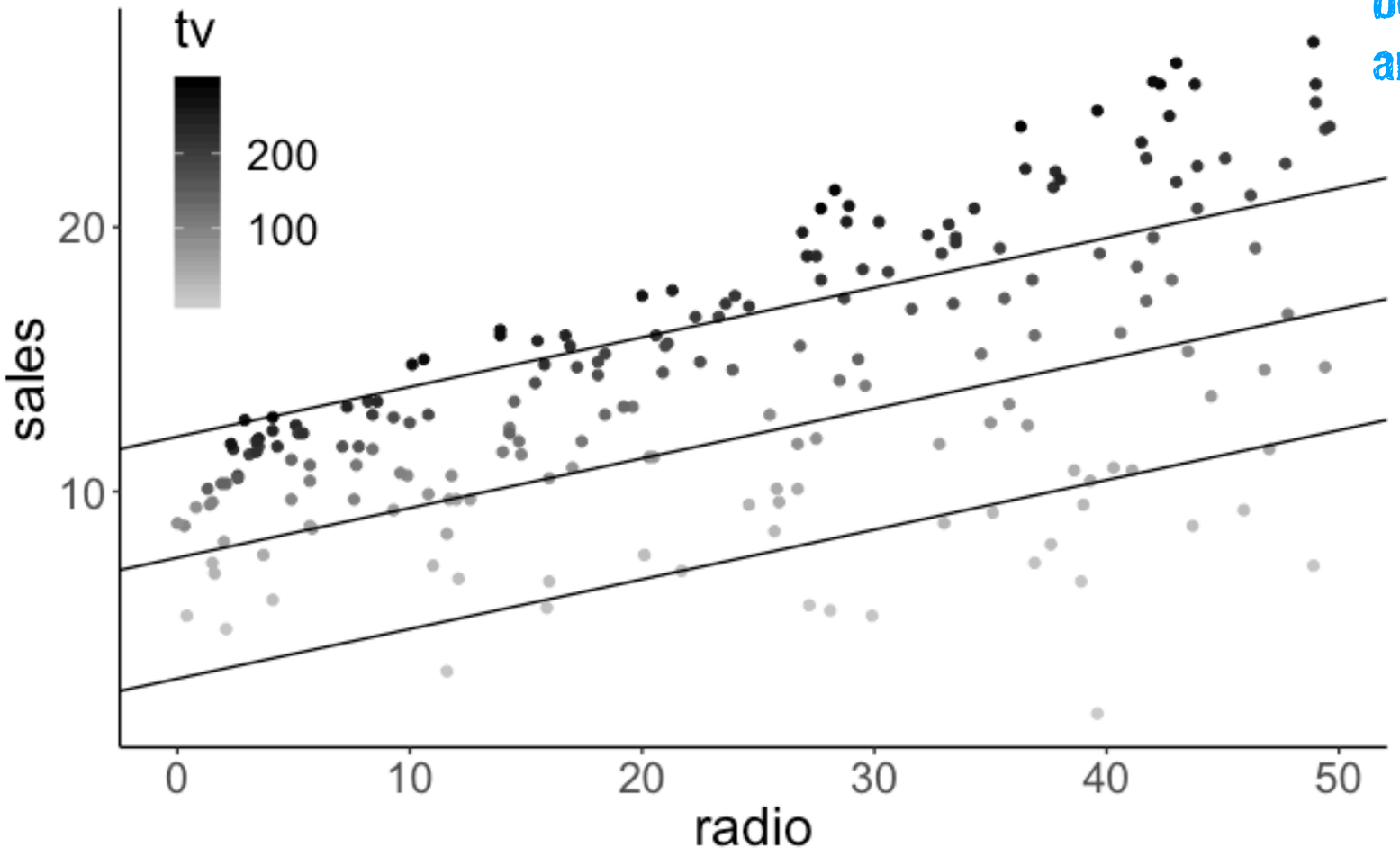
# Reporting results




There is a significant relationship between sales and radio ads, controlling for TV ads $F(1, 197) = 546.74$, $p < .001$.

Holding TV ads fixed, an increase in $1000 on radio ads is associated with an increase in sales by 190 units [170, 200] (95% confidence intervals).

# Visualizing the results




predicted relationship between radio and sales when:

tv = 200

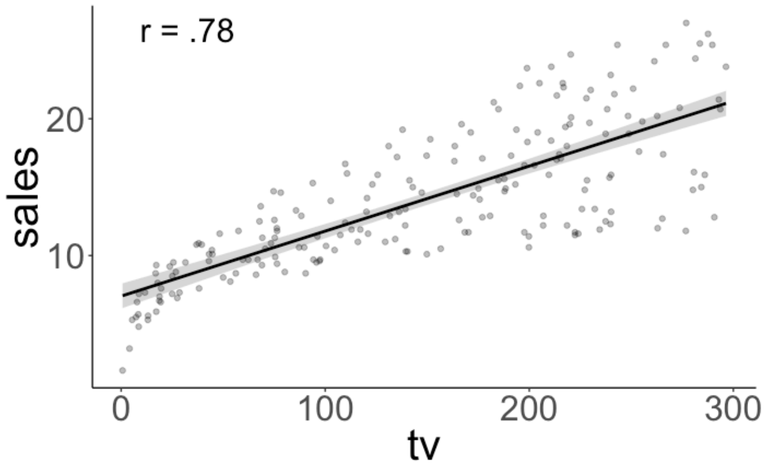
tv = 100

tv = 0

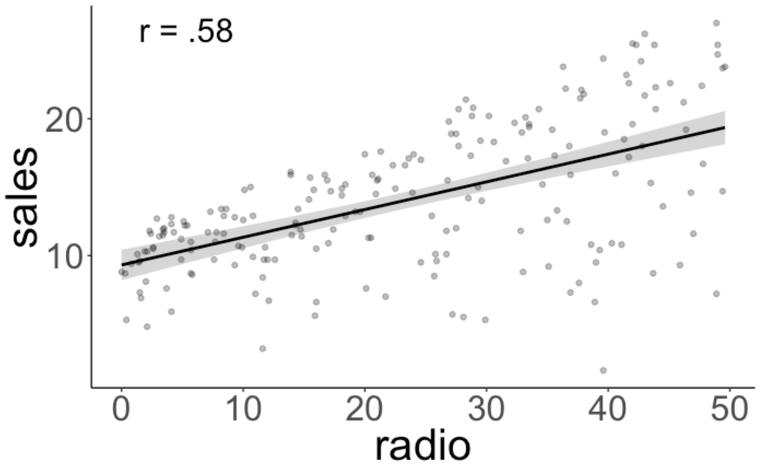
just imagine infinitely many more lines ...

# Why can't I just run several simple regressions?

Relationship between TV ads and sales




Relationship between radio ads and sales




We found that both TV ads and radio ads were related to sales.

But did we need to run a multiple regression? Could we not just have looked at correlations?

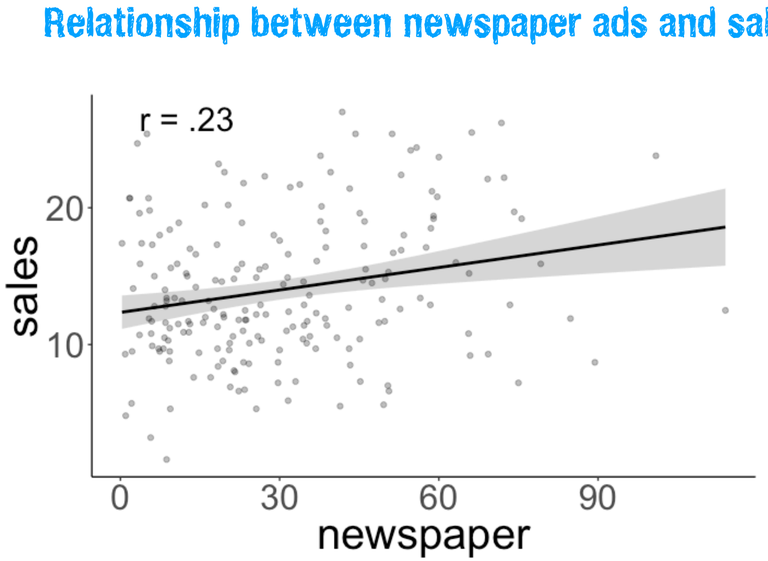
# Advertising data set

money spent on different media (x $1000)

sales (x1000)

| index | tv | radio | newspaper | sales |
|---|---|---|---|---|
| 1 | 230.1 | 37.8 | 69.2 | 22.1 |
| 2 | 44.5 | 39.3 | 45.1 | 10.4 |
| 3 | 17.2 | 45.9 | 69.3 | 9.3 |
| 4 | 151.5 | 41.3 | 58.5 | 18.5 |
| 5 | 180.8 | 10.8 | 58.4 | 12.9 |
| 6 | 8.7 | 48.9 | 75.0 | 7.2 |
| 7 | 57.5 | 32.8 | 23.5 | 11.8 |
| 8 | 120.2 | 19.6 | 11.6 | 13.2 |
| 9 | 8.6 | 2.1 | 1.0 | 4.8 |
| 10 | 199.8 | 2.6 | 21.2 | 10.6 |

# Are newspaper ads and sales related when controlling for radio ads and TV ads?




**this is significant**

```r
# fit the models
fit_c = lm(sales ~ 1 + tv + radio, data = df.ads)
fit_a = lm(sales ~ 1 + tv + radio + newspaper, data = df.ads)

# do the F test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: sales ~ 1 + tv + radio
Model 2: sales ~ 1 + tv + radio + newspaper
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    197 556.91
2    196 556.83  1  0.088717 0.0312 0.8599
```

it's not worth it

```r
sales ~ 1 + tv
sales ~ 1 + tv + newspaper
```

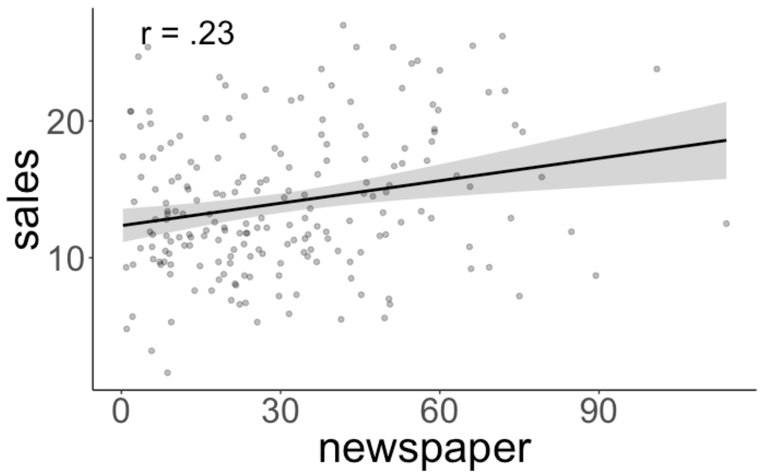
it's worth it

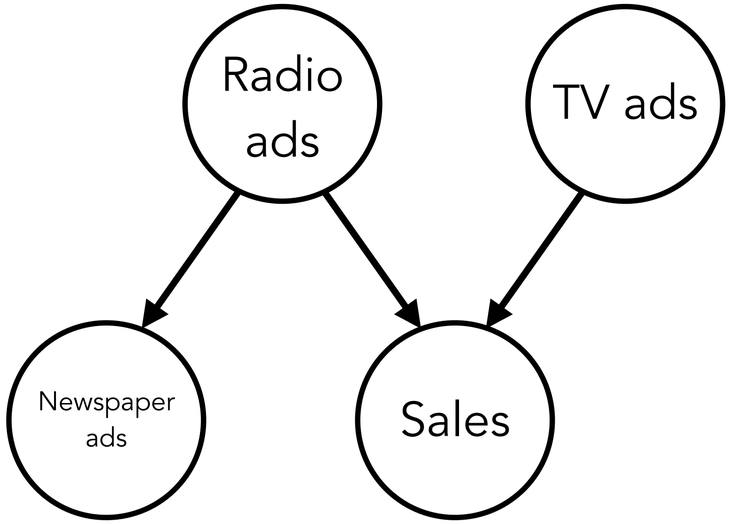
```r
sales ~ 1 + radio
sales ~ 1 + radio + newspaper
```

it's not worth it

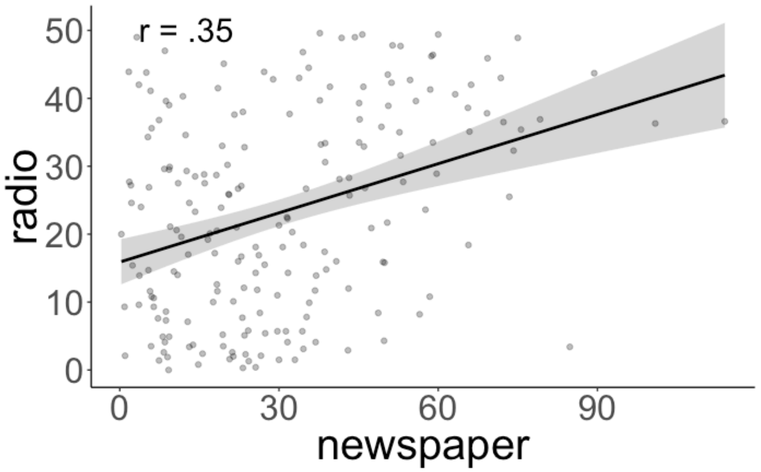
# Are newspaper ads and sales related when controlling for radio ads and TV ads?

## Relationship between newspaper ads and sales

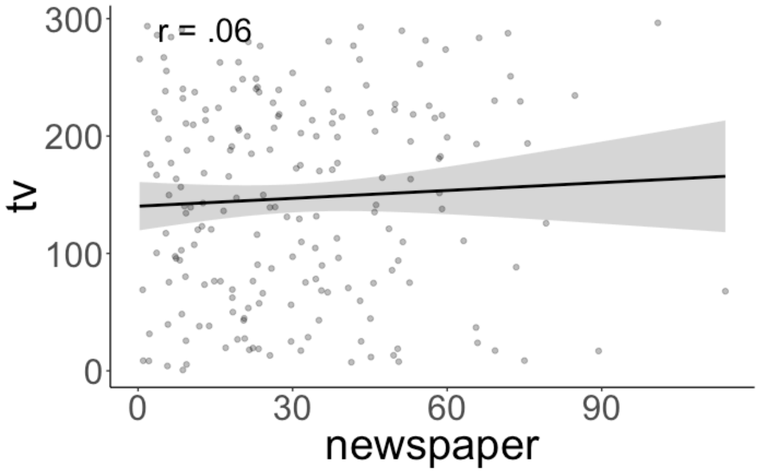



## Relationship between newspaper and radio ads




## Relationship between newspaper and TV ads



# Categorical predictors

# Credit data set

Categorical/Factor Variable

Continuous Variable

`df.credit`

| index | income | limit | rating | cards | age | education | gender | student | married | ethnicity | balance |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 14.89 | 3606 | 283 | 2 | 34 | 11 | Male | No | Yes | Caucasian | 333 |
| 2 | 106.03 | 6645 | 483 | 3 | 82 | 15 | Female | Yes | Yes | Asian | 903 |
| 3 | 104.59 | 7075 | 514 | 4 | 71 | 11 | Male | No | No | Asian | 580 |
| 4 | 148.92 | 9504 | 681 | 3 | 36 | 11 | Female | No | No | Asian | 964 |
| 5 | 55.88 | 4897 | 357 | 2 | 68 | 16 | Male | No | Yes | Caucasian | 331 |
| 6 | 80.18 | 8047 | 569 | 4 | 77 | 10 | Male | No | No | Caucasian | 1151 |
| 7 | 21.00 | 3388 | 259 | 2 | 37 | 12 | Female | No | No | African American | 203 |
| 8 | 71.41 | 7114 | 512 | 2 | 87 | 9 | Male | No | No | Asian | 872 |
| 9 | 15.12 | 3300 | 266 | 5 | 66 | 13 | Female | No | No | Caucasian | 279 |
| 10 | 71.06 | 6819 | 491 | 3 | 41 | 19 | Female | Yes | Yes | African American | 1350 |

```
nrow(df.credit) = 400
```

| variable | description |
|---|---|
| income | in thousand dollars |
| limit | credit limit |
| rating | credit rating |
| cards | number of credit cards |
| age | in years |
| education | years of education |
| gender | male or female |
| student | student or not |
| married | married or not |
| ethnicity | African American, Asian, Caucasian |
| balance | average credit card debt in dollars |

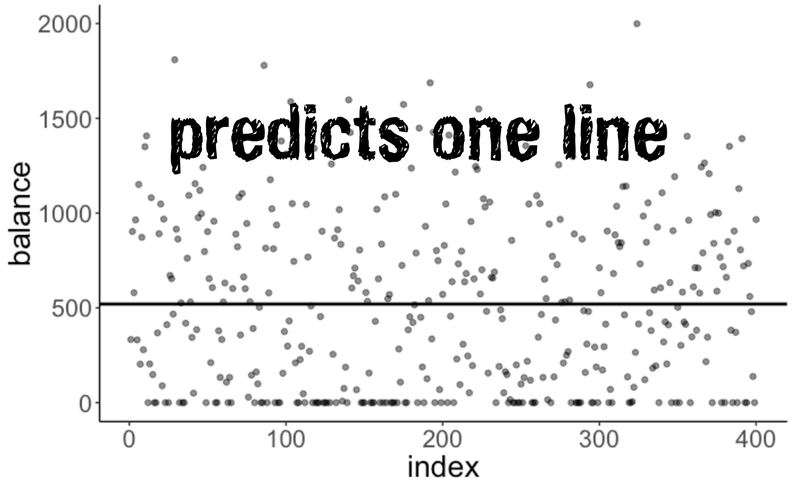
**Do students have a different average credit card balance from non-students?**

$H_0$: Students and non-students have the same balance.

## Model C

$$Y_i = \beta_0 + \epsilon_i$$

## Model prediction




## Fitted model

$$Y_i = 520.02 + e_i$$

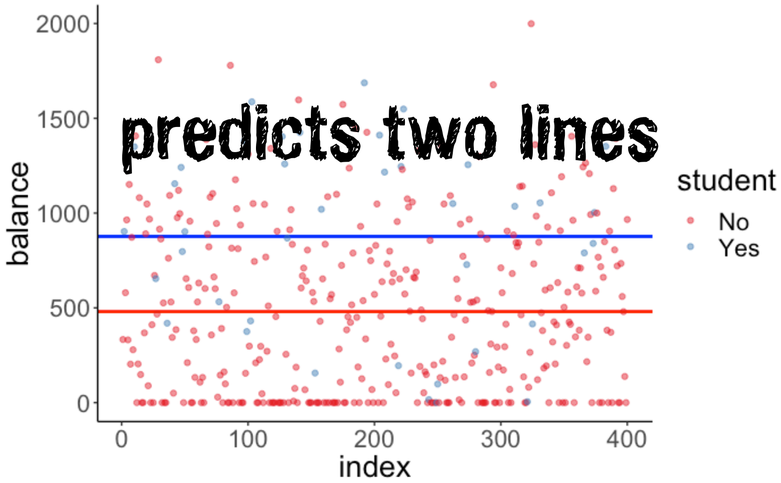
$H_1$: Students and non-students have different balances.

## Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

student (2 categories)

## Model prediction




## Fitted model

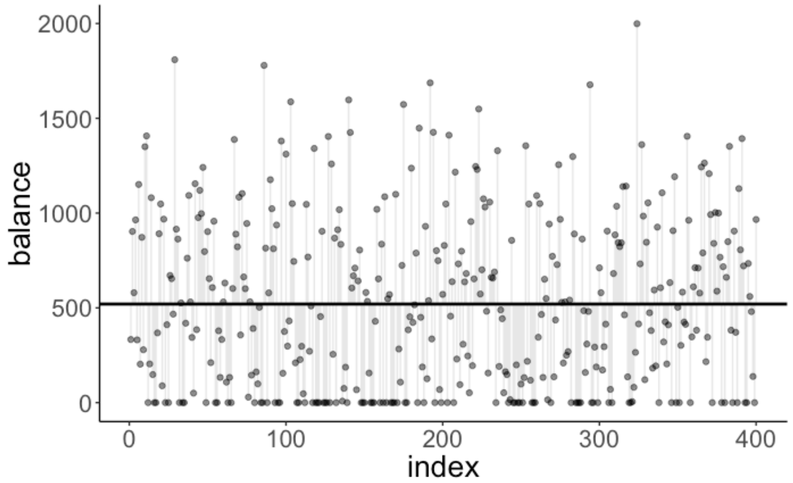
$$Y_i = 480.37 + 396.46 X_i + e_i$$

$H_0$: Students and non-students have the same balance.

## Model C

$$Y_i = \beta_0 + \epsilon_i$$

## Model prediction




## Fitted model

$$Y_i = 520.02 + e_i$$

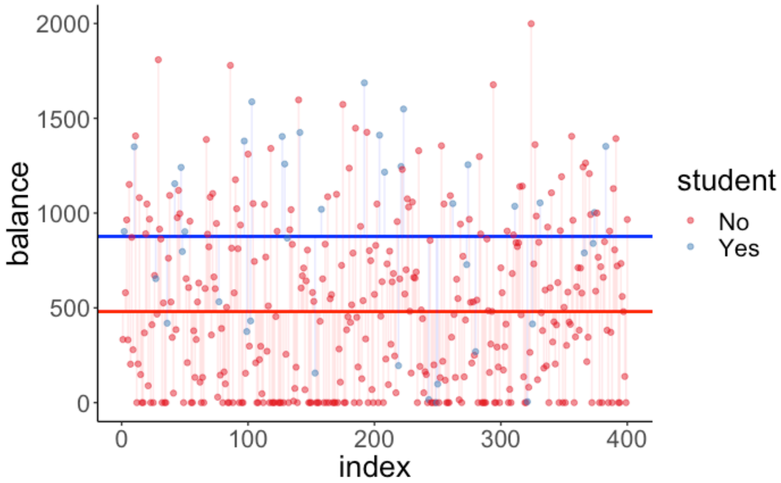
$H_1$: Students and non-students have different balances.

## Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

student (2 categories)

## Model prediction




## Fitted model

$$Y_i = 480.37 + 396.46 X_i + e_i$$

# Worth it?

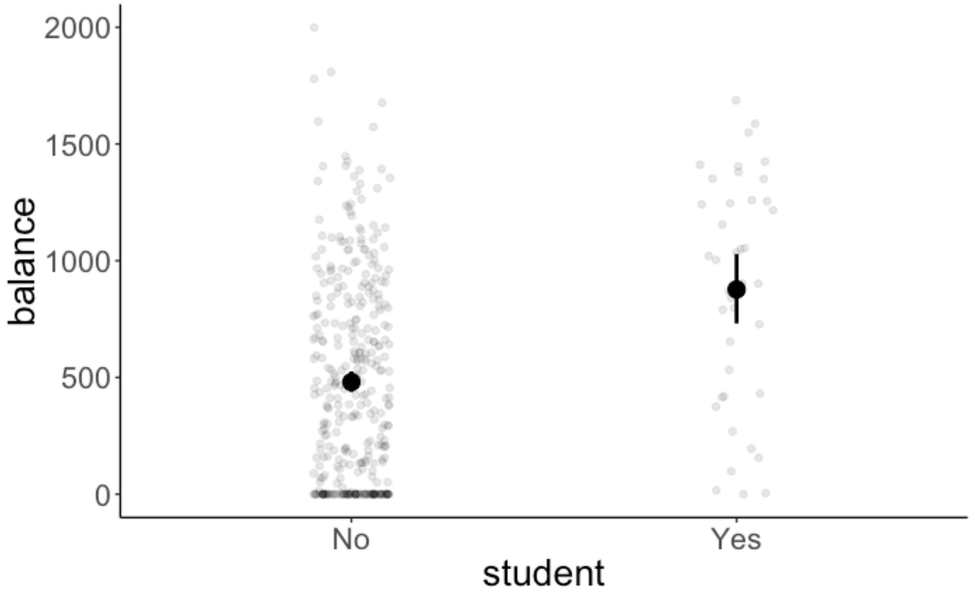
```r
# fit the models
fit_c = lm(balance ~ 1, data = df.credit)
fit_a = lm(balance ~ 1 + student, data = df.credit)

# run the F test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ student
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    399 84339912
2    398 78681540  1   5658372 28.622 1.488e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

**Worth it!**

Two sample t-test (with independent groups)

# Reporting the results




Students have a significantly higher average credit card balance (*Mean* = 876.83, *SD* = 490.00) than non-students (*Mean* = 480.37, *SD* = 439.41), $F(1, 398) = 28.622$, $p < .001$.

# Interpreting the model

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

```
1 fit_a = lm(balance ~ 1 + student, data = df.credit)
2 fit_a %>%
3   summary()
```

```
Call:
lm(formula = balance ~ student, data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-876.82 -458.82  -40.87  341.88 1518.63

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   480.37      23.43   20.50  < 2e-16 ***
studentYes    396.46      74.10    5.35 1.49e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 444.6 on 398 degrees of freedom
Multiple R-squared:  0.06709,  Adjusted R-squared:  0.06475
F-statistic: 28.62 on 1 and 398 DF,  p-value: 1.488e-07
```

# Dummy coding

# Dummy coding

$$\hat{Y}_i = 480.37 + 396.46 \cdot \text{student\_dummy}_i$$

if student = "No" $\hat{Y}_i = 480.37$

if student = "Yes" $\hat{Y}_i = 480.37 + 396.46 = 876.83$

| student | student_dummy |
|---|---|
| No | 0 |
| Yes | 1 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| Yes | 1 |

- Reference category is coded as 0, the other category is coded as 1
- When thrown into an `lm()`, R automatically turns character columns into factors, and determines the reference category alphabetically

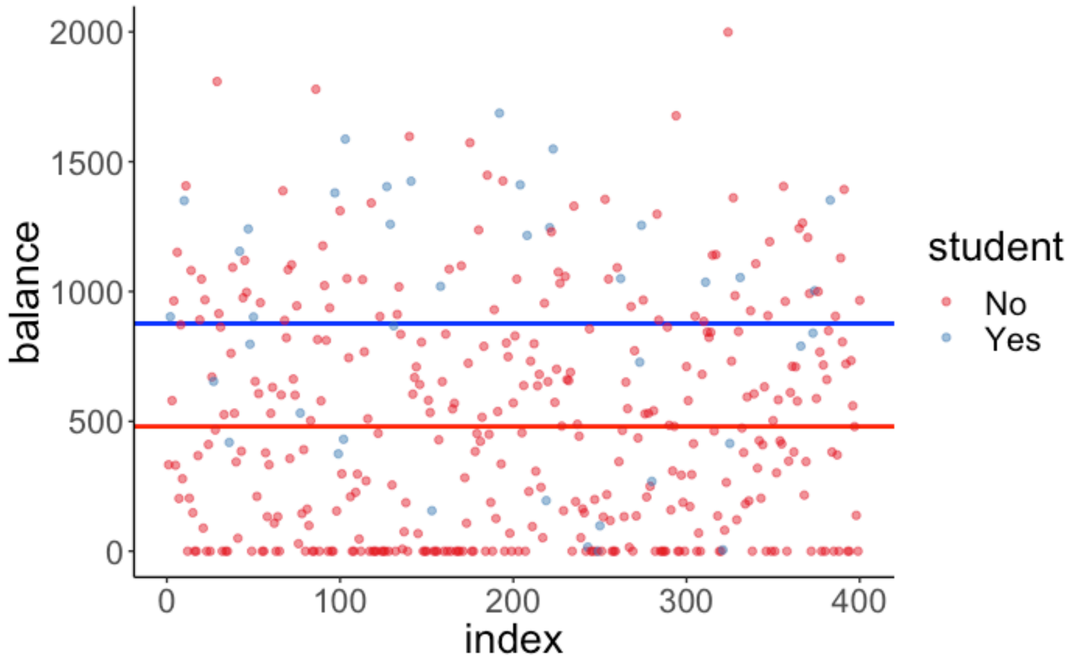
# Dummy coding

$$\hat{Y}_i = 480.37 + 396.46 \cdot \text{student\_dummy}_i + 0_i$$

if student = "No" $\hat{Y}_i = 480.37$

if student = "Yes" $\hat{Y}_i = 480.37 + 396.46 = 876.83$

| student | student_dummy |
|---|---|
| No | 0 |
| Yes | 1 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| No | 0 |
| Yes | 1 |



# Categorical and continuous predictor

# Credit data set

Continuous Variable

Categorical/Factor Variable

Continuous Variable

`df.credit`

| index | income | limit | rating | cards | age | education | gender | student | married | ethnicity | balance |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 14.89 | 3606 | 283 | 2 | 34 | 11 | Male | No | Yes | Caucasian | 333 |
| 2 | 106.03 | 6645 | 483 | 3 | 82 | 15 | Female | Yes | Yes | Asian | 903 |
| 3 | 104.59 | 7075 | 514 | 4 | 71 | 11 | Male | No | No | Asian | 580 |
| 4 | 148.92 | 9504 | 681 | 3 | 36 | 11 | Female | No | No | Asian | 964 |
| 5 | 55.88 | 4897 | 357 | 2 | 68 | 16 | Male | No | Yes | Caucasian | 331 |
| 6 | 80.18 | 8047 | 569 | 4 | 77 | 10 | Male | No | No | Caucasian | 1151 |
| 7 | 21.00 | 3388 | 259 | 2 | 37 | 12 | Female | No | No | African American | 203 |
| 8 | 71.41 | 7114 | 512 | 2 | 87 | 9 | Male | No | No | Asian | 872 |
| 9 | 15.12 | 3300 | 266 | 5 | 66 | 13 | Female | No | No | Caucasian | 279 |
| 10 | 71.06 | 6819 | 491 | 3 | 41 | 19 | Female | Yes | Yes | African American | 1350 |

| variable | description |
|---|---|
| income | in thousand dollars |
| limit | credit limit |
| rating | credit rating |
| cards | number of credit cards |
| age | in years |
| education | years of education |
| gender | male or female |
| student | student or not |
| married | married or not |
| ethnicity | African American, Asian, Caucasian |
| balance | average credit card debt in dollars |

```
nrow(df.credit) = 400
```

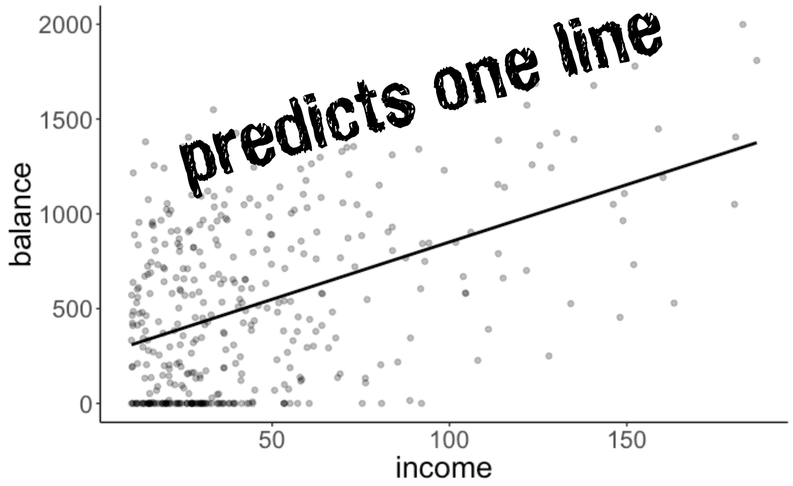
**Do students have a different average credit card balance from non-students, when controlling for income?**

$H_0$: Students and non-students have the same balance, when controlling for income.

## Model C

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \epsilon_i$$

## Model prediction




## Fitted model

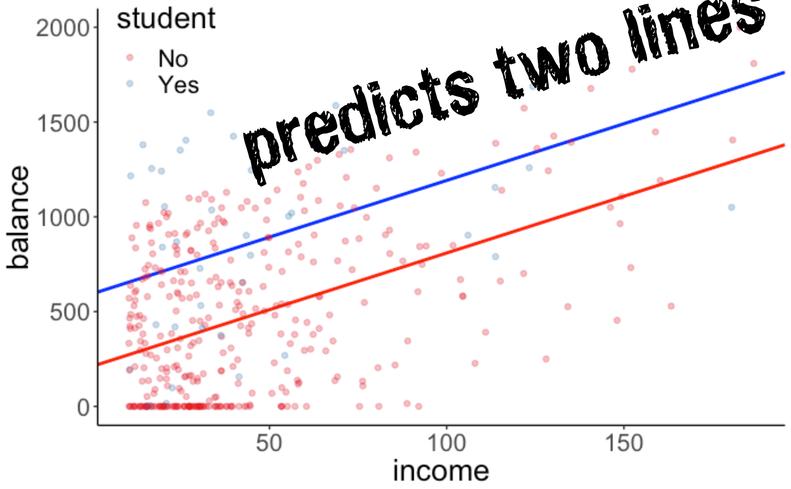
$$\widehat{\text{balance}}_i = 246.515 + 6.048 \cdot \text{income}_i + 0$$

$H_1$: Students and non-students have different balances, when controlling for income.

## Model A

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \beta_2 \text{student}_i + \epsilon_i$$

## Model prediction




## Fitted model

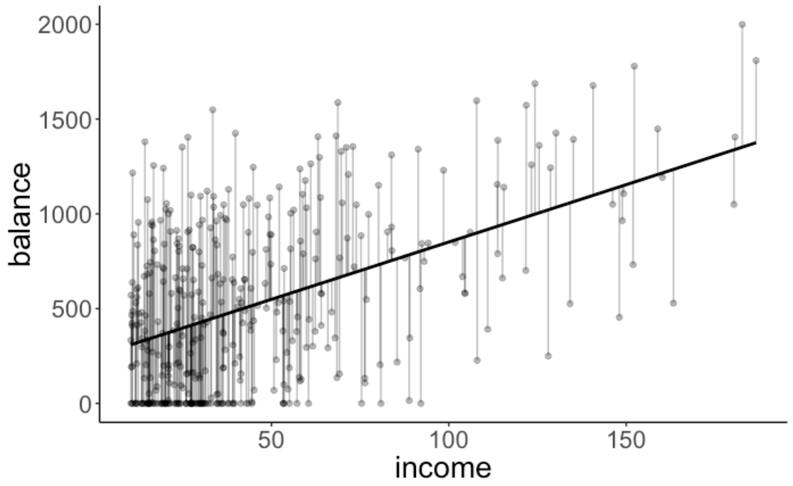
$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i + 0$$

$H_0$: Students and non-students have the same balance, when **controlling for income**.

## Model C

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \epsilon_i$$

## Model prediction




## Fitted model

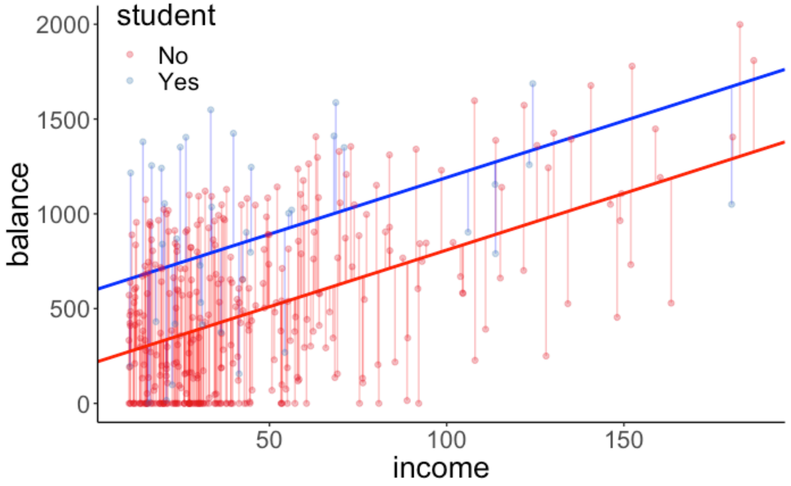
$$\widehat{\text{balance}}_i = 246.515 + 6.048 \cdot \text{income}_i + 0_i$$

$H_1$: Students and non-students have different balances, when **controlling for income**.

## Model A

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \beta_2 \text{student}_i + \epsilon_i$$

## Model prediction




## Fitted model

$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i + 0_i$$

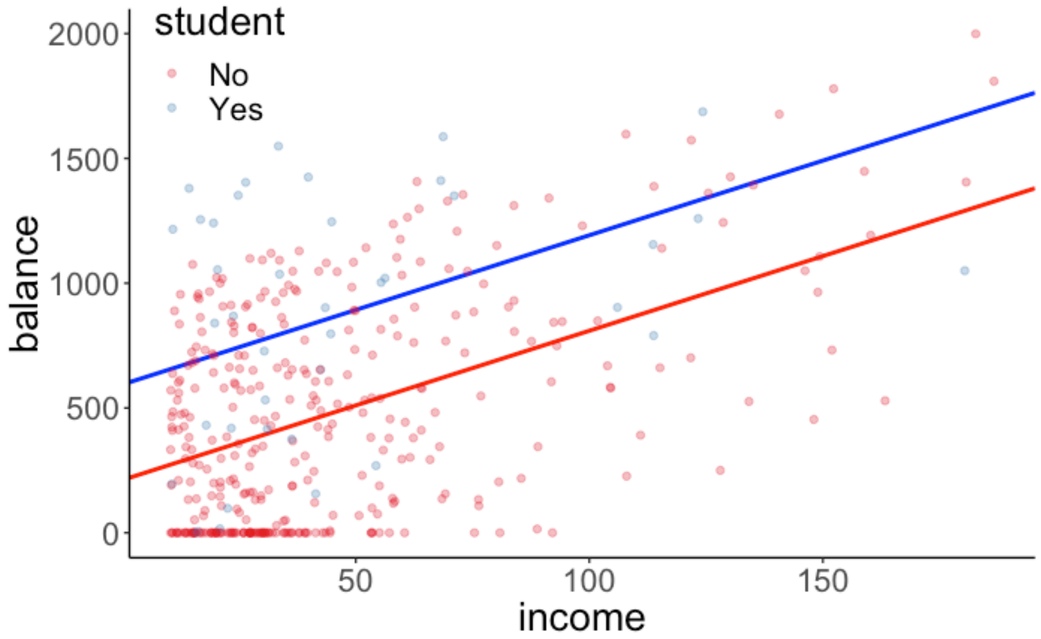
# Worth it?

```r
# fit the models
fit_c = lm(balance ~ 1 + income, df.credit)
fit_a = lm(balance ~ 1 + income + student, df.credit)

# run the F test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1 + income
Model 2: balance ~ 1 + income + student
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    398 66208745
2    397 60939054  1   5269691 34.331 9.776e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

**Worth it!**

# Interpretation



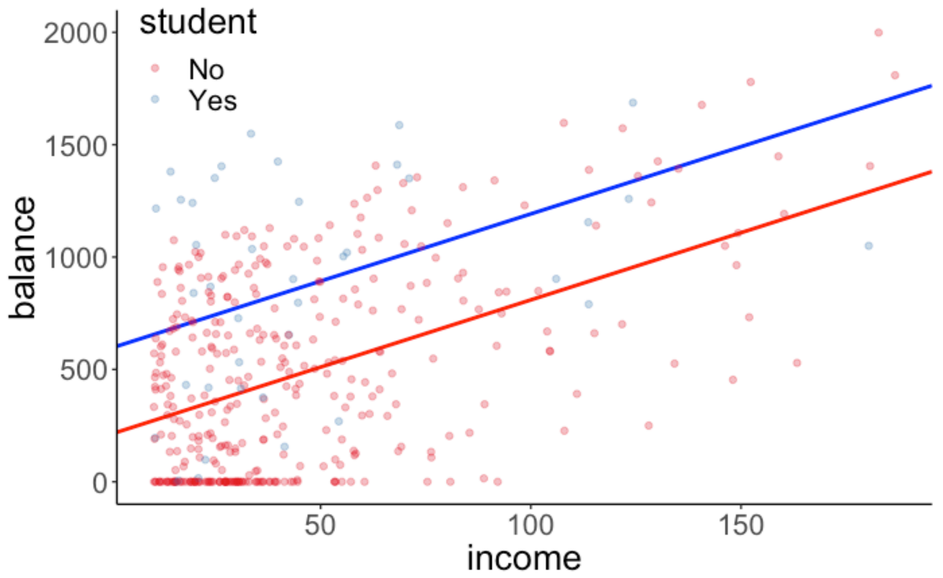

$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i + 0$$

**if student = "No"** $\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + (382.67 \cdot 0)$

**if student = "Yes"**

$$\begin{aligned}\widehat{\text{balance}}_i &= 211.14 + 5.98 \cdot \text{income}_i + (382.67 \cdot 1) \\ &= 211.14 + 382.67 + 5.98 \cdot \text{income}_i \\ &= 593.81 + 5.98 \cdot \text{income}_i\end{aligned}$$

# Reporting the results




Controlling for income, students have a significantly higher average credit card balance (*Mean* = 876.83, *SD* = 490.00) than non-students (*Mean* = 480.37, *SD* = 439.41), $F(1, 397) = 34.331$, $p < .001$.

# Interactions

Is the ***relationship*** between level of income and balance **different** for students than it is for non-students?

**Compact Model**

$$\widehat{\text{balance}}_i = b_0 + b_1 \text{income}_i + b_2 \text{student}_i + \epsilon_i$$

**R-formula for these?**

**Augmented Model**

$$\widehat{\text{balance}}_i = b_0 + b_1 \text{income}_i + b_2 \text{student}_i + b_3 (\text{income}_i \times \text{student}_i) + \epsilon_i$$

**Interaction ~ Moderation**

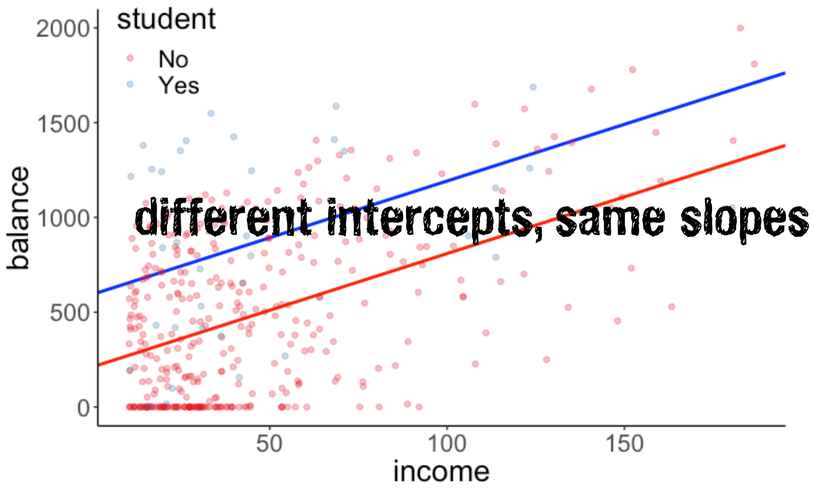
Does student status moderate the relationship between level of income and balance?

$H_0$: The relationship between income and balance is the same for students and non-students.

## Model C

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \beta_2 \text{student}_i + \epsilon_i$$

## Model prediction




## Fitted model

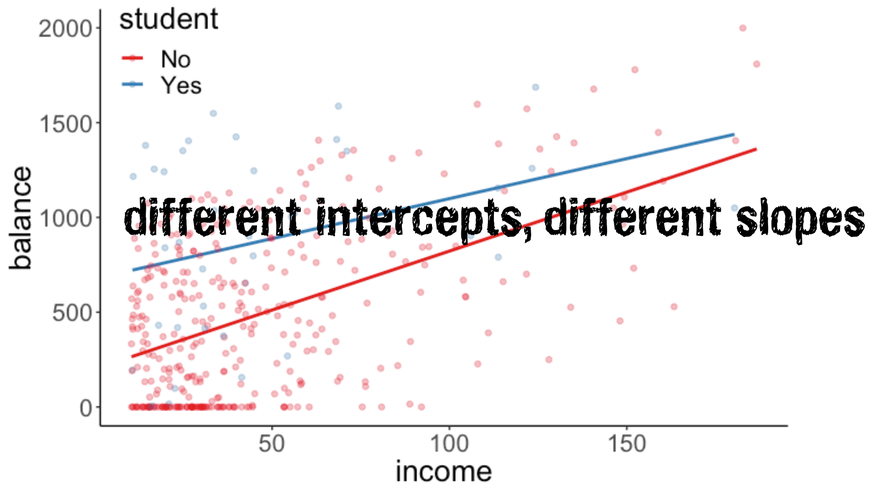
$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i$$

$H_1$: The relationship between income and balance differs between students and non-students.

## Model A

$$\widehat{\text{balance}}_i = b_0 + b_1 \text{income}_i + b_2 \text{student}_i + b_3(\text{income}_i \times \text{student}_i) + \epsilon_i$$

## Model prediction




## Fitted model

$$\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot \text{student}_i - 2.00 \cdot (\text{income}_i \times \text{student}_i)$$

# Worth it?

Is the relationship between level of income and balance different for students than it is for non-students?

Equations for these?

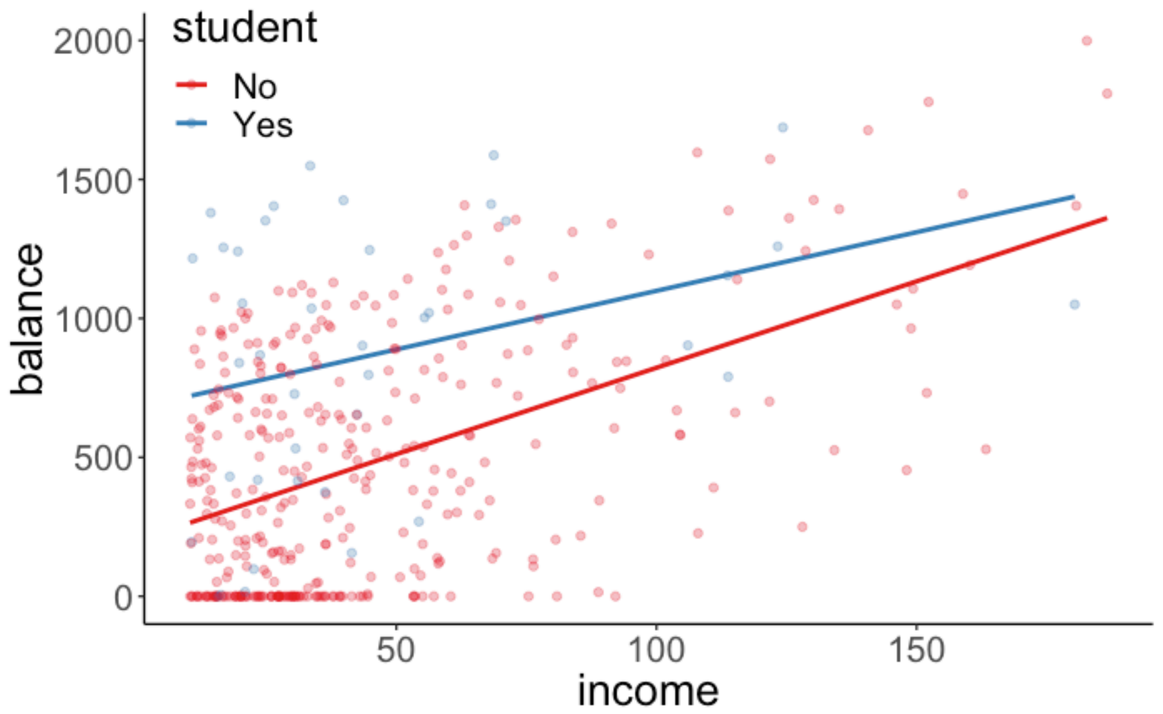
```
1 # fit models
2 fit_c = lm(formula = balance ~ 1 + income + student, data = df.credit)
3 fit_a = lm(formula = balance ~ 1 + income * student, data = df.credit)
4
5 # F-test
6 anova(fit_c, fit_a)
```

not worth it!

```
Analysis of Variance Table

Model 1: balance ~ income + student
Model 2: balance ~ income * student
  Res.Df      RSS Df Sum of Sq      F Pr(>F)
1    397 60939054
2    396 60734545  1    204509 1.3334 0.2489
```

```
1 # alternative coding for model a
2 fit_a = lm(formula = balance ~ 1 + income + student + income:student, data = df.credit)
```

# Interpretation




$$\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot \text{student}_i - 2.00 \cdot (\text{income}_i \times \text{student}_i) + 0$$

**if student = "No"** $\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + (476.68 \cdot 0) - 2.00 \cdot (\text{income}_i \times 0)$

**if student = "Yes"**

$$\begin{aligned}\widehat{\text{balance}}_i &= 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot 1 - 2.00 \cdot (\text{income}_i \times 1) \\ &= 677.3 + 6.22 \cdot \text{income}_i - 2.00 \cdot \text{income}_i \\ &= 677.3 + 4.22 \cdot \text{income}_i\end{aligned}$$

# Interpretation

```
fit1 = lm(formula = balance ~ income + student + income:student, data = df.credit)
```

**Explicitly encode the interaction**

```
1 df.credit %>%
2   mutate(student_dummy = ifelse(student == "No", 0, 1)) %>%
3   mutate(income_student = income * student_dummy) %>%
4   select(balance, income, student, student_dummy, income_student)
```

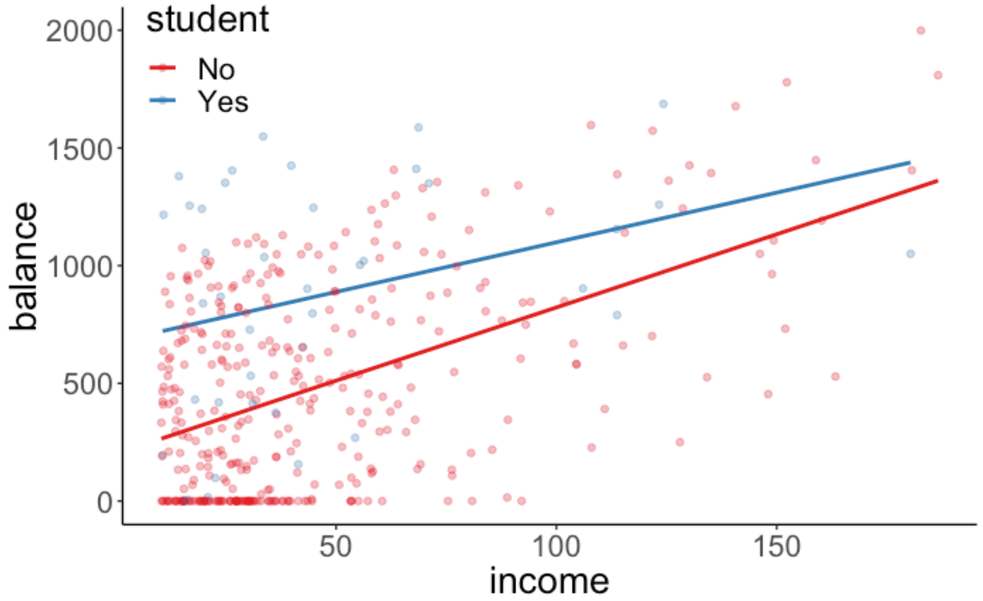
| balance | income | student | student_dummy | income_student |
|---|---|---|---|---|
| 333 | 14.89 | No | 0 | 0.00 |
| 903 | 106.03 | Yes | 1 | 106.03 |
| 580 | 104.59 | No | 0 | 0.00 |
| 964 | 148.92 | No | 0 | 0.00 |
| 331 | 55.88 | No | 0 | 0.00 |
| 1151 | 80.18 | No | 0 | 0.00 |
| 203 | 21.00 | No | 0 | 0.00 |
| 872 | 71.41 | No | 0 | 0.00 |
| 279 | 15.12 | No | 0 | 0.00 |
| 1350 | 71.06 | Yes | 1 | 71.06 |

```
fit2 = lm(formula = balance ~ income + student + income_student, data = df.credit)
```

`fit1` **and** `fit2` **are identical!**

# How to report results of interaction

There is no significant difference in the relationship between income and balance for students versus non-students, $F(1, 396) = 1.33$, $p = 0.25$.




For *students*, an increase in $1000 income is associated with an increase in $4.21 of average credit card balance.

For *non-students*, an increase in $1000 income is associated with an increase in $6.22 of average credit card balance.

There was no evidence that student status *moderated* the relationship between income and balance, $F(1, 396) = 1.33$, $p = 0.25$.

# `lm()` output

# lm() output

```
1 lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
2   summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
studentYes        476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes  -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’
1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799,  Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

```
1 fit_c = lm(formula = balance ~ student + income:student, data = df.credit)
2 fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)
3
4 anova(fit_c, fit_a)
```

```
1 fit_c = lm(formula = balance ~ income + student, data = df.credit)
2 fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)
3
4 anova(fit_c, fit_a)
```

# lm() output

```
1 lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
2   summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
studentYes        476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes  -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’
1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799,  Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

```
1 fit_c = lm(formula = balance ~ 1, data = df.credit)
2 fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)
3
4 anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ 1 + income
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    399 84339912
2    398 66208745  1  18131167 108.99 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’
0.1 ‘ ’ 1
```

**deterministic mapping between t and F**

$$t^2 = F$$

$$10.44^2 = 108.99$$

`anova()` gives me *F*s

but `lm()` gives me *t*s

```
Call:
lm(formula = balance ~ 1 + income, data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-803.64 -348.99  -54.42  331.75 1100.25

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 246.5148    33.1993   7.425  6.9e-13 ***
income        6.0484     0.5794  10.440  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 407.9 on 398 degrees of freedom
Multiple R-squared:  0.215,       Adjusted R-squared:  0.213
F-statistic:   109 on 1 and 398 DF,  p-value: < 2.2e-16
```

# lm() output

```
1 lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
2   summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student, data =
df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
studentYes        476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes  -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799,   Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

- runs many hypothesis tests at the same time
- increases the danger of making a type-I error (incorrectly rejecting the $H_0$)
- will not give us p-values for mixed effects models ...

## The model comparison approach

- allows to formulate hypotheses as specific comparisons between candidate models
- is more flexible: we could test a model with 2 predictors vs. a model with 4 predictors (= 2 df difference)
- gives us insight into the underlying statistical procedure

# `lm()` output

very important

```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
studentYes        476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes  -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’
1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799,  Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

what does this mean?

**not** the overall effect of income!

**instead** the predicted effect of income for non-students (student = 0)

we'll talk more about the difference between simple/conditional effects and main effects next time!

# Summary

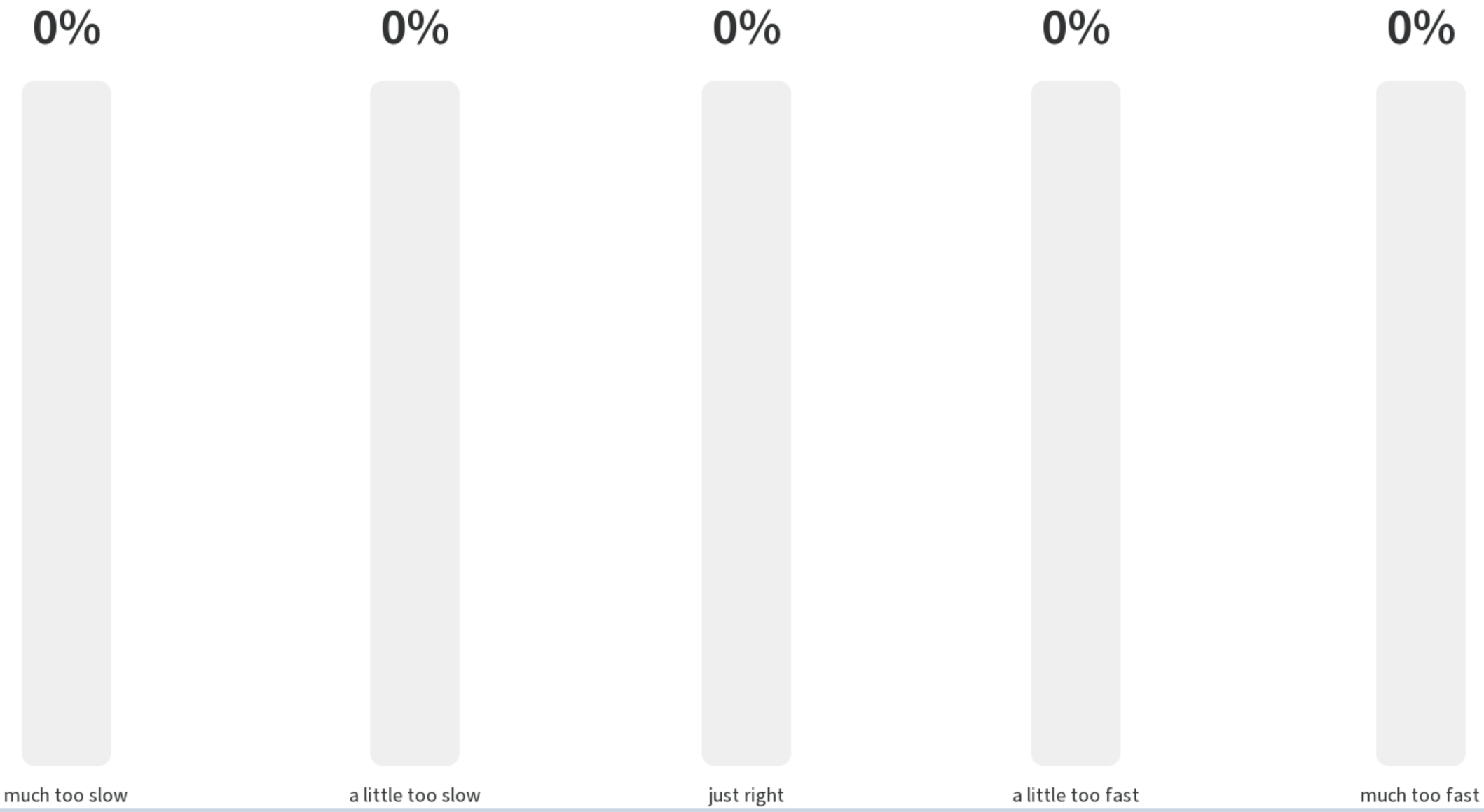
- Quick recap
- Multiple regression
  - two continuous predictors
  - one categorical predictor
  - one continuous and one categorical predictor
- Interactions
- `lm()` output

# Feedback

0

Start the presentation to see live content. For screen share software, share the entire screen. Get help at **pollev.com/app**

0




Start the presentation to see live content. For screen share software, share the entire screen. Get help at **pollev.com/app**

Start the presentation to see live content. For screen share software, share the entire screen. Get help at **pollev.com/app**

# Thank you!