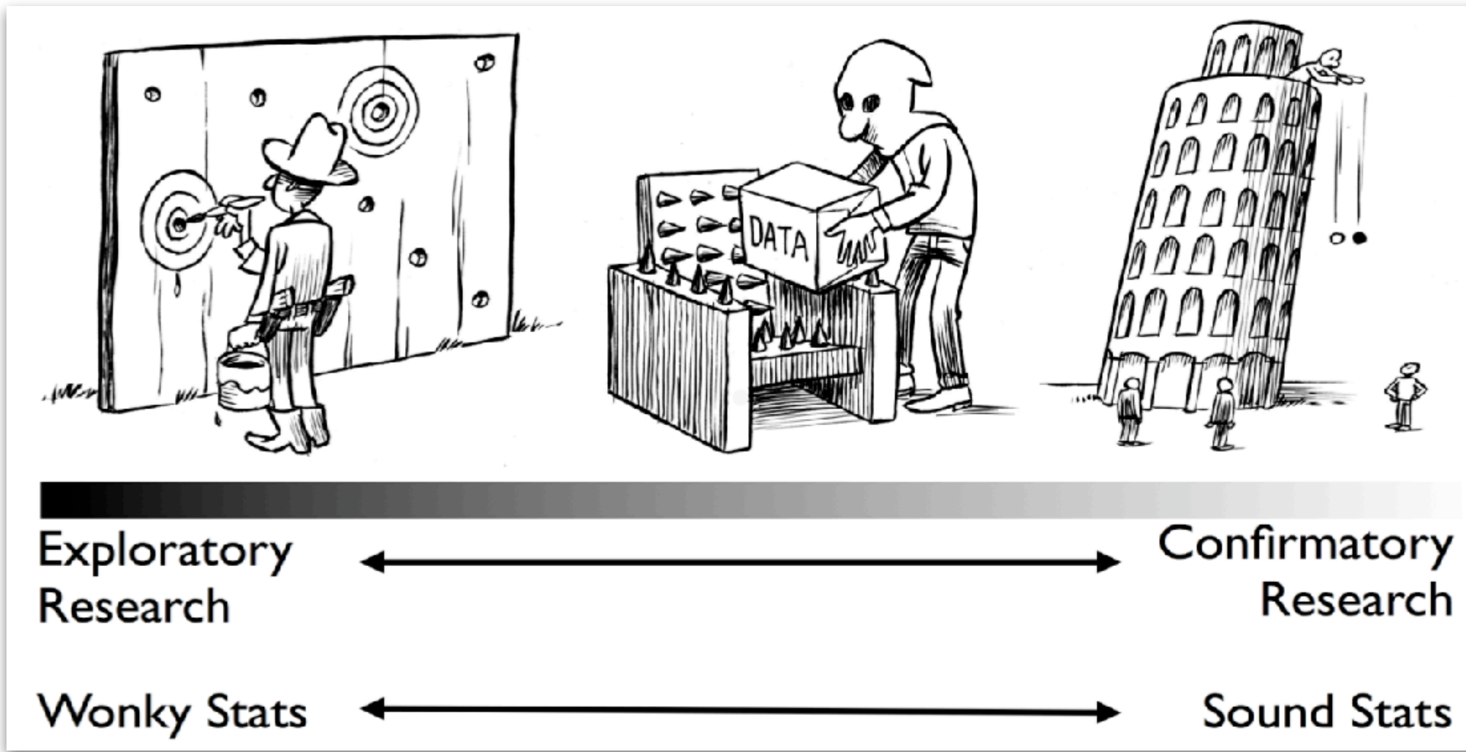
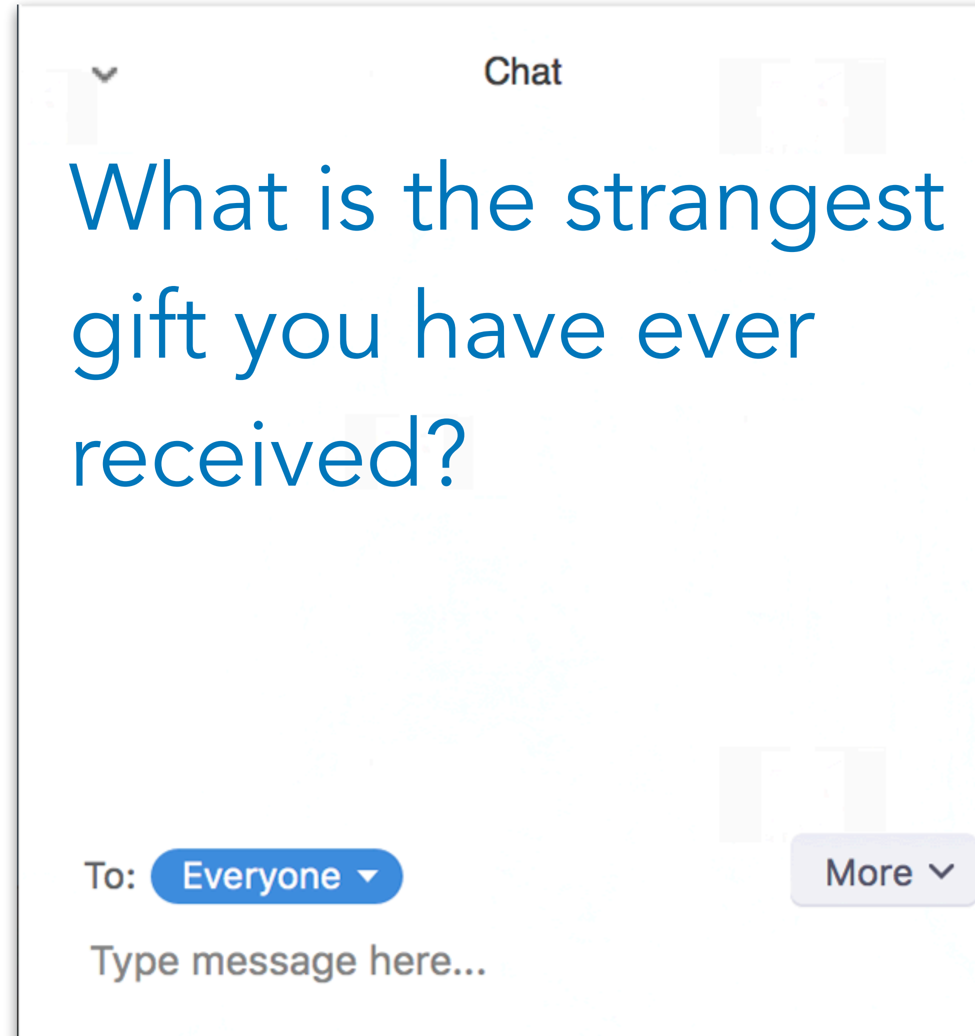
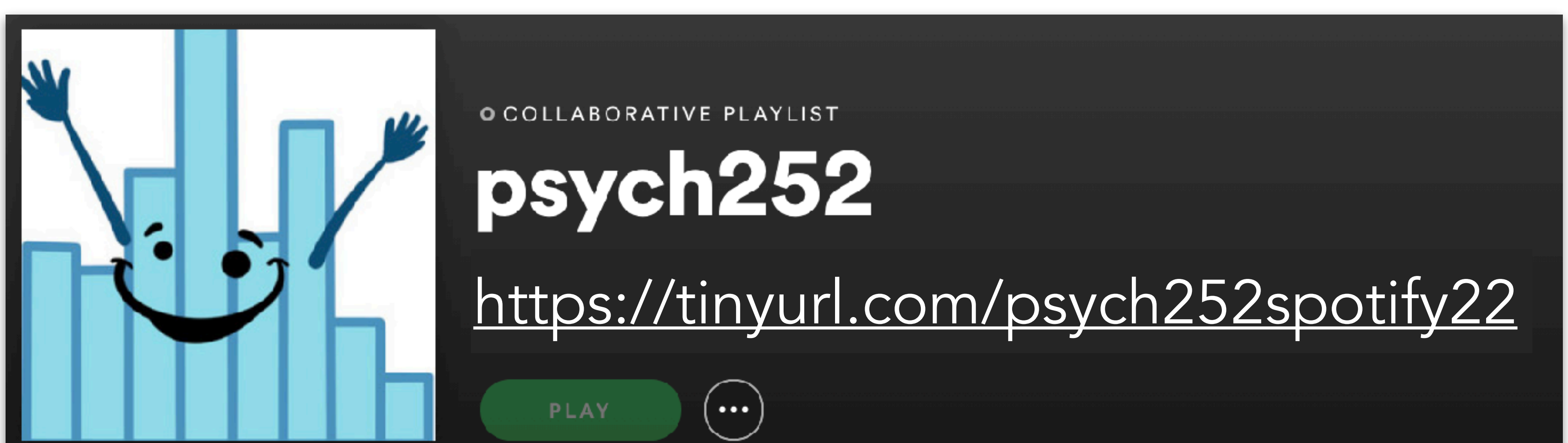
PSYCH 252: Statistical Methods for Behavioral and Social Sciences

Power analysis







We're listening to "Fleur tropicale" by "Francis Bebey" submitted by Tobi

02/04/2022

Logistics

Midterm

will be available a little bit after class today

Psych 252 Midterm

My name goes here

2021-02-12 12:04:21

Introduction

This is a take-home exam. The exam is open notes and open book (in short, you can use any source of information you like as long as you work on the exam by yourself). The maximum score is 120 points. Please adhere to the honor code. Submit the midterm as a PDF on the canvas 'midterm' assignment by **Thursday, February 18th, 8pm**.

The late policy submission policy is:

- We will subtract 2% from your points for each hour that the midterm is submitted late but before midnight. For example, 2% will be subtracted if you submit between 8pm and 9pm, or 8% if you submit between 11pm and midnight.
- 20% will be subtracted if you submit after midnight on Thursday but before 8pm on Friday, February 19th.
- No points will be granted if you submit later than 8pm on Friday, February 19th.

For questions that require written responses, please make sure to show any relevant tables, summaries (e.g. from `lm()` or `anova()`), or visualizations. Some of the code chunks have existing code that you can use to build your code around.

When asked to report results, please do so like you would in a scientific article (see examples from class).

- Please leave the `\clearpage` commands where they are. This makes sure that each question is printed on a separate page in the pdf.
- Some code chunks are set to `eval=F`, make sure to set these to `eval=T` before knitting the final version.
- We note for each question how many points you can get. You can get up to 120 points in total.
- Good coding style matters! We will add or subtract up to 5 points depending on style.

If you have any questions about the midterm, please post them on Piazza addressed to the instructors only. We will answer your question and may choose to share both your question and our answer with the rest of the group.

Best of luck with the midterm!

Honor Code

The Honor Code is the University's statement on academic integrity written by students in 1921. It articulates University expectations of students and faculty in establishing and maintaining the highest standards in academic work:

1. The Honor Code is an undertaking of the students, individually and collectively:

a. that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;
b. that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.

Part 1: Wash your hands!

A student investigated how effective different methods are for eliminating bacteria. She tested four different methods: (1) washing her hands with water only, (2) with regular soap, (3) with antibacterial soap (ABS), or (4) using an antibacterial spray (AS). She suspected that the number of bacteria on her hands might vary considerably from day to day. To account for this, she generated random numbers to determine on what day she would use which treatment. After each treatment, she placed her right hand on a sterile media plate to measure bacteria growth. She incubated each plate for 2 days after which she counted the bacteria colonies. She replicated this procedure 9 times for each of the four treatments.

> Note: For statistical analysis purposes, we make the assumption that the individual measurements are independent from each other.

Part 2: Life satisfaction

In this exercise, we are interested in seeing what affects life satisfaction. We have a (fake) data set with the following variables:

Table 1: Variables in the satisfaction data set.

variable	description
`id`	participant id
`age`	age in years
`kids`	number of kids
`jobsatis`	job satisfaction (1 = not at all, 7 = very much)
`marsatis`	marital satisfaction (1 = not at all, 7 = very much)
`lifsatis`	life satisfaction (1 = not at all, 7 = very much)

Part 3: You've got the power!

In this exercise, we'll take a look at determining what group sample size we would need in order to achieve adequate statistical power to test our research hypothesis of interest. We will be using the data from "data/power.csv" and you can see its visualization in "figure/df_power.png".

> Tip: We will provide saved checkpoints for each problem. Feel free to look ahead to compare your results with ours.

Next week

no class on Wednesday (more time to work on midterm)

no sections

no office hours (at least not for questions about the midterm)

due **Friday 11th before 8pm** (one extra day)

Plan for today

- Quick recap
- Linear contrasts
  - Testing specific hypotheses with linear contrasts
  - `emmeans` for handling linear contrasts in R
- Unbalanced designs
- Power analysis
  - Making decisions
  - Calculating power
  - Effect sizes
  - Determining sample size
- Learn about more advanced simulation techniques in R
  - `map()`
  - list columns: `nest()`, `unnest()`
- Simulating a power analysis in R

will share a 25 minute video lecture

Quick recap

Quick recap: lm() output

`lm()` output

```r
lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
  summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)        200.6232    33.6984   5.953 5.79e-09 ***
income               6.2182     0.5921  10.502  < 2e-16 ***
studentYes         476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes   -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799, Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

?

```r
fit_c = lm(formula = balance ~ student + income:student, data = df.credit)
fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)

anova(fit_c, fit_a)
```

```r
fit_c = lm(formula = balance ~ income + student, data = df.credit)
fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)

anova(fit_c, fit_a)
```

10

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ 1 + income
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    399 84339912
2    398 66208745  1  18131167 108.99 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05
'.' 0.1 ' ' 1
```

`anova()` gives me *Fs*
but `lm()` gives me *ts* ?

```
Call:
lm(formula = balance ~ 1 + income, data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-803.64 -348.99  -54.42  331.75 1100.25

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 246.5148    33.1993   7.425  6.9e-13 ***
income        6.0484     0.5794  10.440  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 407.9 on 398 degrees of freedom
Multiple R-squared:  0.215,    Adjusted R-squared:  0.213
F statistic:   109 on 1 and 398 DF,  p value: < 2.2e 16
```

deterministic mapping between t and F

$$t^2 = F$$

$$10.44^2 = 108.99$$

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`lm()` output


```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)        200.6232    33.6984   5.953 5.79e-09 ***
income               6.2182     0.5921  10.502  < 2e-16 ***
studentYes         476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes   -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799, Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

what does this mean?

not the overall effect of income!!

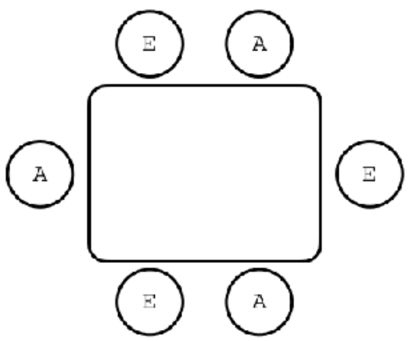
instead the predicted effect of income for non-students!

Quick recap: Categorial predictors

What's the role of skill vs. chance in poker?

Abstract

Adopting a quasi-experimental approach, the present study examined the extent to which the influence of poker playing skill was more important than card distribution. Three average players and three experts sat down at a six-player table and played **60 computer-based** hands of the poker variant "Texas Hold'em" for money. In each hand, one of the average players and one expert received (a) better-than-average cards (winner's box), (b) average cards (neutral box) and (c) worse-than-average cards (loser's box). The standardized manipulation of the card distribution controlled the factor of chance to determine differences in performance between the average and expert groups. Overall, 150 individuals participated in a "fixed- limit" game variant, and 150 individuals participated in a "no-limit" game variant.



During the game, one expert player and one average player received

(a) the winning hand 15 times and the losing hand 5 times (winner's box condition)

(b) the winning hand 10 times and the losing hand 10 times (neutral box condition)

(c) the winning hand 5 times and the losing hand 15 times (loser's box condition)

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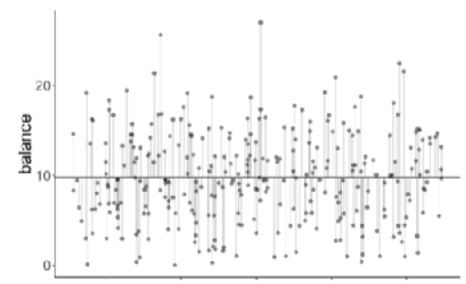
H_0: Card quality does not affect the final balance.

Model C

$\text{balance}_i = \beta_0 + \epsilon_i$

Model prediction



SSE(C) = 7580

Fitted model

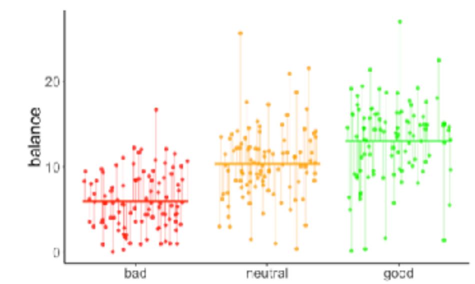
$\widehat{\text{balance}_i} = 9.77$

H_1: Card quality affects the final balance.

Model A

$\text{balance}_i = \beta_0 + \beta_1 \text{hand\_neutral}_i + \beta_2 \text{hand\_good}_i + \epsilon$

Model prediction



SSE(A) = 5021

Fitted model

$\widehat{\text{balance}_i} = 5.94 + 4.41 \cdot \text{hand\_neutral}_i + 7.08 \cdot \text{hand\_good}_i$

21

Does card quality affect the final balance?

SSE(C) = 7580

SSE(A) = 5021

$$\text{PRE} = 1 - \frac{\text{SSE}(A)}{\text{SSE}(C)}$$ **worth it?**

$$= 1 - \frac{5021}{7580} \approx 0.34$$

```
# fit the models
fit_c = lm(formula = balance ~ 1, data = df.poker)
fit_a = lm(formula = balance ~ hand, data = df.poker)

# compare via F-test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ hand
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    299 7580.0
2    297 5020.6  2    2559.4 75.703 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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Interpreting the results

```
lm(formula = balance ~ 1 + hand, data = df.poker)
```

```
Call:
lm(formula = balance ~ hand, data = df.poker)

Residuals:
     Min       1Q   Median       3Q      Max
-12.9264  -2.5902  -0.0115   2.6573  15.2834

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   5.9415     0.4111  14.451  < 2e-16 ***
handneutral   4.4051     0.5815   7.576 4.55e-13 ***
handgood      7.0849     0.5815  12.185  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.111 on 297 degrees of freedom
Multiple R-squared:  0.3377, Adjusted R-squared:  0.3332
F-statistic:  75.7 on 2 and 297 DF,  p-value: < 2.2e-16
```

Quick recap: Categorial predictors

One-way ANOVA

```
lm(formula = balance ~ hand, data = df.poker) %>%
  anova()
```

```
Analysis of Variance Table

Response: balance
           Df Sum Sq Mean Sq F value    Pr(>F)
hand        2 2559.4  1279.7  75.703 < 2.2e-16 ***
Residuals 297 5020.6    16.9
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

What do these mean?

```
# fit the models
fit_c = lm(formula = balance ~ 1, data = df.poker)
fit_a = lm(formula = balance ~ hand, data = df.poker)

# compare via F-test
anova(fit_c, fit_a)
```

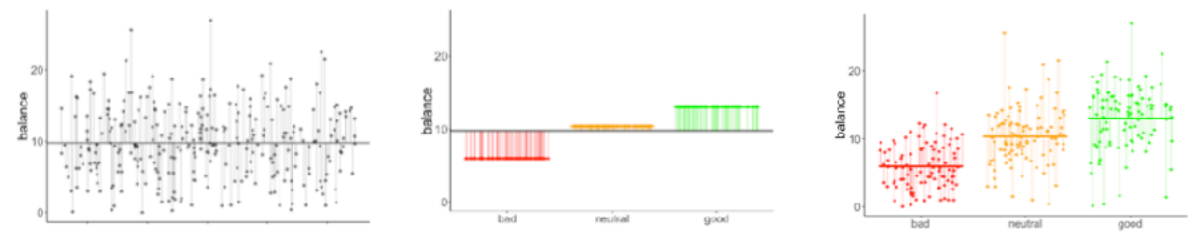
```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ hand
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    299 7580.0
2    297 5020.5  2    2559.4 75.703 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

One-way ANOVA

Variance decomposition

Total variance | Model variance | Residual variance



SS_{total} | SS_{model} | $SS_{residual}$

variance_total	variance_model	variance_residual
7580	2559	5021

Asking more specific questions

Is there a difference in the final balance between neutral hands and good hands?

```
df.poker %>%
  filter(hand %in% c("neutral", "good")) %>%
  lm(formula = balance ~ hand,
     data = .) %>%
  summary()
```

```
Call:
lm(formula = balance ~ hand, data = .)

Residuals:
     Min       1Q   Median       3Q      Max
-12.9264  -2.7141   0.2585   2.7184  15.2834

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  10.3466     0.4448   23.26  < 2e-16 ***
handgood      2.6798     0.6291    4.26 3.16e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.448 on 198 degrees of freedom
Multiple R-squared:  0.08396,	Adjusted R-squared:  0.07933
F-statistic: 18.15 on 1 and 198 DF,  p-value: 3.158e-05
```

Analysis of variance

```
lm(formula = balance ~ hand * skill, data = df.poker) %>%
  anova()
```

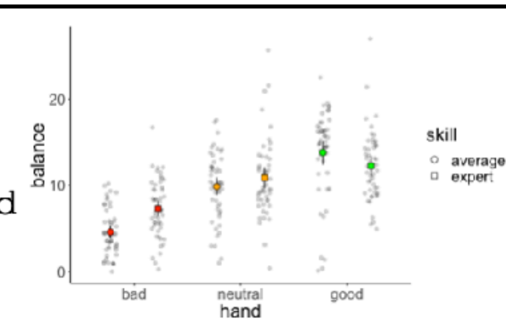
```
Analysis of Variance Table

Response: balance
            Df Sum Sq Mean Sq F value    Pr(>F)
hand         2 2559.4 1279.70 79.1692 < 2.2e-16 ***
skill        1   39.3   39.35  2.4344 0.1197776
hand:skill   2  229.0  114.49  7.0830 0.0009901 ***
Residuals  294 4752.3   16.16
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

main effect of `hand`

no main effect of `skill`

interaction between `hand` and `skill`



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Parameter interpretation

```
lm(formula = balance ~ hand * skill, data = df.poker) %>%
  summary()
```

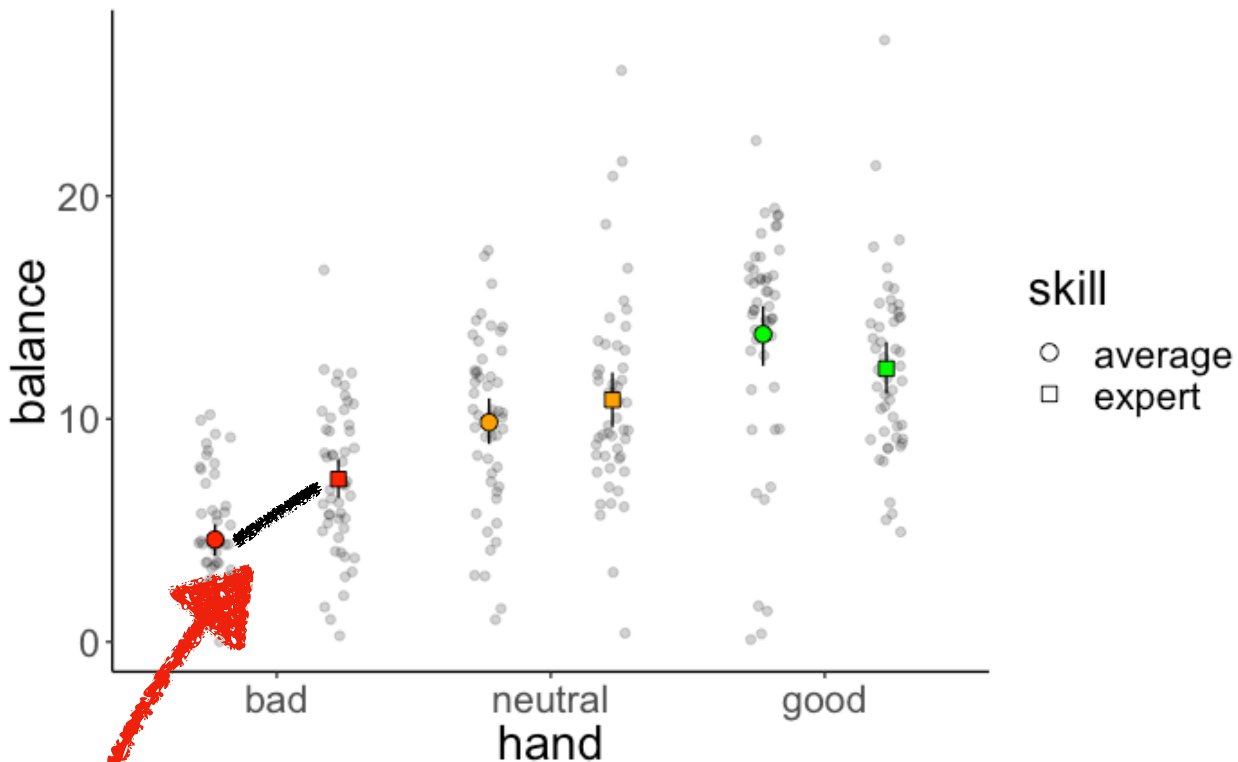
```
Call:
lm(formula = balance ~ hand * skill, data = df.poker)

Residuals:
     Min       1Q   Median       3Q      Max
-13.6976  -2.4740   0.0348   2.4644  14.7806

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)               4.5866     0.5686   8.067 1.85e-14 ***
handneutral               5.2572     0.8041   6.538 2.75e-10 ***
handgood                  9.2110     0.8041  11.455  < 2e-16 ***
skillexpert               2.7098     0.8041   3.370 0.000852 ***
handneutral:skillexpert  -1.7042     1.1372  -1.499 0.135038
handgood:skillexpert     -4.2522     1.1372  -3.739 0.000222 ***
---
Sig                                                          ' 1

Residual standard error: 4.02 on 294 degrees of freedom
Multiple R-squared:  0.3731,Adjusted R-squared:  0.3624
F-statistic: 34.99 on 5 and 294 DF,  p-value: < 2.2e-16
```

acute danger of misinterpretation!!

~~there was a significant effect of skill~~

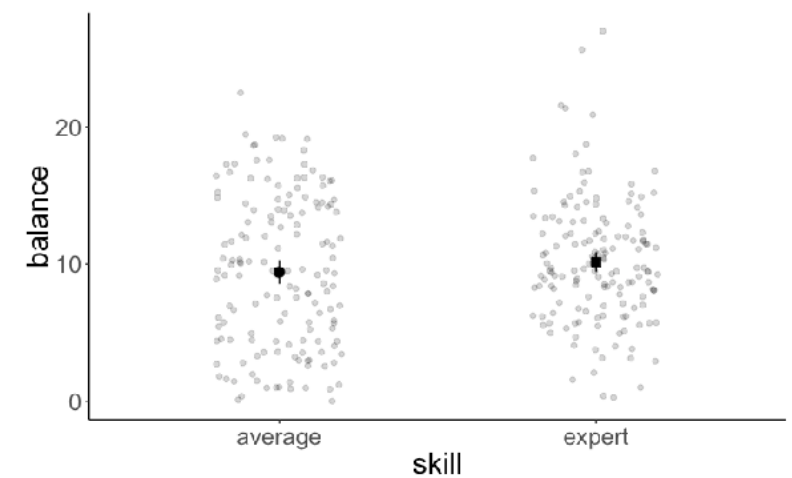
Parameter interpretation

```
lm(formula = balance ~ hand * skill,
   data = df.poker) %>%
  anova()
```




```
Analysis of Variance Table

Response: balance
            Df Sum Sq Mean Sq F value    Pr(>F)
hand         2 2559.4 1279.70 79.1692 < 2.2e-16 ***
skill        1   39.3   39.35  2.4344 0.1197776
hand:skill   2  229.0  114.49  7.0830 0.0009901 ***
Residuals  294 4752.3   16.16
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.'
0.1 ' ' 1
```

there was no main effect of skill!




```
Call:
lm(formula = balance ~ hand * skill, data = df.poker)

Residuals:
     Min       1Q   Median       3Q      Max
-13.6976  -2.4740   0.0348   2.4644  14.7806

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)               4.5866     0.5686   8.067 1.85e-14 ***
handneutral               5.2572     0.8041   6.538 2.75e-10 ***
handgood                  9.2110     0.8041  11.455  < 2e-16 ***
skillexpert               2.7098     0.8041   3.370 0.000852 ***
handneutral:skillexpert  -1.7042     1.1372  -1.499 0.135038
handgood:skillexpert     -4.2522     1.1372  -3.739 0.000222 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.02 on 294 degrees of freedom
Multiple R-squared:  0.3731,	Adjusted R-squared:  0.3624
F-statistic: 34.99 on 5 and 294 DF,  p-value: < 2.2e-16
```

is this difference significantly different from 0?

hand	average	expert	difference
bad	4.59	7.3	2.71

Effects in an ANOVA

- **main effect**: effect of one independent variable on the dependent variable
- **interaction effect**: when the effect of one independent variable depends on the level of another
- **simple effect**: comparison between two specific cell means

https://opentext.wsu.edu/carriecuttler/chapter/9-2-main-effects-and-interactions/

Interpreting parameters

`lm()` gives simple effects

```
lm(formula = balance ~ hand * skill,
   data = df.poker)
```

```
Call:
lm(formula = balance ~ hand * skill, data = df.poker)

Residuals:
     Min       1Q   Median       3Q      Max
-13.6976  -2.4740   0.0348   2.4644  14.7806

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)              4.5866     0.5686   8.067 1.85e-14 ***
handneutral              5.2572     0.8041   6.538 2.75e-10 ***
handgood                 9.2110     0.8041  11.455  < 2e-16 ***
skillexpert              2.7098     0.8041   3.370 0.000852 ***
handneutral:skillexpert -1.7042     1.1372  -1.499 0.135038
handgood:skillexpert    -4.2522     1.1372  -3.739 0.000222 ***
---
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Residual standard error: 4.02 on 294 degrees of freedom
Multiple R-squared:  0.3731,	Adjusted R-squared:  0.3624
F-statistic: 34.99 on 5 and 294 DF,  p-value: < 2.2e-16
```

`anova()` gives main effects, and interactions

```
lm(formula = balance ~ hand * skill,
   data = df.poker) %>%
  anova()
```

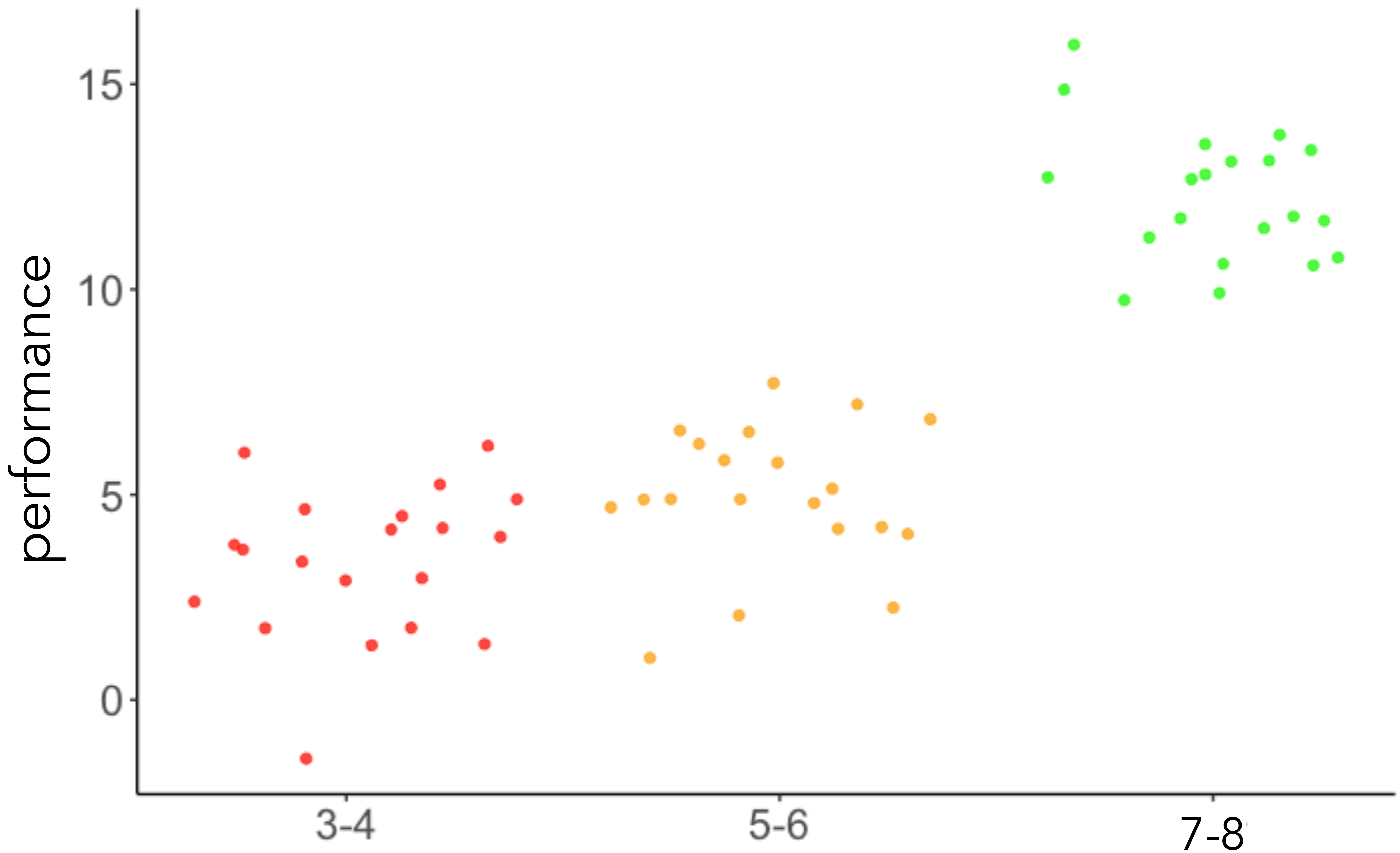
```
Analysis of Variance Table

Response: balance
            Df Sum Sq Mean Sq F value    Pr(>F)
hand         2 2559.4 1279.70 79.1692 < 2.2e-16 ***
skill        1   39.3   39.35  2.4344 0.1197776
hand:skill   2  229.0  114.49  7.0830 0.0009901 ***
Residuals  294 4752.3   16.16
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
' ' 1
```

Linear contrasts

Testing (more) specific hypotheses with linear contrasts

Contrasts

Does performance increase with age?



Data from a hypothetical developmental study

Do better hands win more money?

ANOVA

Does card quality affect the final balance?

post- ts

3-4 vs. 5-6

5-6 vs. 7-8

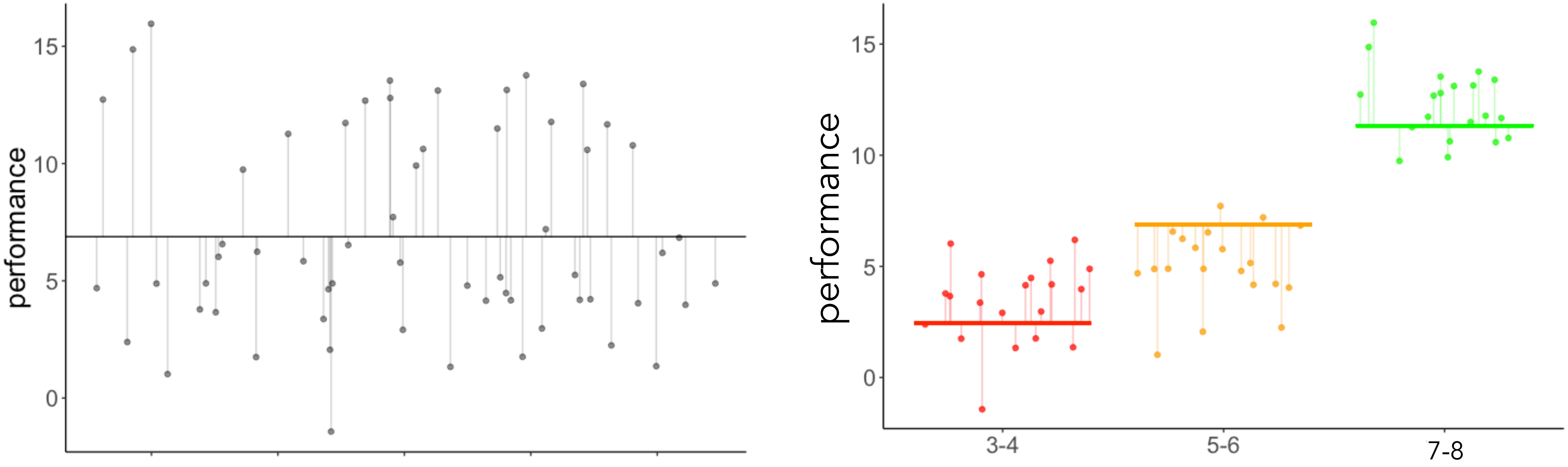
Is there are more direct way of asking this question with a statistical model?

Contrasts

Does performance increase with age?

```
contrasts = c(-1, 0, 1)
```

Compact model **Augmented model**



Model comparison

$p < .001$

`emmeans` for handling linear contrasts in R

Linear contrasts

~~How to use contrasts in R~~

In short: don't bother.[1]

Like many before me, one of my stats classes technically "taught" me contrasts. But I didn't get the point and using them was cumbersome, so I promptly ignored them for years.

Luckily for me, someone came along and fixed the situation: `emmeans`. `emmeans` frames contrasts as a question you pose to a model: you can ask for all pairwise comparisons and get back that. `lm` and `summary` treat the same problem as fitting abstract coefficients, and you are left to answer your own question.

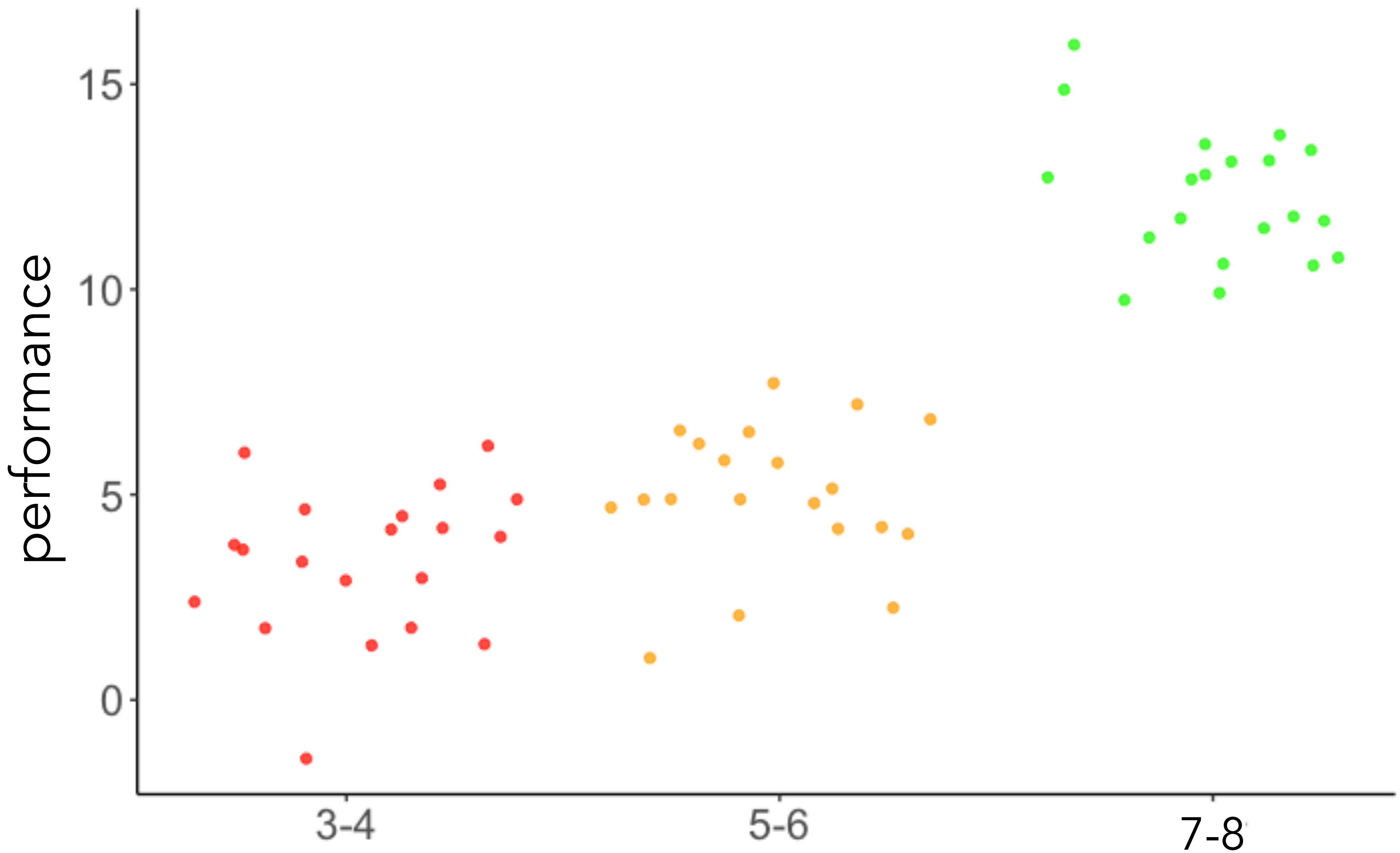
`emmeans` works with `lm`, `glm`, and the Bayesian friends in `brms` and `rstanarm`, so the process is applicable no matter the tool.

And you don't have to learn (much) about contrasts to take advantage of it.

https://timmastny.rbind.io/blog/tests-pairwise-categorical-mean-emmeans-contrast/

Contrasts

Does performance increase with age?



Data from a hypothetical developmental study

Linear contrasts in R

```
library("emmeans") # for calculating contrasts

#  fit the linear model
fit = lm(formula = performance ~ group,
         data = df.development)
```

fit linear model

Linear contrasts in R

```
library("emmeans") # for calculating contrasts

#  fit the linear model
fit = lm(formula = performance ~ group,
         data = df.development)

# check factor levels
levels(df.development$group)
```

```
[1] "3-4" "5-6" "7-8"
```

check factor levels before defining contrasts

Linear contrasts in R

```
library("emmeans") # for calculating contrasts

#  fit the linear model
fit = lm(formula = performance ~ group,
         data = df.development)

# check factor levels
levels(df.development$group) [1] "3-4" "5-6" "7-8"

# define the contrasts of interest
contrasts = list(young_vs_old = c(-0.5, -0.5, 1),
                 three_vs_five = c(-0.5, 0.5, 0))
```

set up linear contrasts

Linear contrasts in R

```r
library("emmeans") # for calculating contrasts

#  fit the linear model
fit = lm(formula = performance ~ group,
         data = df.development)

# check factor levels
levels(df.development$group) [1] "3-4" "5-6" "7-8"

# define the contrasts of interest
contrasts = list(young_vs_old = c(-0.5, -0.5, 1),
                 three_vs_five = c(-0.5, 0.5, 0))

# compute significance test on contrasts
fit %>%
  emmeans("group",
          contr = contrasts,
          adjust = "bonferroni") %>%
  pluck("contrasts")
```

compute the results

```
[1] "3-4" "5-6" "7-8"
 contrast       estimate        SE df t.ratio p.value
 young_vs_old  16.093541 0.4742322 57  33.936  <.0001
 three_vs_five  1.606009 0.5475962 57   2.933  0.0097

P value adjustment: bonferroni method for 2 tests
```

Linear contrasts in R

```r
library("emmeans") # for calculating contrasts

#  fit the linear model
fit = lm(formula = performance ~ group,
         data = df.development)

# check factor levels
levels(df.development$group)

# define the contrasts of interest
contrasts = list(young_vs_old = c(-1, -1, 2),
                 three_vs_five = c(-1, 1, 0))

# compute significance test on contrasts
fit %>%
  emmeans("group",
          contr = contrasts,
          adjust = "bonferroni") %>%
  pluck("contrasts")
```

```
[1] "3-4" "5-6" "7-8"
 contrast      estimate    SE df t.ratio p.value
 young_vs_old    32.187 0.948 57  33.936 <.0001
 three_vs_five    0.803 0.274 57   2.933 0.0097

P value adjustment: bonferroni method for 2 tests
```

hypothesis tests are the same!

Defining contrasts

- groups that we don't want to include in the comparison get a 0
- groups that we want to compare with one another should sum to 0
- this also means that all the contrasts together should sum to 0

Example:

```
contrasts = list(young_vs_old = c(-1, -1, 2),
                 three_vs_five = c(-1, 1, 0))
```

Post hoc tests

```r
fit = lm(formula = performance ~ group,
         data = df.development)

# pairwise differences between all the groups
fit %>%
  emmeans(pairwise ~ group) %>%
  pluck("contrasts")
```

all pairwise tests between groups

```
contrast    estimate        SE df t.ratio p.value
 3-4 - 5-6  -1.606009 0.5475962 57  -2.933  0.0145
 3-4 - 7-8 -16.896546 0.5475962 57 -30.856  <.0001
 5-6 - 7-8 -15.290537 0.5475962 57 -27.923  <.0001

P value adjustment: bonferroni method for 3 tests
```

Post hoc tests

```r
# fit the model
fit = lm(formula = balance ~ hand + skill,
         data = df.poker)

# post hoc tests
fit %>%
  emmeans(pairwise ~ hand + skill,
          adjust = "bonferroni") %>%
  pluck("contrasts")
```

the poker data

```
contrast                               estimate        SE  df t.ratio p.value
bad,average - neutral,average         -4.381023 0.6051766 286  -7.239  <.0001
bad,average - good,average            -7.060823 0.6051766 286 -11.667  <.0001
bad,average - bad,expert              -0.740385 0.4896119 286  -1.512  1.0000
bad,average - neutral,expert          -5.121408 0.7611327 286  -6.729  <.0001
bad,average - good,expert             -7.801208 0.7611327 286 -10.249  <.0001
neutral,average - good,average        -2.679800 0.5884403 286  -4.554  0.0001
neutral,average - bad,expert           3.640638 0.7953578 286   4.577  0.0001
neutral,average - neutral,expert      -0.740385 0.4896119 286  -1.512  1.0000
neutral,average - good,expert         -3.420185 0.7654945 286  -4.468  0.0002
good,average - bad,expert              6.320438 0.7953578 286   7.947  <.0001
good,average - neutral,expert          1.939415 0.7654945 286   2.534  0.1774
good,average - good,expert            -0.740385 0.4896119 286  -1.512  1.0000
bad,expert - neutral,expert           -4.381023 0.6051766 286  -7.239  <.0001
bad,expert - good,expert              -7.060823 0.6051766 286 -11.667  <.0001
neutral,expert - good,expert          -2.679800 0.5884403 286  -4.554  0.0001

P value adjustment: bonferroni method for 15 tests
```

that's a lot of tests!

... not

all pairwise tests between groups

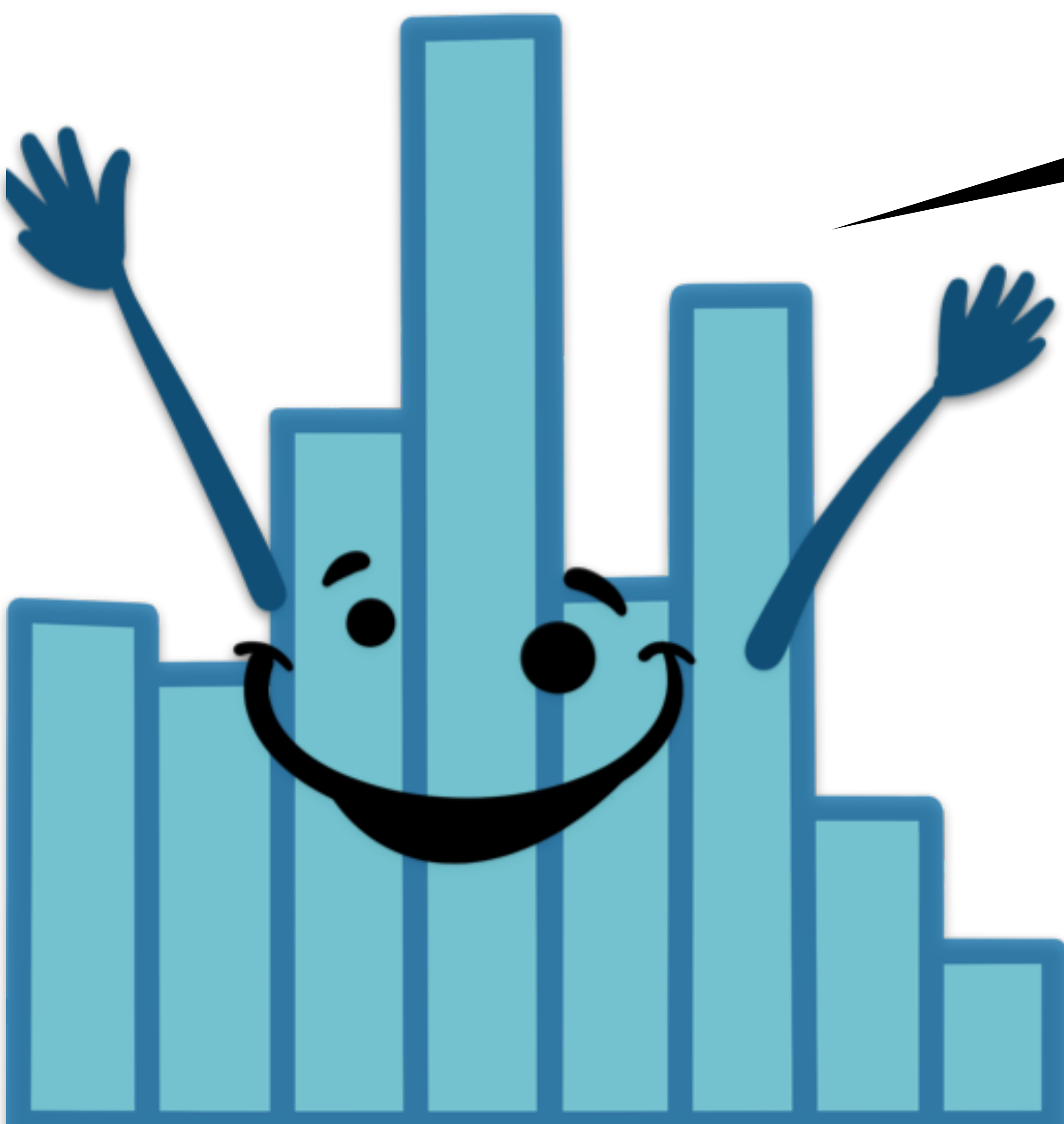
Contrasts

- linear contrasts allow us to ask more specific questions of our data
- rather than asking whether any of the group means are significantly different from each other (ANOVA), we can ask questions such as:
  - Does performance increase with age?
  - Is the overall performance in Condition B and C better from the performance in Condition A?

We're listening to "Wake up" by "Sudan Archives" submitted by Tobi

02:00

stretch break!



Unbalanced designs

not the same number of
participants in each cell

ANOVA

- for all the examples so far, I've assumed a balanced design (i.e. the same number of observations in each of the different factor levels)
- things get funky when we have an unbalanced design

https://towardsdatascience.com/anovas-three-types-of-estimating-sums-of-squares-don-t-make-the-wrong-choice-91107c77a27a

Beware of unbalanced designs

```r
lm(formula = balance ~ skill + hand, data = df.poker.unbalanced) %>%
  anova()
```

```
Analysis of Variance Table

Response: balance
           Df Sum Sq Mean Sq F value  Pr(>F)    
skill       1   74.3   74.28  4.2904 0.03922 *  
hand        2 2385.1 1192.57 68.8827 < 2e-16 ***
Residuals 286 4951.5   17.31                    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

flipped the order

```r
lm(formula = balance ~ hand + skill, data = df.poker.unbalanced) %>%
  anova()
```

```
Analysis of Variance Table

Response: balance
           Df Sum Sq Mean Sq F value Pr(>F)    
hand        2 2419.8 1209.92 69.8845 <2e-16 ***
skill       1   39.6   39.59  2.2867 0.1316    
Residuals 286 4951.5   17.31                   
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

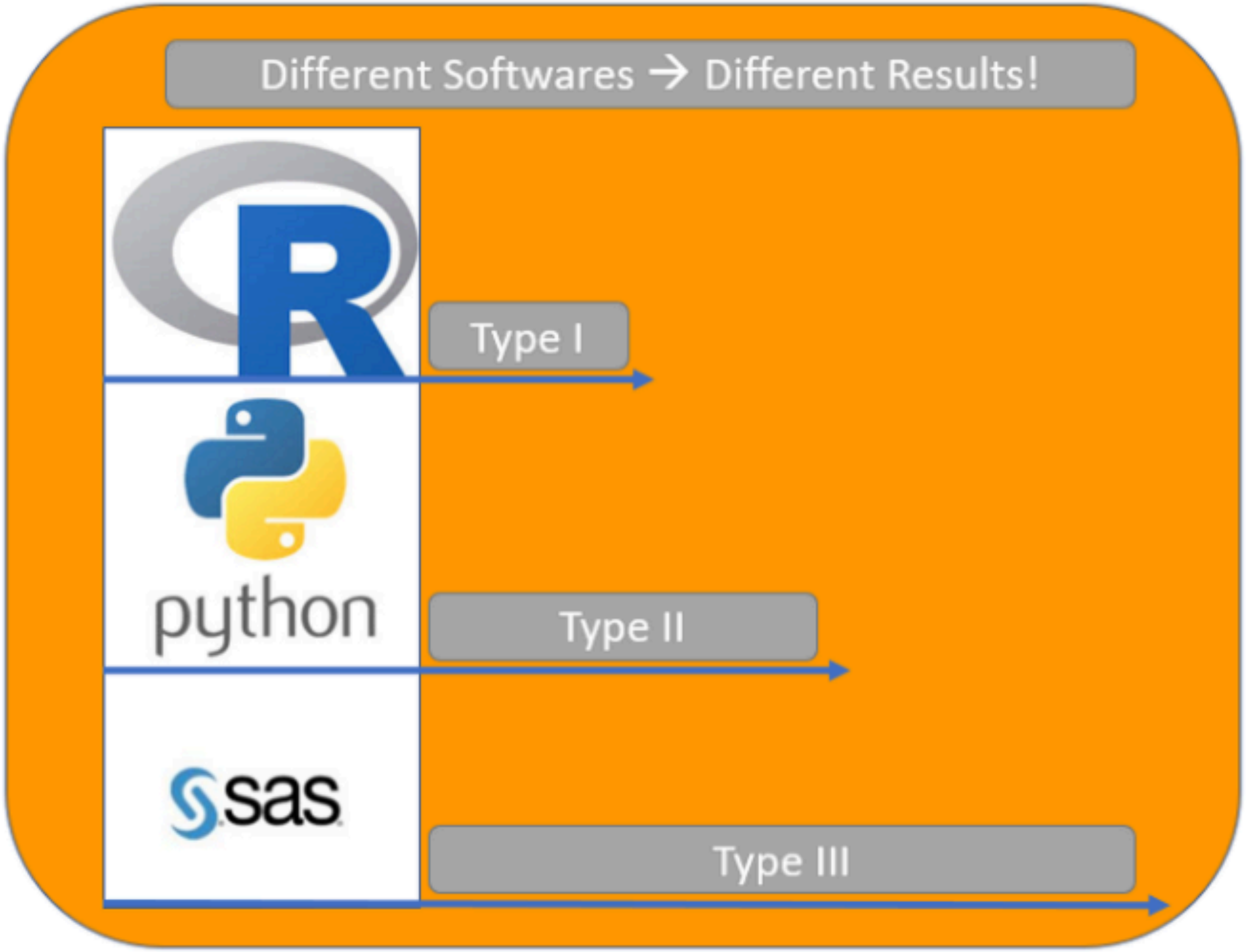
The different sums of squares

Three different methodologies for splitting variation exist: Type I, Type II and Type III Sums of Squares. They do not give the same result in case of unbalanced data.

Type I, Type II and Type III ANOVA have different outcomes!

https://towardsdatascience.com/anovas-three-types-of-estimating-sums-of-squares-don-t-make-the-wrong-choice-91107c77a27a

Default sums of squares ...



Default Types of Sums of Squares for different programming languages

not great for reproducibility ...

Type I Sums of Squares

Type I Sums of Squares are Sequential, so the order of variables in the models makes a difference. This is rarely what we want in practice!

Sums of Squares are Mathematically defined as:

- SS(A) for independent variable A
- SS(B | A) for independent variable B
- SS(AB | B, A) for the interaction effect

caution: this is what `anova()` uses by default

https://towardsdatascience.com/anovas-three-types-of-estimating-sums-of-squares-don-t-make-the-wrong-choice-91107c77a27a

Type II Sums of Squares

Type II Sums of Squares should be used if there is no interaction between the independent variables.

Sums of Squares are Mathematically defined as:

- SS(A | B) for independent variable A
- SS(B | A) for independent variable B
- No interaction effect

however, often not used in practice ...
(mostly because we are interested in interaction effects)

Type III Sums of Squares

The Type III Sums of Squares are also called partial sums of squares again another way of computing Sums of Squares:

- Like Type II, the Type III Sums of Squares are not sequential, so the order of specification does not matter.
- Unlike Type II, the Type III Sums of Squares do specify an interaction effect.

Sums of Squares are Mathematically defined as:

- SS(A | B, AB) for independent variable A
- SS(B | A, AB) for independent variable B

this is the default in the literature (e.g. SPSS uses it)

Route I: Using "afex"

```r
library("afex")

fit = aov_ez(id = "participant",
             dv = "balance",
             data = df.poker.unbalanced,
             between = c("hand", "skill"))
fit$Anova
```

```
Contrasts set to contr.sum for the following variables: hand, skill
Anova Table (Type III tests)

Response: dv
             Sum Sq  Df   F value    Pr(>F)    
(Intercept) 27781.3   1 1676.9096 < 2.2e-16 ***
hand         2285.3   2   68.9729 < 2.2e-16 ***
skill          48.9   1    2.9540 0.0867525 .  
hand:skill    246.5   2    7.4401 0.0007089 ***
Residuals    4705.0 284                        
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Route II: Using "emmeans"

preferred route!!

```r
library("emmeans")

lm(formula = balance ~ hand + skill,
   data = df.poker.unbalanced) %>%
  joint_tests()
```

very handy function

model term	df1	df2	F.ratio	p.value
hand	2	284	68.973	<.0001
skill	1	284	2.954	0.0868
hand:skill	2	284	7.440	0.0007

Same same ...

Route I: Using "afex"

```r
library("afex")

fit = aov_ez(id = "participant",
             dv = "balance",
             data = df.poker.unbalanced,
             between = c("hand", "skill"))
fit$Anova
```

```
Contrasts set to contr.sum for the following variables: hand, skill
Anova Table (Type III tests)

Response: dv
             Sum Sq  Df   F value    Pr(>F)    
(Intercept) 27781.3   1 1676.9096 < 2.2e-16 ***
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skill          48.9   1    2.9540 0.0867525 .  
hand:skill    246.5   2    7.4401 0.0007089 ***
Residuals    4705.0 284                        
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Route II: Using "emmeans"

preferred route!!

```r
library("emmeans")

lm(formula = balance ~ hand + skill,
   data = df.poker.unbalanced) %>%
  joint_tests()
```

very handy function

```
model term df1 df2 F.ratio p.value
 hand         2 284  68.973  <.0001
 skill        1 284   2.954  0.0868
 hand:skill   2 284   7.440  0.0007
```

... but different

can come apart when we deal with repeated observations, but we'll deal with that later!

Unbalanced design

- There are different kinds of ANOVAs, for which the sums of squares are calculated differently.
- This makes a difference when we have an unbalanced design (i.e. the number of participants is not the same for each cell in our design).

`joint_tests()` is your friend!

Power analysis

Making decisions

Type I Error	Type II Error
	

H_0: Not pregnant. **H_1**: Pregnant.

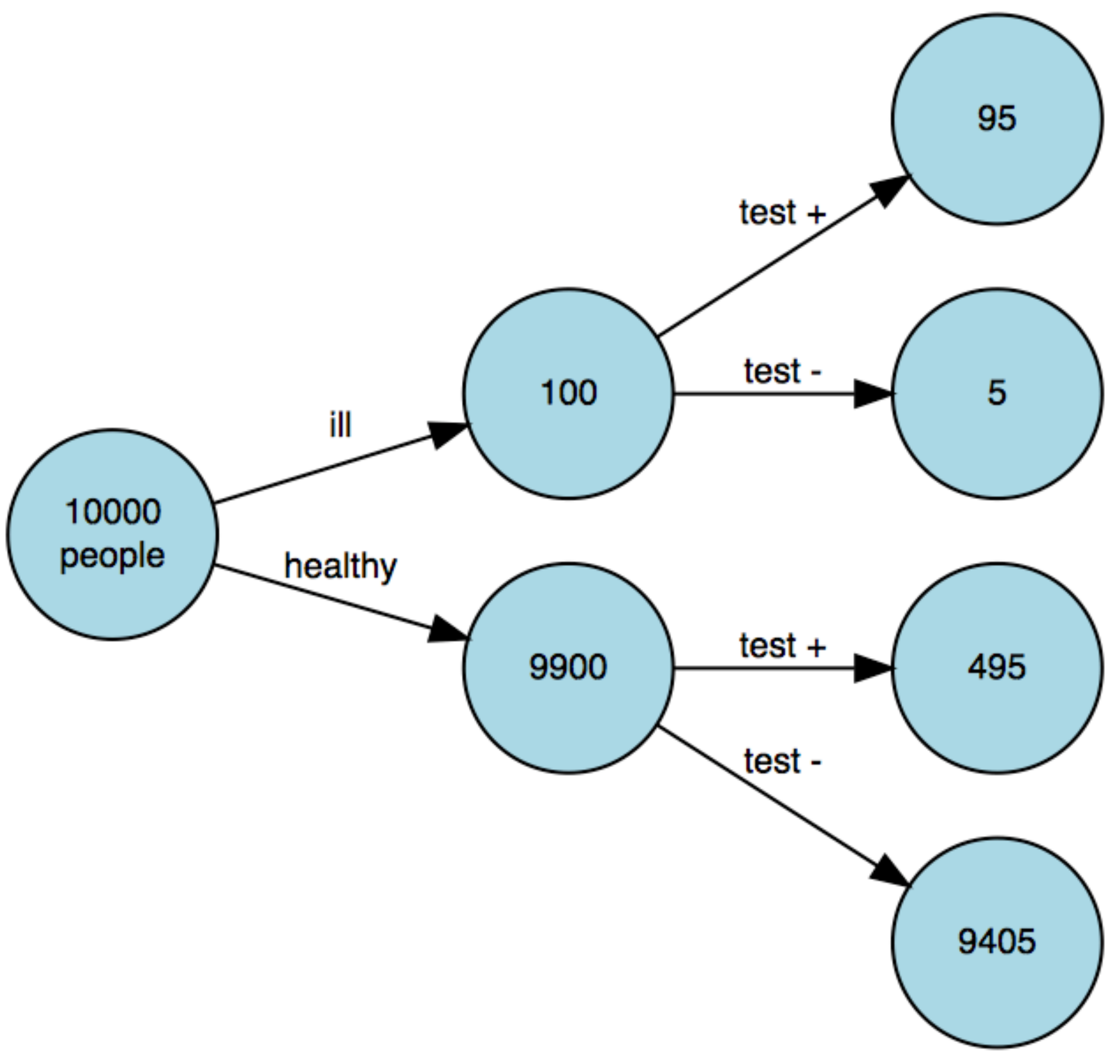
Type I Error: Falsely rejecting the null hypothesis (even though it is true).

Type II Error: Failing to reject the null hypothesis (even though it is false).

Clue guide to probability

H$_0$: The person is healthy.

H$_1$: The person is ill.



Power

Sensitivity

true positive $p(\text{reject } H_0 \mid H_1 \text{ is true})$ $\boxed{1-\beta}$

Type II error

false negative $p(\text{not reject } H_0 \mid H_1 \text{ is true})$ β

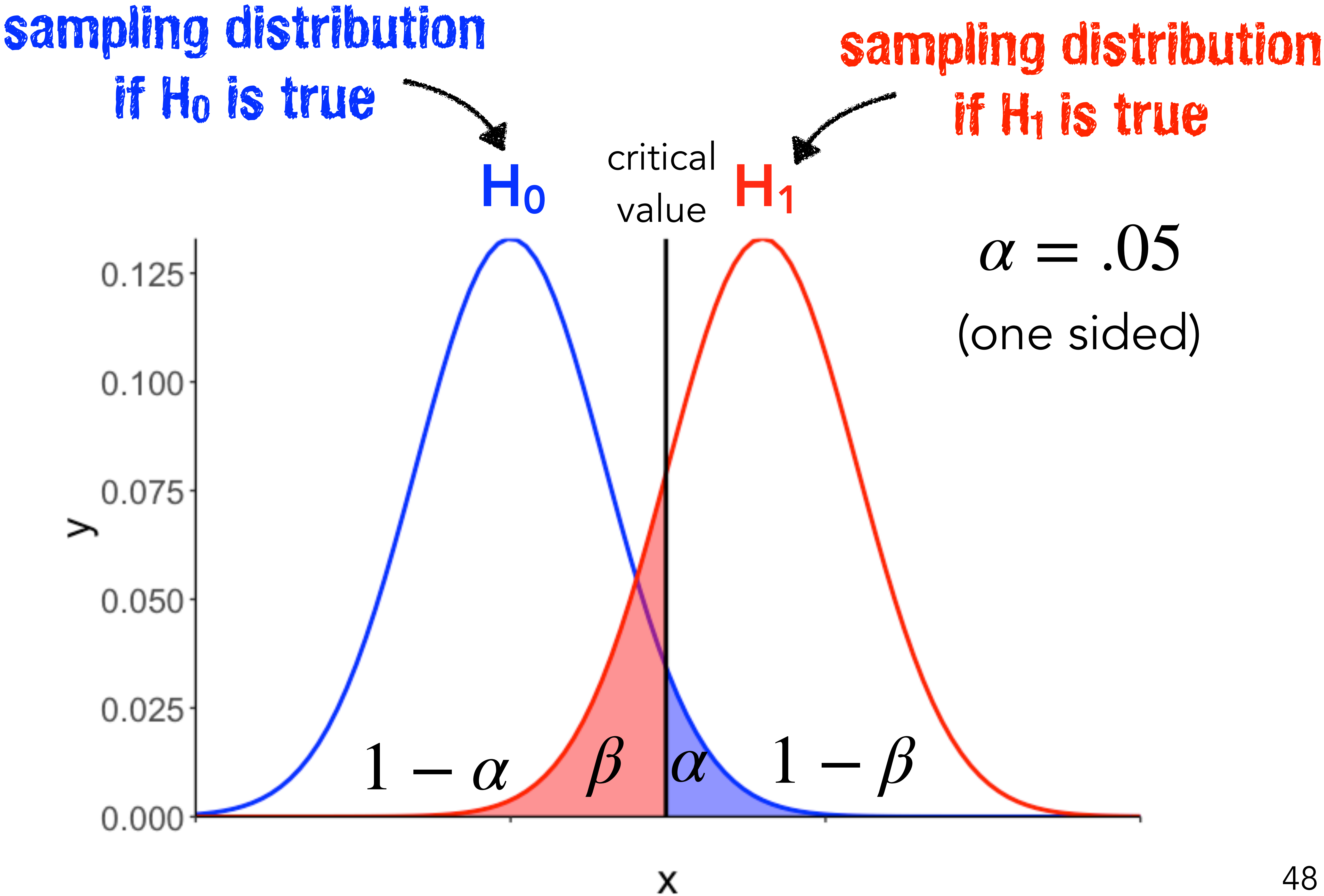
Type I error

false positive $p(\text{reject } H_0 \mid H_0 \text{ is true})$ α

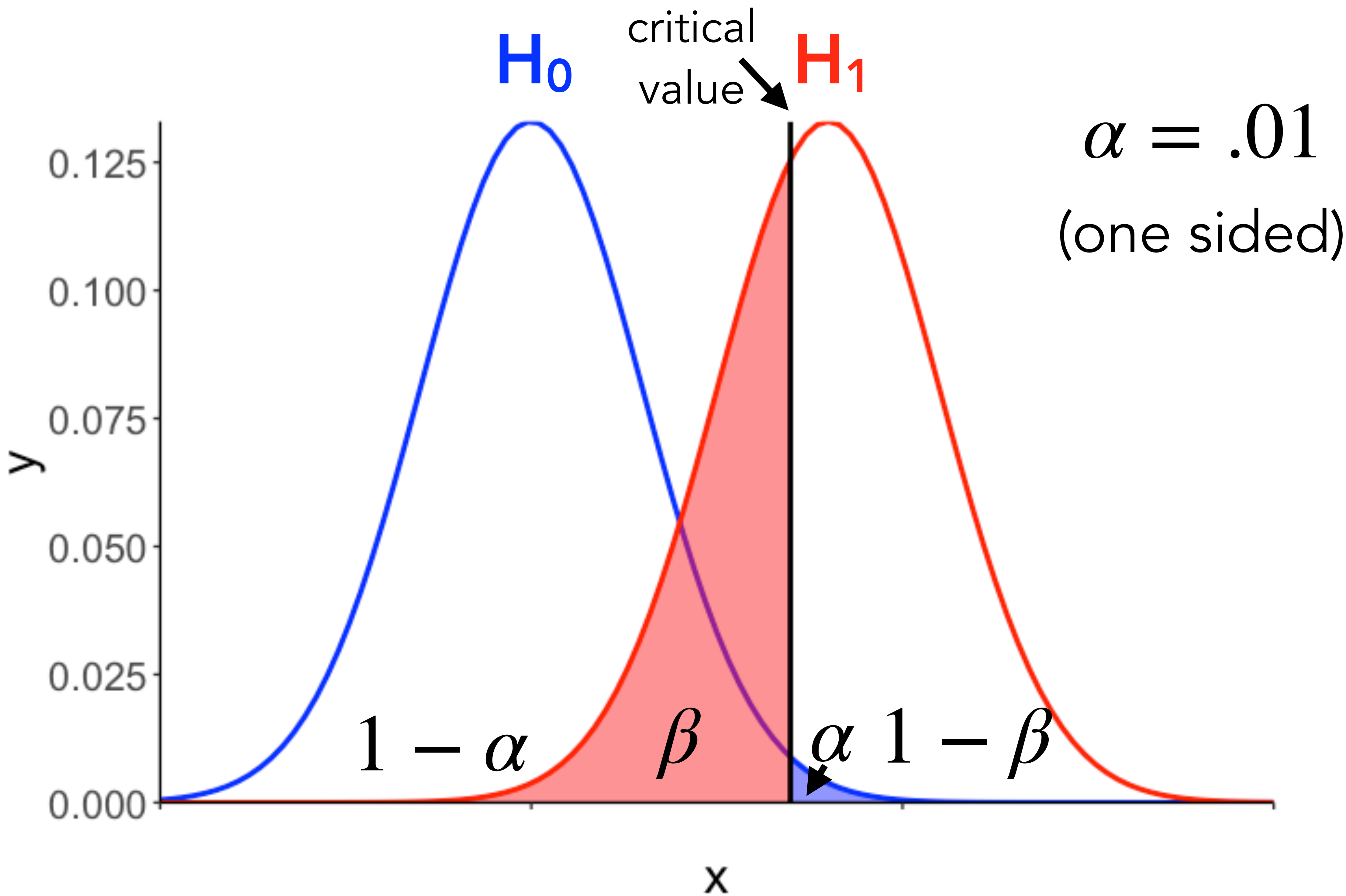
true negative $p(\text{not reject } H_0 \mid H_0 \text{ is true})$ $1-\alpha$

Specificity

What affects power?

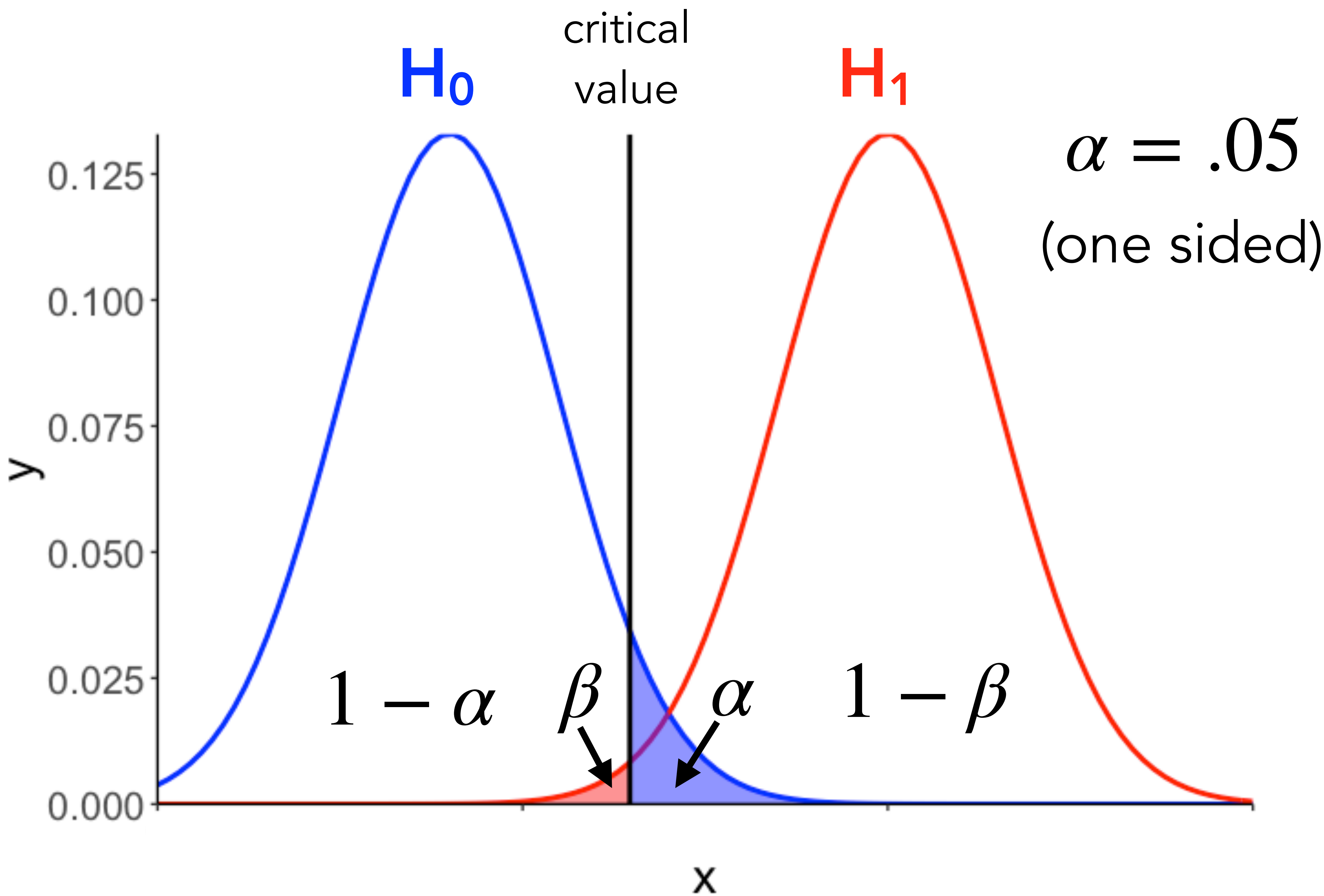


What affects power?



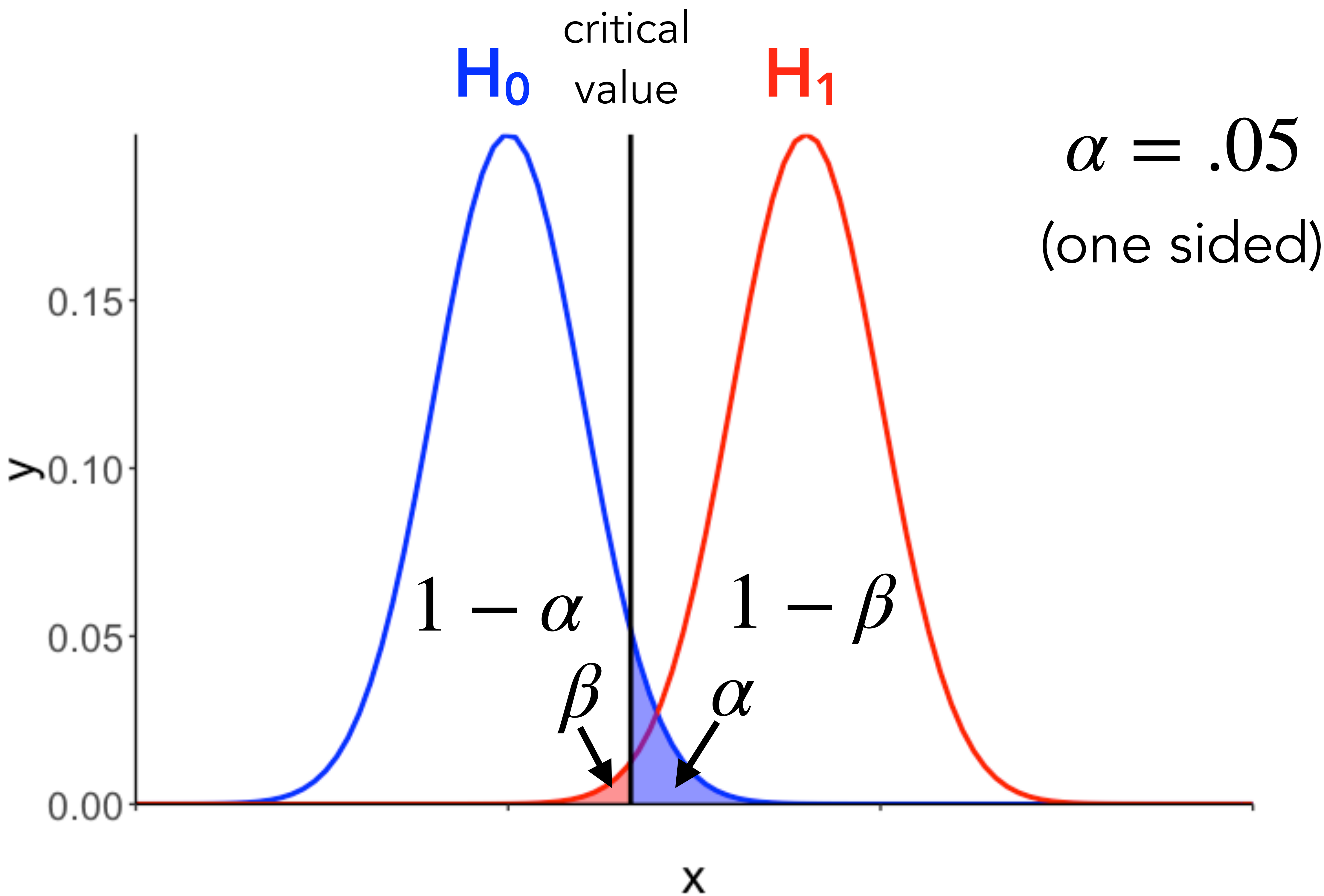
α affects power

What affects power?



distance between means affects power

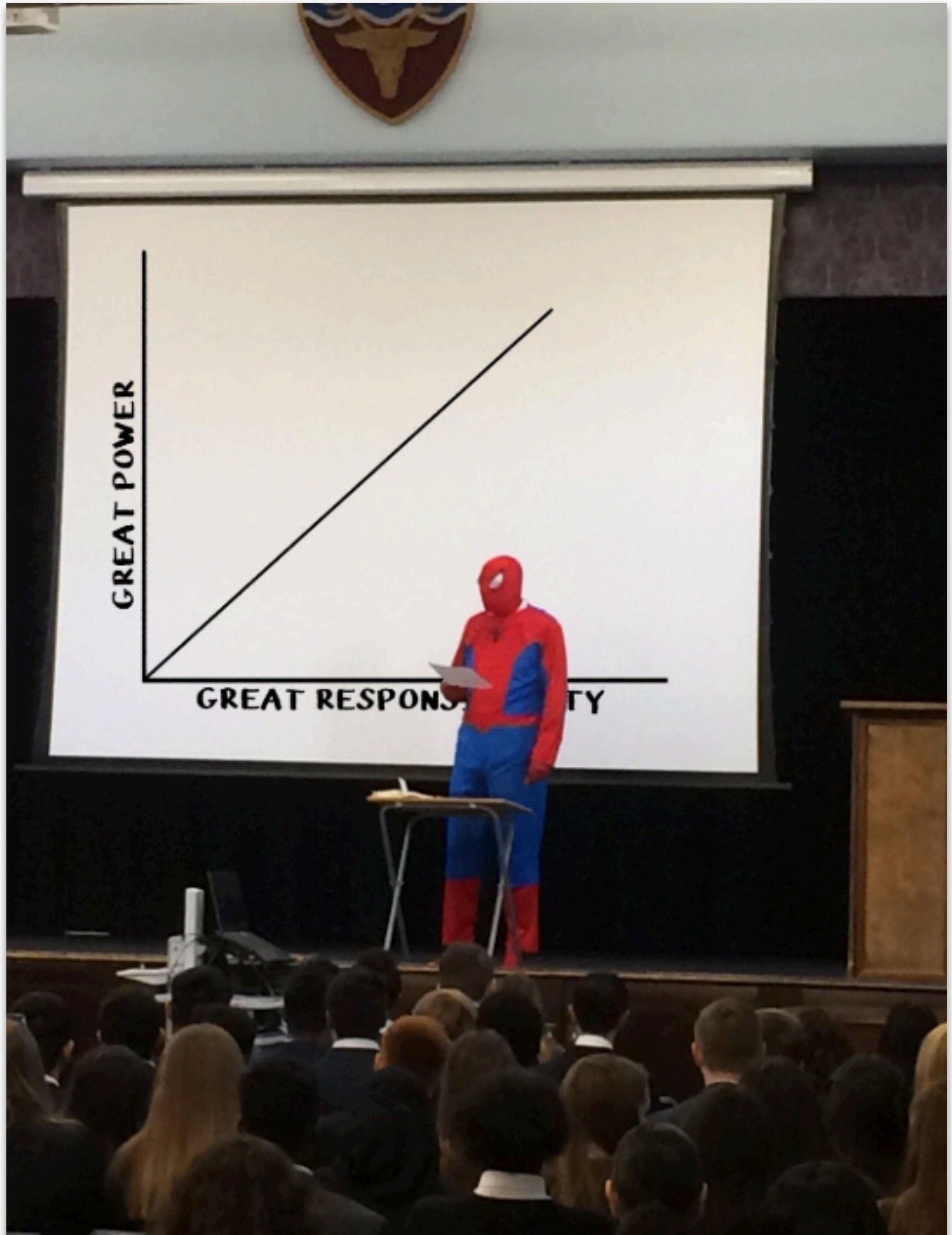
What affects power?



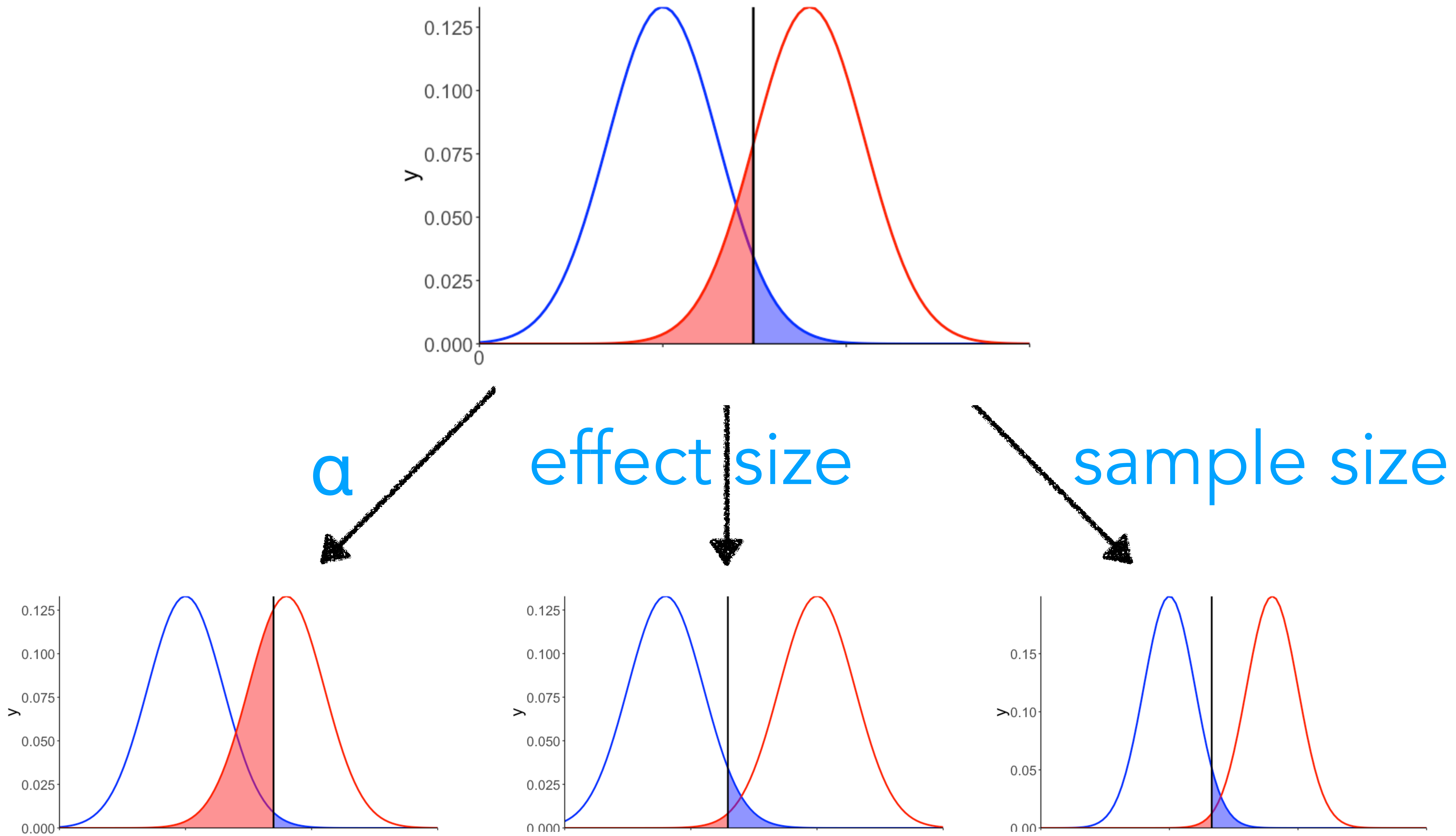
variance affects power

Calculating power

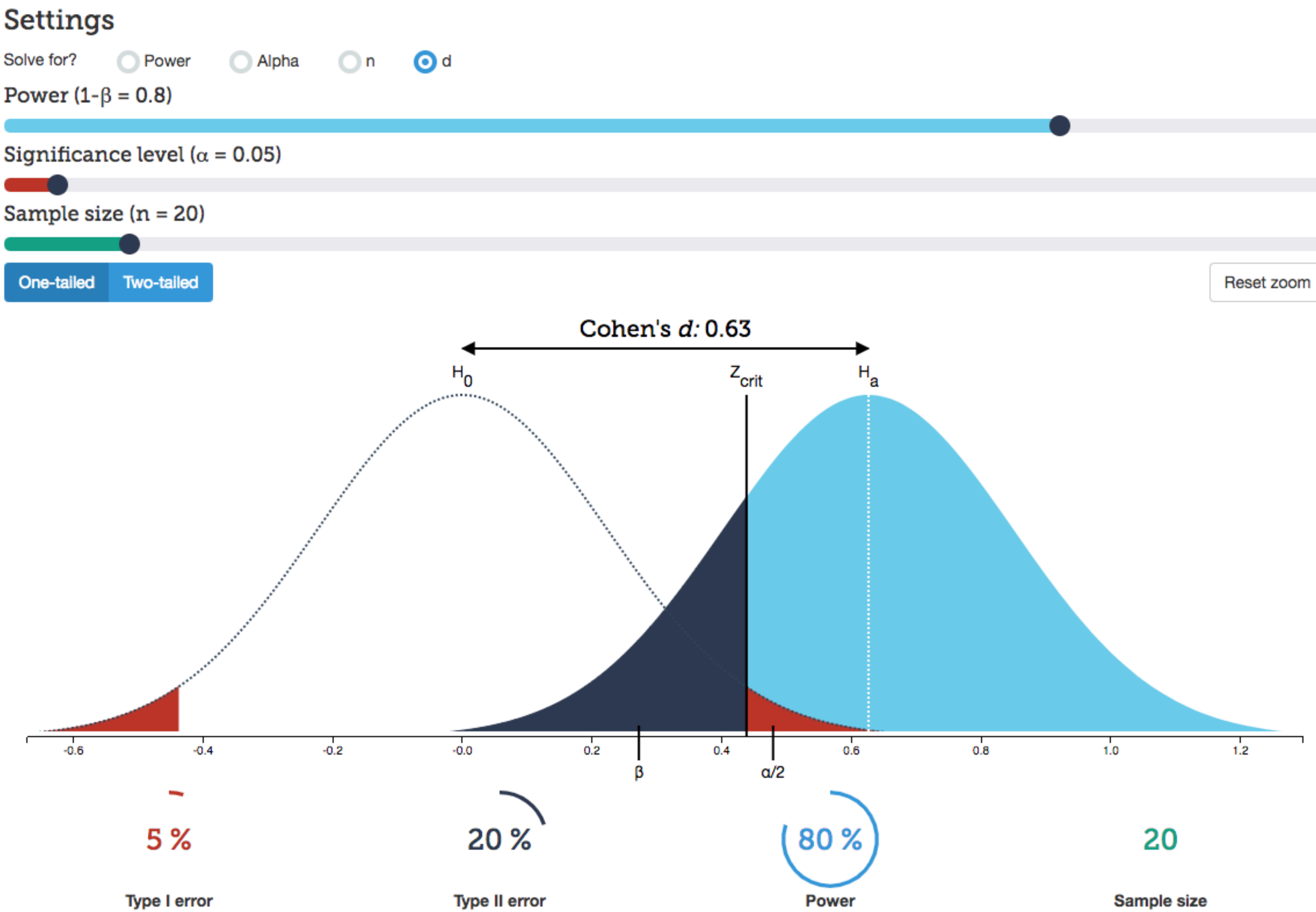
With great power comes ...



The knobs we can turn to affect power



Visualization demo



https://rpsychologist.com/d3/NHST/

The **power** of a binary hypothesis test is the probability that the test rejects the null hypothesis (H_0) when a **specific** alternative hypothesis (H_1) is true.

H_0: Students and non-students have the same balance.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

$$\beta_1 = 0$$

H_1: Students and non-students have different balances.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\beta_1 \neq 0$$

We cannot calculate power in this case.
We need a specific alternative hypothesis!

The **power** of a binary hypothesis test is the probability that the test rejects the null hypothesis (H_0) when a **specific** alternative hypothesis (H_1) is true.

H_0: Students and non-students have the same balance.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

$$\beta_1 = 0$$

H_1: Students and non-students have different balances.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\beta_1 = 300$$

We can calculate power in this case (since we have a specific alternative hypothesis)!

Effect sizes

Effect sizes

- a p-value tells us whether we can reject the H_0
- effect sizes is a measure of the strength of the actual effect

Why can't we just use p-values as a measure of the effect size?

$$F = \frac{\mathrm{PRE}/(\mathrm{PA} - \mathrm{PC})}{(1 - \mathrm{PRE})/(n - \mathrm{PA})}$$

any PRE will become significant if *n* gets large enough

PRE = proportional reduction in error
PA = # parameters in the augmented model
PC = # parameters in the compact model
n = sample size

statistical vs. practical significance

Effect sizes

PRE = **p**roportional **r**eduction in **e**rror

Compact model

SSE(C)

Augmented model

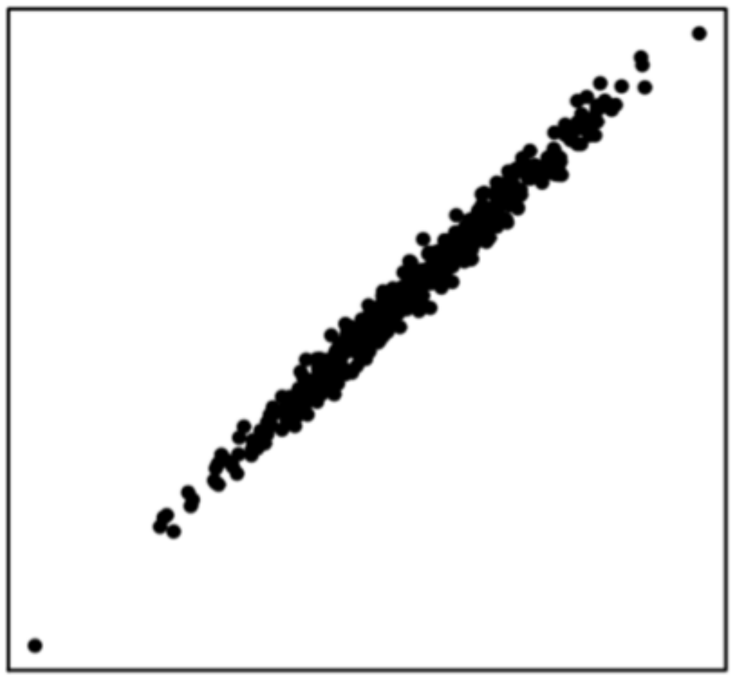
SSE(A)

$$\text{PRE} = 1 - \frac{\text{SSE}(A)}{\text{SSE}(C)}$$

SSE = sum of squared errors

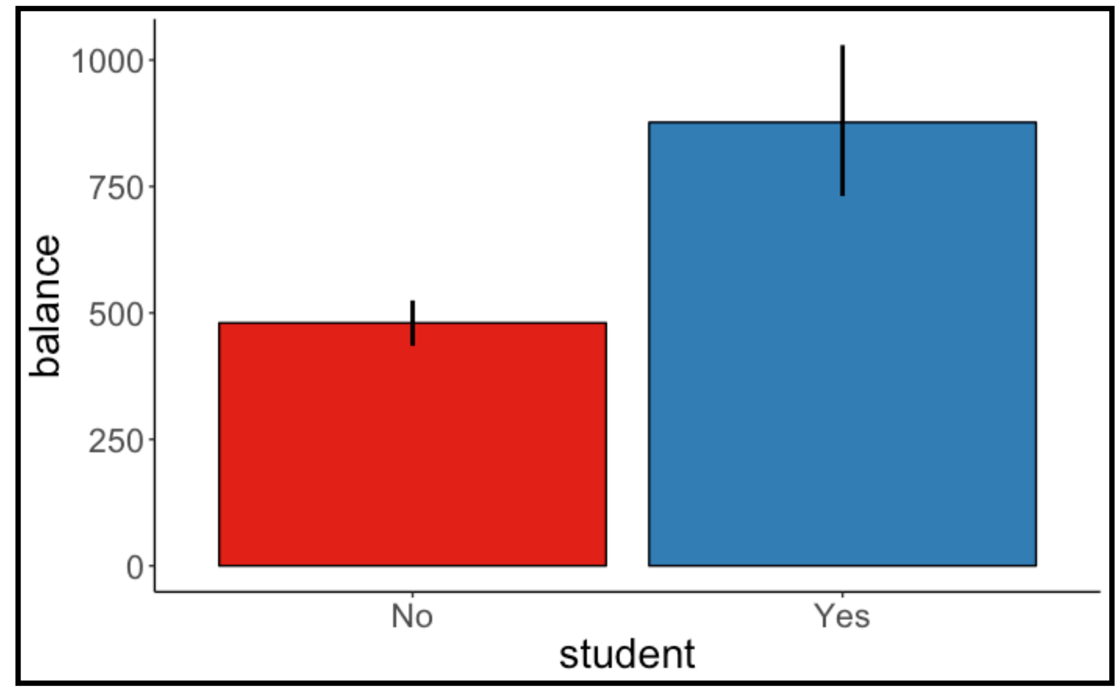
Common effect sizes

Relationships between variables



r correlation

Differences between groups



Cohen's d

Correlation

Pearson correlation

$$r(X, Y) = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2} \cdot \sqrt{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}}$$

Cohen's guidelines for the social sciences

Effect size	*r*
Small	0.1
Medium	0.3
Large	0.5

depends very much on the domain

Cohen's *d*

- standardized difference between two means

absolute difference between means

$$d = \frac{|\bar{y}_1 - \bar{y}_2|}{s_p}$$

pooled standard variation

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Effect size	***d***
Very small	0.01
Small	0.20
Medium	0.50
Large	0.80
Very large	1.20
Huge	2.0

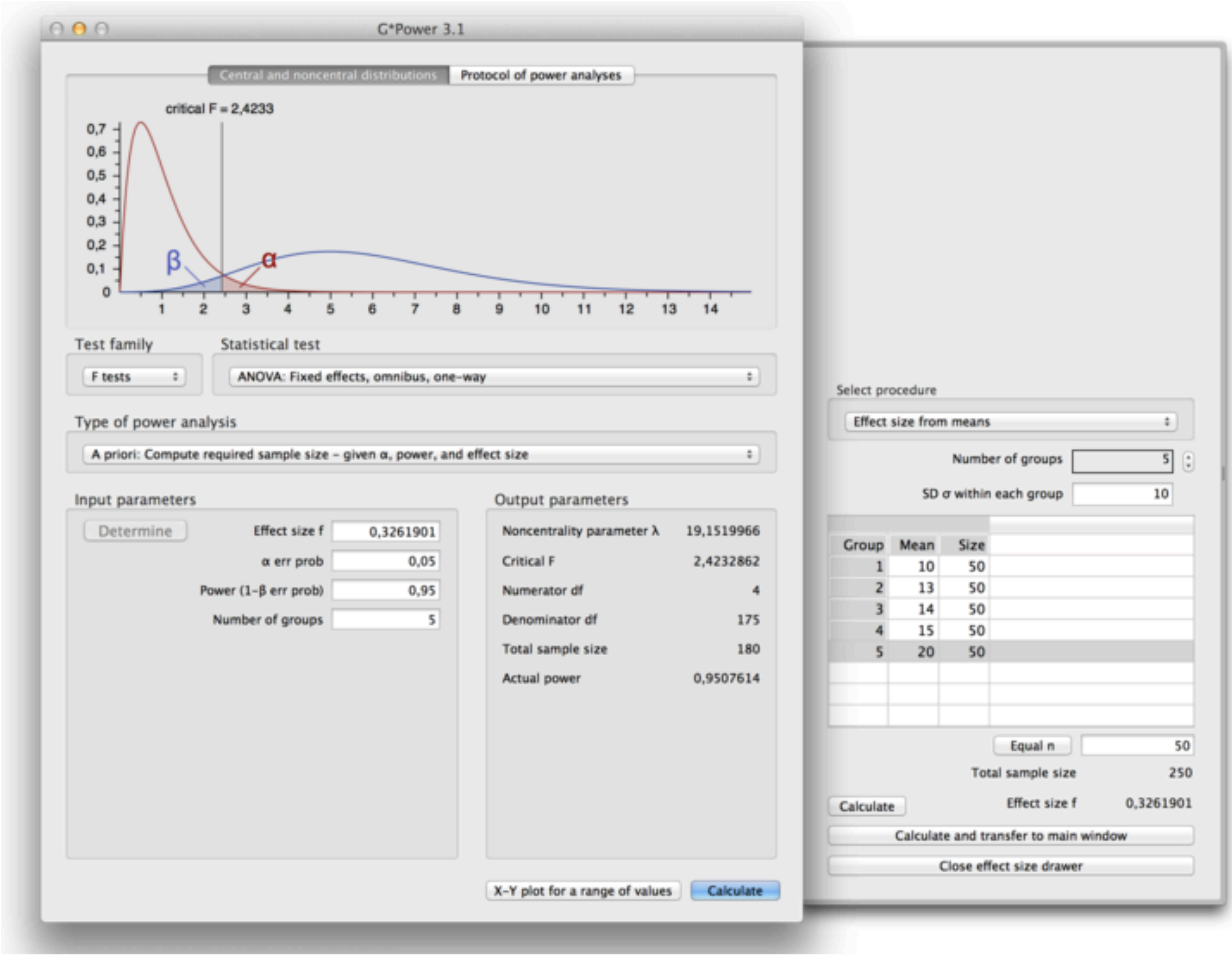
Difference between two means in pooled standard deviation

Determining sample size

How many participants do I need to run to have a good chance of detecting a true effect?

G*Power 3.1: Alternative software for power calculations

Option 1

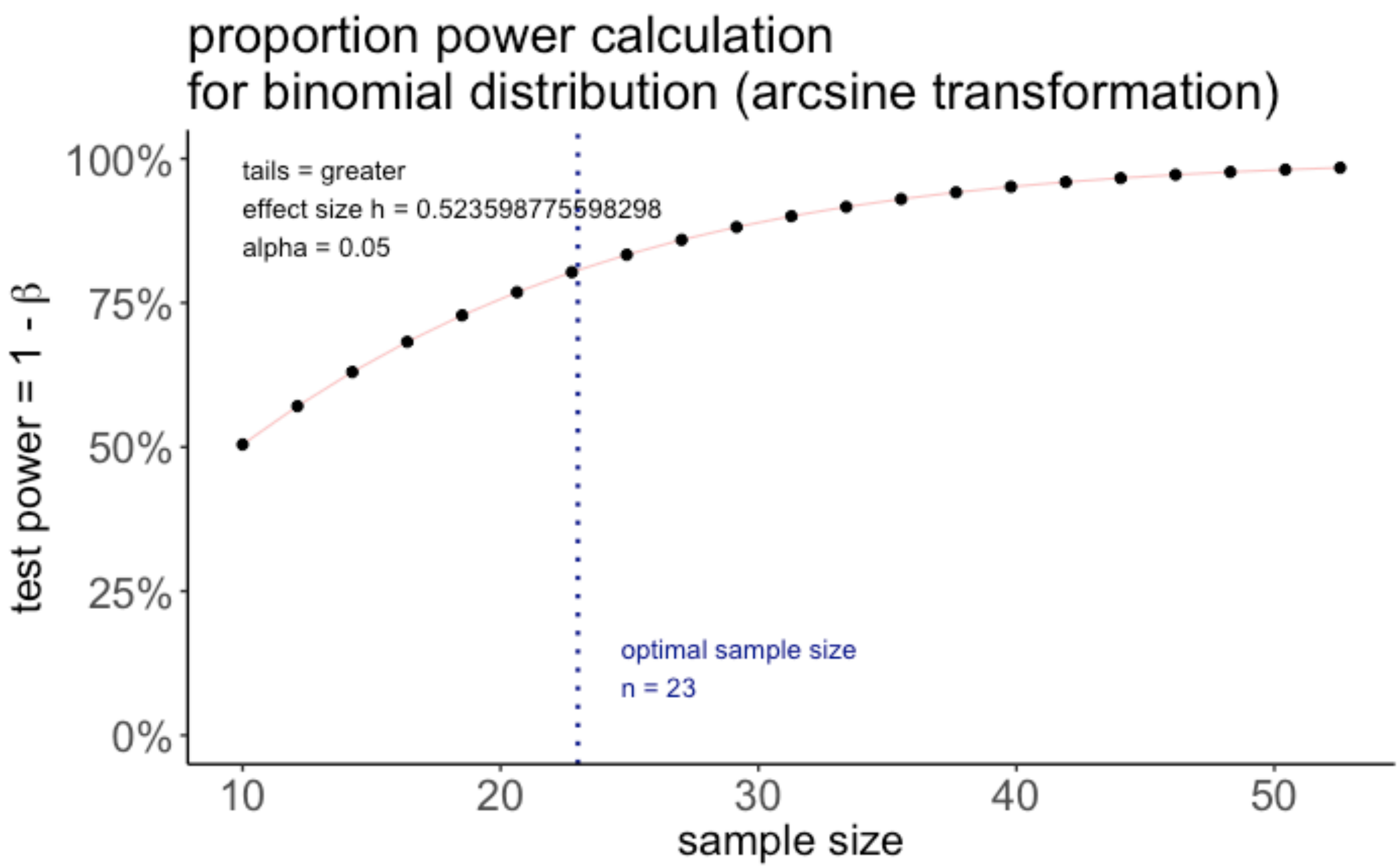


http://www.gpower.hhu.de/

"pwr" package in R

Option 2

```r
library("pwr")
pwr.p.test(h = ES.h(p1 = 0.75, p2 = 0.50),
                 sig.level = 0.05,
                 power = 0.80,
                 alternative = "greater") %>%
 plot()
```



Option 3

Power simulation recipe

- assume:
  - α, *n*, effect size
- simulate a large number of data sets of size *n* with the specified effect size
- for each data set, run a statistical test to calculate the p-value
- determine the probability of rejecting the H_0 (given that H_1 is true)

Power analysis via simulation

Let's simulate

```
library("purrr")
```

automatically loaded with

```
library("tidyverse")
```

map()

map()

vector or list

function

```
map(.x = 1:3, .f = ~ .^2)
```

outputs a list

```
[[1]]
[1] 1

[[2]]
[1] 4

[[3]]
[1] 9
```

- **map**(list, function) applies a function to each element of the list
- it's a unified version of the many different **apply**() functions in base R
- you already know a cousin of **map**(): **replicate**()
- use **map**(), don't write for(){} loops!
- it's extremely powerful in combination with data frames

`map()`

same same but different

```r
map(.x = 1:3, .f = ~ .x^2)

map(1:3, ~ .x^2)

map(1:3, ~ .^2)

map(.x = 1:3, .f = function(.x) .x^2)

# using a function
square = function(x){x^2}
map(1:3, square)
```

Power analysis example

Power simulation recipe

- assume:
  - α, *n*, effect size
- simulate a large number of data sets of size *n* with the specified effect size
- for each data set, run a statistical test to calculate the p-value
- determine the probability of rejecting the H_0 (given that H_1 is true)

Let's simulate ...

```
# make reproducible
set.seed(1)

# number of simulations
n_simulations = 5

# run simulation
expand_grid(n = seq(10, 40, 2),
            simulation = 1:n_simulations,
            p = 0.75) %>%
  mutate(index = 1:n(),
         .before = n) %>%
  group_by(index, n, simulation) %>%
  mutate(response = rbinom(n = n(),
                           size = n,
                           prob = p)) %>%
  group_by(index, simulation, p) %>%
  nest() %>%
  mutate(fit = map(.x = data,
                   .f = ~ binom.test(x = .$response,
                                     n = .$n,
                                     p = 0.5,
                                     alternative = "two.sided"))) %>%
  mutate(coef = map(.x = fit,
                    .f = ~ tidy(.))) %>%
  select(simulation, p, index, coef) %>%
  unnest(cols = coef) %>%
  rename(n = parameter) %>%
  group_by(n, p) %>%
  summarize(power = sum(p.value < 0.05) / n()) %>%
  ungroup()
```

Let's simulate ...

```r
# make reproducible
set.seed(1)

# number of simulations
n_simulations = 5

# run simulation
expand_grid(n = seq(10, 40, 2),
            simulation = 1:n_simulations,
            p = 0.75) %>%
  mutate(index = 1:n(),
         .before = n) %>%
```

index	n	simulation	p
1	10	1	0.75
2	10	2	0.75
3	10	3	0.75
4	10	4	0.75
5	10	5	0.75
6	12	1	0.75
7	12	2	0.75
8	12	3	0.75
9	12	4	0.75
10	12	5	0.75

Let's simulate ...

```
# make reproducible
set.seed(1)

# number of simulations
n_simulations = 5

# run simulation
expand_grid(n = seq(10, 40, 2),
            simulation = 1:n_simulations,
            p = 0.75) %>%
  mutate(index = 1:n(),
         .before = n) %>%
  group_by(index, n, simulation) %>%
  mutate(response = rbinom(n = n(),
                           size = n,
                           prob = p)) %>%
```

index	n	simulation	p	response
1	10	1	0.75	8
2	10	2	0.75	7
3	10	3	0.75	8
4	10	4	0.75	8
5	10	5	0.75	7
6	12	1	0.75	10
7	12	2	0.75	8
8	12	3	0.75	11
9	12	4	0.75	10
10	12	5	0.75	11

Let's simulate ...

```r
# make reproducible
set.seed(1)

# number of simulations
n_simulations = 5

# run simulation
expand_grid(n = seq(10, 40, 2),
            simulation = 1:n_simulations,
            p = 0.75) %>%
  mutate(index = 1:n(),
         .before = n) %>%
  group_by(index, n, simulation) %>%
  mutate(response = rbinom(n = n(),
                           size = n,
                           prob = p)) %>%
  group_by(index, simulation, p) %>%
  nest() %>%
```

index	simulation	p	data
1	1	0.75	10, 8
2	2	0.75	10, 6
3	3	0.75	10, 8
4	4	0.75	10, 6
5	5	0.75	10, 6
6	1	0.75	12, 9
7	2	0.75	12, 11
8	3	0.75	12, 10
9	4	0.75	12, 8
10	5	0.75	12, 10

Let's simulate ...

```r
# make reproducible
set.seed(1)

# number of simulations
n_simulations = 5

# run simulation
expand_grid(n = seq(10, 40, 2),
            simulation = 1:n_simulations,
            p = 0.75) %>%
  mutate(index = 1:n(),
         .before = n) %>%
  group_by(index, n, simulation) %>%
  mutate(response = rbinom(n = n(),
                           size = n,
                           prob = p)) %>%
  group_by(index, simulation, p) %>%
  nest() %>%
  mutate(fit = map(.x = data,
                   .f = ~ binom.test(x = .$response,
                                     n = .$n,
                                     p = 0.5,
                                     alternative = "two.sided"))) %>%
  mutate(coef = map(.x = fit,
                    .f = ~ tidy(.))) %>%
  select(simulation, p, index, coef) %>%
  unnest(cols = coef) %>%
```

simulation	p	index	estimate	statistic	p.value	n	conf.low	conf.high	method	alternative
1	0.75	1	1.00	10	0.00	10	0.69	1.00	Exact binomial test	two.sided
2	0.75	2	0.70	7	0.34	10	0.35	0.93	Exact binomial test	two.sided
3	0.75	3	0.60	6	0.75	10	0.26	0.88	Exact binomial test	two.sided
4	0.75	4	0.70	7	0.34	10	0.35	0.93	Exact binomial test	two.sided
5	0.75	5	0.80	8	0.11	10	0.44	0.97	Exact binomial test	two.sided

Let's simulate ...

```r
# make reproducible
set.seed(1)

# number of simulations
n_simulations = 5

# run simulation
expand_grid(n = seq(10, 40, 2),
            simulation = 1:n_simulations,
            p = 0.75) %>%
  mutate(index = 1:n(),
         .before = n) %>%
  group_by(index, n, simulation) %>%
  mutate(response = rbinom(n = n(),
                           size = n,
                           prob = p)) %>%
  group_by(index, simulation, p) %>%
  nest() %>%
  mutate(fit = map(.x = data,
                   .f = ~ binom.test(x = .$response,
                                     n = .$n,
                                     p = 0.5,
                                     alternative = "two.sided"))) %>%
  mutate(coef = map(.x = fit,
                    .f = ~ tidy(.))) %>%
  select(simulation, p, index, coef) %>%
  unnest(cols = coef) %>%
  rename(n = parameter) %>%
  group_by(n, p) %>%
  summarize(power = sum(p.value < 0.05) / n()) %>%
  ungroup()
```

n	p	power
10	0.75	0.2
12	0.75	0.2
14	0.75	0.4
16	0.75	0.2
18	0.75	0.6
20	0.75	0.8
22	0.75	0.6
24	0.75	0.4
26	0.75	0.6
28	0.75	0.8
30	0.75	0.8

Let's simulate ...

- here, I've used a simple example (Binomial test)
- but: we can use the same recipe for any statistical test that we are planning on running

Power simulation recipe

- assume:
  - α, *n*, effect size
- simulate a large number of data sets of size *n* with the specified effect size
- for each data set, run a statistical test to calculate the p-value for a given α
- determine the probability of rejecting the H_0 (given that H_1 is true)

Plan for today

- Quick recap
- Linear contrasts
  - Testing specific hypotheses with linear contrasts
  - `emmeans` for handling linear contrasts in R
- Unbalanced designs
- Power analysis
  - Making decisions
  - Calculating power
  - Effect sizes
  - Determining sample size
- Learn about more advanced simulation techniques in R
  - `map()`
  - list columns: `nest()`, `unnest()`
- Simulating a power analysis in R

will share a 25 minute video lecture

Feedback

How was the pace of today's class?

How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?

Thank you!