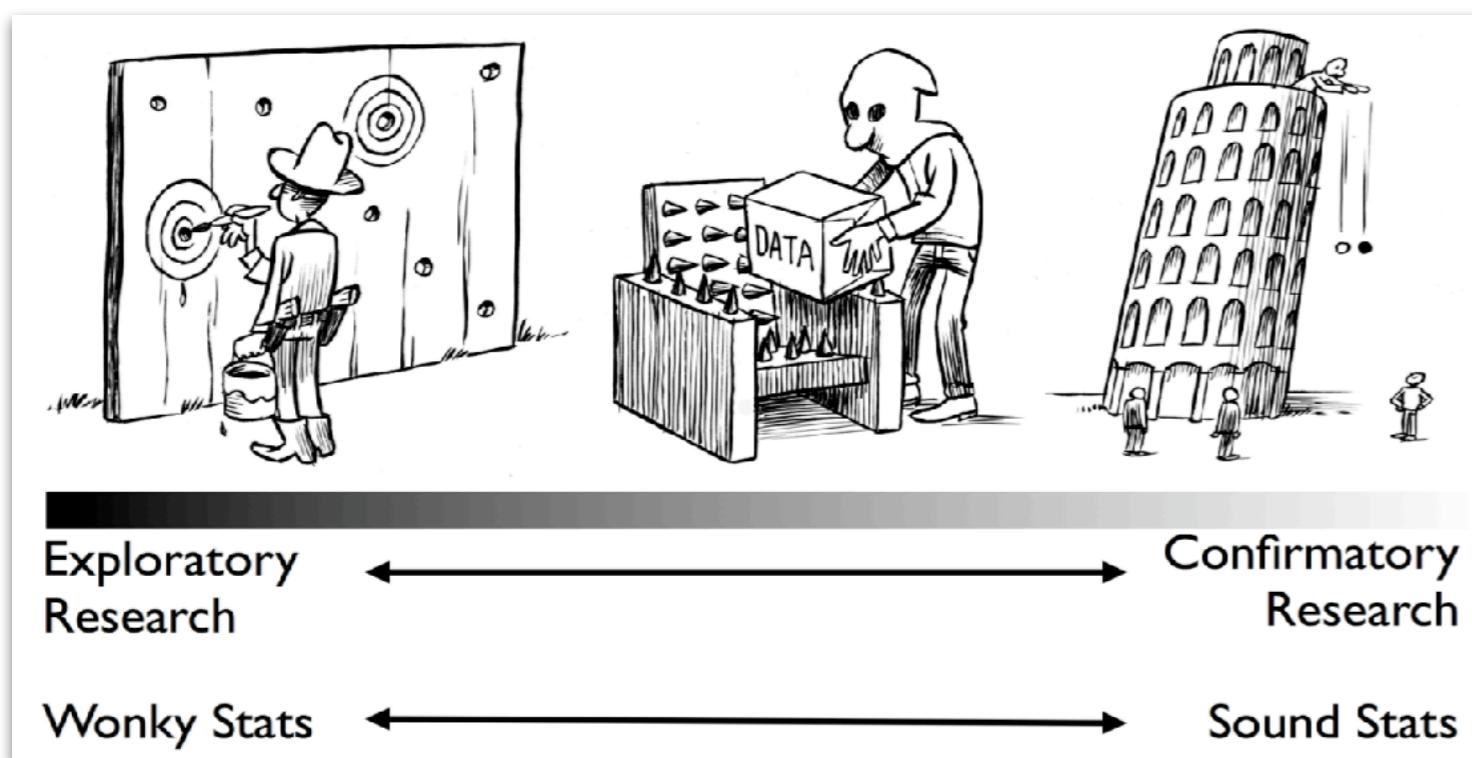


# Power analysis



Chat

What is the strangest gift you have ever received?

To: Everyone ▾

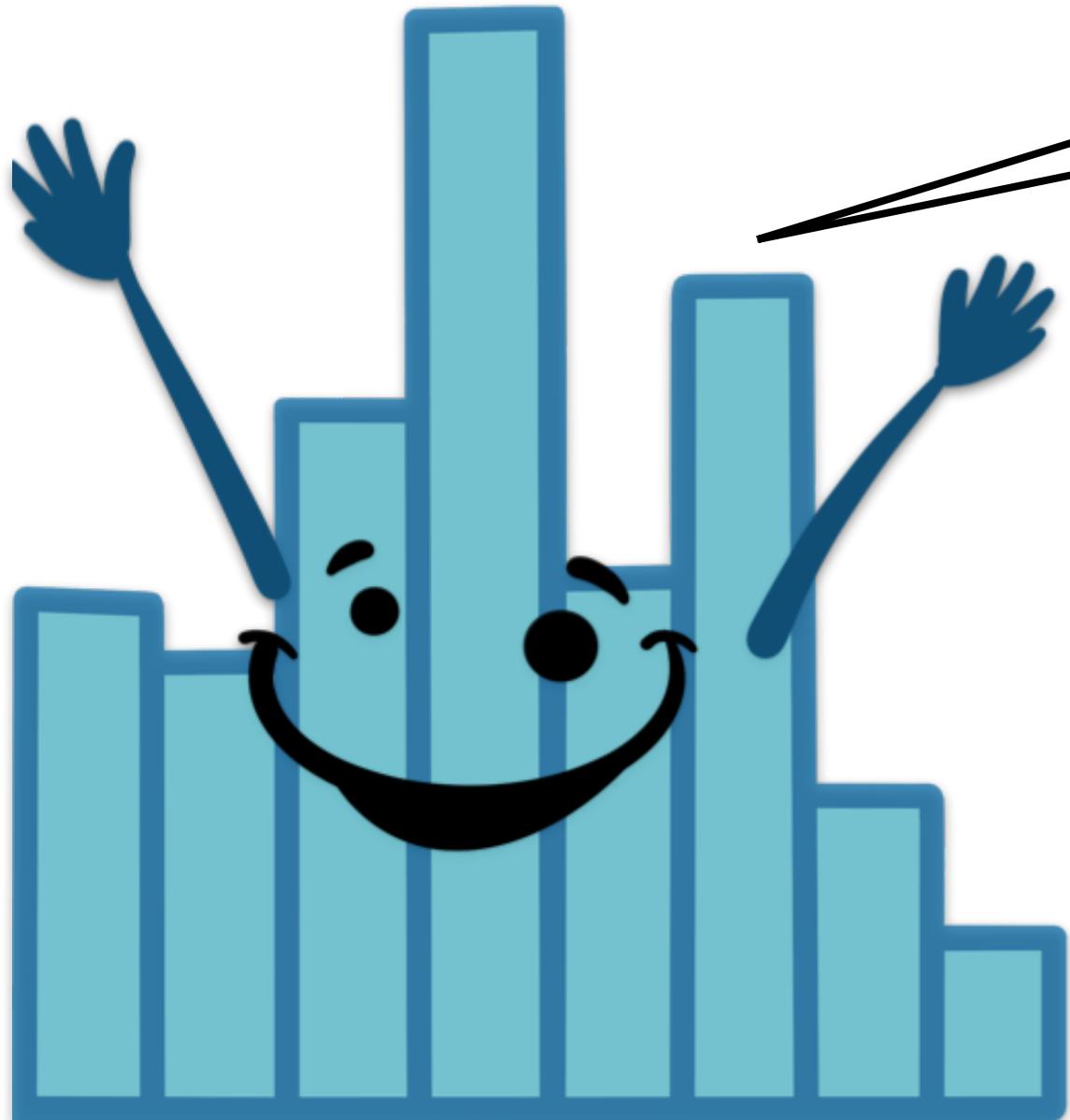
Type message here...

More ▾

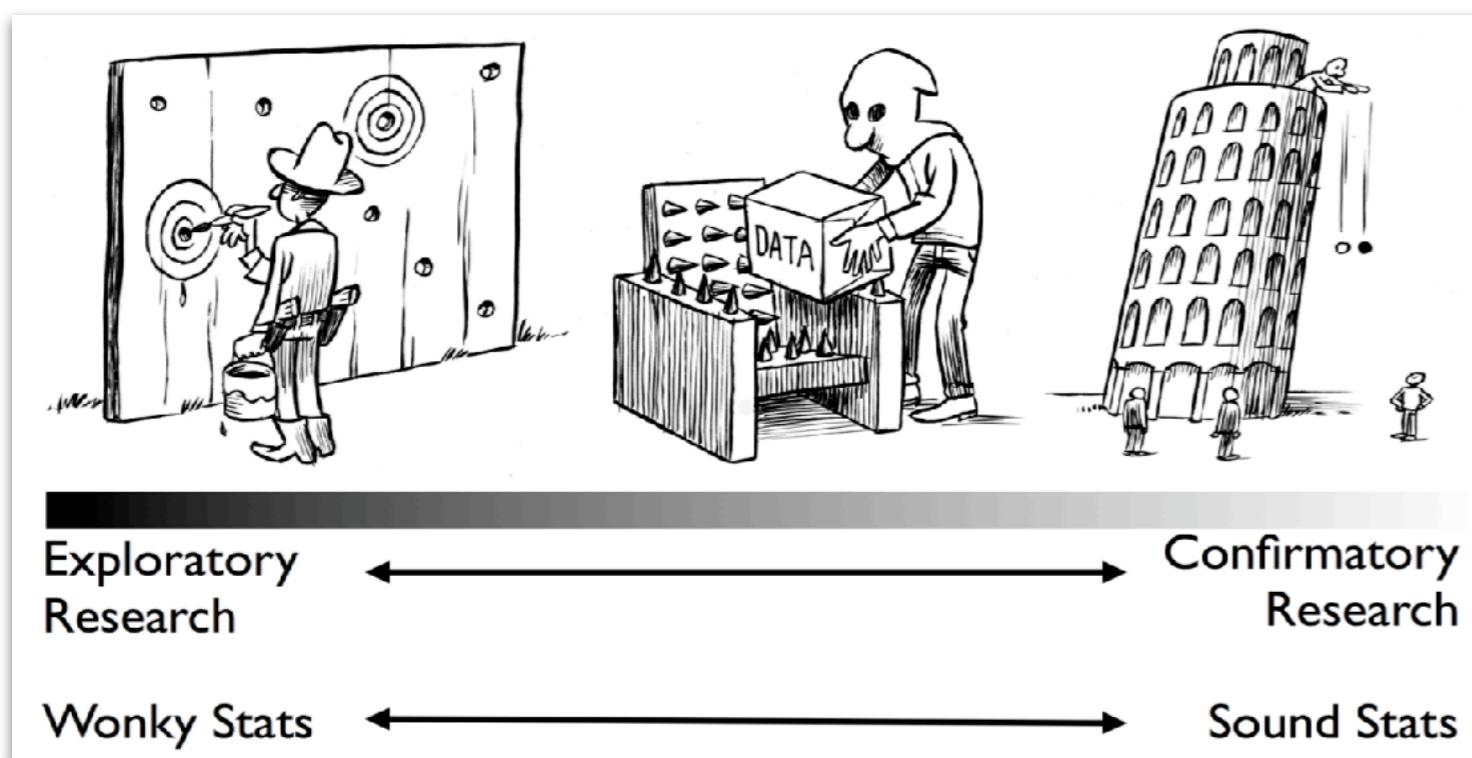


02/12/2021

Remember to  
record the  
lecture!



# Power analysis



Chat

What is the strangest gift you have ever received?

To: Everyone ▾

Type message here...

More ▾



02/12/2021

# **Things that came up**

# Go! Go! Linear model



Indrajeet Patil  
@patilindrajeets

...

Your periodic reminder that the most common statistical tests are nothing but special cases of linear models 🕵️

[lindeloev.github.io/tests-as-linear...](https://lindeloev.github.io/tests-as-linear/)

I wish someone had pointed out this beautiful simplicity and parsimony when I was learning statistics 🧑

#rstats

**Common statistical tests are linear models**  
Last updated: 28 June, 2019. Also check out the [Python version](#).

Common name	Built-in function in R	Equivalent linear model in R	Exact?	The linear model in words	Icon
<b>y ~ independent of x</b> P: One-sample t-test N: Wilcoxon signed-rank	t.test(y) wilcox.test(y)	lm(y ~ 1) lm(signed_rank(y) ~ 1)	✓ for N > 14	One number (intercept, i.e., the mean) predicts y. -(Same, but it predicts the signed rank of y.)	
P: Paired-sample t-test N: Wilcoxon matched pairs	t.test(y1, y2, paired=TRUE) wilcox.test(y1, y2, paired=TRUE)	lm(y2 - y1 ~ 1) lm(signed_rank(y2 - y1) ~ 1)	✓ for N > 14	One intercept predicts the pairwise y1-y2 differences. -(Same, but it predicts the signed rank of y2-y1.)	
<b>y ~ continuous x</b> P: Pearson correlation N: Spearman correlation	cor.test(x, y, method='Pearson') cor.test(x, y, method='Spearman')	cor.test(x, y, method='Pearson') cor.test(x, y, method='Spearman')		lm(y ~ 1 + x) lm(rank(y) ~ 1 + rank(x))	
<b>y ~ discrete x</b> P: Two-sample t-test P: Welch's t-test N: Mann-Whitney U	t.test(y1, y2, var.equal=TRUE) t.test(y1, y2, var.equal=FALSE) wilcox.test(y1, y2)	t.test(y1, y2, var.equal=TRUE) t.test(y1, y2, var.equal=FALSE) wilcox.test(y1, y2)	✓ ✓ for N > 11	lm(y ~ 1 + G <sub>2</sub> ) <sup>a</sup> gls(y ~ 1 + G <sub>2</sub> , weights=...) <sup>b</sup> lm(signed_rank(y) ~ 1 + G <sub>2</sub> ) <sup>a</sup>	
<b>Simple regression: lm(y ~ 1 + x)</b>					
<b>Multiple regression: lm(y ~ 1 + x<sub>1</sub> + x<sub>2</sub> + ...)</b>					

See worked examples and more details at the accompanying notebook: <https://lindeloev.github.io/tests-as-linear/>

<sup>a</sup> Note: G<sub>2</sub> = 0 or 1 indicator of x<sub>2</sub> for each non-missing values of the group variable. Similarly for S<sub>2</sub> = 0 or 1 for sex. The first line with G<sub>2</sub> is main effect of group, the second with S<sub>2</sub> for sex and the third is the group × sex interaction. For two levels (e.g. male/female), lm(x) would just be "x", and lm(x) would be x<sub>1</sub> interacted with each level.

<sup>b</sup> Note: Run gls using the following arguments: `glsm(y ~ 1 + G2 + ... + GN + S2 + S3 + ... + SK + G2*S2 + G3*S3 + ... + GN*SK, family=...)`. As linear-model, the Chi-square test and Goodness-of-fit for the binomial test. The signed rank function is `signed_rank = function(x) -x` coded" indicator variables (either 0 or 1) exploiting the fact that when Δx = 1 between categories the difference equals the slope. Subscripts (e.g., G<sub>i</sub> or y<sub>i</sub>) indicate different columns in data. lm requires long-format data for all non-continuous models. All of this is exposed in greater detail and worked examples at <https://lindeloev.github.io/tests-as-linear/>.

<sup>c</sup> See the note to the two-way ANOVA for explanation of the notation.

<sup>d</sup> Same model, but with one variance per group: `gls(value ~ 1 + G2, weights = varIdent(form = ~1|group), method="ML")`.

Jonas K. Lindeløv

12:04 AM · Feb 12, 2021 · Twitter Web App

102 Retweets 6 Quote Tweets 373 Likes

## Common statistical tests are linear models

Last updated: 28 June, 2019. Also check out the [Python version](#).

See worked examples and notebook: <https://lindeloev.github.io/tests-as-linear/>

Common name	Built-in function in R	Equivalent linear model in R	Exact?	The linear model in words
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<b>y ~ discrete x</b> P: Two-sample t-test P: Welch's t-test N: Mann-Whitney U	t.test(y1, y2, var.equal=TRUE) t.test(y1, y2, var.equal=FALSE) wilcox.test(y1, y2)	t.test(y1, y2, var.equal=TRUE) t.test(y1, y2, var.equal=FALSE) wilcox.test(y1, y2)	✓ ✓ for N > 11	lm(y ~ 1 + G <sub>2</sub> ) <sup>a</sup> gls(y ~ 1 + G <sub>2</sub> , weights=...) <sup>b</sup> lm(signed_rank(y) ~ 1 + G <sub>2</sub> ) <sup>a</sup>
<b>Simple regression: lm(y ~ 1 + x)</b>				
<b>Multiple regression: lm(y ~ 1 + x<sub>1</sub> + x<sub>2</sub> + ...)</b>				

List of common parametric (P) non-parametric (N) tests and equivalent linear models. The notation  $y \sim 1 + x$  is R shorthand for  $y = 1 \cdot b + a \cdot x$  which most of us learned in school. Models in similar colors are highly similar, but really, notice how similar they *all* are across colors! For non-parametric models, the linear models are reasonable approximations for non-small sample sizes (see "Exact" column and click links to see simulations). Other less accurate approximations exist, e.g., Wilcoxon for the sign test and Goodness-of-fit for the binomial test. The signed rank function is `signed_rank = function(x) -x`. The variables G<sub>i</sub> and S<sub>i</sub> are "dummy coded" indicator variables (either 0 or 1) exploiting the fact that when  $\Delta x = 1$  between categories the difference equals the slope. Subscripts (e.g., G<sub>i</sub> or y<sub>i</sub>) indicate different columns in data. lm requires long-format data for all non-continuous models. All of this is exposed in greater detail and worked examples at <https://lindeloev.github.io/tests-as-linear/>.

<sup>a</sup> See the note to the two-way ANOVA for explanation of the notation.

<sup>b</sup> Same model, but with one variance per group: `gls(value ~ 1 + G2, weights = varIdent(form = ~1|group), method="ML")`.

<https://lindeloev.github.io/tests-as-linear/>

# Excellent online book!

Preface
I Preliminaries
1 General Introduction
2 Basics of R
II Data
3 Data, variables & experimental desi...
4 Data Wrangling
5 Summary statistics
6 Data Visualization
III Bayesian Data Analysis
7 Basics of Probability Theory
8 Statistical models
9 Bayesian parameter estimation
10 Model Comparison
11 Bayesian hypothesis testing
IV Applied (generalized) linear mod...
12 Linear regression
13 Bayesian regression in practice
14 Categorical predictors
15 Generalized linear model
V Frequentist statistics
16 Null Hypothesis Significance Testing
17 Comparing frequentist and Bayesi...

☰    ⌂    A    ⬇    ⌂

# An Introduction to Data Analysis

Michael Franke

*last rendered at: 2021-01-26 08:56:45*

## Preface

This book provides basic reading material for an introduction to data analysis. It uses R to handle, plot and analyze data. After covering the use of R for data wrangling and plotting, the book introduces key concepts of data analysis from a Bayesian and a frequentist tradition. This text is intended for use as a first introduction to statistics for an audience with some affinity towards programming, but no prior exposition to R.

Many people have supported this project actively by providing text, examples, code or technical support. Many thanks for their support to (in alphabetic order): Tobias Anton, Florence Bockting, Noa Kallioinen, Minseok Kang, Marcel Klehr, Özge Özenoglu, Maria Pershina, Timo Roettger, Polina Tsvilodub and Inga Wohlert.

# An Introduction to Data Analysis



# **Logistics**

# Midterm

will be available shortly after class today

Psych 252 Midterm

My name goes here

2021-02-12 12:04:21

## Introduction

This is a take-home exam. The exam is open notes and open book (in short, you can use any source of information you like as long as you work on the exam by yourself). The maximum score is 120 points. Please adhere to the honor code. Submit the midterm as a PDF on the canvas 'midterm' assignment by **Thursday, February 18th, 8pm**.

The late policy submission policy is:

- We will subtract 2% from your points for each hour that the midterm is submitted late but before midnight. For example, 2% will be subtracted if you submit between 8pm and 9pm, or 8% if you submit between 11pm and midnight.
- 20% will be subtracted if you submit after midnight on Thursday but before 8pm on Friday, February 19th.
- No points will be granted if you submit later than 8pm on Friday, February 19th.

For questions that require written responses, please make sure to show any relevant tables, summaries (e.g. from `lm()` or `anova()`), or visualizations. Some of the code chunks have existing code that you can use to build your code around.

When asked to report results, please do so like you would in a scientific article (see examples from class).

- Please leave the `\clearpage` commands where they are. This makes sure that each question is printed on a separate page in the pdf.
- Some code chunks are set to `eval=F`, make sure to set these to `eval=T` before knitting the final version.
- We note for each question how many points you can get. You can get up to 120 points in total.
- Good coding style matters! We will add or subtract up to 5 points depending on style.

If you have any questions about the midterm, please post them on Piazza addressed to the instructors only. We will answer your question and may choose to share both your question and our answer with the rest of the group.

Best of luck with the midterm!

## Honor Code

The Honor Code is the University's statement on academic integrity written by students in 1921. It articulates University expectations of students and faculty in establishing and maintaining the highest standards in academic work:

1. The Honor Code is an undertaking of the students, individually and collectively:
  - a. that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;
  - b. that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.

## Part 1: Wash your hands!

A student investigated how effective different methods are for eliminating bacteria. She tested four different methods: (1) washing her hands with water only, (2) with regular soap, (3) with antibacterial soap (ABS), or (4) using an antibacterial spray (AS). She suspected that the number of bacteria on her hands might vary considerably from day to day. To account for this, she generated random numbers to determine on what day she would use which treatment. After each treatment, she placed her right hand on a sterile media plate to measure bacteria growth. She incubated each plate for 2 days after which she counted the bacteria colonies. She replicated this procedure 9 times for each of the four treatments.

Note: For statistical analysis purposes, we make the assumption that the individual measurements are independent from each other.

## Part 2: Life satisfaction

In this exercise, we are interested in seeing what affects life satisfaction. We have a (fake) data set with the following variables:

Table 1: Variables in the satisfaction data set.

variable	description
id	participant id
age	age in years
kids	number of kids
jobsatis	job satisfaction (1 = not at all, 7 = very much)
marsatis	marital satisfaction (1 = not at all, 7 = very much)
lifsatis	life satisfaction (1 = not at all, 7 = very much)

## Part 3: You've got the power!

In this exercise, we'll take a look at determining what group sample size we would need in order to achieve adequate statistical power to test our research hypothesis of interest. We will be using the data from "data/power.csv" and you can see its visualization in "figure/df\_power.png".

Tip: We will provide saved checkpoints for each problem. Feel free to look ahead to compare your results with ours.

# Next week

no class on Monday (President's Day)

no class on Wednesday (more time to work on midterm)

no sections

office hours are open (but won't answer questions about the midterm)

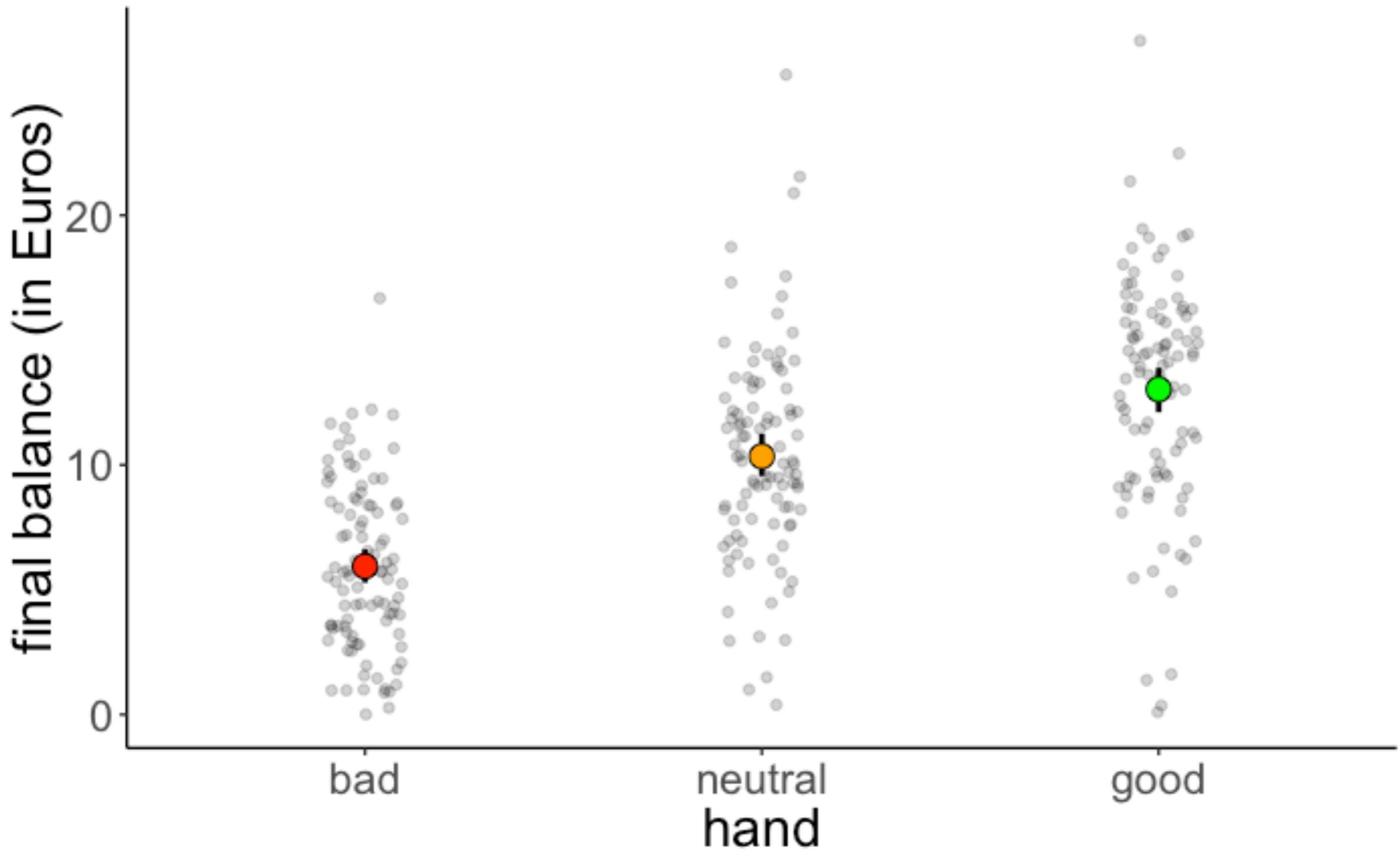
# Plan for today

- Linear contrasts
  - Testing specific hypotheses with linear contrasts
  - emmeans for handling linear contrasts in R
- Power analysis
  - Making decisions
  - Calculating power
  - Effect sizes
  - Determining sample size
- Learn about more advanced simulation techniques in R
  - `map()`
  - list columns: `nest()`, `unnest()`

# **Linear contrasts**

# **Testing (more) specific hypotheses with linear contrasts**

# Do better hands win more money?



# Do better hands win more money?



ANOVA

# Does card quality affect the final balance?



post-hoc tests

bad vs. neutral

neutral vs. good

Is there are more direct way of asking this question with a statistical model?

# Contrasts

```
1 df.poker = df.poker %>%
2   mutate(hand_contrast = factor(hand,
3                                 levels = c("bad", "neutral", "good"),
4                                 labels = c(-1, 0, 1)),
5   hand_contrast = hand_contrast %>% as.character() %>% as.numeric())
```

participant	hand	balance	hand_contrast
1	bad	4.00	-1
2	bad	5.55	-1
3	bad	9.45	-1
51	neutral	11.74	0
52	neutral	10.04	0
53	neutral	9.49	0
101	good	10.86	1
102	good	8.68	1
103	good	14.36	1

# Contrasts

```
1 df.poker = df.poker %>%
2   mutate(hand_contrast = factor(hand,
3                                 levels = c("bad", "neutral", "good"),
4                                 labels = c(-1, 0, 1)),
5   hand_contrast = hand_contrast %>% as.character() %>% as.numeric())
6
7 fit = lm(formula = balance ~ hand_contrast,
8         data = df.poker)
```

```
Call:
lm(formula = balance ~ hand_contrast, data = df.fit)

Residuals:
    Min      1Q  Median      3Q     Max
-13.214 -2.684 -0.019  2.444 15.858

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 9.7715    0.2381   41.03 <2e-16 ***
hand_contrast 3.5424    0.2917   12.14 <2e-16 ***
---
Signif. codes:  0 '****' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.125 on 298 degrees of freedom
Multiple R-squared:  0.3311, Adjusted R-squared:  0.3289
F-statistic: 147.5 on 1 and 298 DF,  p-value: < 2.2e-16
```

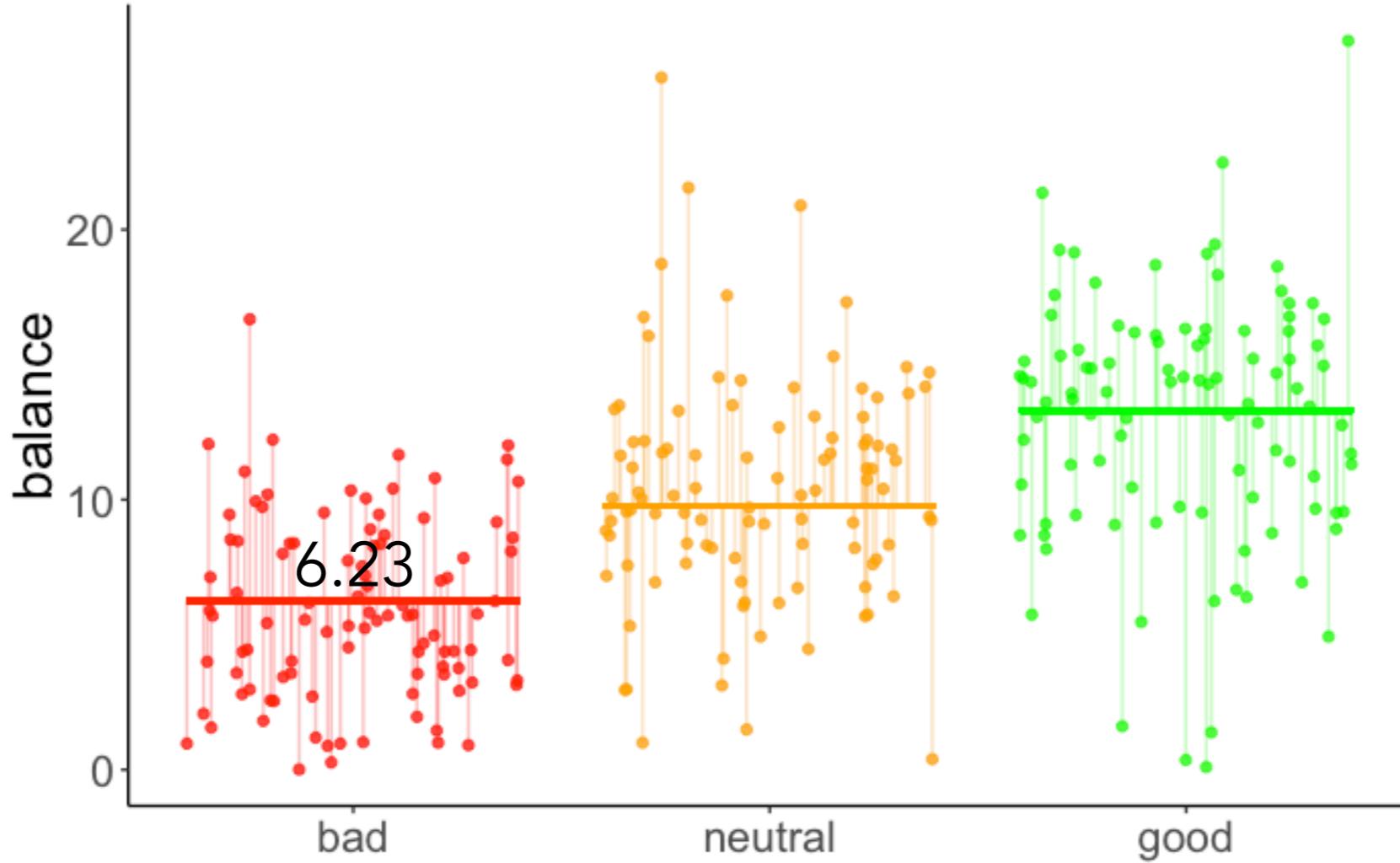
mean in neutral condition

significant contrast

# Contrasts

name	estimate	std.error	statistic	p.value
intercept	9.77	0.24	41.03	0
contrast	3.54	0.29	12.15	0

$$\widehat{\text{balance}}_i = b_0 + b_1 \cdot \text{hand\_contrast}$$



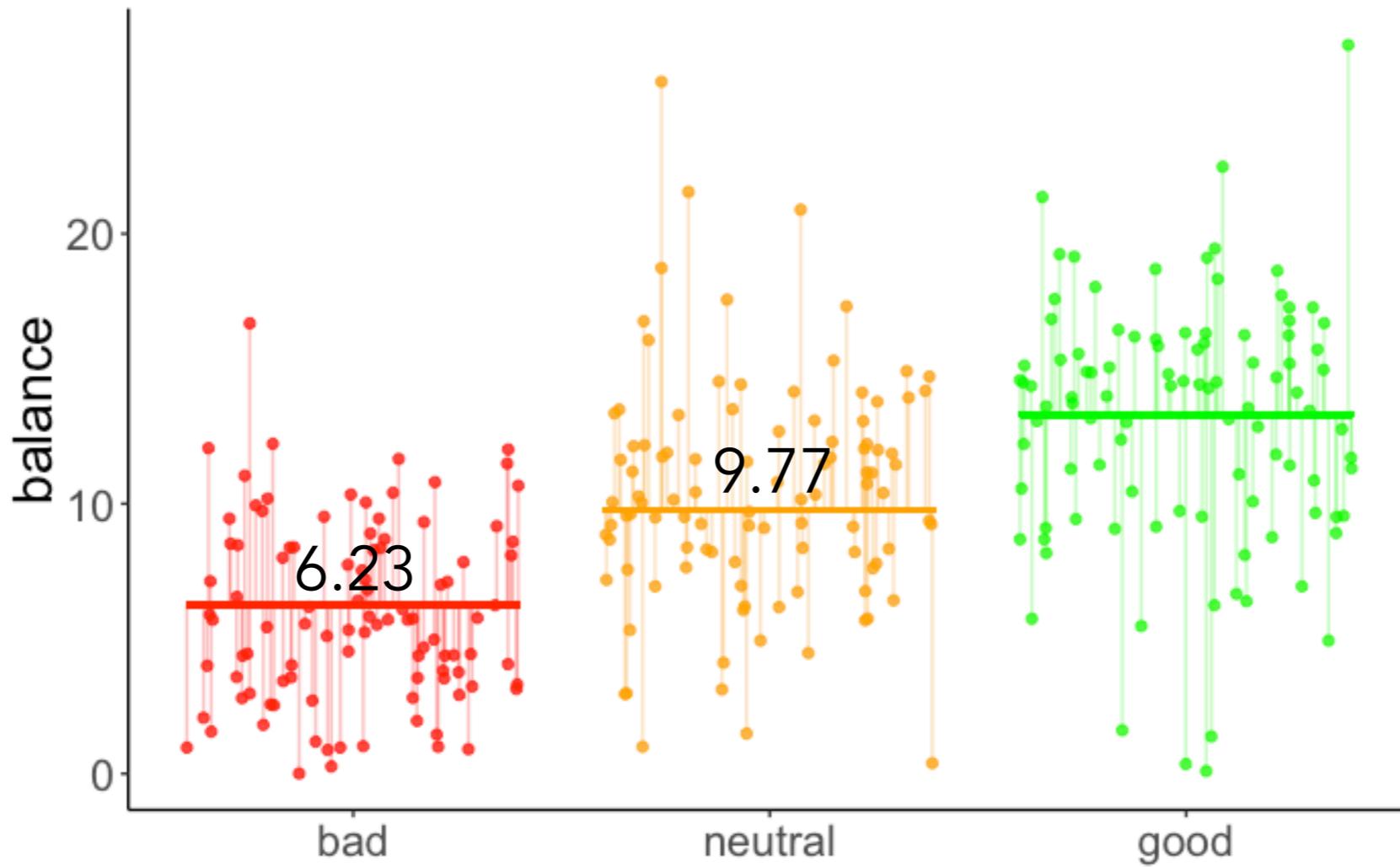
**if contrast == -1**

$$\begin{aligned}\widehat{\text{balance}}_i &= b_0 + b_1 \cdot \text{hand\_contrast}_i \\ &= 9.77 + (-1) \cdot 3.54 = 6.23\end{aligned}$$

# Contrasts

name	estimate	std.error	statistic	p.value
intercept	9.77	0.24	41.03	0
contrast	3.54	0.29	12.15	0

$$\widehat{\text{balance}}_i = b_0 + b_1 \cdot \text{hand\_contrast}$$



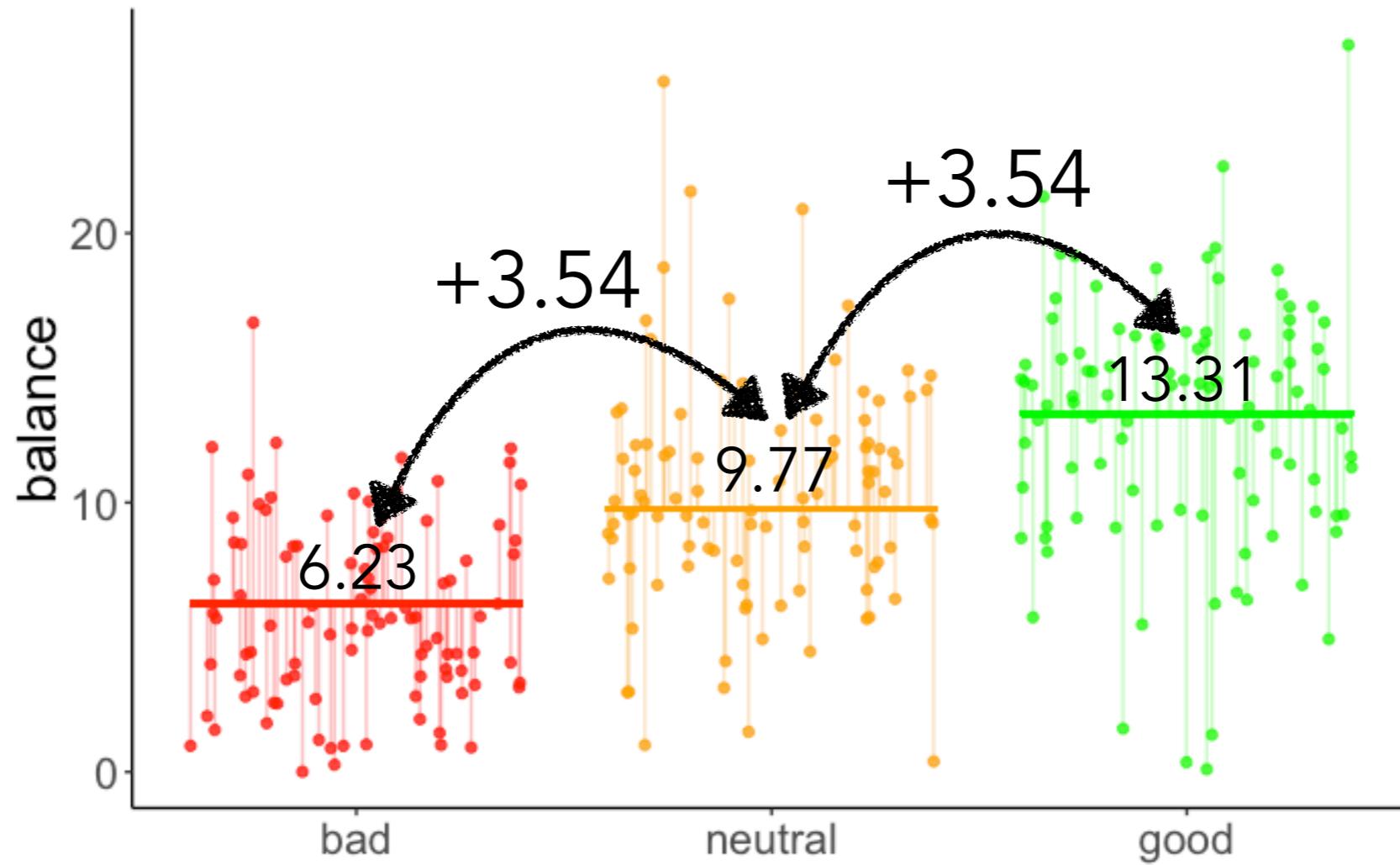
if  $\text{contrast} == 0$

$$\begin{aligned}\widehat{\text{balance}}_i &= b_0 + b_1 \cdot \text{hand\_contrast}_i \\ &= 9.77 + 0 \cdot 3.54 = 9.77\end{aligned}$$

# Contrasts

name	estimate	std.error	statistic	p.value
intercept	9.77	0.24	41.03	0
contrast	3.54	0.29	12.15	0

$$\widehat{\text{balance}}_i = b_0 + b_1 \cdot \text{hand\_contrast}$$

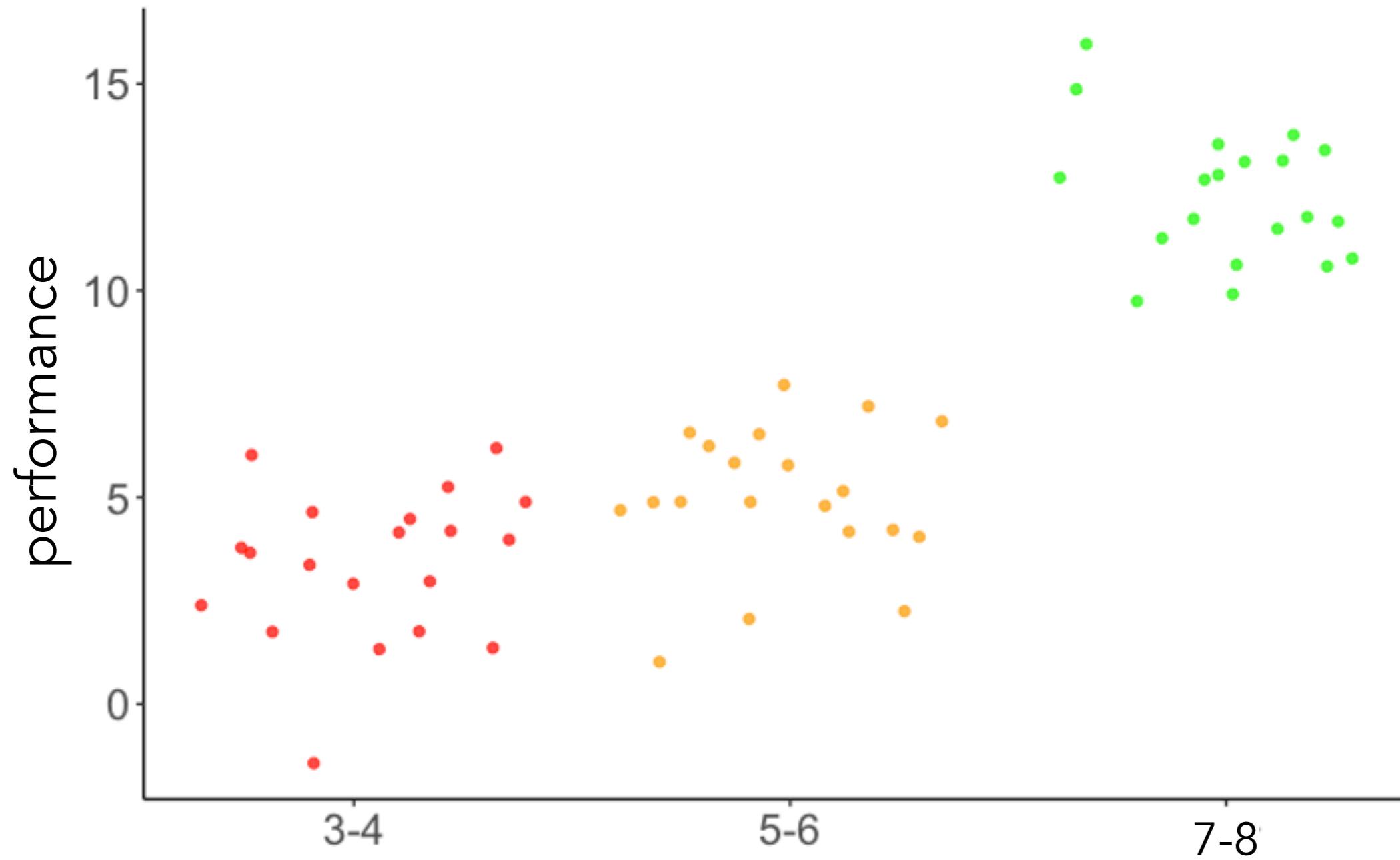


if contrast == 1

$$\begin{aligned}\widehat{\text{balance}}_i &= b_0 + b_1 \cdot \text{hand\_contrast}_i \\ &= 9.77 + 1 \cdot 3.54 = 13.31\end{aligned}$$

# Contrasts

**Does performance increase with age?**



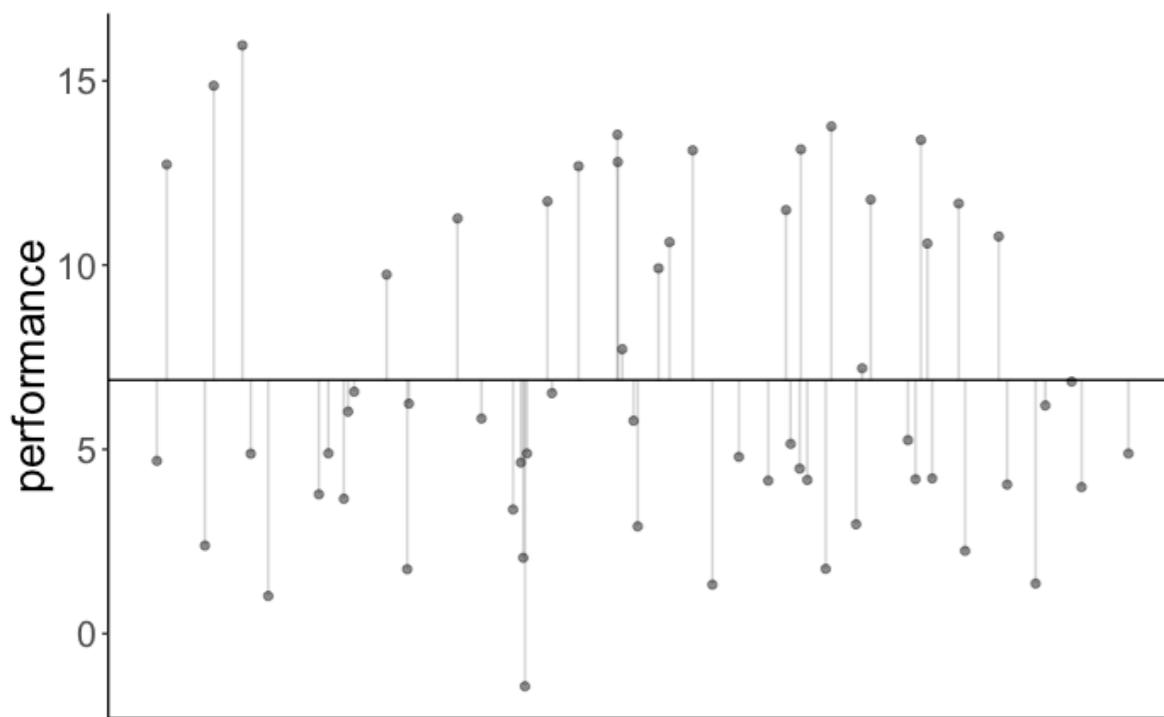
**Data from a hypothetical developmental study**

# Contrasts

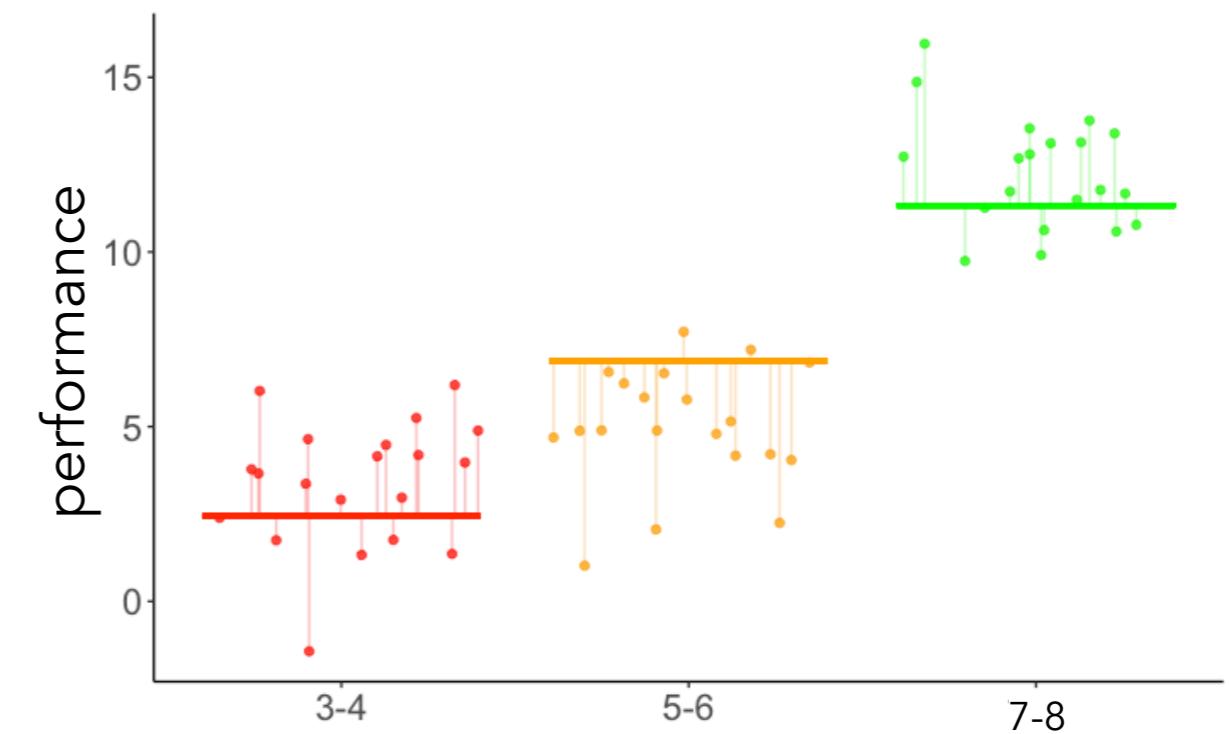
**Does performance increase with age?**

contrasts = c(-1, 0, 1)

**Compact model**



**Augmented model**

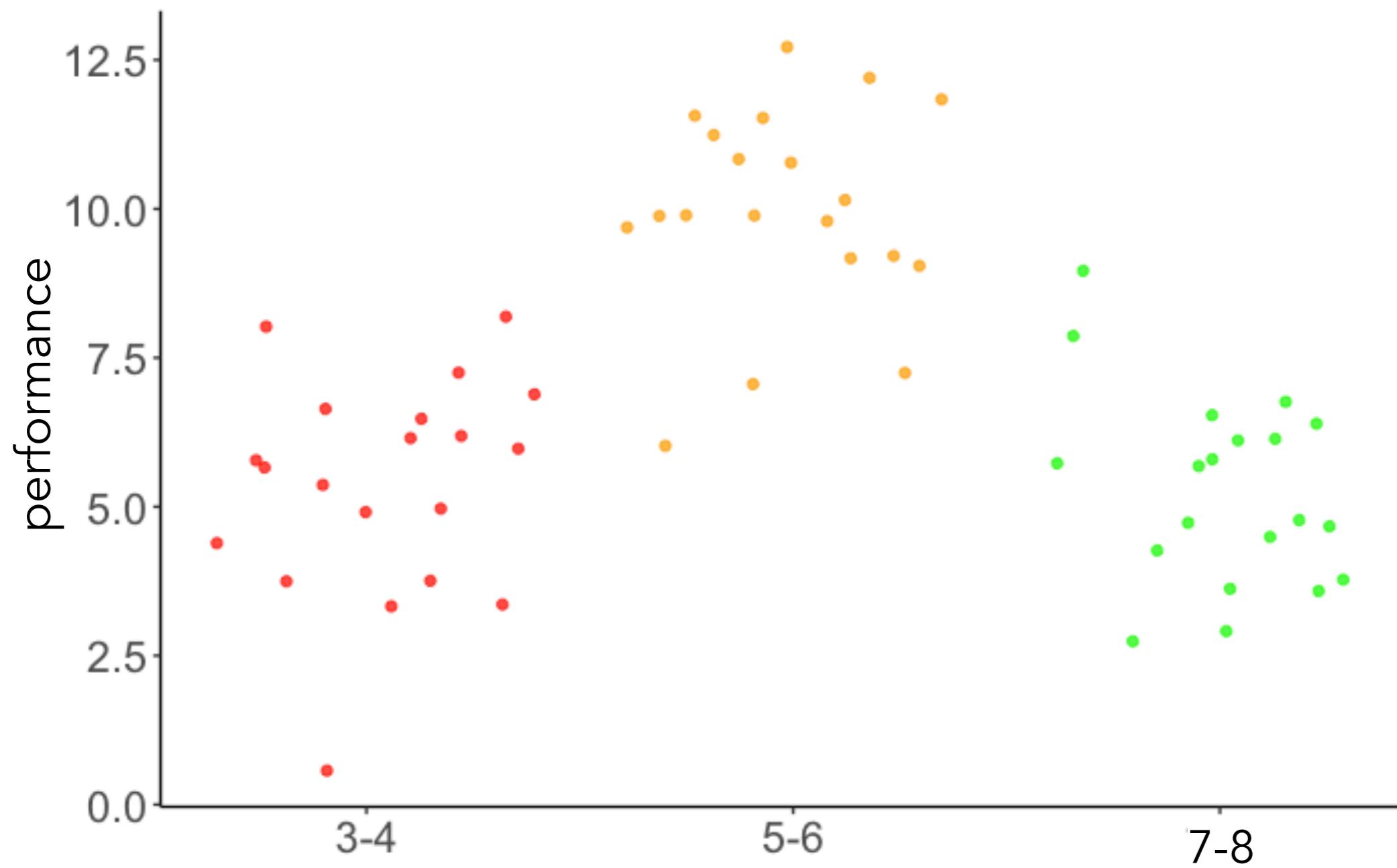


**Model comparison**

$p < .001$

# Contrasts

**Does performance increase with age?**



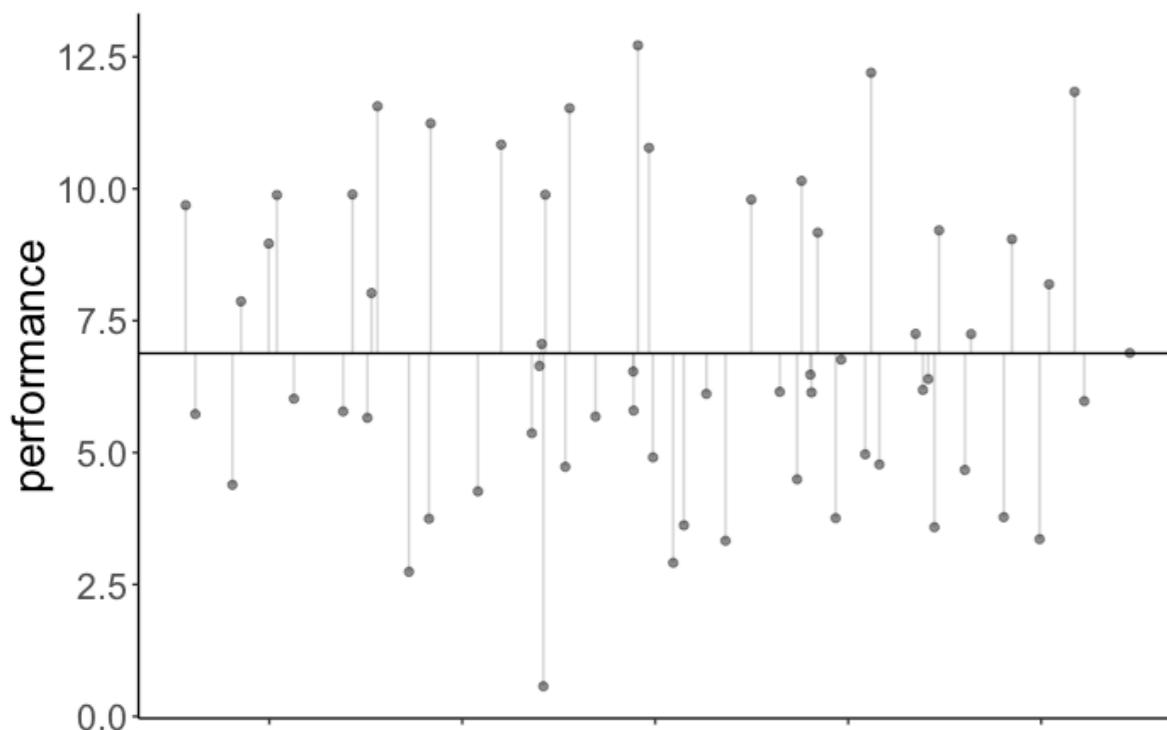
**Data from another hypothetical developmental study**

# Contrasts

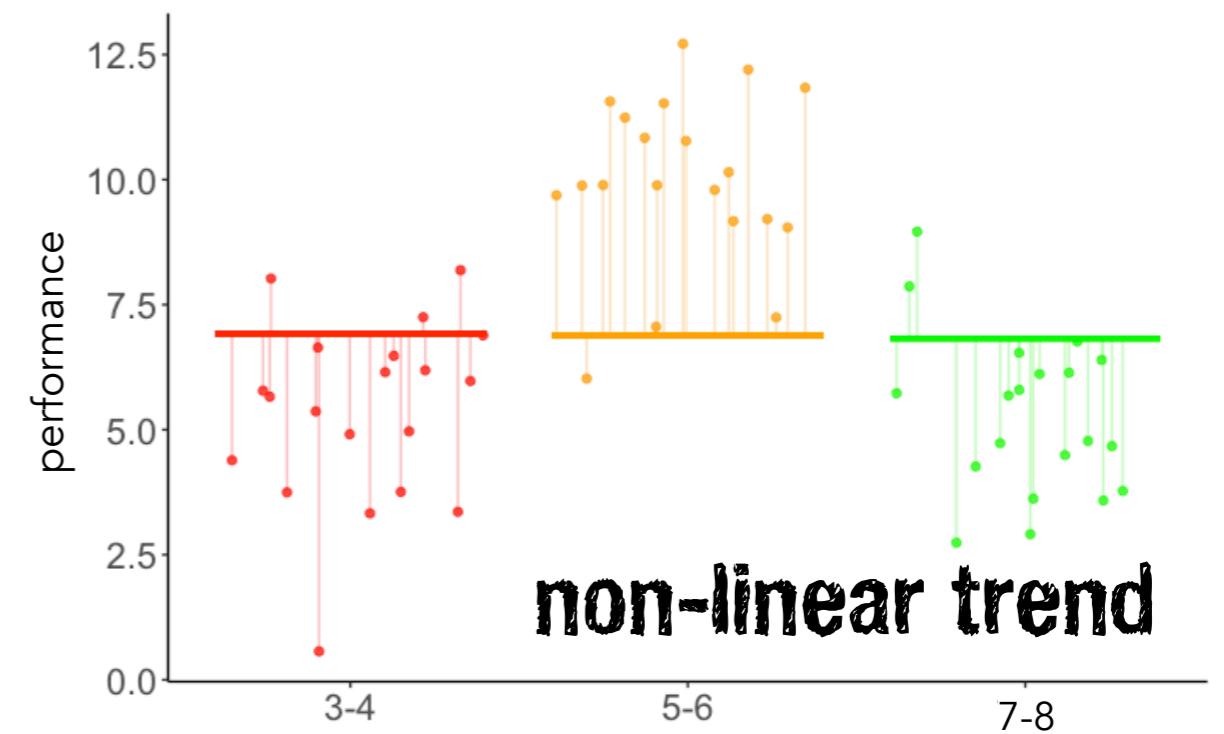
**Does performance increase with age?**

contrasts = c(-1, 0, 1)

**Compact model**



**Augmented model**



**Model comparison**

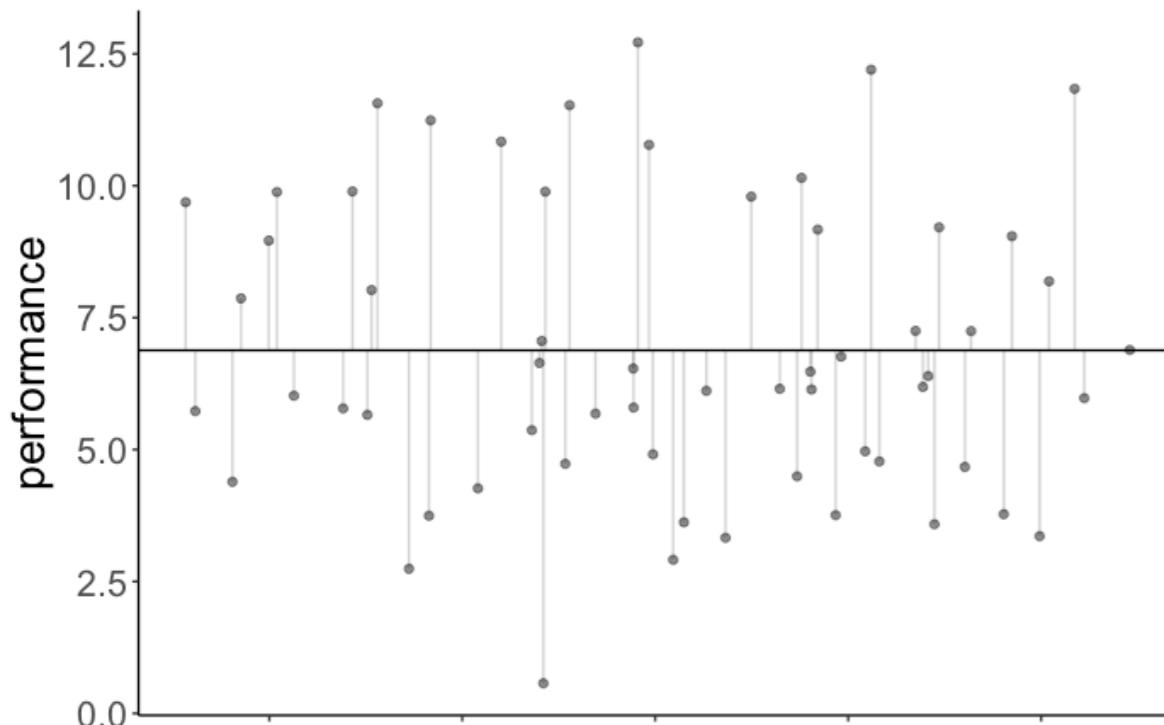
**p = .8508**

# Contrasts

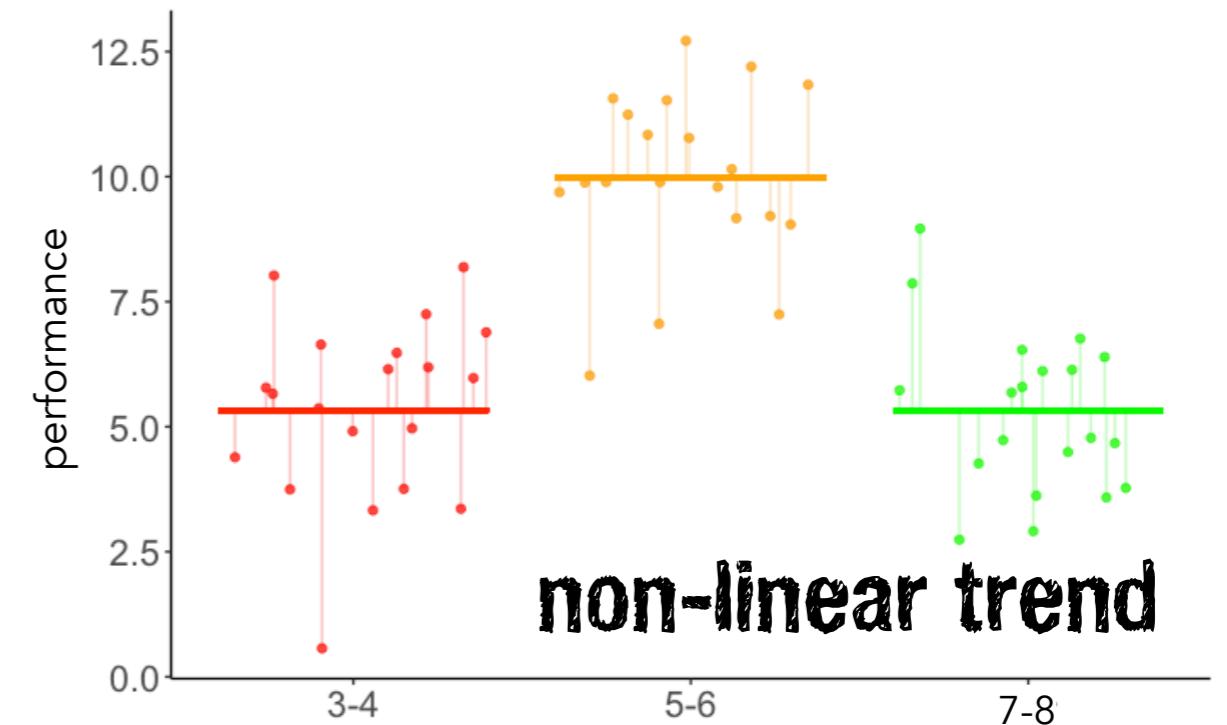
**Does performance increase with age?**

contrasts = c(-1, 2, -1)

**Compact model**



**Augmented model**



**Model comparison**

$p < .001$

# **emmeans for handling linear contrasts in R**

# Linear contrasts

## ~~How to use contrasts in R~~

In short: don't bother.<sup>1</sup>

Like many before me, one of my stats classes technically “taught” me contrasts. But I didn’t get the point and using them was cumbersome, so I promptly ignored them for years.

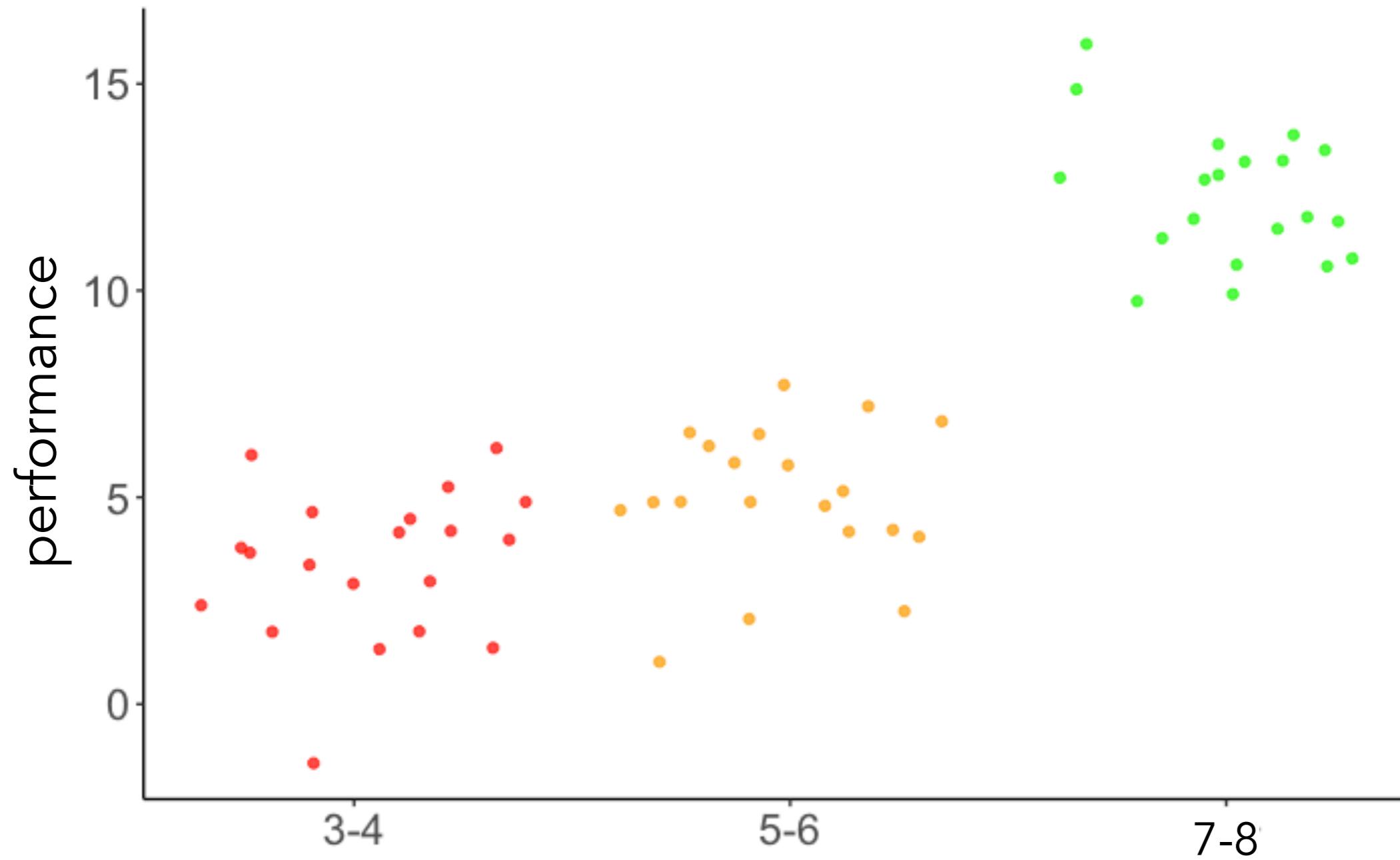
Luckily for me, someone came along and fixed the situation: [emmeans](#). emmeans frames contrasts as a question you pose to a model: you can ask for all pairwise comparisons and get back that. `lm` and `summary` treat the same problem as fitting abstract coefficients, and you are left to answer your own question.

`emmeans` works with `lm`, `glm`, and the Bayesian friends in [brms](#) and [rstanarm](#), so the process is applicable no matter the tool.

And you don't have to learn (much) about contrasts to take advantage of it.

# Contrasts

**Does performance increase with age?**



**Data from a hypothetical developmental study**

# Linear contrasts in R

```
1 library("emmeans") # for calculating contrasts  
2  
3 # fit the linear model  
4 fit = lm(formula = performance ~ group,  
5           data = df.development)
```

fit linear model

# Linear contrasts in R

```
1 library("emmeans") # for calculating contrasts  
2  
3 # fit the linear model  
4 fit = lm(formula = performance ~ group,  
5           data = df.development)  
6  
7 # check factor levels  
8 levels(df.development$group) [1] "3-4" "5-6" "7-8"
```

check factor levels before  
defining contrasts

# Linear contrasts in R

```
1 library("emmeans") # for calculating contrasts
2
3 # fit the linear model
4 fit = lm(formula = performance ~ group,
5           data = df.development)
6
7 # check factor levels
8 levels(df.development$group) [1] "3-4" "5-6" "7-8"
9
10 # define the contrasts of interest
11 contrasts = list(young_vs_old = c(-0.5, -0.5, 1),
12                   three_vs_five = c(-0.5, 0.5, 0)))
```

set up linear contrasts

# Linear contrasts in R

```
1 library("emmeans") # for calculating contrasts
2
3 # fit the linear model
4 fit = lm(formula = performance ~ group,
5           data = df.development)
6
7 # check factor levels
8 levels(df.development$group)
9
10 # define the contrasts of interest
11 contrasts = list(young_vs_old = c(-0.5, -0.5, 1),
12                   three_vs_five = c(-0.5, 0.5, 0))
13
14 # compute significance test on contrasts
15 fit %>%
16   emmeans("group",
17           contr = contrasts,
18           adjust = "bonferroni") %>%
19   pluck("contrasts")
```

**compute the results**

	[1] "3-4" "5-6" "7-8"	contrast	estimate	SE	df	t.ratio	p.value
young_vs_old	16.093541	young_vs_old	0.4742322	57	33.936	<.0001	
three_vs_five	1.606009	three_vs_five	0.5475962	57	2.933	0.0097	

P value adjustment: bonferroni method for 2 tests

# Linear contrasts in R

```
1 library("emmeans") # for calculating contrasts
2
3 # fit the linear model
4 fit = lm(formula = performance ~ group,
5           data = df.development)
6
7 # check factor levels
8 levels(df.development$group)
9
10 # define the contrasts of interest
11 contrasts = list(young_vs_old = c(-1, -1, 2),
12                   three_vs_five = c(-1, 1, 0))
13
14 # compute significance test on contrasts
15 fit %>%
16   emmeans("group",
17           contr = contrasts,
18           adjust = "bonferroni") %>%
19   pluck("contrasts")
```

hypothesis tests  
are the same!

contrast	estimate	SE	df	t.ratio	p.value
young_vs_old	32.187	0.948	57	33.936	<.0001
three_vs_five	0.803	0.274	57	2.933	0.0097

P value adjustment: bonferroni method for 2 tests

# Interpreting the coefficients

```
1 fit = lm(formula = performance ~ group,  
2           data = df.development)  
3  
4 # check factor levels  
5 levels(df.development$group)  
6  
7 # define the contrasts of interest  
8 contrasts = list(young_vs_old = c(-1, -1, 2),  
9                   three_vs_five = c(-0.5, 0.5, 0))  
10  
11 # compute estimated marginal means  
12 leastsquare = emmeans(fit, "group")  
13  
14 # run analyses  
15 contrast(leastsquare,  
16             contrasts,  
17             adjust = "bonferroni")
```

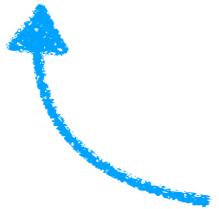
hypothesis tests  
are the same!

contrast	estimate	SE	df	t.ratio	p.value
young_vs_old	32.187	0.948	57	33.936	<.0001
three_vs_five	0.803	0.274	57	2.933	0.0097

P value adjustment: bonferroni method for 2 tests

# Post hoc tests

```
1 fit = lm(formula = performance ~ group,  
2           data = df.development)  
3  
4 # pairwise differences between all the groups  
5 fit %>%  
6   emmeans(pairwise ~ group) %>%  
7   pluck("contrasts")
```



all pairwise tests between groups

contrast	estimate	SE	df	t.ratio	p.value
3-4 - 5-6	-1.606009	0.5475962	57	-2.933	0.0145
3-4 - 7-8	-16.896546	0.5475962	57	-30.856	<.0001
5-6 - 7-8	-15.290537	0.5475962	57	-27.923	<.0001

P value adjustment: bonferroni method for 3 tests

# Post hoc tests

```
1 # fit the model  
2 fit = lm(formula = balance ~ hand + skill,  
3           data = df.poker)  
4  
5 # post hoc tests  
6 fit %>%  
7   emmeans(pairwise ~ hand + skill,  
8             adjust = "bonferroni") %>%  
9   pluck("contrasts")
```

the poker data

contrast	estimate	SE	df	t.ratio	p.value
bad,average - neutral,average	-4.381023	0.6051766	286	-7.239	<.0001
bad,average - good,average	-7.060823	0.6051766	286	-11.667	<.0001
bad,average - bad,expert	-0.740385	0.4896119	286	-1.512	1.0000
bad,average - neutral,expert	-5.121408	0.7611327	286	-6.729	<.0001
bad,average - good,expert	-7.801208	0.7611327	286	-10.249	<.0001
neutral,average - good,average	-2.679800	0.5884403	286	-4.554	0.0001
neutral,average - bad,expert	3.640638	0.7953578	286	4.577	0.0001
neutral,average - neutral,expert	-0.740385	0.4896119	286	-1.512	1.0000
neutral,average - good,expert	-3.420185	0.7654945	286	-4.468	0.0002
good,average - bad,expert	6.320438	0.7953578	286	7.947	<.0001
good,average - neutral,expert	1.939415	0.7654945	286	2.534	0.1774
good,average - good,expert	-0.740385	0.4896119	286	-1.512	1.0000
bad,expert - neutral,expert	-4.381023	0.6051766	286	-7.239	<.0001
bad,expert - good,expert	-7.060823	0.6051766	286	-11.667	<.0001
neutral,expert - good,expert	-2.679800	0.5884403	286	-4.554	0.0001

that's a lot of tests!

... not

P value adjustment: bonferroni method for 15 tests

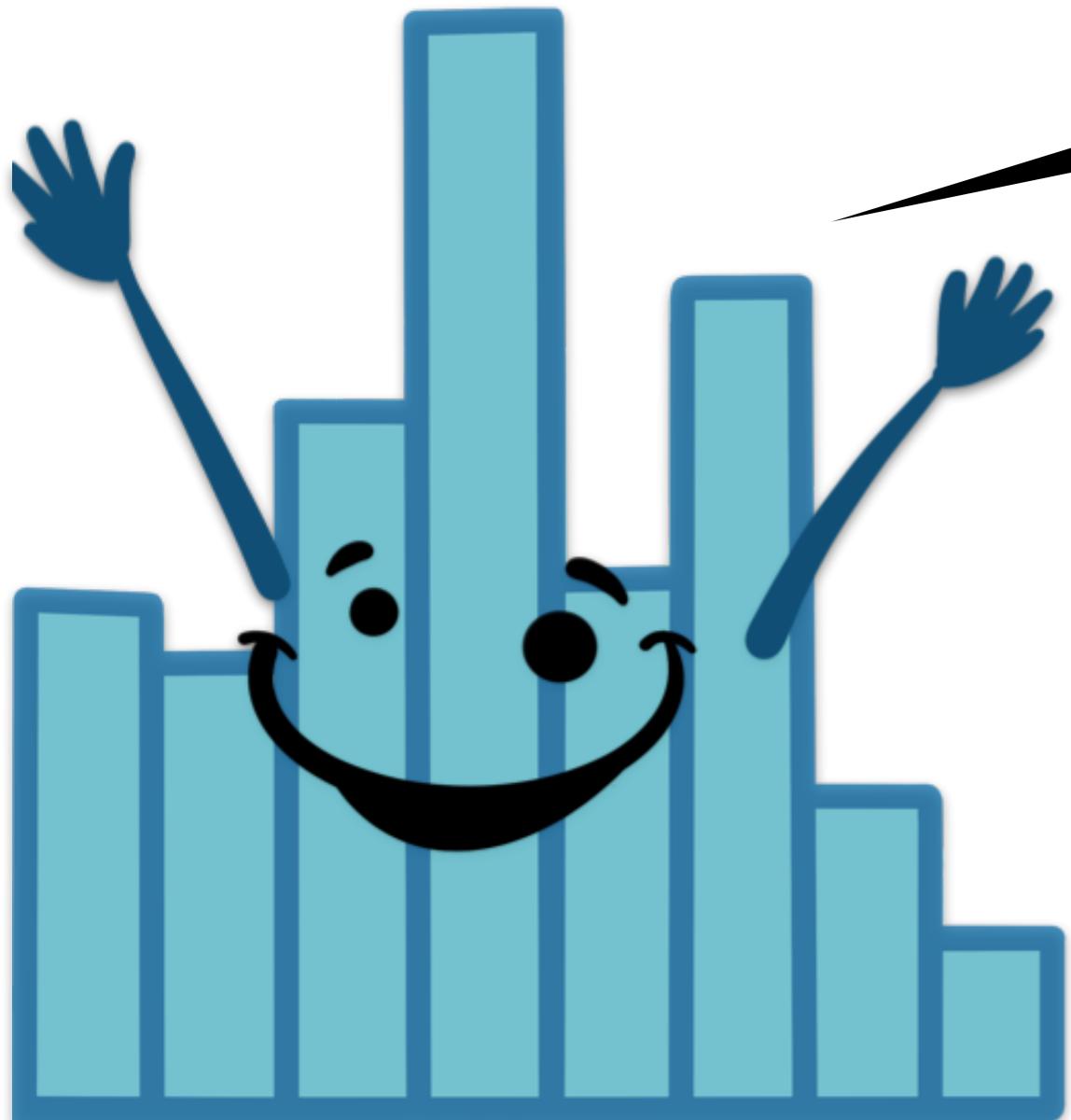
all pairwise tests between groups

# Contrasts

- linear contrasts allow us to ask more specific questions of our data
- rather than asking whether any of the group means are significantly different from each other (ANOVA), we can ask questions such as:
  - Does performance increase with age?
  - Is the overall performance in Condition B and C better from the performance in Condition A?

01:00

stretch break!



# **Power analysis**

# Making decisions

## Type I Error



## Type II Error



$H_0$ : Not pregnant.  $H_1$ : Pregnant.

**Type I Error:** Falsely rejecting the null hypothesis (even though it is true).

**Type II Error:** Failing to reject the null hypothesis (even though it is false).

# Clue guide to probability

$H_0$ : The person is healthy.

$H_1$ : The person is ill.

Power

$$1 - \beta$$

Sensitivity

$$p(\text{reject } H_0 | H_1 \text{ is true})$$

$$\beta$$

Type II error

$$p(\text{not reject } H_0 | H_1 \text{ is true})$$

$$\alpha$$

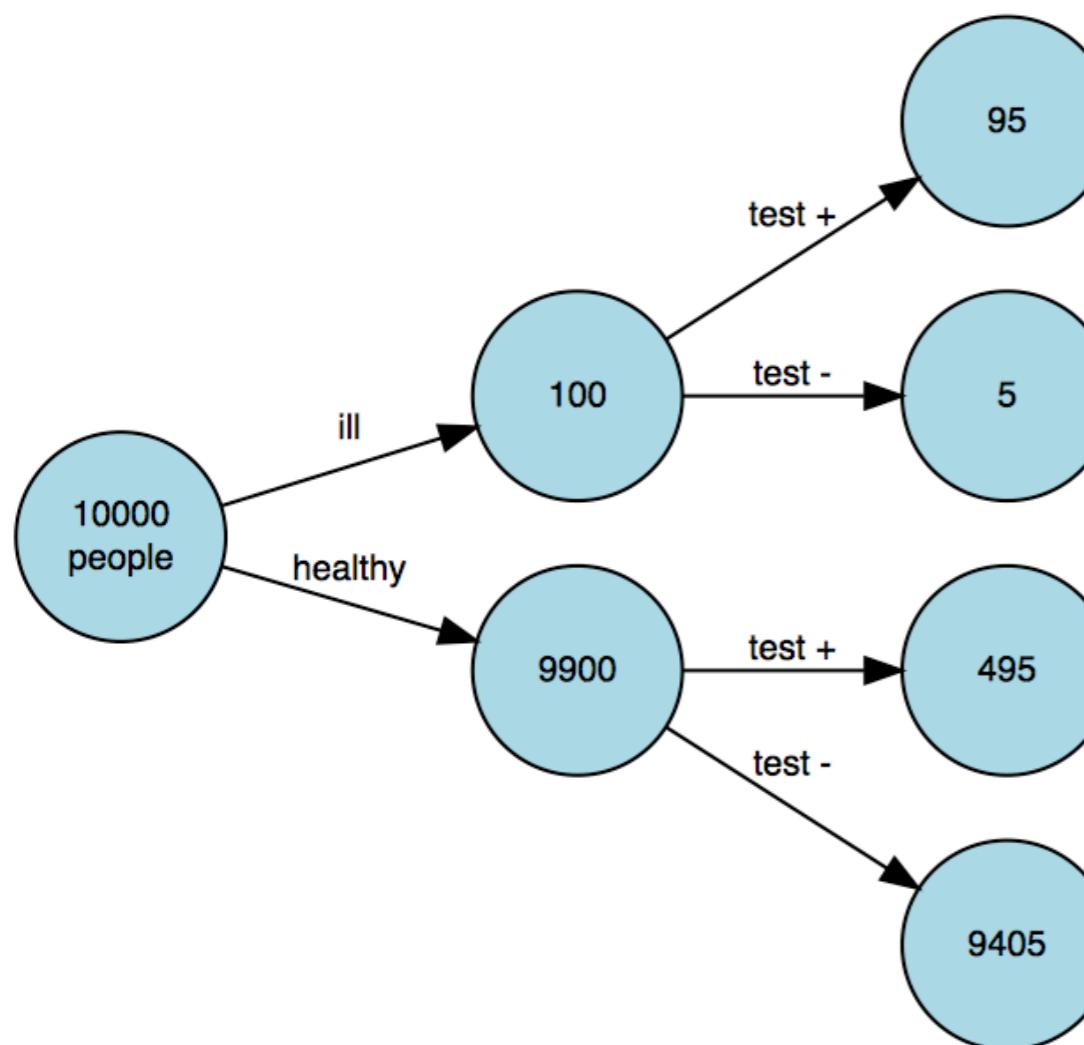
Type I error

$$p(\text{reject } H_0 | H_0 \text{ is true})$$

$$1 - \alpha$$

Specificity

$$p(\text{not reject } H_0 | H_0 \text{ is true})$$



true positive

$$p(\text{reject } H_0 | H_1 \text{ is true})$$

false negative

$$p(\text{not reject } H_0 | H_1 \text{ is true})$$

false positive

$$p(\text{reject } H_0 | H_0 \text{ is true})$$

true negative

$$p(\text{not reject } H_0 | H_0 \text{ is true})$$

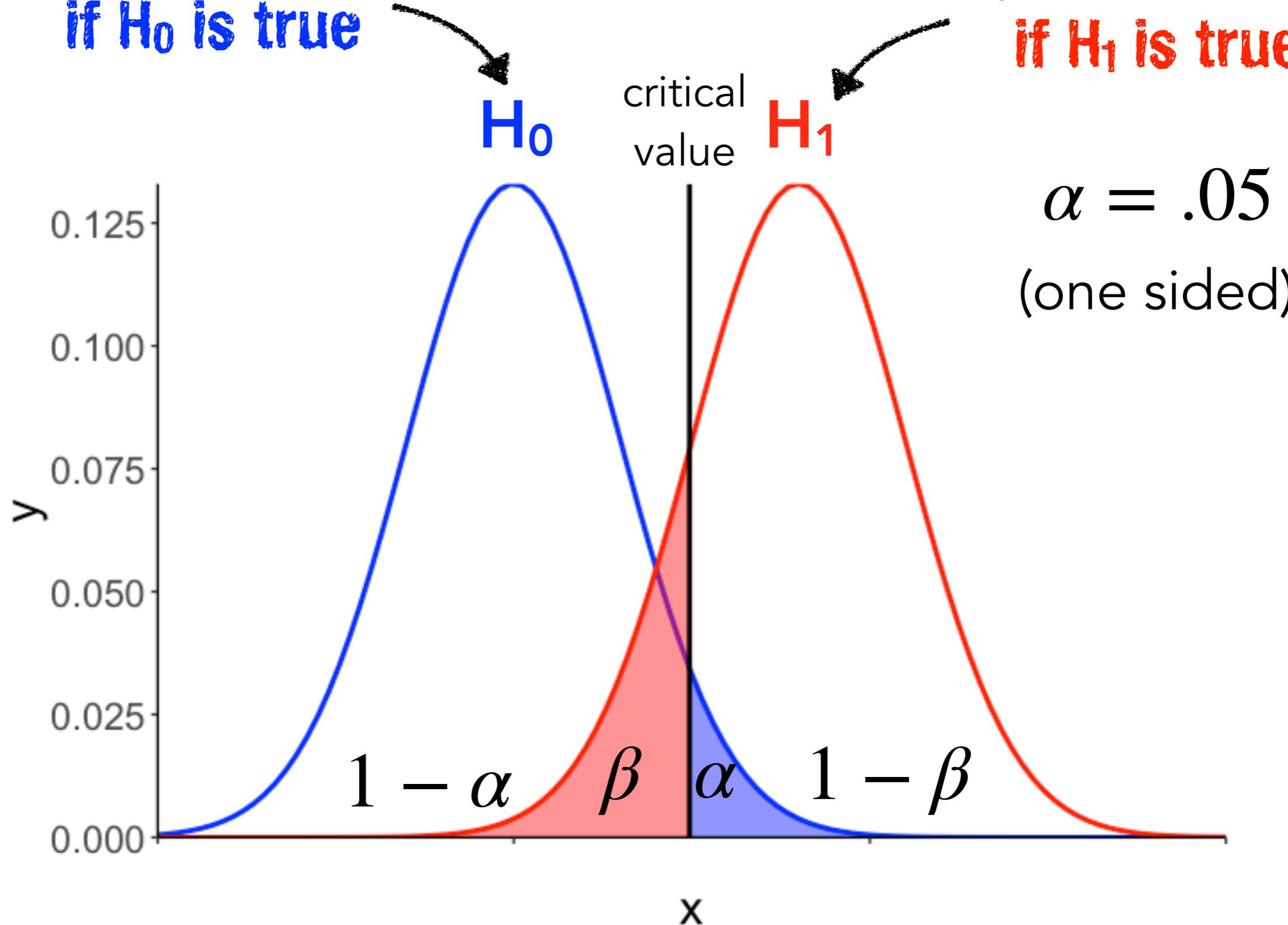
# What affects power?

sampling distribution

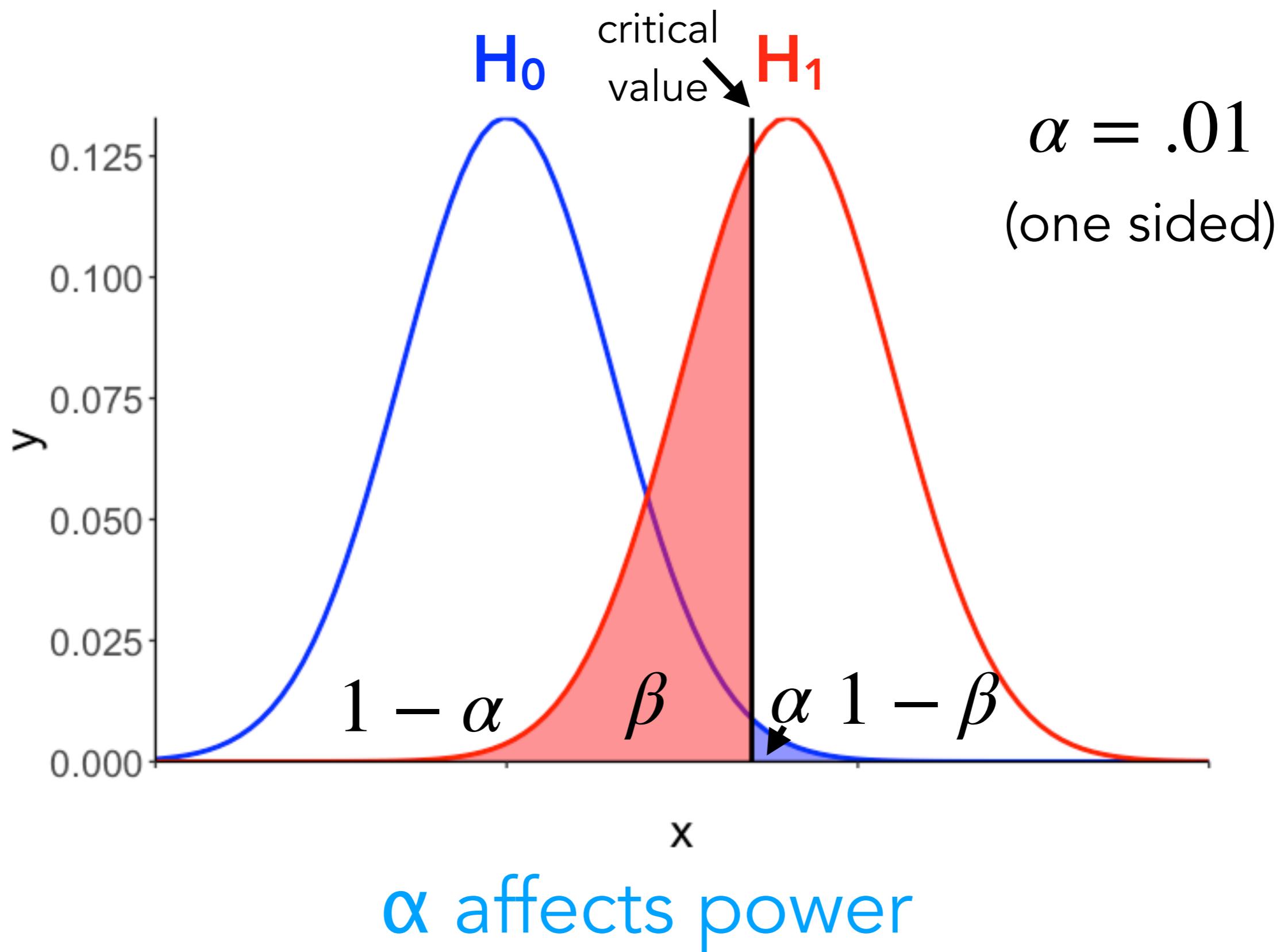
if  $H_0$  is true

sampling distribution

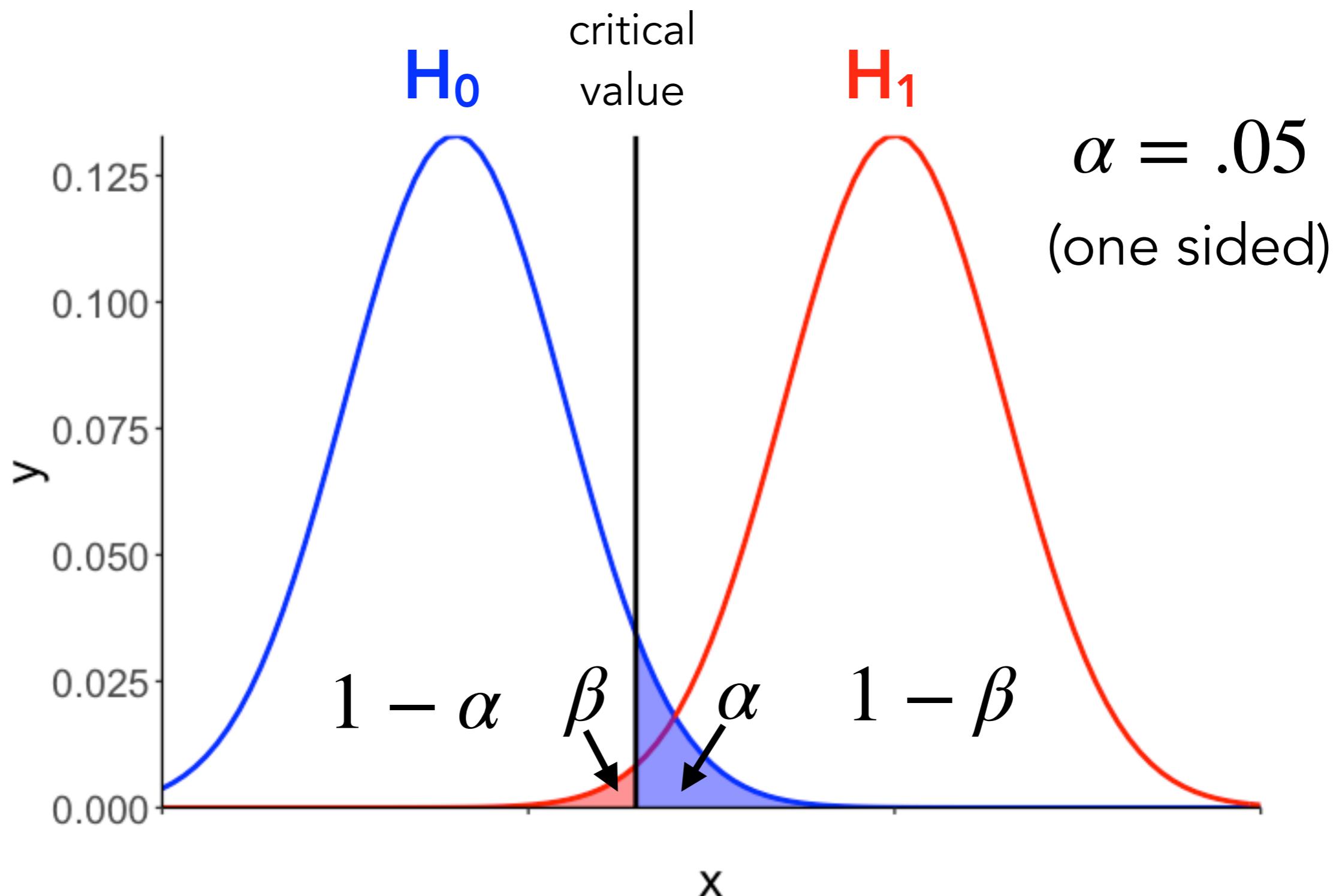
if  $H_1$  is true



# What affects power?

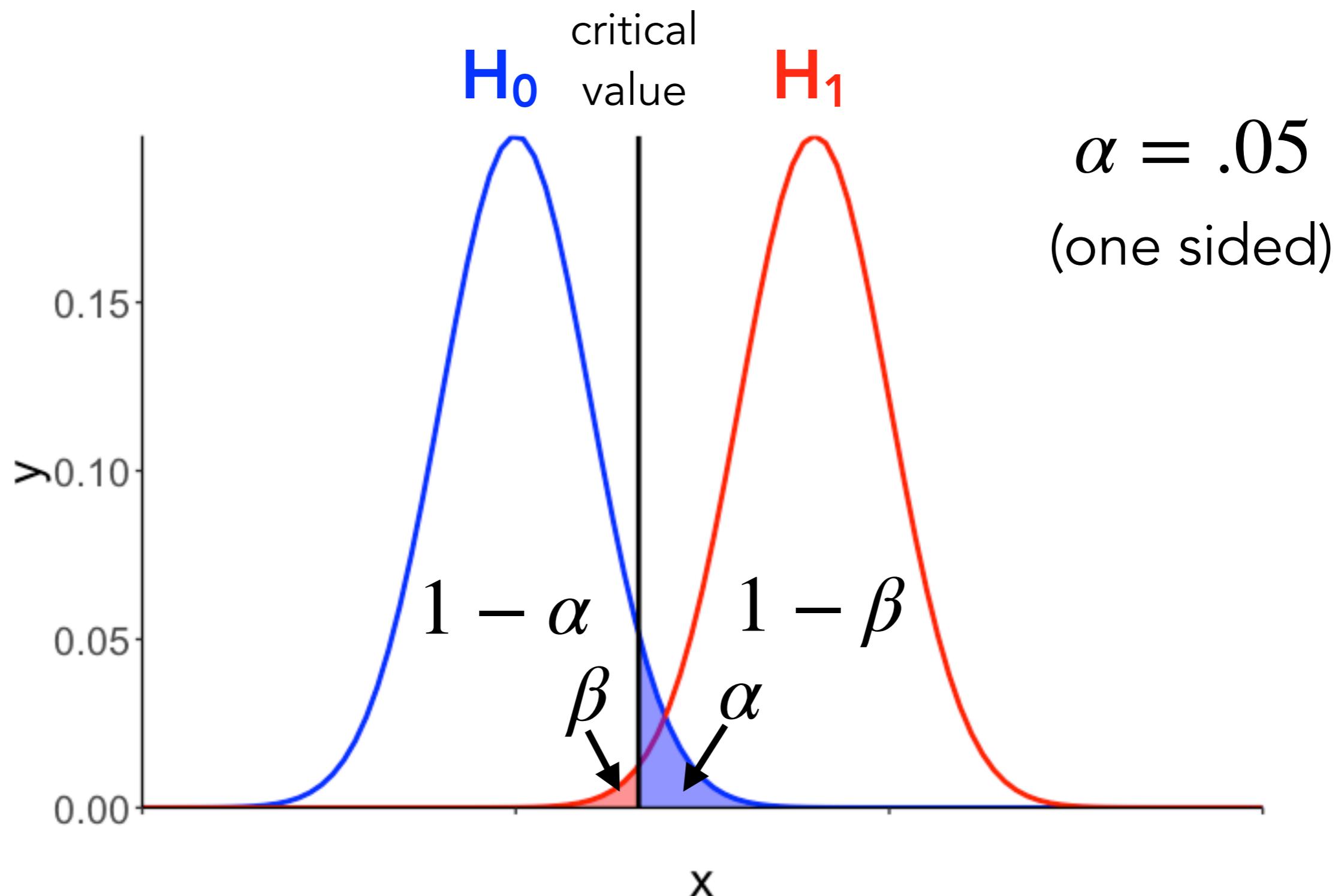


# What affects power?



distance between means affects power

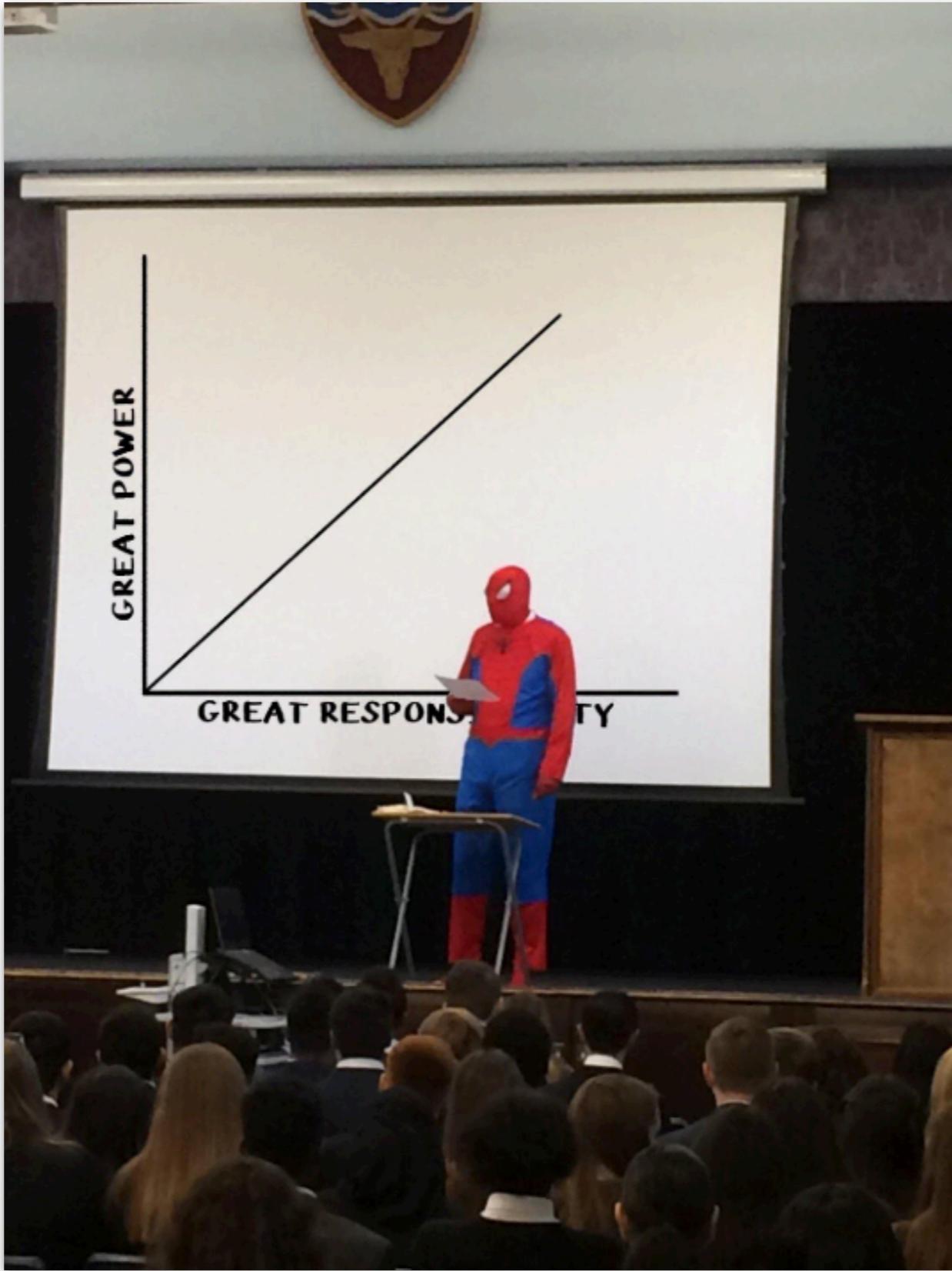
# What affects power?



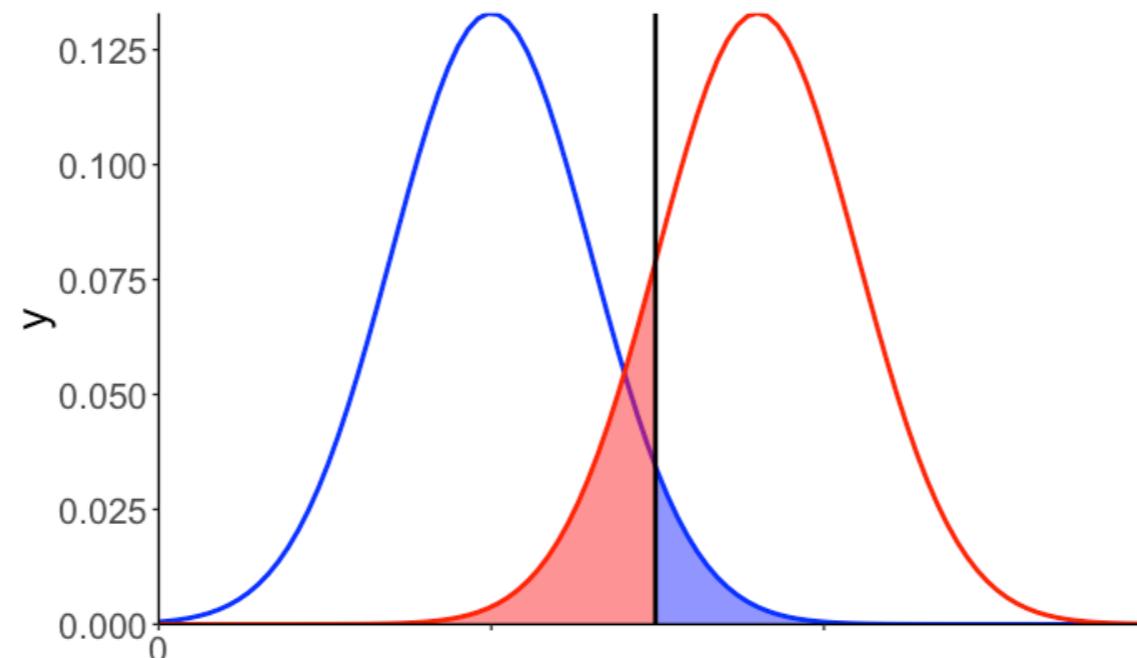
variance affects power

# **Calculating power**

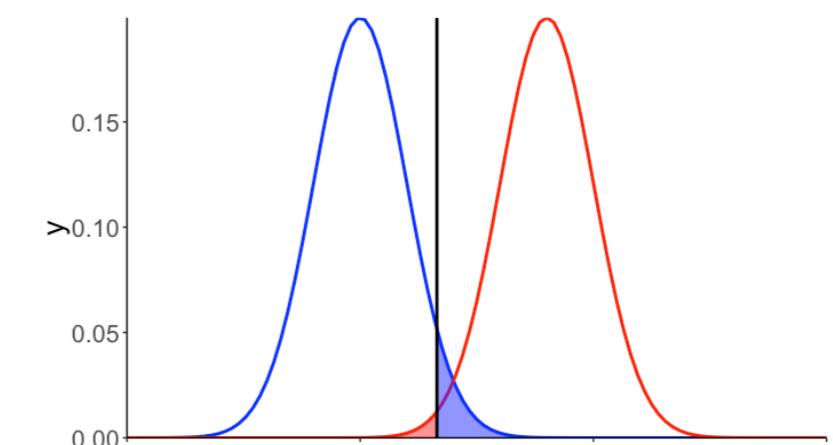
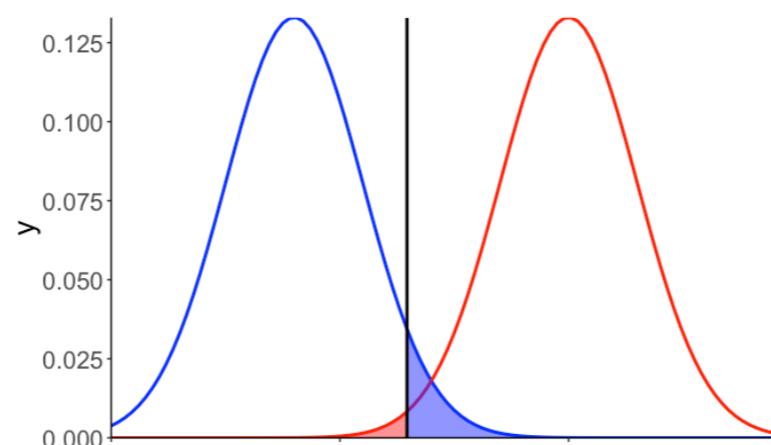
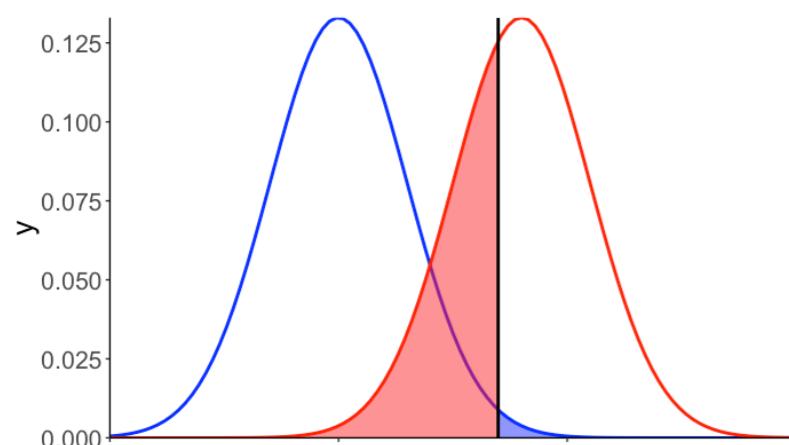
# With great power comes ...



# The knobs we can turn to affect power



$\alpha$       effect size      sample size



# Visualization demo

## Settings

Solve for?  Power  Alpha  n  d

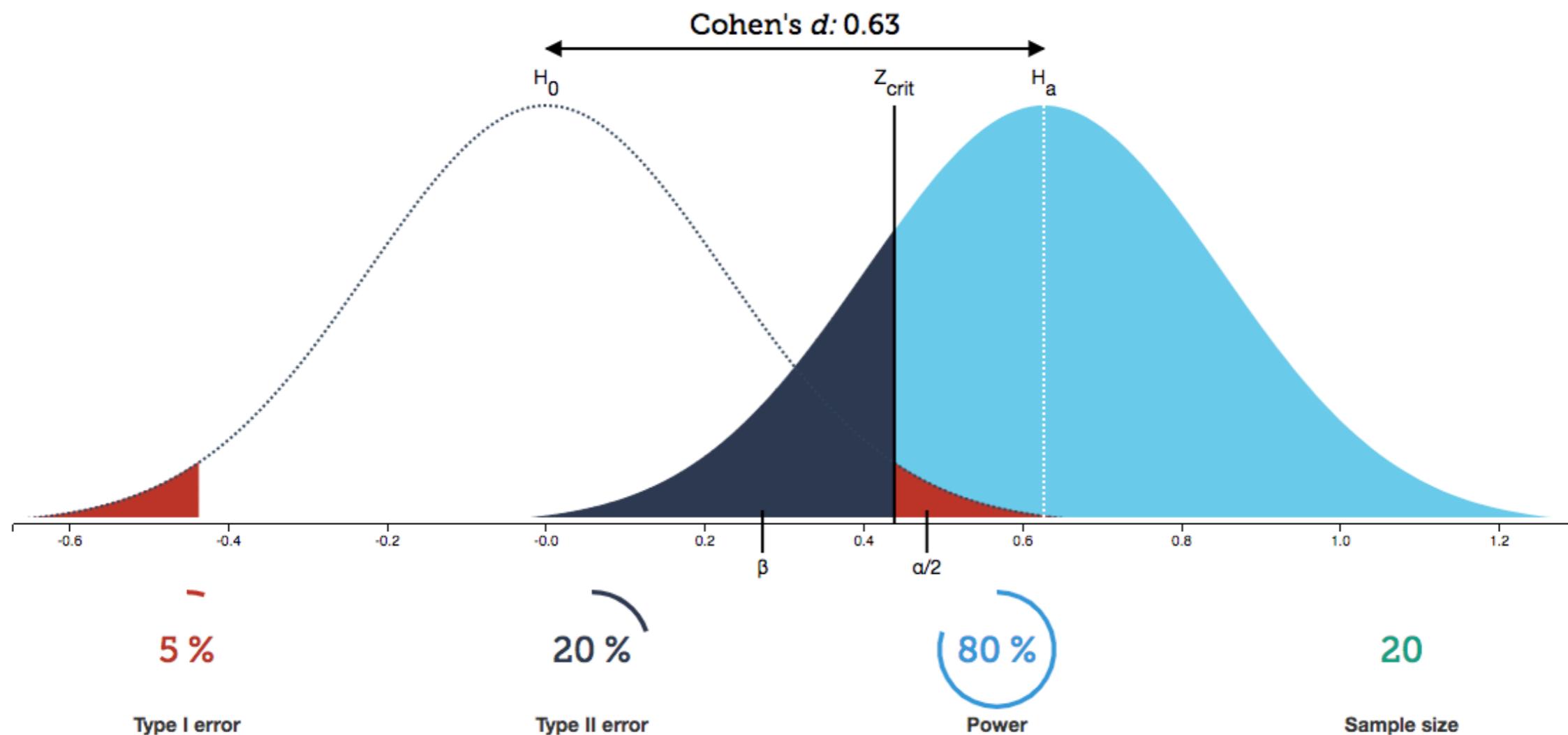
Power ( $1-\beta = 0.8$ )

Significance level ( $\alpha = 0.05$ )

Sample size ( $n = 20$ )

One-tailed  Two-tailed

Reset zoom



<https://rpsychologist.com/d3/NHST/>

The **power** of a binary hypothesis test is the probability that the test rejects the null hypothesis ( $H_0$ ) when a **specific** alternative hypothesis ( $H_1$ ) is true.

---

$H_0$ : Students and non-students have the same balance.

**Model C**

$$Y_i = \beta_0 + \epsilon_i$$

$$\beta_1 = 0$$

$H_1$ : Students and non-students have different balances.

**Model A**

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\beta_1 \neq 0$$

We cannot calculate power in this case.  
We need a specific alternative hypothesis!

The **power** of a binary hypothesis test is the probability that the test rejects the null hypothesis ( $H_0$ ) when a **specific** alternative hypothesis ( $H_1$ ) is true.

---

$H_0$ : Students and non-students have the same balance.

**Model C**

$$Y_i = \beta_0 + \epsilon_i$$

$$\beta_1 = 0$$

$H_1$ : Students and non-students have different balances.

**Model A**

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\beta_1 = 300$$

We can calculate power in this case (since we have a specific alternative hypothesis)!

# **Effect sizes**

# Effect sizes

- a p-value tells us whether we can reject the  $H_0$
- effect sizes is a measure of the strength of the actual effect

**Why can't we just use p-values  
as a measure of the effect size?**

$$F = \frac{\text{PRE}/(\text{PA} - \text{PC})}{(1 - \text{PRE})/(n - \text{PA})}$$

PRE = proportional reduction in error

PA = # parameters in the augmented model

PC = # parameters in the compact model

n = sample size

any PRE will become significant if n gets large enough

**statistical vs.  
practical significance**

# Effect sizes

**PRE** = proportional reduction in error

**Compact model**

SSE(C)

**Augmented model**

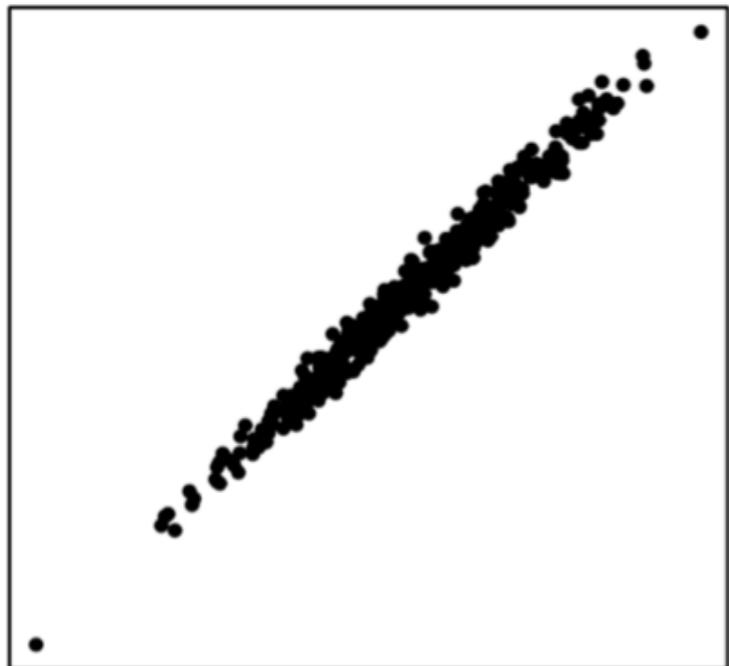
SSE(A)

$$\text{PRE} = 1 - \frac{\text{SSE}(A)}{\text{SSE}(C)}$$

SSE = sum of squared errors

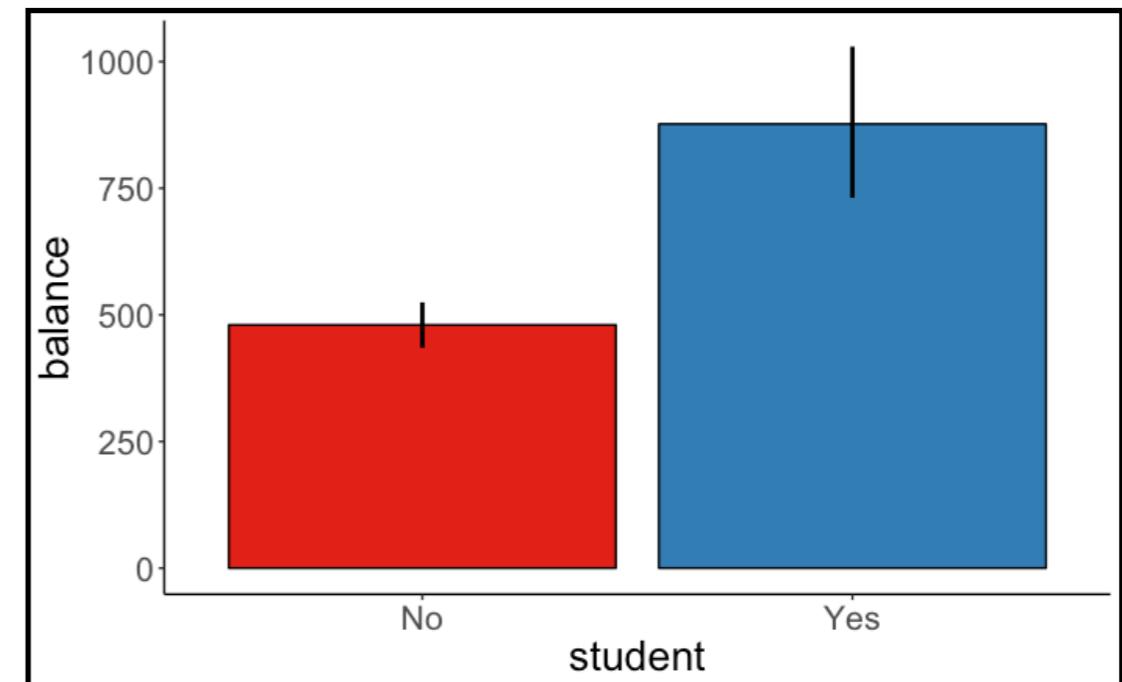
# Common effect sizes

Relationships between variables



$r$  correlation

Differences between groups



Cohen's  $d$

# Correlation

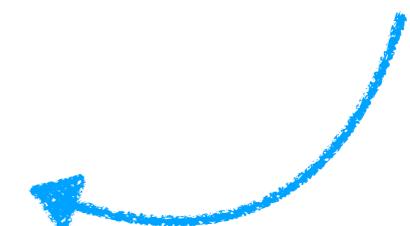
## Pearson correlation

$$r(X, Y) = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \cdot \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}}$$

Cohen's guidelines for the social sciences

Effect size	$r$
Small	0.1
Medium	0.3
Large	0.5

depends very  
much on the  
domain



# Cohen's $d$

- standardized difference between two means

absolute difference between means

$$d = \frac{|\bar{y}_1 - \bar{y}_2|}{s_p}$$

pooled standard variation

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

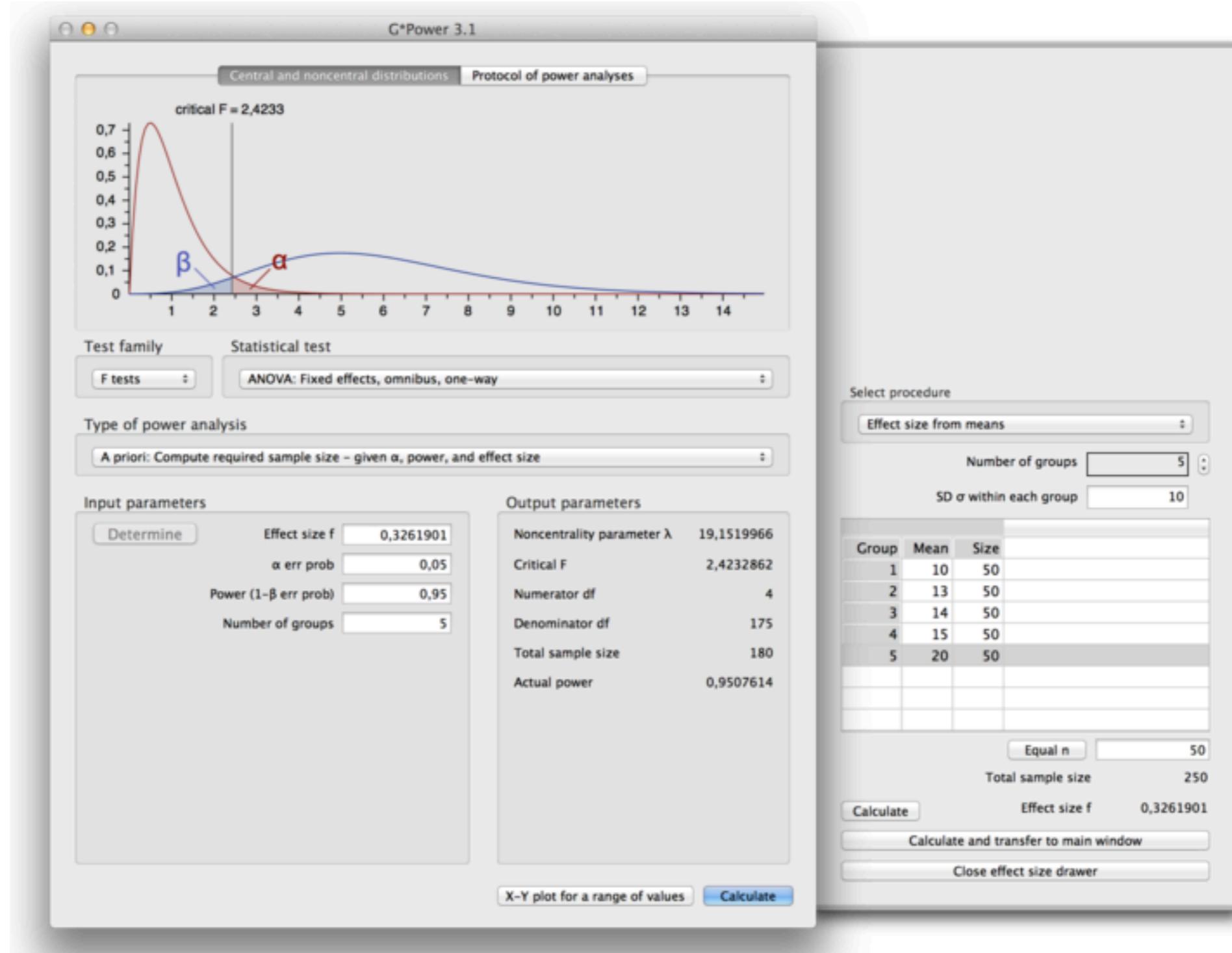
Effect size	$d$
Very small	0.01
Small	0.20
Medium	0.50
Large	0.80
Very large	1.20
Huge	2.0

Difference between two means in pooled standard deviation

# Determining sample size

**How many participants do I need to run to have a good chance of detecting a true effect?**

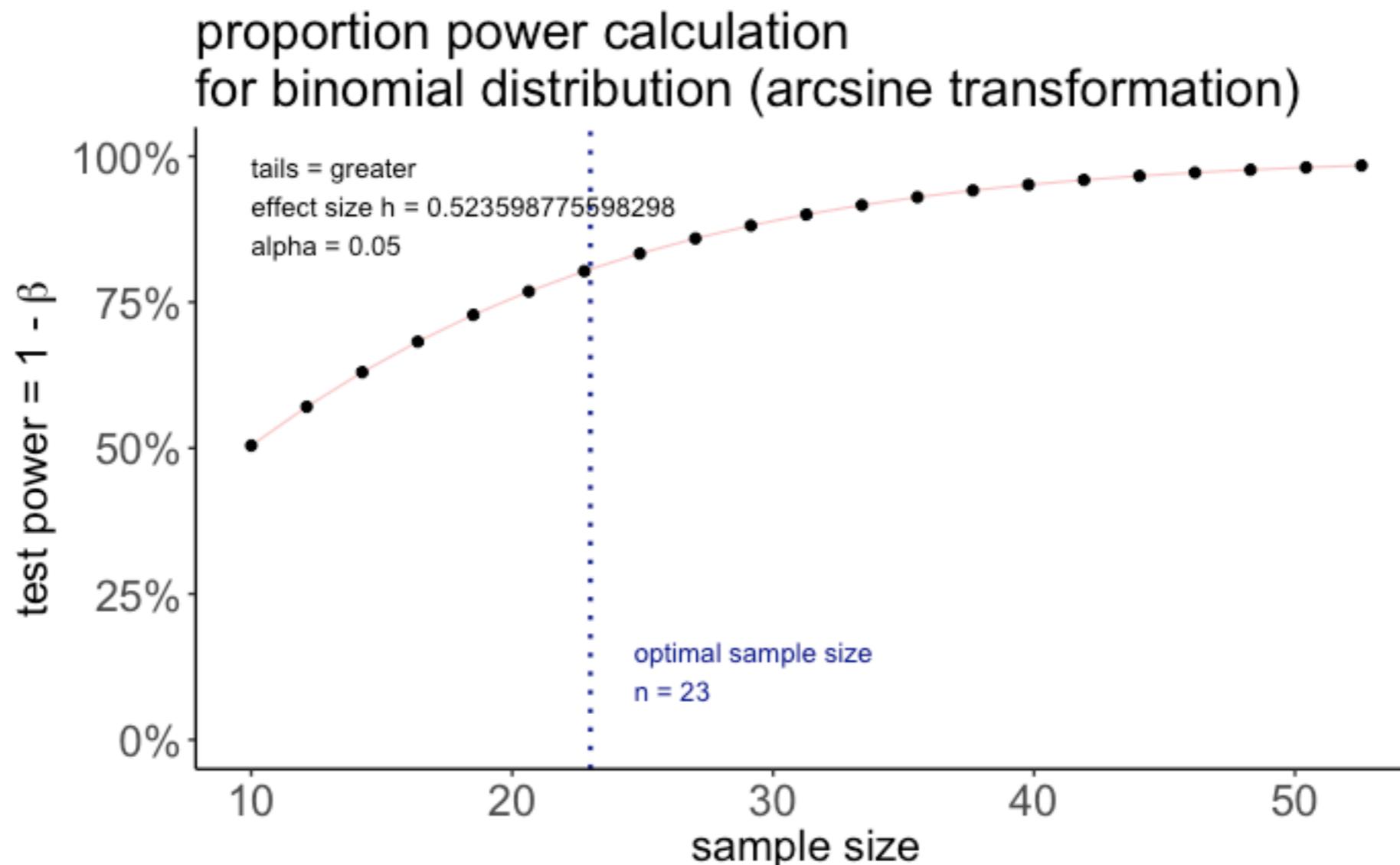
# G\*Power 3.1: Alternative software for power calculations



<http://www.gpower.hhu.de/>

# Example

```
1 library("pwr")
2 pwr.p.test(h = ES.h(p1 = 0.75, p2 = 0.50),
3             sig.level = 0.05,
4             power = 0.80,
5             alternative = "greater") %>%
6   plot()
```



# Power simulation recipe

- assume:
  - $\alpha$ ,  $n$ , effect size
- simulate a large number of data sets of size  $n$  with the specified effect size
- for each data set, run a statistical test to calculate the p-value
- determine the probability of rejecting the  $H_0$  (given that  $H_1$  is true)

**Learn about more advanced  
simulation techniques in R**

# Let's simulate

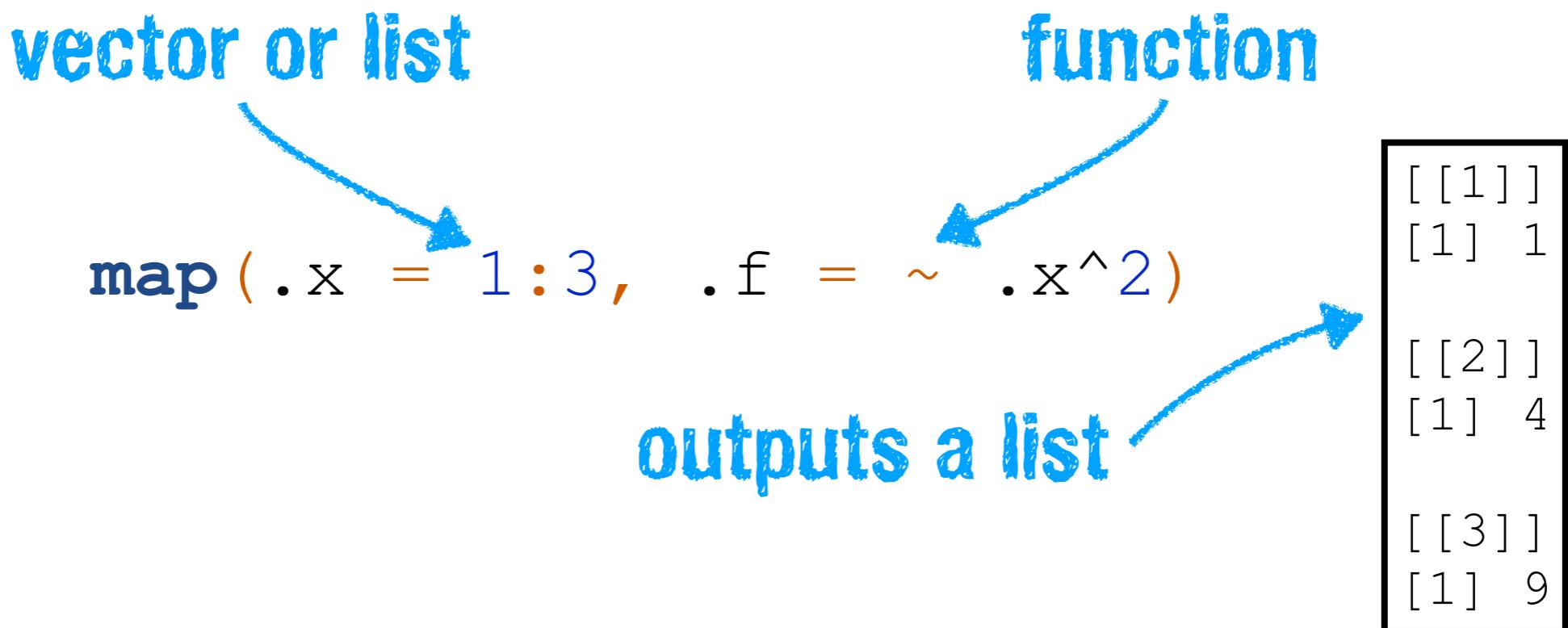
```
library("purrr")
```



automatically loaded with  
**library**("tidyverse")

**map ( )**

# map()



- `map(list, function)` applies a function to each element of the list
- it's a unified version of the many different `apply()` functions in base R
- you already know a cousin of `map()`: `replicate()`
- use `map()`, don't write `for () {}` loops!
- it's extremely powerful in combination with data frames

# map ()

same same but different

```
map (.x = 1:3, .f = ~ .x^2)
```

```
map (1:3, ~ .x^2)
```

```
map (1:3, ~ .^2)
```

```
map (.x = 1:3, .f = function (.x) .x^2)
```

# using a function

```
square = function (x) { x^2 }
```

```
map (1:3, square)
```



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time

# Plan for today

- Linear contrasts
  - Testing specific hypotheses with linear contrasts
  - emmeans for handling linear contrasts in R
- Power analysis
  - Making decisions
  - Calculating power
  - Effect sizes
  - Determining sample size
- Learn about more advanced simulation techniques in R
  - `map()`
  - list columns: `nest()`, `unnest()`

# **Feedback**

# How was the pace of today's class?

much      a little      just      a little      much  
too      too      right      too      too  
slow      slow

# How happy were you with today's class overall?



**What did you like about today's class? What could be improved next time?**

# Thank you!