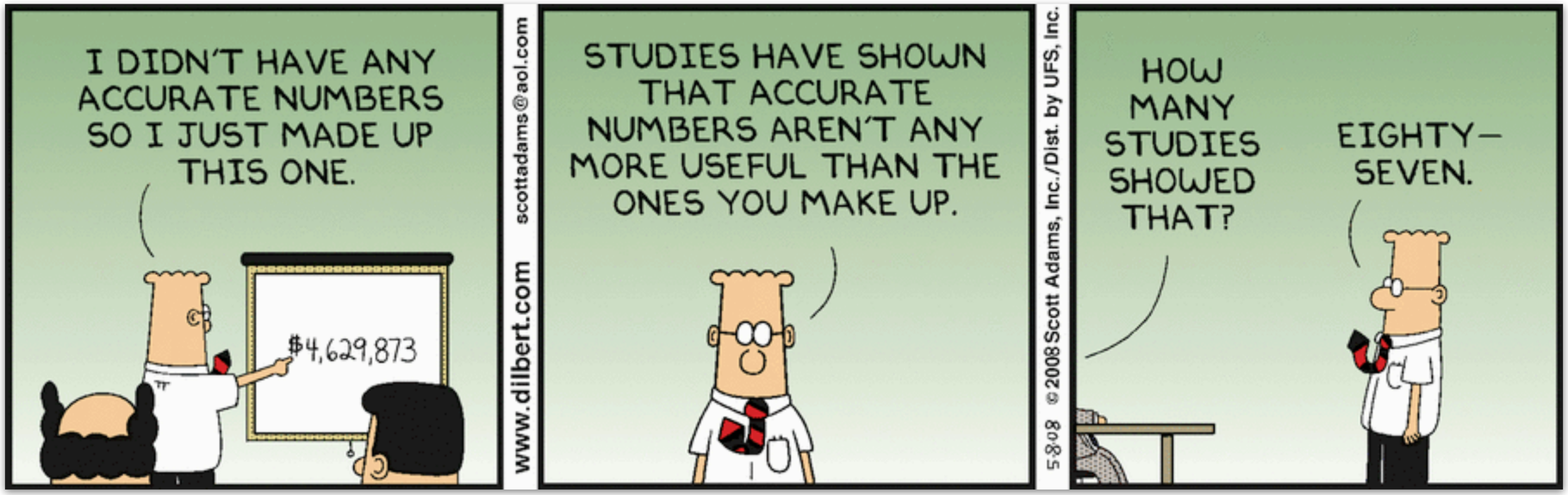
PSYCH 252: Statistical Methods for Behavioral and Social Sciences

Linear model 2



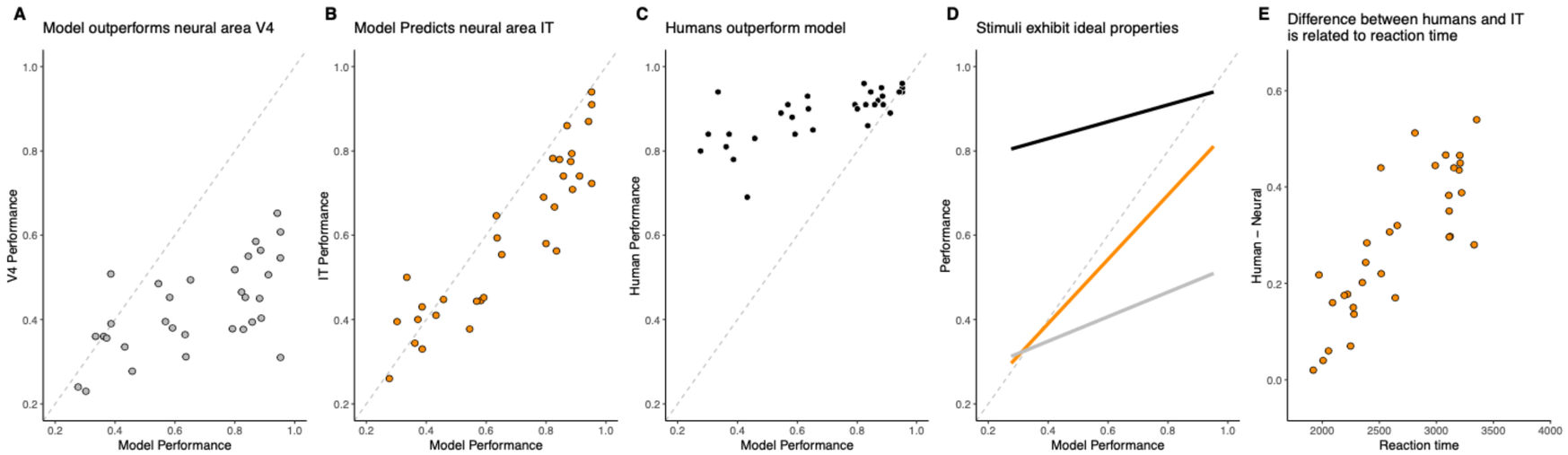
01/31/2025

Logistics

Homework 2

Homework 2

Grades are posted, and solutions are on Canvas.



Homework 4

Homework 4: Modeling data

Homework 4

My name goes here

Collaborator 1 Collaborator 2

January 30, 2025

This homework is due by **Thursday, February 6th, 8:00pm**. Upload both the rendered pdf file (`4_modeling_data.pdf`) and the Rmarkdown code file (`4_modeling_data.rmd`) to Canvas.

I. Simple linear regression and prediction

In this section and the next, we'll be revisiting the credit dataset.

```
# Load data
df.credit = read_csv("data/credit.csv") %>%
  clean_names()
```

1. Explore and visualize the data

Use the `ggpairs()` function from the `GGally` package to make scatterplots of all pairwise combinations of the numeric variables.

Tip: use `progress = FALSE` within the function to suppress unwanted progress bars of each plot

```
### YOUR CODE HERE ###

######################
```

That's perhaps a bit too congested to be useful. Recreate the plot, focusing on just ~5 of the variables that seem interesting to you.

```
### YOUR CODE HERE ###

######################
```

Does your plot tell you anything interesting about the data? Briefly describe one observation.

YOUR ANSWER HERE:

2. Simple linear regression

You decide you want to test the relationship between credit limit (`limit`) and average credit card debt (`balance`).

a) First, let's visualize this relationship. Create a scatterplot of `balance` (on the y-axis) as a function of `limit` (on the x-axis). Set the transparency of the points to 0.3.

```
### YOUR CODE HERE ###

######################
```

```
  ### YOUR CODE HERE ###

    ######################
  return(r2)
}
```

Use your function to calculate r-squared for the data.

```
### YOUR CODE HERE ###

######################
r_squared
```

```
## function(Y_true, Y_pred) {
##   # Input: Y_true and Y_pred should each be a numeric vector of the same length
##
##   ### YOUR CODE HERE ###
##
##     ######################
##   return(r2)
## }
```

d) In theory, what is the minimum and maximum value r-squared can be on **train data**, and why? What does it mean if r-squared is 0?

YOUR ANSWER HERE:

II. Multiple linear regression and controls

In psychological research, people often run linear regressions in which the goal is to assess the relationship between two variables (x and y) while "controlling" for other variables (z1, z2, ...). These control variables could, for example, be age and gender. But how should we decide whether and which variables to control for? In this exercise, we will see what potential effects controlling for variables can have in different situations.

Now you are interested in whether age is a significant predictor of credit limit.

4. Interpreting model parameters

a) Build a simple linear regression model to predict `limit` from `age`. Is age a significant predictor of credit limit? What is the interpretation of the intercept parameter? What is the interpretation of the slope parameter for age?

```
# Simple regression without control variables
### YOUR CODE HERE ###

######################
```

YOUR ANSWER HERE:

After reading another paper in the literature you realize that age is actually related to income, which is a strong predictor of credit limit, so you are interested in seeing whether age is related to credit limit controlling for income.

b) Build a multiple regression model to predict `limit` using both `age` and `income` as predictors. Is age still a significant predictor? What could be an explanation for the change in significance, if there was any?

```
# Multiple regression with control variables
### YOUR CODE HERE ###

######################
```

YOUR ANSWER HERE:

III. Interactions

We will be using the following dataset.

families.csv:

Data from a study of 68 companies, examining relationships between the quality of family-friendly programs at each company, the percentage of employees with families who use these programs, and employee satisfaction (all continuous variables).

Fields:

famprog: the amount of family-friendly programs from (1 = Nothing at all to 9 = Amazing family-friendliness) perfam: the percentage of employees with families in the organization (from 0% to 100%) empastis: the average rating of employee satisfaction (1 = Extremely unsatisfied to 7 = Extremely satisfied)

```
df.families = read_csv("data/families.csv")
```

5. Visualizing simple relationships

a) First, let's visualize a few relationships, plotting regression lines with confidence intervals over scatterplots.

Does employee satisfaction depend on the amount of family programs?

```
### YOUR CODE HERE ###

######################
```

Does employee satisfaction depend upon the percentage of employees with families?

```
### YOUR CODE HERE ###

######################
```

Does the number of family programs depend on the percentage of employees with families?

```
### YOUR CODE HERE ###

######################
```

Doesn't seem like there's much going on in this dataset so far...

Let's try visualizing all three variables at once, mapping one of the three variables to `color`.

```
### YOUR CODE HERE ###

######################
```

Still not super clear.

6. Visualizing interactions

Sometimes the easiest way to visualize continuous data with 3 (or more) continuous variables is to break some of them down into categorical variables, for example by doing a median split, or breaking into quantiles. Let's break the `perfam` and `famprog` columns into terciles (thirds).

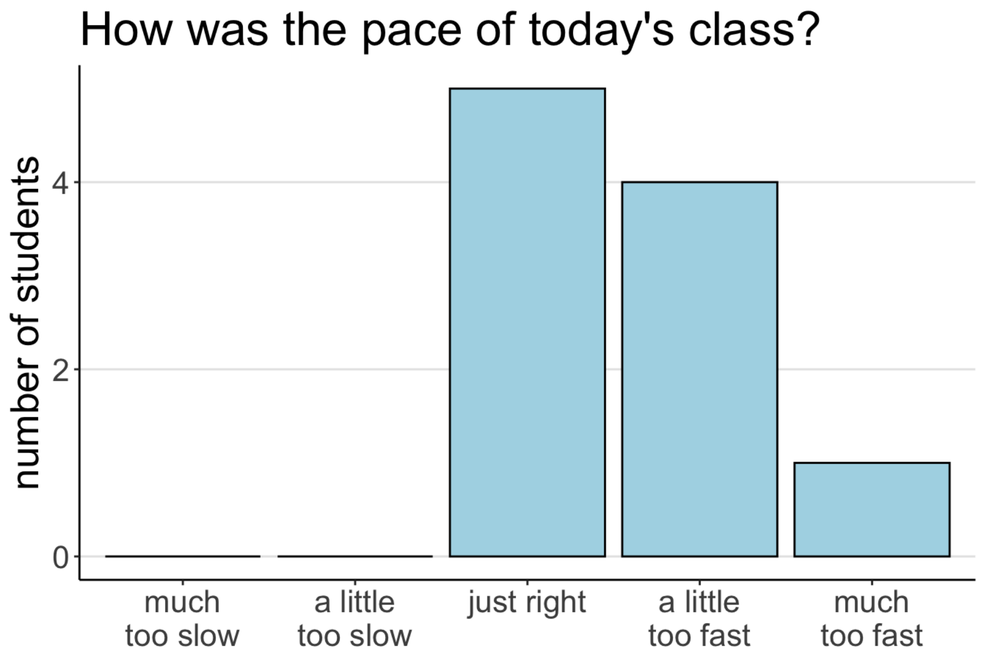
Midterm

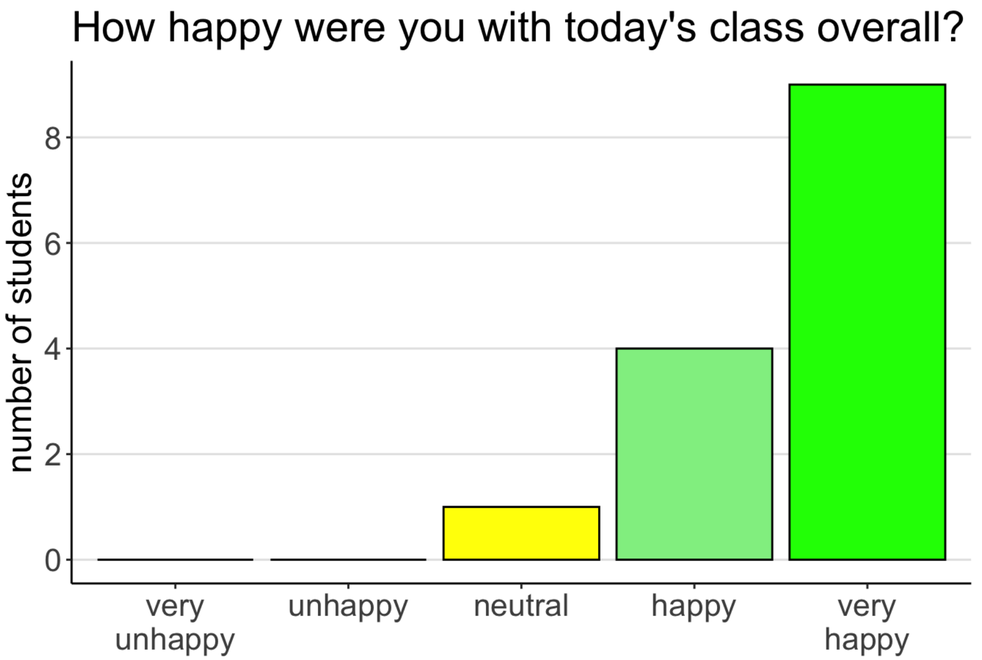
Midterm

- will be released Friday next week
- midterm is take-home, open exam (you can use all the course notes, all of the internet, etc.)
- is due on **Friday, February 14th at 8pm** (stretch days can't be used for the midterm)
- midterm is like the other homeworks but:
  - it's longer (there won't be class on Wednesday that week)
  - you'll need to work on it on your own
- there won't be office hours or sections in the week that the midterm is due
- ask questions about the midterm on Ed Discussion (please **ask private first** -- we will then respond and, if appropriate, check with you whether we can share with the rest of the class)

Feedback

Your feedback





The explanations of concepts and examples are very clear and widely accessible to students with varying stats and programming background. I appreciate the focus on developing both intuition and practical skills, in contrast to many other stats class that seem to prioritize one or the other.

The phase was a **tad too fast**. Also I really really **appreciate the jokes and analogies**! (I really hope they continue) That's the only way I can get through all these STATS :(

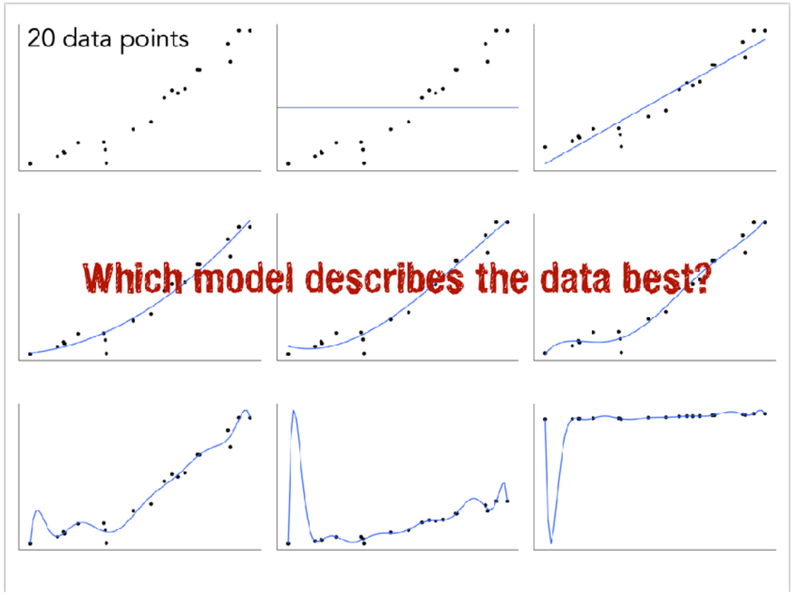
Please **standardize grading** among TAs. maybe TAs can proactively walk around during section and offer help. Please **write in plain speak what the equations are** - helpful to have them in English

Plan for today

- Quick recap
- Generating a sampling distribution for PRE
- Correlation
- Regression
  - The conceptual tour
  - The R route

Quick recap

Quick recap: Linear model 1



Compact model

model_C: $Y_i = \beta_0 + \text{ERROR}$

Augmented model

model_A: $Y_i = \beta_0 + \beta_1 X_i + \text{ERROR}$

$$\text{ERROR}(C) \geq \text{ERROR}(A)$$

Proportional reduction in error (PRE)

$$\text{PRE} = \frac{\text{ERROR}(C) - \text{ERROR}(A)}{\text{ERROR}(C)}$$

- to answer the **worth it?** question we need inferential statistics (define ERROR, sampling distributions, etc.)
- more likely to be **worth it** if:
  1. **PRE** is high
  2. the number of additional parameters in A compared to C is low (**PRE per additional parameter**)
  3. the number of parameters that could have been added to model_C to create model_A but were not is high

more impressed if the number of observations n is much greater than the number of parameters

Quick recap: Linear model 1

Procedure

1. Start with research question
2. Formulate hypothesis as a comparison between compact and augmented model
3. Fit parameters in each model
4. Calculate the proportional reduction of error (PRE)
5. Decide whether PRE is **worth it**

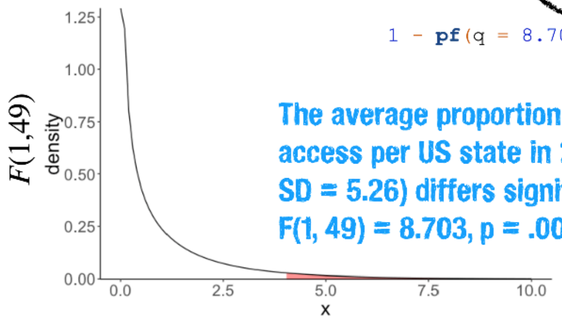
Decide whether it's **worth it**

Note: I've used the approximated PRE here (PRE = 0.15), but I'm reporting the F value for the non-approximated PRE value.

$$F = \frac{\text{PRE}/(\text{PA} - \text{PC})}{(1 - \text{PRE})/(n - \text{PA})}$$

$$= \frac{.15/(1 - 0)}{(1 - .15)/(50 - 1)} = 8.703 \quad p = .00486$$

```
1 - pf(q = 8.703, df1 = 1, df2 = 49)
```



The average proportion of internet access per US state in 2003 (M = 72.8, SD = 5.26) differs significantly from 75%, F(1, 49) = 8.703, p = .005.

Research question and hypotheses

Is the average percentage of internet users per state significantly different from 75%?

Model$_C$: $Y_i = B_0 + \epsilon_i$

0 parameters

$$Y_i = 75 + e_i$$

Model$_A$: $Y_i = \beta_0 + \epsilon_i$

1 parameter

$$Y_i = b_0 + e_i$$

$$= \overline{Y} + e_i$$

```
t.test(df.internet$internet, mu = 75)
```

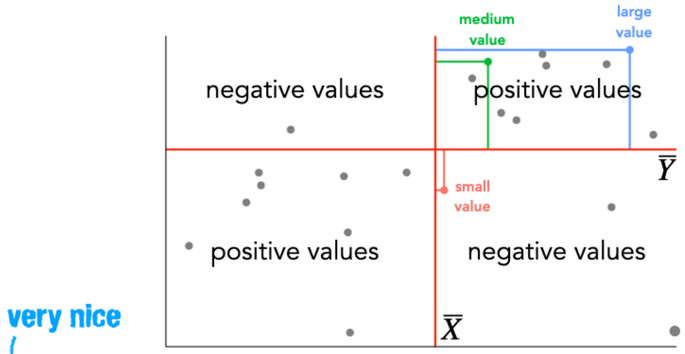
One Sample t-test

data: df.internet$internet
t = -2.9502, df = 49, p-value = **0.00486**
alternative hypothesis: true mean is not equal to 75

Quick recap: Linear model 1

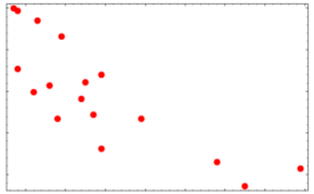
sample covariance

$$Cov(X, Y) = \sum_{i=1}^{n} \frac{(X_i - \overline{X})(Y_i - \overline{Y})}{n - 1}$$



https://tinystats.github.io/teacups-giraffes-and-statistics/05_correlation.html

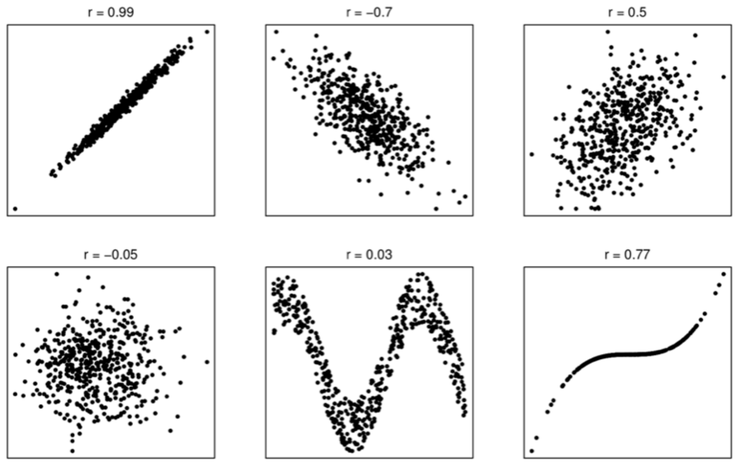
Who is the correlation champion?



Winner gets chocolate!

-1 : -0.75
-0.75 : -0.5
-0.5 : -0.25
-0.25 : 0
0 : 0.25
0.25 : 0.5
0.5 : 0.75
0.75 : 1

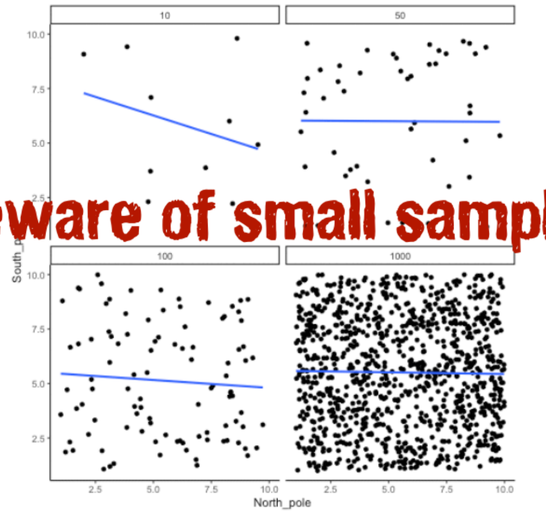
Solution



$n = [10, 50, 100, 1000]$

$X \sim \mathcal{U}(\text{min} = 0, \text{max} = 10)$

$Y \sim \mathcal{U}(\text{min} = 0, \text{max} = 10)$



Beware of small samples!

Generating a sampling distribution for PRE

Fit parameters and calculate PRE

C: $Y_i = 75 + e_i$ **A**: $Y_i = \overline{Y} + e_i$

i	state	internet	compact_b	compact_se	augmented_b	augmented_se
1	AK	79.0	75	16.00	72.81	38.37
2	AL	63.5	75	132.25	72.81	86.60
3	AR	60.9	75	198.81	72.81	141.75
4	AZ	73.9	75	1.21	72.81	1.20
5	CA	77.9	75	8.41	72.81	25.95
6	CO	79.4	75	19.36	72.81	43.48
7	CT	77.5	75	6.25	72.81	22.03
8	DE	74.5	75	0.25	72.81	2.87
9	FL	74.3	75	0.49	72.81	2.23
10	GA	72.2	75	7.84	72.81	0.37

$$\text{PRE} = 1 - \frac{\text{ERROR}(A)}{\text{ERROR}(C)}$$

$$= 1 - \frac{1355}{1595} \approx .15$$

Model A has 15% less error than Model C.

SSE(C) = 1595 SSE(A) = 1355

Decide whether it's **worth it**

- we have to construct a sampling distribution of PRE assuming that H_0 is true
- and then compare the observed value of PRE to that distribution

Population distribution

$$Y_i = 75 + \epsilon_i \qquad \epsilon_i \sim \mathcal{N}(\mu = 0,\ \sigma = 5)$$

Model C

$$Y_i = 75 + e_i$$

0 parameters

Model A

$$Y_i = \overline{Y} + e_i$$

1 parameter

Sampling distribution of PRE

```r
# simulation parameters
n_samples = 1000
sample_size = 50
mu = 75 # true mean of the distribution
sigma = 5 # true standard deviation of the errors

# function to draw samples from the population distribution
fun.draw_sample = function(sample_size, mu, sigma){
  sample = mu + rnorm(sample_size, mean = 0, sd = sigma)
}

# draw samples
samples = n_samples %>%
  replicate(fun.draw_sample(sample_size, mu, sigma)) %>%
  t() # transpose the resulting matrix (i.e. flip rows and columns)
```

sample	index	number
1	1	75.30
1	2	72.06
1	3	77.66
1	4	67.41
1	5	76.53
1	6	67.32
1	7	73.50
1	8	72.36
1	9	71.74
1	10	74.72

⋮

Sampling distribution of PRE

```
# put samples in data frame and compute PRE
df.samples = samples %>%
  as_tibble(.name_repair = ~ 1:ncol(samples)) %>%
  mutate(sample = 1:n()) %>%
  pivot_longer(cols = -sample,
               names_to = "index",
               values_to = "value") %>%
  mutate(compact = 75) %>%
  group_by(sample) %>%
  mutate(augmented = mean(value))
```

sample	index	value	compact	augmented
1	1	73.43	75	74.75
1	2	76.38	75	74.75
1	3	79.92	75	74.75
1	4	72.33	75	74.75
1	5	77.75	75	74.75
2	1	79.84	75	73.92
2	2	78.44	75	73.92
2	3	79.49	75	73.92
2	4	71.81	75	73.92
2	5	79.57	75	73.92
3	1	78.99	75	74.93
3	2	67.28	75	74.93
3	3	77.74	75	74.93
3	4	73.73	75	74.93
3	5	73.49	75	74.93

Sampling distribution of PRE

```
# put samples in data frame and compute PRE
df.samples = samples %>%
  as_tibble(.name_repair = ~ 1:ncol(samples)) %>%
  mutate(sample = 1:n()) %>%
  pivot_longer(cols = -sample,
               names_to = "index",
               values_to = "value") %>%
  mutate(compact = 75) %>%
  group_by(sample) %>%
  mutate(augmented = mean(value)) %>%
  summarize(sse_compact = sum((value - compact)^2),
            sse_augmented = sum((value - augmented)^2),
            pre = 1 - sse_augmented/sse_compact)
```

sample	index	value	compact	augmented
1	1	73.43	75	74.75
1	2	76.38	75	74.75
1	3	79.92	75	74.75
1	4	72.33	75	74.75
1	5	77.75	75	74.75

calculate SSE for each model

calculate PRE

$$\text{PRE} = 1 - \frac{\text{SSE}_A}{\text{SSE}_C}$$

sample	sse_compact	sse_augmented	pre
1	1244.66	1241.52	0.00
2	953.25	894.95	0.06
3	1083.60	1083.32	0.00
4	888.06	798.19	0.10
5	1246.57	1233.25	0.01

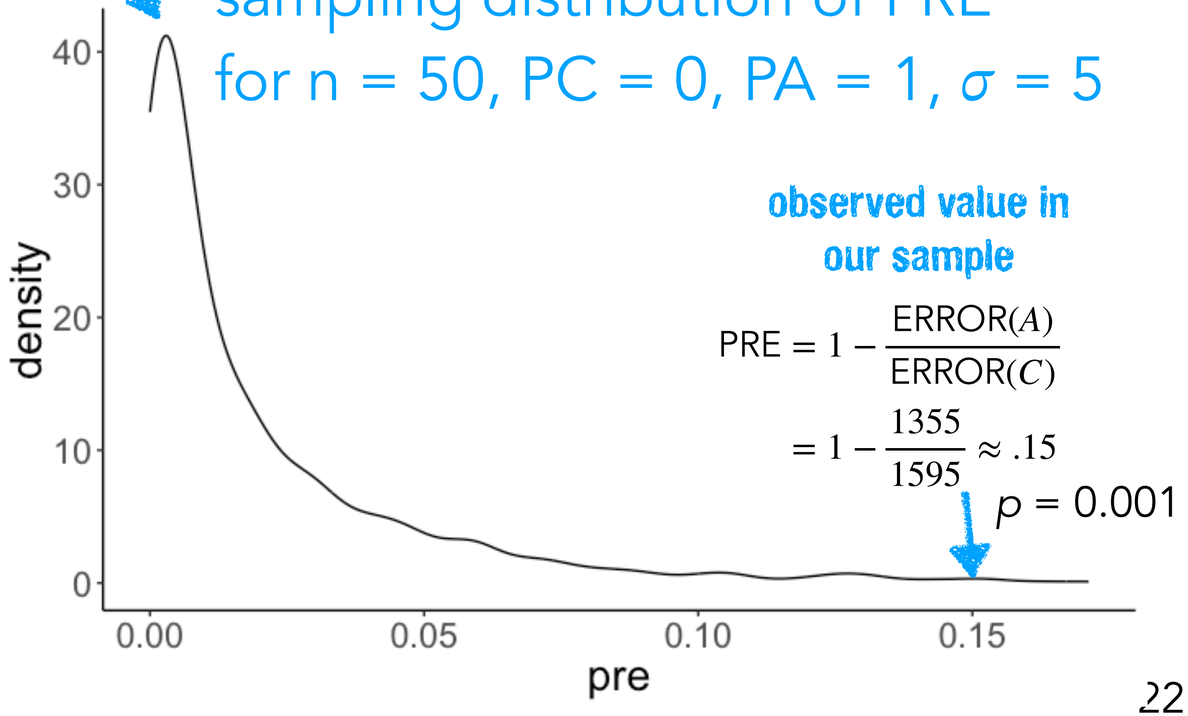
Sampling distribution of PRE

```r
# sampling distribution for PRE
ggplot(data = df.samples,
       mapping = aes(x = pre)) +
  stat_density(geom = "line")

# p-value for our sample
df.samples %>%
  summarize(p_value = sum(pre >= df.summary$pre)/n())
```

sample	sse_compact	sse_augmented	pre
1	1244.66	1241.52	0.00
2	953.25	894.95	0.06
3	1083.60	1083.32	0.00
4	888.06	798.19	0.10
5	1246.57	1233.25	0.01

sampling distribution of PRE
for n = 50, PC = 0, PA = 1, $\sigma = 5$



observed value in our sample

$$\text{PRE} = 1 - \frac{\text{ERROR}(A)}{\text{ERROR}(C)}$$

$$= 1 - \frac{1355}{1595} \approx .15$$

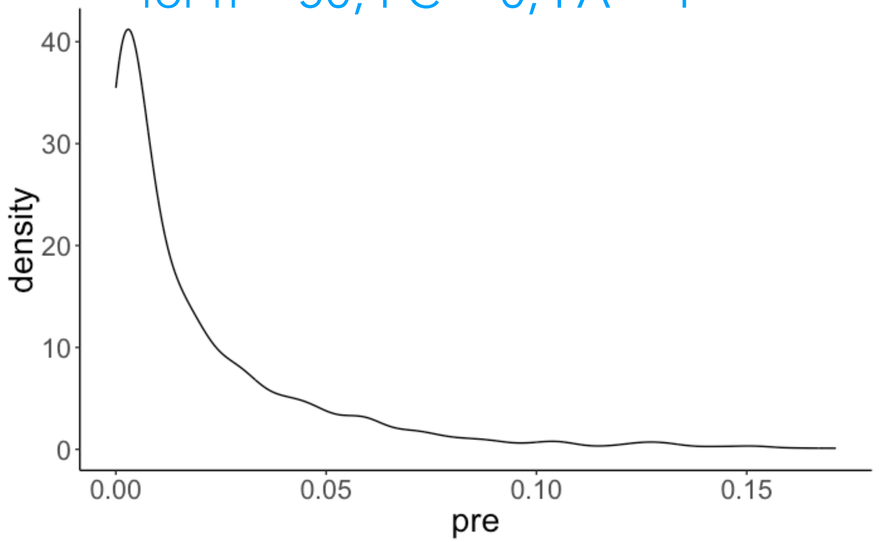
$p = 0.001$

Sampling distribution of PRE

deterministic mapping

sampling distribution of PRE

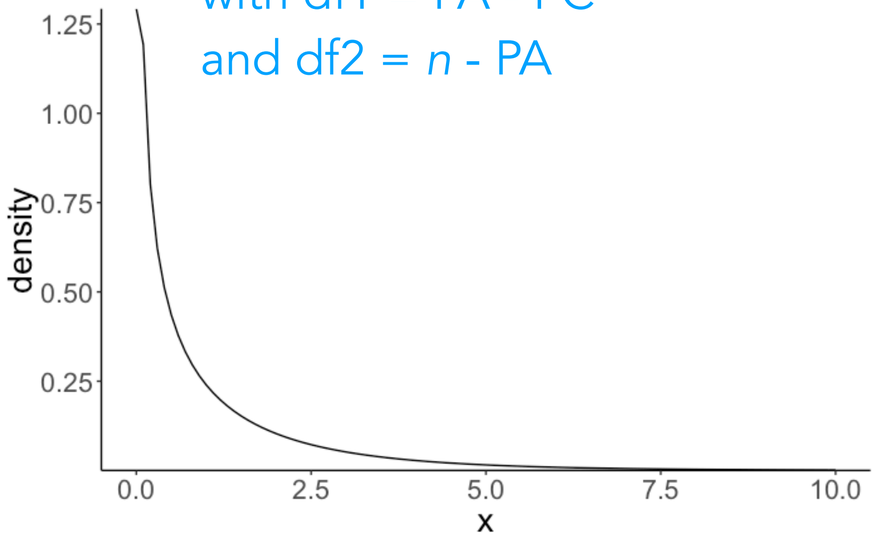
for n = 50, PC = 0, PA = 1



***F*(df1, df2) distribution**

with df1 = PA - PC

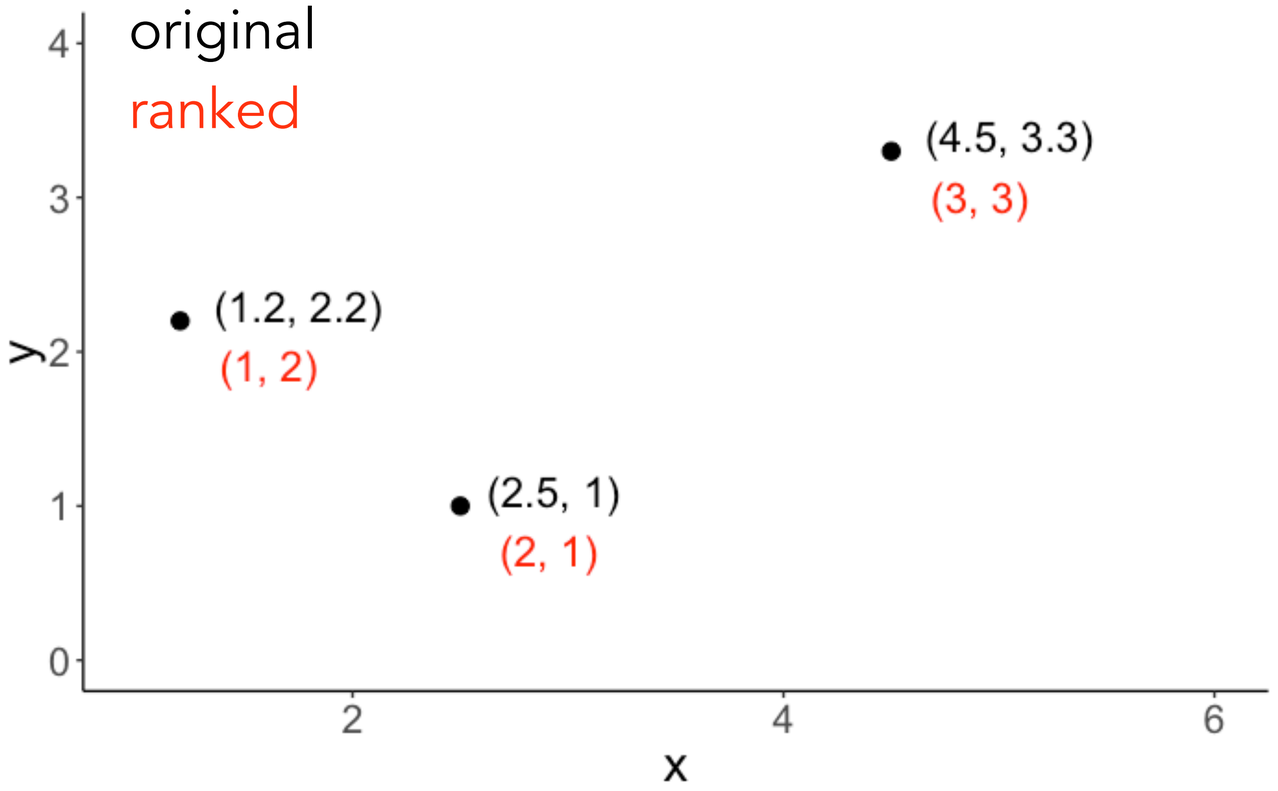
and df2 = *n* - PA



Correlation

Spearman rank order correlation

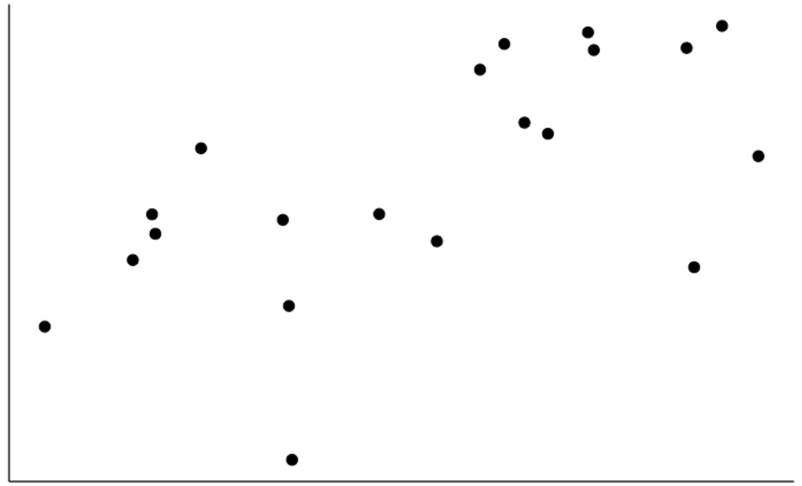
- transform original data into ranks
- calculate correlation on the ranked data



Spearman rank order correlation

- transform original data into ranks
- calculate correlation on the ranked data

x	y	x_rank	y_rank
0.27	1.14	5	12
0.37	0.97	6	8
0.57	0.92	10	6
0.91	0.85	18	4
0.20	0.98	3	9
0.90	1.39	17	17
0.94	1.44	19	20
0.66	1.40	12	18
0.63	1.33	11	15
0.06	0.71	1	2



r	spearman	r_ranks
0.609	0.595	0.595

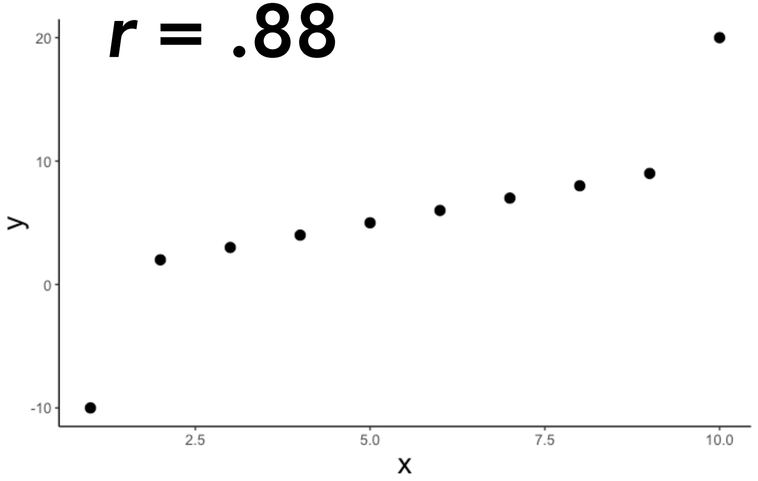
```
1 # correlation
2 df.spearman %>%
3   summarize(r = cor(x, y, method = "pearson"),
4             spearman = cor(x, y, method = "spearman"),
5             r_ranks = cor(x_rank, y_rank, method = "pearson"))
```

Spearman rank order correlation

original

ranked

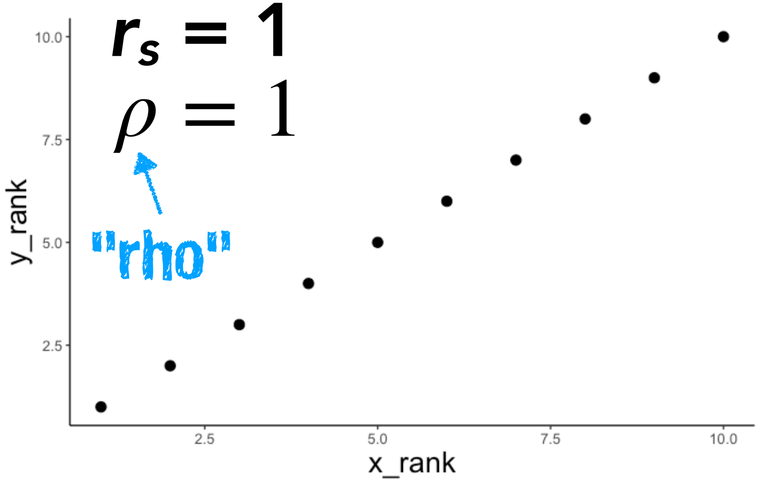
$r = .88$



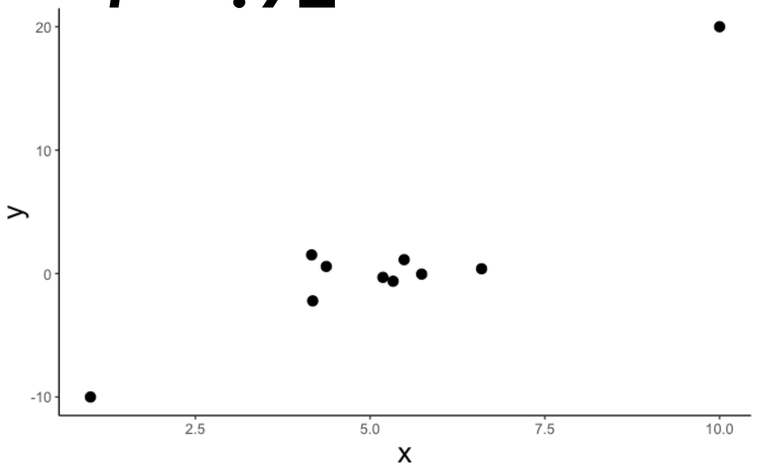
$r_s = 1$

$\rho = 1$

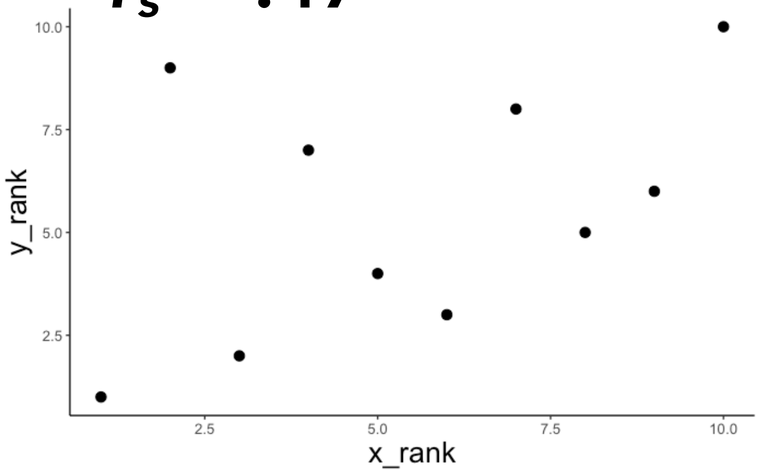
"rho"



$r = .92$



$r_s = .47$



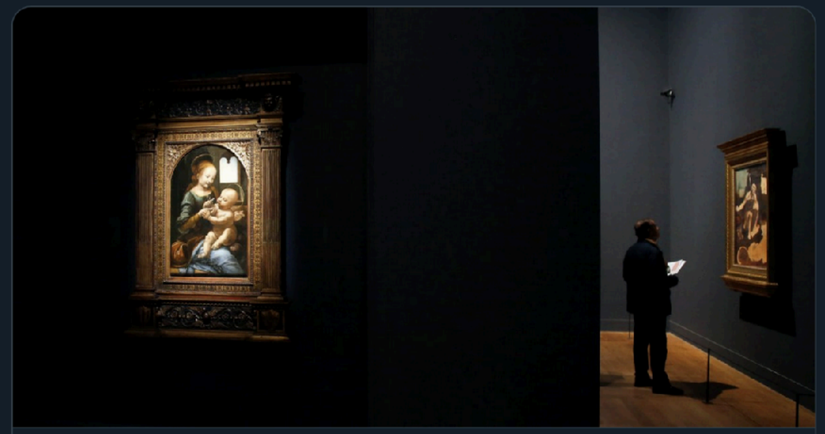
Pearson vs. Spearman

- Pearson's *r* captures the extent to which the relationship between two variable is **linear**
- Spearman's ρ captures the extent to which the relationship between two variables is **monotonic**
- What's better?
  - depends on the context
  - Spearman is robust to outliers, but it throws away (potentially useful) information

CORRELATION IS NOT CAUSATION

NYT Health
@NYTHealth

Want to live longer? Try going to the opera. Researchers in Britain have found that people who reported going to a museum or concert even once a year lived longer than those who didn't.



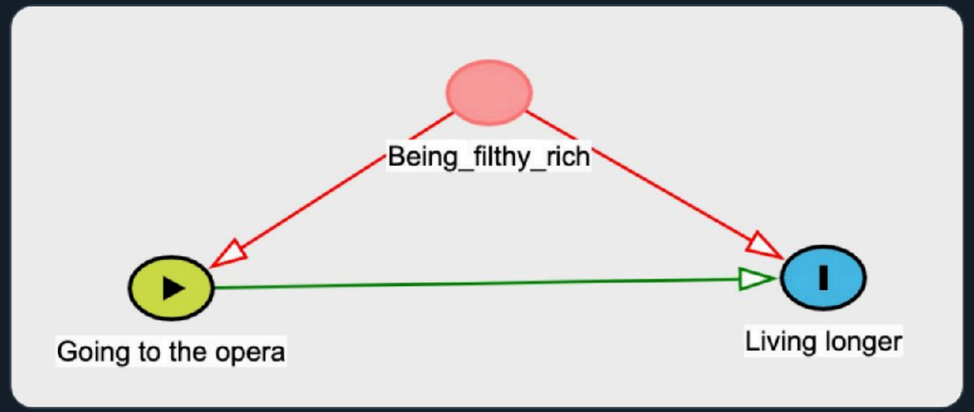
Another Benefit to Going to Museums? You May Live Longer
Researchers in Britain found that people who go to museums, the theater and the opera were less likely to die in the study period than those who didn't.
nytimes.com

9:19 AM · Dec 22, 2019 · SocialFlow

336 Retweets **1.3K** Likes

Andrew Heiss
@andrewheiss

ooh ooh i can draw the dag for this one!

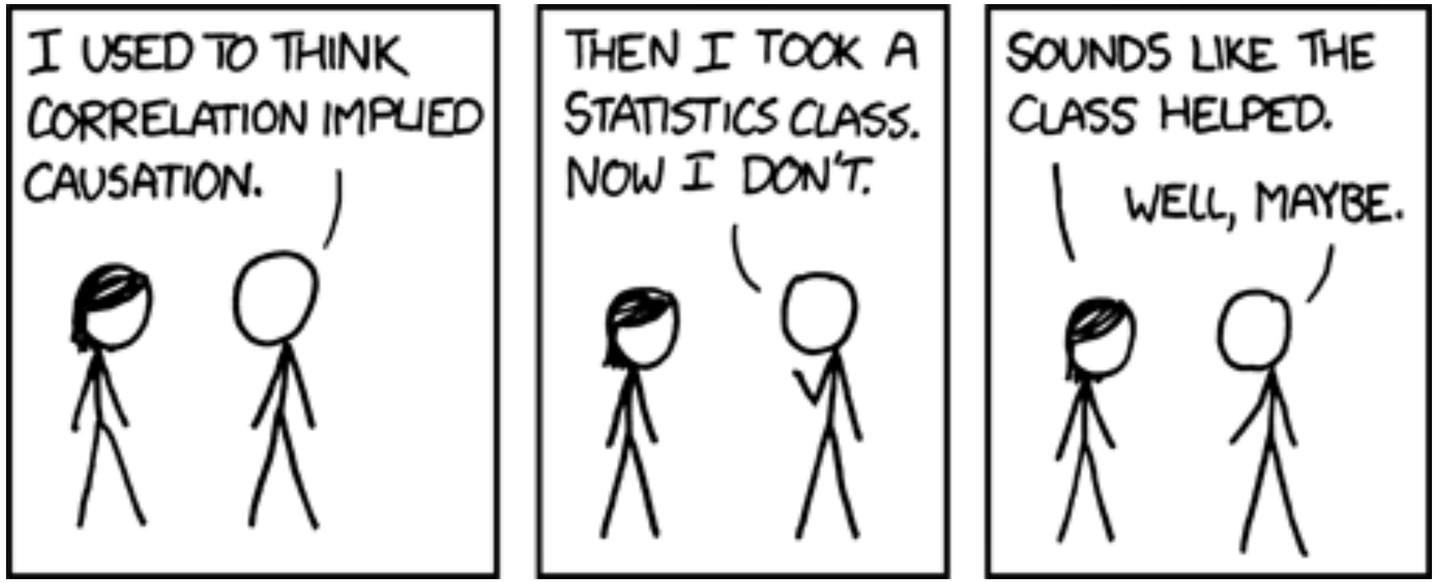


NYT Health @NYTHealth · Dec 22, 2019
Want to live longer? Try going to the opera. Researchers in Britain have found that people who reported going to a museum or concert even once a year lived longer than those who didn't. nyti.ms/2Q9AmZV

2:47 PM · Dec 22, 2019 · Twitter Web App

View Tweet activity

837 Retweets **3.9K** Likes



- correlations suggest that there is some causal relationship
- but this relationship need not be a direct causal relationship from A to B (or from B to A)

more about causation in a later class

Regression

The conceptual tour

Linear model: Simple regression

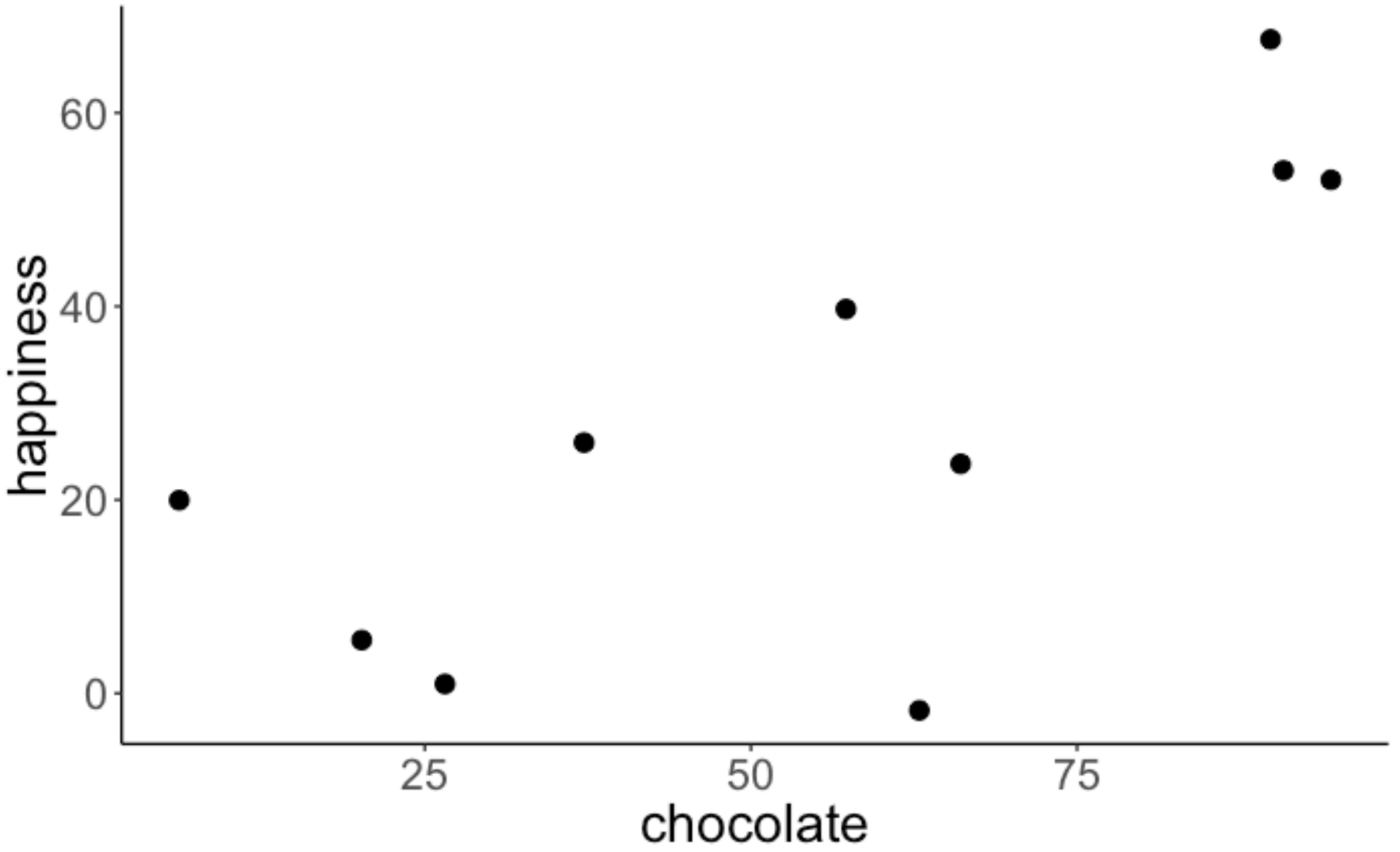
$$\texttt{Data} = \texttt{Model} + \texttt{Error}$$

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\hat{Y}_i = \beta_0 + \beta_1 X_i$$

the model is a linear combination of predictors

Is there a relationship between chocolate consumption and happiness?



The general procedure

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data
3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

The general procedure

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H_0: Chocolate consumption and happiness are unrelated.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

and

$$\beta_1 = 0$$

H_1: Chocolate consumption and happiness are related.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

chocolate consumption

The general procedure

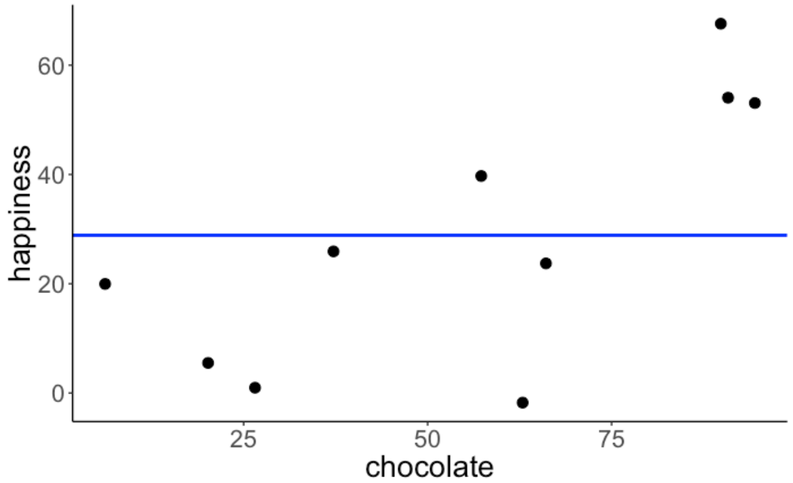
1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. **Fit model parameters to the data**
3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

H_0: Chocolate consumption and happiness are unrelated.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

Model prediction



Fitted model

$$Y_i = 28.88 + e_i$$

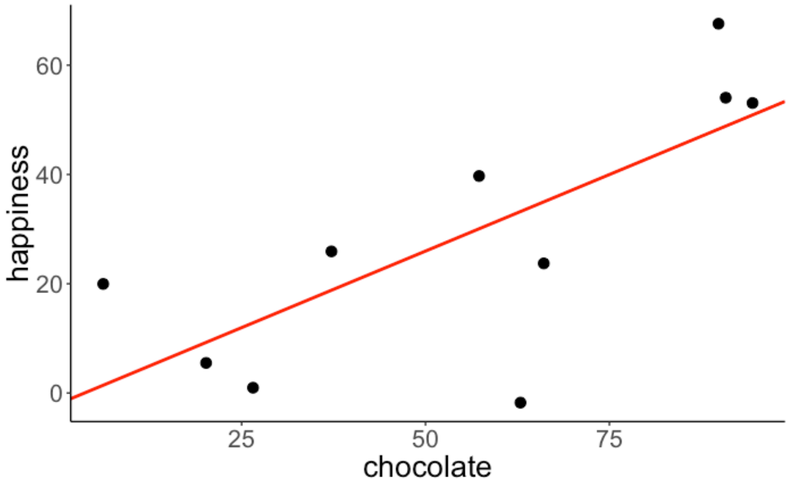
H_1: Chocolate consumption and happiness are related.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

chocolate consumption

Model prediction



Fitted model

$$Y_i = -2.04 + 0.56 X_i + e_i$$

The general procedure

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data
3. **Calculate the proportional reduction of error (PRE) in our sample**
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

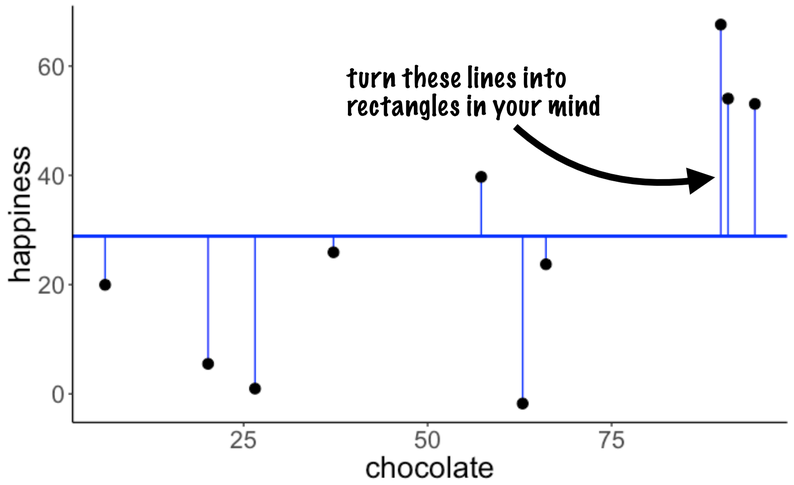
Calculate PRE

$$\text{PRE} = 1 - \frac{\text{ERROR}(A)}{\text{ERROR}(C)}$$

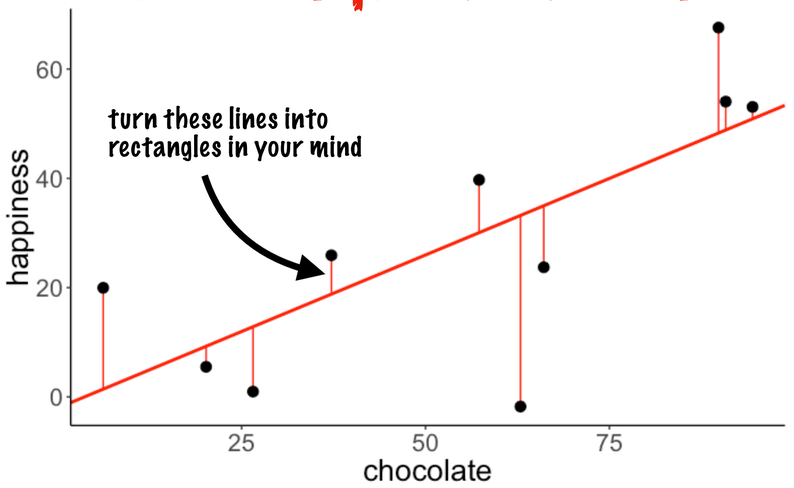
Both models were fit to minimize the sum of squared errors

OLS = Ordinary **least squares** regression

Sum of squared errors



Sum of squared errors



SSE(C) = 5215.016

SSE(A) = 2396.946

$$\text{PRE} = 1 - \frac{2396.946}{5215.016} \approx 0.54$$

The augmented model reduces the error by 54%.

The general procedure

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data
3. Calculate the proportional reduction of error (PRE) in our sample
4. **Decide whether the augmented model is worth it by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)**

Decide whether it's **worth it**

- To compute the *F* statistic, we need:
  - PRE
  - number of parameters in Model C (PC) and Model A (PA)
  - number of observations *n*

- more likely to be **worth it** if:
  1. **PRE** is high
  2. the number of additional parameters in A compared to C is low (**PRE per additional parameter**)
  3. the number of parameters that could have been added to $model_C$ to create $model_A$ but were not

difference in parameters between models A and C

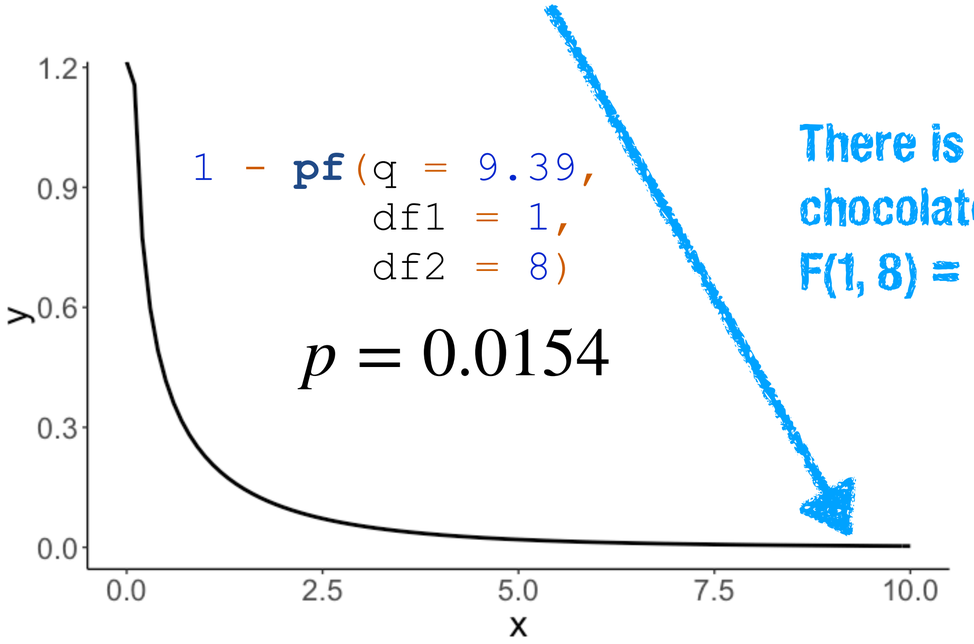
$$F = \frac{\text{PRE}/(\text{PA} - \text{PC})}{(1 - \text{PRE})/(n - \text{PA})}$$

number of observations vs. parameters in Model

Decide whether it's **worth it**

- To compute the *F* statistic, we need:
  - PRE = 0.54
  - PC = 1
  - PA = 2
  - n = 10

$$F = \frac{\text{PRE}/(\text{PA} - \text{PC})}{(1 - \text{PRE})/(n - \text{PA})}$$
$$= \frac{0.54/(2 - 1)}{(1 - 0.54)/(10 - 2)}$$
$$= 9.39$$



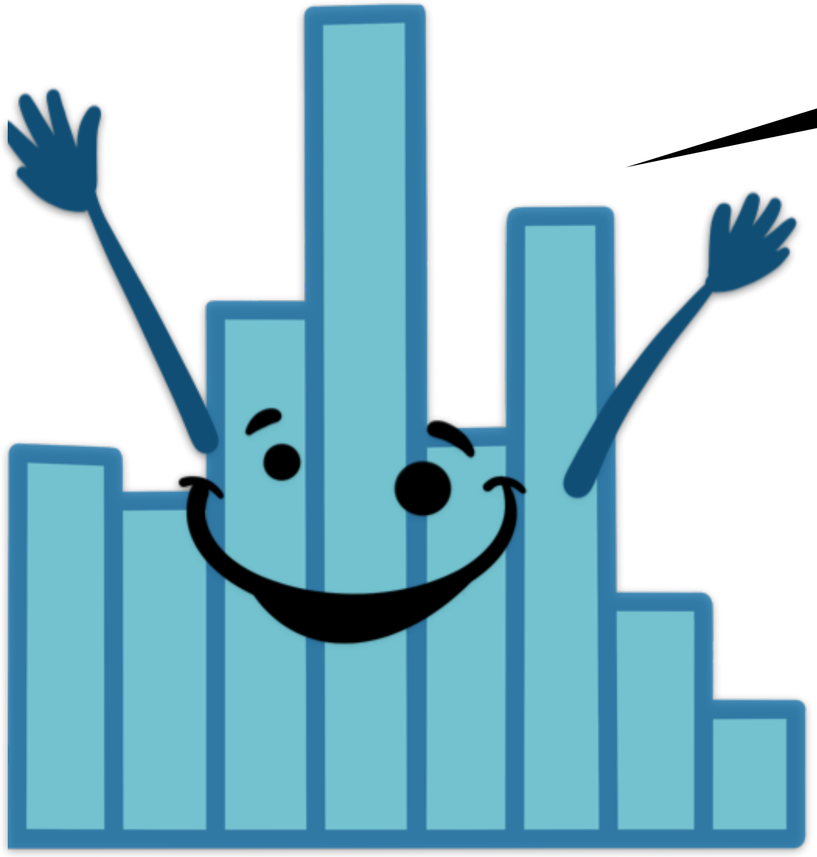

```
1 - pf(q = 9.39,
       df1 = 1,
       df2 = 8)
```

$$p = 0.0154$$

There is a significant relationship between chocolate consumption and happiness F(1, 8) = 9.39, p = .0154.

02:00

stretch break!



The R route

Credit card debt

Credit data set

`df.credit`

index	income	limit	rating	cards	age	education	gender	student	married	ethnicity	balance
1	14.89	3606	283	2	34	11	Male	No	Yes	Caucasian	333
2	106.03	6645	483	3	82	15	Female	Yes	Yes	Asian	903
3	104.59	7075	514	4	71	11	Male	No	No	Asian	580
4	148.92	9504	681	3	36	11	Female	No	No	Asian	964
5	55.88	4897	357	2	68	16	Male	No	Yes	Caucasian	331
6	80.18	8047	569	4	77	10	Male	No	No	Caucasian	1151
7	21.00	3388	259	2	37	12	Female	No	No	African American	203
8	71.41	7114	512	2	87	9	Male	No	No	Asian	872
9	15.12	3300	266	5	66	13	Female	No	No	Caucasian	279
10	71.06	6819	491	3	41	19	Female	Yes	Yes	African American	1350

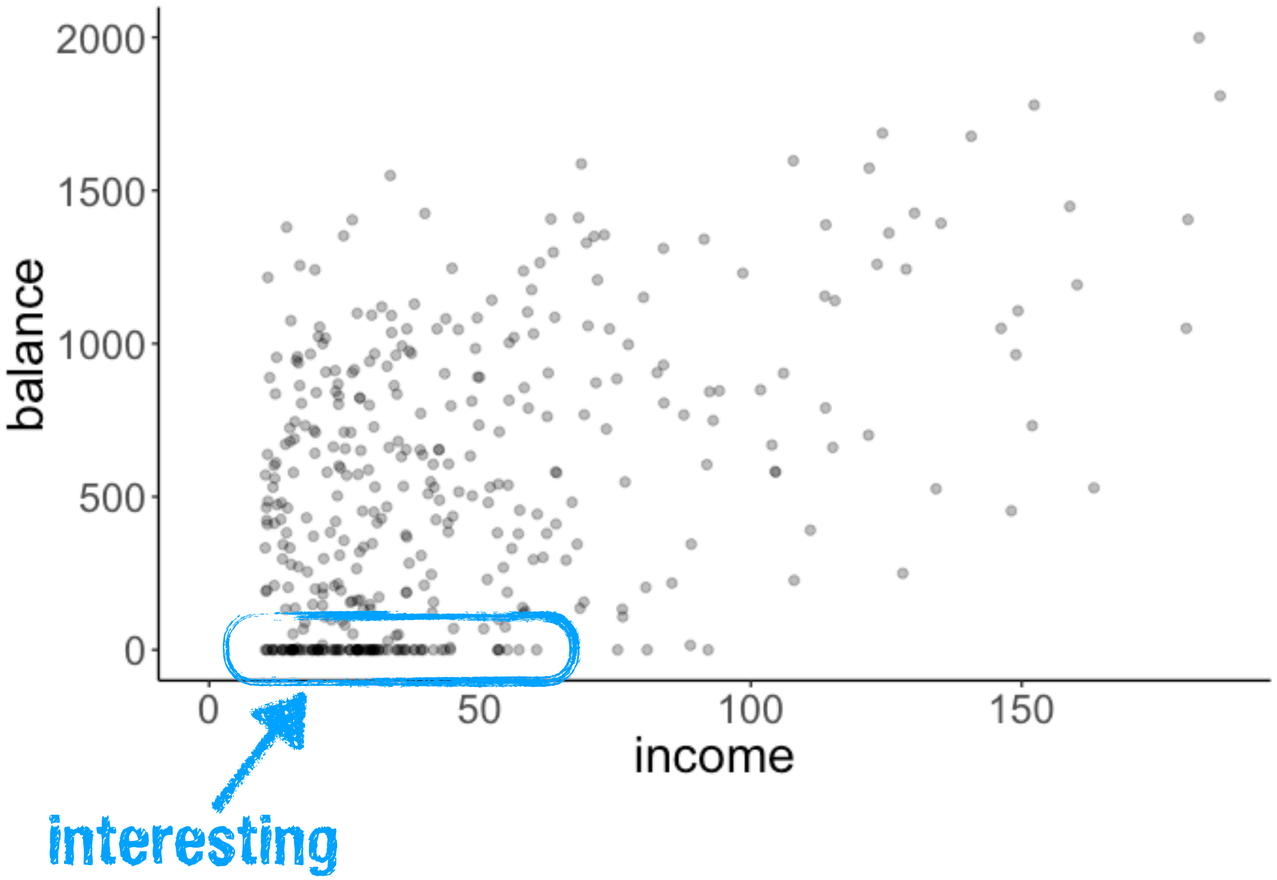
variable	description
income	in thousand dollars
limit	credit limit
rating	credit rating
cards	number of credit cards
age	in years
education	years of education
gender	male or female
student	student or not
married	married or not
ethnicity	African American, Asian, Caucasian
balance	average credit card debt in dollars

```
nrow(df.credit) = 400
```

Is there a relationship between income and the average credit card debt?

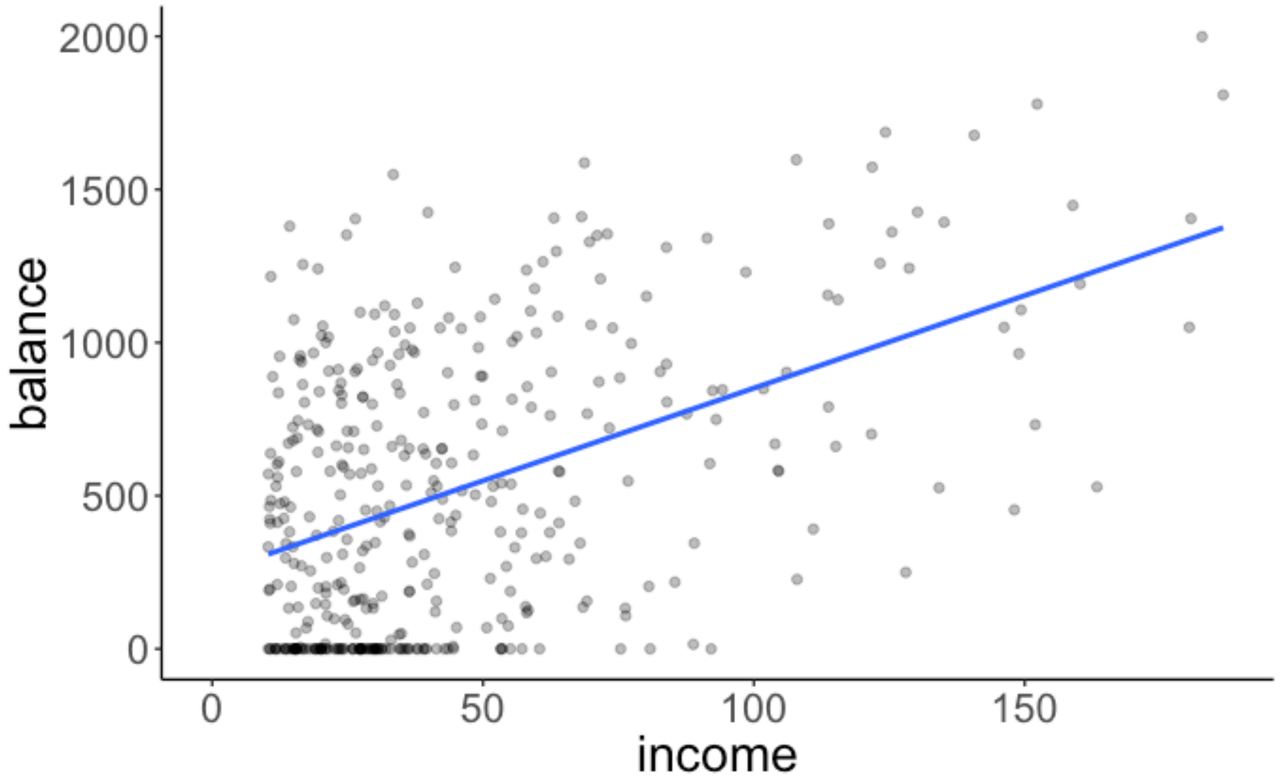
Always plot the data first ...

```
1 ggplot(data = df.credit,
2        mapping = aes(x = income,
3                      y = balance)) +
4   geom_point(alpha = 0.3)
```



Always plot the data first ...

```
1 ggplot(data = df.credit,
2        mapping = aes(x = income,
3                      y = balance)) +
4   geom_point(alpha = 0.3) +
5   geom_smooth(method = "lm", se = F)
```



Linear model

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\text{balance}_i = b_0 + b_1 \cdot \text{income}_i + e_i$$

```
fit = lm(formula = balance ~ 1 + income, data = df.credit)
```

outcome

intercept

predictor

data

(doesn't need to be specified explicitly, but please **do it anyhow**!)

lm()

```
fit = lm(balance ~ 1 + income, data = df.credit)

print(fit)
```

```
Call:
lm(formula = balance ~ 1 + income, data = df.credit)

Coefficients:
(Intercept)       income
    246.515        6.048
```

parameter estimates → which minimize the sum of squared errors between model and data

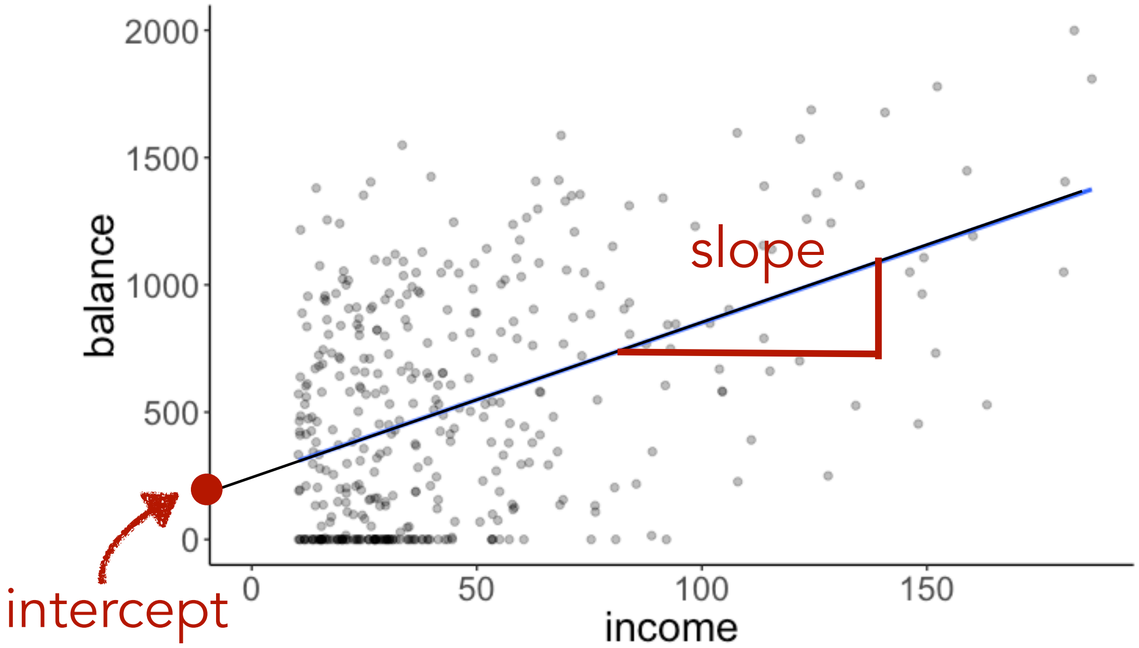
Interpreting regression parameters

```
Coefficients:
(Intercept)         income
    246.515          6.048
```

variable	description
income	in thousand dollars
balance	average credit card debt in dollars

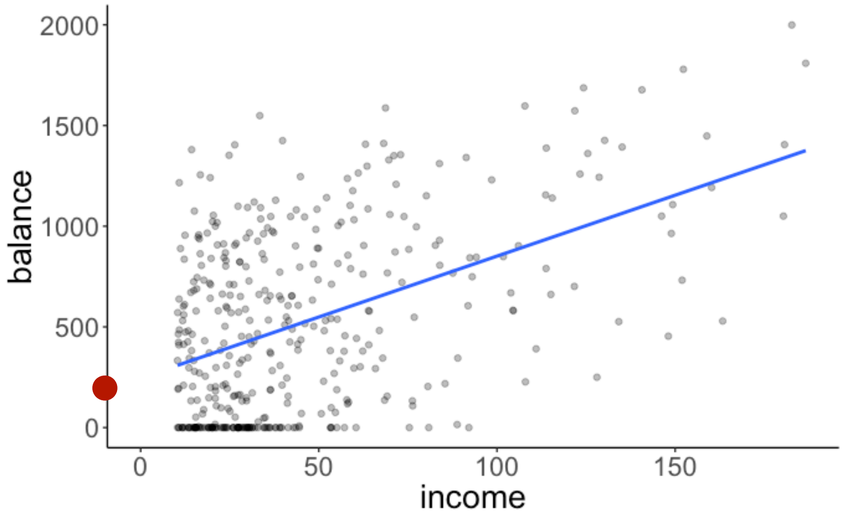
$$\text{balance}_i = b_0 + b_1 \cdot \text{income}_i + e_i$$

$$\text{balance}_i = 246.515 \cdot 1 + 6.048 \cdot \text{income}_i + e_i$$

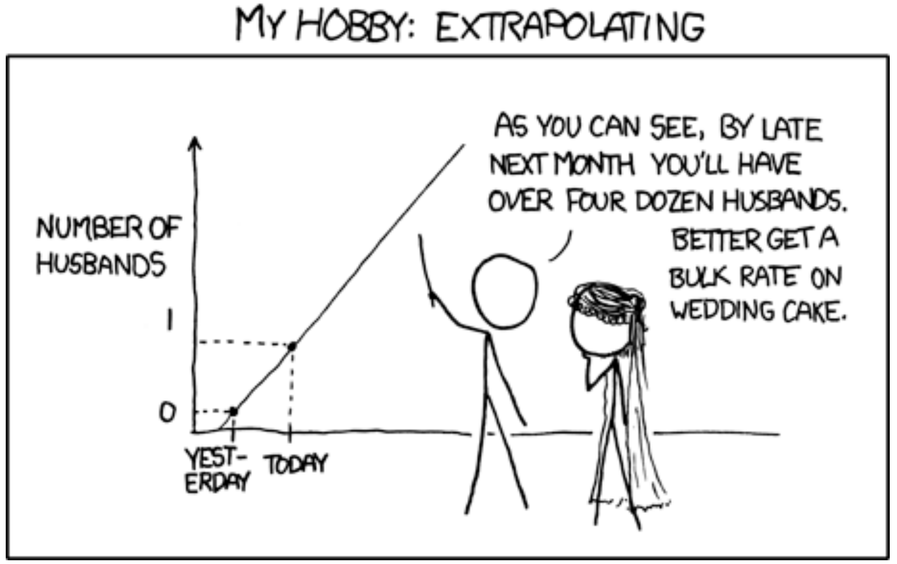


For each additional thousand dollars income, a person's average credit card is predicted to increase by $6.05.

Be careful about extrapolating predictions



- intercept is often outside the range of predictor values
- sometimes doesn't make sense (e.g. age = 0, height = 0, ...)



summary()

```
1 lm(balance ~ 1 + income, data = df.credit) %>%
2   summary()
```

```
Call:
lm(formula = balance ~ 1 + income, data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-803.64 -348.99  -54.42  331.75 1100.25

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 246.5148    33.1993   7.425  6.9e-13 ***
income        6.0484     0.5794  10.440  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 407.9 on 398 degrees of freedom
Multiple R-squared:  0.215,  Adjusted R-squared:  0.213
F-statistic:   109 on 1 and 398 DF,  p-value: < 2.2e-16
```

`library("broom")`

helps with tidying up model objects in R

`augment()` adds columns to the original data such as predictions, residuals and cluster assignments

`tidy()` summarizes a model's statistical findings such as coefficients of a regression

`glance()` provides a one-row summary of model-level statistics


broom: turn messy model outputs into tidy TIBBLES!
nls
glm
lm
ENTER
t.test
broom
EXIT
@allison_horst

augment()

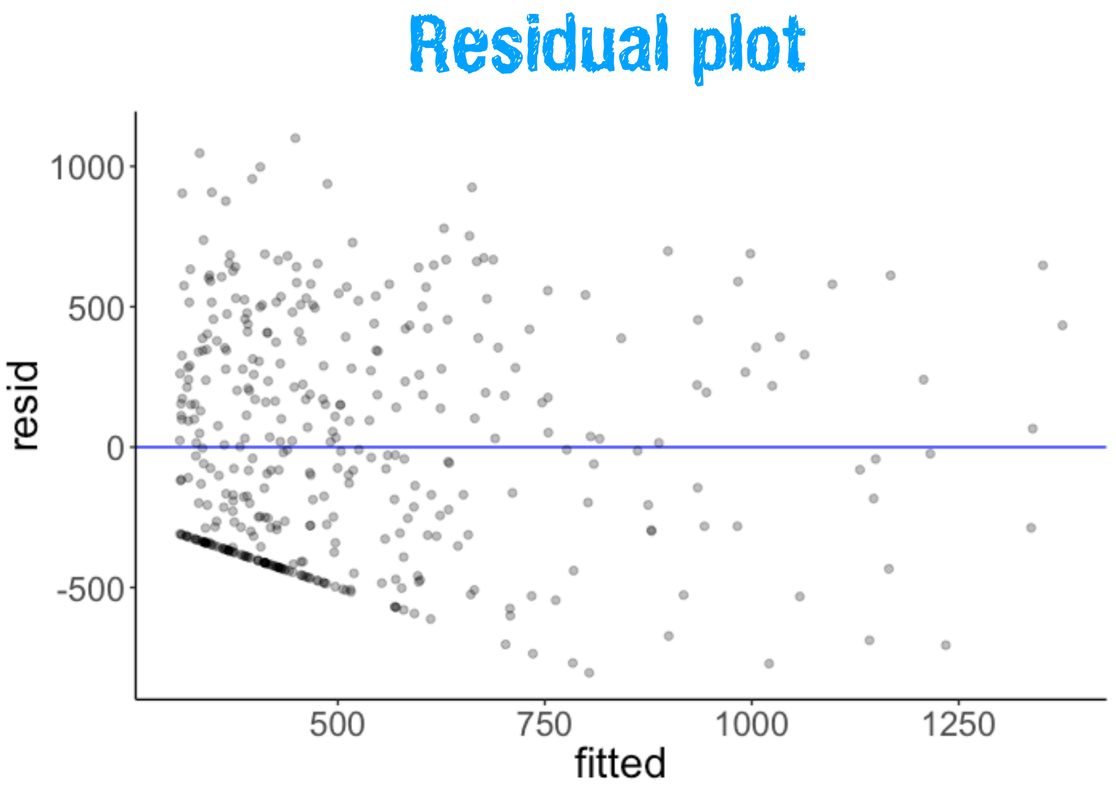
```
fit %>%
  augment()
```

balance	income	.fitted	.se.fit	.resid	.hat	.sigma	.cooksd	.std.resid
333	14.89	336.58	26.92	-3.58	0.00	408.38	0.00	-0.01
903	106.03	887.79	40.71	15.21	0.01	408.38	0.00	0.04
580	104.59	879.13	39.99	-299.13	0.01	408.10	0.00	-0.74
964	148.92	1147.26	63.45	-183.26	0.02	408.27	0.00	-0.45
331	55.88	584.51	21.31	-253.51	0.00	408.18	0.00	-0.62
1151	80.18	731.47	28.74	419.53	0.00	407.83	0.00	1.03
203	21.00	373.51	24.76	-170.51	0.00	408.29	0.00	-0.42
872	71.41	678.42	25.42	193.58	0.00	408.26	0.00	0.48
279	15.12	338.00	26.83	-59.00	0.00	408.37	0.00	-0.14
1350	71.06	676.32	25.30	673.68	0.00	406.97	0.01	1.65

augment()

```
fit %>%
   augment()
```

balance	income	.fitted	.resid
333	14.89	336.58	-3.58
903	106.03	887.79	15.21
580	104.59	879.13	-299.13
964	148.92	1147.26	-183.26
331	55.88	584.51	-253.51
1151	80.18	731.47	419.53
203	21.00	373.51	-170.51
872	71.41	678.42	193.58
279	15.12	338.00	-59.00
1350	71.06	676.32	673.68



tidy()

```
1 lm(balance ~ 1 + income, data = df.credit) %>%
2   summary()
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 246.5148    33.1993   7.425  6.9e-13 ***
income        6.0484     0.5794  10.440  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
1 fit %>%
2   tidy(conf.int = TRUE)
```

a data frame, yay!

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	246.51	33.20	7.43	0	181.25	311.78
income	6.05	0.58	10.44	0	4.91	7.19

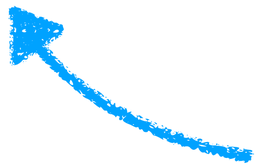
glance()

```
1 lm(balance ~ 1 + income, data = df.credit) %>%
2   summary()
```

```
Residual standard error: 407.9 on 398 degrees of freedom
Multiple R-squared:  0.215,  Adjusted R-squared:  0.213
F-statistic:   109 on 1 and 398 DF,  p-value: < 2.2e-16
```

```
1 fit %>%
2   glance()
```

r.squared	adj.r.squared	sigma	statistic	p.value	df	logLik	AIC	BIC	deviance	df.residual
0.21	0.21	407.86	108.99	0	2	-2970.95	5947.89	5959.87	66208745	398



useful model summary
(we will learn later what
the different values mean)

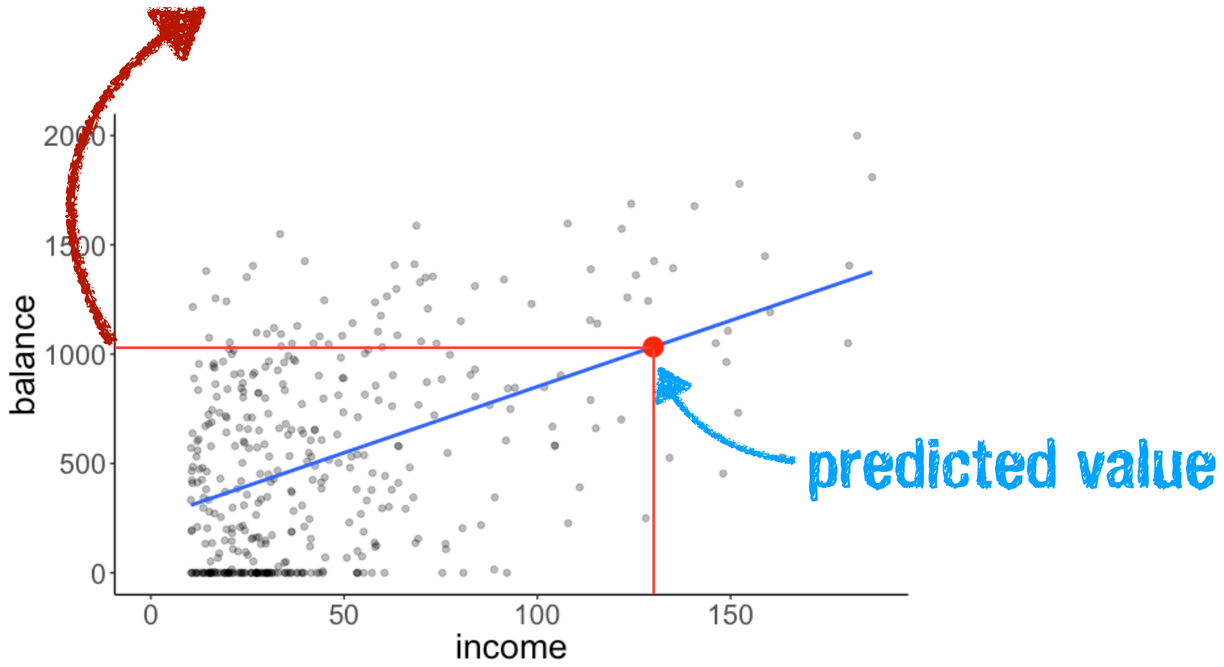
Making predictions

```
fit = lm(balance ~ 1 + income, data = df.credit)
```

$$\text{balance}_i = 246.515 + 6.048 \cdot \text{income}_i + e_i$$

```
augment(fit, newdata = tibble(income = 130))
```

$$\widehat{\text{balance}} = 246.515 + 6.048 \cdot 130$$



Hypothesis test

Compact Model

$$\text{balance}_i = \beta_0 + \epsilon_i$$

```
fit_c = lm(formula = balance ~ 1,
           data = df.credit)
```

Augmented Model

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \epsilon_i$$

```
fit_a = lm(formula = balance ~ 1 + income,
           data = df.credit)
```

```
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ 1 + income
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    399 84339912
2    398 66208745  1  18131167 108.99 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The general procedure

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data

```
fit_c = lm(formula = balance ~ 1,
           data = df.credit)

fit_a = lm(formula = balance ~ 1 + income,
           data = df.credit)
```

---

3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

```
anova(fit_c, fit_a)
```

Hypothesis test

```
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ 1 + income
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    399 84339912
2    398 66208745  1  18131167 108.99 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

$$\text{PRE} = 1 - \frac{66208745}{84339912} \approx 0.215$$

The augmented model reduces the error by 21.5%.

```
lm(balance ~ 1 + income, data = df.credit) %>%
  summary()
```

R^2

```
Residual standard error: 407.9 on 398 degrees of freedom
Multiple R-squared:  0.215,    Adjusted R-squared:  0.213
F-statistic:   109 on 1 and 398 DF,  p-value: < 2.2e-16
```

Hypothesis test

the **compact model** predicts the mean (which doesn't explain any of the variance)

- in the case of a simple regression PRE (proportion of reduced error) is identical to R^2 (variance explained)
- and R^2 is directly related to the correlation coefficient *r*

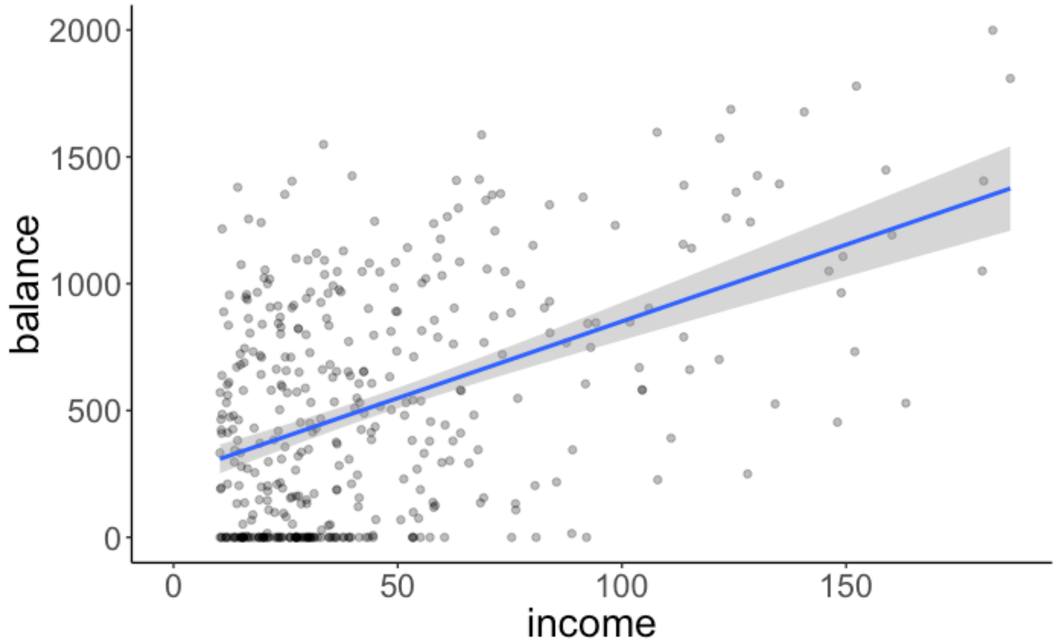
```
cor(df.credit$balance,
    df.credit$income)
```

$$R^2 = 0.215$$

$$r = .463$$

effect size measure

Reporting the results

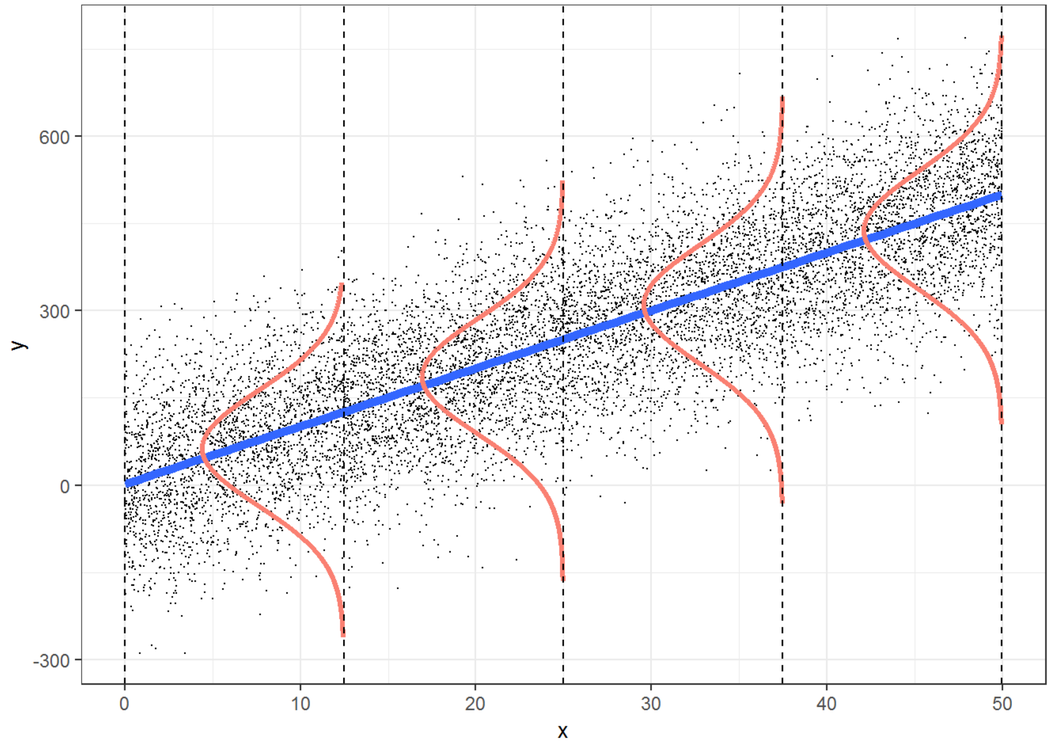


There is a significant relationship between a person's income and the average balance on their credit cards $F(1, 389) = 108.99$, $p < .001$, $r = .463$.

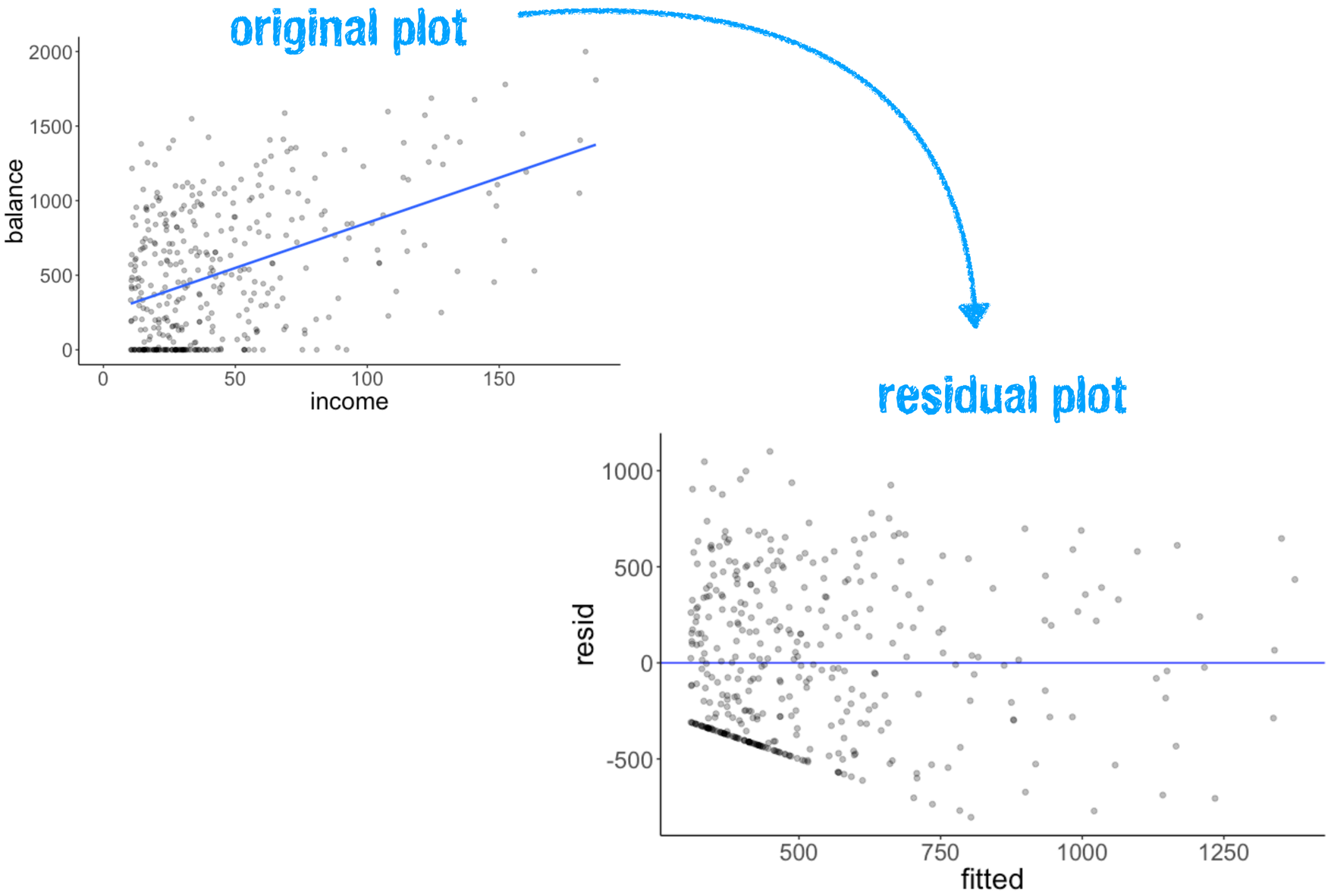
With each additional $1000 of income, the average balance is predicted to increase by $6.05 [4.91, 7.19] (95% CI).

Model assumptions

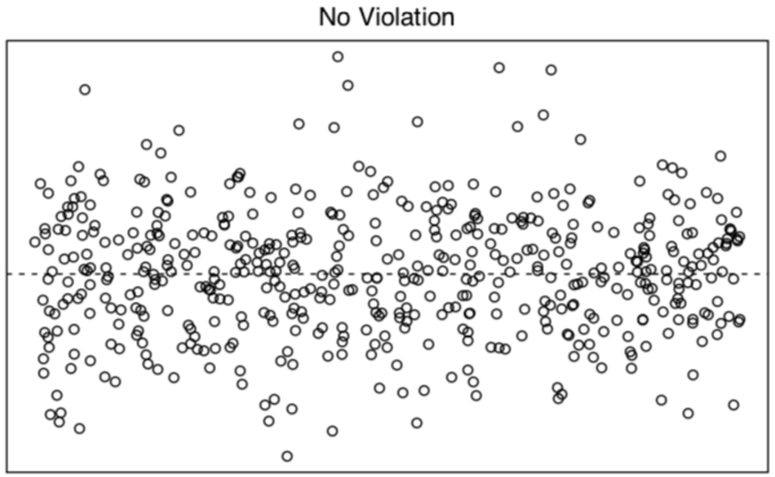
- independent observations
- *Y* is continuous
- errors are normally distributed
- errors have constant variance
- error terms are uncorrelated

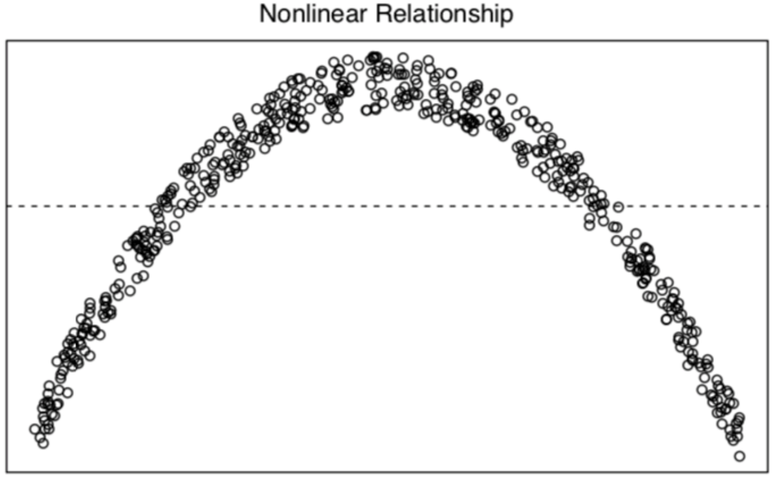


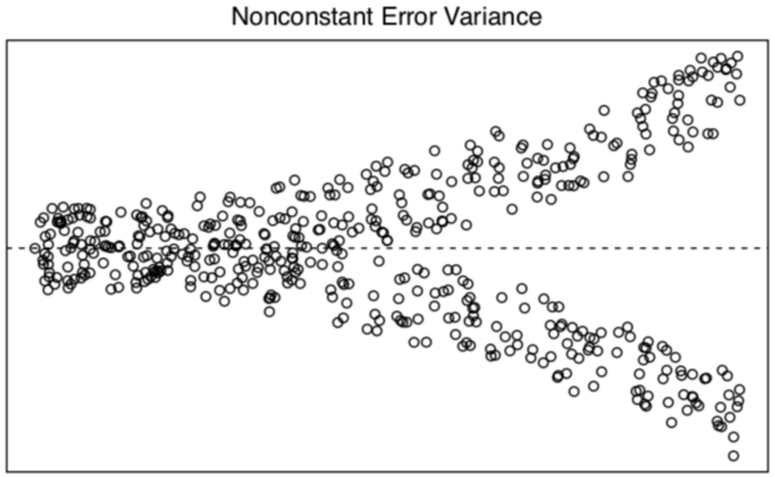
Model assumptions

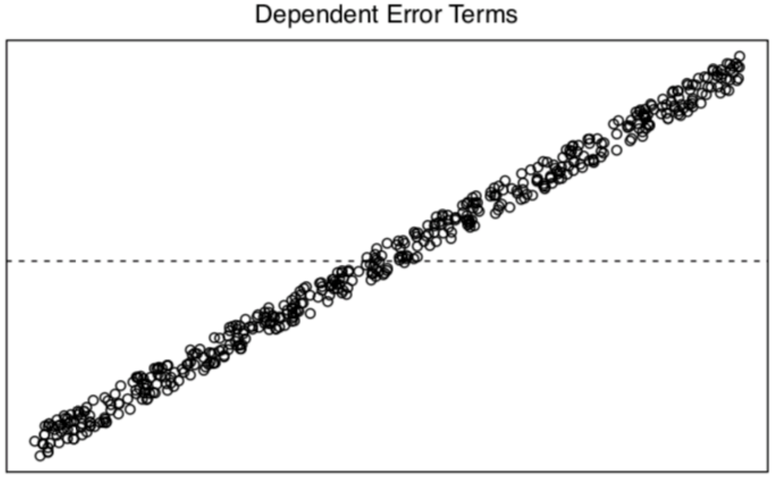


Model assumptions







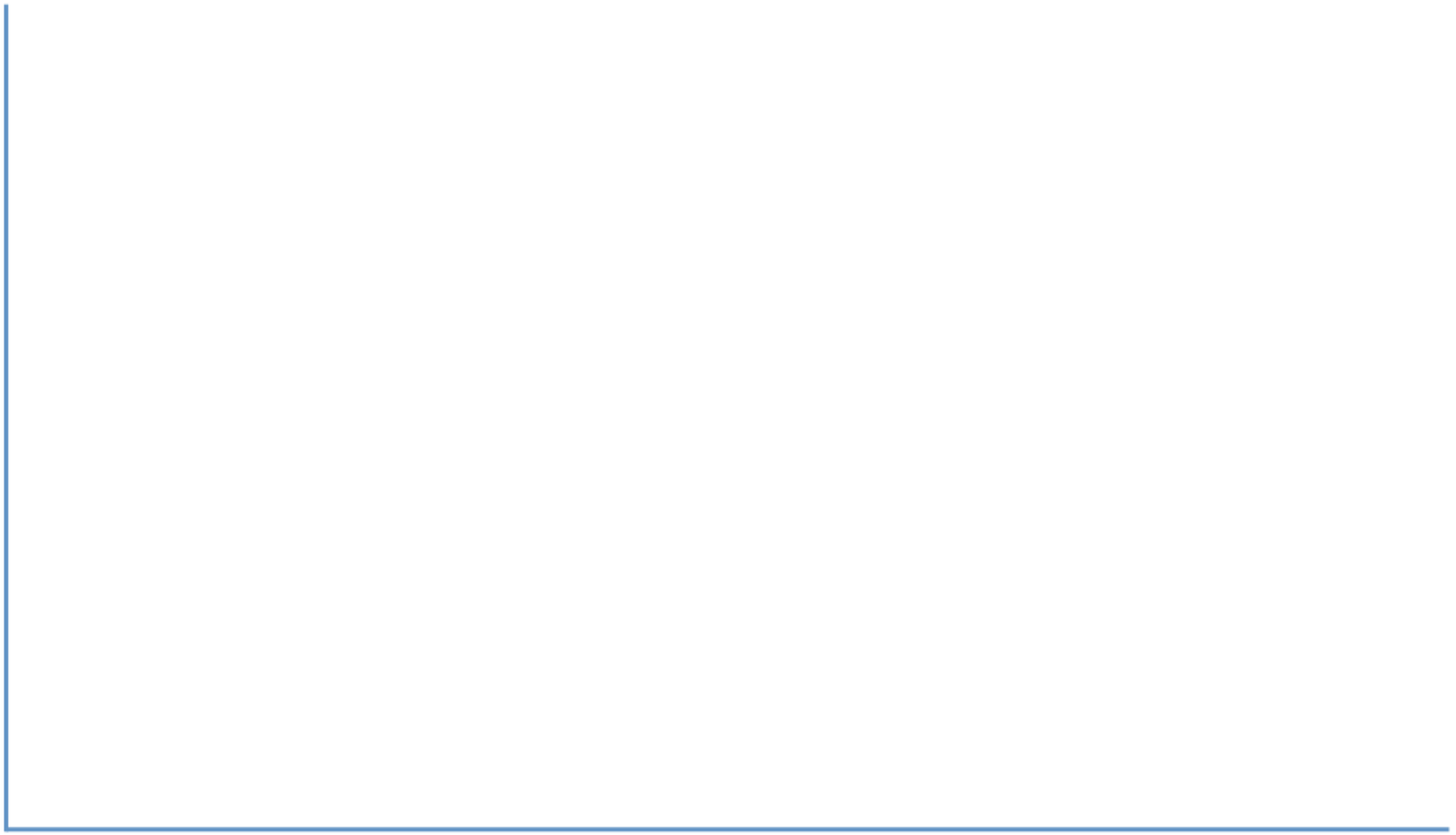


Summary

- Quick recap
- Generating a sampling distribution for PRE
- Correlation
- Regression
  - The conceptual tour
  - The R route

Feedback

How was the pace of today's class?



How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?

Thank you!