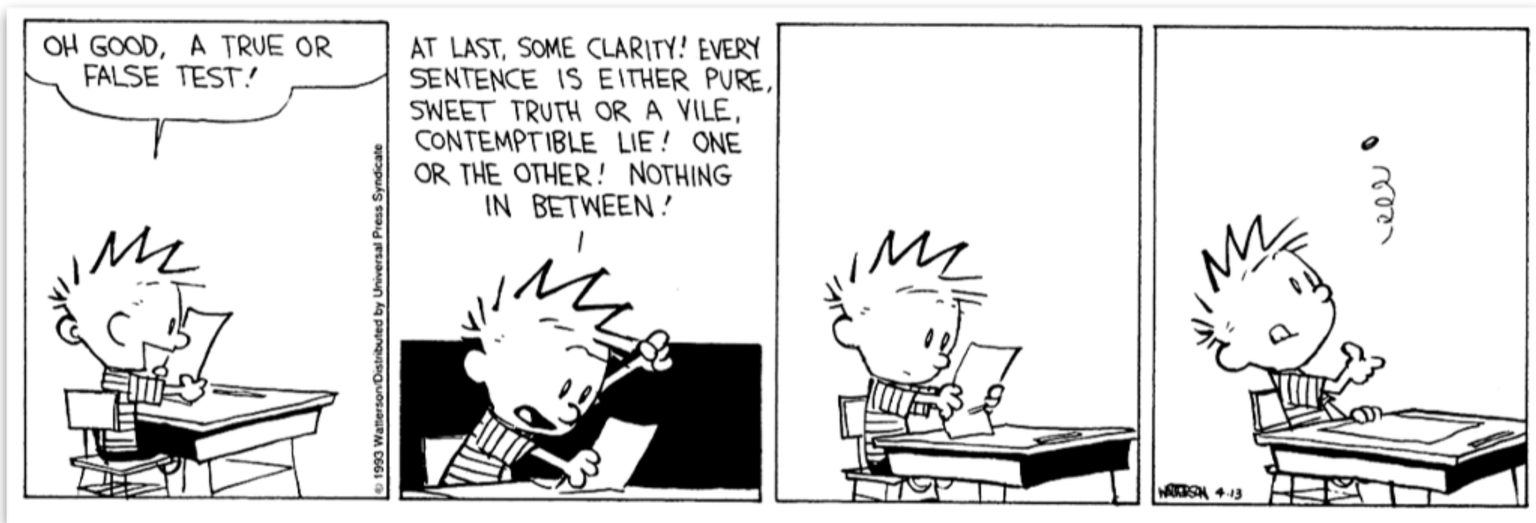
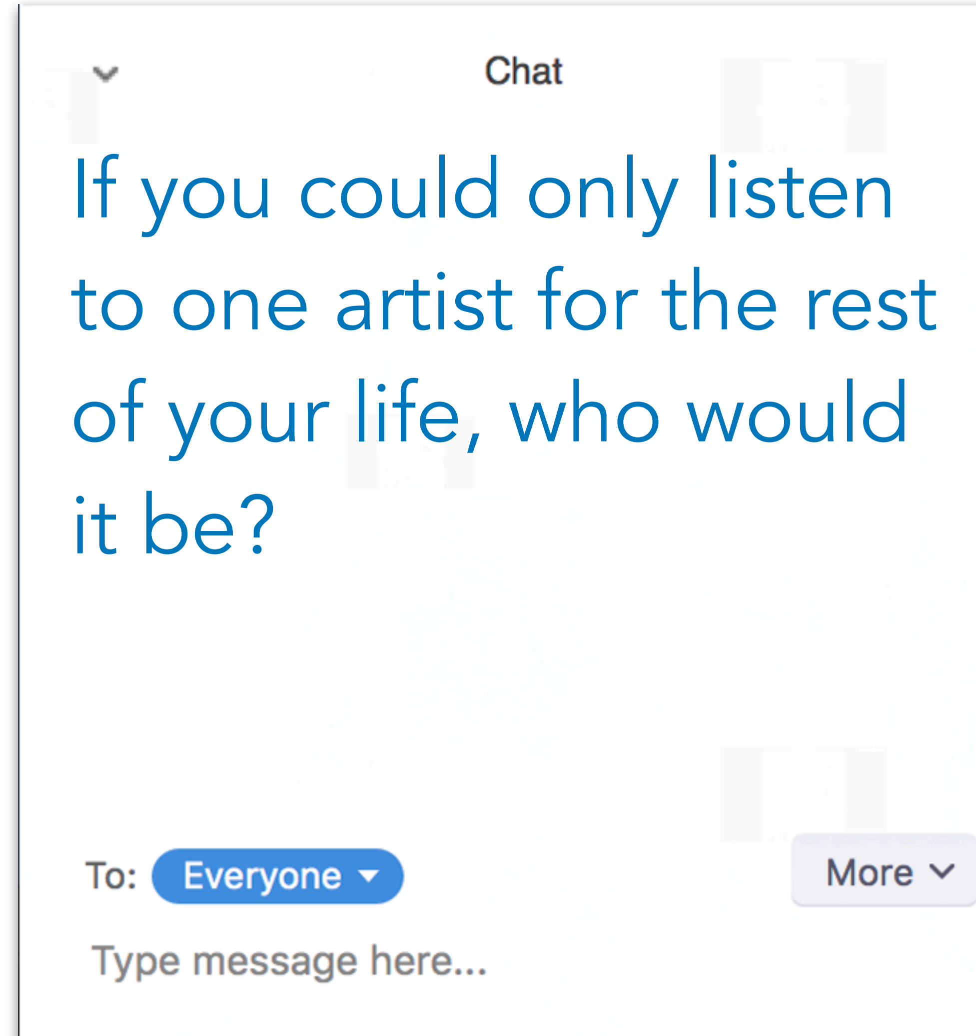
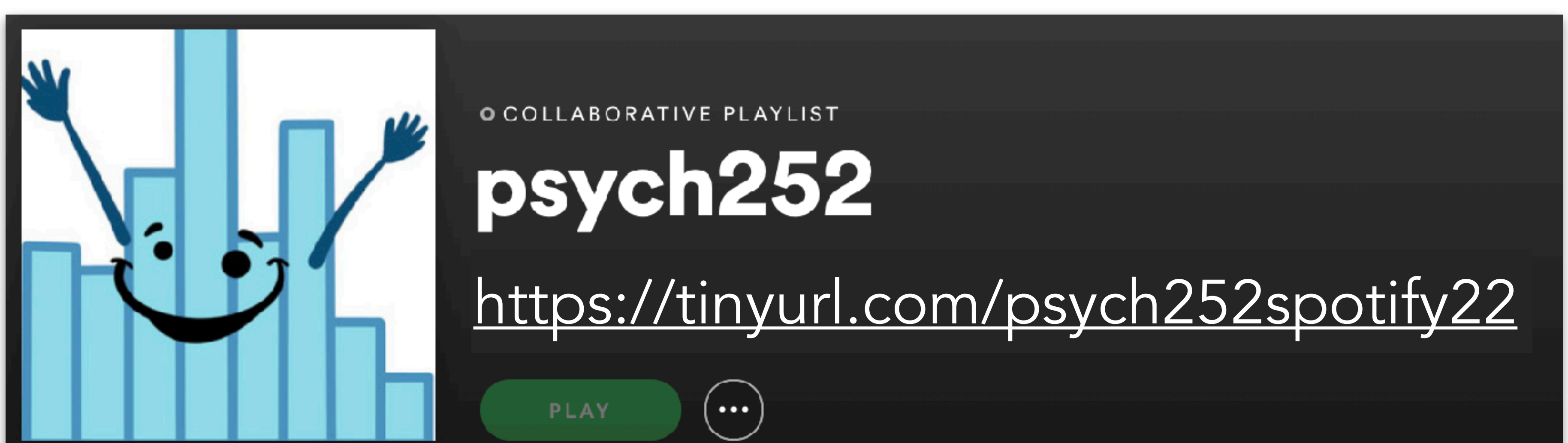
PSYCH 252: Statistical Methods for Behavioral and Social Sciences

Linear mixed effects models 1





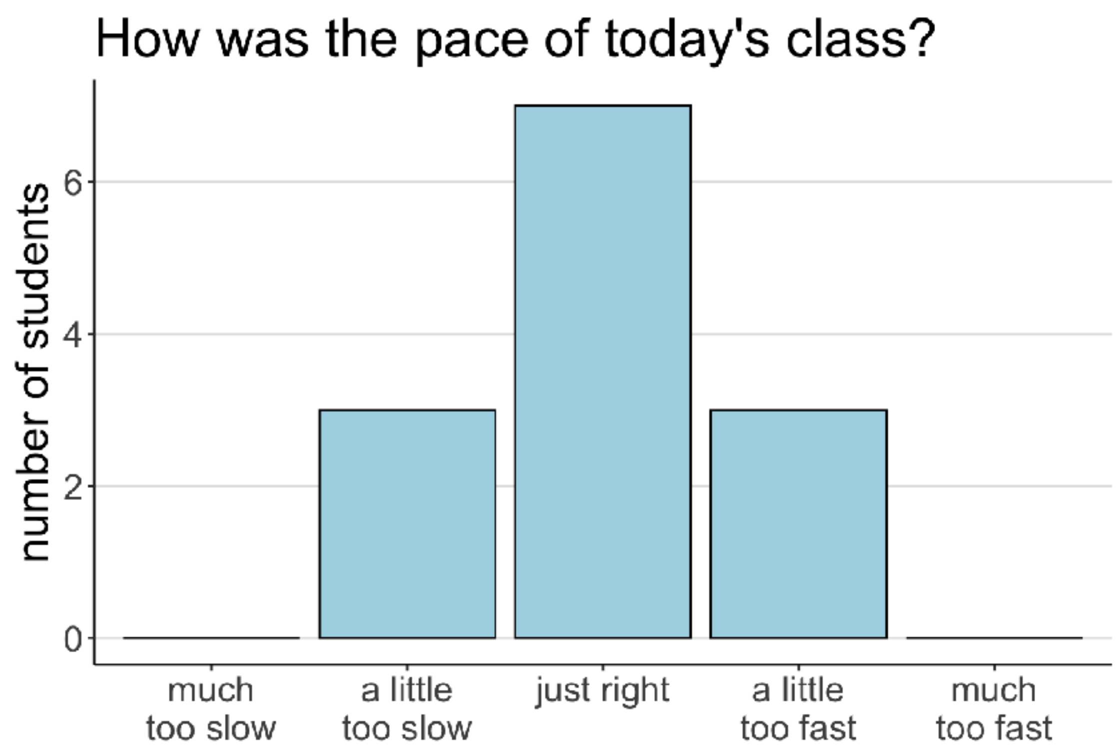
We're listening to "It runs through me" by "Tom Misch, De La Soul" submitted by Thing Thinker

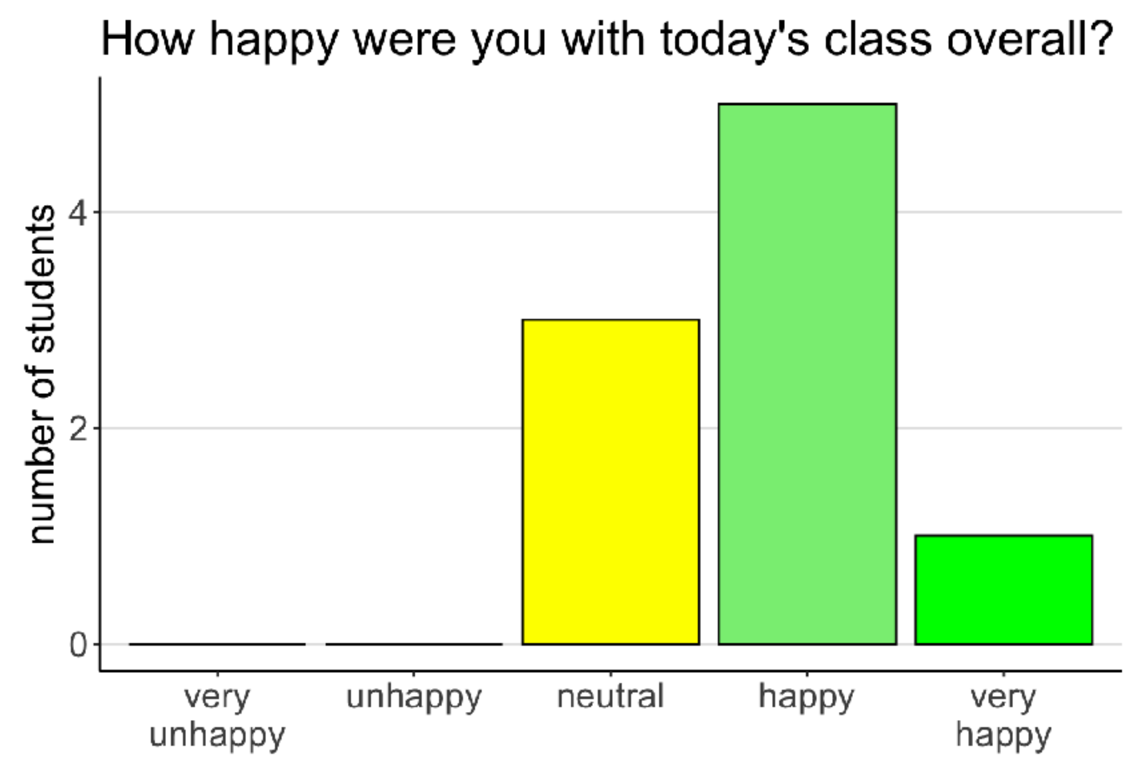


02/14/2022

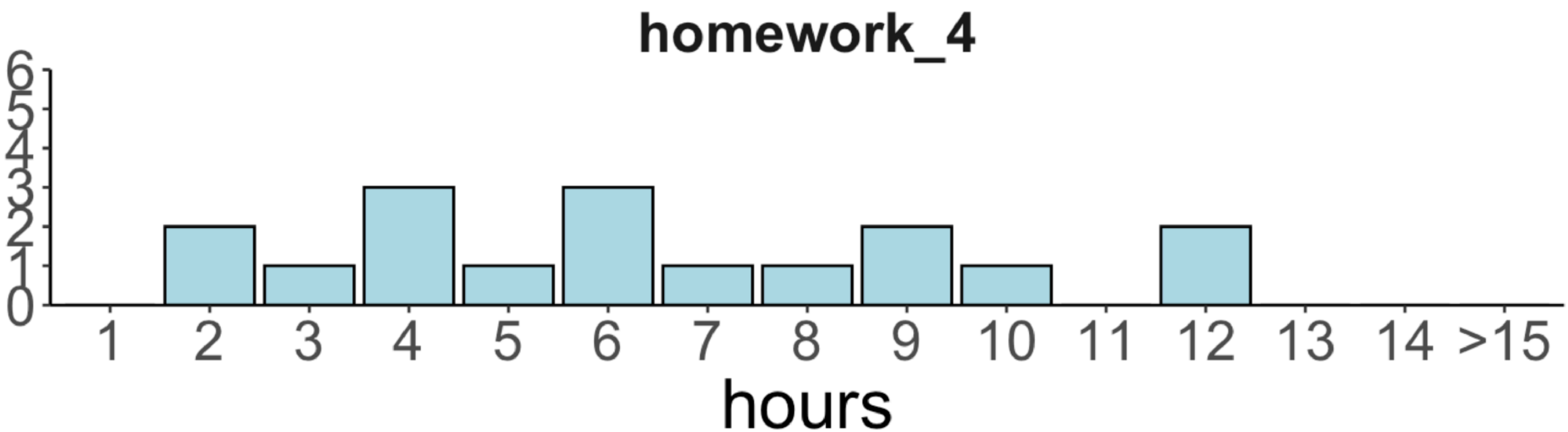
Your feedback

Your feedback





Homework 4



Plan for today

- Quick recap
- Observation, intervention, counterfactual
- Controlling for variables
  - Patterns of inference
  - Should I control?
- Mediation
- Moderation
- Linear mixed effects model
  - Modeling dependence in data

Quick recap

Quick recap: Model comparison

More complex models fit the data better.

We need to trade-off **model fit** and model **complexity**.

Quick recap: Model comparison

1. compare the proportional reduction in error using `anova()`
   - only works for nested models
2. cross-validation
   - works generally, but can take some time ...
   - different cross-validation procedures (LOO, k-fold, Monte Carlo)
3. AIC, BIC
   - works as long as we can compute the likelihood of the data
   - some discussion whether number of parameters is a good measure of model complexity
4. Bayesian model comparison

Quick recap: AIC and BIC

- AIC = Akaike Information Criterion
- BIC = Bayesian Information Criterion

not that much Bayesian about it ...

$$\text{AIC} = 2k - 2\ln(\hat{L})$$

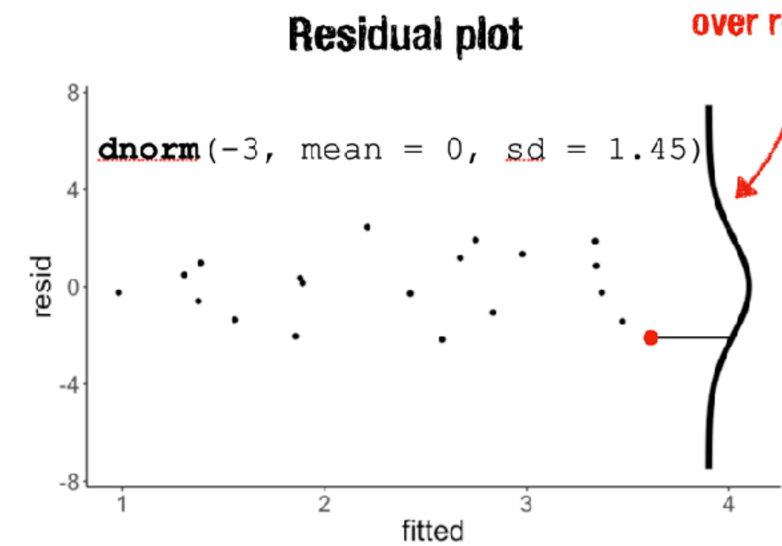
?

$$\text{BIC} = \ln(n)k - 2\ln(\hat{L})$$

$\hat{L}$ = maximized value of the likelihood function of the model
k = number of parameters in the model
n = number of observations

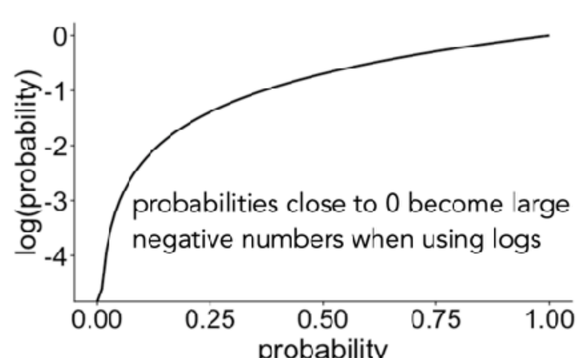
AIC and BIC

normal distribution over residuals



since the data points are independent, we can calculate the overall likelihood by multiplying the likelihood of each observation

`log()` is your friend!



multiplying probabilities

$$0.01 \cdot 0.01 \cdot 0.01 \cdot 0.01 = 0.00000001$$

number becomes extremely small quickly

take log()

`log(0.01)= -4.60517`

number becomes large but that's ok

summing logs

$$(-4.60517) + (-4.60517) + (-4.60517) + (-4.60517) = -18.42068$$

transform back into probability

`exp(-18.42068)= 0.00000001`

often not necessary since we just use logLikelihood

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AIC and BIC

`dnorm(1.88, mean = 0, sd = 1.45) = 0.12`

```r
# generate some data
df.like = tibble(
  x = runif(20, min = 0, max = 1),
  y = 1 + 3 * x + rnorm(20, sd = 2)
)

# fit the model
fit = lm(formula = y ~ x,
         data = df.like)

# model summary
fit %>%
  glance()
```

x	y	fitted	resid	likelihood
0.90	5.22	3.34	1.88	0.12
0.27	0.20	1.56	-1.36	0.18
0.37	-0.18	1.86	-2.04	0.10
0.57	2.14	2.42	-0.28	0.27
0.91	3.13	3.37	-0.24	0.27
0.20	0.78	1.38	-0.59	0.25
0.90	4.20	3.34	0.86	0.23
0.94	2.05	3.47	-1.42	0.17
0.66	3.85	2.67	1.18	0.20
0.63	0.41	2.58	-2.17	0.09

inferred standard deviation of the error

$$\sum_{i=1}^{n} \ln(\text{likelihood})$$

r.squared	adj.r.squared	sigma	statistic	p.value	df	logLik	AIC	BIC	deviance	df.residual
0.25	0.21	1.45	6.16	0.02	2	-34.74	75.47	78.46	37.77	18

$$e \sim \mathcal{N}(\text{mean} = 0, \text{sd} = 1.45)$$

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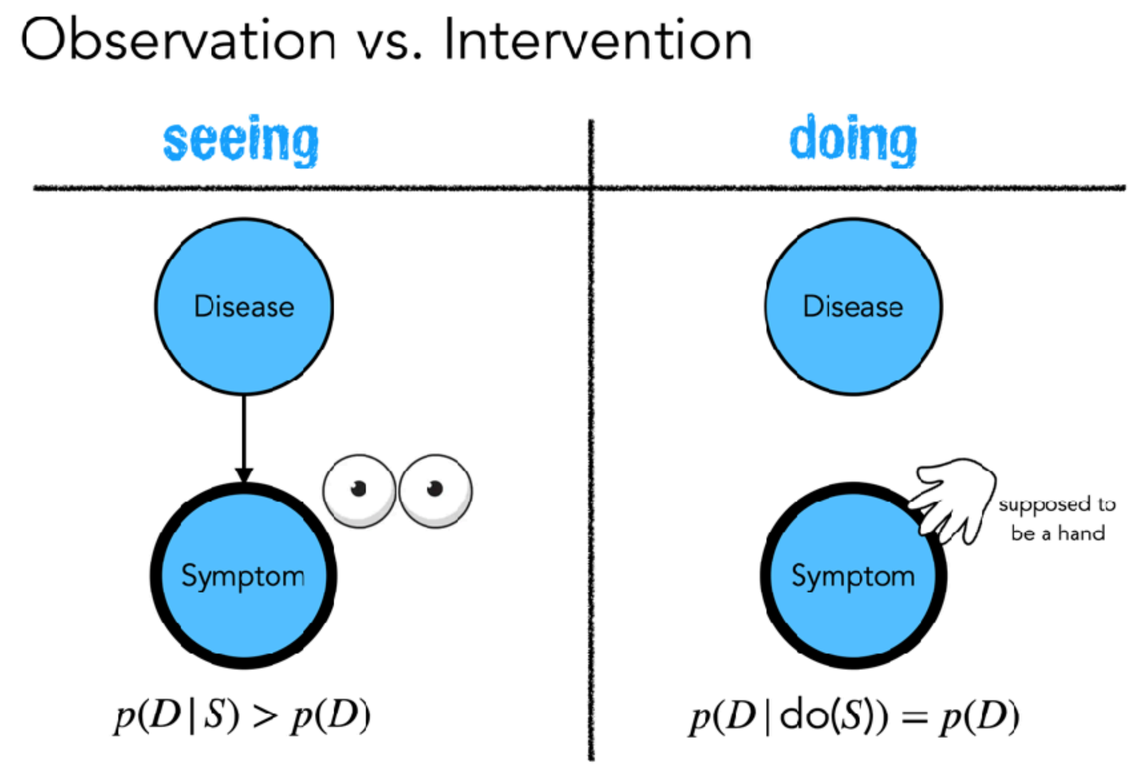
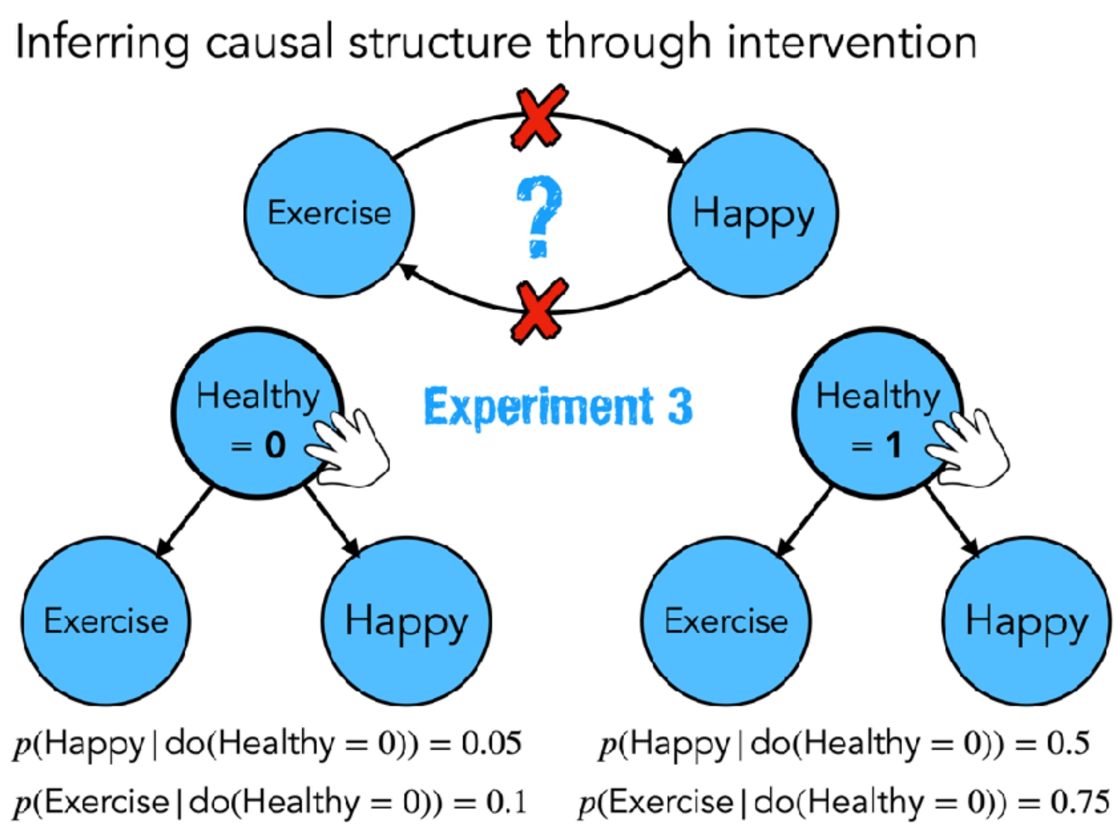
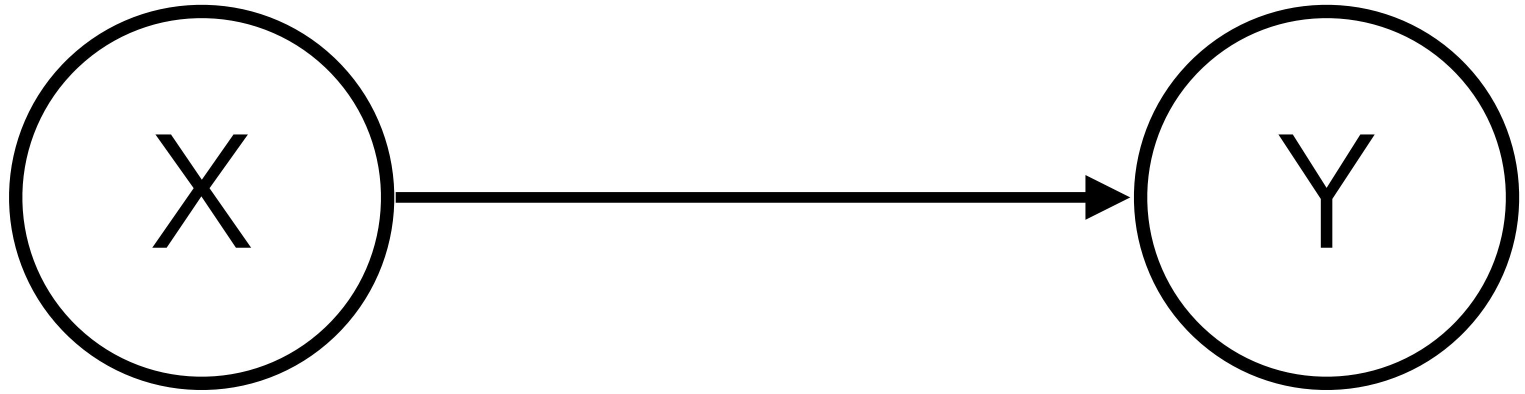
Quick recap: causation vs. correlation

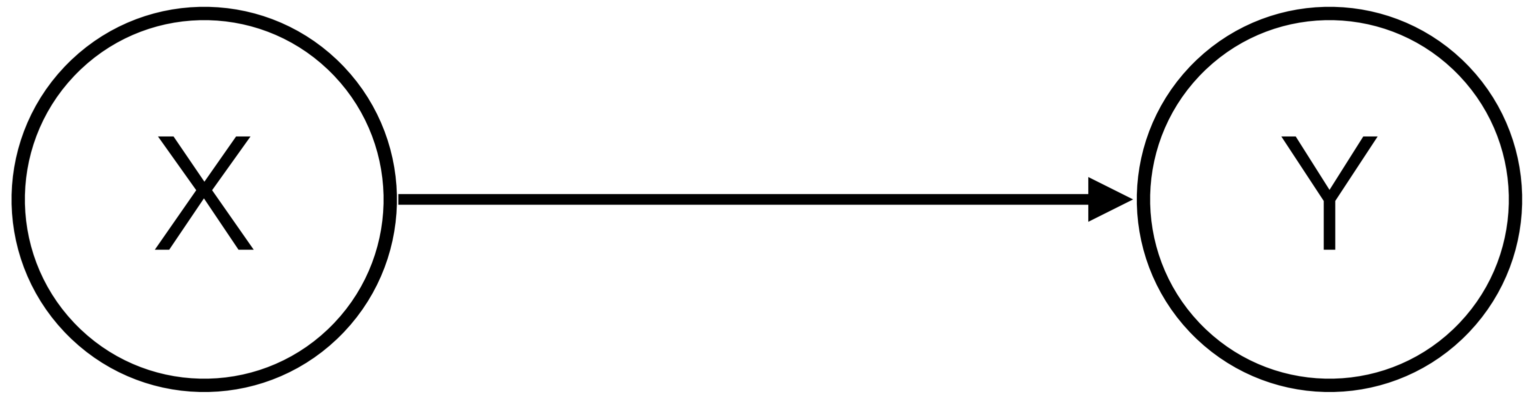




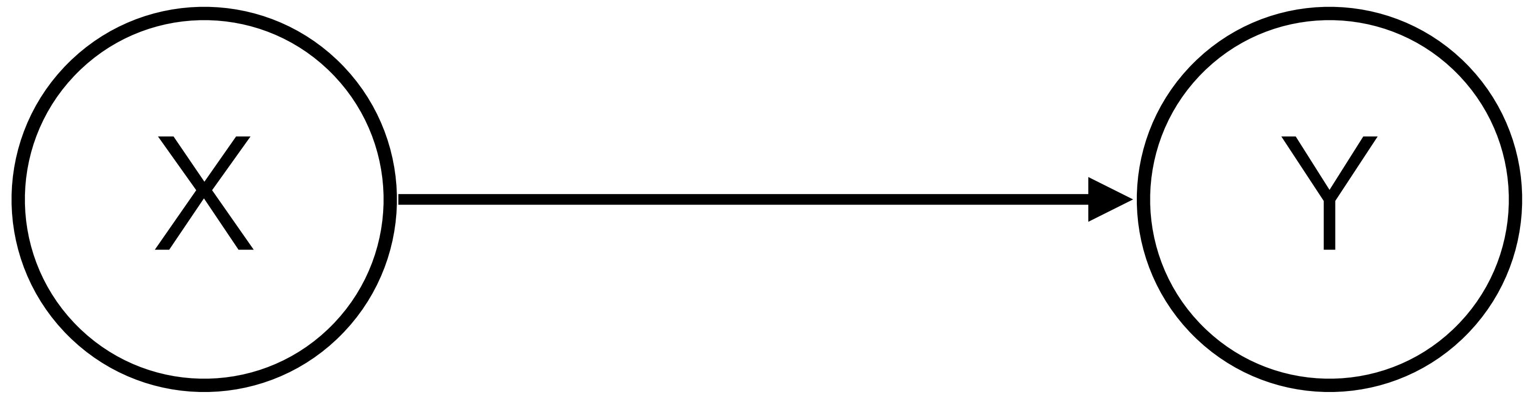
Observation, intervention, counterfactual



Level	Concept	Expression	Activity	Question	Example
I	Observation/ Prediction	$p(y\|x)$	Seeing	How does x change my belief in y?	Would the grass be wet if we *found* the sprinkler off?



Level	Concept	Expression	Activity	Question	Example
I	Observation/ Prediction	$p(y\|x)$	Seeing	How does x change my belief in y?	Would the grass be wet if we *found* the sprinkler off?
II	Intervention/ Hypothetical	$p(y\|\mathrm{do}(x))$	Doing	Would y happen if I did x?	Would the the grass be wet if *made sure* that the sprinkler was off?



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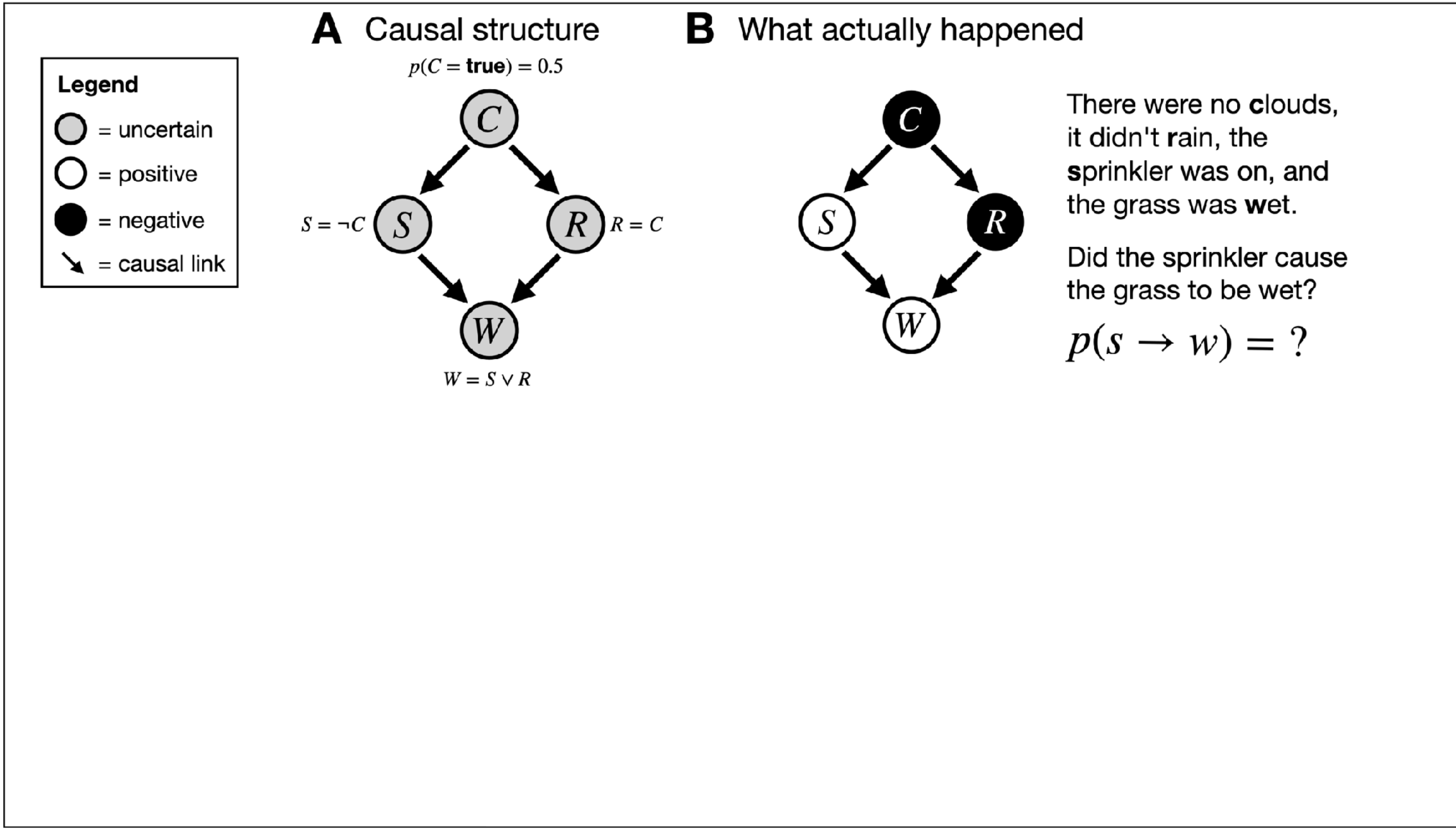
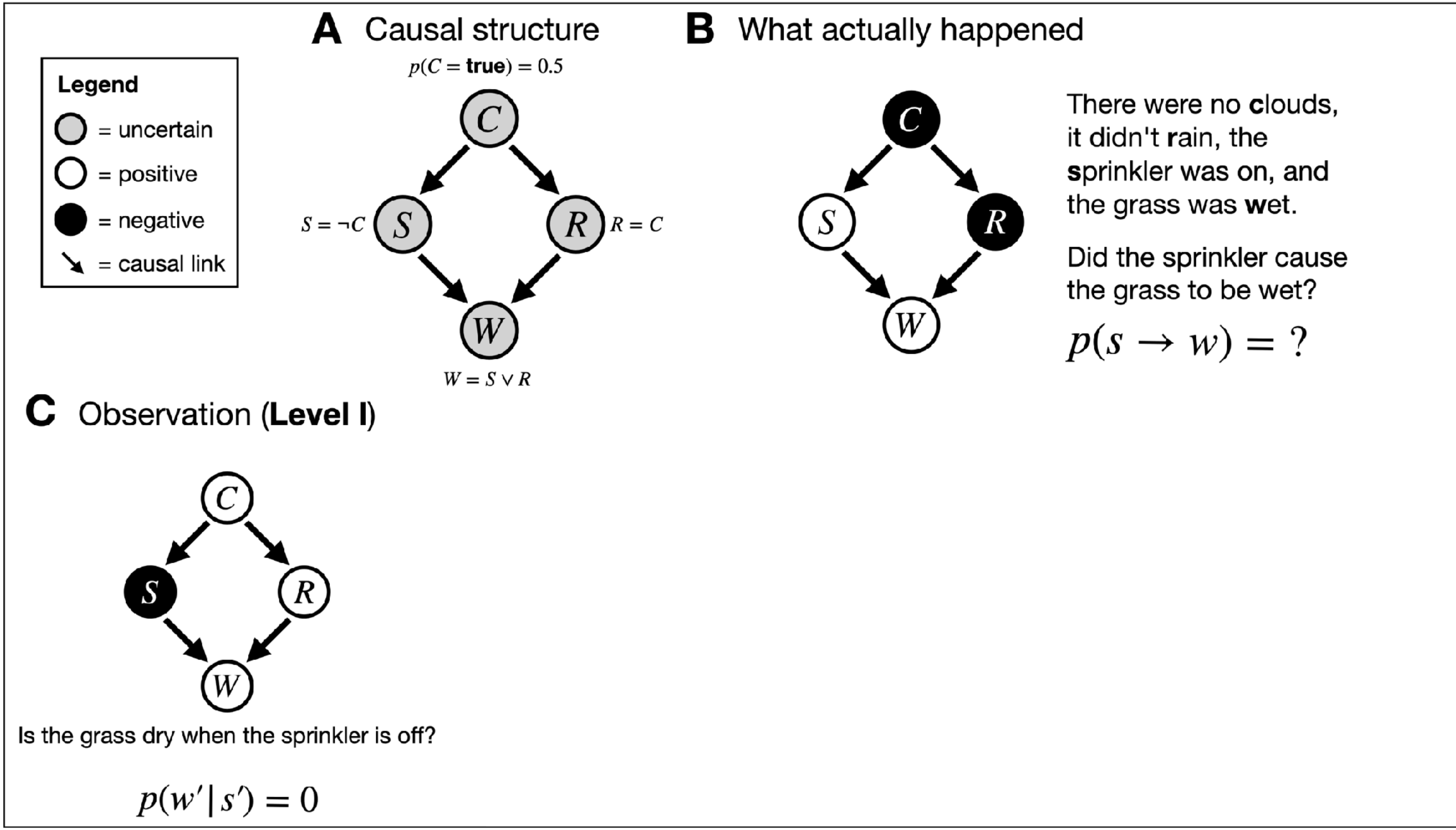
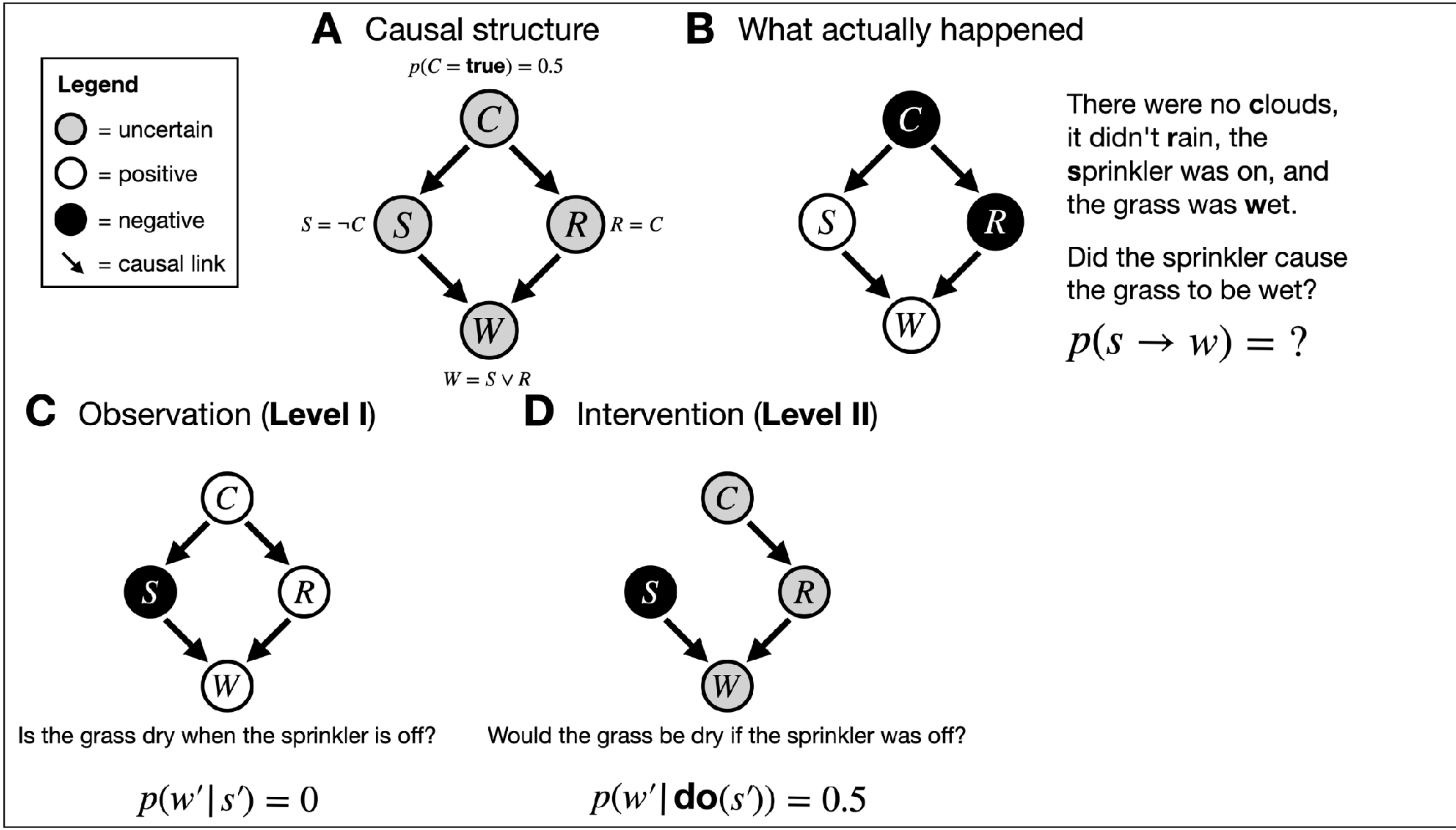
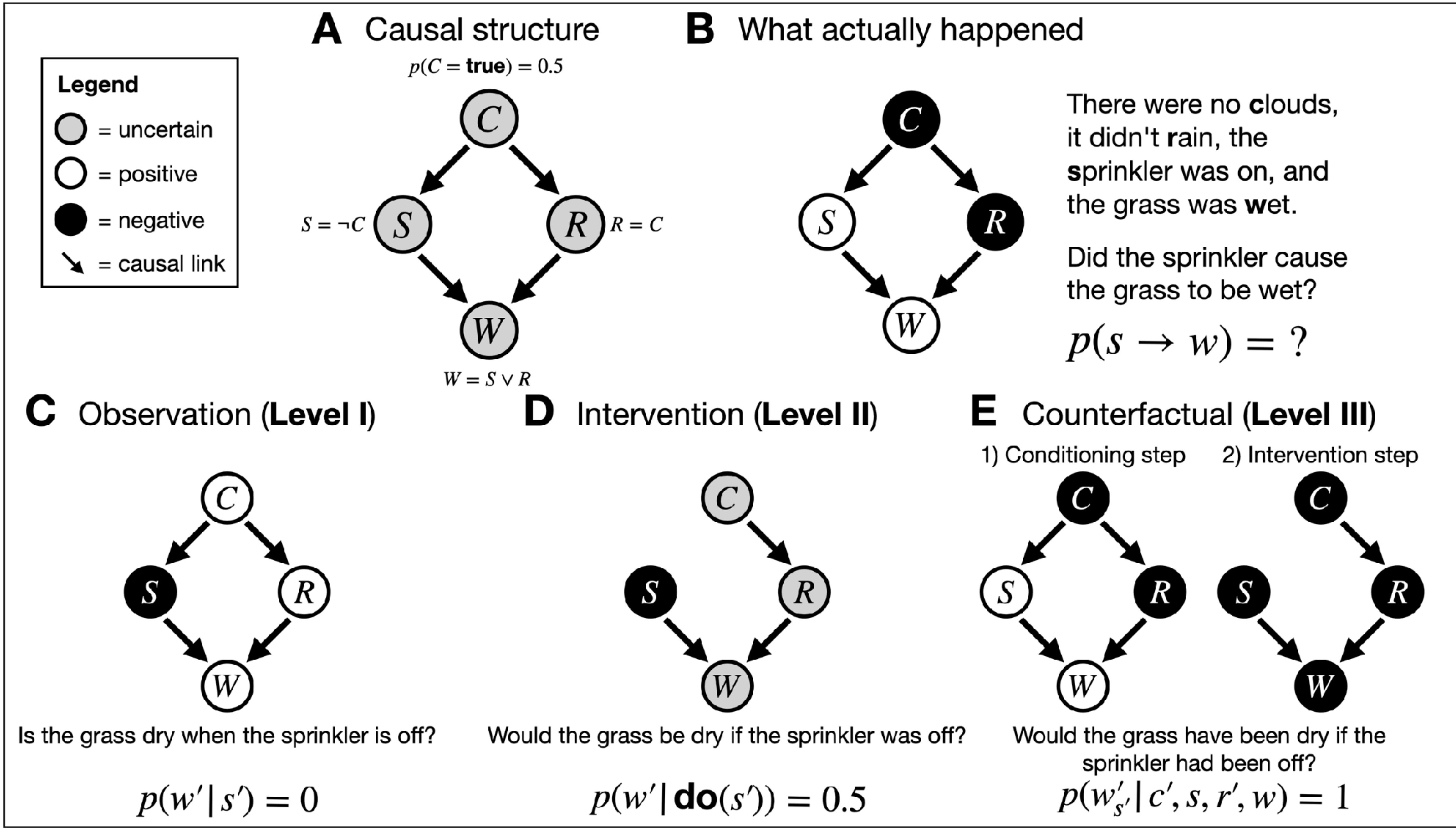
II	Intervention/ Hypothetical	$p(y\|\mathrm{do}(x))$	Doing	Would y happen if I did x?	Would the the grass be wet if *made sure* that the sprinkler was off?
III	Counterfactual	$p(y_x\|x', y')$	Explaining	Would y have happened instead of y', if I had done x instead of x'?	Would the grass have been wet if we *had made sure* that the sprinkler was off, given that the grass is wet and the sprinkler on?


Legend
= uncertain
= positive
= negative
= causal link
A Causal structure
p(C = true) = 0.5
C
S = ¬C
S
R
R = C
W
W = S ∨ R
B What actually happened
C
S
R
W
There were no clouds, it didn't rain, the sprinkler was on, and the grass was wet.
Did the sprinkler cause the grass to be wet?
p(s → w) = ?


A Causal structure
B What actually happened
Legend
= uncertain
= positive
= negative
= causal link
$p(C = \textbf{true}) = 0.5$
$S = \neg C$
$R = C$
$W = S \vee R$
There were no **c**louds, it didn't **r**ain, the **s**prinkler was on, and the grass was **w**et.
Did the sprinkler cause the grass to be wet?
$p(s \rightarrow w) = ?$
C Observation (**Level I**)
Is the grass dry when the sprinkler is off?
$p(w' \mid s') = 0$


A Causal structure
B What actually happened
Legend
= uncertain
= positive
= negative
= causal link
p(C = true) = 0.5
S = ¬C
R = C
W = S ∨ R
There were no clouds, it didn't rain, the sprinkler was on, and the grass was wet.
Did the sprinkler cause the grass to be wet?
p(s → w) = ?
C Observation (Level I)
D Intervention (Level II)
Is the grass dry when the sprinkler is off?
Would the grass be dry if the sprinkler was off?
p(w′ | s′) = 0
p(w′ | do(s′)) = 0.5

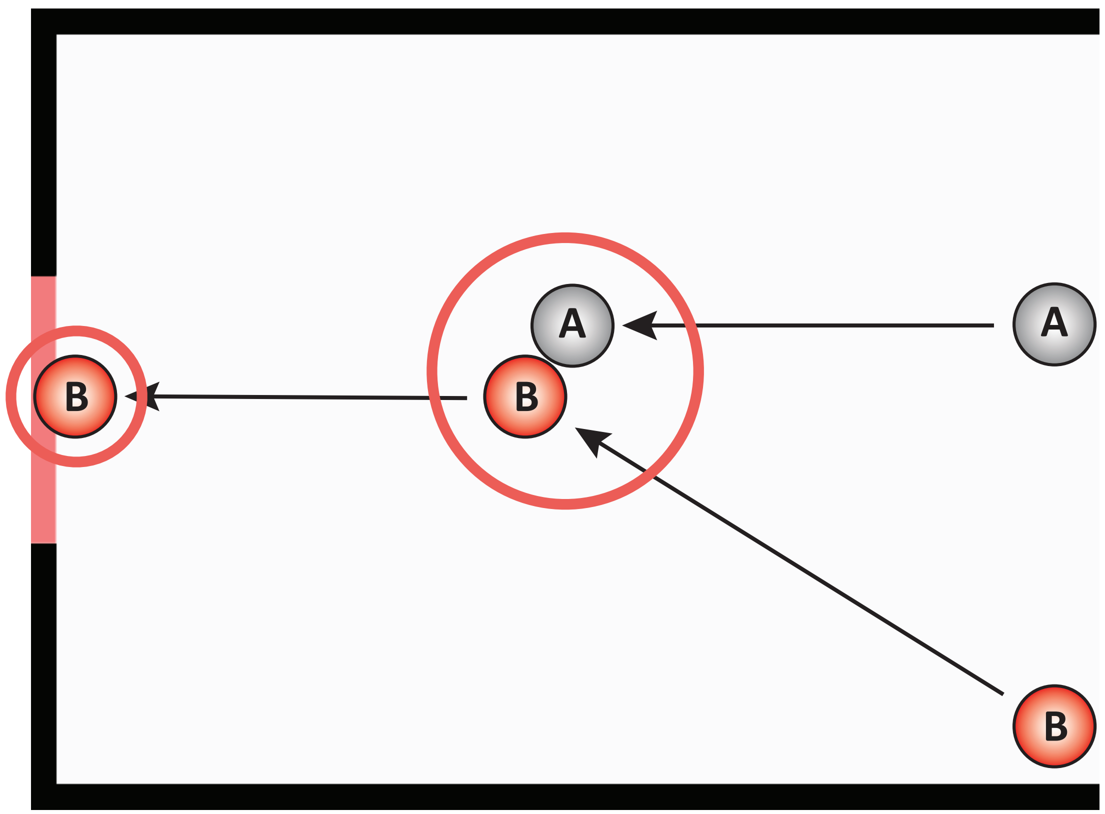


Legend
= uncertain
= positive
= negative
= causal link
A Causal structure
$p(C = \textbf{true}) = 0.5$
$S = \neg C$
$R = C$
$W = S \vee R$
B What actually happened
There were no clouds, it didn't rain, the sprinkler was on, and the grass was wet.
Did the sprinkler cause the grass to be wet?
$p(s \to w) = ?$
C Observation (Level I)
Is the grass dry when the sprinkler is off?
$p(w' \mid s') = 0$
D Intervention (Level II)
Would the grass be dry if the sprinkler was off?
$p(w' \mid \textbf{do}(s')) = 0.5$
E Counterfactual (Level III)
1) Conditioning step
2) Intervention step
Would the grass have been dry if the sprinkler had been off?
$p(w'_{s'} \mid c', s, r', w) = 1$

Did A cause B to go through the gate?

Counterfactual Simulation Model

What happened?



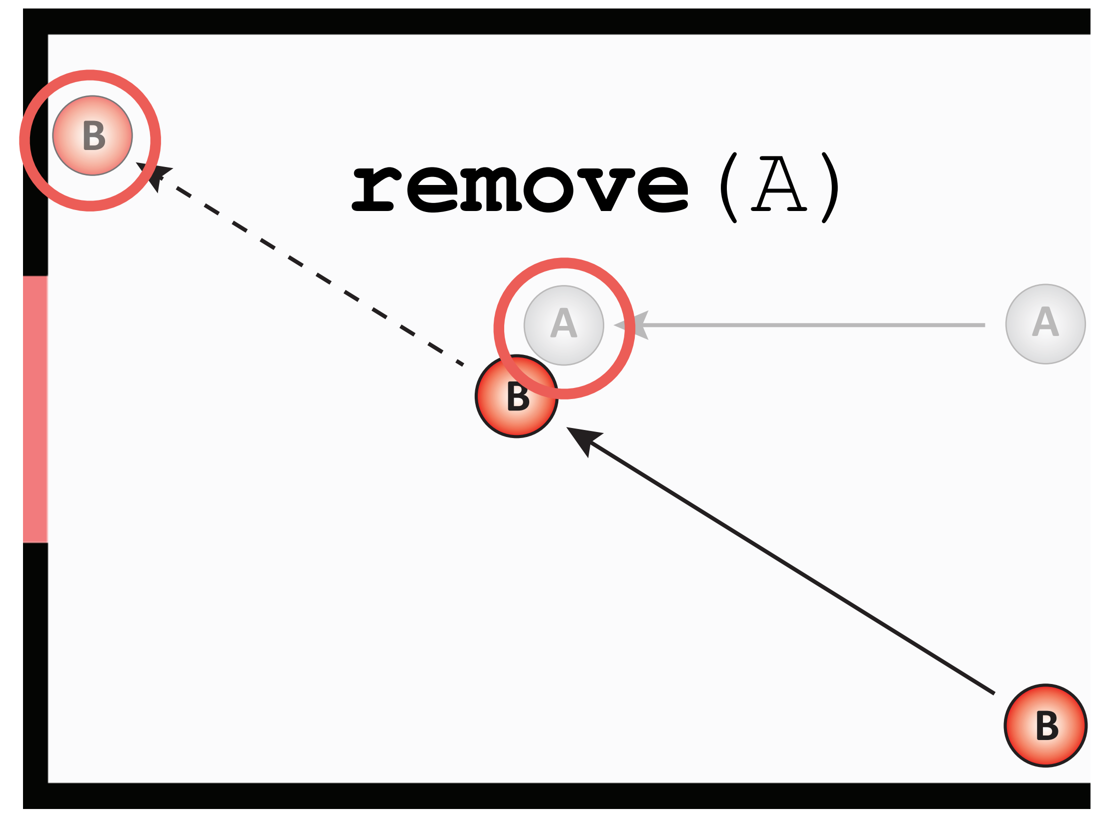
Actual situation

B went through the gate

?

≠

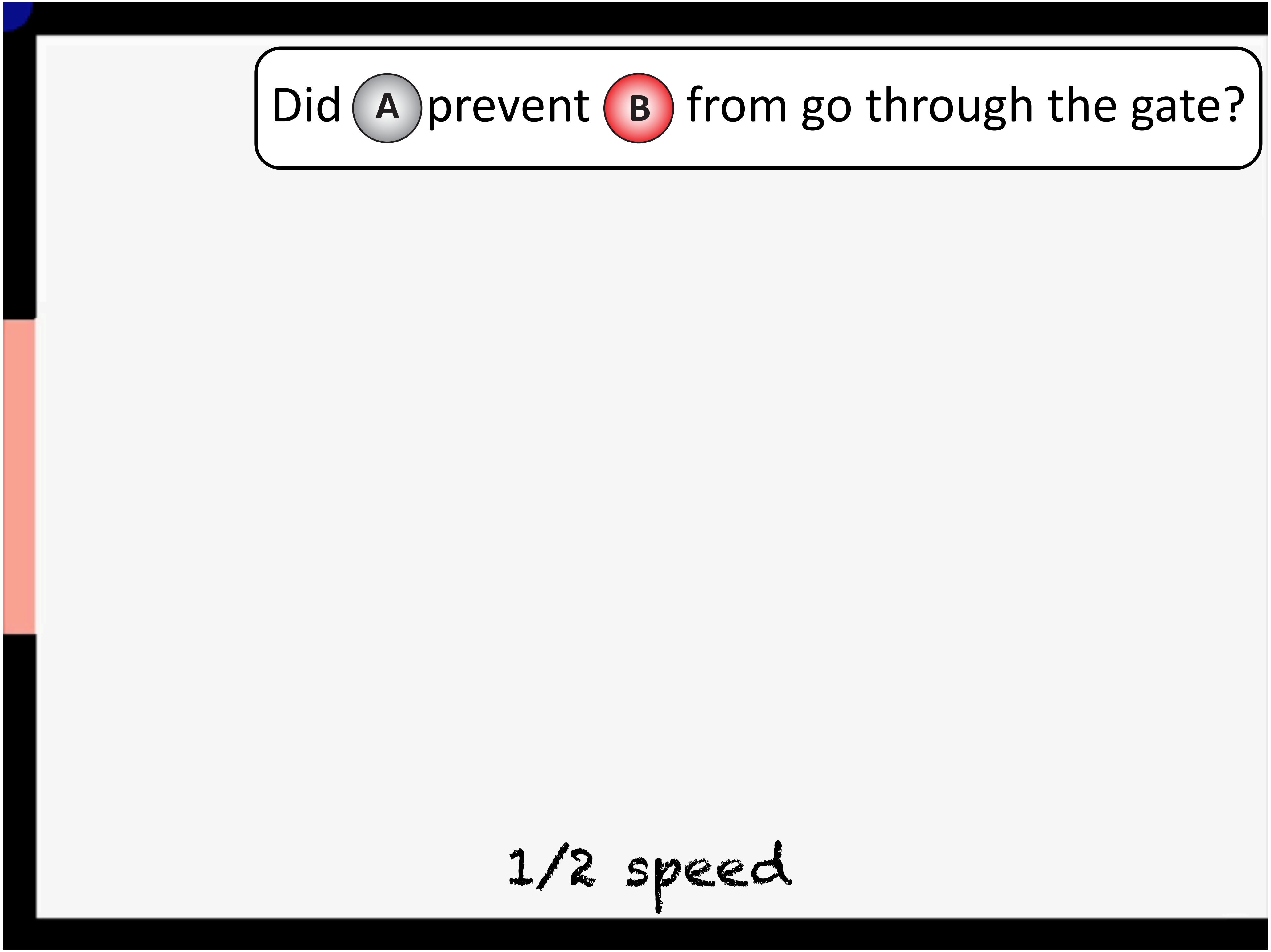
What would have happened?



Counterfactual situation

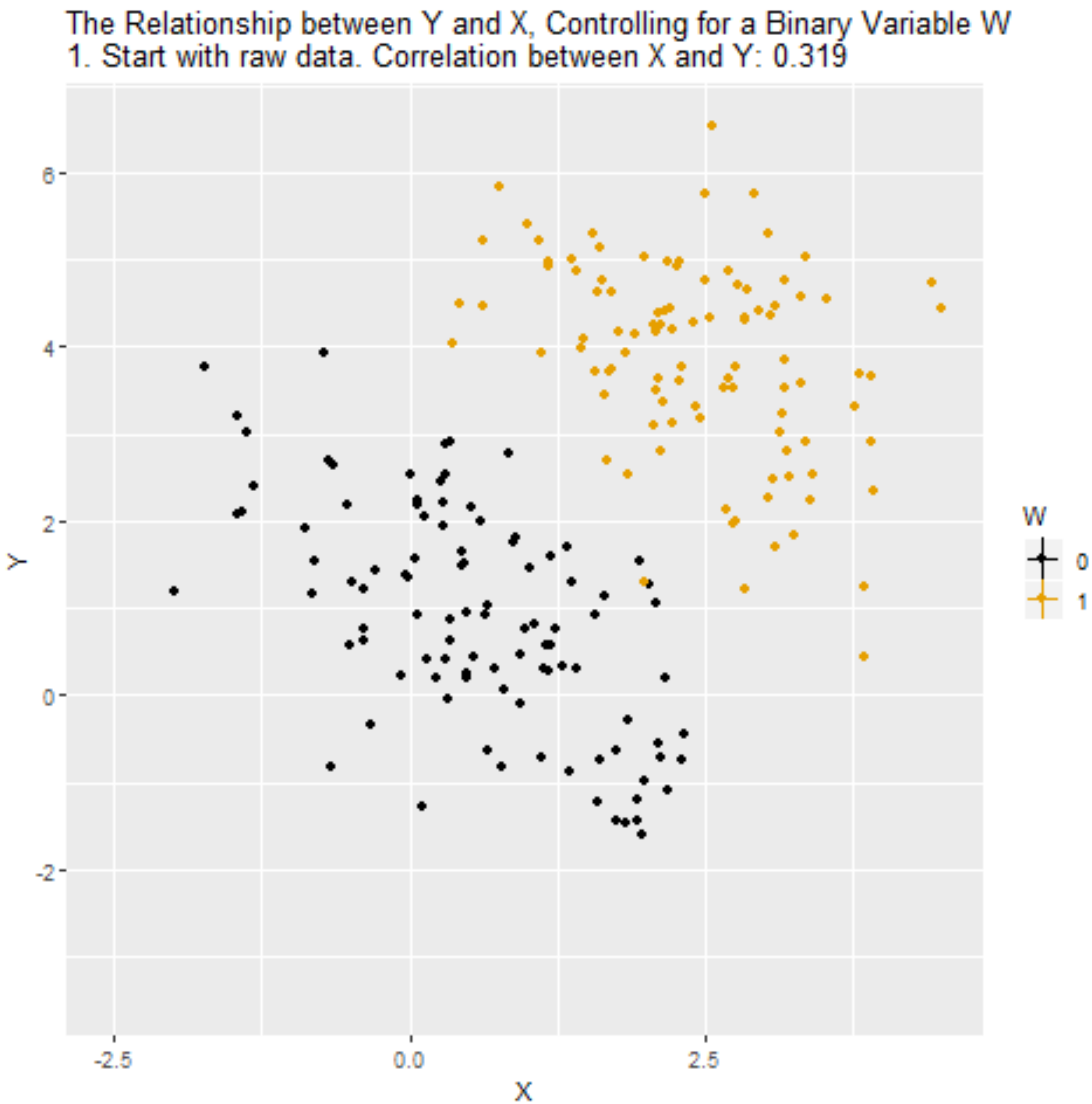
B would have missed the gate

Gerstenberg, Goodman, Lagnado, & Tenenbaum (2021). A counterfactual simulation model of causal judgment for physical events. *Psychological Review*



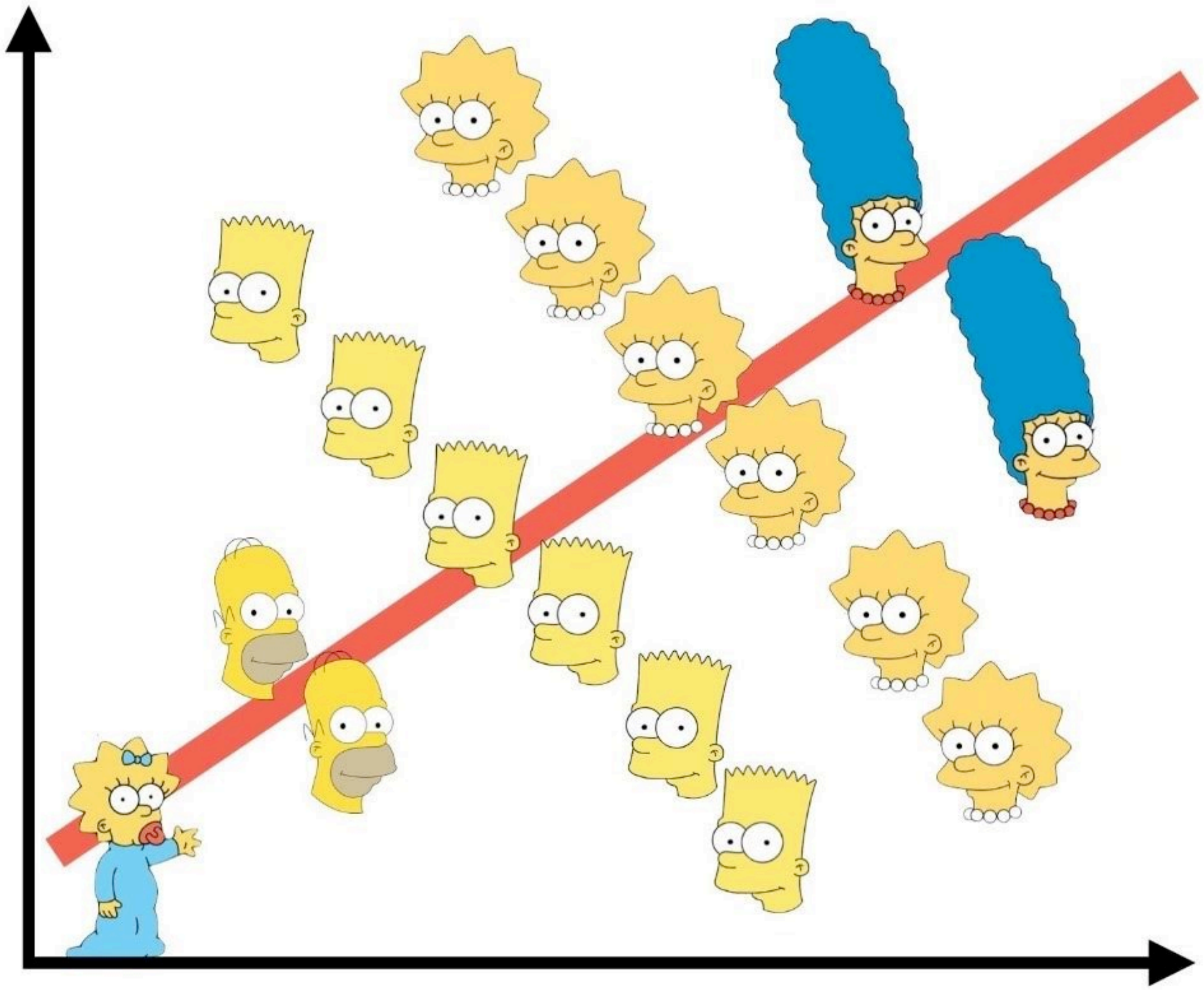
Gerstenberg, Peterson, Goodman, Lagnado, & Tenenbaum (2017) Eye-tracking causality. *Psychological Science*

Controlling for variables



https://nickchk.com/causalgraphs.html

Simpson's paradox



- when the relationship between two variables changes strongly after conditioning on another variable
- interesting real world cases
- **google** it!

What does controlling for variables mean?

we are not actually "**controlling**" the variable

instead, we are taking the variable into consideration when making predictions

the hope is that we get a better estimate of the parameter that we are interested in by taking into account other factors

Patterns of inference

Patterns of inference: **Common cause**

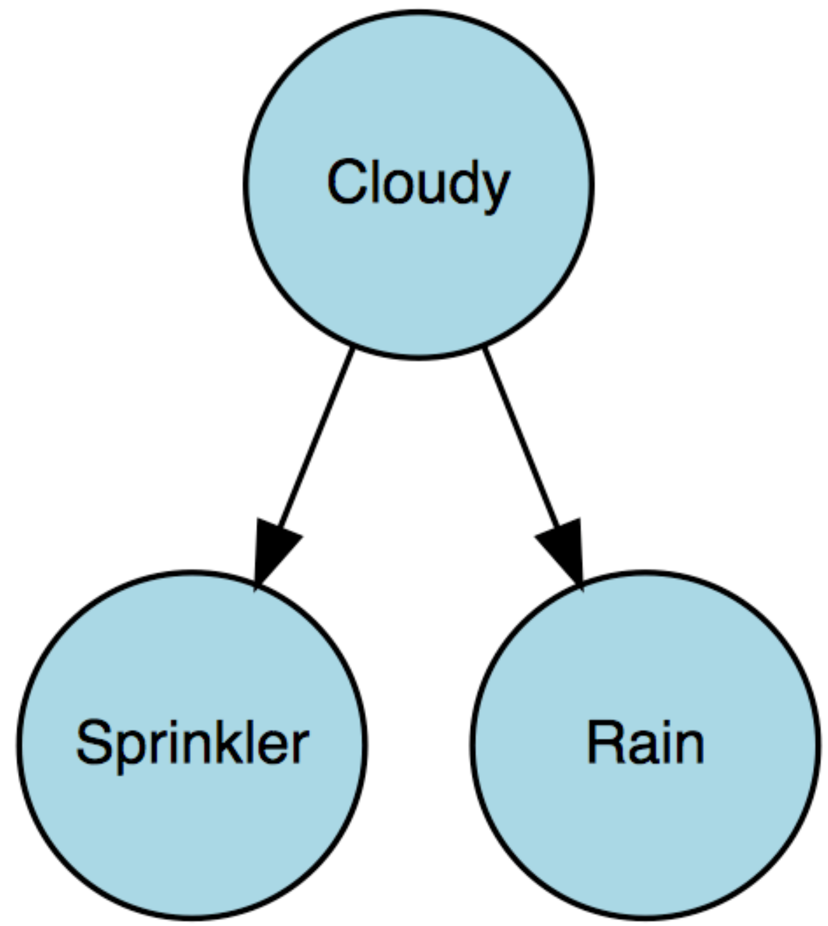
$$p(S \mid R) = p(S)$$

or

$$p(S \mid R) \neq p(S)$$

?

p(C)
0.5



C	p(S)
F	0.5
T	0.1

C	p(R)
F	0
T	0.3

Patterns of inference: **Common cause**

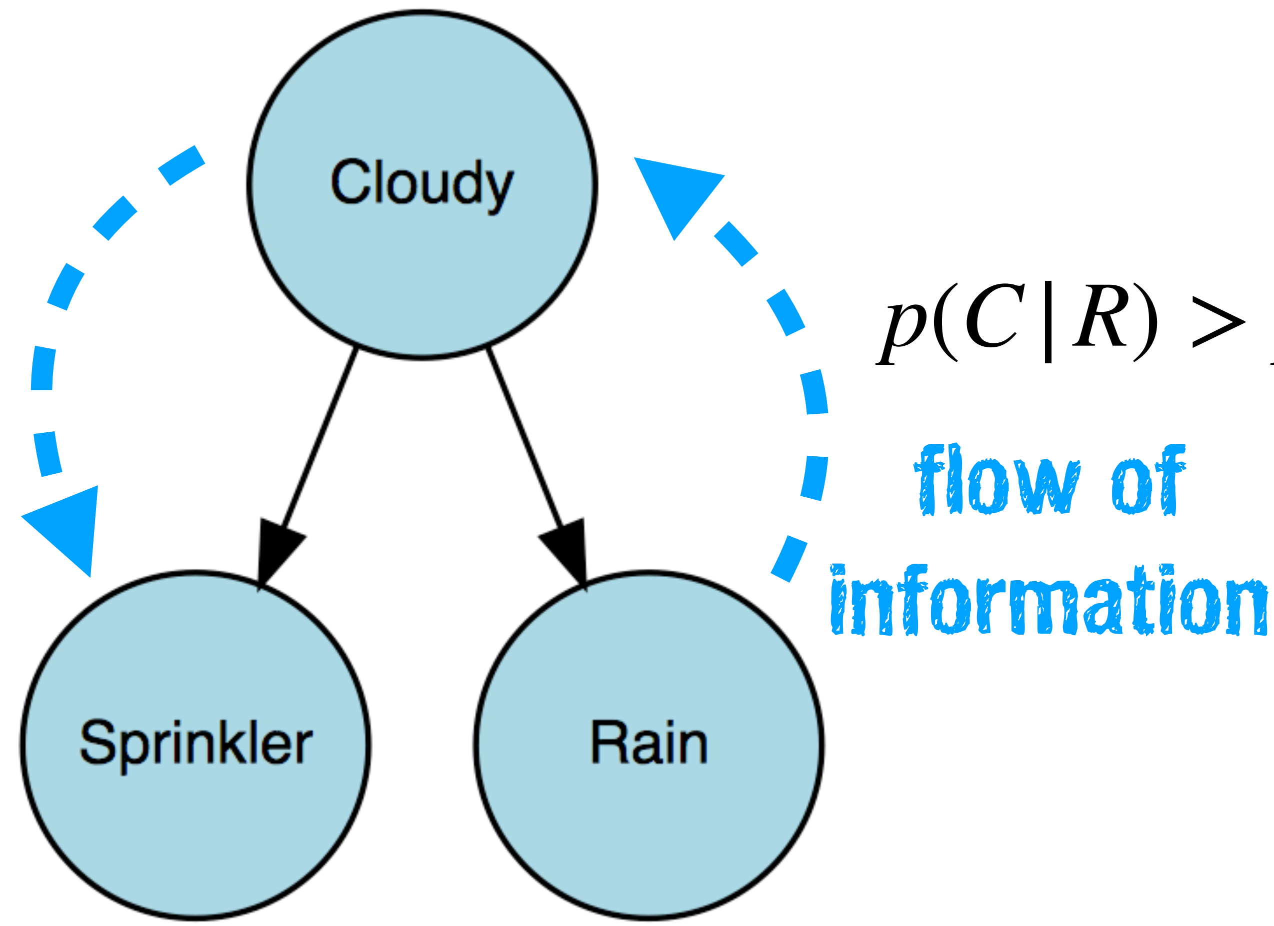
- effects of a common cause are *unconditionally dependent*

$$p(S \mid R) \neq p(S)$$

p(C)
0.5

$$p(S \mid C) < p(S)$$

$$p(C \mid R) > p(C)$$



C	p(S)
F	0.5
T	0.1

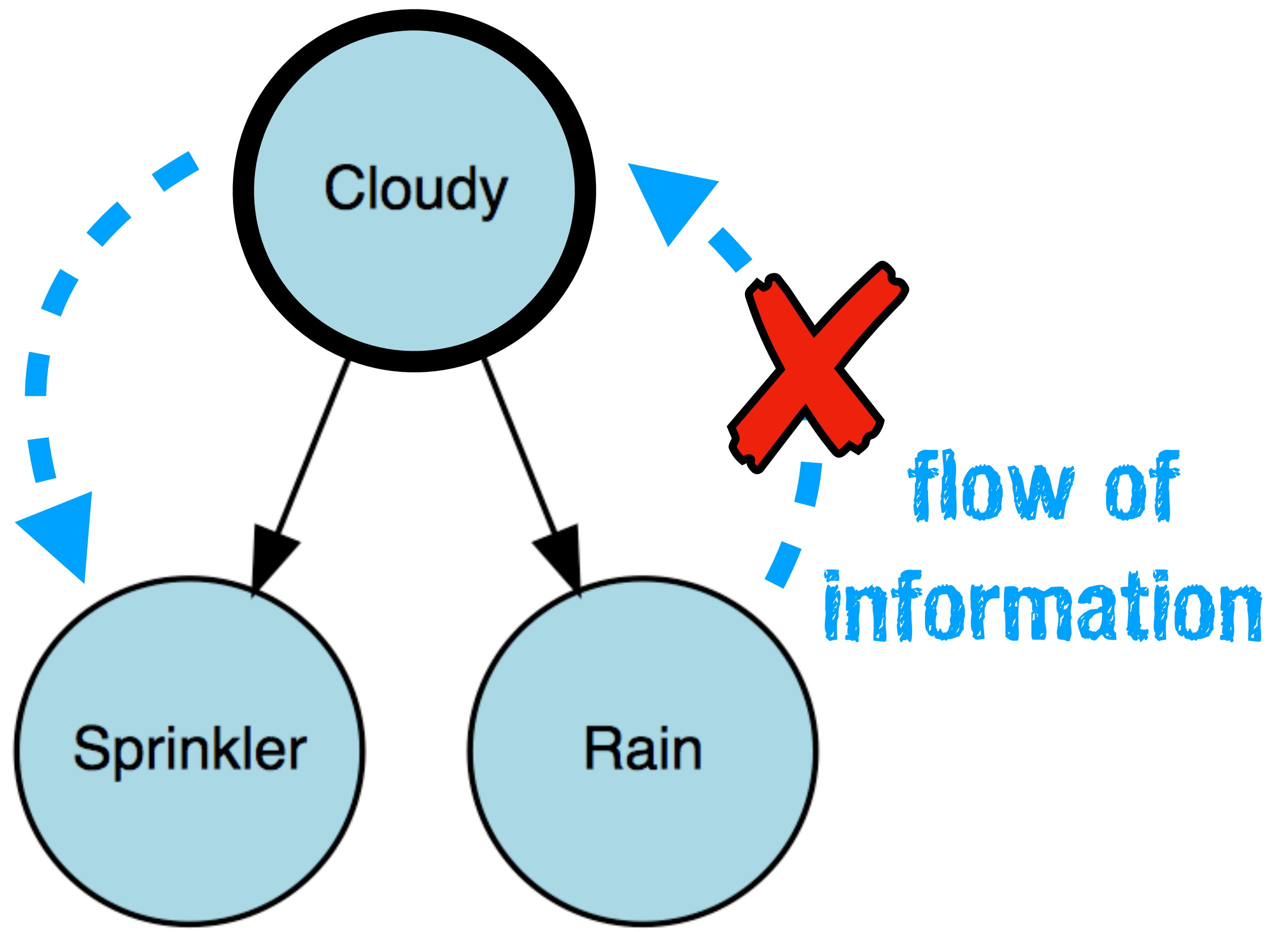
C	p(R)
F	0
T	0.3

Patterns of inference: **Common cause**

- effects of a common cause are *conditionally independent given the cause*

$$p(S \mid R, C) = p(S \mid C)$$

p(C)
1



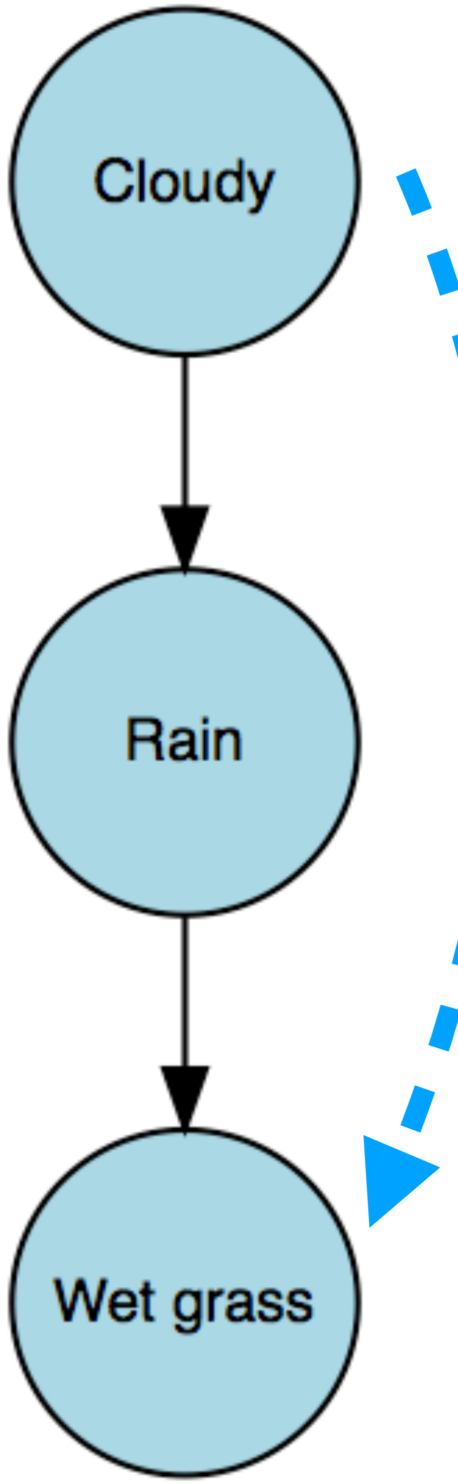
C	p(S)
F	0.5
T	0.1

C	p(R)
F	0
T	0.3

Patterns of inference: **Causal chain**

- cause and effect in a causal chain are *unconditionally dependent*

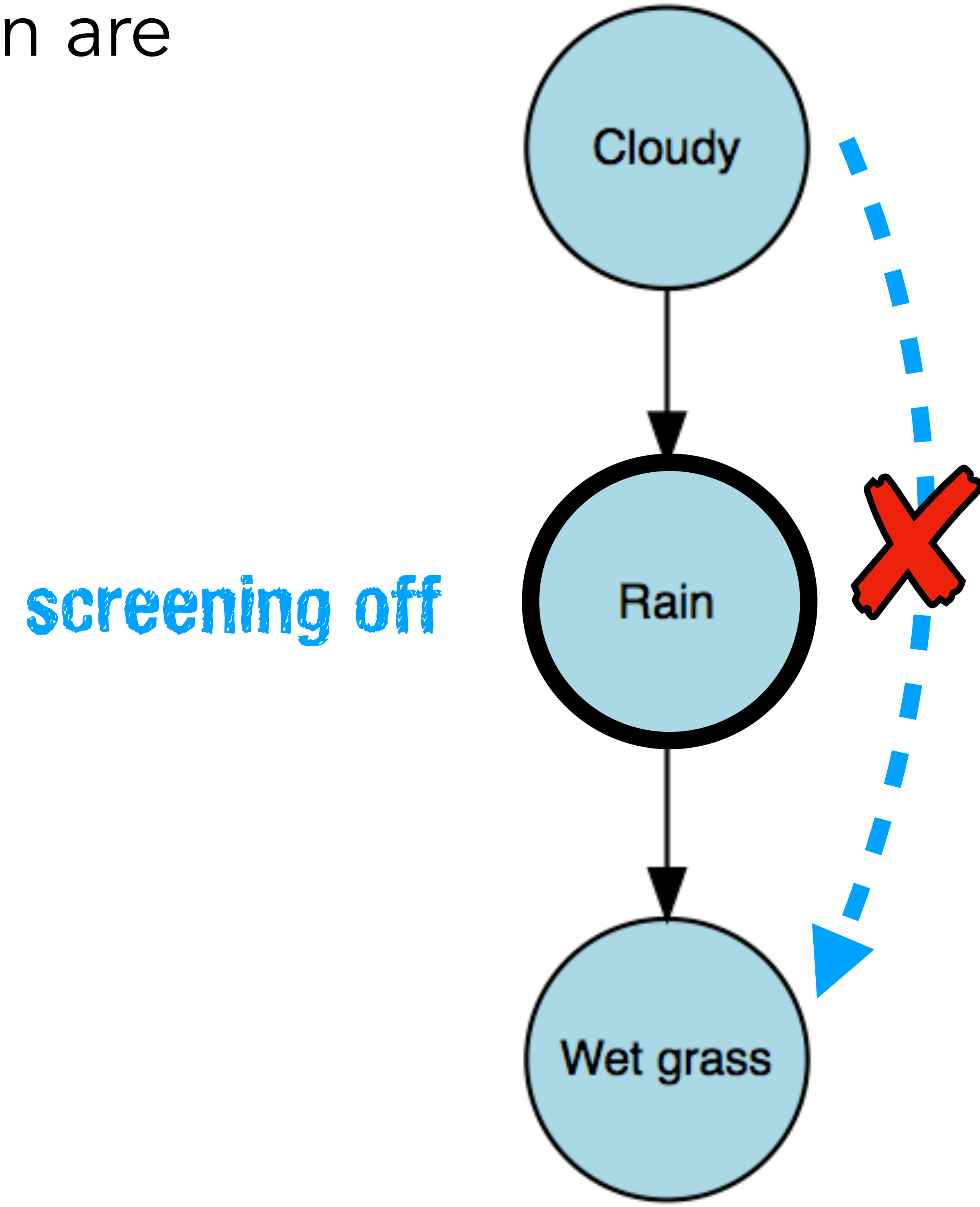
$$p(W \mid C) \neq p(W)$$



Patterns of inference: **Causal chain**

- cause and effect in a causal chain are *conditionally independent*

$$p(W \mid C, R) = p(W \mid R)$$

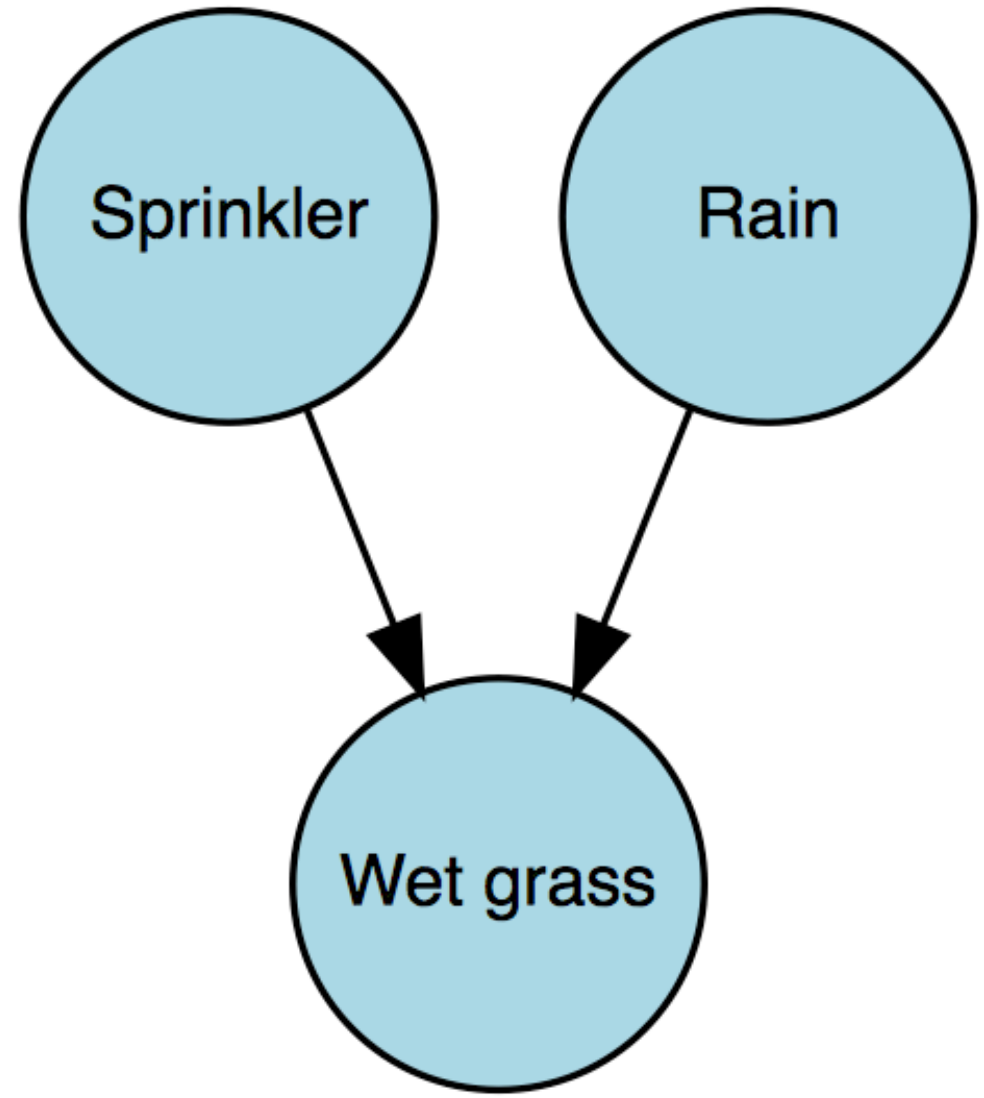


Patterns of inference: **Common effect**

- two causes of a common effect are *unconditionally independent*

$$p(S \mid R) = p(S)$$

(e.g. Sprinkler is set by a timer)

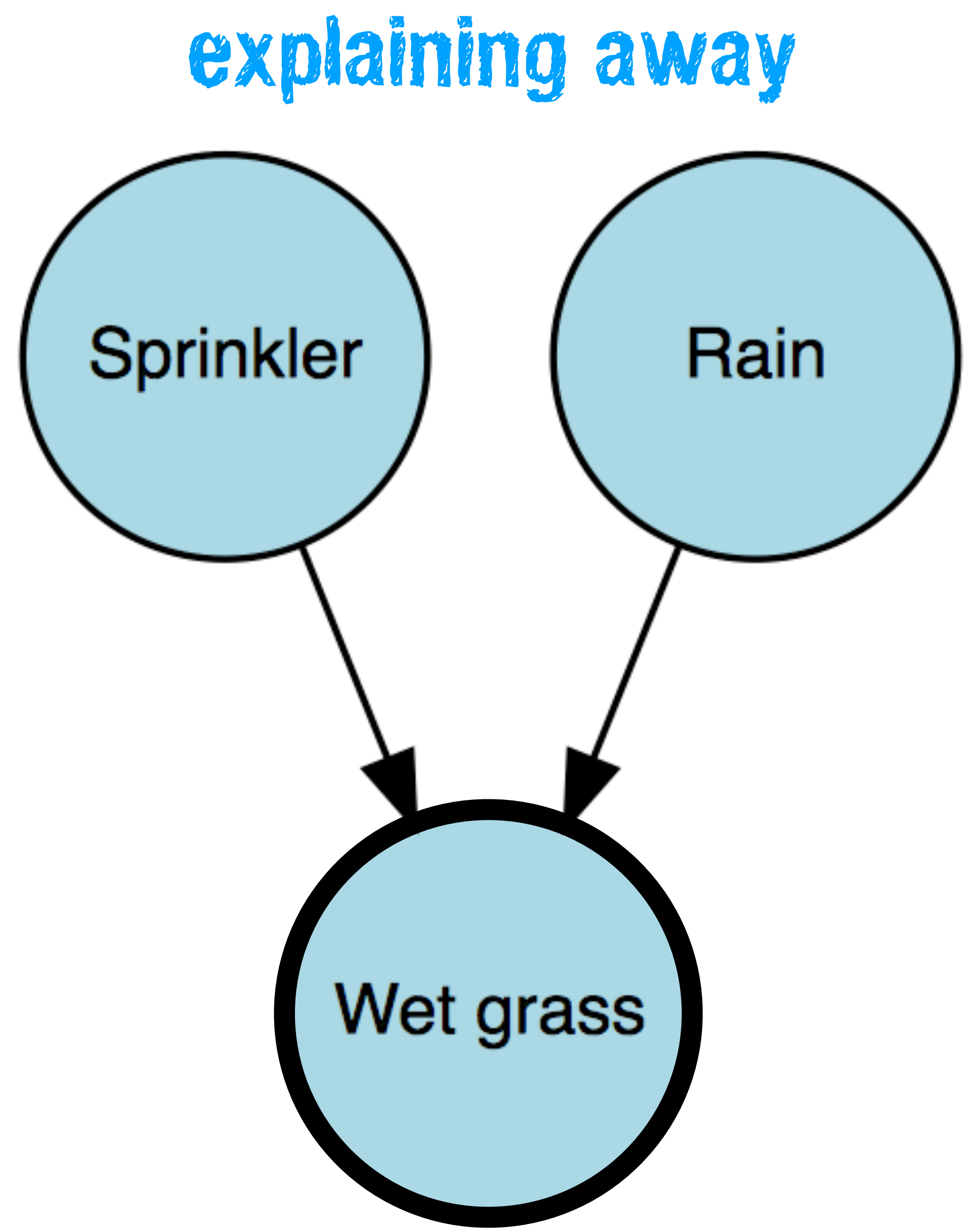


Patterns of inference: **Common effect**

- two causes of a common effect are *conditionally dependent given the effect*

$$p(S \mid R, W) \neq p(S \mid W)$$

- intuitively: both causes compete to explain the effect



Note: The pattern of inference depends on the structural form which captures how `Sprinkler` and `Rain` jointly affect `Wet grass`. Explaining away holds for the commonly used noisy-or integration function.

Should I control?

When should I control for variables?

recent advances in graphical models have produced a way to help distinguish good from bad controls

***d*-separation**

directional

decide from a causal graph whether a set of variables `X` is independent of another set `Y`, given a third set `Z`

Goal: we want a precise (and unbiased) estimate of the predictive relationship between `X` and `Y`

we want to block all other paths from X to Y

http://bayes.cs.ucla.edu/BOOK-09/ch11-1-2-final.pdf

When should I control for variables?

How can I tell whether two variables are independent?

Recipe for independence

1. Draw the ancestral graph
2. "Moralize" the graph by "marrying" the parents
3. "Disorient" the graph by replacing arrows with edges
4. Delete the givens and their edges
5. Read the answer off the graph

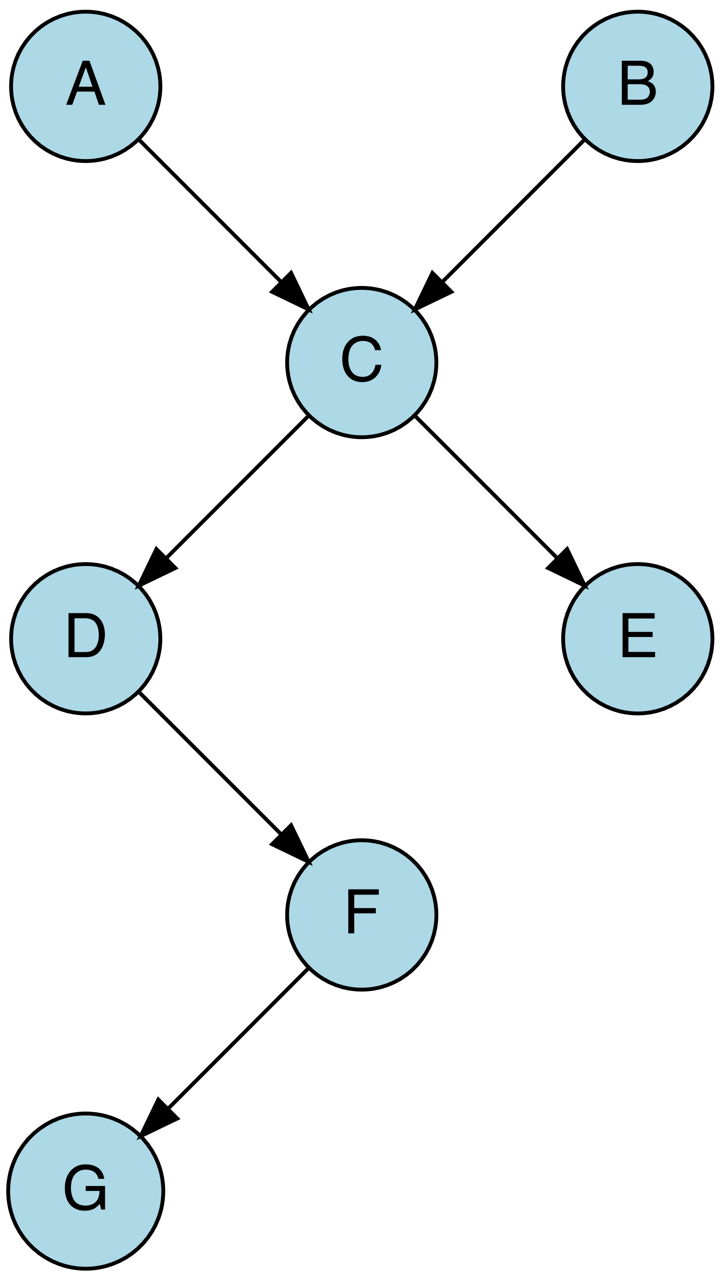
- if variables are **disconnected** they are independent
- if variables are connected (have a path between them) they are not guaranteed to be independent

http://web.mit.edu/jmn/www/6.034/d-separation.pdf

When should I control for variables?

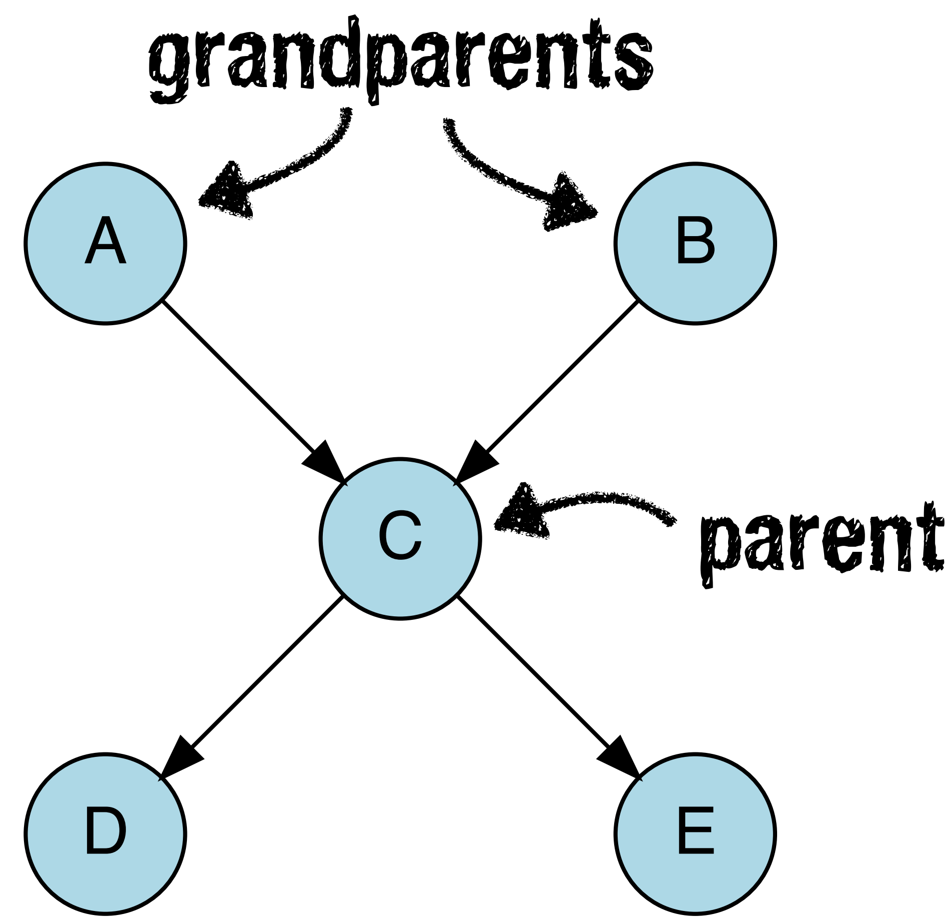
Are D and E independent?

$$p(D \mid E) = p(D) \ ?$$



1. Draw the ancestral graph

Construct the "ancestral graph" of all variables mentioned in the probability expression. This is a reduced version of the original net, consisting only of the variables mentioned and all of their ancestors (parents, parents' parents, etc.)



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Are D and E independent?

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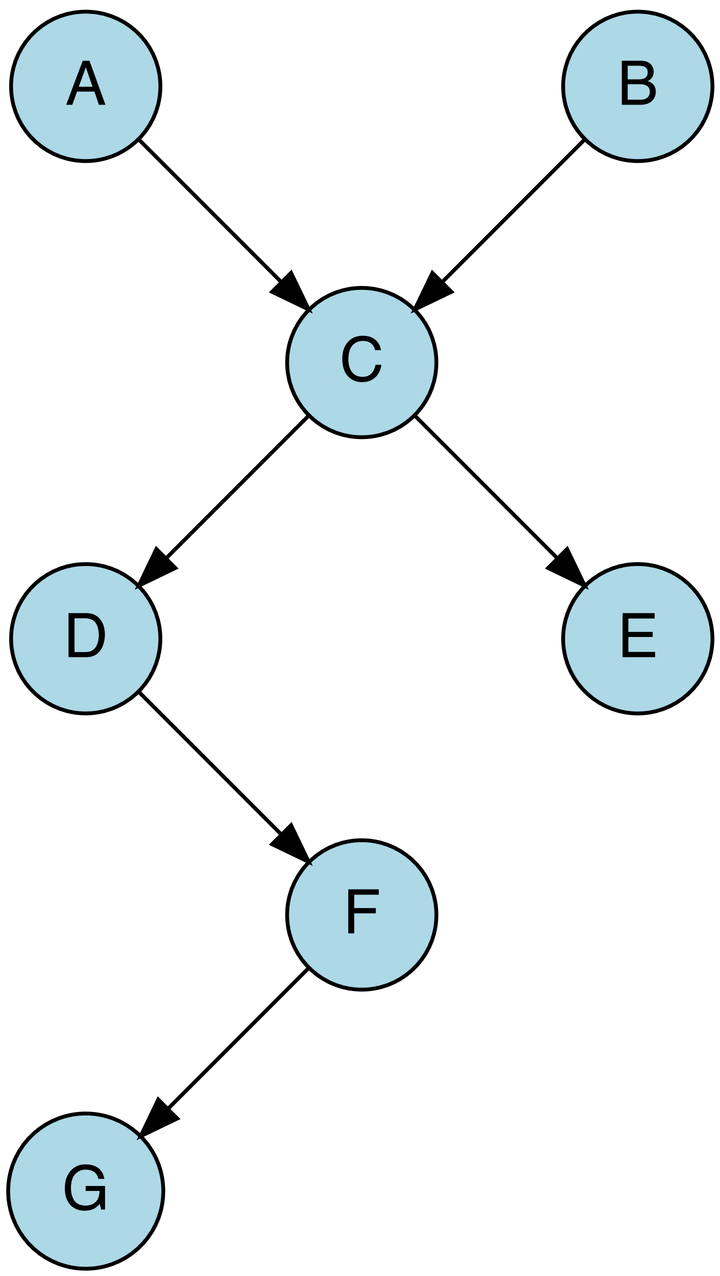
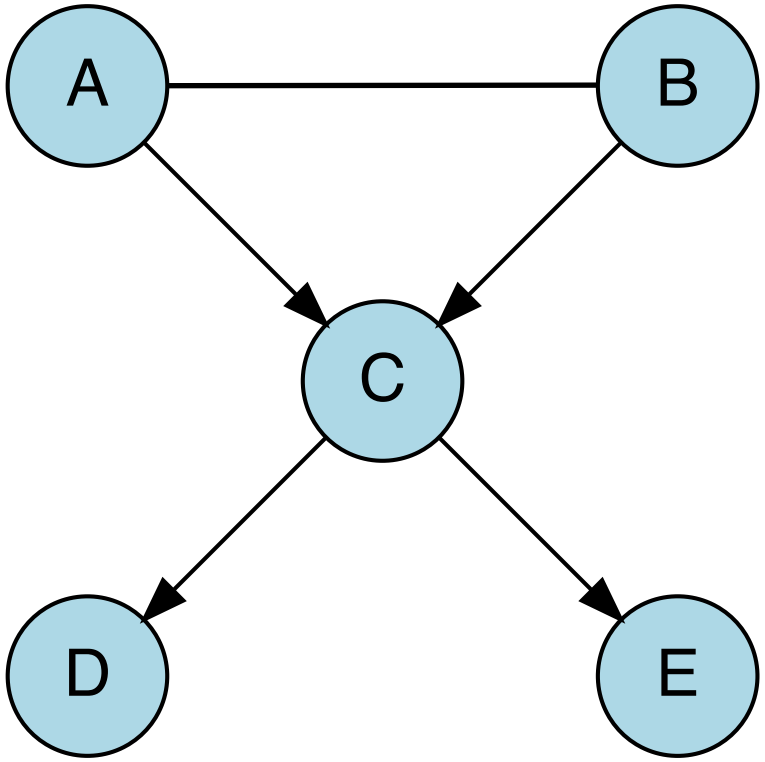


2. "Moralize" the graph

let's get married!

For each pair of variables with a common child, draw an undirected edge (line) between them. (If a variable has more than two parents, draw lines between every pair of parents.)



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Are D and E independent?

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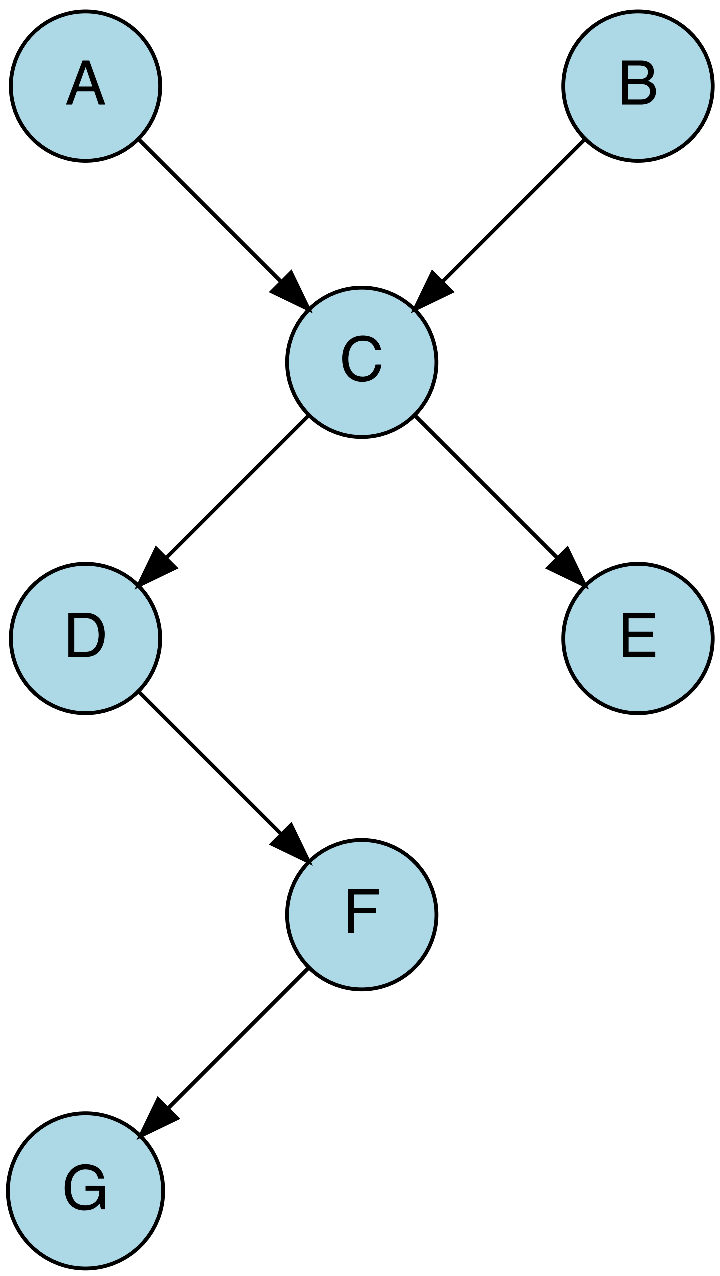
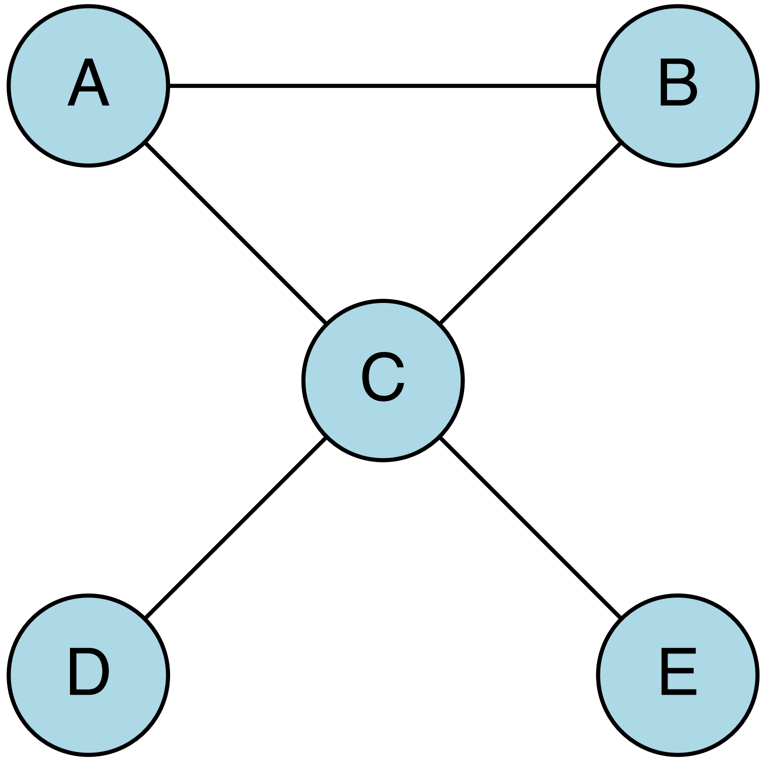


3. "Disorient" the graph

Replace arrows arrows with lines



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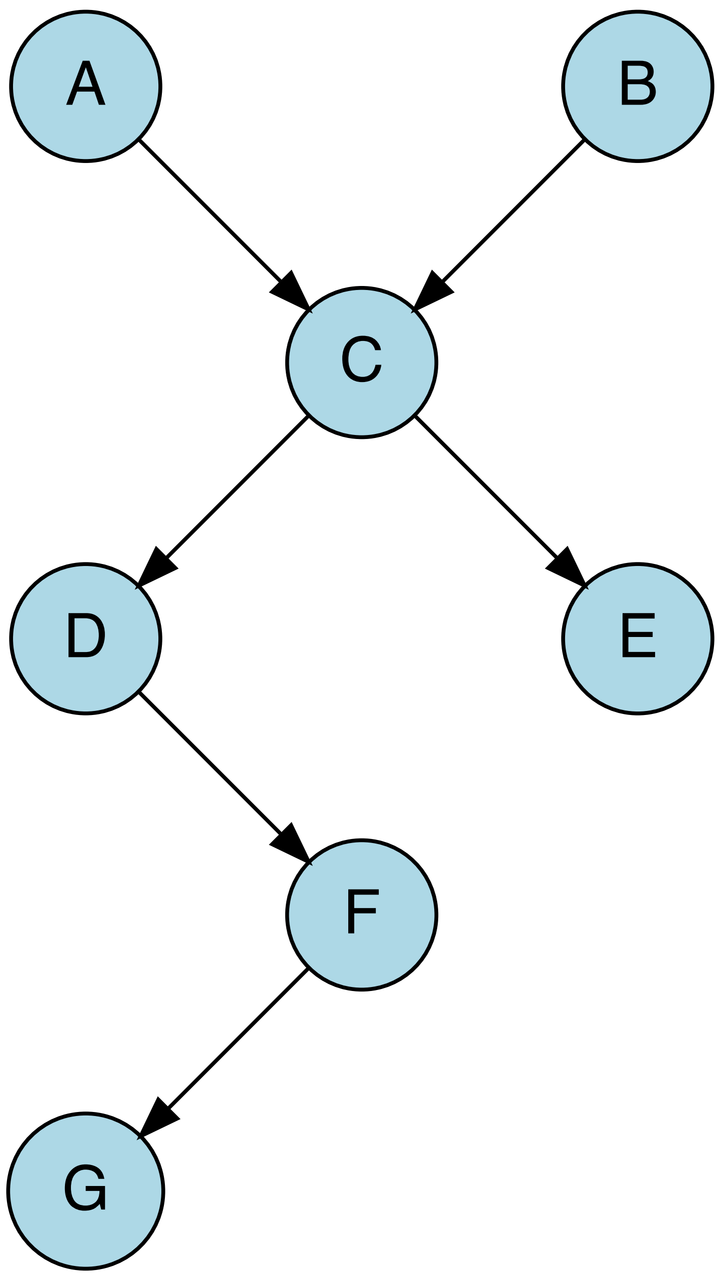
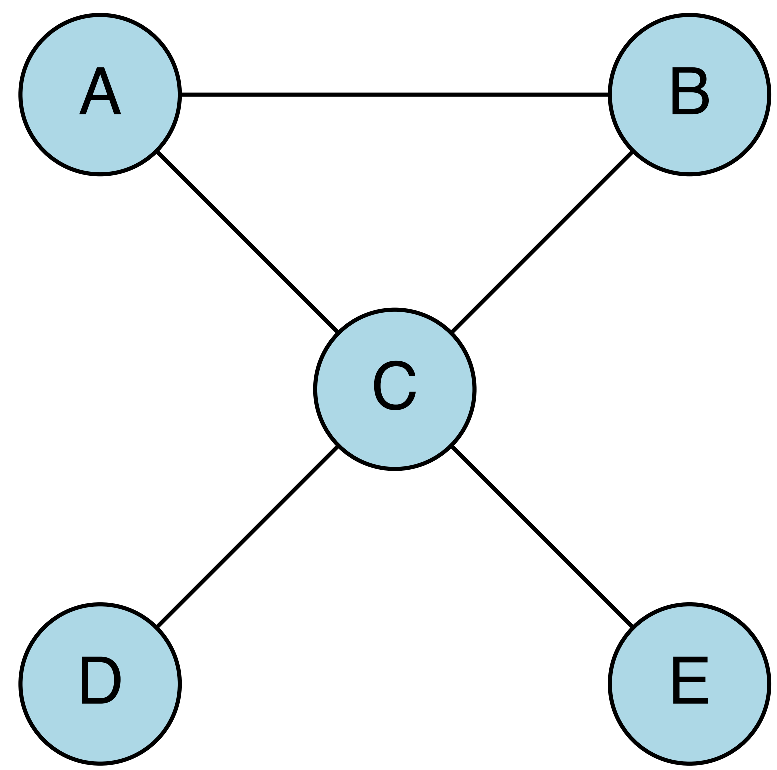


4. Delete the givens

Remove the variables that we condition on, as well as their edges

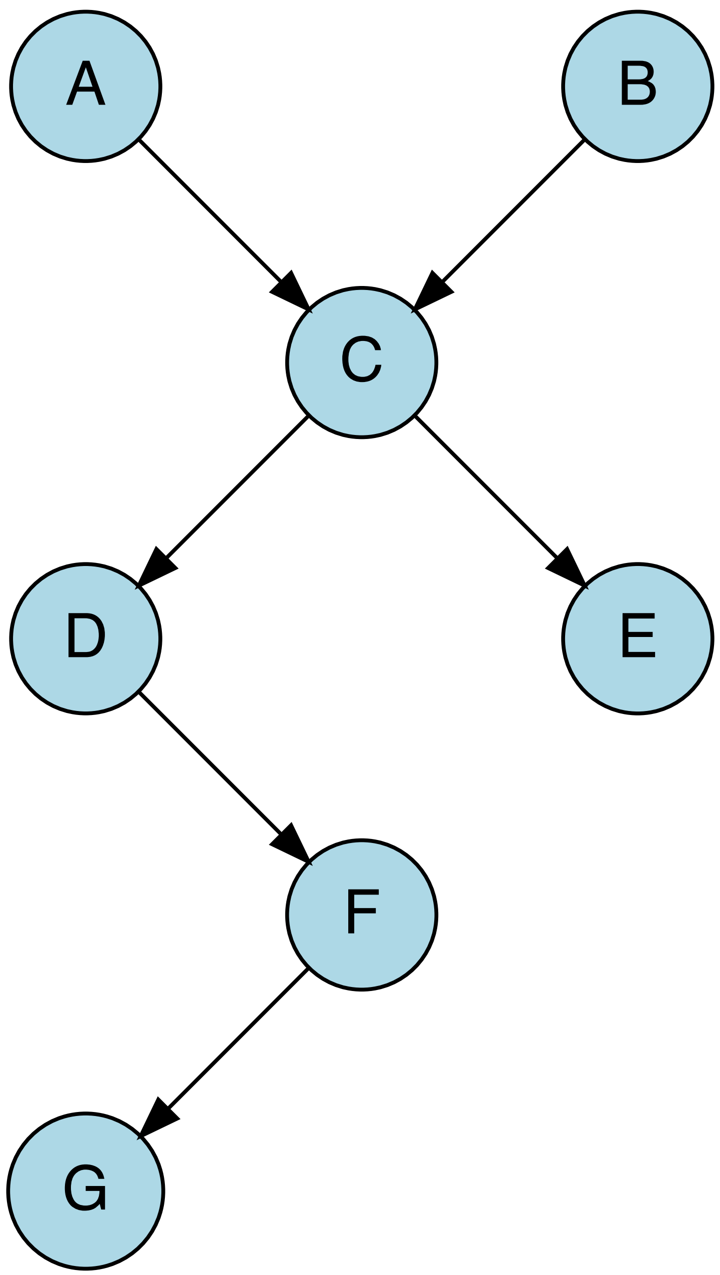
we didn't condition on anything, so there is nothing to delete



When should I control for variables?

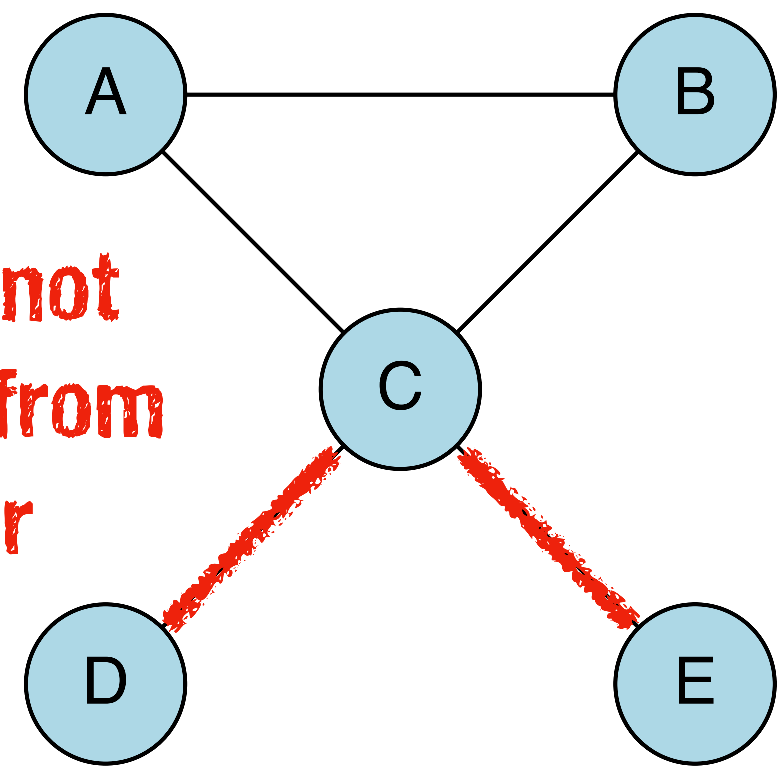
Are D and E independent?

$$p(D \mid E) = p(D) \ ?$$



5. Read answer off the graph

- if variables are **disconnected** they are independent
- if variables are connected (have a path between them) they are not guaranteed to be independent



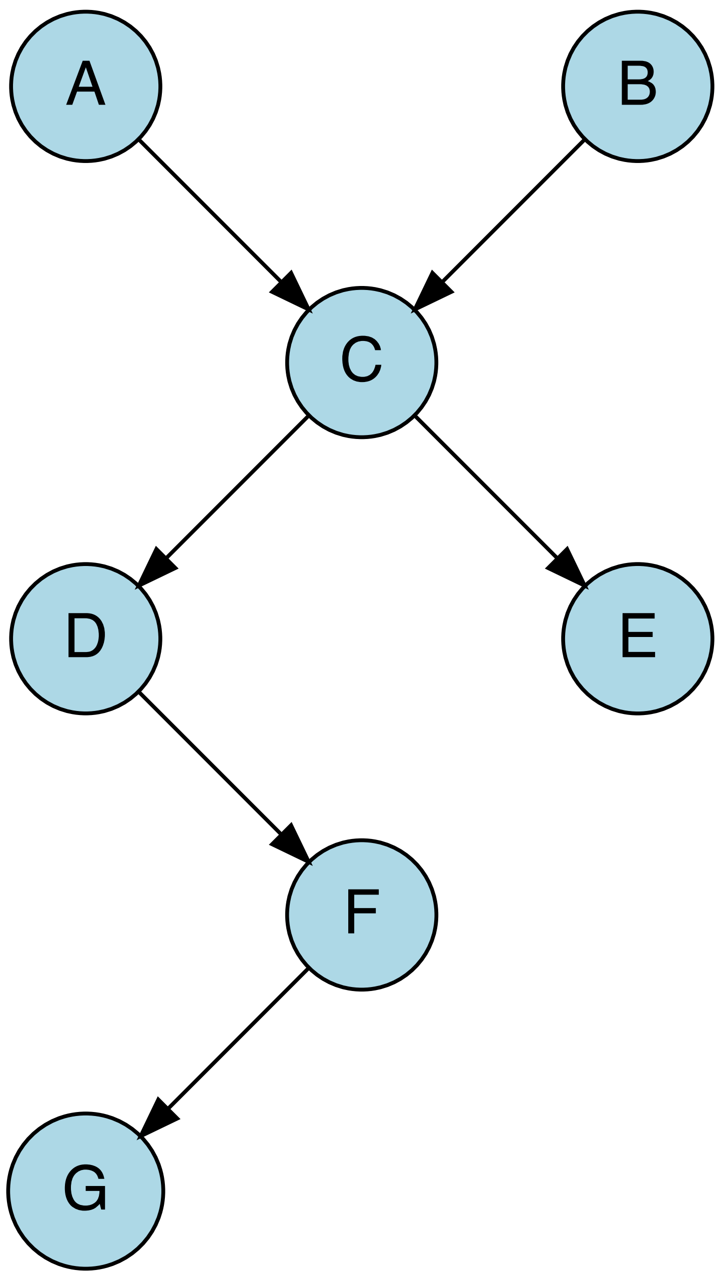
D and E are not independent from each other

they are connected via at least one path

When should I control for variables?

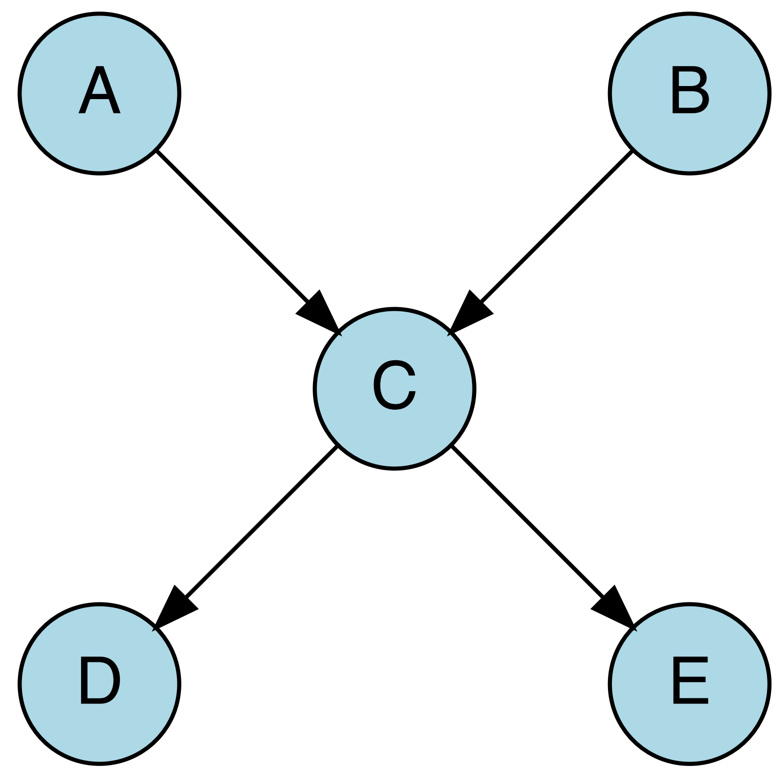
Are D and E independent, given C?

$$p(D \mid E, C) = p(D \mid C) \ ?$$



1. Draw the ancestral graph

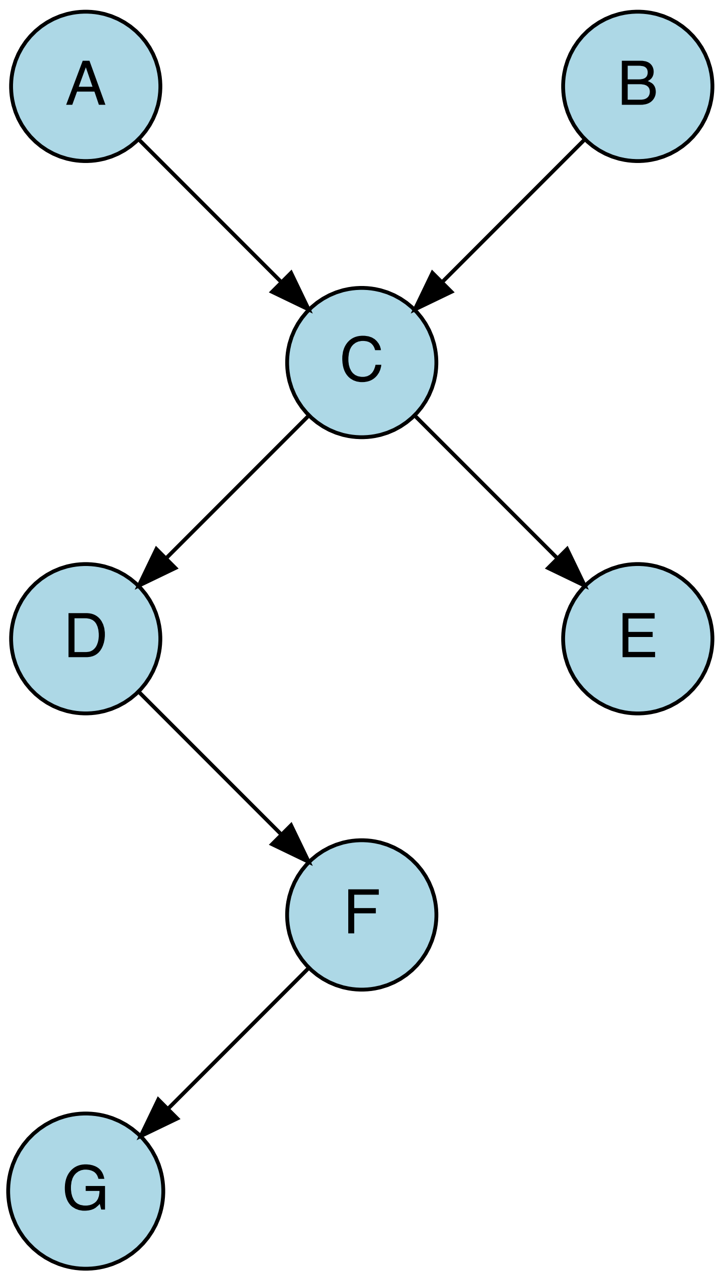
Construct the "ancestral graph" of all variables mentioned in the probability expression. This is a reduced version of the original net, consisting only of the variables mentioned and all of their ancestors (parents, parents' parents, etc.)



When should I control for variables?

Are D and E independent, given C?

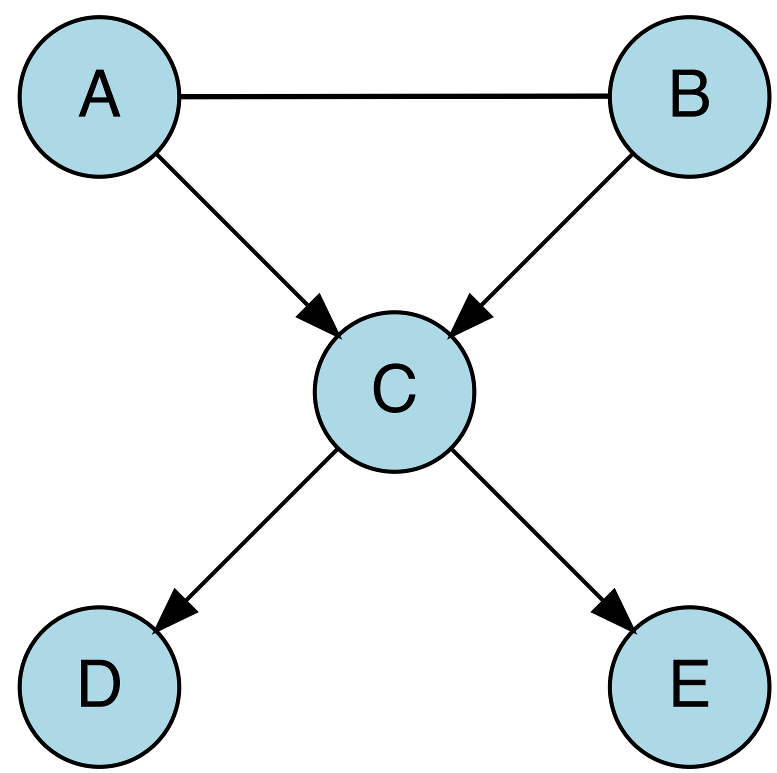
$$p(D \mid E, C) = p(D \mid C) \text{ ?}$$



2. "Moralize" the graph

let's get married!

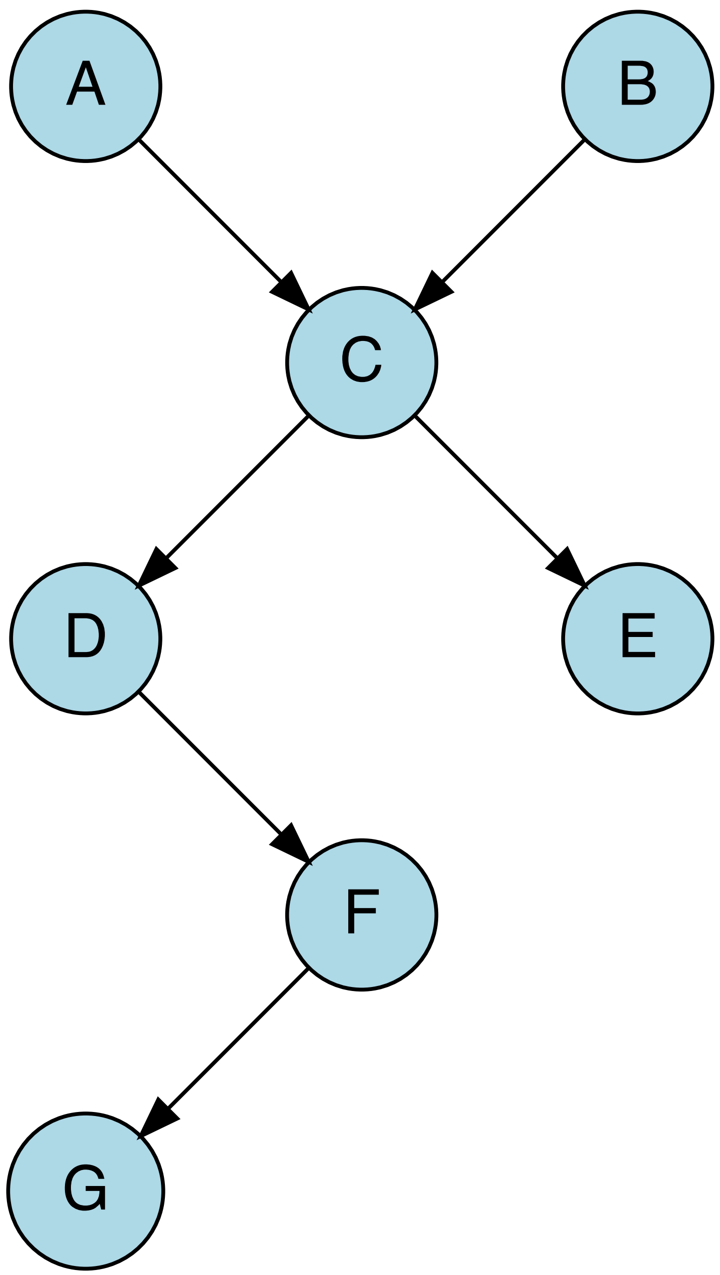
For each pair of variables with a common child, draw an undirected edge (line) between them. (If a variable has more than two parents, draw lines between every pair of parents.)



When should I control for variables?

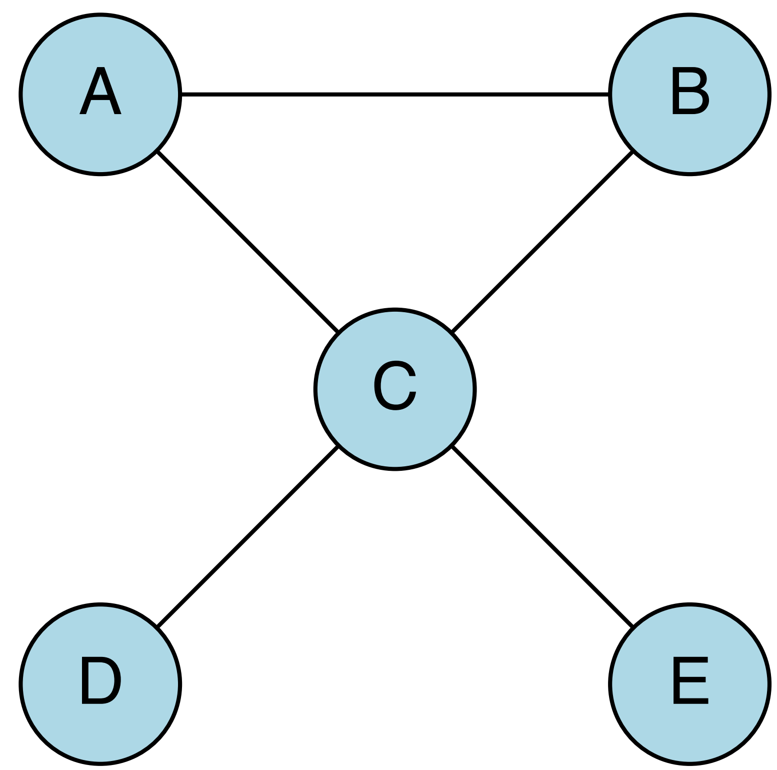
Are D and E independent, given C?

$$p(D \mid E, C) = p(D \mid C) \text{ ?}$$



3. "Disorient" the graph

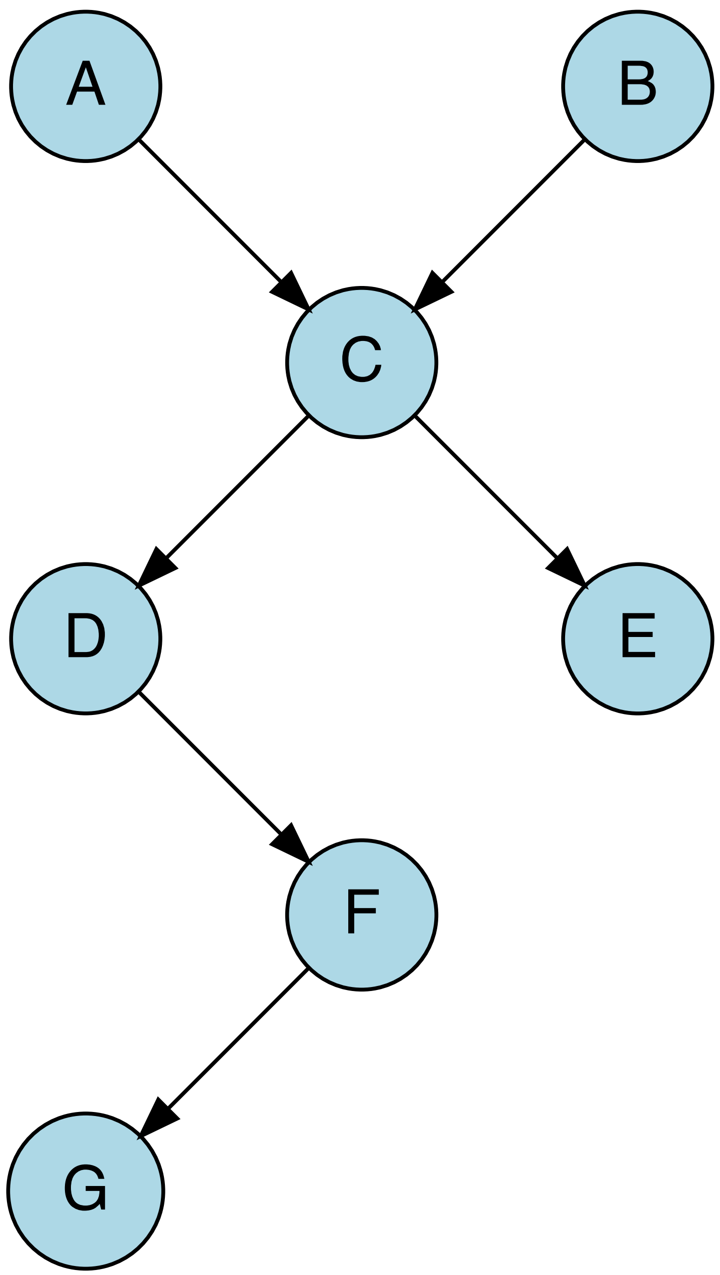
Replace arrows arrows with lines



When should I control for variables?

Are D and E independent, given C?

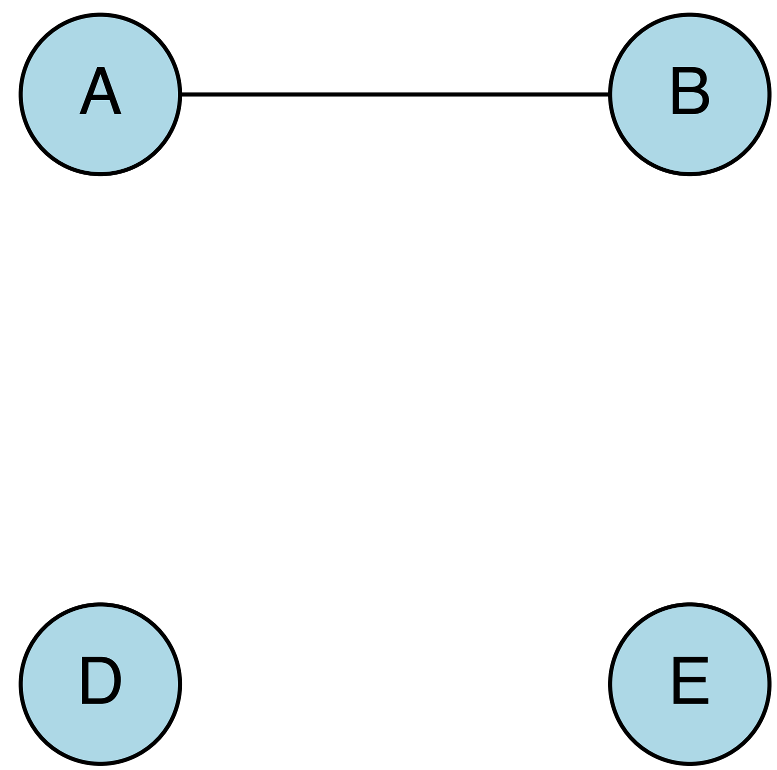
$$p(D \mid E, C) = p(D \mid C)\ ?$$



4. Delete the givens

Remove the variables that we condition on, as well as their edges

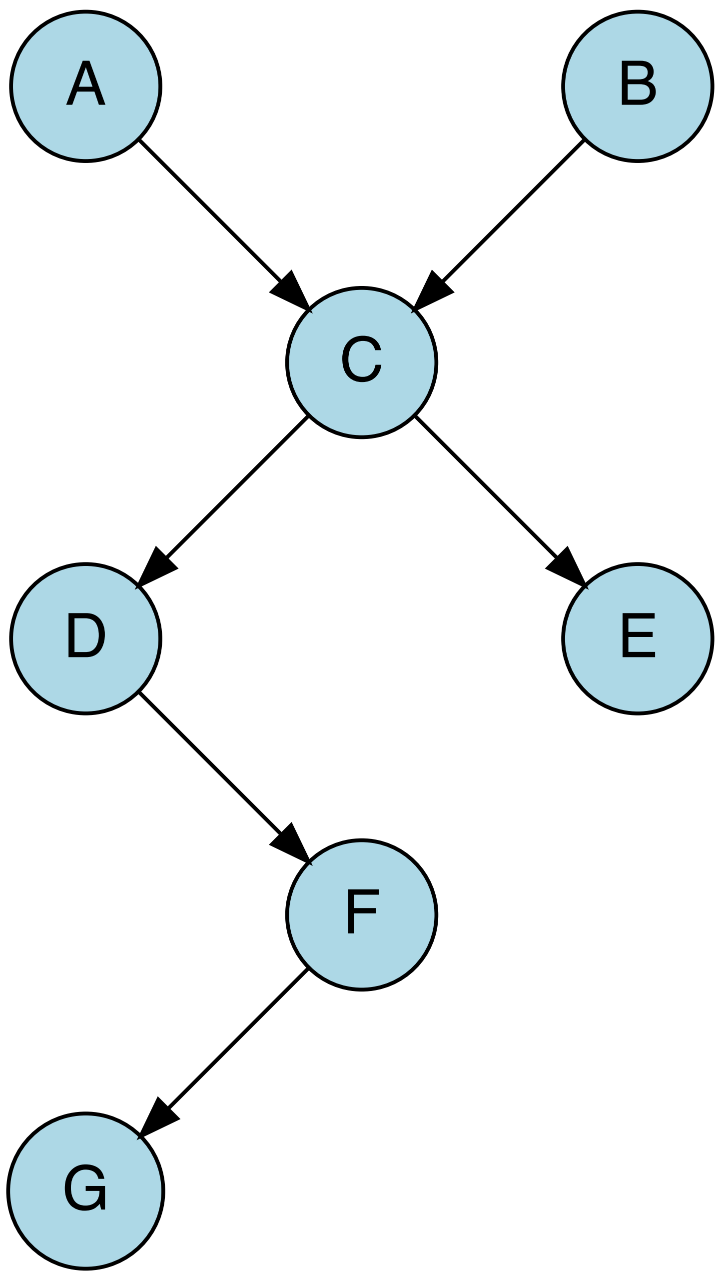
we conditioned on C!



When should I control for variables?

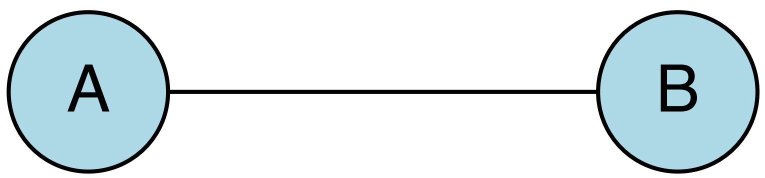
Are D and E independent, given C?

$$p(D \mid E, C) = p(D \mid C) \text{ ?}$$



5. Read answer off the graph

- if variables are **disconnected** they are independent
- if variables are connected (have a path between them) they are not guaranteed to be independent



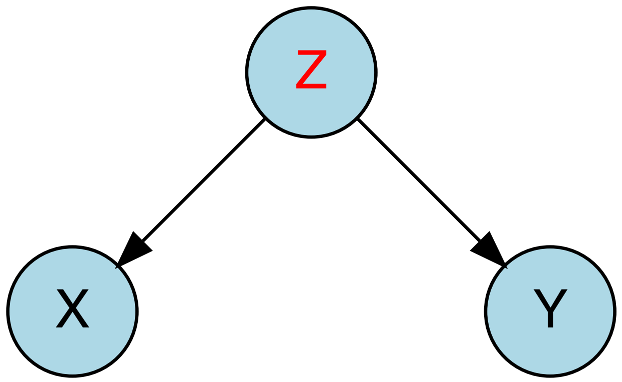
D and E are independent from each other conditioned on C

they aren't connected via a path

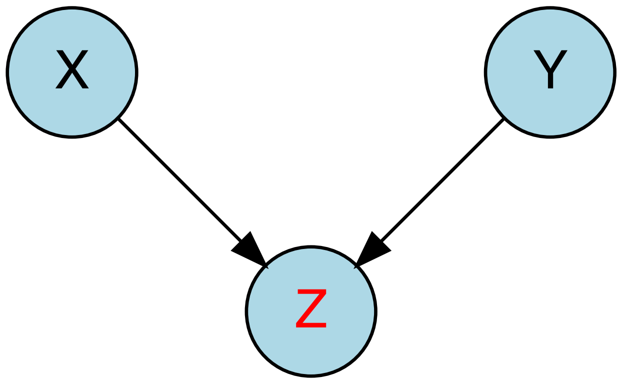
So what?

Patterns of inference

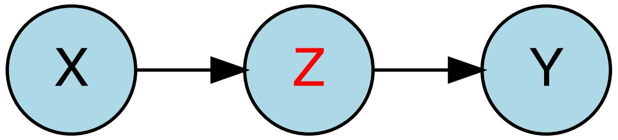
We want to estimate the (causal) relationship between `X` and `Y`



common cause



common effect



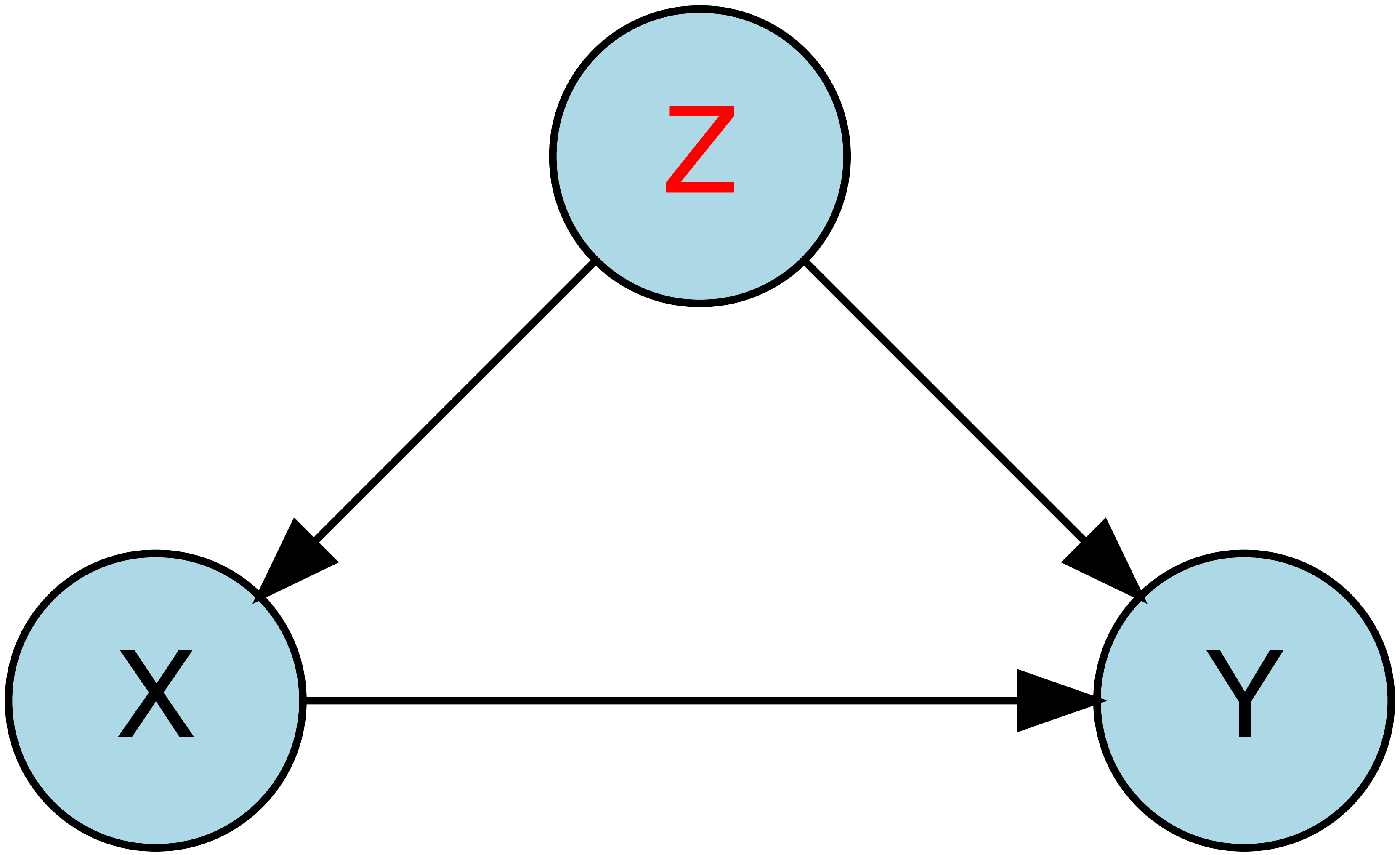
causal chain

by controlling for `Z` we hope to get a better estimate of the relationship between `X` and `Y`

d-separation helps us tell apart **good controls** from **bad controls**

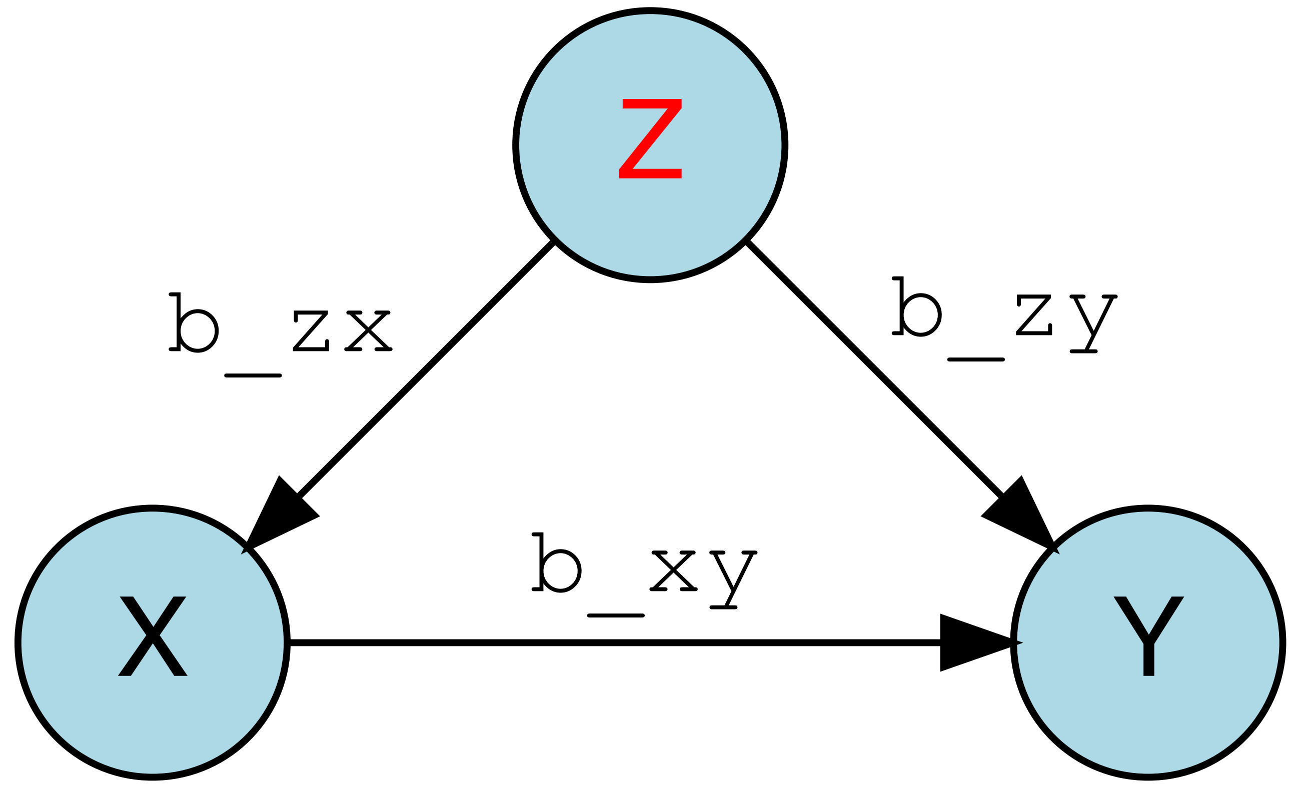
When should I control for variables?

I want to estimate the effect that X has on Y



Is Z a **good** or a **bad** control here?

When should I control for variables?




```r
set.seed(1)

n = 1000
b_zx = 2
b_xy = 2
b_zy = 2
sd = 1

df = tibble(z = rnorm(n = n, sd = sd),
            x = b_zx * z + rnorm(n = n, sd = sd),
            y = b_zy * z + b_xy * x + rnorm(n = n, sd = sd))
```

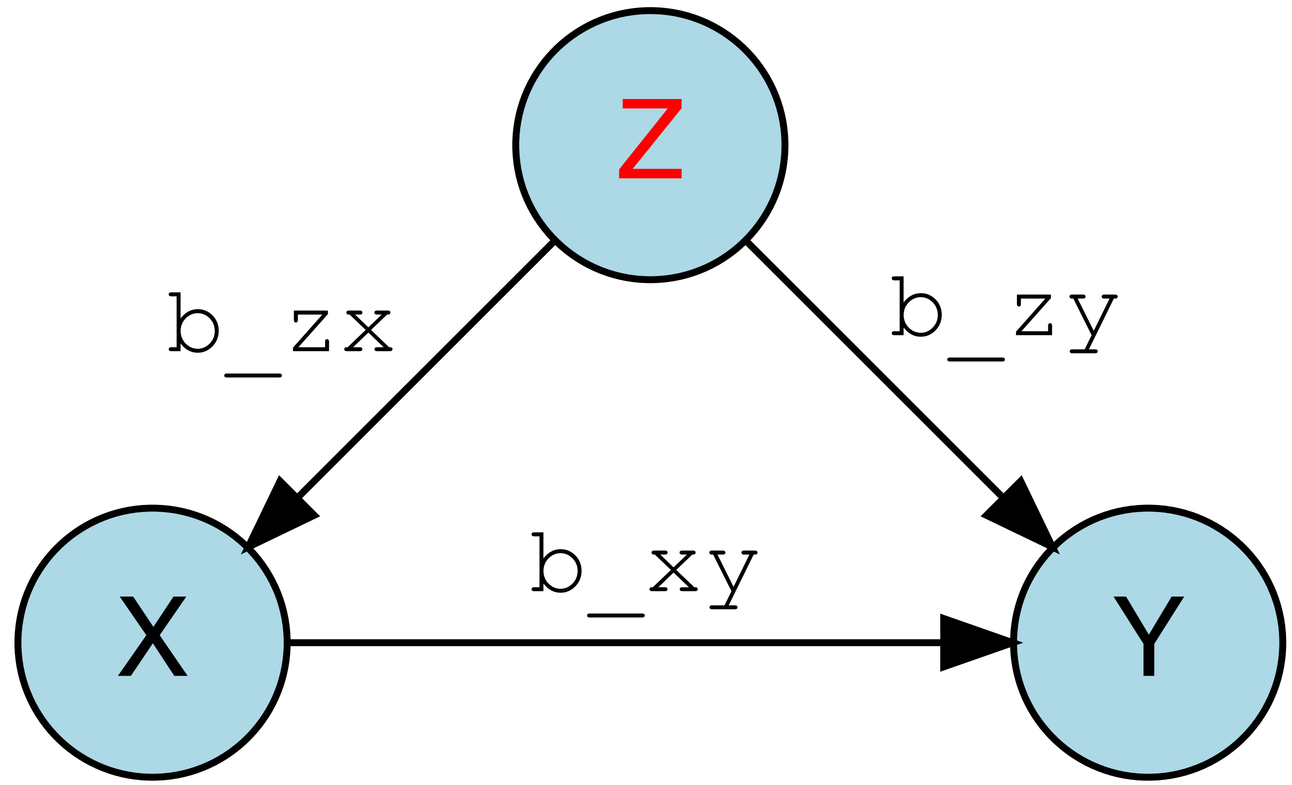
overestimating X's effect on Y

$$Y = b_0 + b_1 \cdot X + e$$

```r
# without control
lm(formula = y ~ x,
   data = df) %>%
  summary()
```

```
Call:
lm(formula = y ~ x, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-4.6011 -0.9270 -0.0506  0.9711  4.0454

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.02449    0.04389   0.558    0.577
x            2.82092    0.01890 149.225   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.388 on 998 degrees of freedom
Multiple R-squared:  0.9571,    Adjusted R-squared:  0.9571
F-statistic: 2.227e+04 on 1 and 998 DF,  p-value: < 2.2e-16
```

When should I control for variables?




```r
set.seed(1)

n = 1000
b_zx = 2
b_xy = 2
b_zy = 2
sd = 1

df = tibble(z = rnorm(n = n, sd = sd),
            x = b_zx * z + rnorm(n = n, sd = sd),
            y = b_zy * z + b_xy * x + rnorm(n = n, sd = sd))
```

$$Y = b_0 + b_1 \cdot X + b_2 \cdot Z + e$$

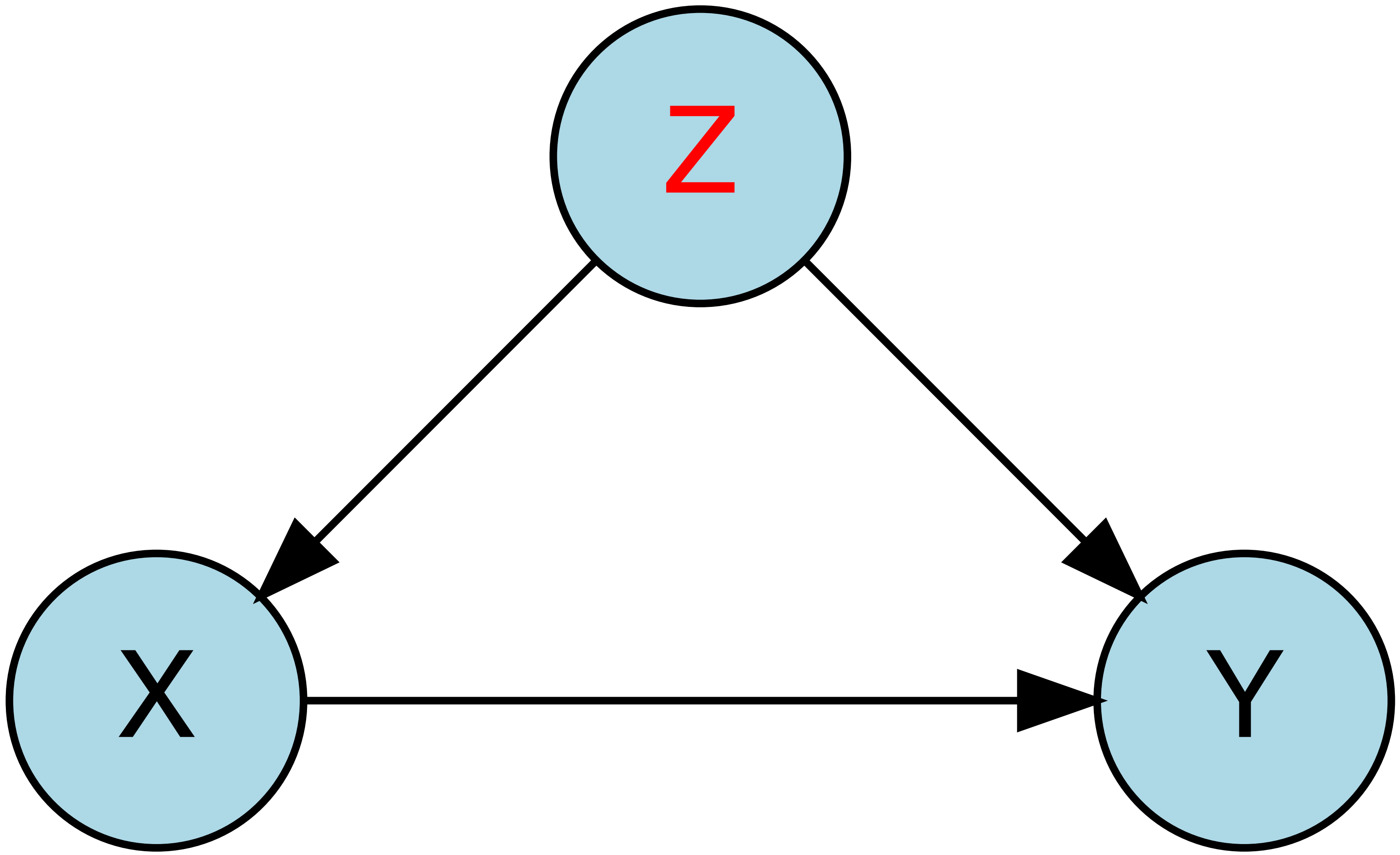
```r
# with control
lm(formula = y ~ x + z,
   data = df) %>%
  summary()
```

accurate estimate of X's effect on Y

```
Call:
lm(formula = y ~ x + z, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-3.6151 -0.6564 -0.0223  0.6815  2.8132

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.01624    0.03260   0.498    0.618
x            2.02202    0.03135  64.489   <2e-16 ***
z            2.00501    0.07036  28.497   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.031 on 997 degrees of freedom
Multiple R-squared:  0.9764,	Adjusted R-squared:  0.9763
F-statistic: 2.059e+04 on 2 and 997 DF,  p-value: < 2.2e-16
```

When should I control for variables?

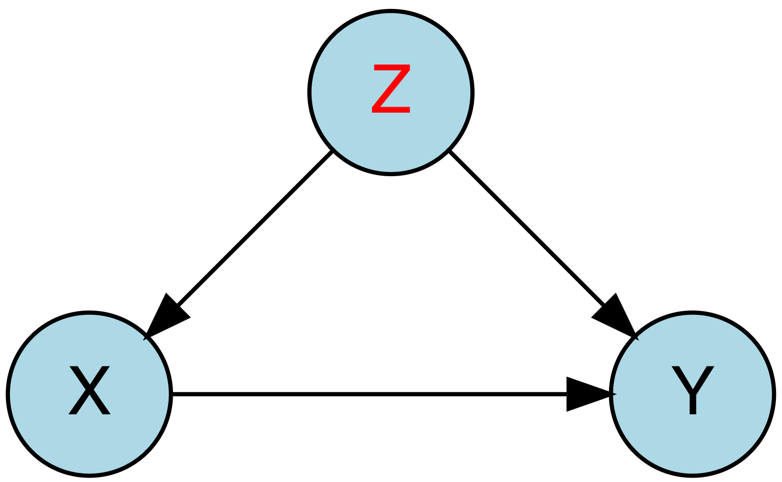
I want to estimate the effect that X has on Y



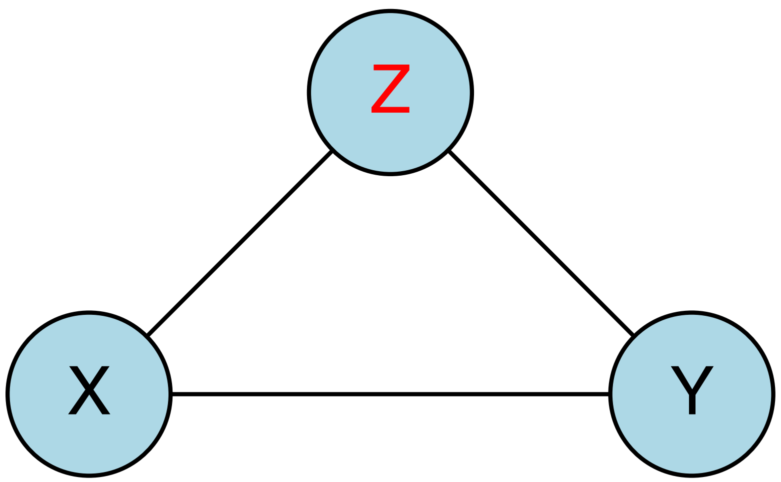
Z is a **good** control here!

When should I control for variables?

I want to estimate the effect that X has on Y

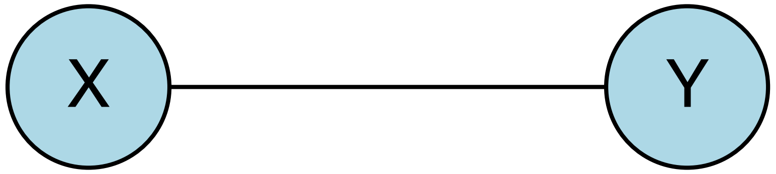


not controlling for Z



multiple paths

controlling for Z

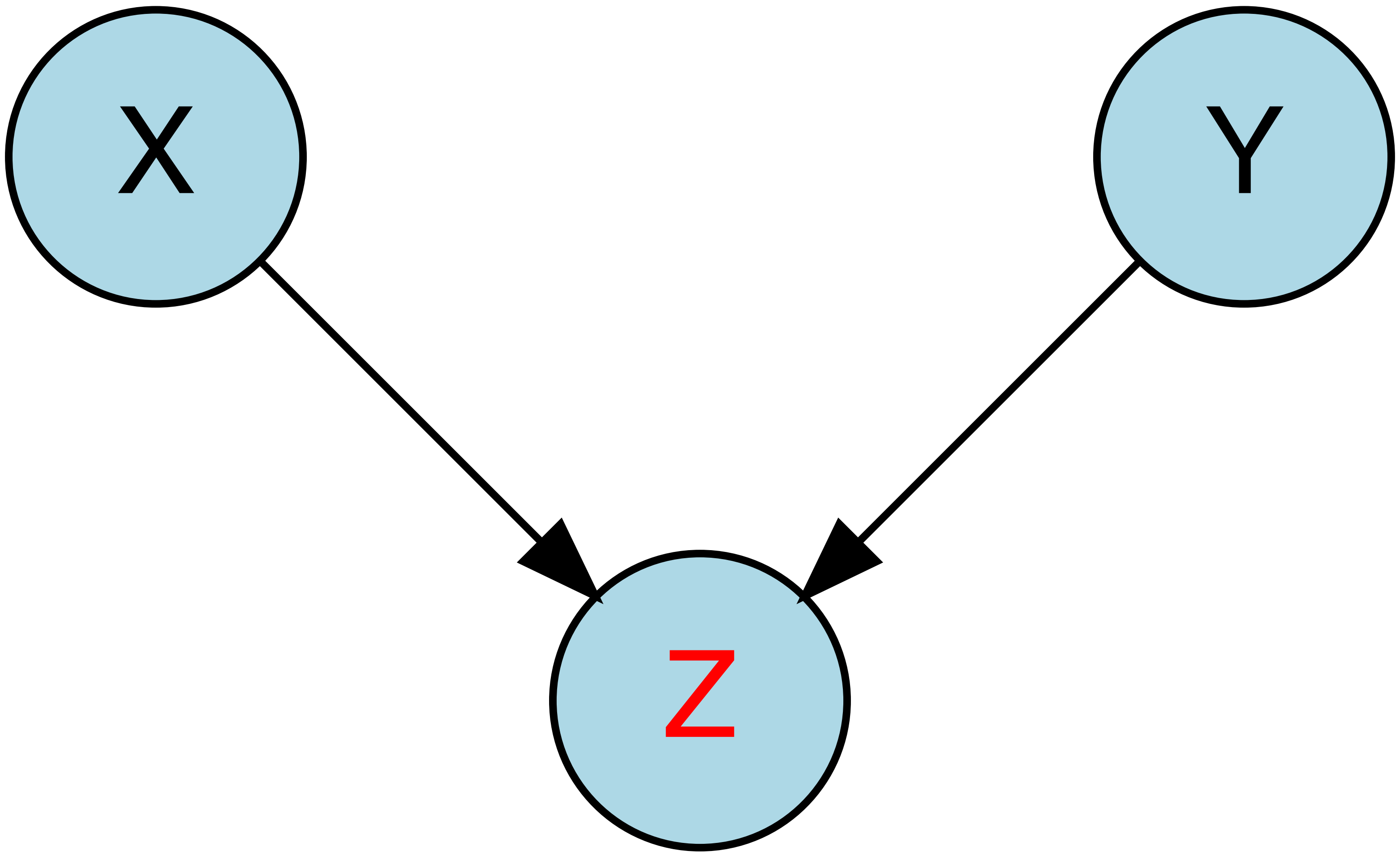


one path

Z is a **good** control here!

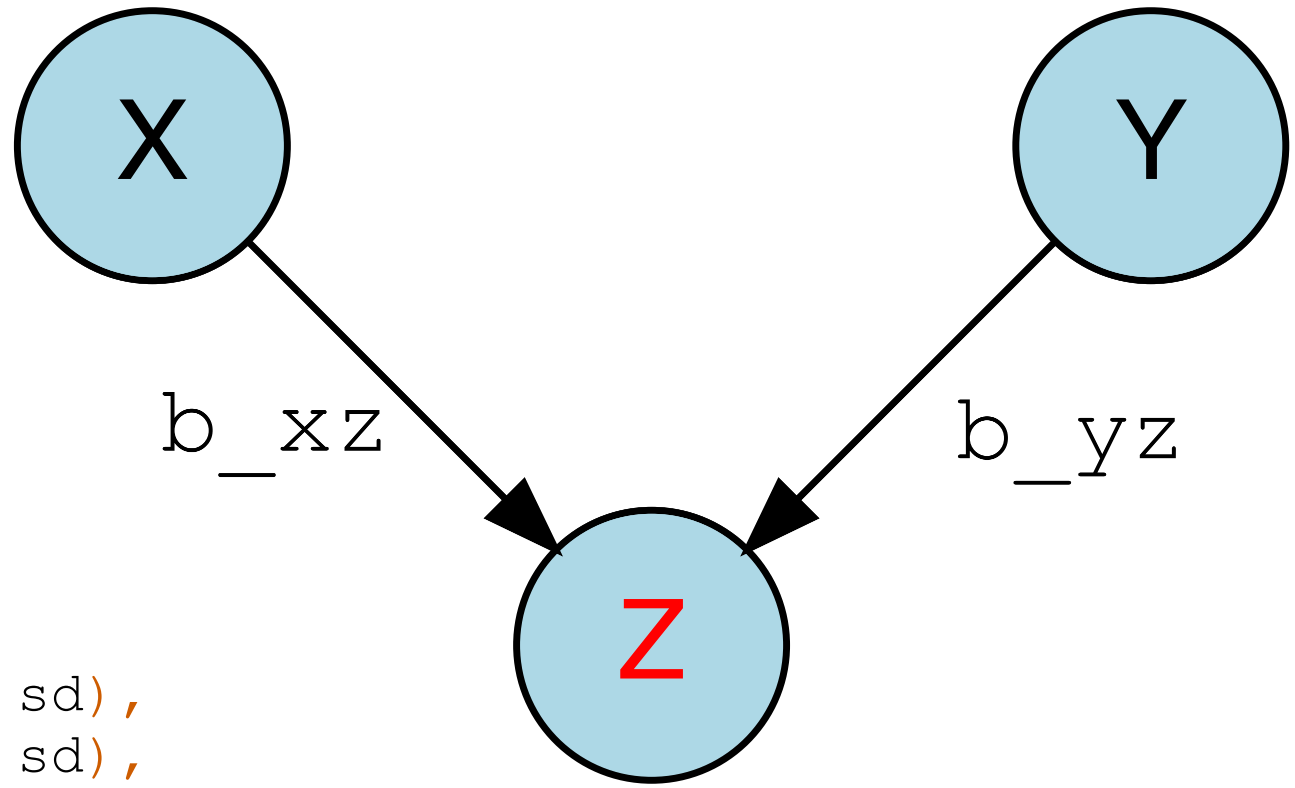
When should I control for variables?

I want to estimate the effect that X has on Y



Is Z a **good** or a **bad** control here?

When should I control for variables?




```r
set.seed(1)

n = 1000
b_xz = 2
b_yz = 2
sd = 1

df = tibble(x = rnorm(n = n, sd = sd),
            y = rnorm(n = n, sd = sd),
            z = x * b_xz + y * b_yz + rnorm(n = n, sd = sd))
```

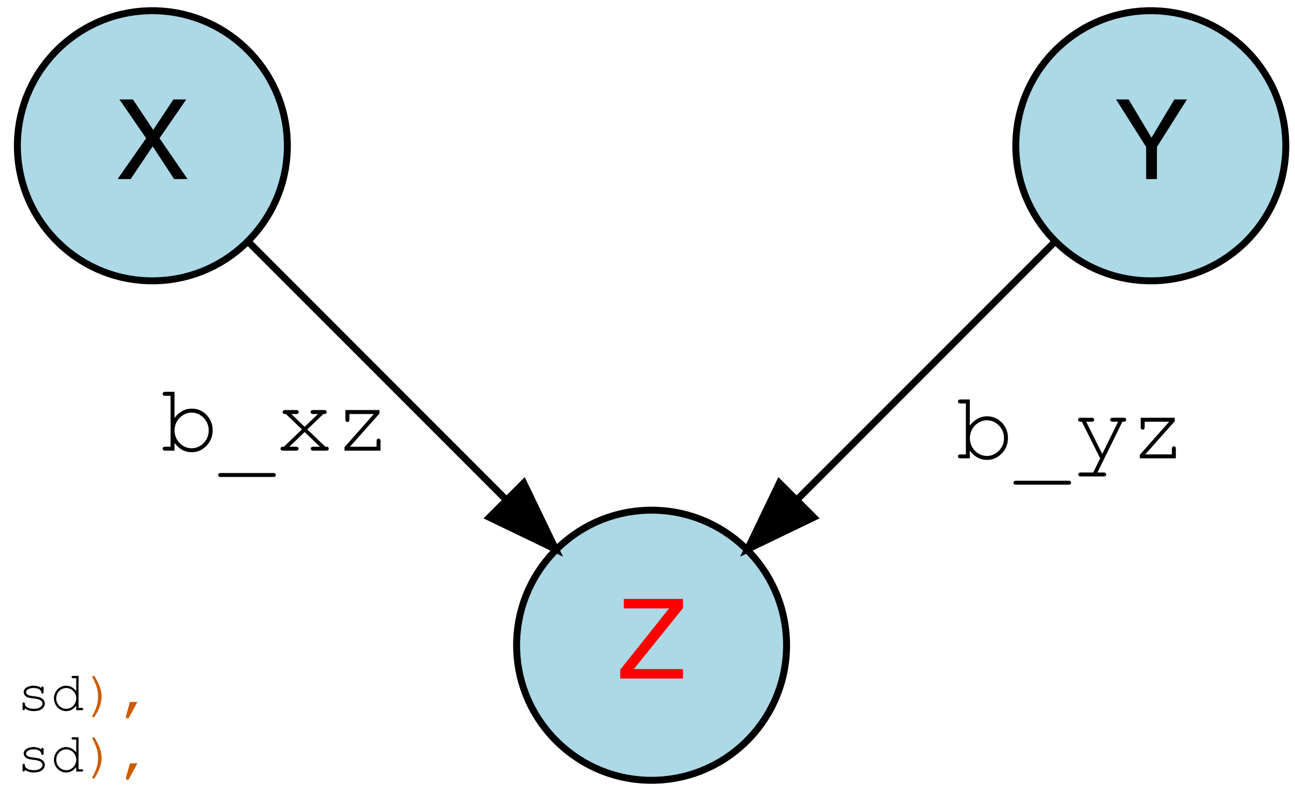
$$Y = b_0 + b_1 \cdot X + e$$

```r
# without control
lm(formula = y ~ x,
   data = df) %>%
  summary()
```

accurate estimate of X's effect on Y

```
Call:
lm(formula = y ~ x, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-3.2484 -0.6720 -0.0138  0.7554  3.6443

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.016187   0.032905  -0.492    0.623
x            0.006433   0.031809   0.202    0.840

Residual standard error: 1.04 on 998 degrees of freedom
Multiple R-squared:  4.098e-05, Adjusted R-squared:  -0.000961
F-statistic: 0.0409 on 1 and 998 DF,  p-value: 0.8398
```

When should I control for variables?




```r
set.seed(1)

n = 1000
b_xz = 2
b_yz = 2
sd = 1

df = tibble(x = rnorm(n = n, sd = sd),
            y = rnorm(n = n, sd = sd),
            z = x * b_xz + y * b_yz + rnorm(n = n, sd = sd))
```

$$Y = b_0 + b_1 \cdot X + b_2 \cdot Z + e$$

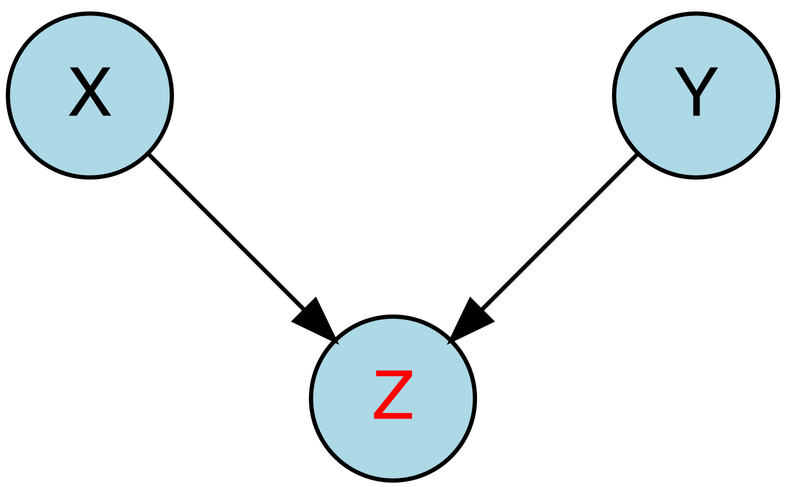
```r
# with control
lm(formula = y ~ x + z,
   data = df) %>%
  summary()
```

inaccurate estimate of X's effect on Y

```
Call:
lm(formula = y ~ x + z, data = df)

Residuals:
     Min       1Q   Median       3Q      Max
-1.35547 -0.30016  0.00298  0.31119  1.73408

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.009608   0.014477  -0.664    0.507
x           -0.816164   0.018936 -43.102   <2e-16 ***
z            0.398921   0.006186  64.489   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4578 on 997 degrees of freedom
Multiple R-squared:  0.8066,    Adjusted R-squared:  0.8062
F-statistic:  2079 on 2 and 997 DF,  p-value: < 2.2e-16
```

When should I control for variables?

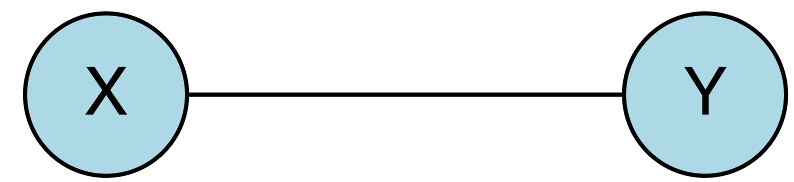
I want to estimate the effect that X has on Y



not controlling for Z

no path

controlling for Z



one path

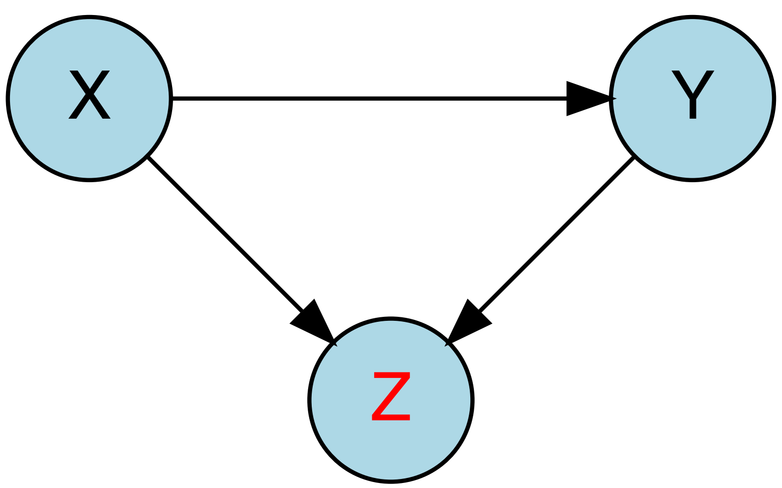
Z is a **bad** control here!

When should I control for variables?

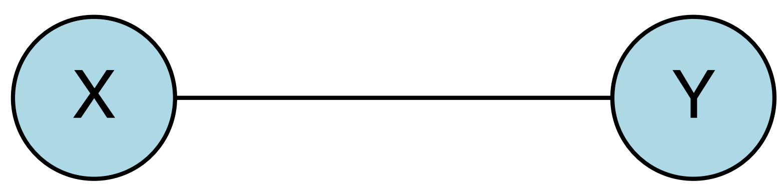
- checking for **d-separation** tells us whether or not variables are (conditionally) independent
- it also tells us whether paths of dependence "open up", or get "closed down"
- the graphical procedure doesn't necessarily reveal whether the dependence between variables changes: it reveals the **structure** of dependence but not the **strength**
- you can always double check via running simulations in R

When should I control for variables?

I want to estimate the effect that X has on Y

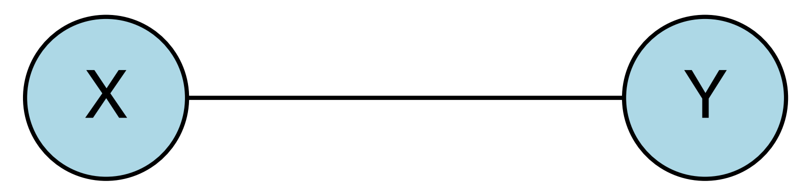


not controlling for Z



one path

controlling for Z



change in dependence

not revealed in the undirected graph

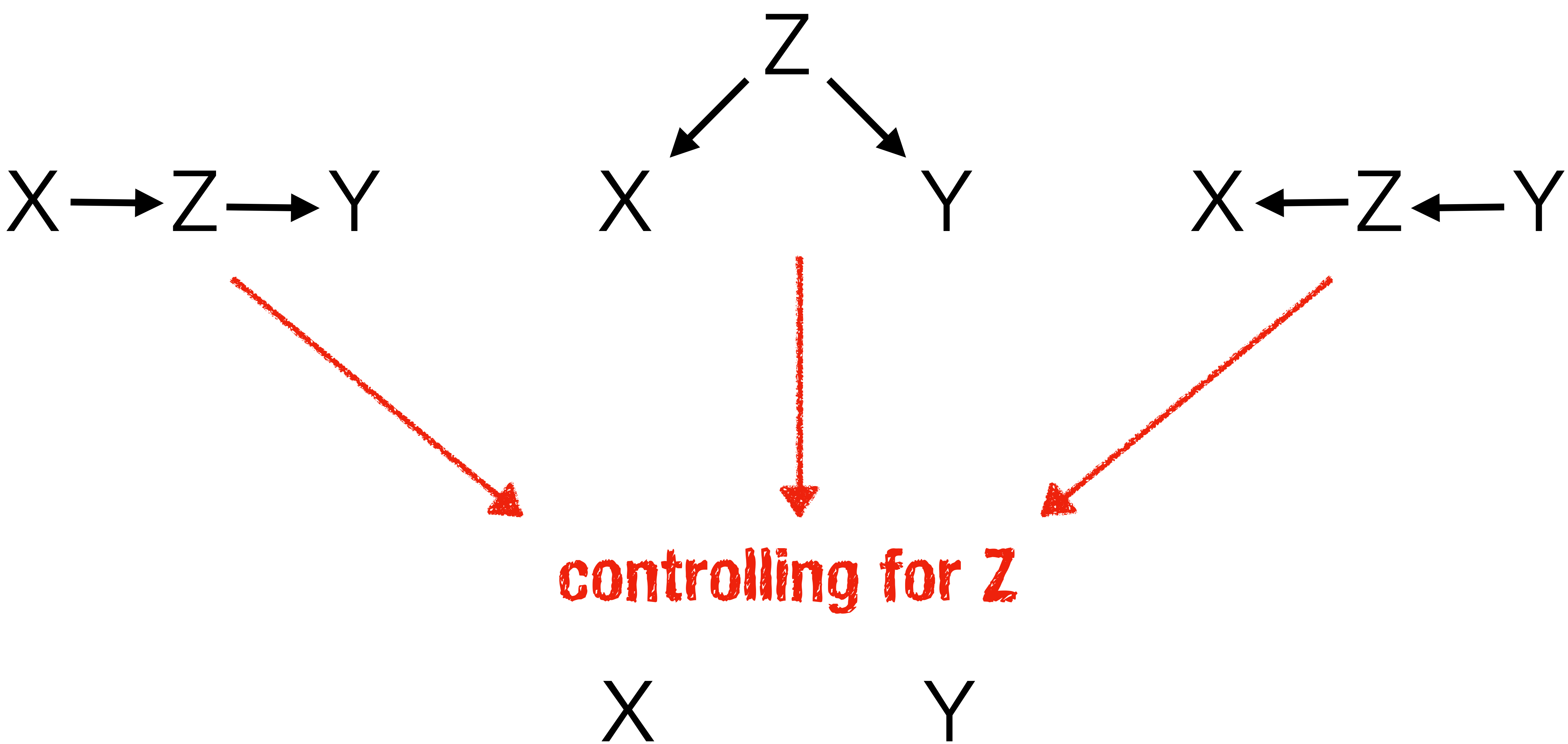
Z is a **bad** control here!

When should I control for variables?

- **good controls** reduce additional paths from `X` to `Y` apart from the direct path we are interested in estimating
- **bad controls** introduce additional paths (or change existing ones) that lead to a biased estimate of the direct path between `X` and `Y`

When should I control for variables?

Problem: We don't know the ground truth ...



we need to manipulate X experimentally to tell these apart*

* sort of (see next slide)

When should I control for variables?

- **causal discovery** is a very active field

what causal claims can we make from observational data?

Identifiability of Gaussian structural equation models with equal error variances

Jonas Peters*
Seminar for Statistics
ETH Zurich
Switzerland

Peter Bühlmann*
Seminar for Statistics
ETH Zurich
Switzerland

October 29, 2018

causal model is fully identifiable if all noise variables have the same variances, and all variables are observed

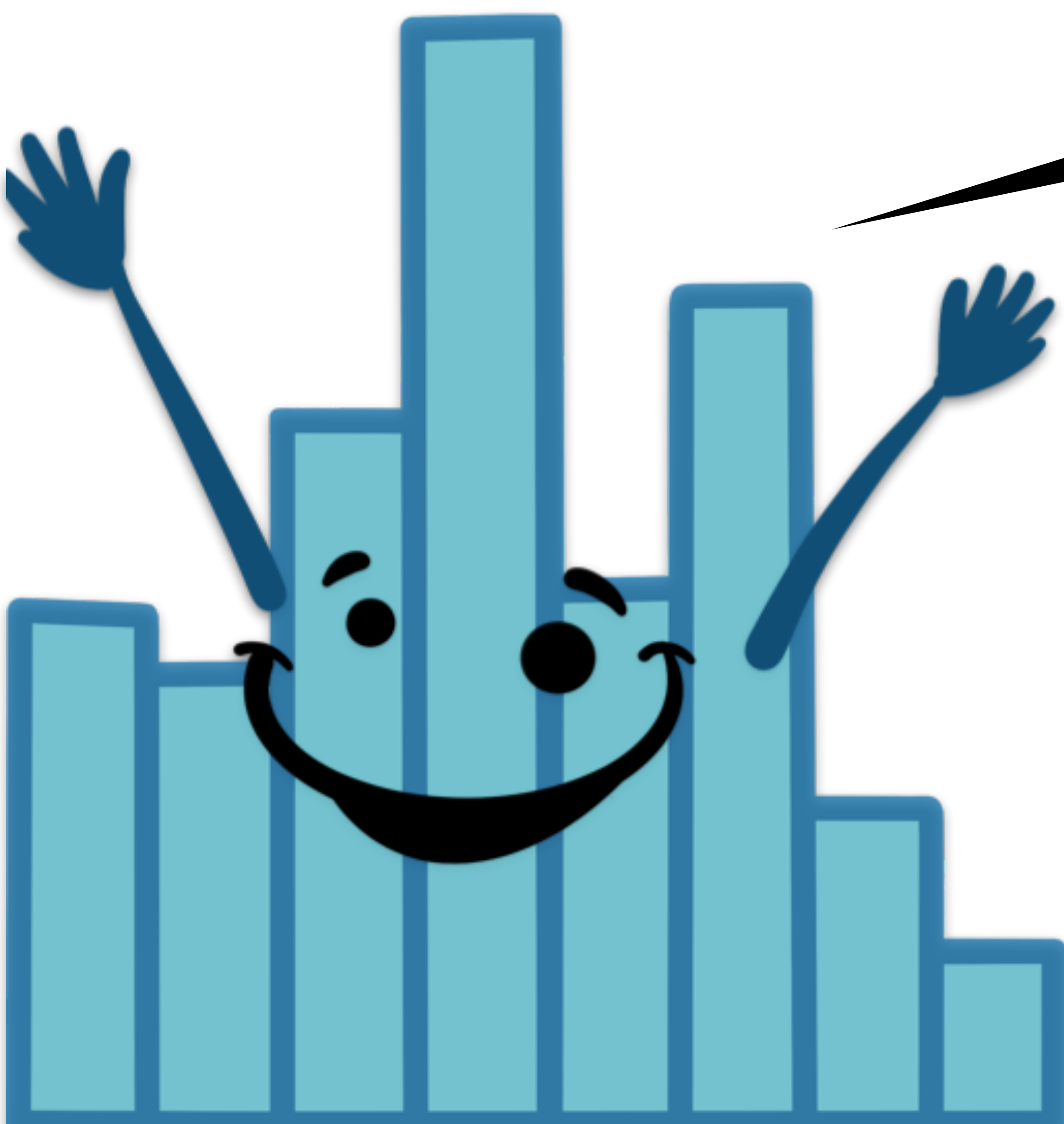
beyond the scope of our class ...

Peters & Buhlmann (2018) Identifiability of Gaussian structural equation models with equal error variances. *Biometrika*

We're listening to "Talk Of The Town" by "Jack Johnson, Kawika Kahiapo" submitted by Thing Thinker

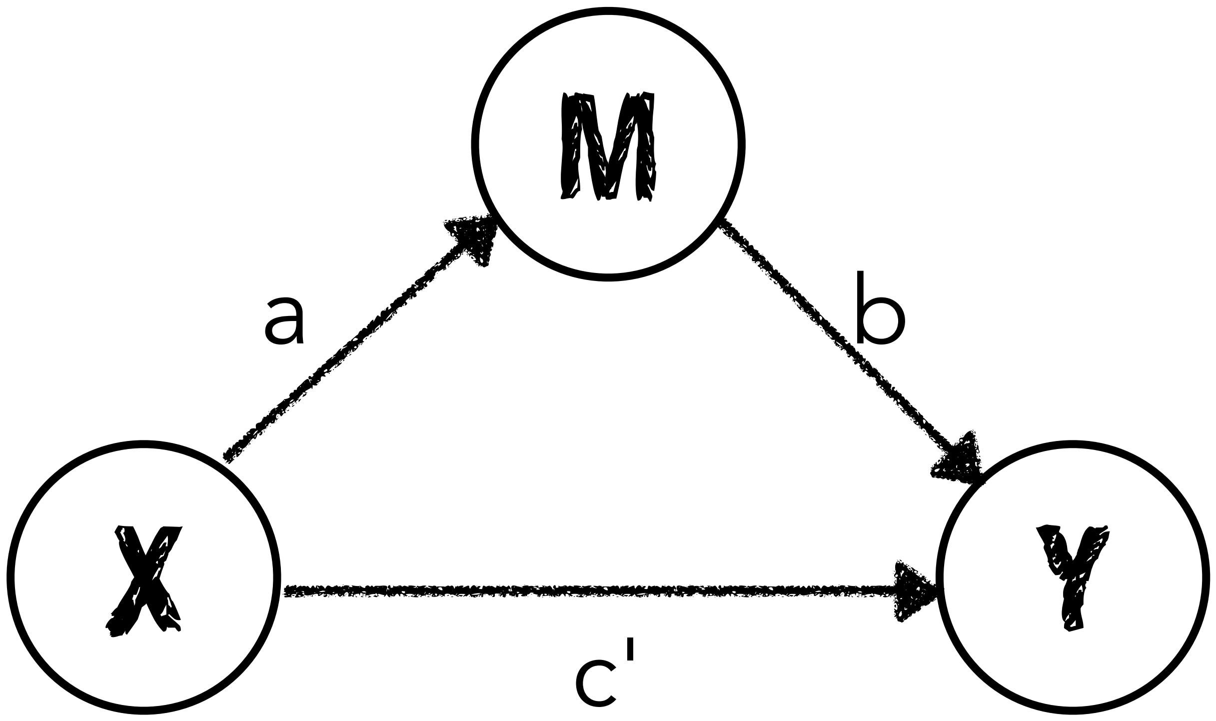
02:00

stretch break!



Mediation

Definition

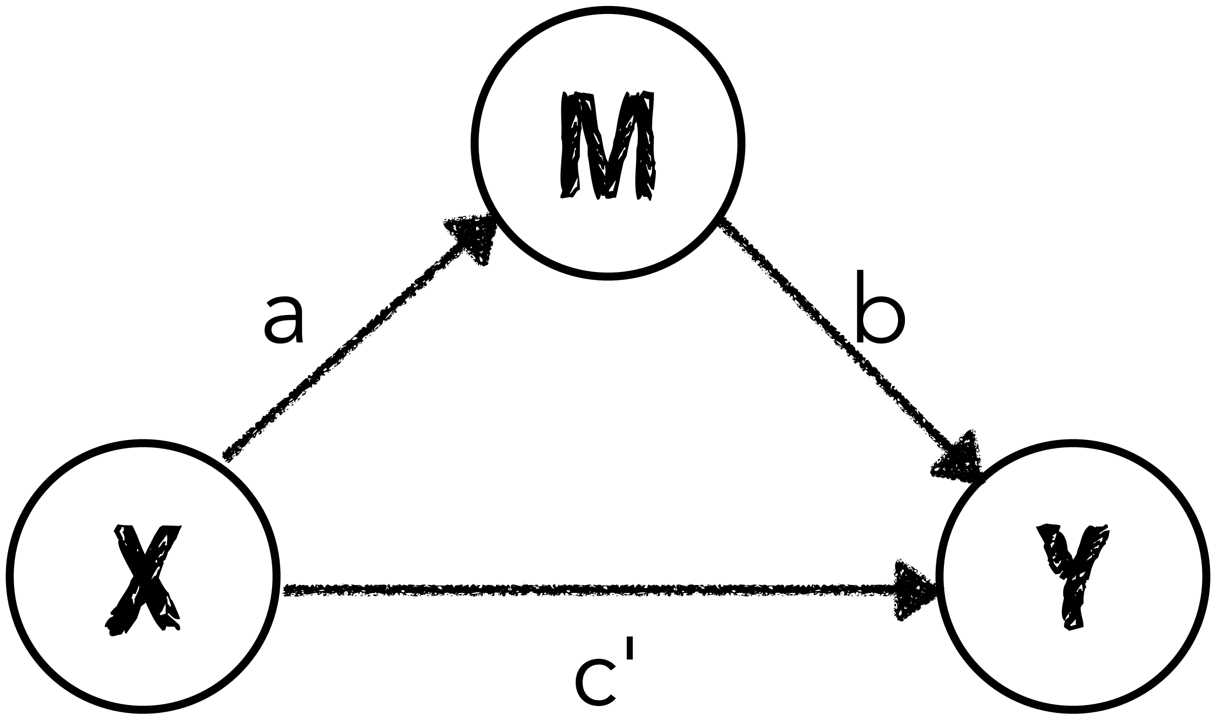


Rather than a direct causal relationship between **X** and **Y**, a mediation model proposes that **X** influences the mediator variable **M**, which in turn influences **Y**. Thus, the mediator variable serves to clarify the nature of the relationship between **X** and **Y**.

Adapted from Wikipedia

https://en.wikipedia.org/wiki/Mediation_(statistics)

Example



X = grades in Psych 252

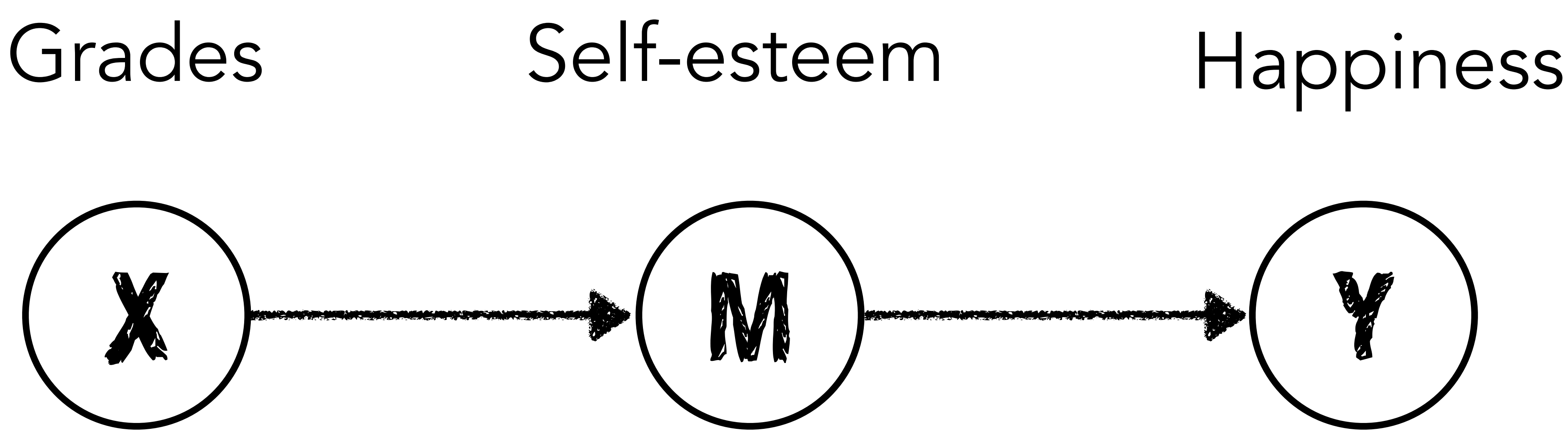
M = feelings of self-esteem

Y = happiness

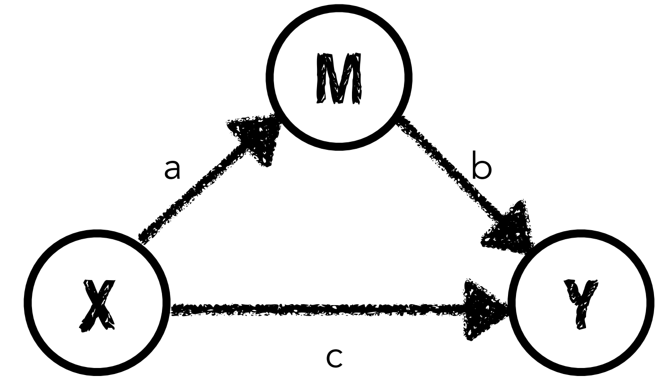
Is the relationship between grades in Psych 252 and happiness mediated by feelings of self-esteem?

Simulate a mediation analysis

```r
# number of participants
n = 100

# generate data
df.mediation = tibble(
  x = rnorm(n, 75, 7),             # grades
  m = 0.7 * x + rnorm(n, 0, 5), # self-esteem
  y = 0.4 * m + rnorm(n, 0, 5)  # happiness
)
```



Bootstrapping




```r
library("mediation")

# bootstrapped mediation
fit.mediation = mediate(model.m = fit.m_x,
                        model.y = fit.y_mx,
                        treat = "x",
                        mediator = "m",
                        boot = T)

# summarize results
fit.mediation %>% summary()
```

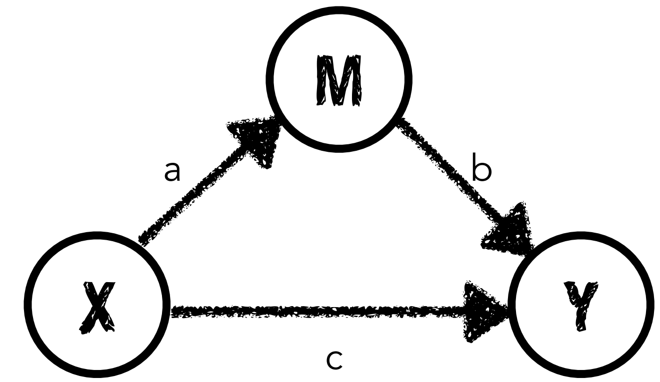
$$\hat{m} = b_0 + b_1 \cdot x$$

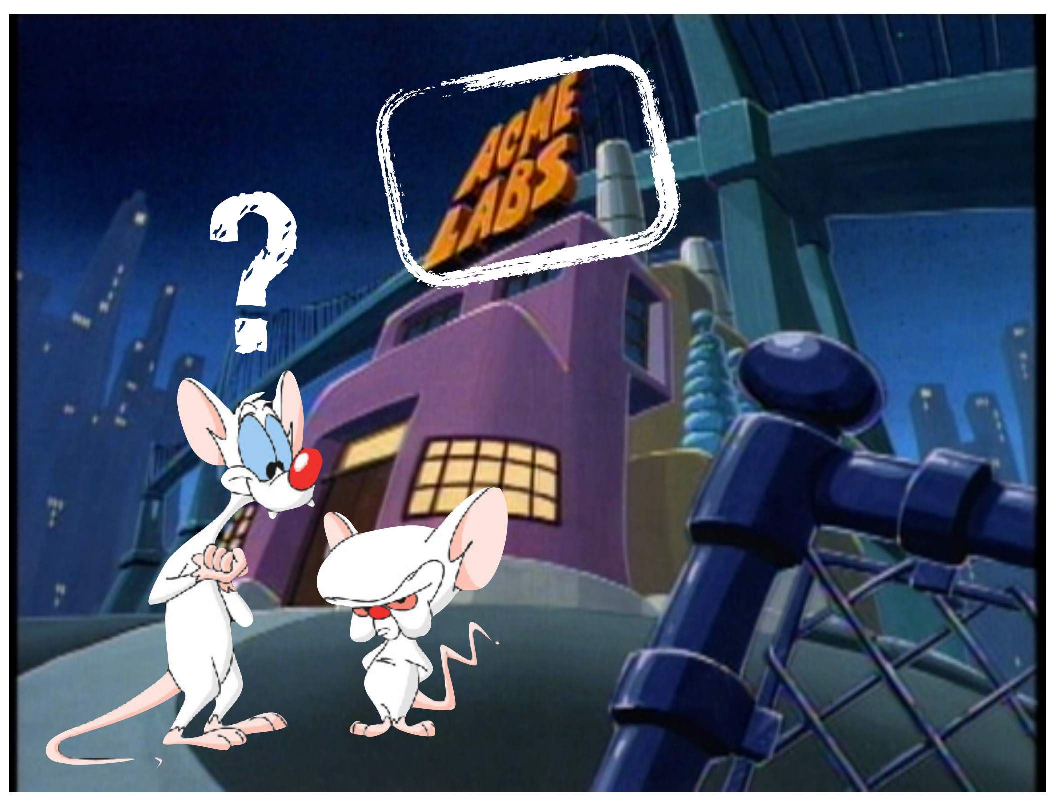
$$\hat{y} = b_0 + b_1 \cdot m + b_2 \cdot x$$

```
Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value    
ACME            0.28078      0.14059         0.42  <2e-16 ***
ADE            -0.11179     -0.29276         0.10   0.272    
Total Effect    0.16899     -0.00415         0.34   0.064 .  
Prop. Mediated  1.66151     -3.22476        11.46   0.064 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100


Simulations: 1000
```

2. Bootstrapping

```
Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

                Estimate 95% CI Lower 95% CI Upper p-value
ACME             0.28078      0.14059         0.42  <2e-16 ***
ADE             -0.11179     -0.29276         0.10   0.272
Total Effect     0.16899     -0.00415         0.34   0.064 .
Prop. Mediated  1.66151     -3.22476        11.46   0.064 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100


Simulations: 1000
```





2. Bootstrapping

```
Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value
ACME            0.28078      0.14059         0.42  <2e-16 ***
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Prop. Mediated  1.66151     -3.22476        11.46   0.064 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Sample Size Used: 100


Simulations: 1000
```
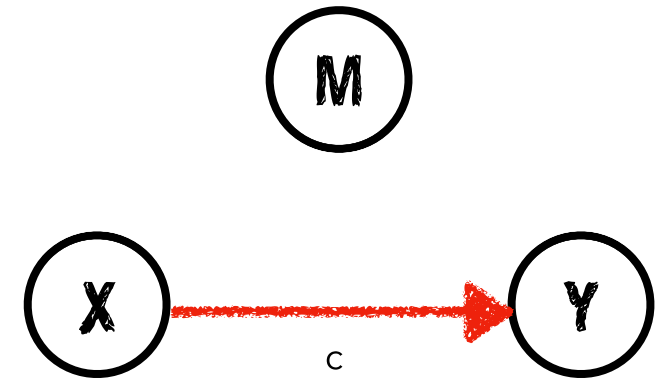
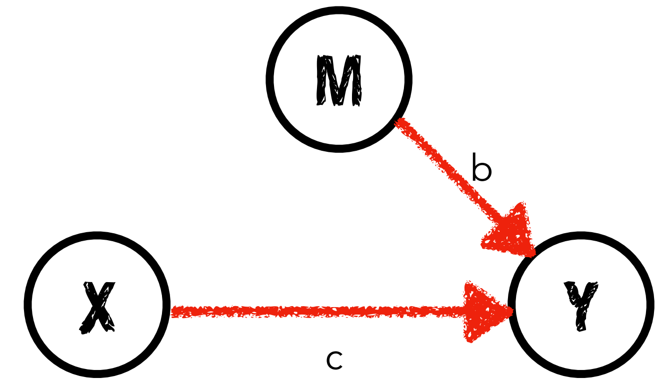



$$\hat{y} = b_0 + b_1 \cdot x$$

```
Call:
lm(formula = y ~ 1 + x, data = df.mediation)

Residuals:
    Min      1Q  Median      3Q     Max
-10.917  -3.738  -0.259   2.910  12.540

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  8.78300    6.16002   1.426   0.1571
x            0.16899    0.08116   2.082   0.0399 *
```

2. Bootstrapping



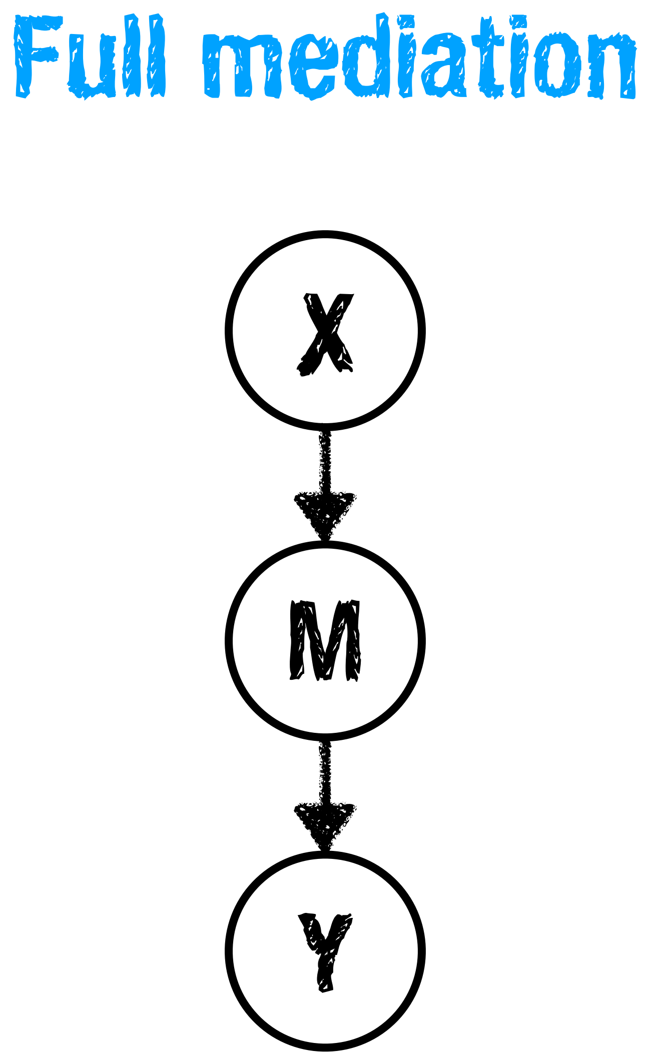

```
Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value    
ACME            0.28078      0.14059         0.42  <2e-16 ***
ADE            -0.11179     -0.29276         0.10   0.272    
Total Effect    0.16899     -0.00415         0.34   0.064 .  
Prop. Mediated  1.66151     -3.22476        11.46   0.064 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100


Simulations: 1000
```

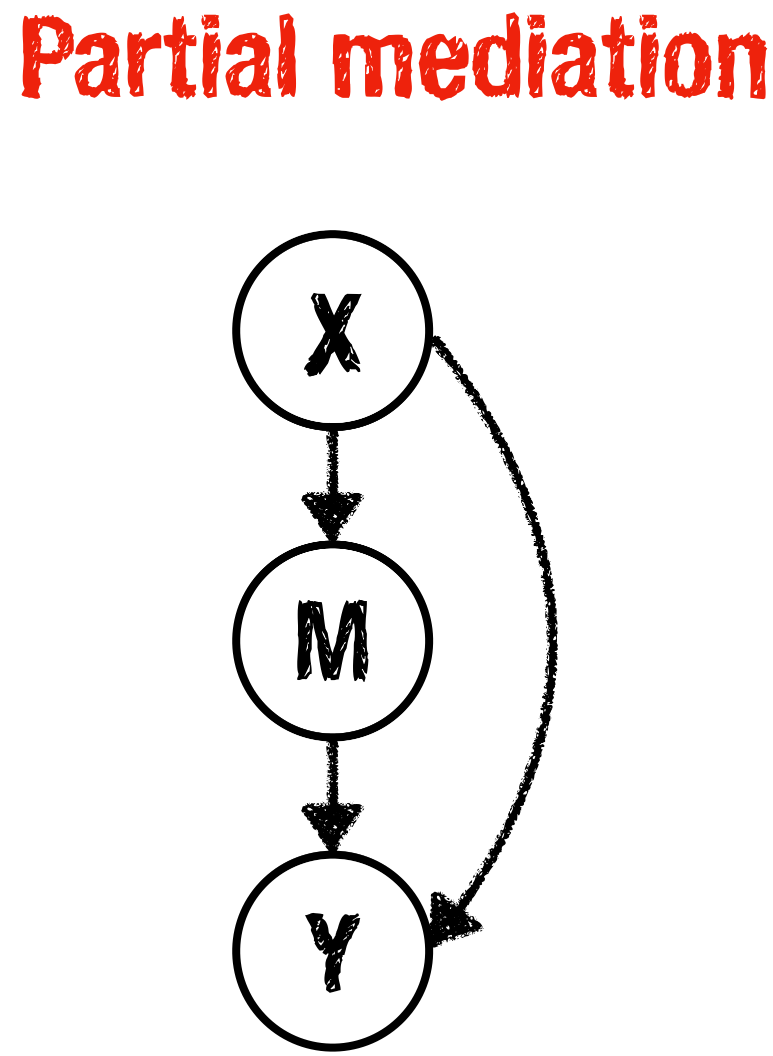
$$\hat{y} = b_0 + b_1 \cdot m + b_2 \cdot x$$

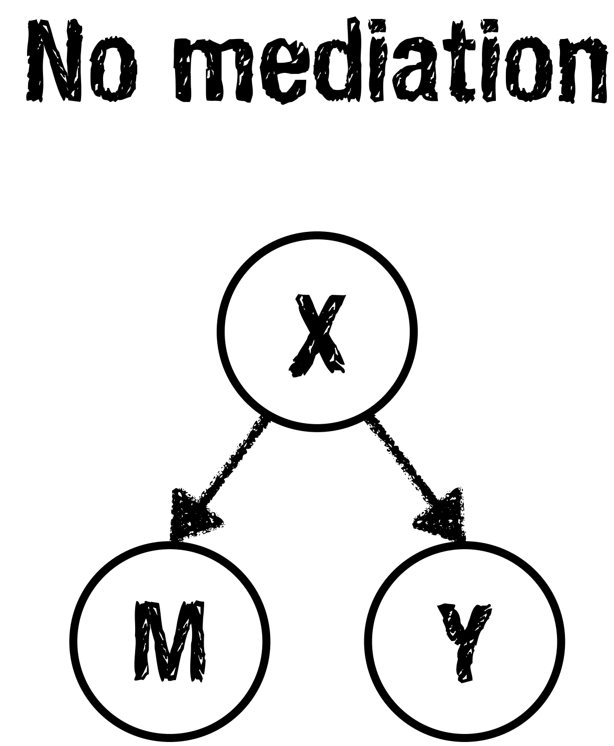
ADE: Average direct effect

```
Call:
lm(formula = y ~ 1 + m + x, data = df.mediation)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.3651 -3.3037 -0.6222  3.1068 10.3991 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  7.80952    5.68297   1.374    0.173    
m            0.42381    0.09899   4.281 4.37e-05 ***
x           -0.11179    0.09949  -1.124    0.264    
```

Underlying causal model





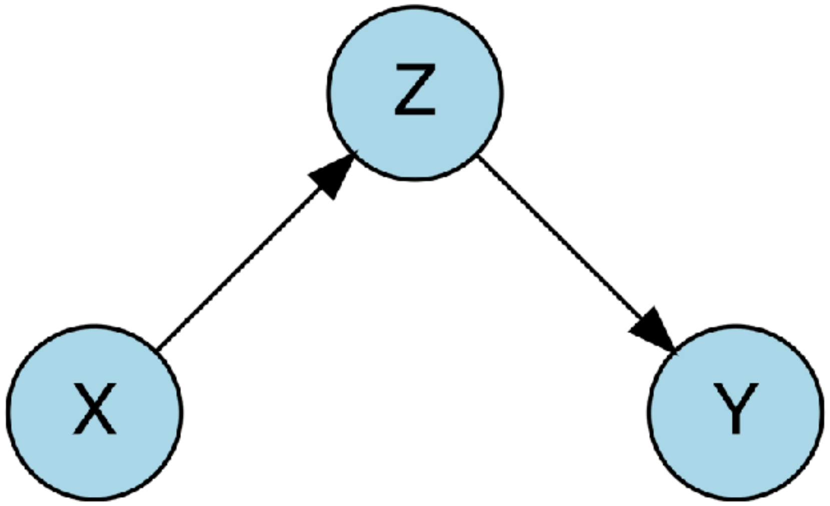


Full mediation: When the effect of **X** on **Y** completely disappears, **M** fully mediates between **X** and **Y**.

Partial mediation: When the effect of **X** on **Y** still exists, but in a smaller magnitude, **M** partially mediates between **X** and **Y**.

Beware of mediation analyses!!!

```
set.seed(1)

n = 100 # number of observations

# causal chain
df.causal_chain = tibble(x = rnorm(n, 0, 1),
                         z = 2 * x + rnorm(n, 0, 1),
                         y = 2 * z + rnorm(n, 0, 1))
```

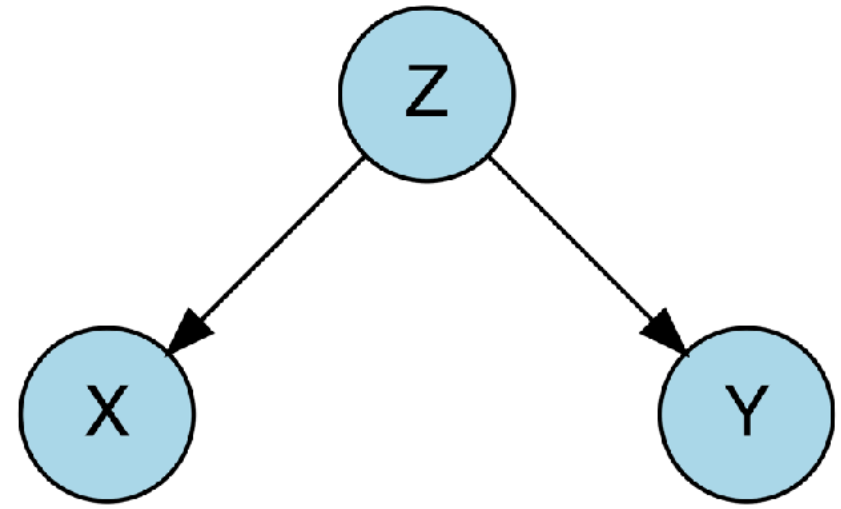



```
Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value
ACME             0.8287       0.6234         1.05  <2e-16 ***
ADE             -0.0535      -0.2548         0.15    0.55
Total Effect     0.7752       0.6391         0.90  <2e-16 ***
Prop. Mediated   1.0690       0.8131         1.35  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100


Simulations: 1000
```

nice mediation result!

Beware of mediation analyses!!!

```r
set.seed(1)

n = 100 # number of observations

# common cause
df.common_cause = tibble(z = rnorm(n, 0, 1),
                         x = 2 * z + rnorm(n, 0, 1),
                         y = 2 * z + rnorm(n, 0, 1))
```

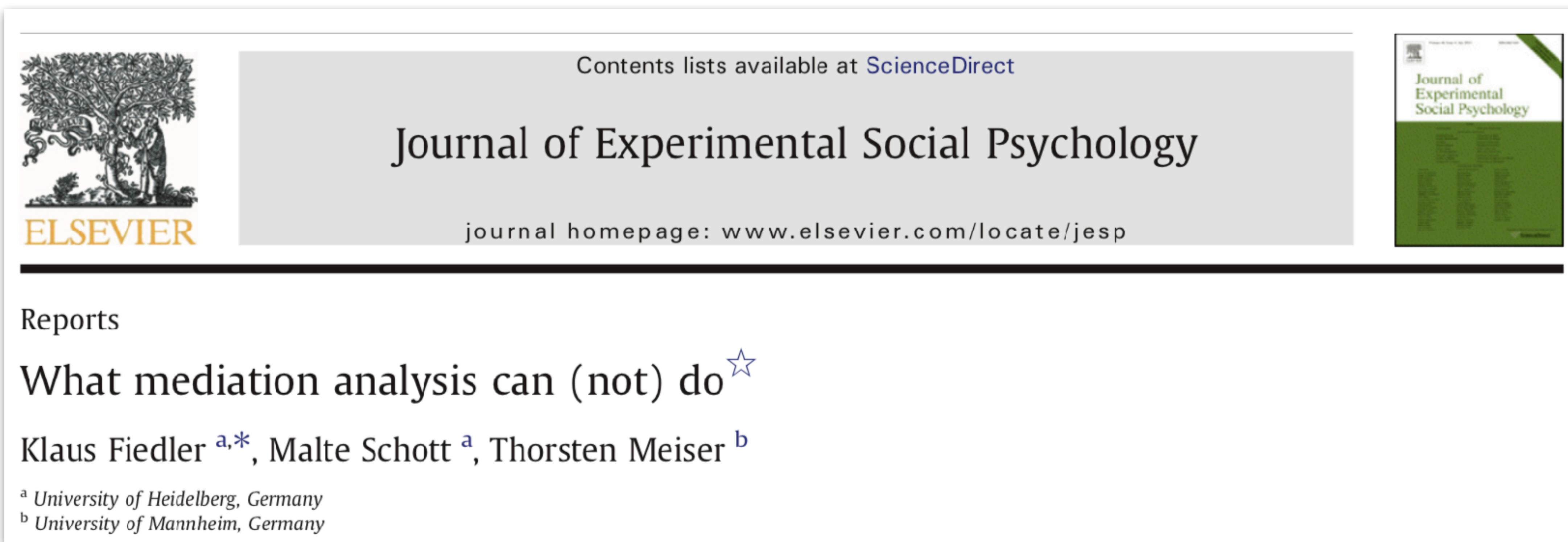



```
Causal Mediation Analysis

Nonparametric Bootstrap Confidence Intervals with the Percentile Method

               Estimate 95% CI Lower 95% CI Upper p-value    
ACME             0.8287       0.6065         1.04  <2e-16 ***
ADE             -0.0535      -0.2675         0.16    0.56    
Total Effect     0.7752       0.6353         0.90  <2e-16 ***
Prop. Mediated   1.0690       0.8134         1.37  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Sample Size Used: 100


Simulations: 1000
```

(not) nice mediation result!

Limitations

- correlational analysis
  - we need theories / experiments to tease apart causes and effects to properly map our variables onto the diagram


Contents lists available at ScienceDirect

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Reports

What mediation analysis can (not) do☆

Klaus Fiedler [a,*], Malte Schott [a], Thorsten Meiser [b]

[a] University of Heidelberg, Germany
[b] University of Mannheim, Germany

Fiedler, K., Schott, M., & Meiser, T. (2011). What mediation analysis can (not) do. *Journal of Experimental Social Psychology*, 47(6), 1231-1236.

Limitations

many-to-one mapping

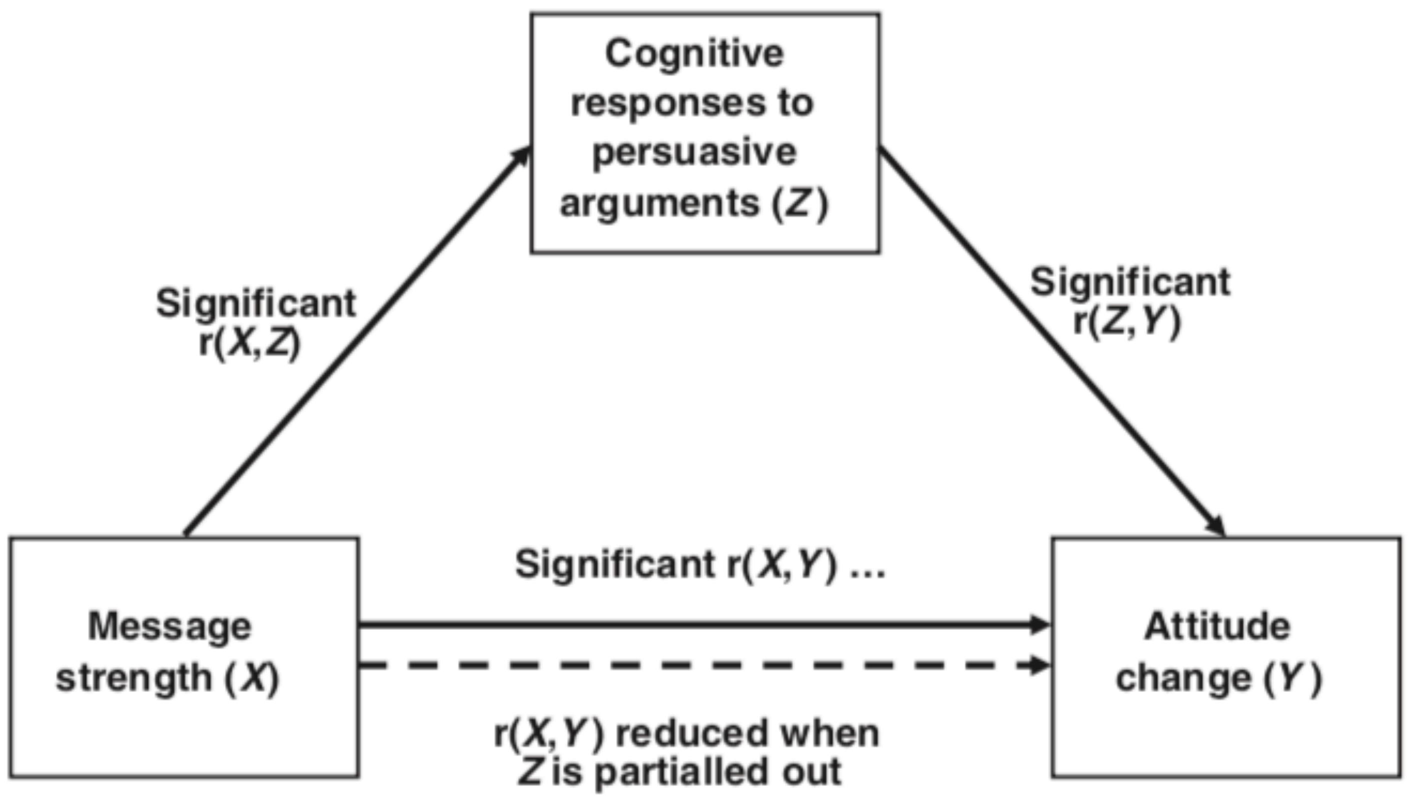


Fig. 1. Illustration of a mediation analysis in the context of persuasion research: The impact of message strength (X) on attitude change (Y) is supposed to be mediated by the recipients' cognitive responses (Z).

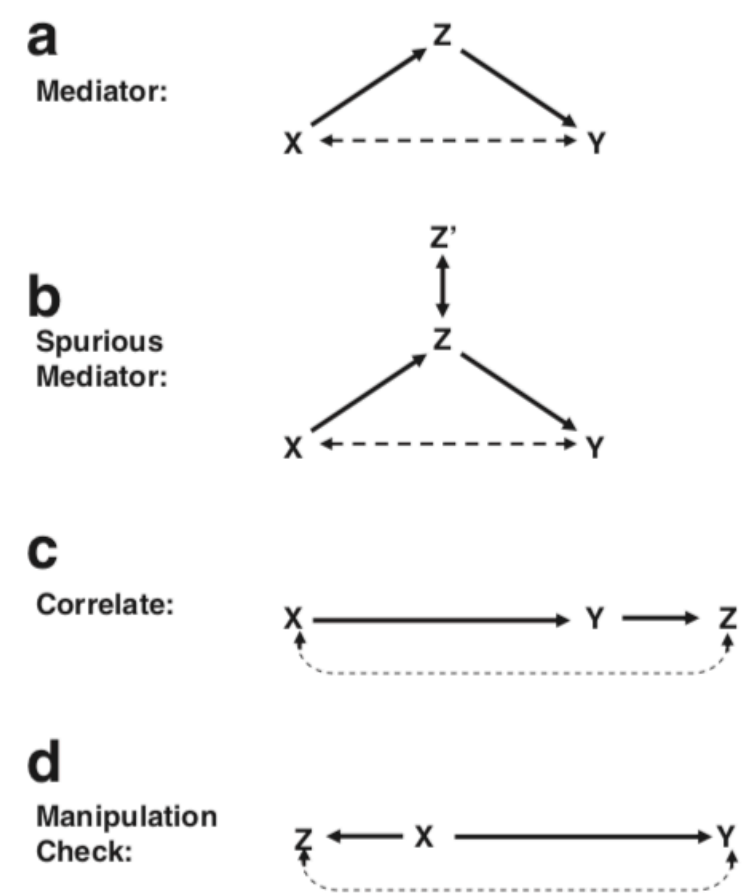


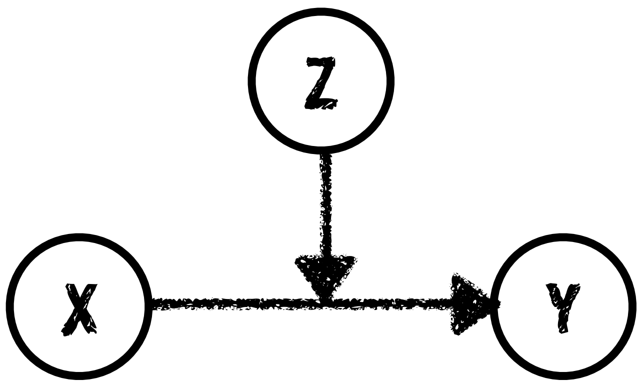
Fig. 2. Different causal models for the interpretation of the same intercorrelations between three variables, X, Y, and Z.

only experiments allow us to tell apart possible causal structures

Fiedler, K., Schott, M., & Meiser, T. (2011). What mediation analysis can (not) do. *Journal of Experimental Social Psychology*, 47(6), 1231-1236.

Moderation

Definition

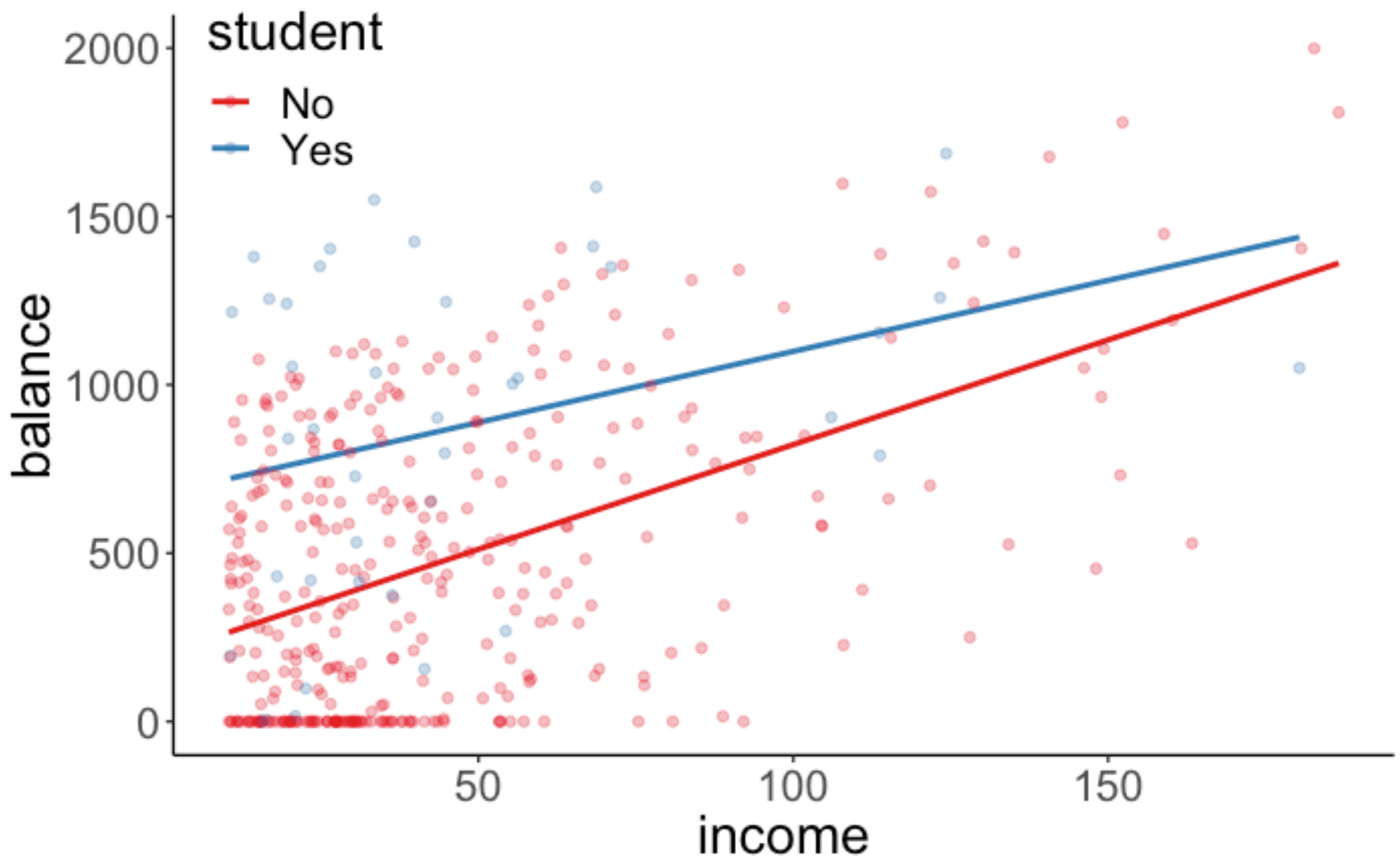


Moderation means that the effect of a predictor depends on the value of another.

Here, the nature of the relationship between **X** and **Y** depends on **Z**.

Have we come across moderation already?

Relationship between credit card balance, income, and whether the person is a student.



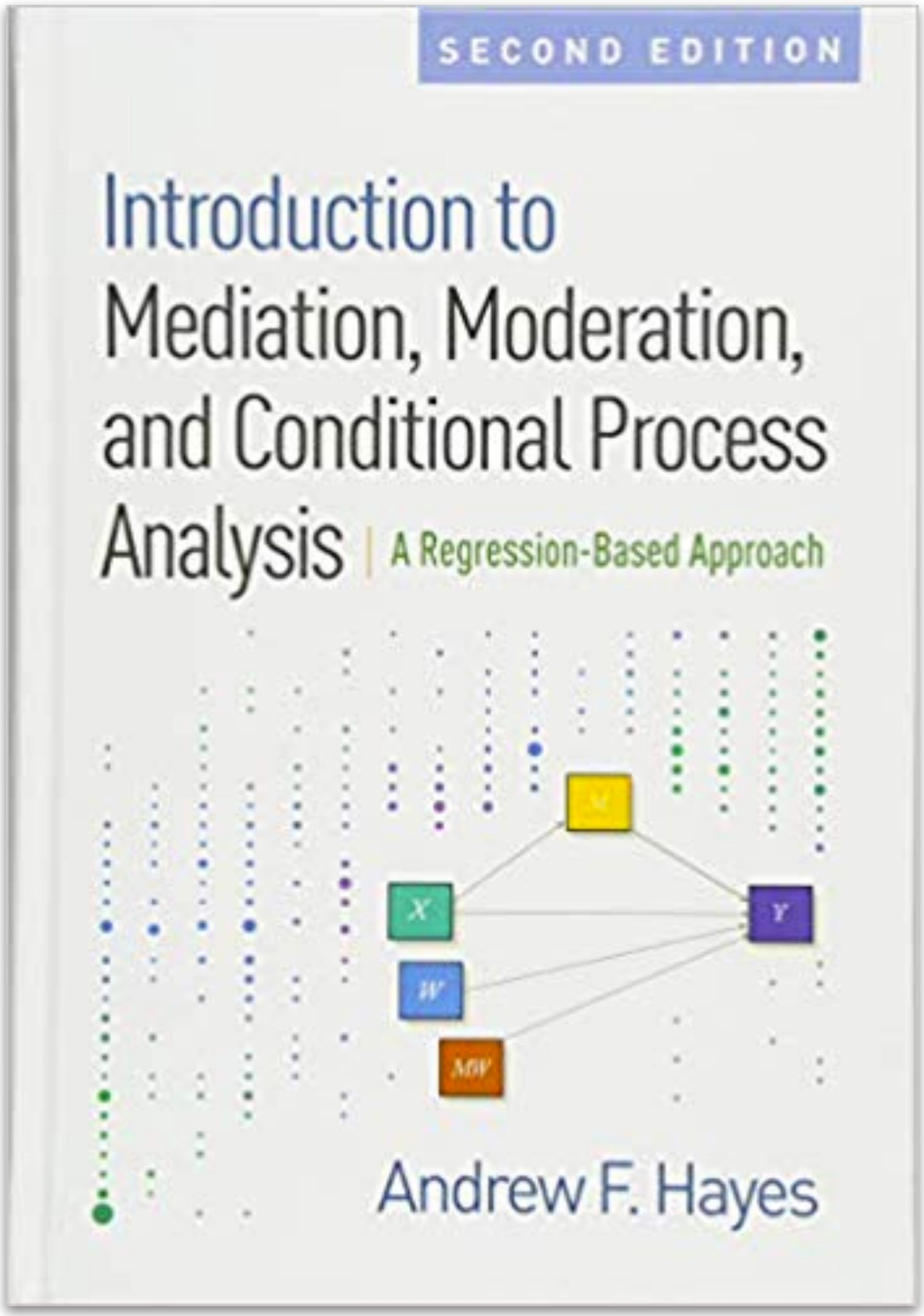
$$\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot \text{student}_i - 2.00 \cdot (\text{income}_i \times \text{student}_i)$$

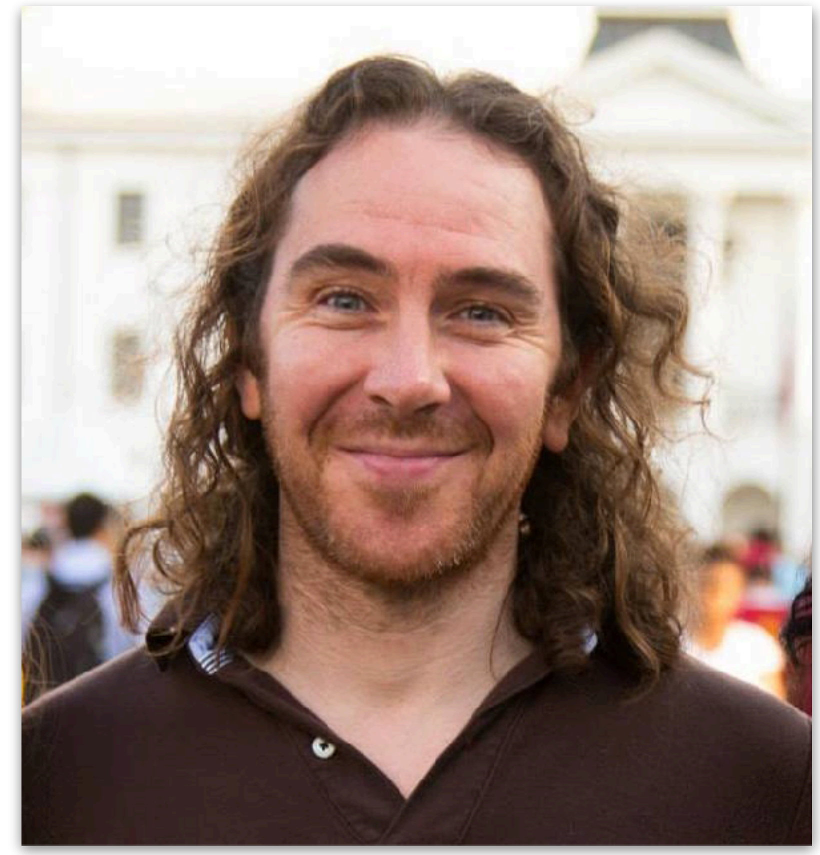
if student = "No" $\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i$

if student = "Yes"

$$\begin{aligned}\widehat{\text{balance}}_i &= 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot 1 - 2.00 \cdot (\text{income}_i \times 1) \\ &= 677.3 + 6.22 \cdot \text{income}_i - 2.00 \cdot \text{income}_i \\ &= 677.3 + 4.22 \cdot \text{income}_i\end{aligned}$$

Learn more about mediation and moderation





Recoded with `brms` by Solomon Kurz here: https://bookdown.org/connect/#/apps/1523/access

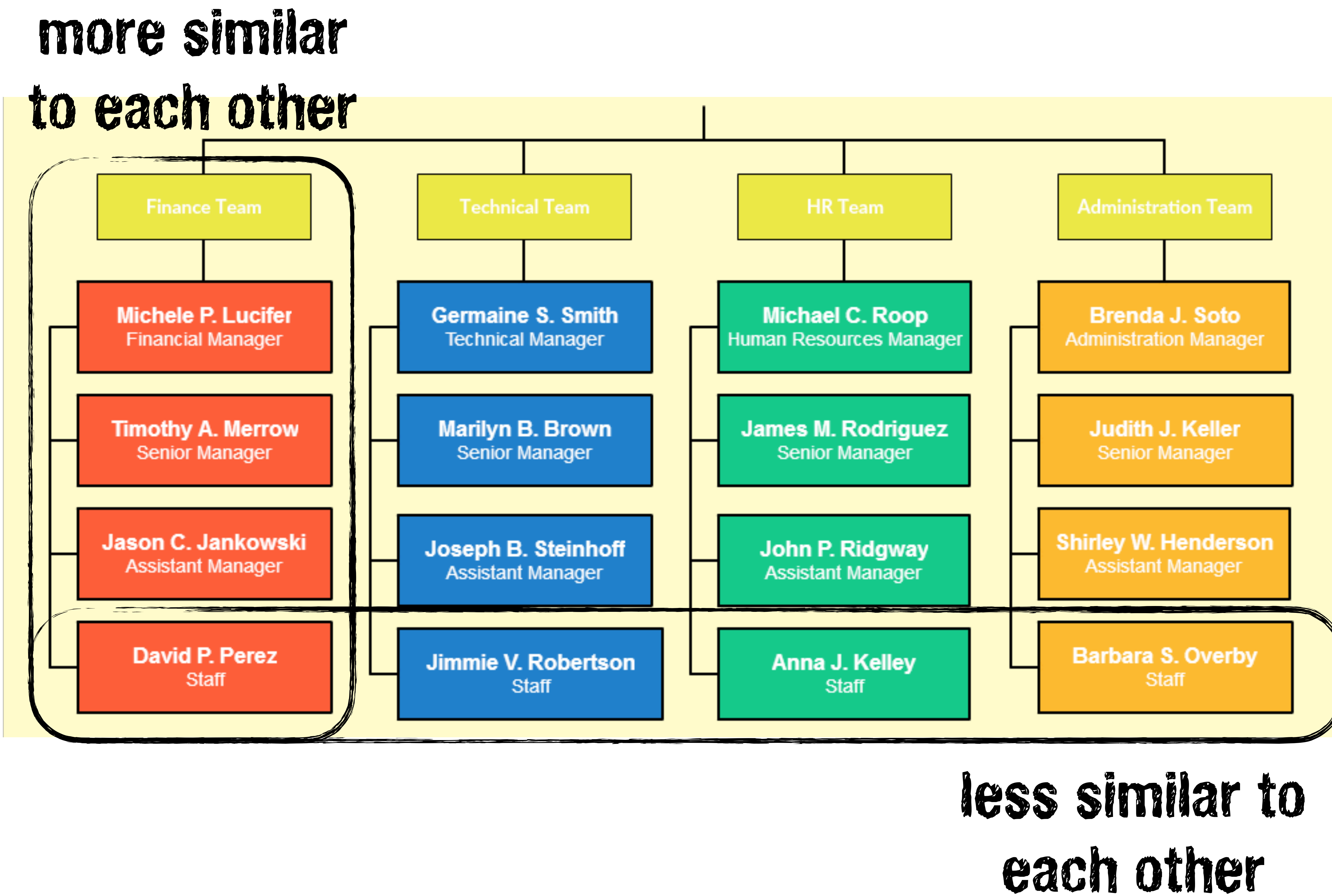
Linear mixed effects models

Dependence

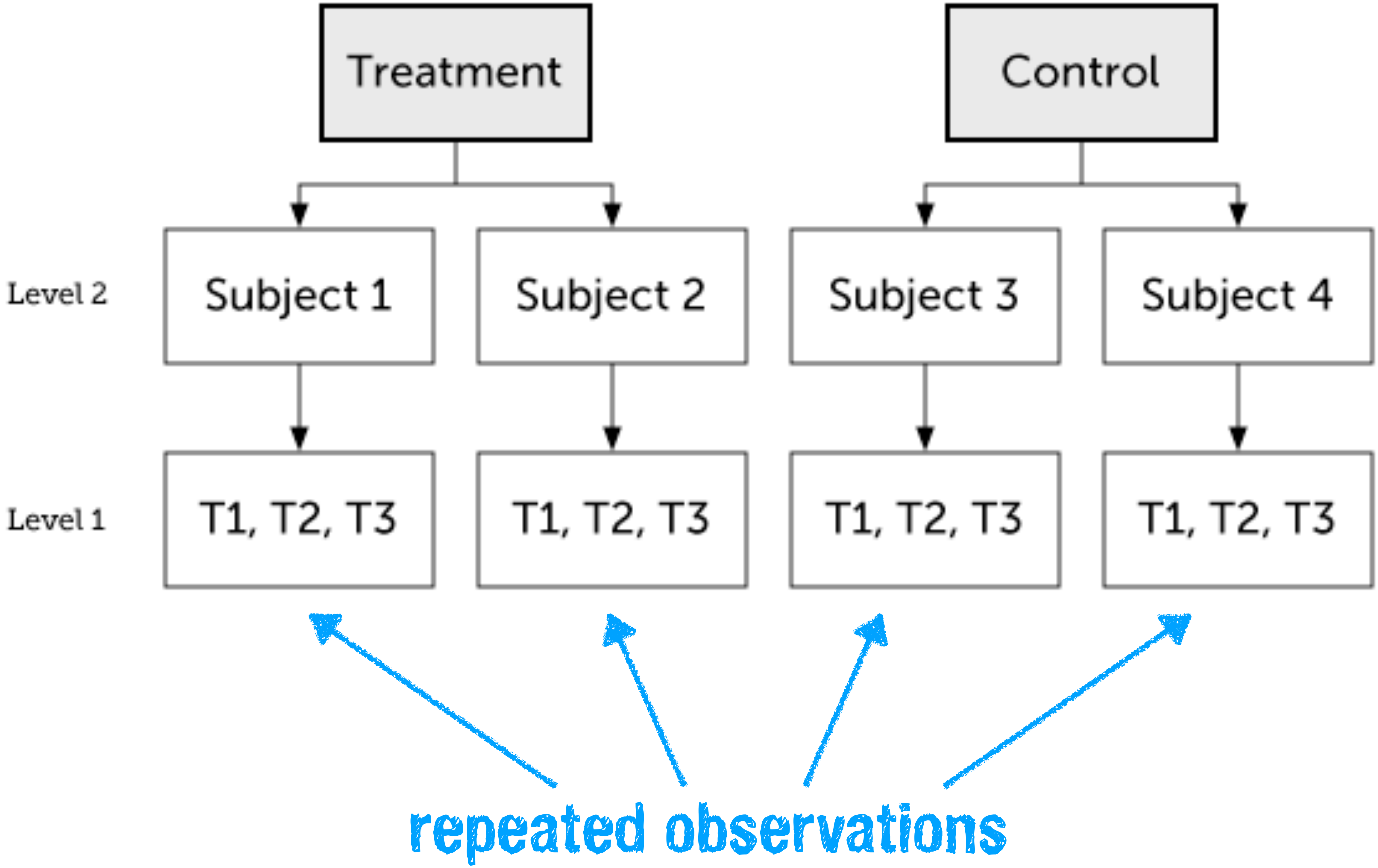
- so far, all the models that we've discussed (linear model with different kinds of predictors and contrasts) make the assumption that the data are **iid** (independent, and identically distributed)

- often this assumption is violated
    - **psychology experiments**: many observations from the same participants
    - **survey data**: different populations between different states in the US
    - **time series**: distribution at $t + 1$ depends on t

Main use cases: **Hierarchical models**

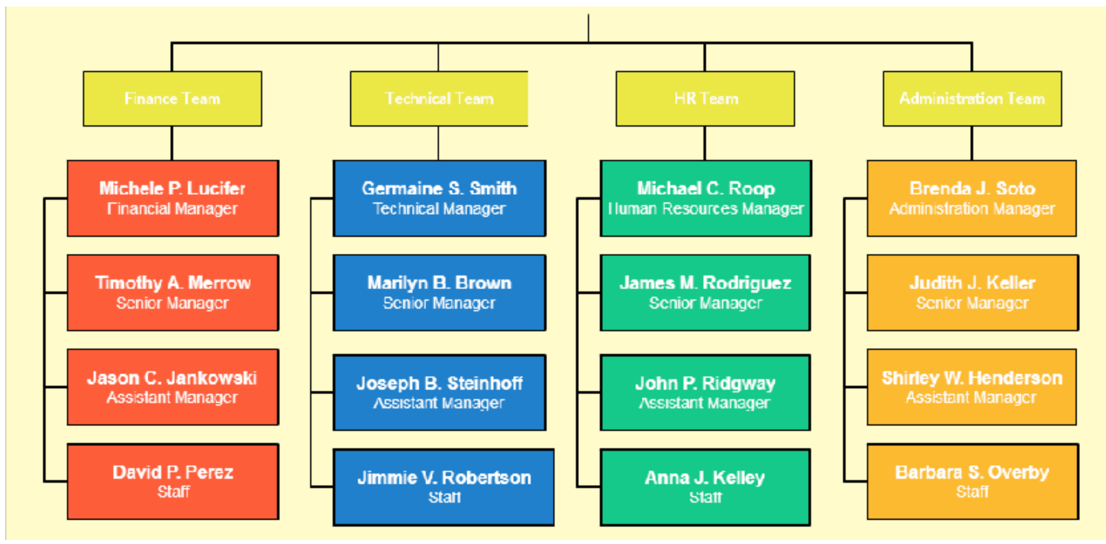


Main use cases: **Longitudinal models**

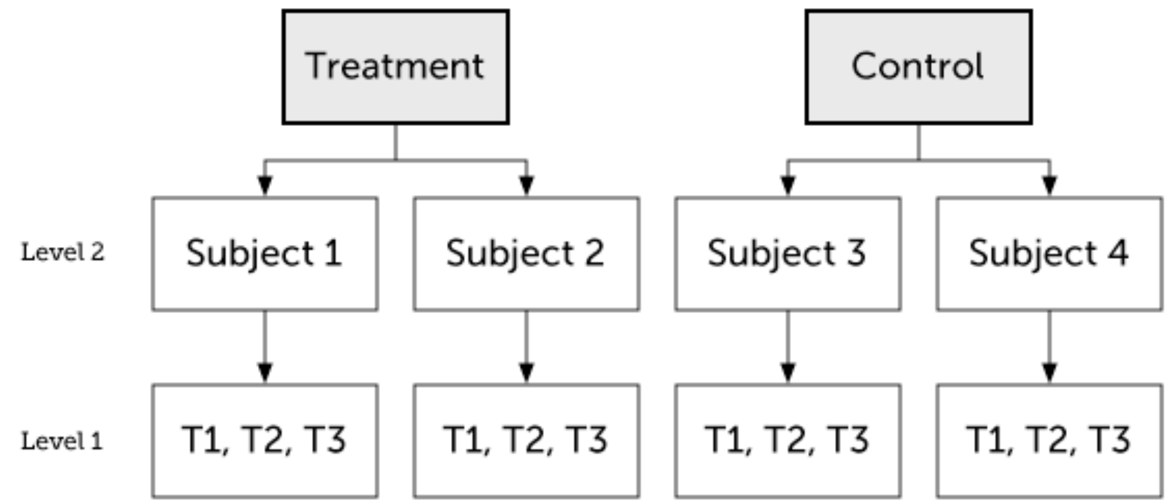


Linear mixed effects models

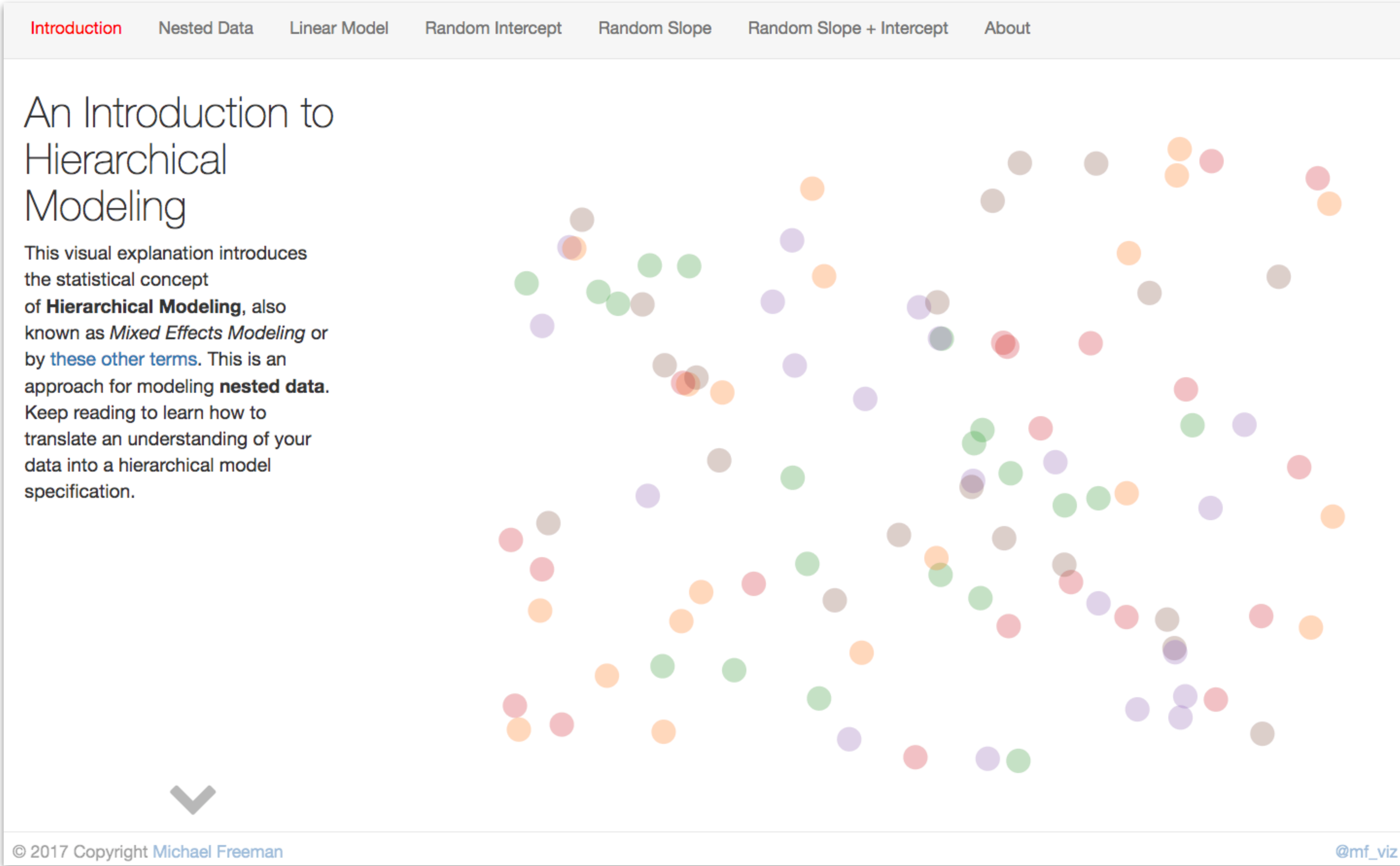
Hierarchical models



Longitudinal models



- allow us to account for dependencies in our data
- **hierarchical models**: schools > teachers > students
- **longitudinal models**: repeated observations from the same people



http://mfviz.com/hierarchical-models/

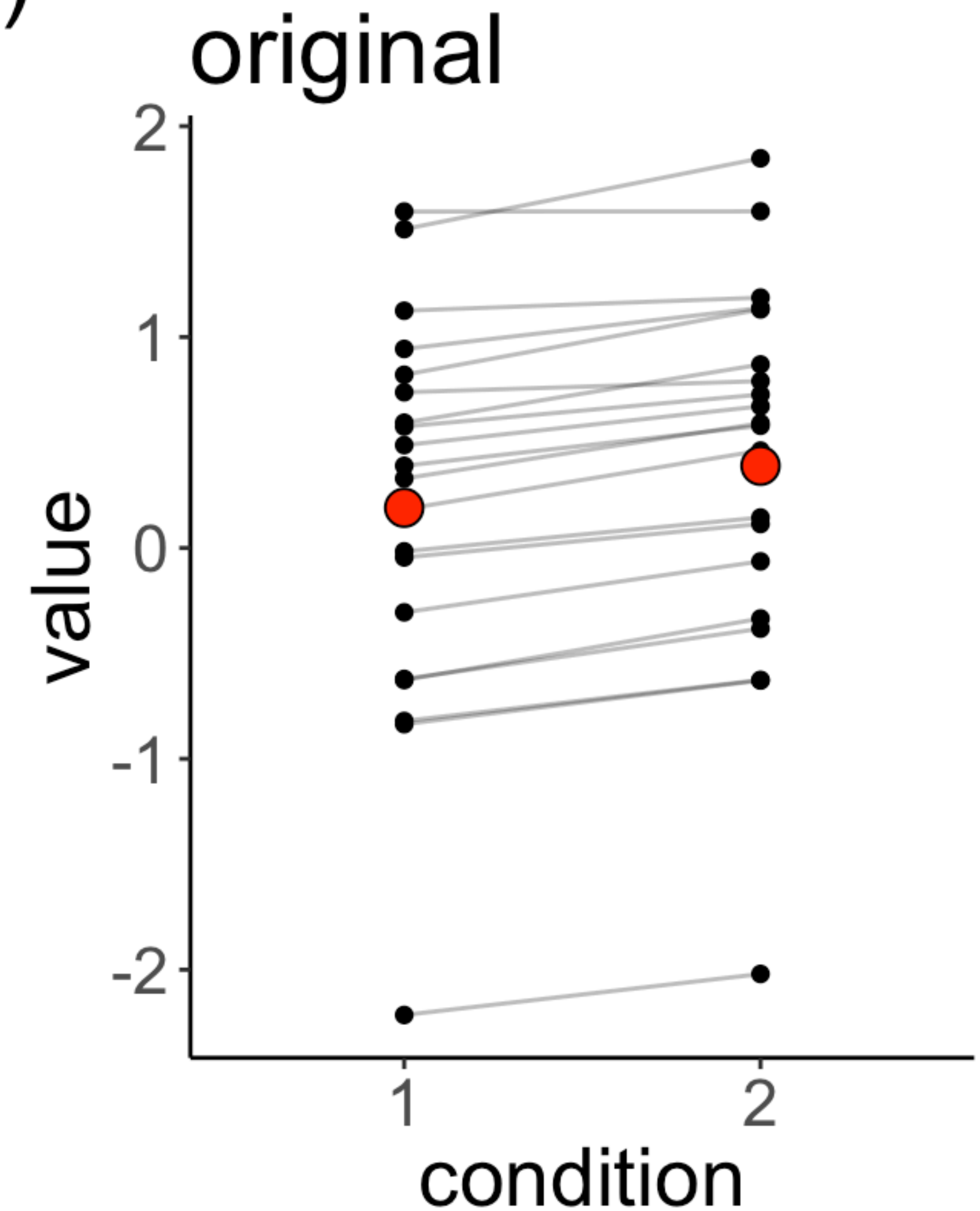
Modeling dependence in data

Dependence

Does it really matter?

Is there a significant difference between conditions 1 and 2?

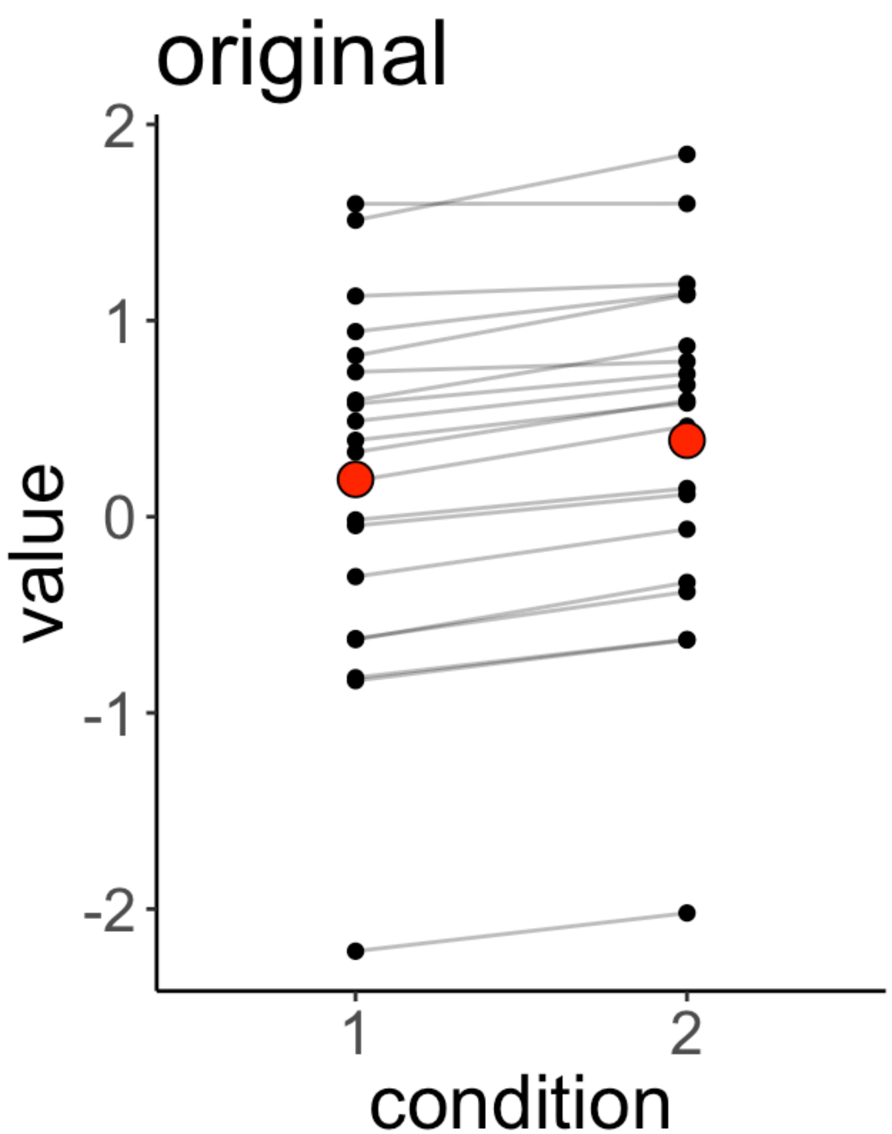
a)



Dependence

assuming independence!

```r
# linear model
lm(formula = value ~ condition,
   data = df.original) %>%
  summary()
```




```
Call:
lm(formula = value ~ condition, data = df.original)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.4100 -0.5530  0.1945  0.5685  1.4578 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.1905     0.2025   0.941    0.353
condition2    0.1994     0.2864   0.696    0.491

Residual standard error: 0.9058 on 38 degrees of freedom
Multiple R-squared:  0.01259,	Adjusted R-squared:  -0.0134 
F-statistic: 0.4843 on 1 and 38 DF,  p-value: 0.4907
```

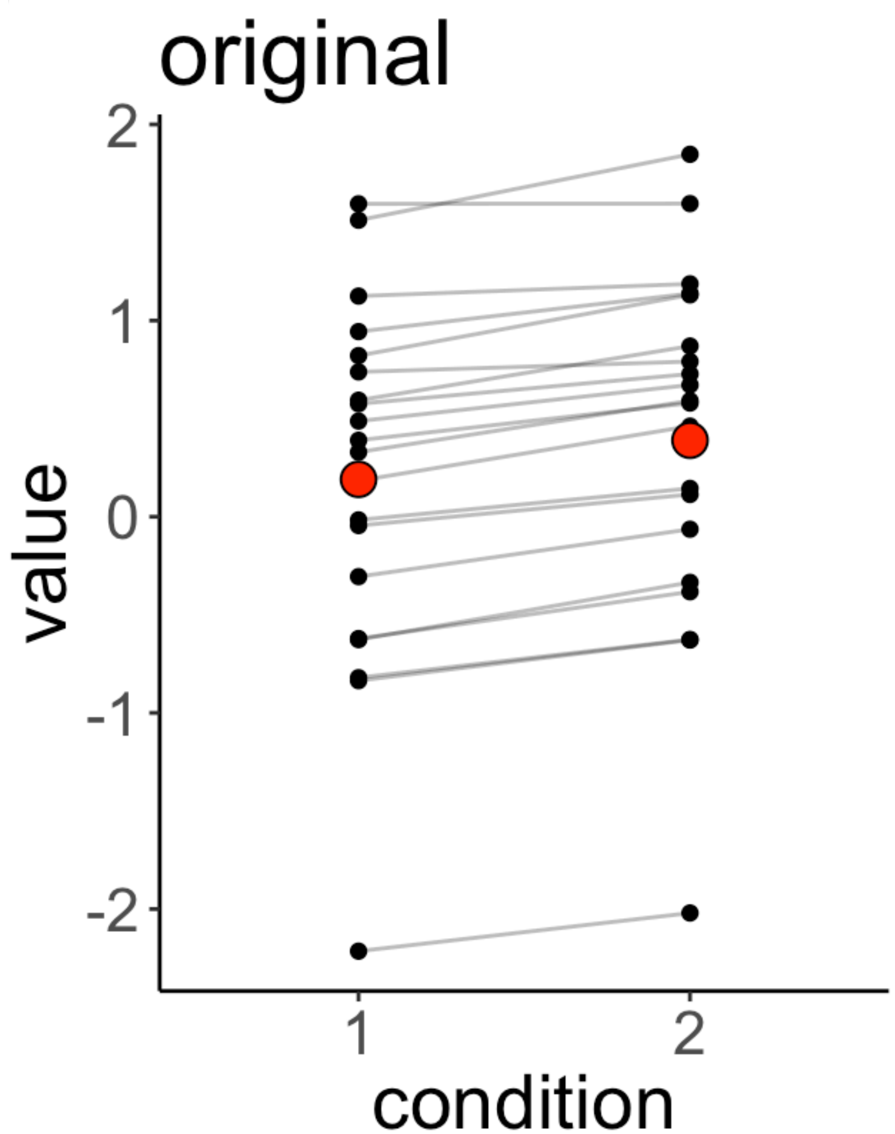
- we ignore the fact that we have repeated observations from the same participants
- in the data it looks like there is a small but consistent effect of condition

meet **`lmer`**()

Dependence

new syntax

```r
# fit a linear mixed effects model
lmer(formula = value ~ condition + (1 | participant),
                      data = df.original) %>%
  summary()
```




```
Linear mixed model fit by REML ['lmerMod']
Formula: value ~ condition + (1 | participant)
   Data: df.original

REML criterion at convergence: 17.3

Scaled residuals:
     Min       1Q   Median       3Q      Max
-1.55996 -0.36399 -0.03341  0.34400  1.65823

Random effects:
 Groups      Name        Variance Std.Dev.
 participant (Intercept) 0.816722 0.90373
 Residual                0.003796 0.06161
Number of obs: 40, groups:  participant, 20

Fixed effects:
            Estimate Std. Error t value
(Intercept)  0.19052    0.20255   0.941
condition2   0.19935    0.01948  10.231

Correlation of Fixed Effects:
           (Intr)
condition2 -0.048
```

no p-value!

NO P-VALUE

Dependence

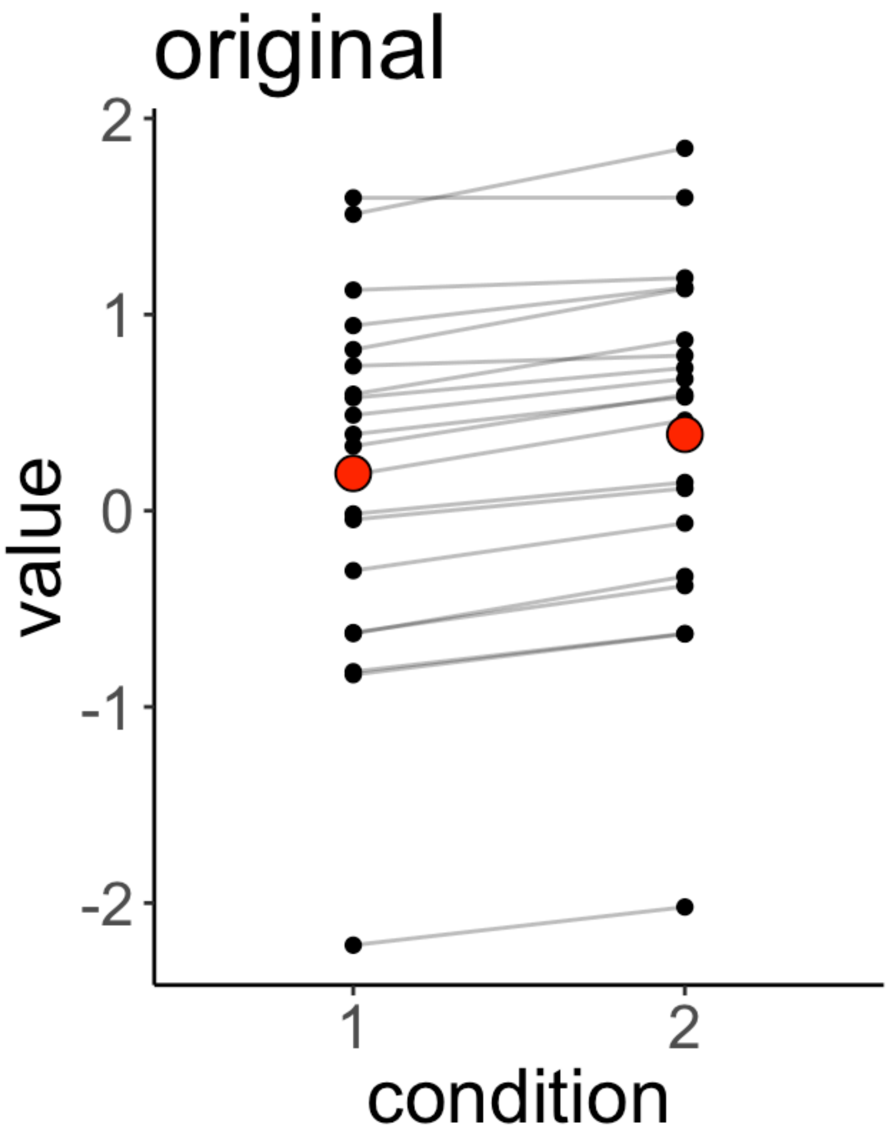
we can still do our good ol model comparison trick

```
# fit models
fit.compact = lmer(formula = value ~ 1 + (1 | participant),
                   data = df.original)

fit.augmented = lmer(formula = value ~ 1 + condition + (1 | participant),
                     data = df.original)

# compare via Chisq-test
anova(fit.compact, fit.augmented)
```

```
refitting model(s) with ML (instead of REML)
Data: df.original
Models:
fit.compact: value ~ 1 + (1 | participant)
fit.augmented: value ~ 1 + condition + (1 | participant)
              Df    AIC    BIC   logLik deviance  Chisq Chi Df Pr(>Chisq)
fit.compact    3 53.315 58.382 -23.6575   47.315
fit.augmented  4 17.849 24.605  -4.9247    9.849 37.466      1  9.304e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Dependence

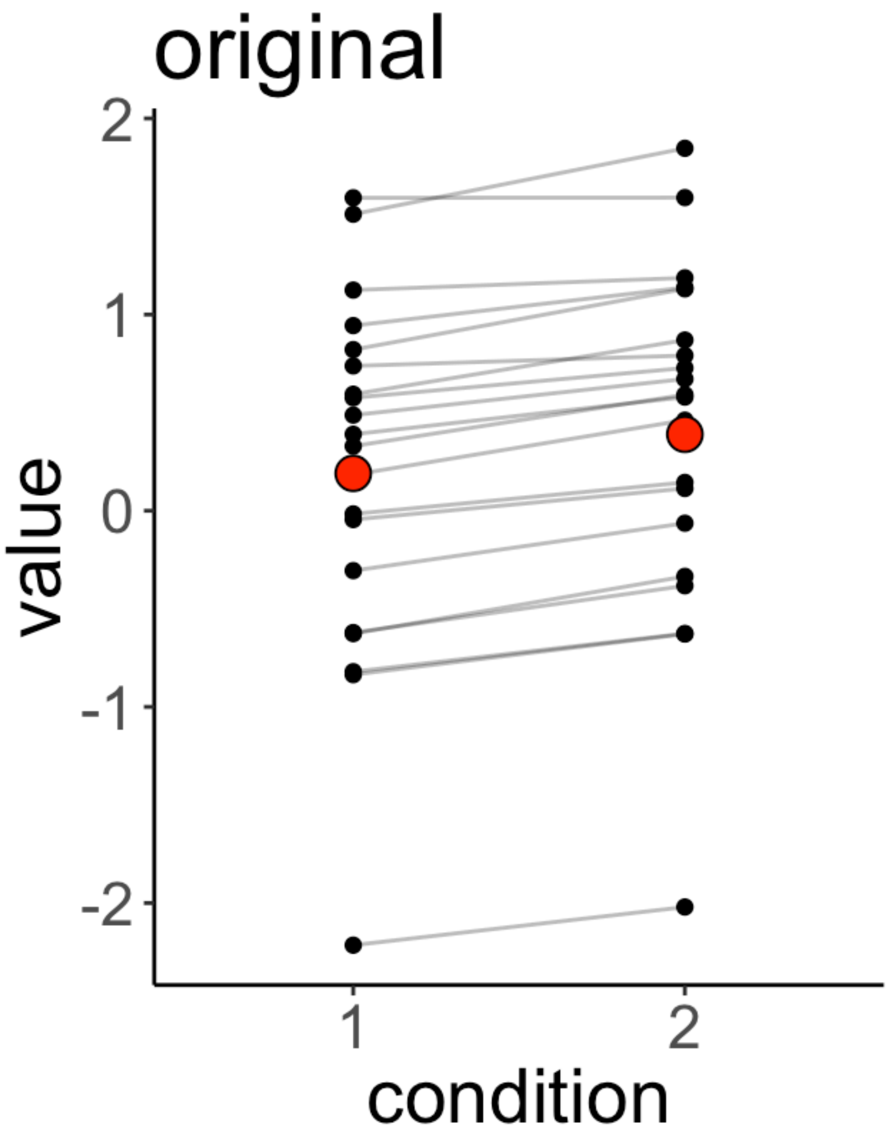
Why is the effect of condition significant when we account for the dependence in the data?



- there are large interindividual differences in the baseline
- the variance explained by the effect of condition is (much) smaller than the interindividual variance
- **but**: the effect of condition is highly consistent

Dependence

Why is the effect of condition significant when we account for the dependence in the data?



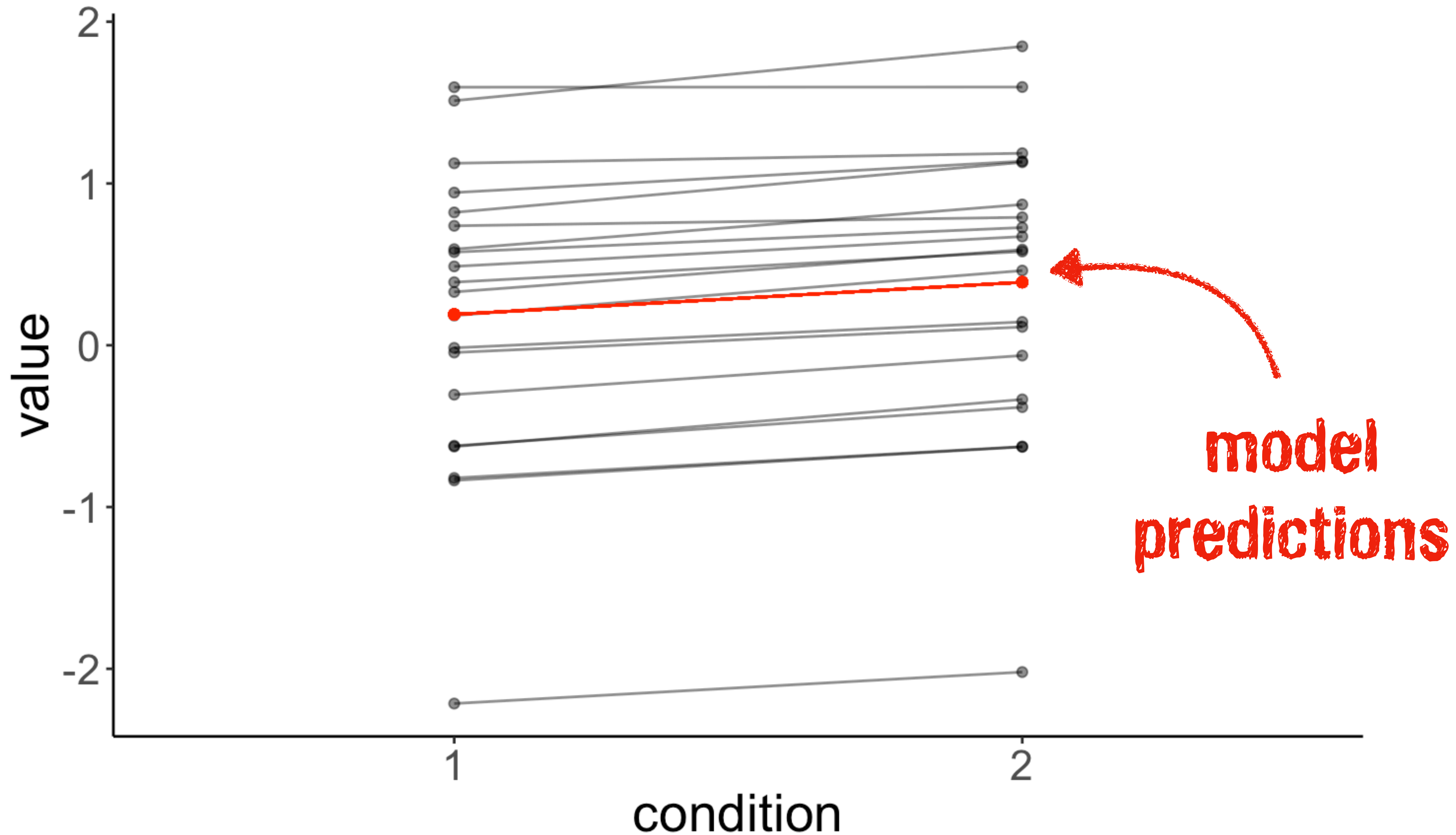
- by explicitly modeling the dependence in the data, we account for the interindividual differences

let's visualize the model predictions!

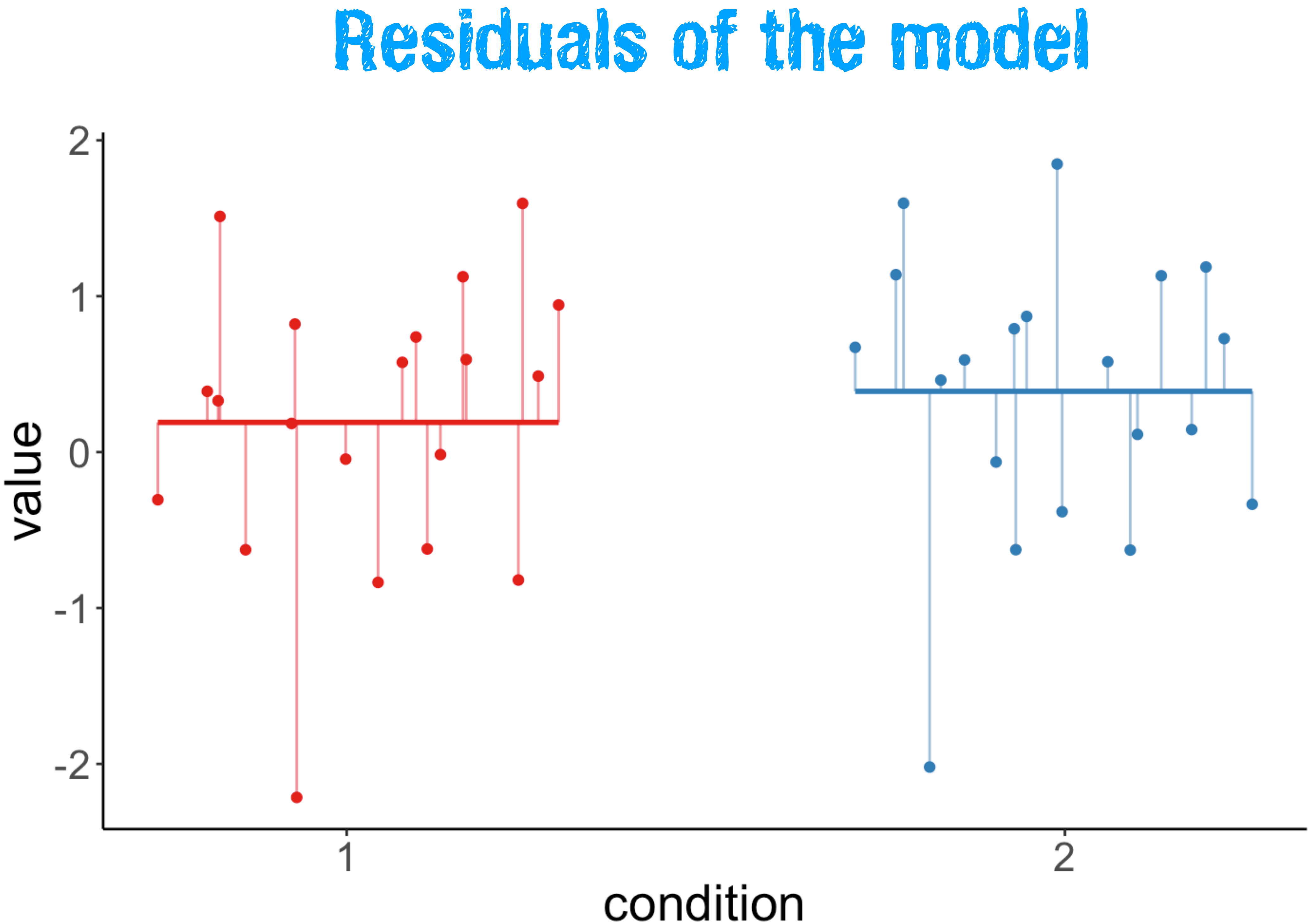
Linear model (assuming independence)

Predictions by the linear model which assumes independence

```
lm(formula = value ~ condition,
   data = df.original)
```



Linear model (assuming independence)

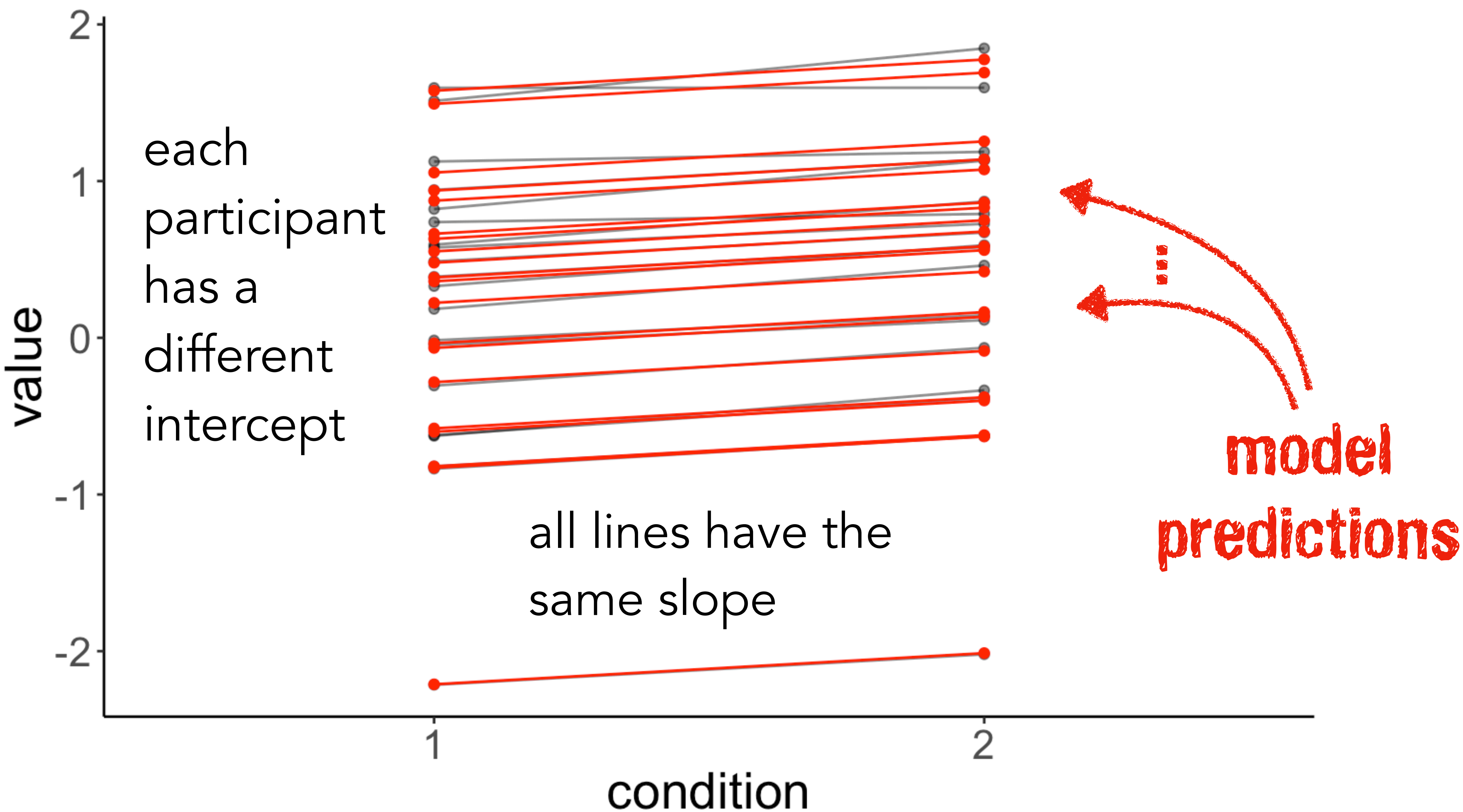


This is not much better than fitting a single line (point).

Linear mixed effects model (accounting for dependence)

Predictions by the linear mixed effects model

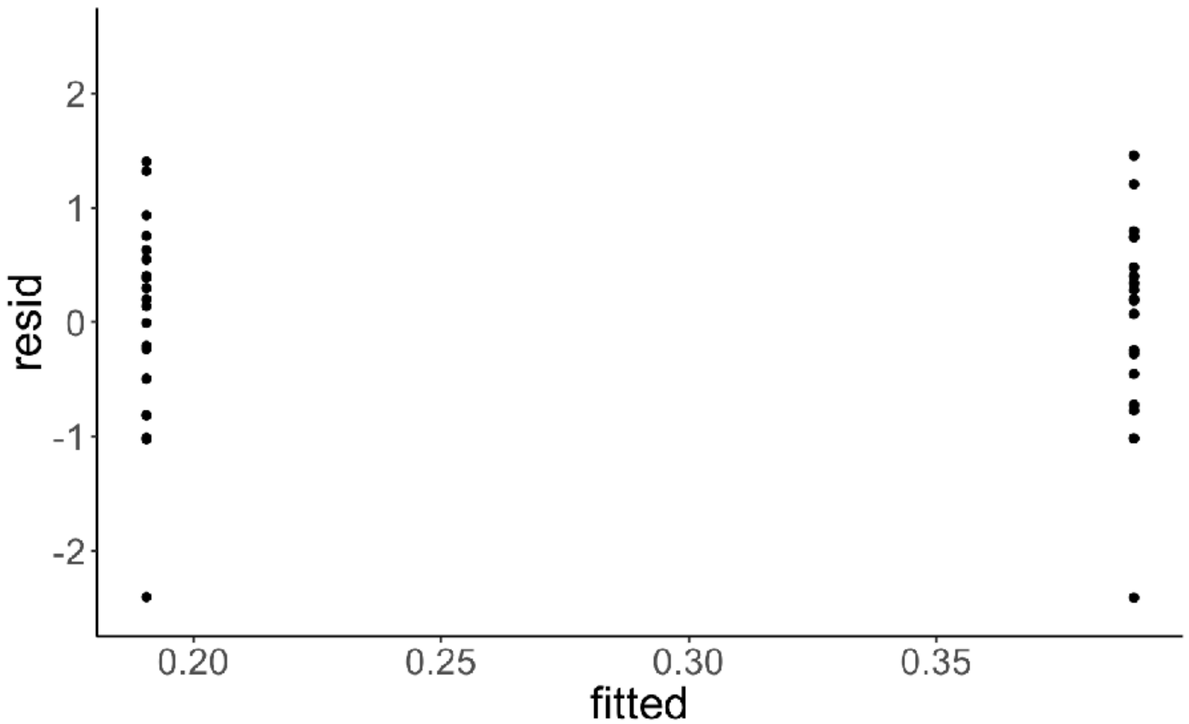
```
lmer(formula = value ~ condition + (1 | participant),
     data = df.original)
```



Model comparison

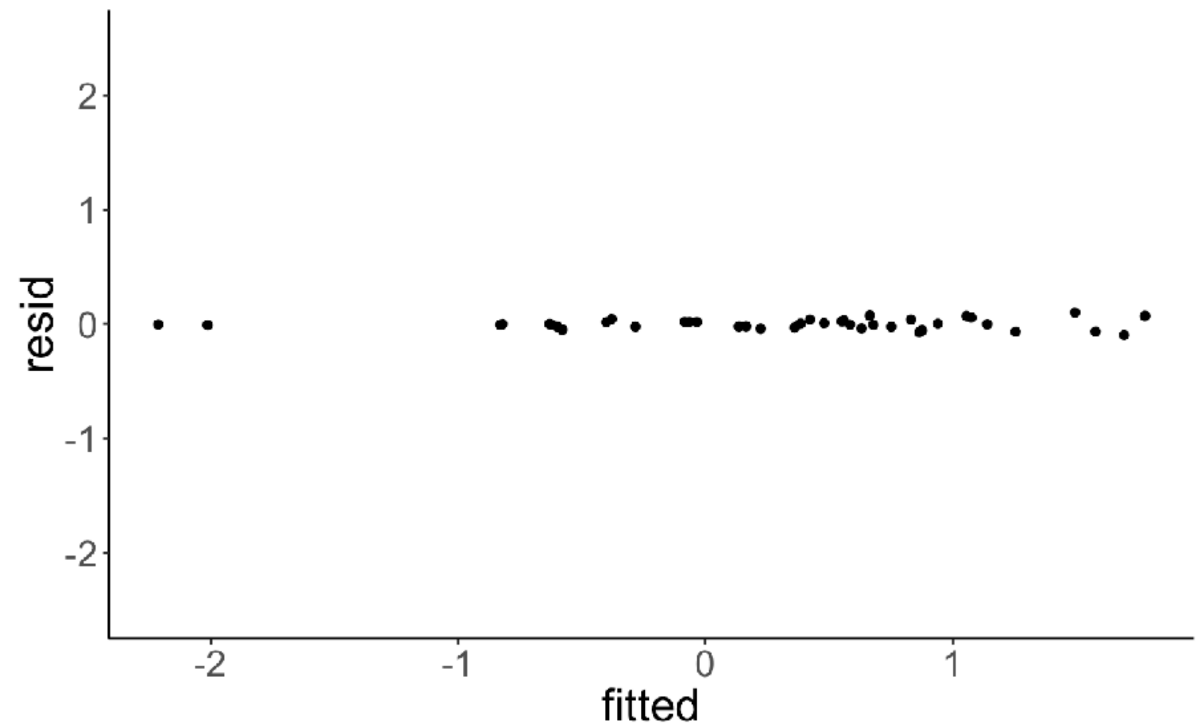
Residual plots

```
lm(formula = value ~ 1 + condition,
   data = df.original)
```



much variance left
to be explained

```
lmer(formula = value ~ 1 + condition +
                   (1 | participant),
     data = df.original)
```



almost all variance
explained

Model comparison

Hypothesis test

Is taking into account individual differences worth it?

```r
# fit models (without and with dependence)
fit.compact = lm(formula = value ~ 1 + condition,
                 data = df.original)

fit.augmented = lmer(formula = value ~ 1 + condition + (1 | participant),
                     data = df.original)

# compare models
# note: the lmer model has to be supplied first
anova(fit.augmented, fit.compact)
```

```
refitting model(s) with ML (instead of REML)
Data: df.original
Models:
fit.compact: value ~ 1 + condition
fit.augmented: value ~ 1 + condition + (1 | participant)
              Df     AIC     BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
fit.compact    3 109.551 114.617 -51.775  103.551
fit.augmented  4  17.849  24.605  -4.925    9.849 93.701      1  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Linear model

```r
lm(formula = value ~ 1 + condition,
   data = df.original)
```

$$\text{value}_i = b_0 + b_1 \cdot \text{condition}_i + e_i$$

i = observation

$$e_i \sim \mathcal{N}(\text{mean} = 0, \text{sd} = s_{\text{error}})$$

3 parameters: b_0, b_1, s_{error}

Linear mixed effects model

```r
lmer(formula = value ~ 1 + condition +
                 (1 | participant),
     data = df.original)
```

$$\text{value}_{i,j} = b_0 + b_1 \cdot \text{condition}_{i,j} + U_i + e_i$$

i = participant,

j = time point

$$e_i \sim \mathcal{N}(\text{mean} = 0, \text{sd} = s_{\text{error}})$$

$$U_i \sim \mathcal{N}(\text{mean} = 0, \text{sd} = s_{\text{U}})$$

b_0, b_1 = fixed effects

U_i = random effect

here: random intercept

4 parameters: b_0, b_1, s_{error}, s_U

Model coefficients

Linear model

```
fit = lm(formula = value ~ 1 + condition,
         data = df.original)
coef(fit)
```

```
(Intercept)  condition2
  0.1905239   0.1993528
```

- one intercept
- one slope for condition

Linear mixed effects model

```
fit = lmer(formula = value ~ 1 + condition +
                     (1 | participant),
           data = df.original)
coef(fit)
```

```
$participant
   (Intercept) condition2
1  -0.57839428  0.1993528
2   0.22299824  0.1993528
3  -0.82920677  0.1993528
4   1.49310938  0.1993528
5   0.36042775  0.1993528
6  -0.82060123  0.1993528
7   0.47929171  0.1993528
8   0.66401020  0.1993528
9   0.55135879  0.1993528
10 -0.28306703  0.1993528
11  1.57681676  0.1993528
12  0.38457642  0.1993528
13 -0.59969682  0.1993528
14 -2.21148391  0.1993528
15  1.05439374  0.1993528
16 -0.06476643  0.1993528
17 -0.03505690  0.1993528
18  0.93945348  0.1993528
19  0.87495531  0.1993528
20  0.63135911  0.1993528

attr(,"class")
[1] "coef.mer"
```

- different intercept for each participant
- one slope for condition

Summary

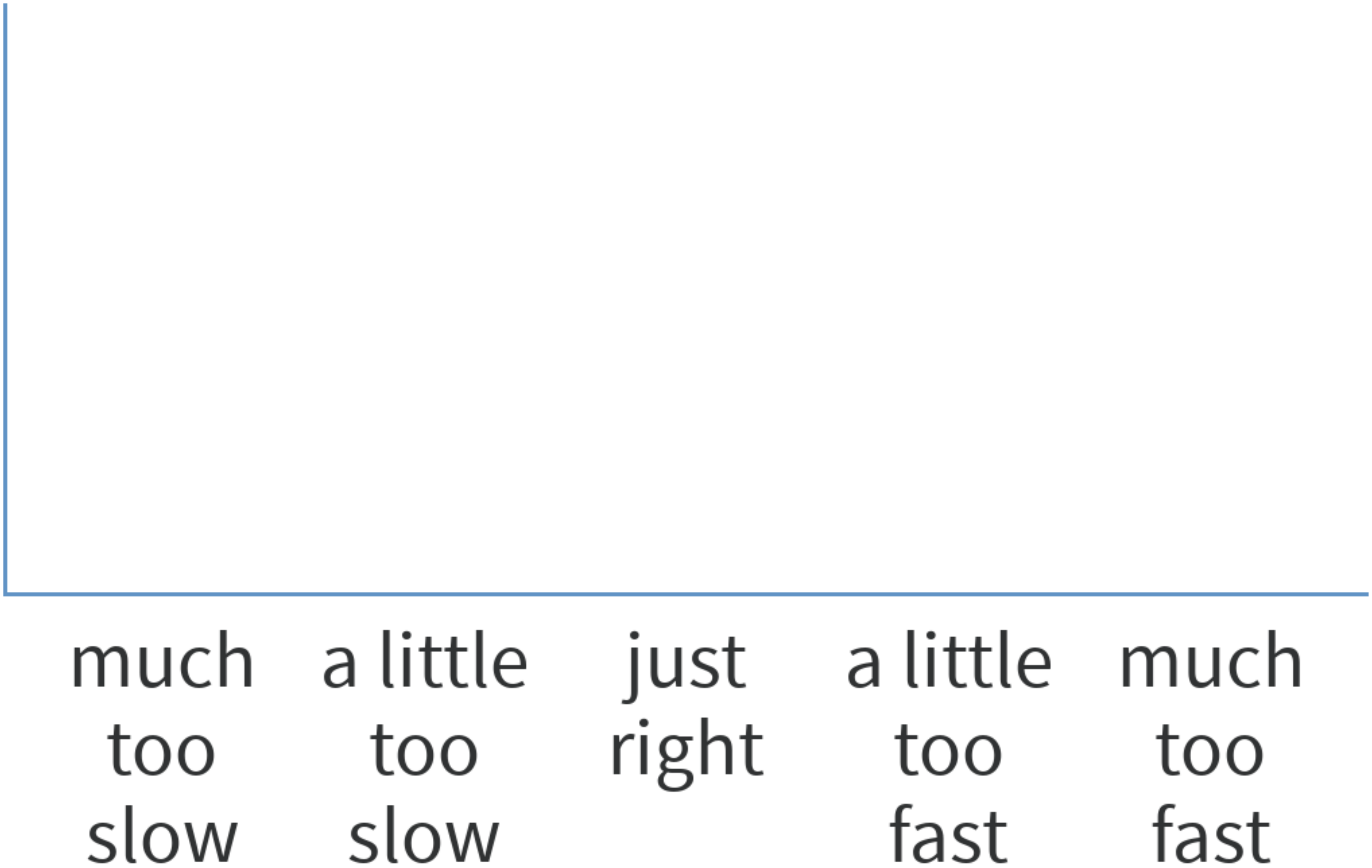
- Quick recap
- Observation, intervention, counterfactual
- Controlling for variables
  - Patterns of inference
  - Should I control?
- Mediation
- Moderation
- Linear mixed effects model
  - Modeling dependence in data

Thank you!

Happy Valentine's Day

Feedback

How was the pace of today's class?



How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?