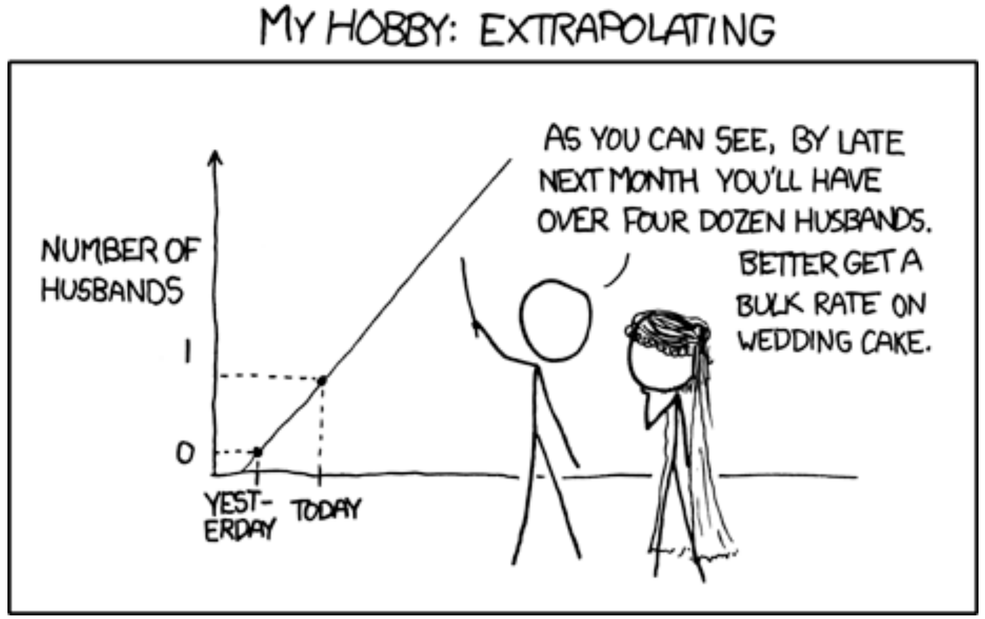
PSYCH 252: Statistical Methods for Behavioral and Social Sciences

Linear model 1



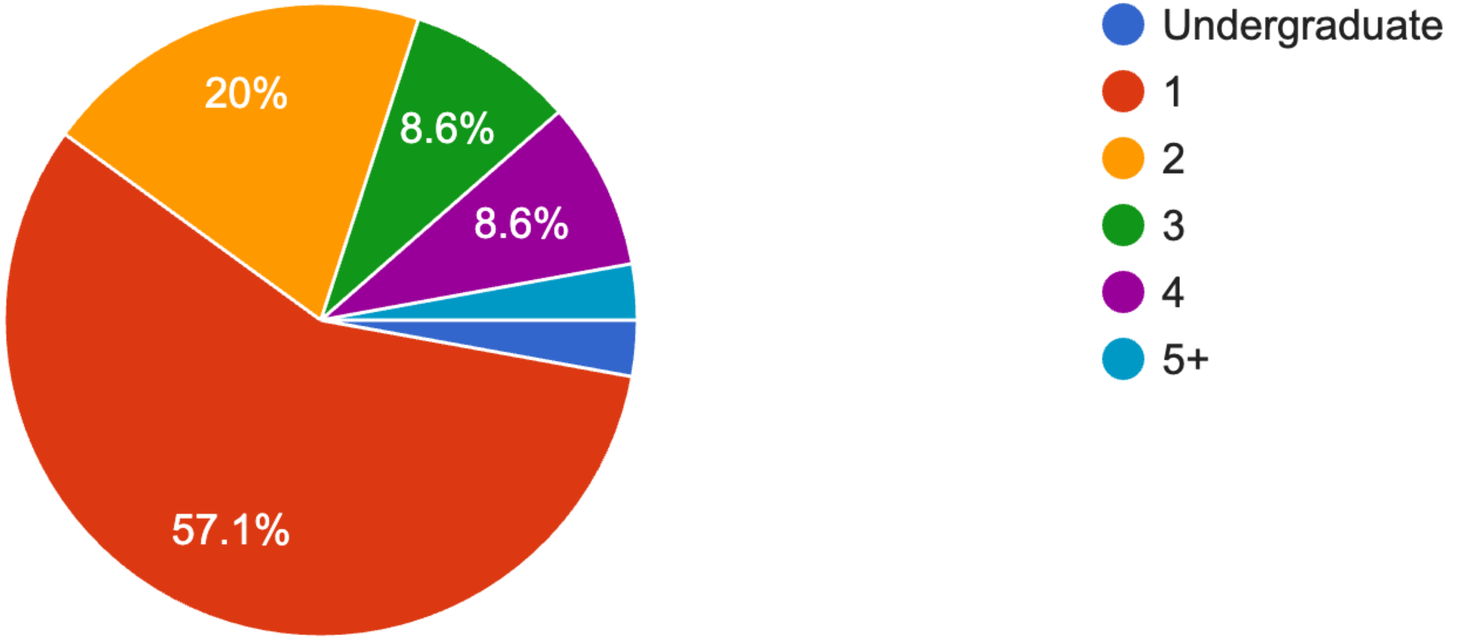
01/29/2025

Things that came up

Survey

What year of graduate school are you in?

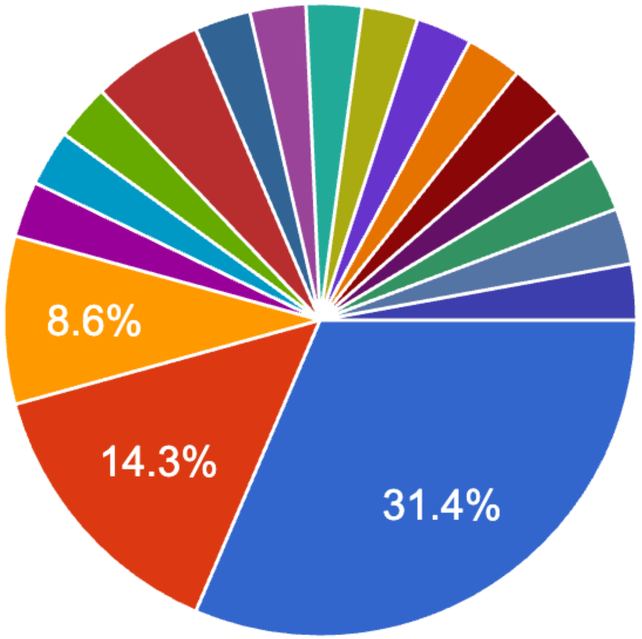
35 responses



Survey

What department are you in?

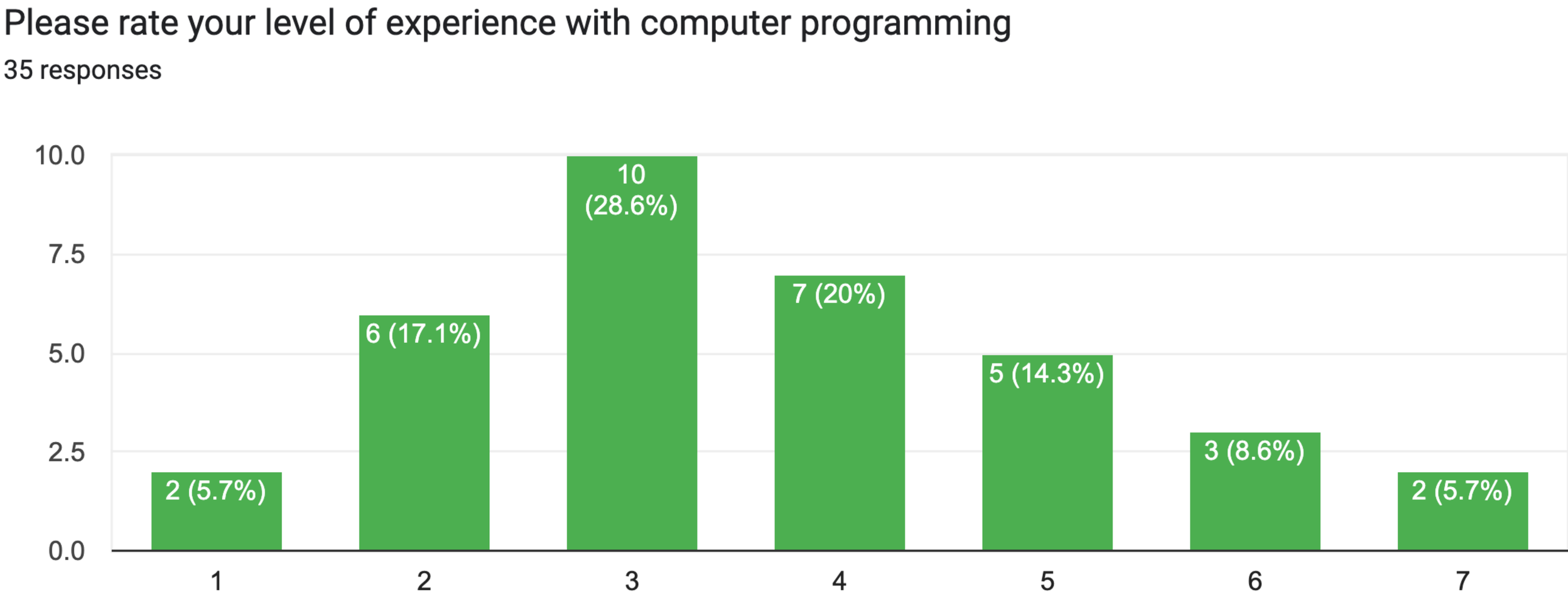
35 responses



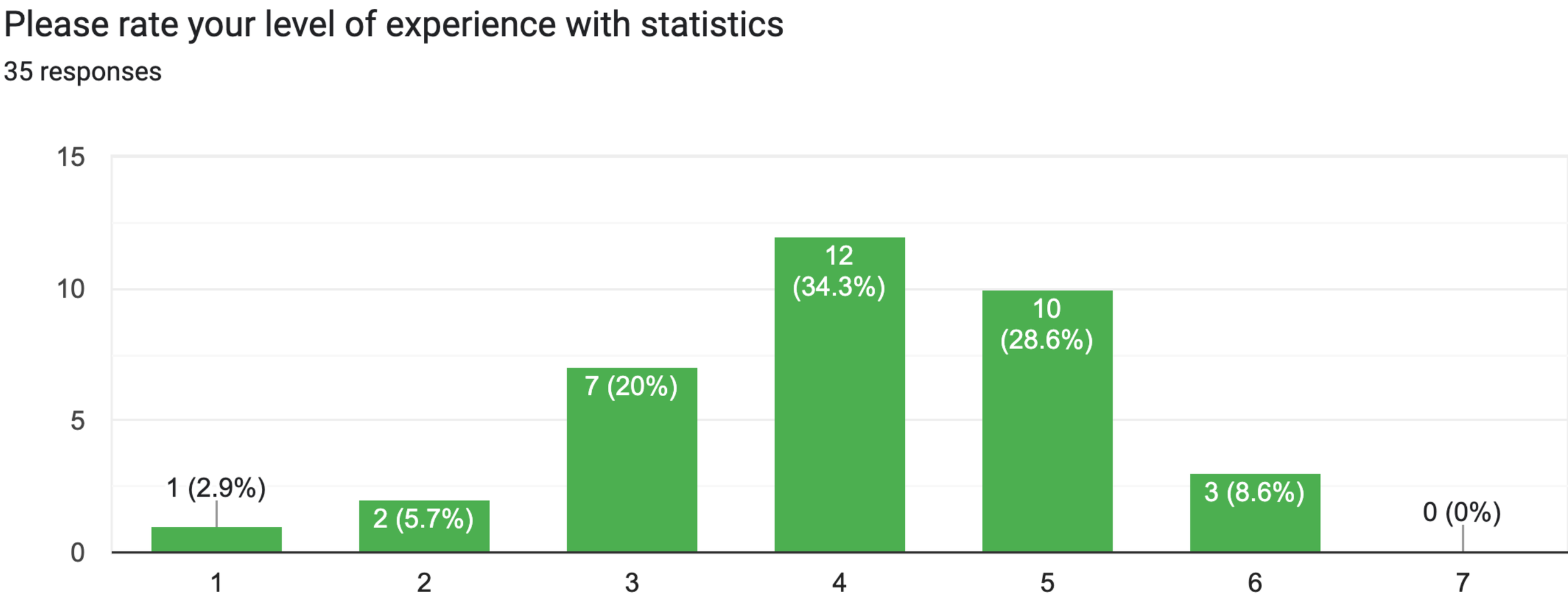
- Neuroscience
- Music
- Earth Systems
- Sustainability
- Neuroscience (School of Medicine)
- Engineering: CME, ME
- Communication
- Community Health and Prevention Re…
- MS&E: Organizations, Technology, Entrepreneurship
- Communication PhD
- Symbolic System
- MS&E
- Psychology
- Education
- GSB: Organizational Behavior
- GSB: Other
- Linguistics
- Computer Science: HCI
- Computer Science: Other
- Economics

1/3

Survey

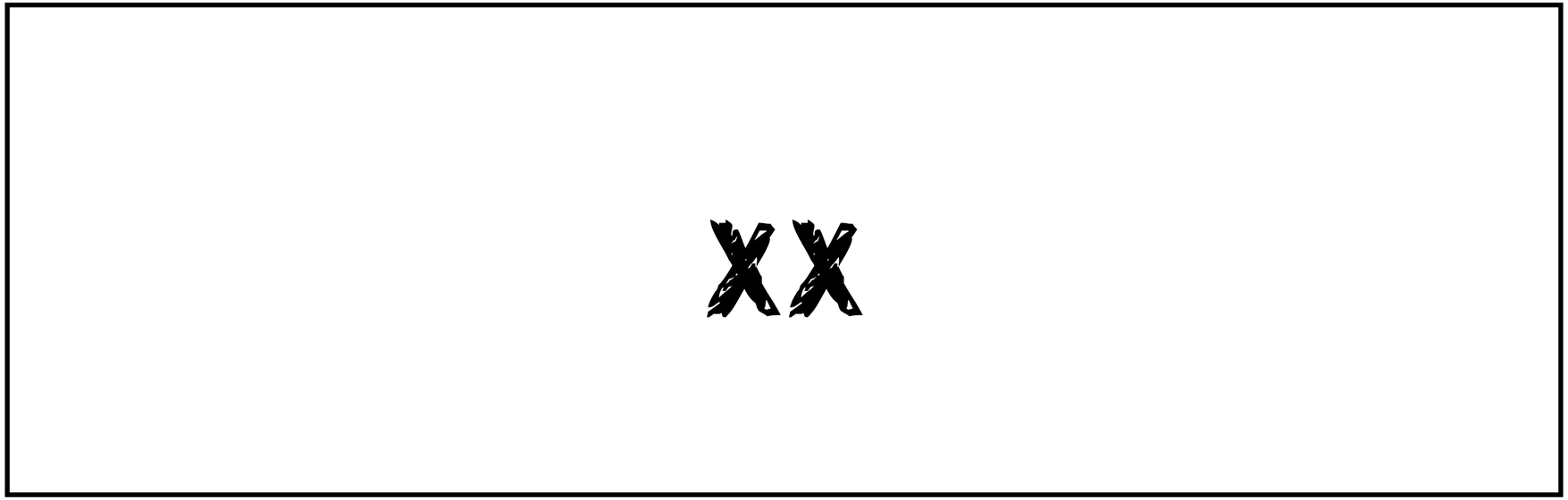

Please rate your level of experience with computer programming
35 responses
10.0
7.5
5.0
2.5
0.0
2 (5.7%)
6 (17.1%)
10 (28.6%)
7 (20%)
5 (14.3%)
3 (8.6%)
2 (5.7%)
1
2
3
4
5
6
7

Survey


Please rate your level of experience with statistics
35 responses
15
10
5
0
1 (2.9%)
2 (5.7%)
7 (20%)
12 (34.3%)
10 (28.6%)
3 (8.6%)
0 (0%)
1
2
3
4
5
6
7

Daniel Litt
@littmath

You are given an urn containing 100 balls; n of them are red, and 100-n are green, where n is chosen uniformly at random in [0, 100]. You take a random ball out of the urn—it's red—and discard it. The next ball you pick (out of the 99 remaining) is:



8:50 AM · Jan 28, 2024 · **489.2K** Views

291 202 827 702

More likely to be red

0%

More likely to be green

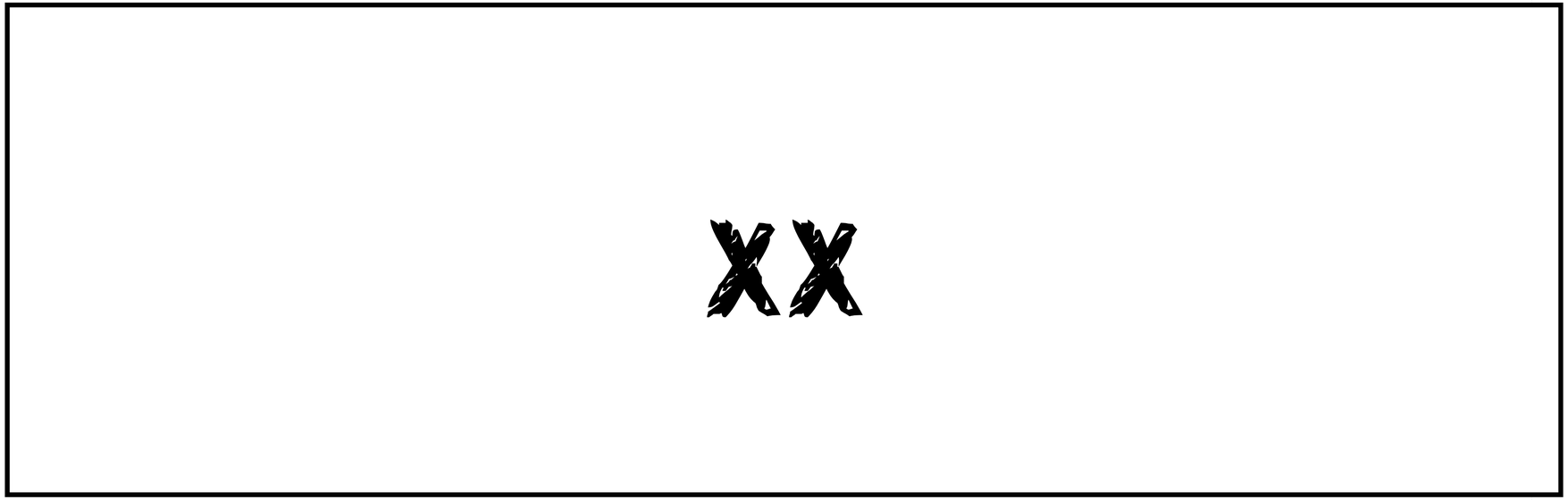
0%

Equally likely

0%

Daniel Litt
@littmath

You are given an urn containing 100 balls; n of them are red, and 100-n are green, where n is chosen uniformly at random in [0, 100]. You take a random ball out of the urn—it's red—and discard it. The next ball you pick (out of the 99 remaining) is:



8:50 AM · Jan 28, 2024 · **489.2K** Views

291 202 827 702

guy whose whole personality is duckdb 🦫

@tjmahr

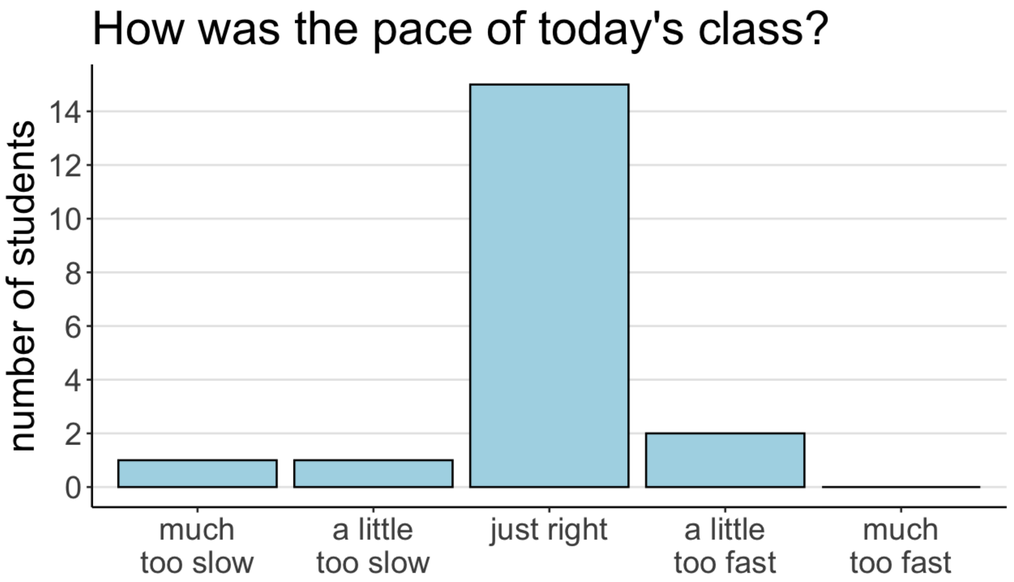
```r
f <- function(n = 100) {
  n_red <- sample.int(n, 1)
  p1 <- n_red / n
  p2 <- (n_red - 1) / (n - 1)
  draw <- sample(c("r", "g"), 1, prob = c(p1, 1 - p1))
  if (draw == "g") {
    "skip"
  } else {
    sample(c("r", "g"), 1, prob = c(p2, 1 - p2))
  }
}
counts <- replicate(100000, f()) |> table()
counts
#>
#>     g     r  skip
#> 16843 33661 49496

# p(color | first red)
counts[1:2] |> proportions()
#>
#>         g         r
#> 0.3334983 0.6665017
```

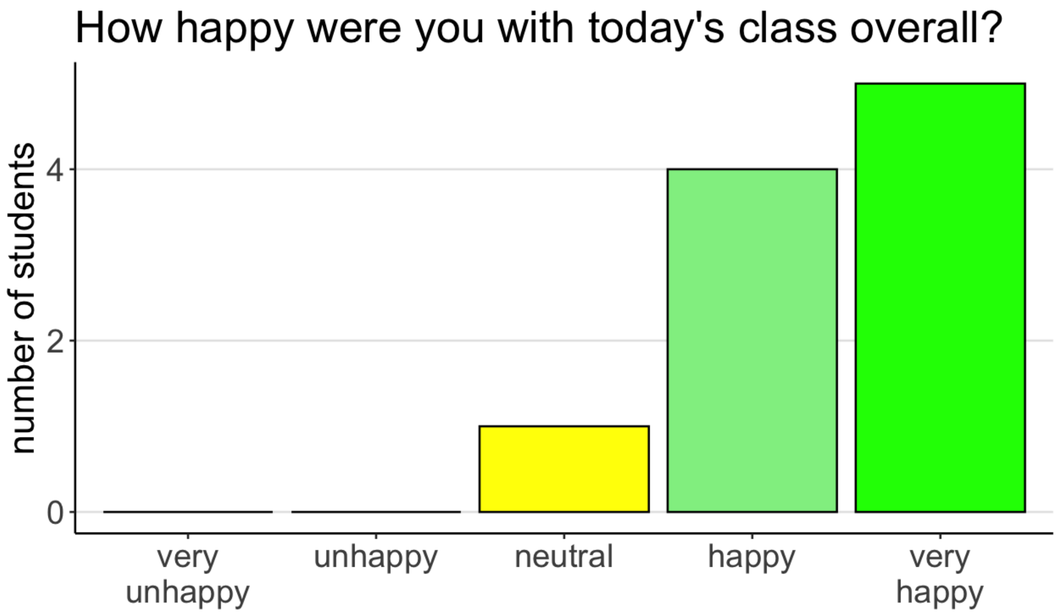
ALT

Created on 2024-01-29 with reprex v2.1.0

Your feedback

Your feedback





- Wonderful
- Showing the code/implementation behind mean_cl_boot was very helpful!
- It would be ideal to have a worked example to apply the different concepts throughout the class
- I like that the lecture focuses on intuition building. Sometimes the analogies/jokes are excessive
- I get it more but I'm lost still

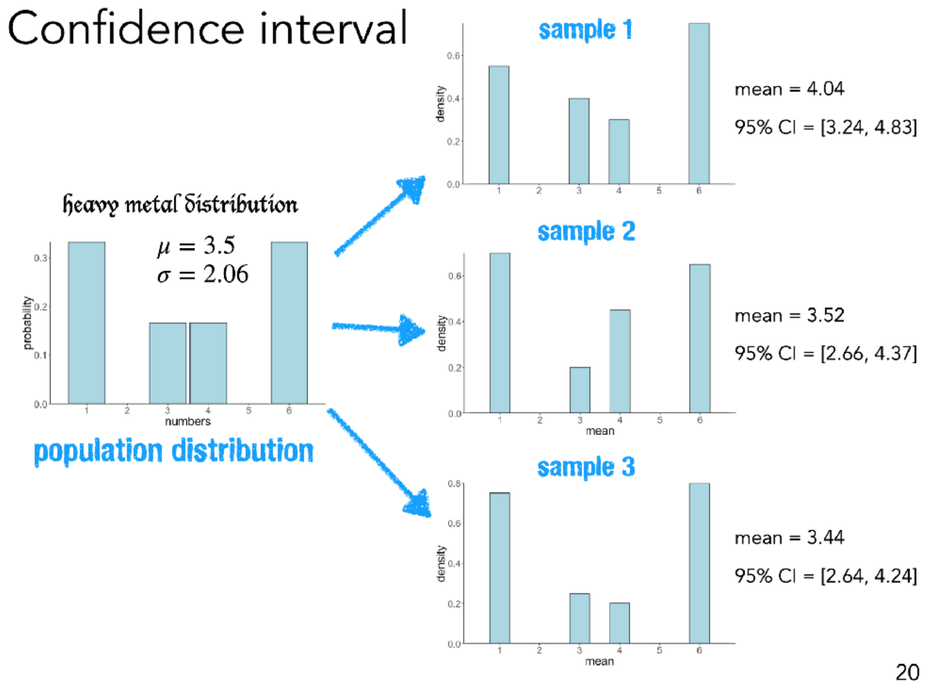
Plan for today

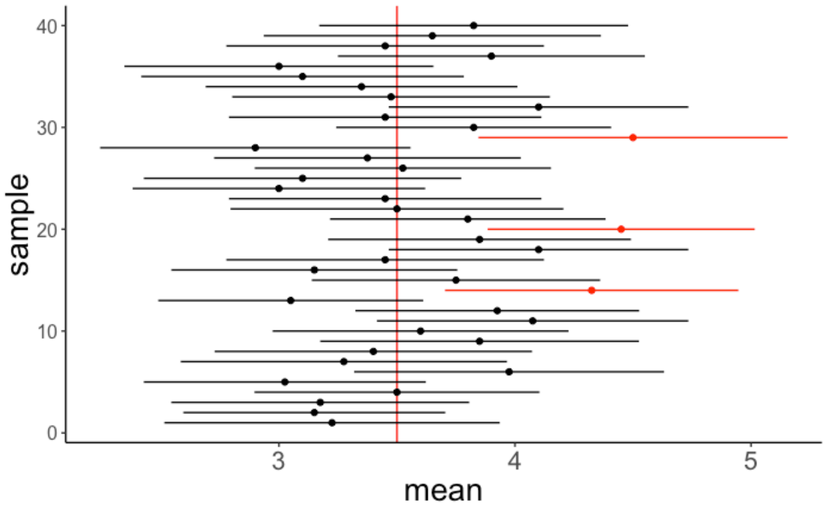
- Quick recap
- Modeling data
- Hypothesis testing as model comparison
- Correlation
  - Pearson's moment correlation
  - Spearman's rank correlation
- Regression

Quick recap

Quick recap: Confidence intervals

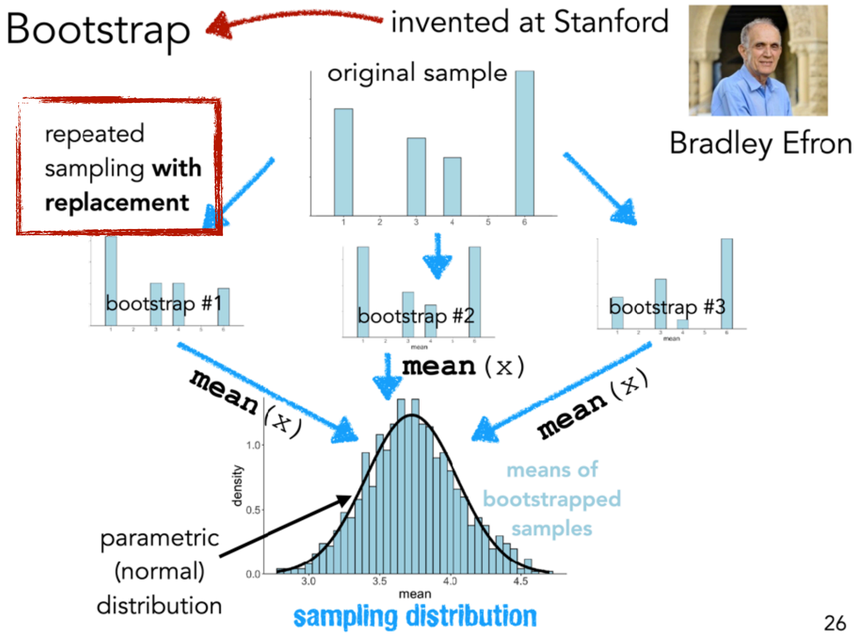
"If we were to repeat the experiment over and over, then 95% of the time the confidence intervals contain the estimate of interest."



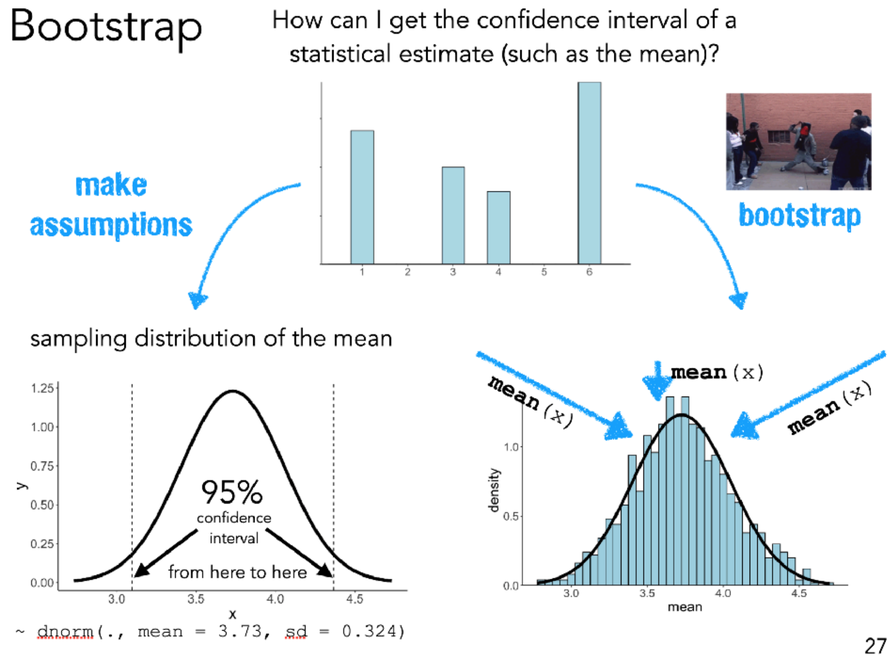


https://www.the100.ci/2024/12/05/why-you-are-not-allowed-to-say-that-your-95-confidence-interval-contains-the-true-parameter-with-a-probability-of-95/

Quick recap: Bootstrapping



26



27

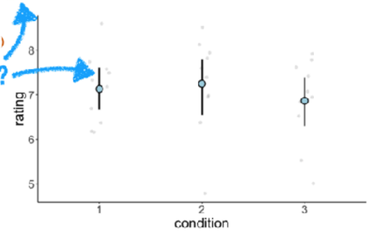
make sure to set the seed!

`mean_cl_boot()` explained

```
set.seed(1)

n = 10 # sample size per group
k = 3 # number of groups

df.data = tibble(participant = 1:(n*k),
                 condition = as.factor(rep(1:k, each = n)),
                 rating = rnorm(n*k, mean = 7, sd = 1))

ggplot(data = df.data,
       mapping = aes(x = condition,
                     y = rating)) +
  geom_point(alpha = 0.1,
             position = position_jitter(width = 0.1, height = 0)) +
  stat_summary(fun.data = "mean_cl_boot",
               shape = 21,
               size = 1,
               fill = "lightblue")
```

participant	condition	rating
1	1	6.37
2	1	7.18
3	1	6.16
4	1	6.60
5	1	7.33

what is this magic?



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Quick recap: Modeling data

$$\text{Data} = \text{Model} + \text{Error}$$

what makes for a good model?

- we build models with parameters, and fit those parameters to **minimize error**
- adding additional parameters to the model will always improve the model fit and reduce error
- fundamental trade-off between **simplicity** and **accuracy**

Assumption of normal distribution

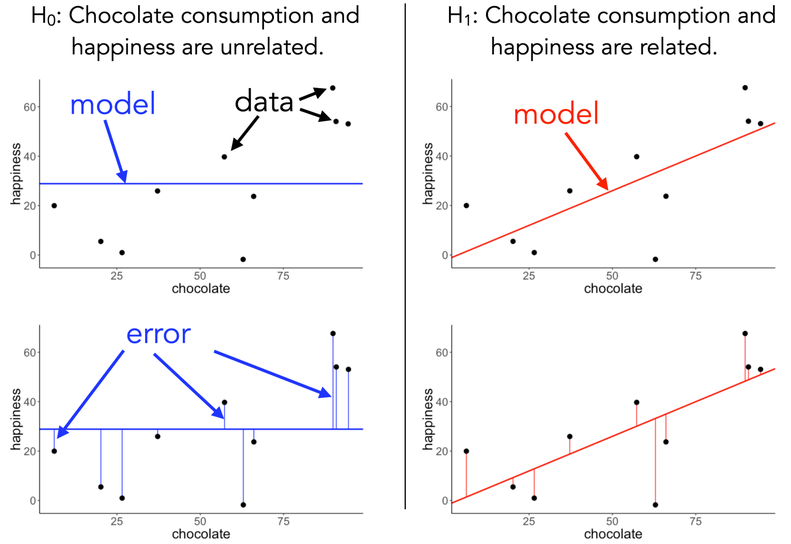
$$\text{Error} = \text{Data} - \text{Model}$$

assumed to be normally distributed

don't need to be normally distributed!!

very common misconception!!!

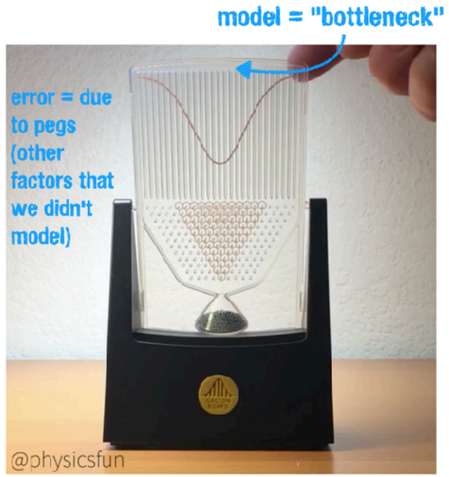
$$\text{Data} = \text{Model} + \text{Error}$$



ERROR

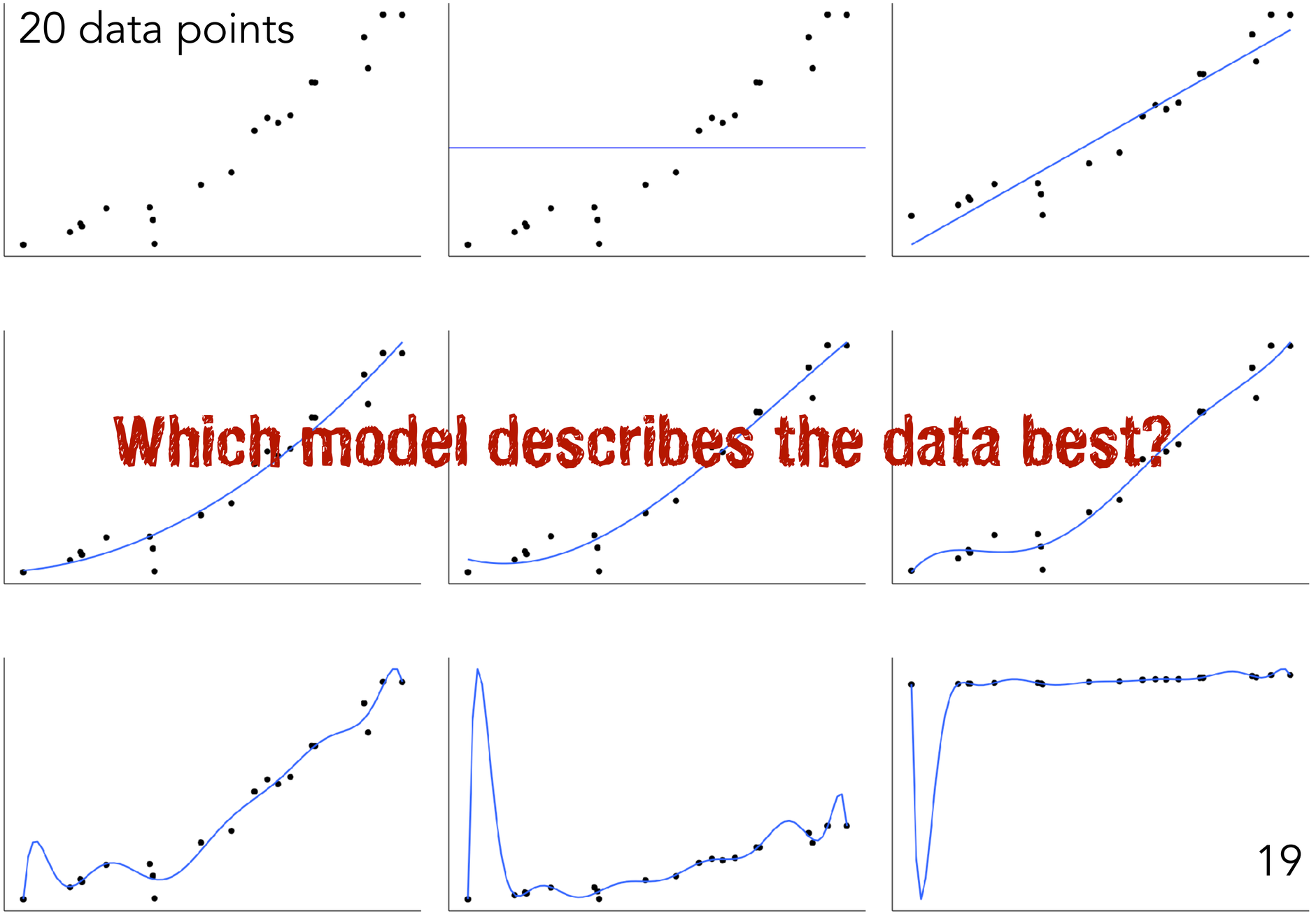
1. We assume that the error between model and data is due to (a potentially large number of) factors that we didn't take into account.

2. We assume that each of these factors influences the data in **an additive way** (some pulling in one, others pulling in another direction).



Result: normal distribution

Modeling data


20 data points
Which model describes the data best?
19 parameters


THE BEST WAY TO
EXPLAIN OVERFITTING
imgflip.com

Example

Percentage of households that had internet access in the year 2013 by US state

i	internet	state	college	auto	density
1	79.0	AK	28.0	1.2	1.2
2	63.5	AL	23.5	1.3	94.4
3	60.9	AR	20.6	1.7	56.0
4	73.9	AZ	27.4	1.3	56.3
5	77.9	CA	31.0	0.8	239.1
6	79.4	CO	37.8	1.0	48.5
7	77.5	CT	37.2	1.0	738.1
8	74.5	DE	29.8	1.1	460.8
9	74.3	FL	27.2	1.2	350.6
10	72.2	GA	28.3	1.1	168.4

set before looking at the data

$\text{model}_1: \quad Y_i = 75 + \text{ERROR}$

worth it?

fit to the data

$\text{model}_2: \quad Y_i = \beta_0 + \text{ERROR}$

worth it?

additional predictor

$\text{model}_3: \quad Y_i = \beta_0 + \beta_1 X_i + \text{ERROR}$

fit to the data

Compact model

model_C: $Y_i = \beta_0 + \text{ERROR}$

Augmented model

model_A: $Y_i = \beta_0 + \beta_1 X_i + \text{ERROR}$

$$\text{ERROR}(C) \geq \text{ERROR}(A)$$

Proportional reduction in error (PRE)

$$\text{PRE} = \frac{\text{ERROR}(C) - \text{ERROR}(A)}{\text{ERROR}(C)}$$

Compact model

model$_C$: $Y_i = \beta_0 + \text{ERROR}$

$$\text{ERROR}(C) = 50$$

Augmented model

model$_A$: $Y_i = \beta_0 + \beta_1 X_i + \text{ERROR}$

$$\text{ERROR}(A) = 30$$

Proportional reduction in error (PRE)

$$\text{PRE} = 1 - \frac{\text{ERROR}(A)}{\text{ERROR}(C)}$$

$$= 1 - \frac{30}{50} = .40$$

Increasing the complexity of the model reduced the error by 40%. **worth it?**

worth it?

Compact model

$$\text{model}_C: \; Y_i = \beta_0 + \text{ERROR}$$

Augmented model

$$\text{model}_A: \; Y_i = \beta_0 + \beta_1 X_i + \text{ERROR}$$

Proportional reduction in error (PRE)

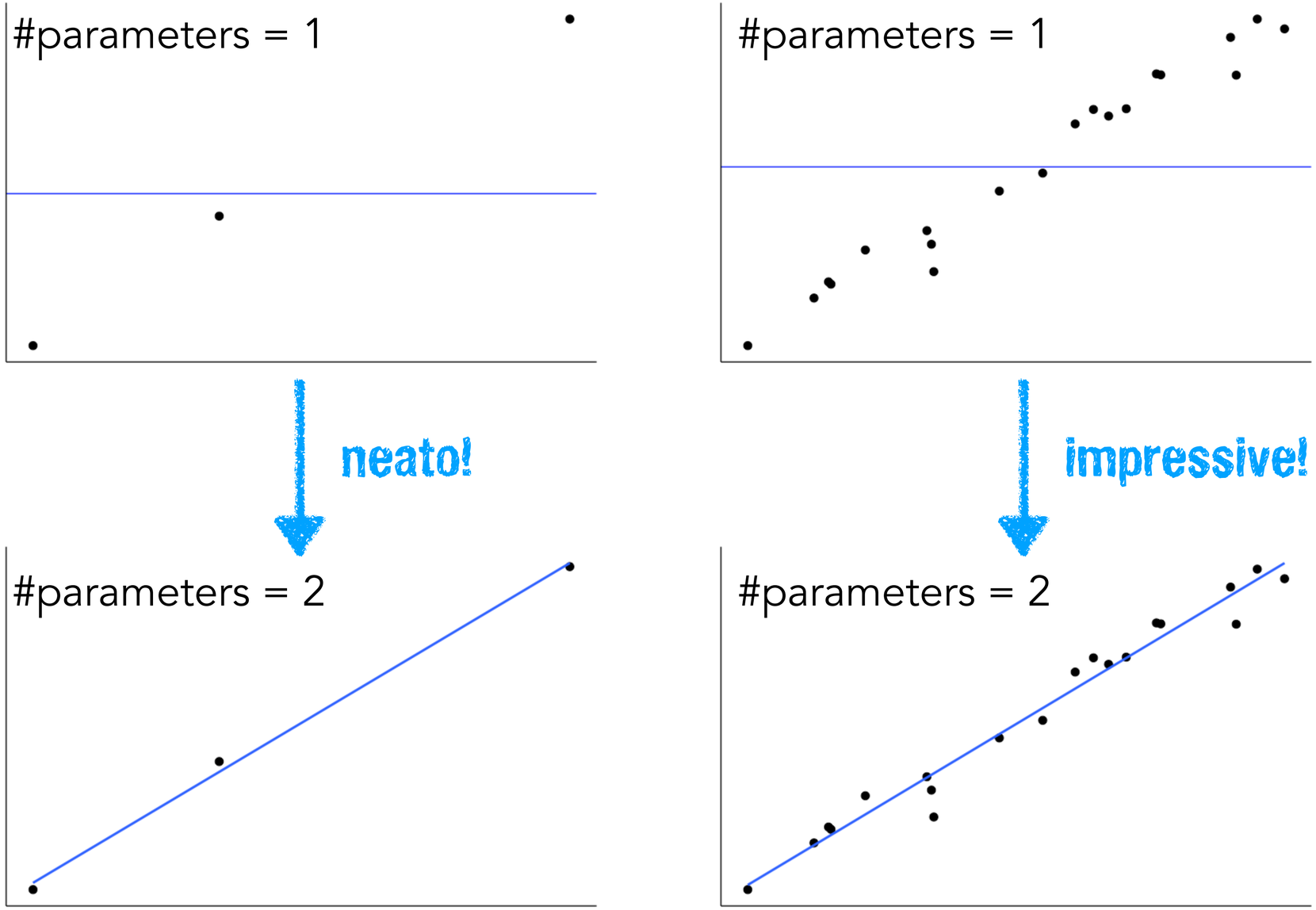
$$\text{PRE} = 1 - \frac{\text{ERROR}(A)}{\text{ERROR}(C)}$$

$$= 1 - \frac{30}{50} = .40$$

- to answer the **worth it?** question we need inferential statistics (define ERROR, sampling distributions, etc.)
- more likely to be **worth it** if:
  1. **PRE** is high
  2. the number of additional parameters in A compared to C is low (**PRE per additional parameter**)
  3. the number of parameters that could have been added to model_C to create model_A but were not is high

more impressed if the number of observations n is much greater than the number of parameters

PRE per parameter for different *n*



General procedure

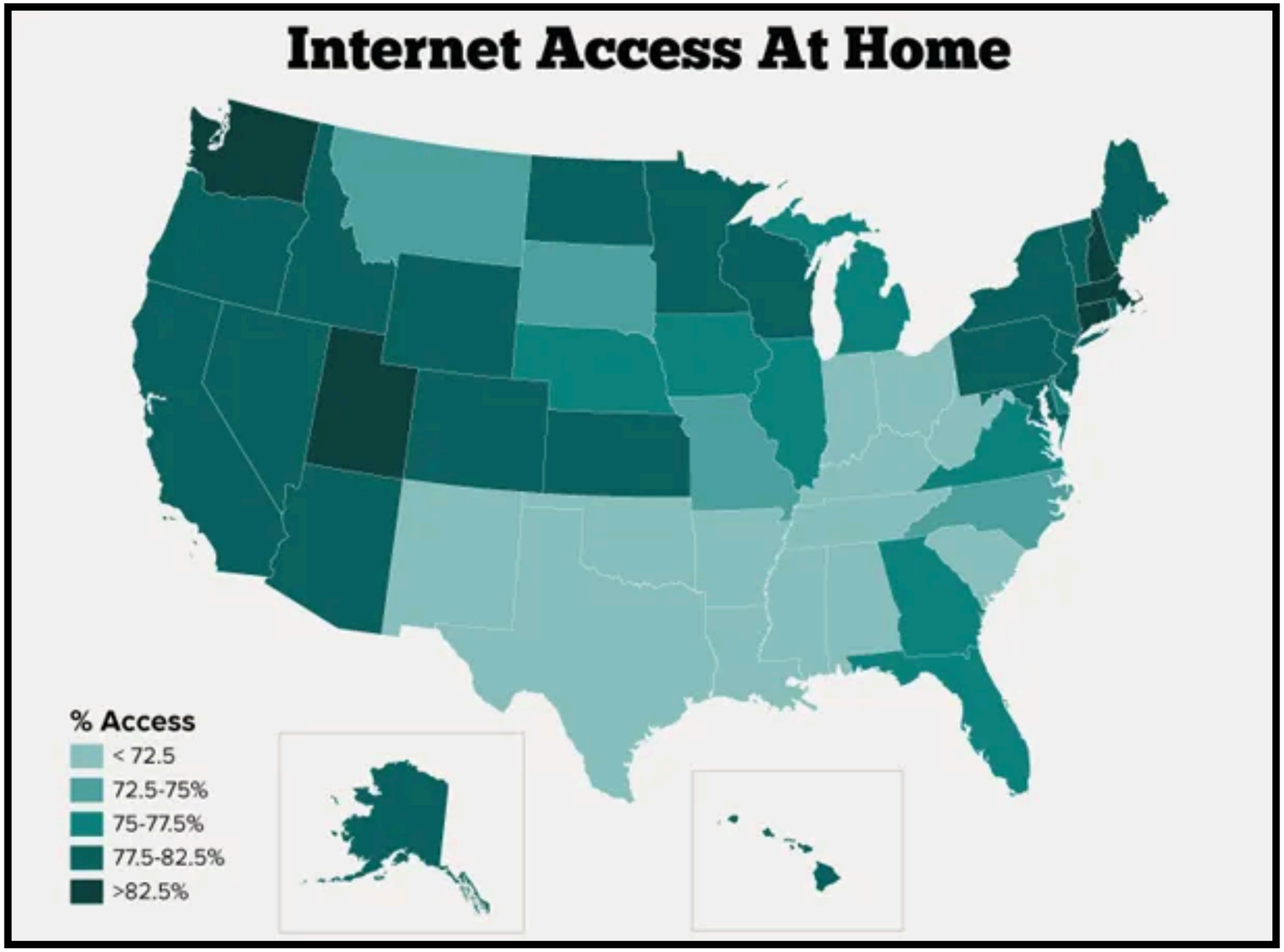
- for any question we want to ask about our DATA
  - we define $model_C$ and $model_A$
  - compare the models using PRE
  - determine whether PRE is **worth it**
- in standard frequentist lingo:
  - $model_C = H_0$ (null hypothesis)
  - $model_A = H_1$ (alternative hypothesis)

- hypothesis test:
  - H_0: **all** the parameters that are included in $model_A$ but not in $model_C$ are 0
  - H_1: **not all** the parameters that are included in $model_A$ but not in $model_C$ are 0

Hypothesis testing as model comparison

Procedure

1. Start with research question
2. Formulate hypothesis as a comparison between compact and augmented model
3. Fit parameters in each model
4. Calculate the proportional reduction of error (PRE)
5. Decide whether PRE is **worth it**



http://thedailyviz.com/2012/07/19/mapping-internet-access-in-u-s-homes/

Notation

DATA = MODEL + ERROR

$$Y_i = \beta_0 + \epsilon_i$$ simple model (true parameters)

$$Y_i = b_0 + e_i$$ simple model (estimated parameters)

$$\hat{Y}_i = b_0$$

college

$$Y_i = b_0 + b_1 X_{i1} + e_i$$ more complex model

Percentage of households that had internet access in the year 2013 by US state

i	internet	state	college	auto	density
1	79.0	AK	28.0	1.2	1.2
2	63.5	AL	23.5	1.3	94.4
3	60.9	AR	20.6	1.7	56.0
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9	74.3	FL	27.2	1.2	350.6
10	72.2	GA	28.3	1.1	168.4

Greek letters β or ε represent the true but unknowable parameters in the population.

Roman letters ***b*** or ***e*** represent estimates of these parameters using our DATA.

Research question and hypotheses

Is the average percentage of internet users per state significantly different from 75%?

Model$_C$: $Y_i = B_0 + \epsilon_i$

0 parameters

$$Y_i = 75 + e_i$$

Model$_A$: $Y_i = \beta_0 + \epsilon_i$

1 parameter

$$Y_i = b_0 + e_i$$

$$= \overline{Y} + e_i$$

i	internet	state	college	auto	density
1	79.0	AK	28.0	1.2	1.2
2	63.5	AL	23.5	1.3	94.4
3	60.9	AR	20.6	1.7	56.0
4	73.9	AZ	27.4	1.3	56.3
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9	74.3	FL	27.2	1.2	350.6
10	72.2	GA	28.3	1.1	168.4

Fit parameters and calculate PRE

C: $Y_i = 75 + e_i$ **A**: $Y_i = \overline{Y} + e_i$

i	state	internet	compact_b	compact_se	augmented_b	augmented_se
1	AK	79.0	75	16.00	72.81	38.37
2	AL	63.5	75	132.25	72.81	86.60
3	AR	60.9	75	198.81	72.81	141.75
4	AZ	73.9	75	1.21	72.81	1.20
5	CA	77.9	75	8.41	72.81	25.95
6	CO	79.4	75	19.36	72.81	43.48
7	CT	77.5	75	6.25	72.81	22.03
8	DE	74.5	75	0.25	72.81	2.87
9	FL	74.3	75	0.49	72.81	2.23
10	GA	72.2	75	7.84	72.81	0.37

$$\text{PRE} = 1 - \frac{\text{ERROR}(A)}{\text{ERROR}(C)}$$

$$= 1 - \frac{1355}{1595} \approx .15$$

Model A has 15% less error than Model C.

SSE(C) = 1595 SSE(A) = 1355

Decide whether it's **worth it**

- we need a sampling distribution of PRE
  - a distribution of what PRE would look like if Model C (our H_0) were true
- PRE is closely related to the *F* statistic!

Decide whether it's **worth it**

- To compute the *F* statistic, we need:
  - PRE
  - number of parameters in Model C (PC) and Model A (PA)
  - number of observations *n*

more likely to be **worth it** if:

1. **PRE** is high
2. the number of additional parameters in A compared to C is low (**PRE per additional parameter**)
3. the number of parameters that could have been added to $model_C$ to create $model_A$ but were not

difference in parameters between models A and C

$$F = \frac{\text{PRE}/(\text{PA} - \text{PC})}{(1 - \text{PRE})/(n - \text{PA})}$$

number of observations vs. parameters in Model A

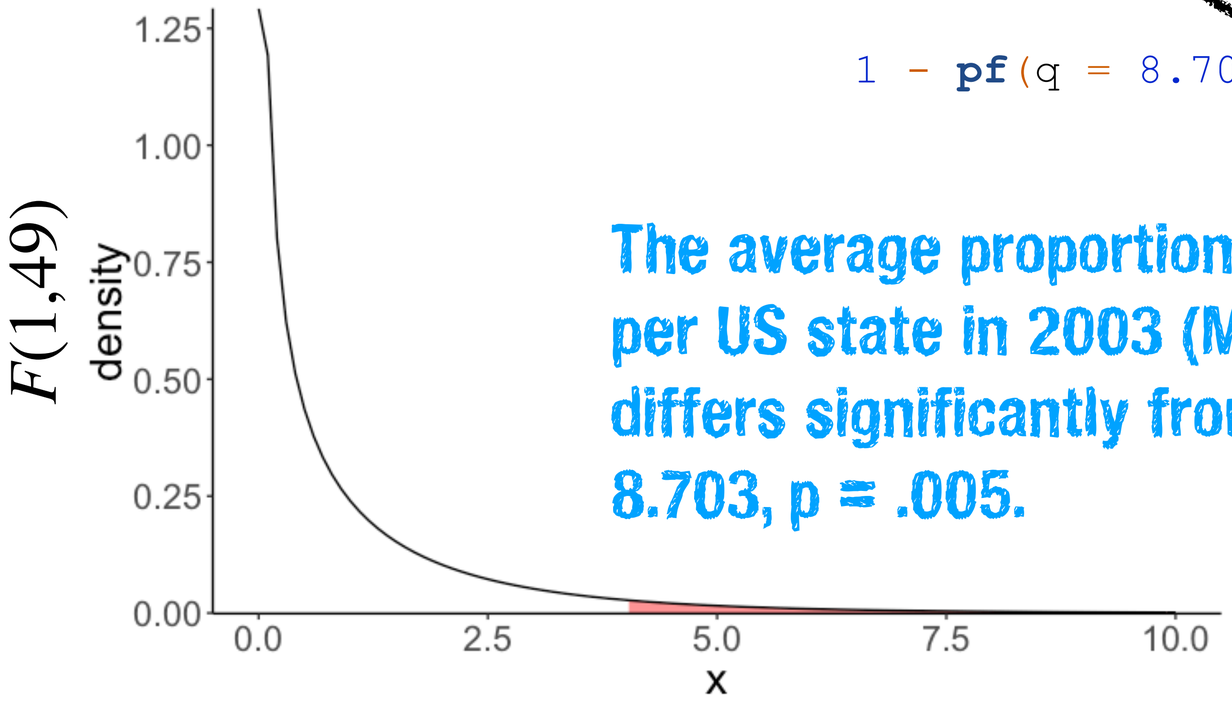
Decide whether it's worth it

Note: I've used the approximated PRE here (PRE = 0.15), but I'm reporting the F value for the non-approximated PRE value.

$$F = \frac{\text{PRE}/(\text{PA} - \text{PC})}{(1 - \text{PRE})/(n - \text{PA})}$$

$$= \frac{.15/(1 - 0)}{(1 - .15)/(50 - 1)} = 8.703 \quad p = .00486$$

```
1 - pf(q = 8.703, df1 = 1, df2 = 49)
```



The average proportion of internet access per US state in 2003 (M = 72.8, SD = 5.26) differs significantly from 75%, F(1, 49) = 8.703, p = .005.

we just performed a one sample t-test ...

```
t.test(df.internet$internet, mu = 75)
```

One Sample t-test

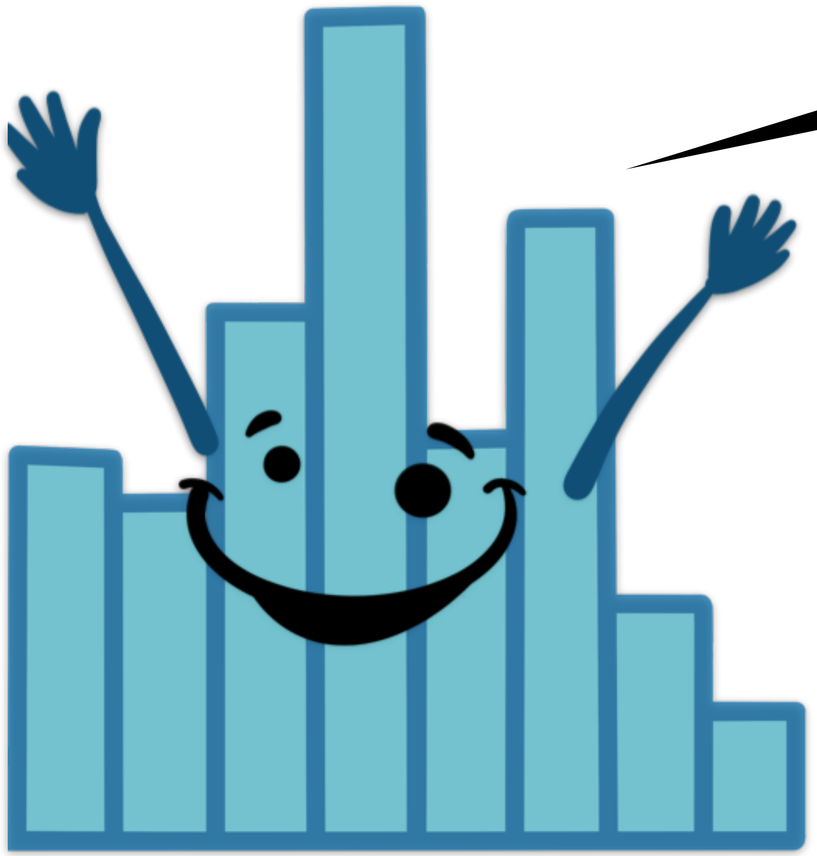
data: df.internet$internet
t = -2.9502, df = 49, p-value = **0.00486**
alternative hypothesis: true mean is not equal to 75

but ...

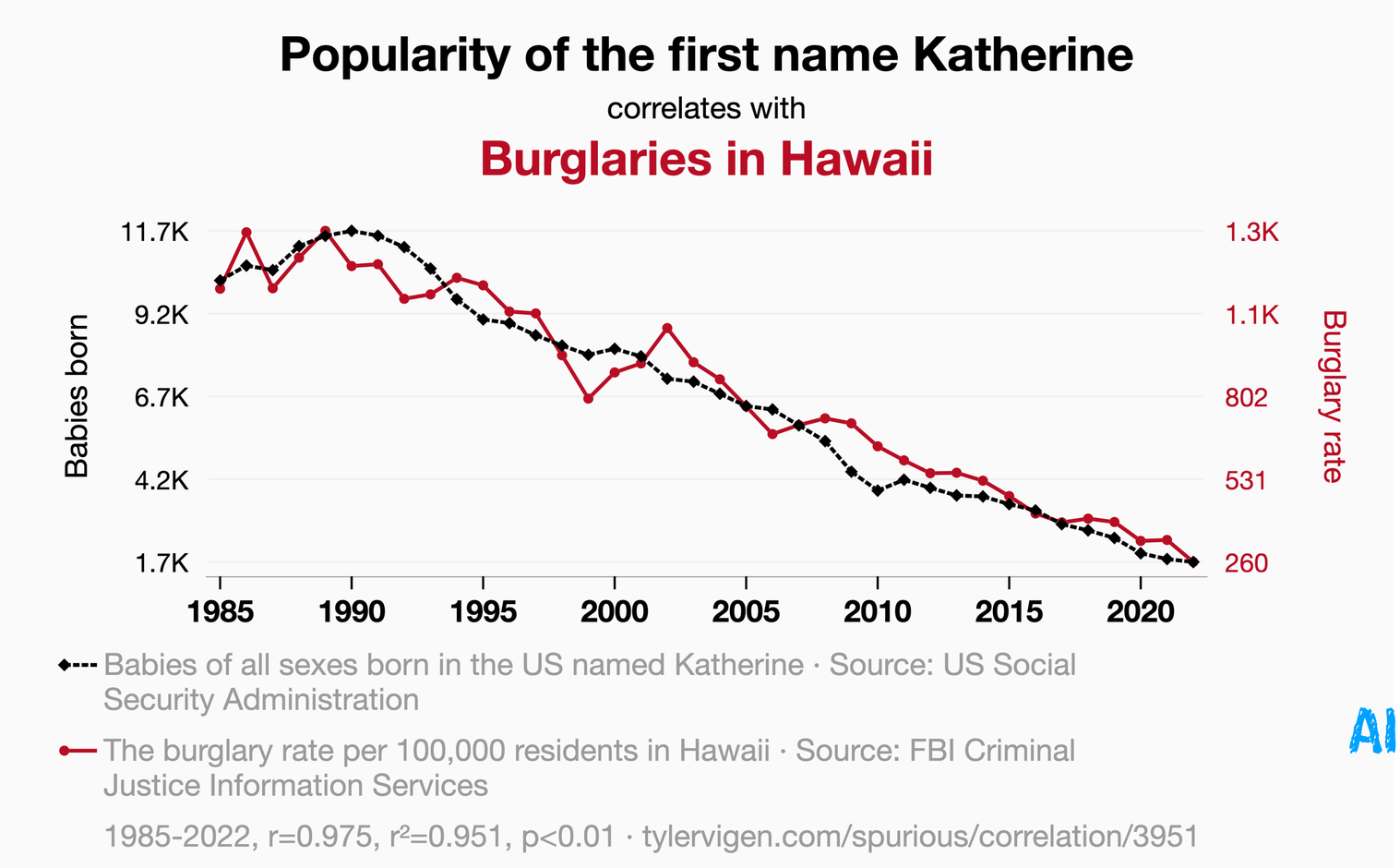
- we will apply the general procedure we went through to day to a large range of different situations
- thinking of hypothesis testing as model comparison is (hopefully more) intuitive
- this approach still works even if we can't find a recipe in our favorite stats cook book

02:00

stretch break!



Correlation

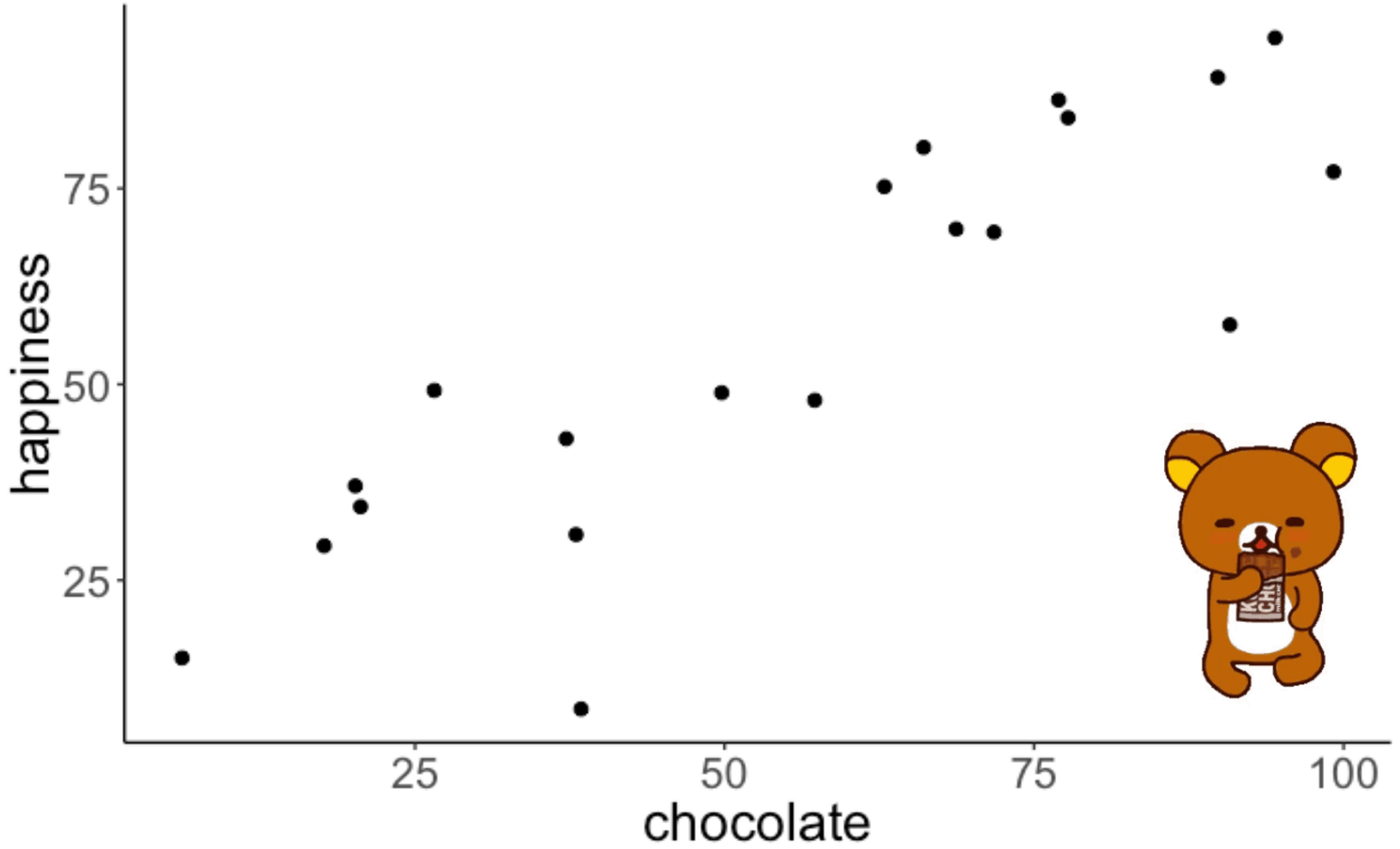


AI generated

As the saying goes, "Kat's out of the bag," and it seems that also applies to burglars in Hawaii! With fewer Katherines around, there were less Kat burglars trying to pull off heists in the sunny state. It appears that the name Katherine was previously a common alias for cat burglars with a penchant for pilfering pineapples. However, with this name falling out of favor, it seems the purr-petrators have also disappeared, leading to a decrease in burglaries. It's a feline mystery, but it looks like Hawaii can rest easy knowing that the Katherine connection has been pawsitively purr-vented!

https://tylervigen.com/spurious-correlations

How to best characterize the relationship between x and y by a single number?

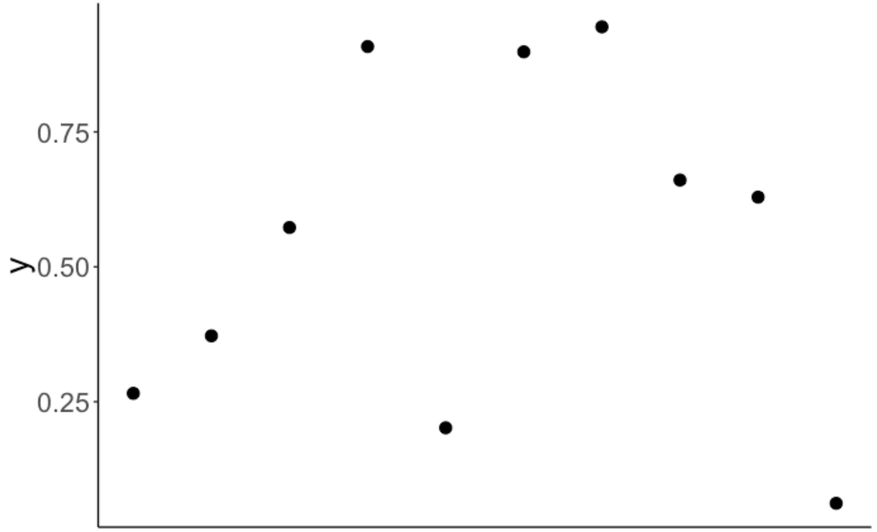


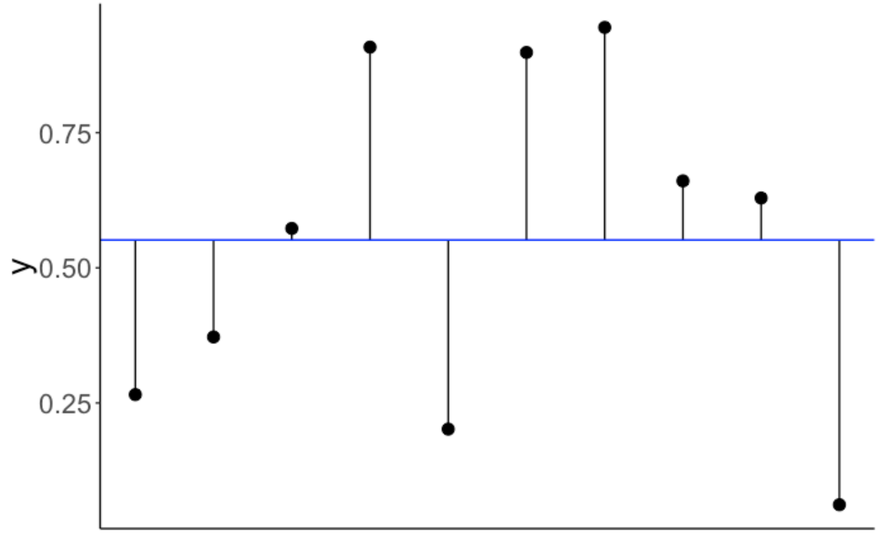
correlation = a measure of the relationship between two variables

sample variance

$$Var(Y) = \sum_{i=1}^{n} \frac{(Y_i - \overline{Y})^2}{n-1}$$

sum of squared errors



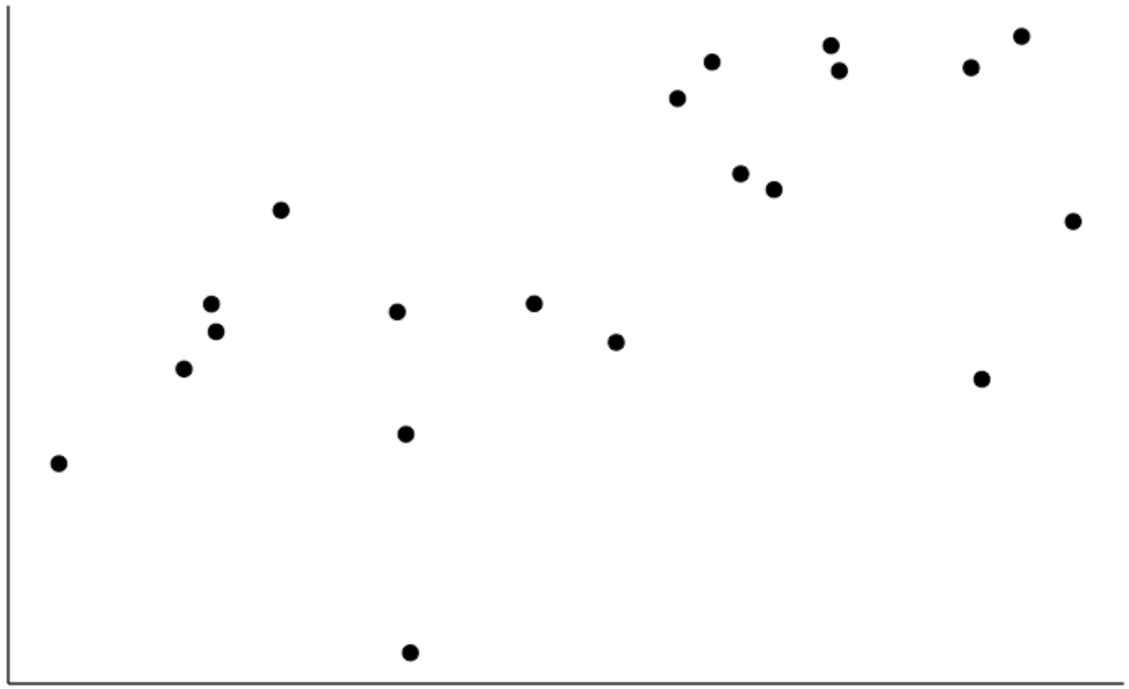


(I was too lazy to draw rectangles ...)

How well does the mean capture the data?

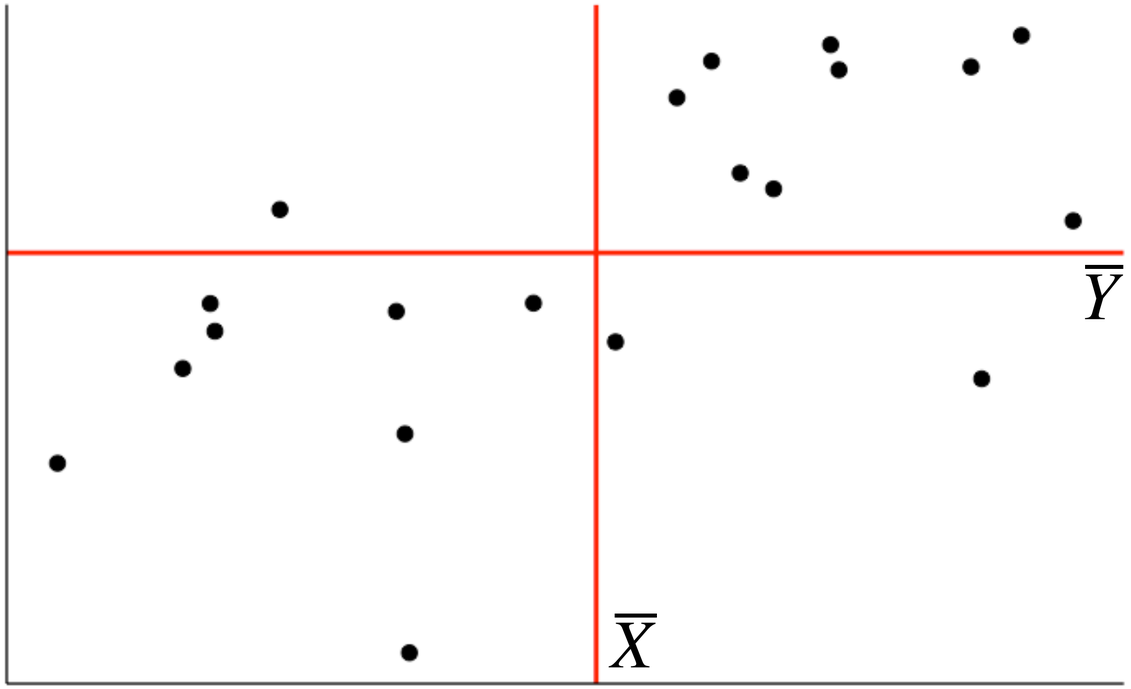
sample covariance

$$Cov(X, Y) = \sum_{i=1}^{n} \frac{(X_i - \overline{X})(Y_i - \overline{Y})}{n - 1}$$



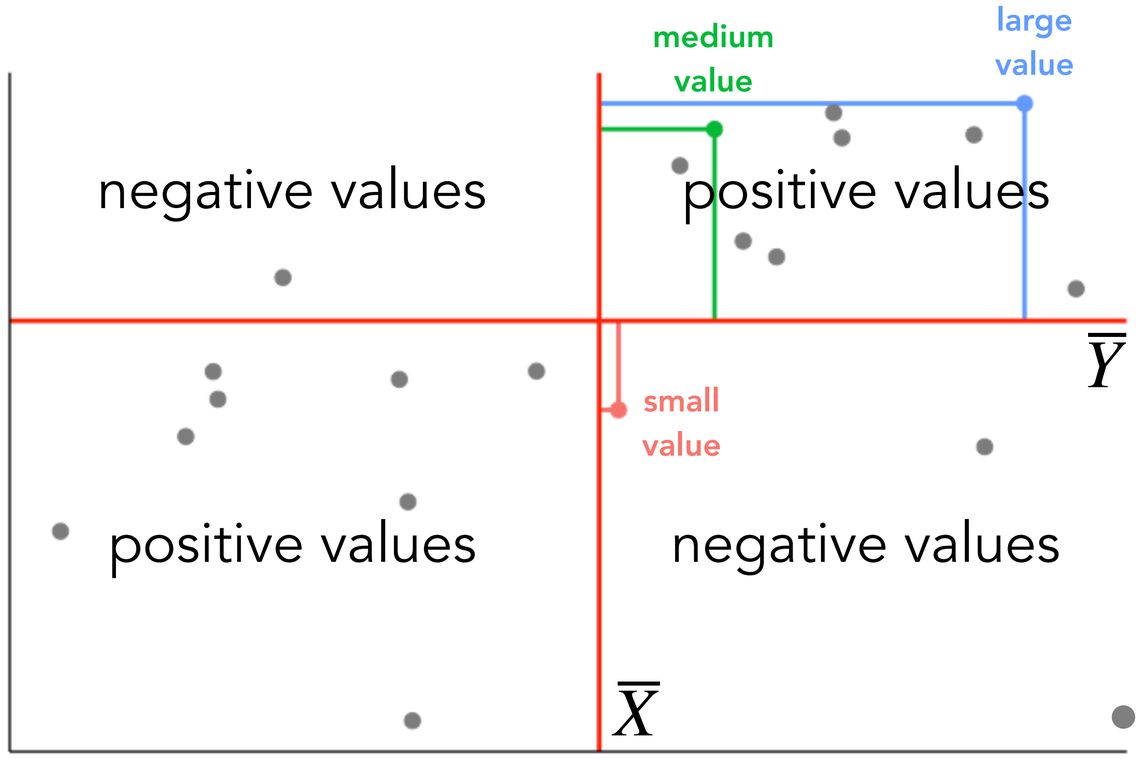
sample covariance

$$Cov(X, Y) = \sum_{i=1}^{n} \frac{(X_i - \overline{X})(Y_i - \overline{Y})}{n - 1}$$



sample covariance

$$Cov(X, Y) = \sum_{i=1}^{n} \frac{(X_i - \overline{X})(Y_i - \overline{Y})}{n - 1}$$



very nice

beware of outliers

https://tinystats.github.io/teacups-giraffes-and-statistics/05_correlation.html

sample covariance

$$Cov(X, Y) = \sum_{i=1}^{n} \frac{(X_i - \overline{X})(Y_i - \overline{Y})}{n - 1}$$

depends on the scale of the variables

the $n - 1$s cancel out

sample correlation coefficient

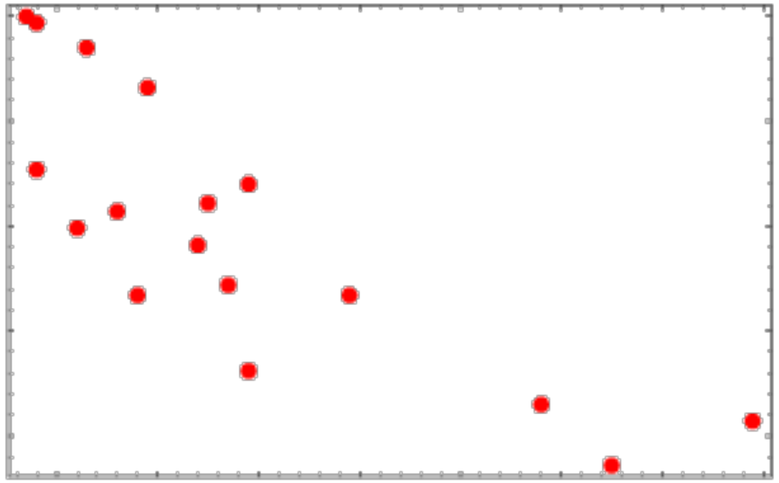
$$r(X, Y) = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2} \cdot \sqrt{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}}$$

standardized covariation
(dividing by the standard deviations)

Properties of the Pearson correlation

- standardized: $-1 \leq r \leq 1$
- scale independent (for both X and Y)
- commutativity: $r(X, Y) = r(Y, X)$ ← association not causation
- sign determines the direction of dependence
- captures **linear dependence** only

Who is the correlation champion?

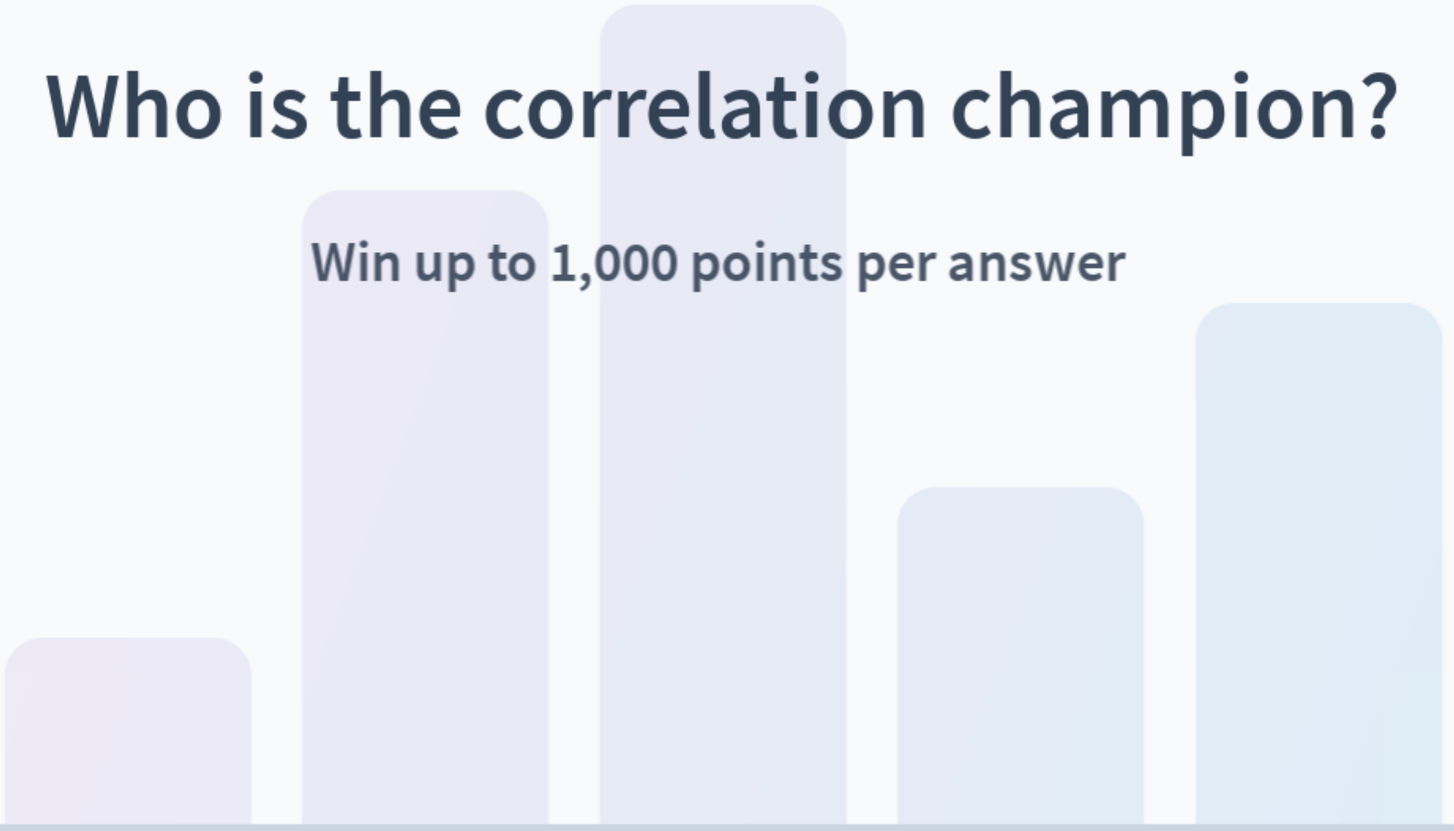


Winner gets chocolate!

-1 : -0.75
-0.75 : -0.5
-0.5 : -0.25
-0.25 : 0
0 : 0.25
0.25 : 0.5
0.5 : 0.75
0.75 : 1

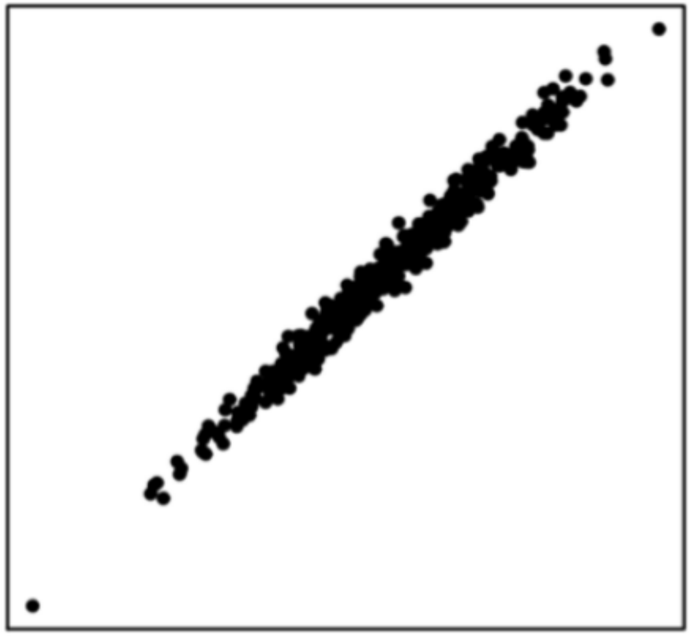
Who is the correlation champion?

Win up to 1,000 points per answer



In what range is the correlation coefficient?

0



15

-1 : -0.75

-0.75 : -0.5

-0.5 : -0.25

-0.25 : 0

0 : 0.25

0.25 : 0.5

0.5 : 0.75

0.75 : 1

0

-1 : -0.75

-0.75 : -0.5

-0.5 : -0.25

-0.25 : 0

0 : 0.25

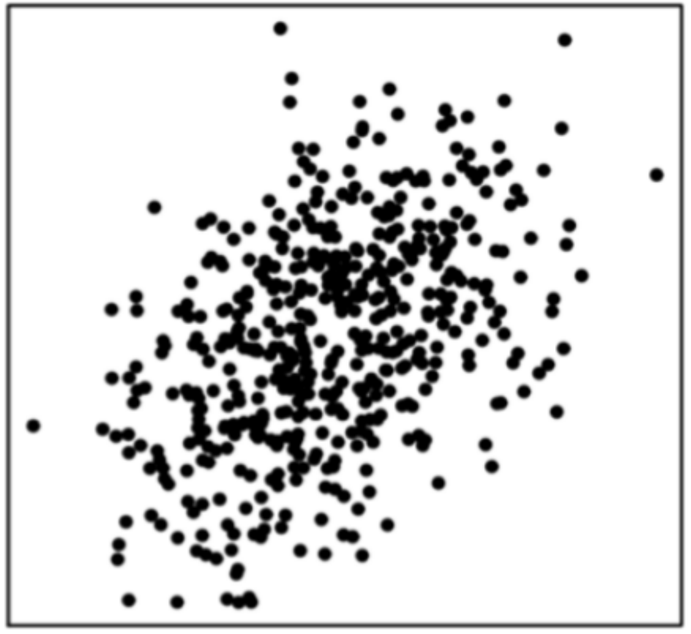
0.25 : 0.5

0.5 : 0.75

0.75 : 1

15

0



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-1 : -0.75

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-0.5 : -0.25

-0.25 : 0

0 : 0.25

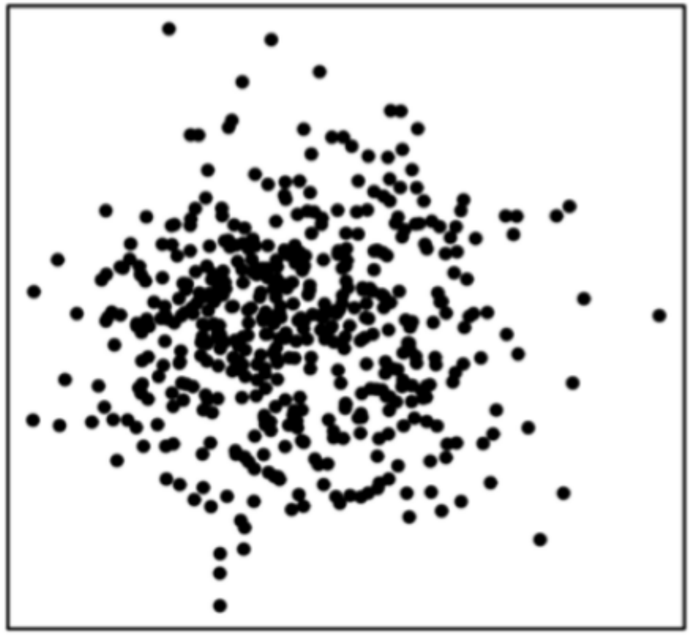
0.25 : 0.5

0.5 : 0.75

0.75 : 1

Leaderboard

0



15

-1 : -0.75

-0.75 : -0.5

-0.5 : -0.25

-0.25 : 0

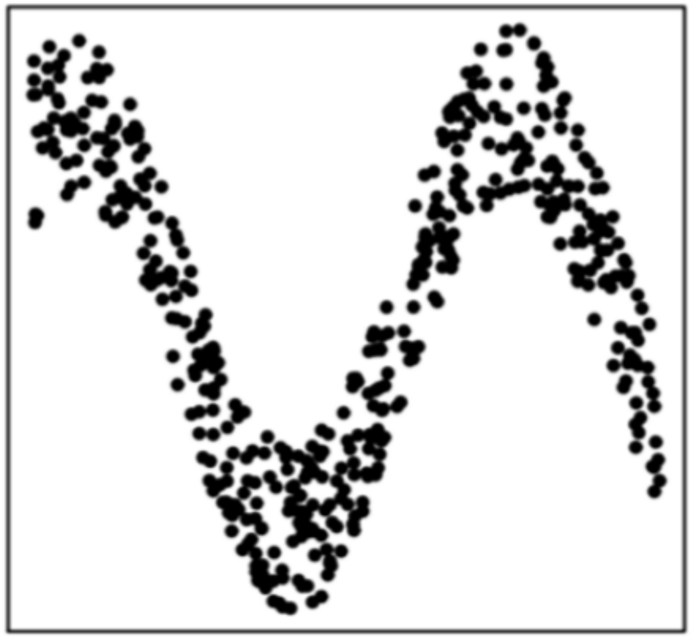
0 : 0.25

0.25 : 0.5

0.5 : 0.75

0.75 : 1

0



15

-1 : -0.75

-0.75 : -0.5

-0.5 : -0.25

-0.25 : 0

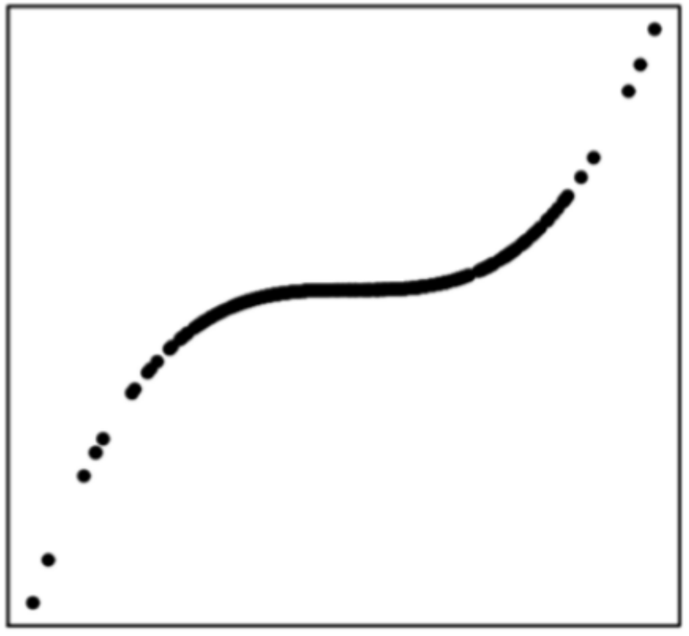
0 : 0.25

0.25 : 0.5

0.5 : 0.75

0.75 : 1

0



15

-1 : -0.75

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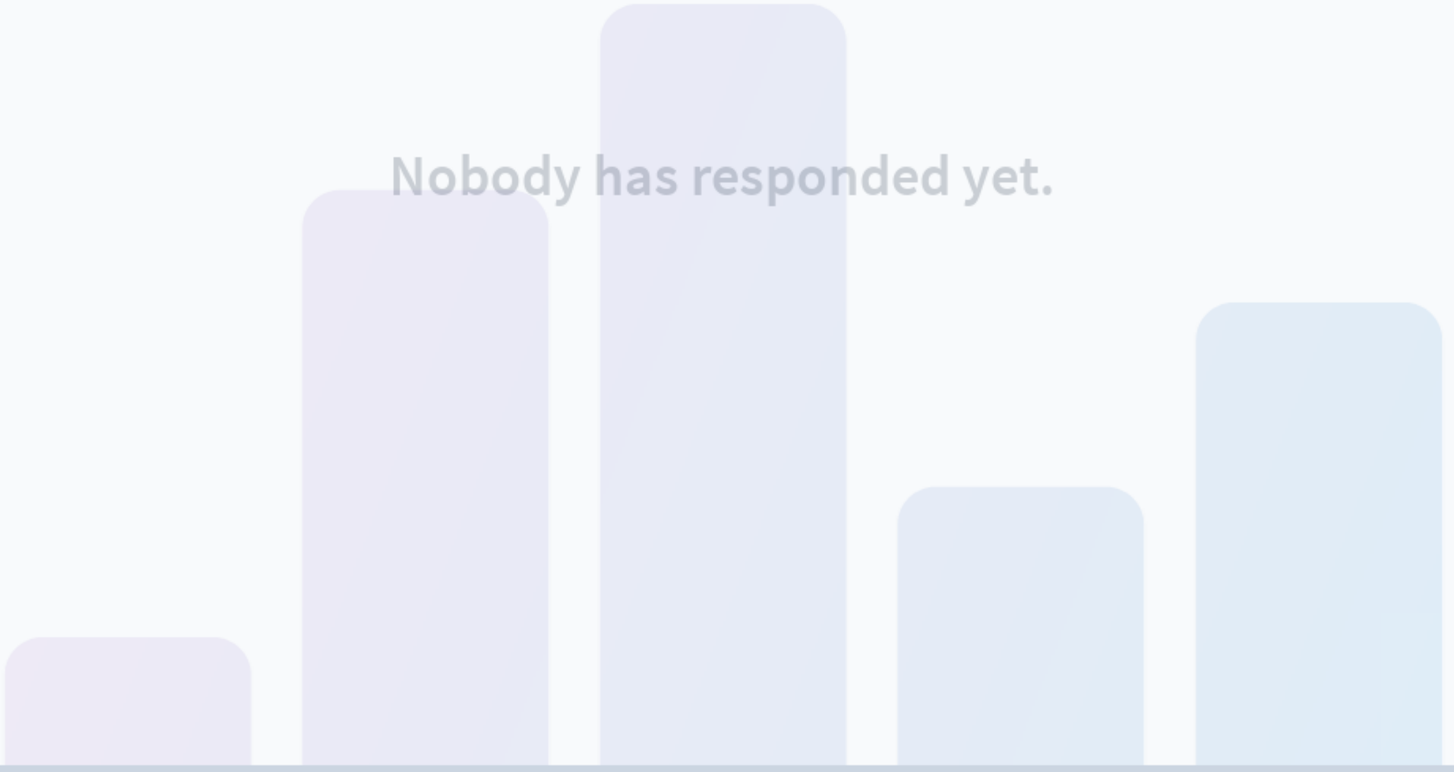
0.25 : 0.5

0.5 : 0.75

0.75 : 1

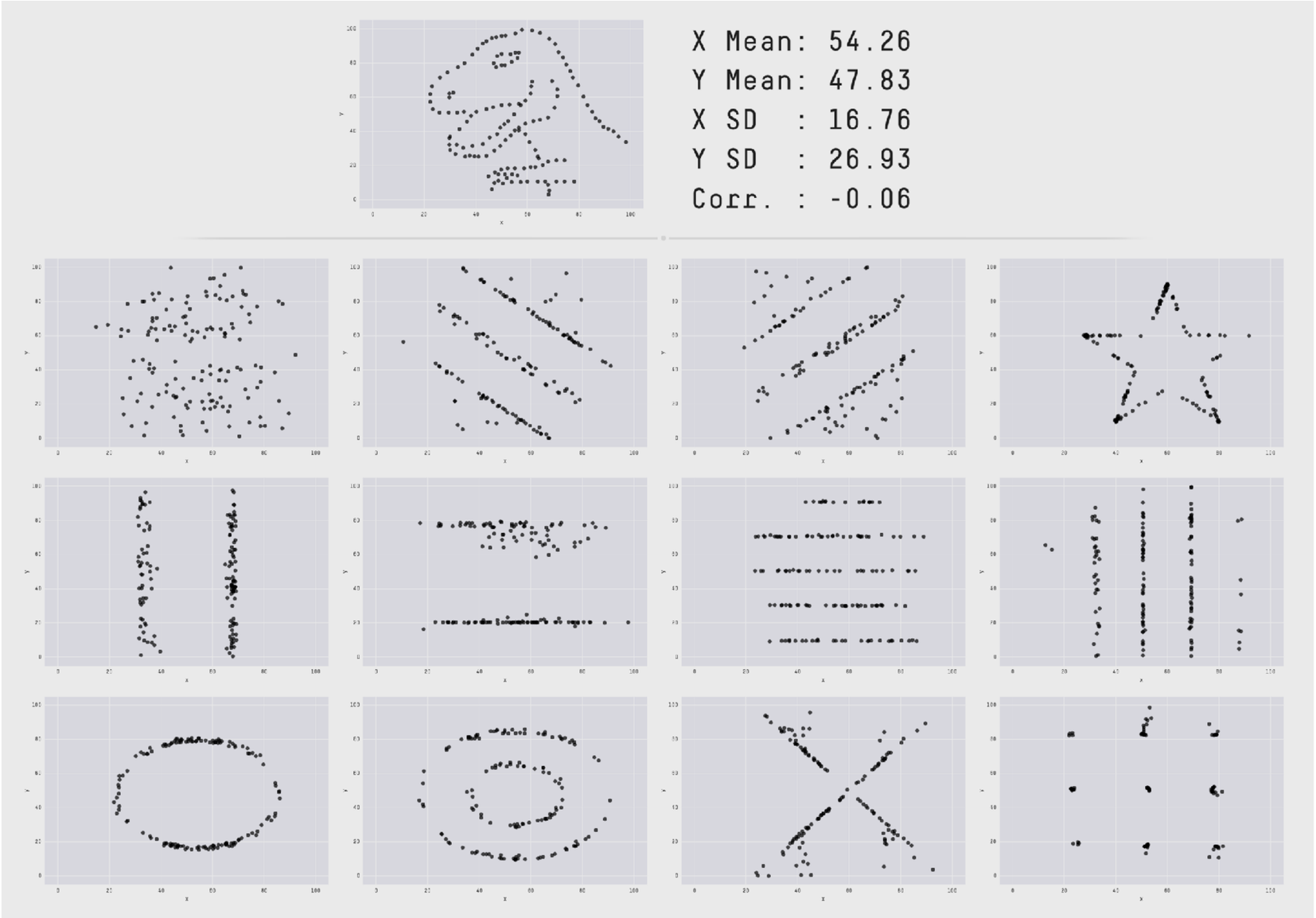
Leaderboard



Solution

XX

Be careful about interpreting correlations!

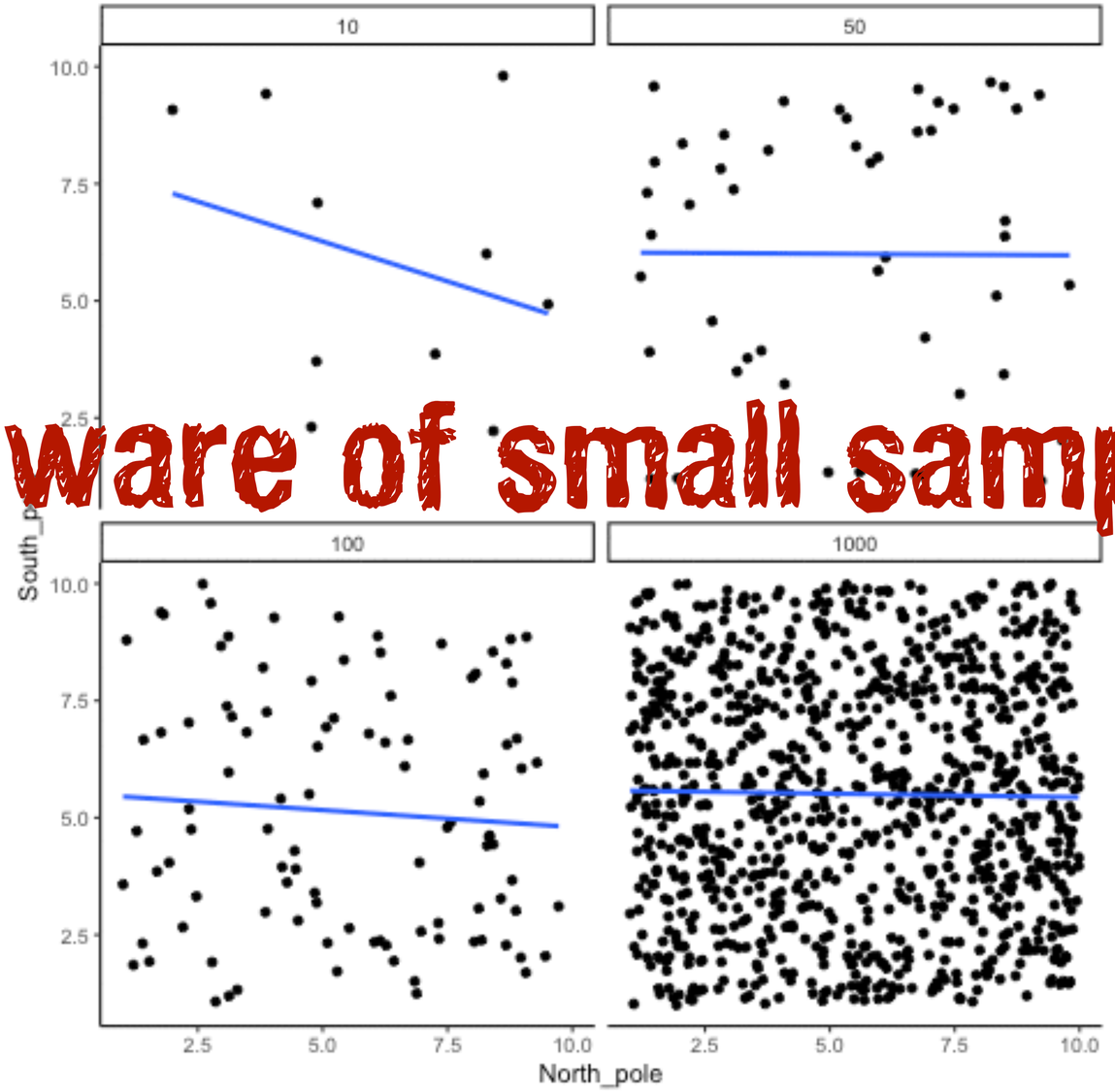


always visualize the data ...

$n = [10, 50, 100, 1000]$

$X \sim \mathcal{U}(\text{min} = 0, \text{max} = 10)$

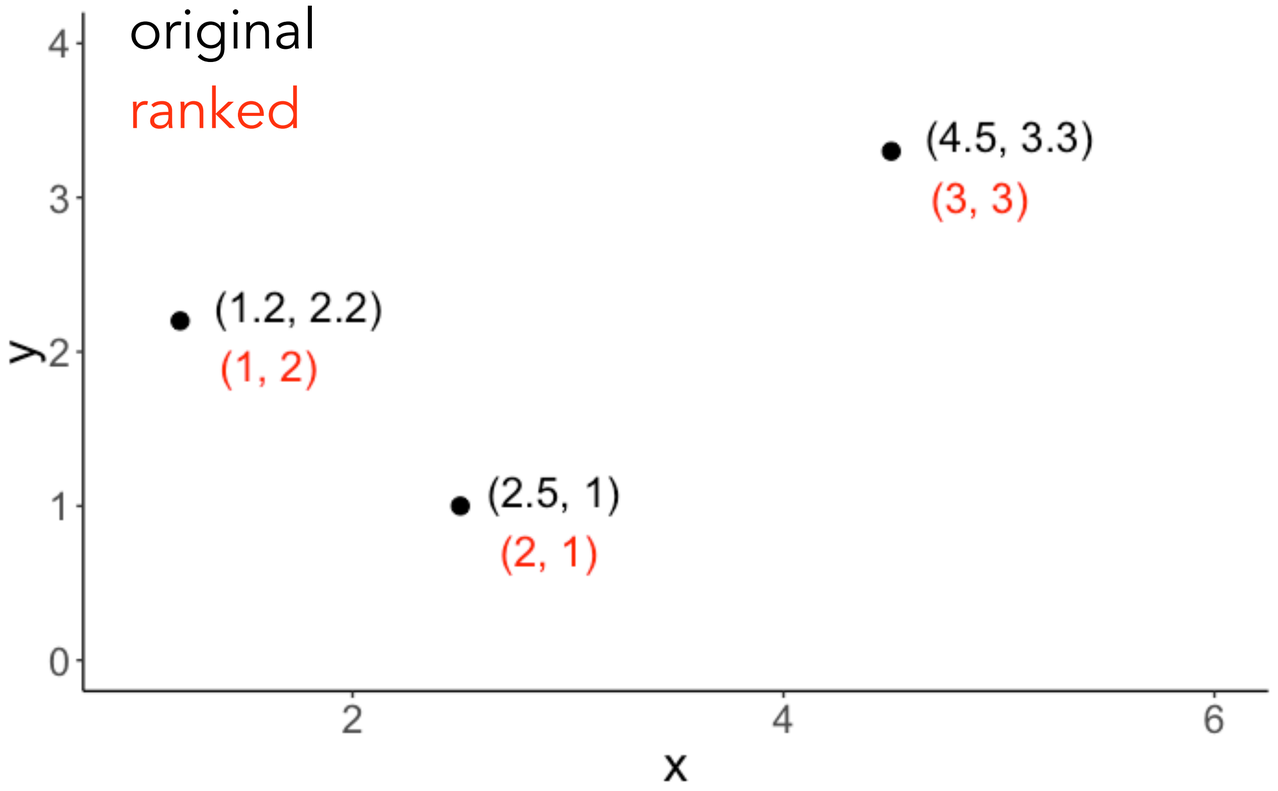
$Y \sim \mathcal{U}(\text{min} = 0, \text{max} = 10)$



Beware of small samples!

Spearman rank order correlation

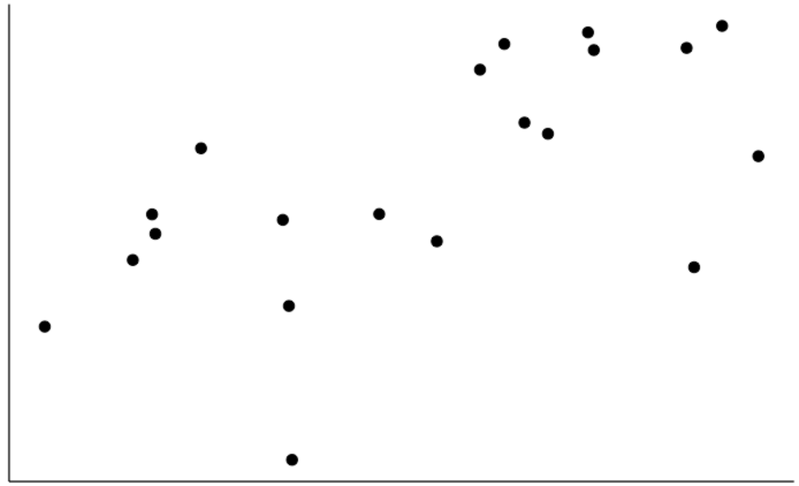
- transform original data into ranks
- calculate correlation on the ranked data



Spearman rank order correlation

- transform original data into ranks
- calculate correlation on the ranked data

x	y	x_rank	y_rank
0.27	1.14	5	12
0.37	0.97	6	8
0.57	0.92	10	6
0.91	0.85	18	4
0.20	0.98	3	9
0.90	1.39	17	17
0.94	1.44	19	20
0.66	1.40	12	18
0.63	1.33	11	15
0.06	0.71	1	2



r	spearman	r_ranks
0.609	0.595	0.595

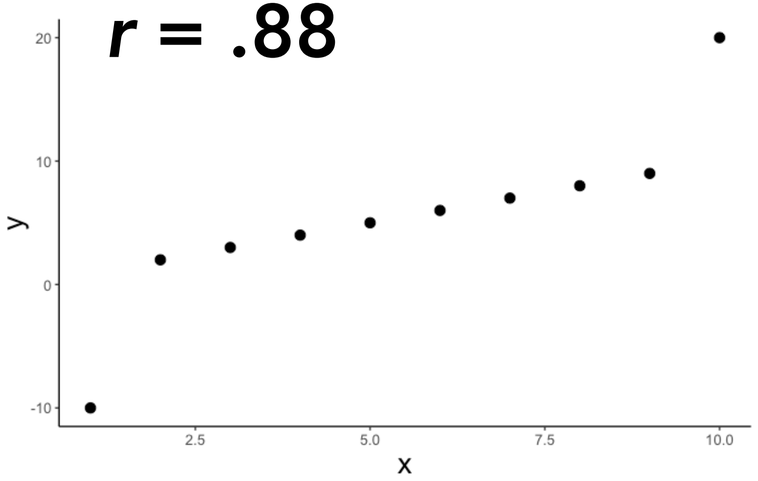
```
1 # correlation
2 df.spearman %>%
3   summarize(r = cor(x, y, method = "pearson"),
4             spearman = cor(x, y, method = "spearman"),
5             r_ranks = cor(x_rank, y_rank, method = "pearson"))
```

Spearman rank order correlation

original

ranked

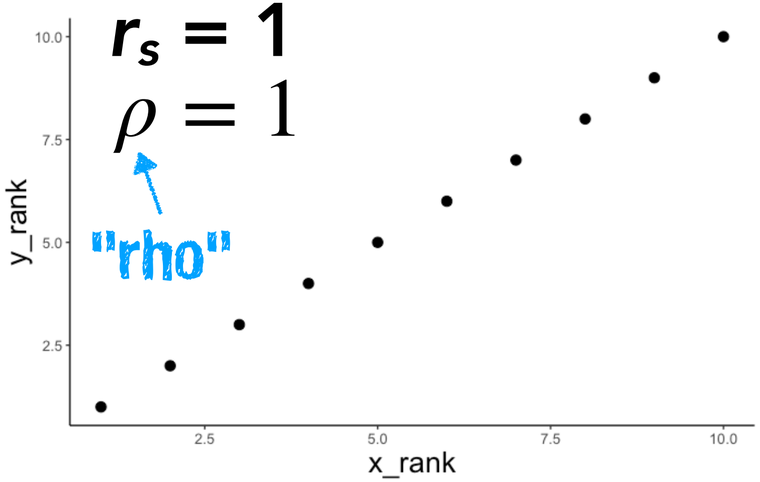
$r = .88$



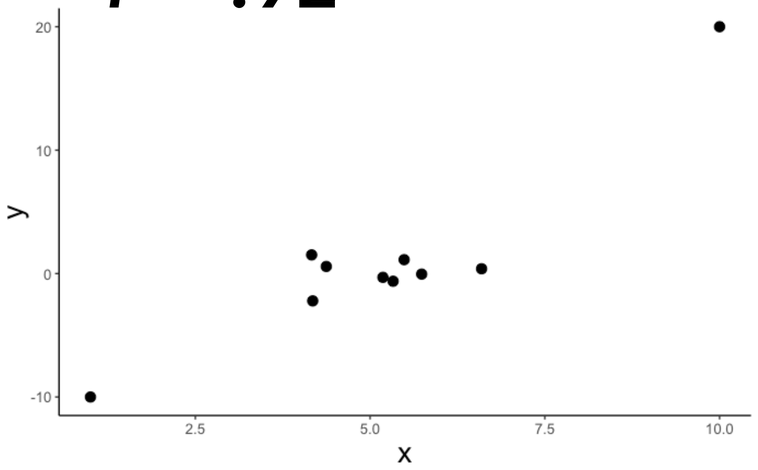
$r_s = 1$

$\rho = 1$

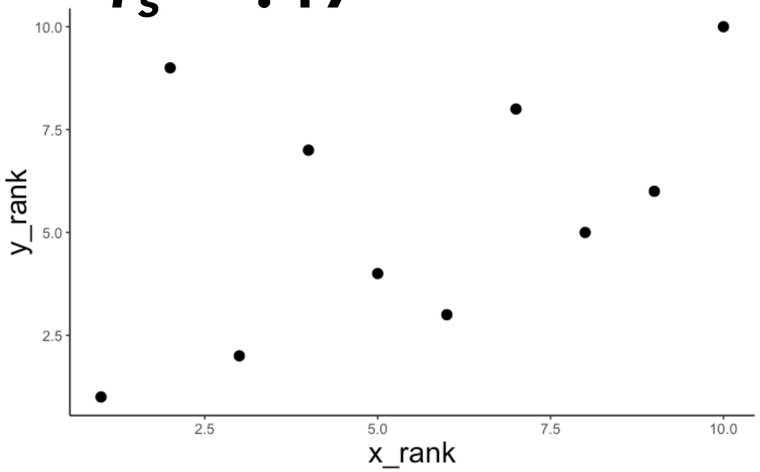
"rho"



$r = .92$



$r_s = .47$



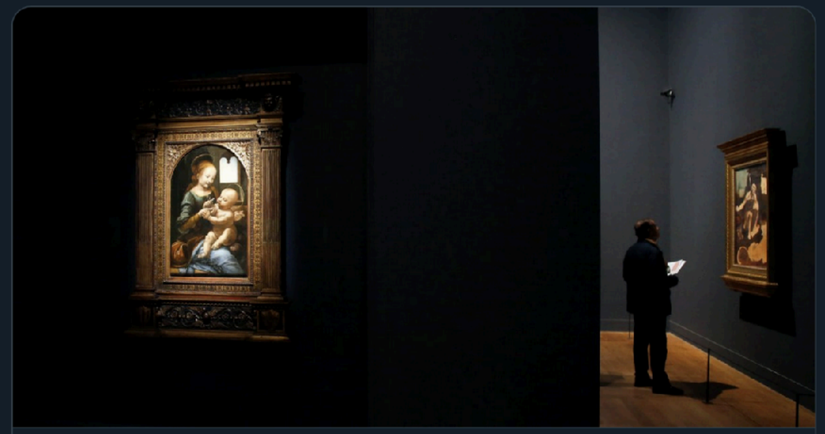
Pearson vs. Spearman

- Pearson's *r* captures the extent to which the relationship between two variable is **linear**
- Spearman's ρ captures the extent to which the relationship between two variables is **monotonic**
- What's better?
  - depends on the context
  - Spearman is robust to outliers, but it throws away (potentially useful) information

CORRELATION IS NOT CAUSATION

NYT Health
@NYTHealth

Want to live longer? Try going to the opera. Researchers in Britain have found that people who reported going to a museum or concert even once a year lived longer than those who didn't.



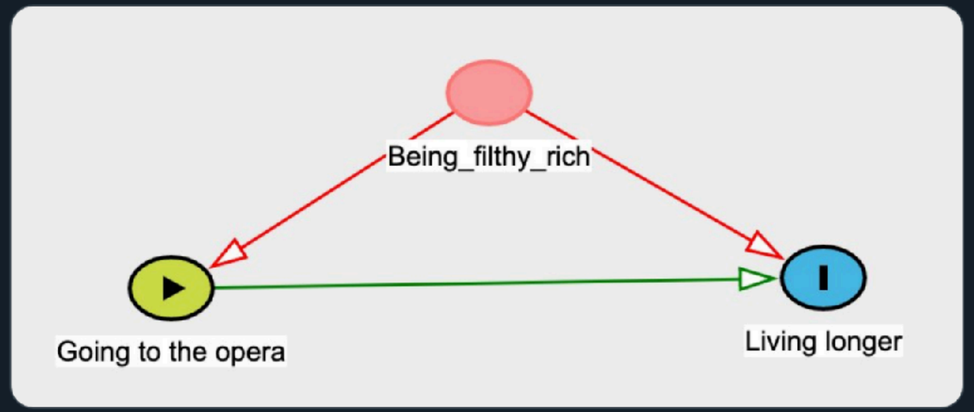
Another Benefit to Going to Museums? You May Live Longer
Researchers in Britain found that people who go to museums, the theater and the opera were less likely to die in the study period than those who didn't.
nytimes.com

9:19 AM · Dec 22, 2019 · SocialFlow

336 Retweets **1.3K** Likes

Andrew Heiss
@andrewheiss

ooh ooh i can draw the dag for this one!

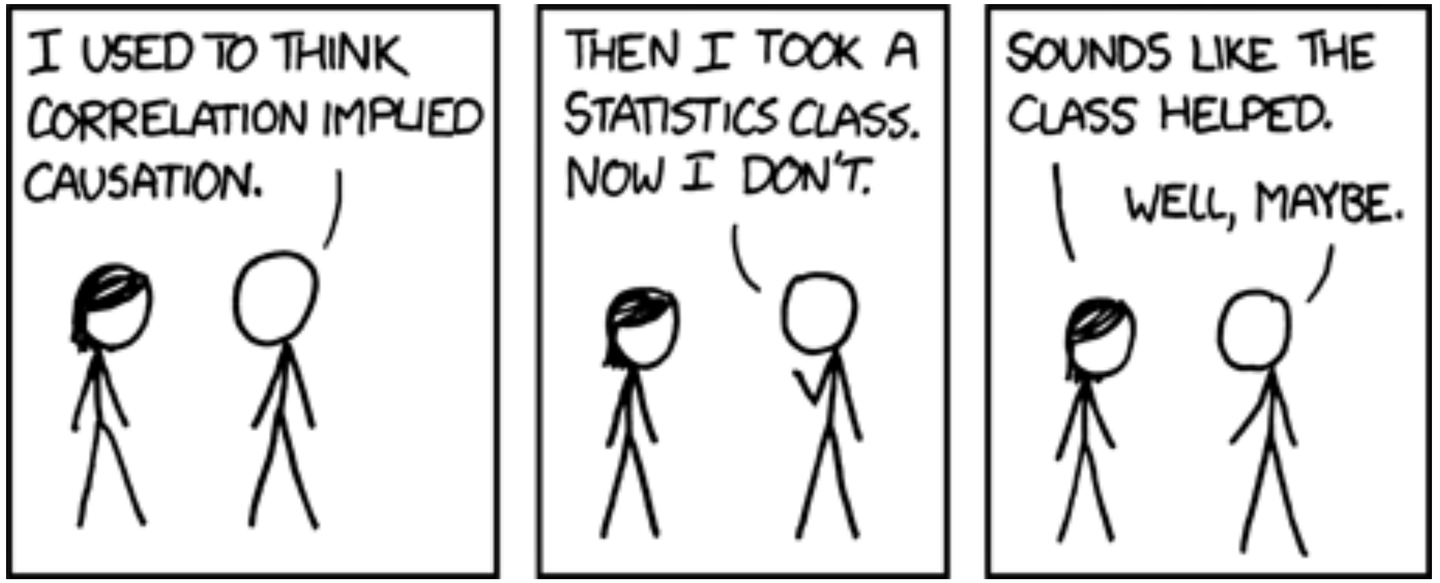


NYT Health @NYTHealth · Dec 22, 2019
Want to live longer? Try going to the opera. Researchers in Britain have found that people who reported going to a museum or concert even once a year lived longer than those who didn't. nyti.ms/2Q9AmZV

2:47 PM · Dec 22, 2019 · Twitter Web App

View Tweet activity

837 Retweets **3.9K** Likes



- correlations suggest that there is some causal relationship
- but this relationship need not be a direct causal relationship from A to B (or from B to A)

more about causation in a later class

Regression

The conceptual tour

Linear model: Simple regression

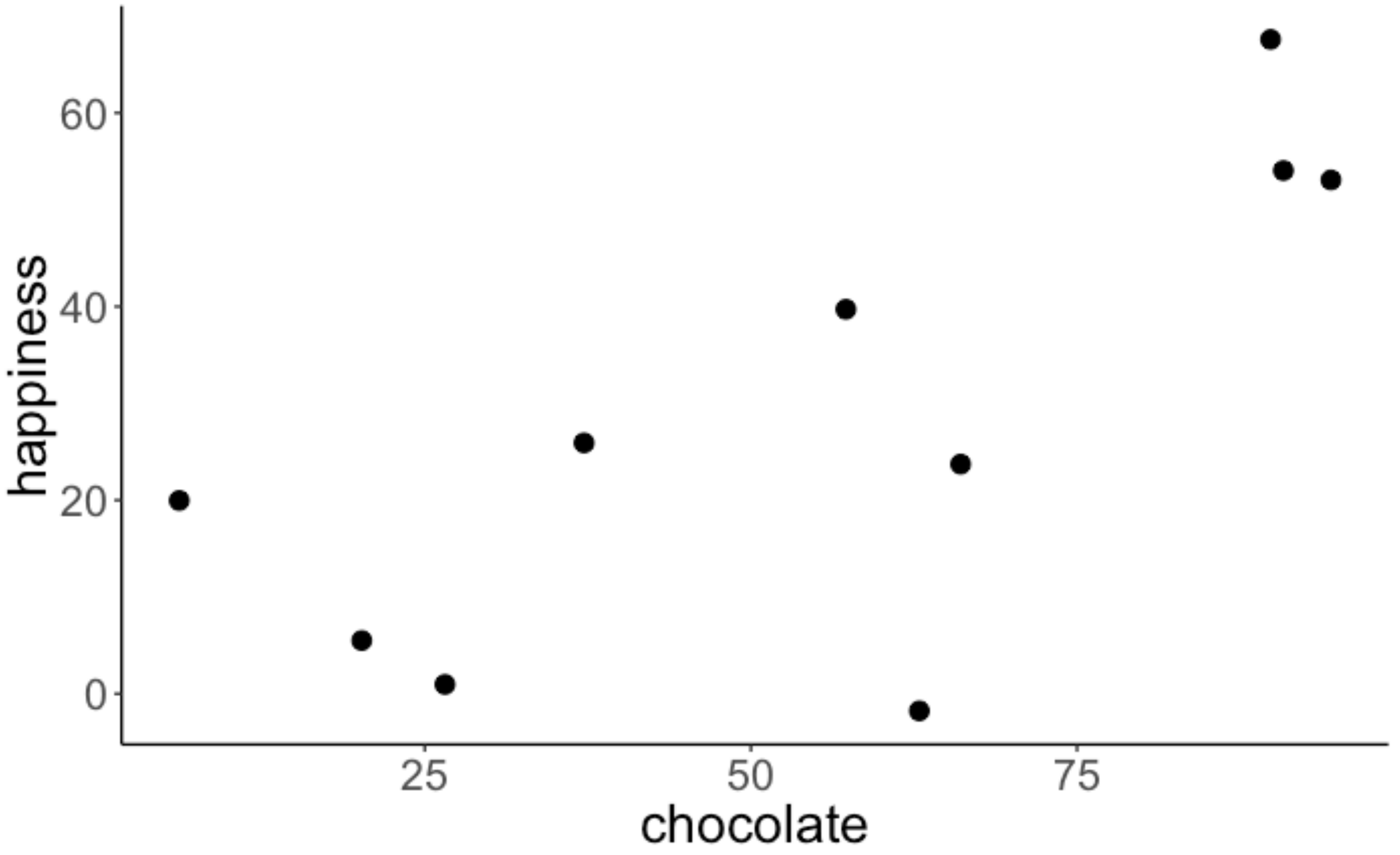
$$\texttt{Data} = \texttt{Model} + \texttt{Error}$$

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

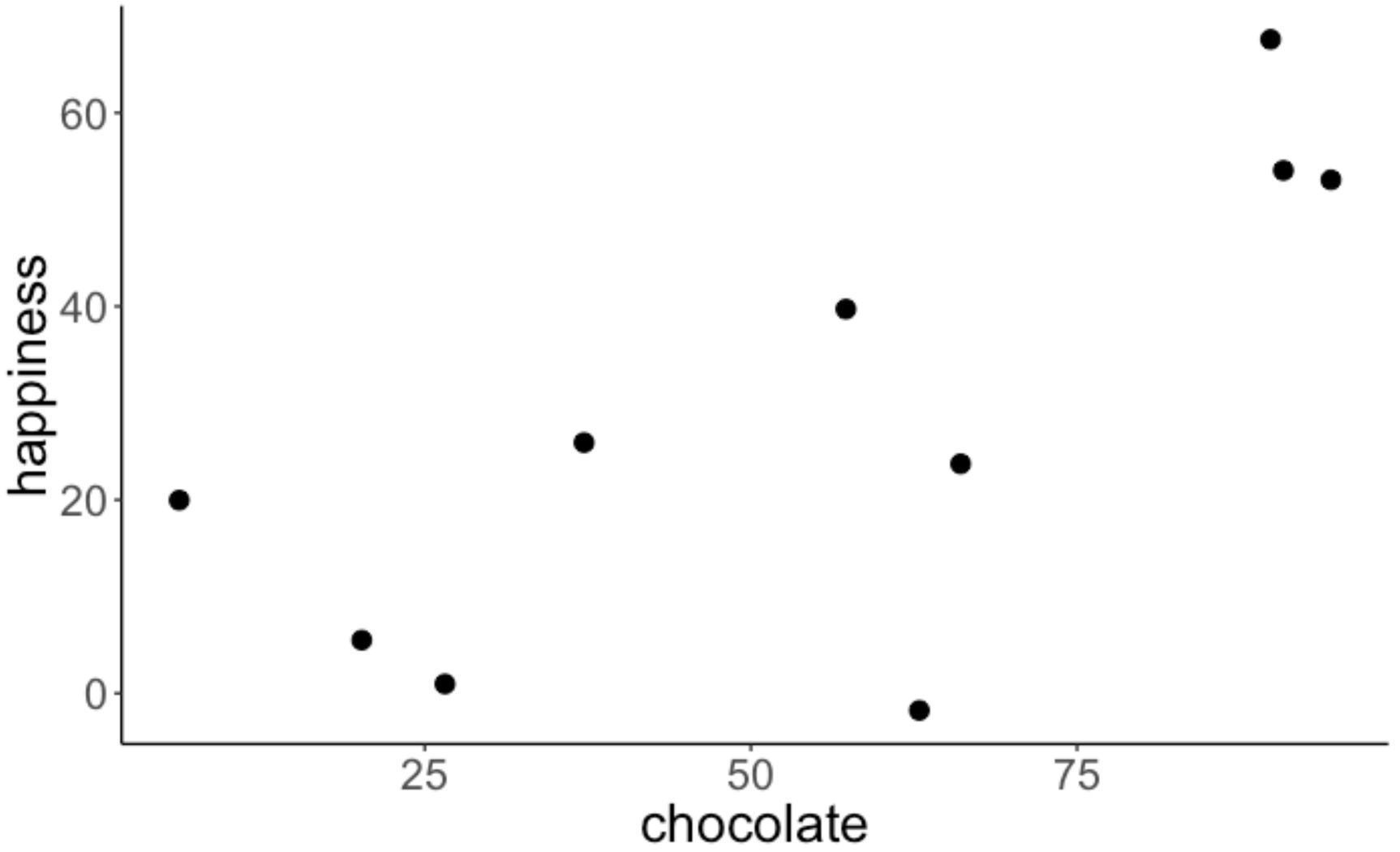
$$\hat{Y}_i = \beta_0 + \beta_1 X_i$$

the model is a linear combination of predictors

Does chocolate make us happy?



Is there a relationship between chocolate consumption and happiness?



The general procedure

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data
3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

The general procedure

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H_0: Chocolate consumption and happiness are unrelated.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

and

$$\beta_1 = 0$$

H_1: Chocolate consumption and happiness are related.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

chocolate consumption

The general procedure

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. **Fit model parameters to the data**
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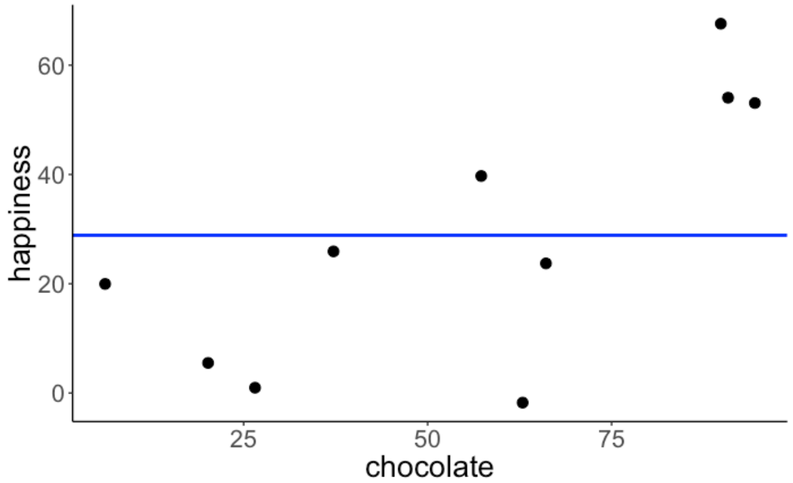
H_0: Chocolate consumption and happiness are unrelated.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

Model prediction



Fitted model

$$Y_i = 28.88 + e_i$$

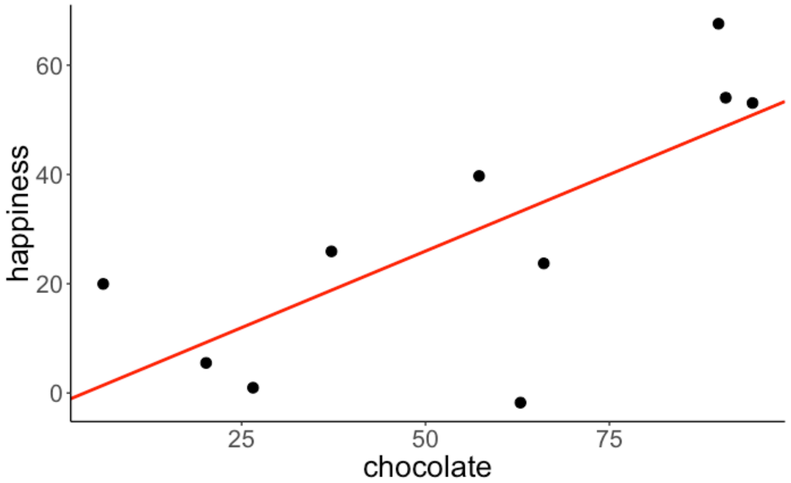
H_1: Chocolate consumption and happiness are related.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

chocolate consumption

Model prediction



Fitted model

$$Y_i = -2.04 + 0.56 X_i + e_i$$

The general procedure

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data
3. **Calculate the proportional reduction of error (PRE) in our sample**
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

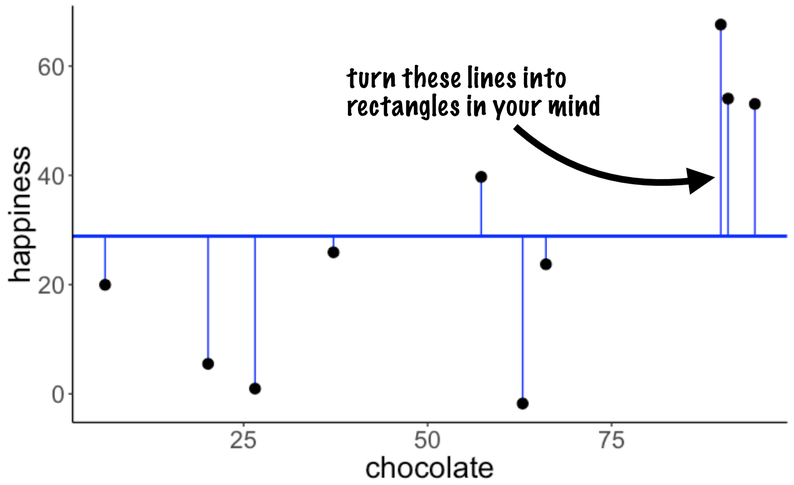
Calculate PRE

$$\text{PRE} = 1 - \frac{\text{ERROR}(A)}{\text{ERROR}(C)}$$

Both models were fit to minimize the sum of squared errors

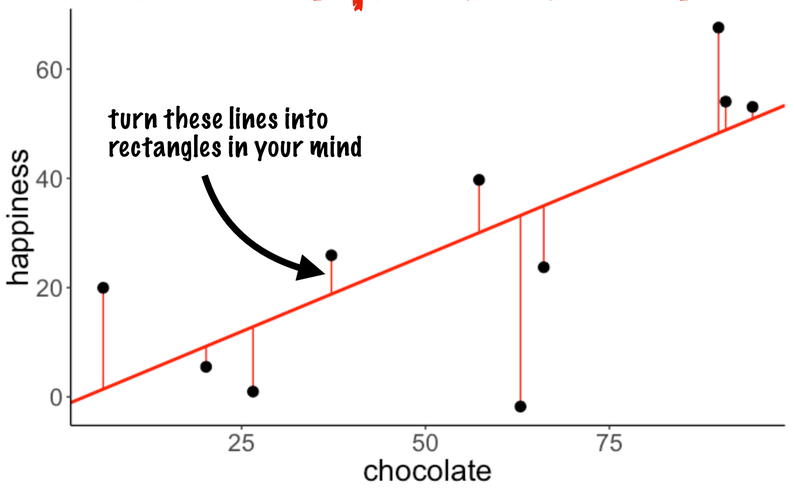
OLS = Ordinary **least squares** regression

Sum of squared errors



SSE(C) = 5215.016

Sum of squared errors

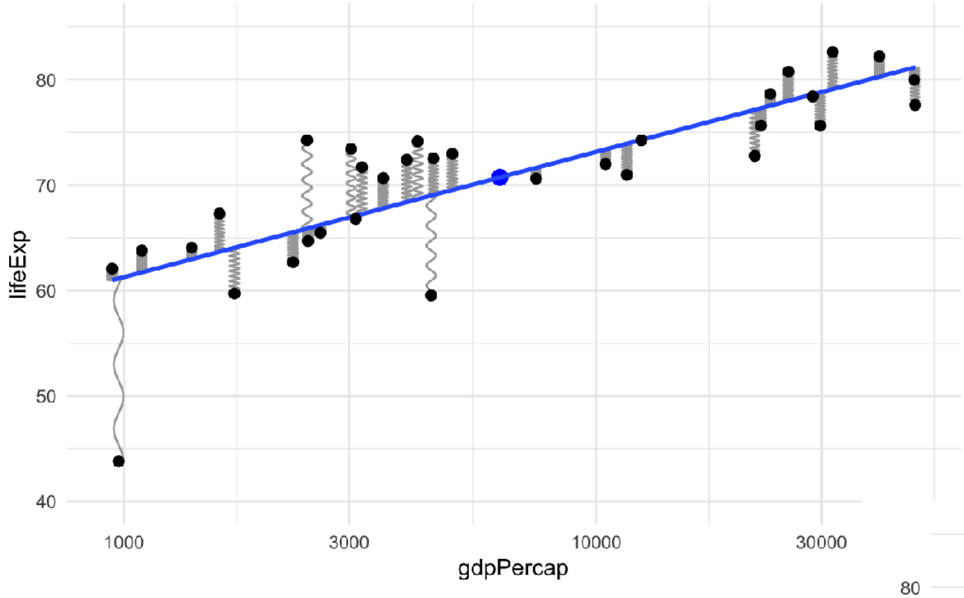


SSE(A) = 2396.946

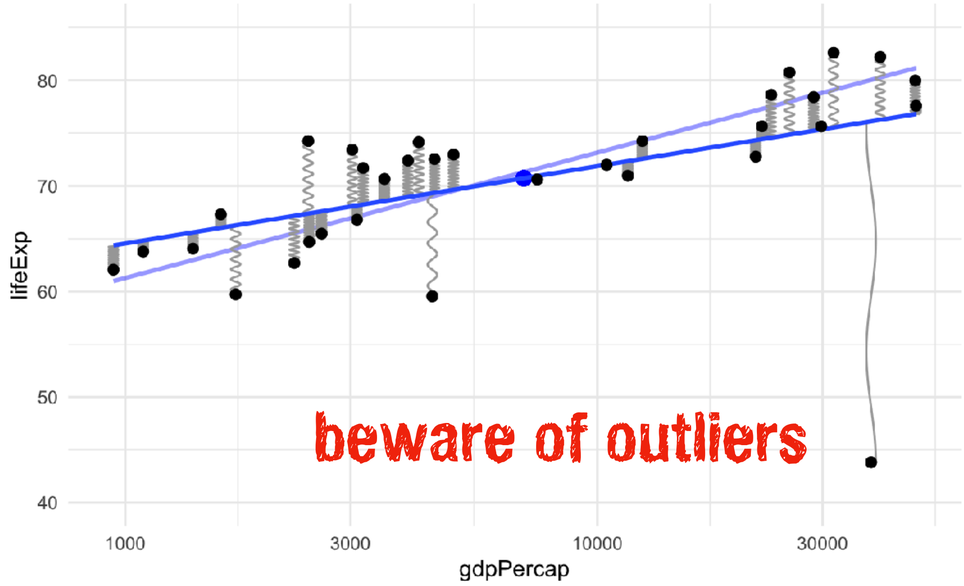
$$\text{PRE} = 1 - \frac{2396.946}{5215.016} \approx 0.54$$

The augmented model reduces the error by 54%.

Least squares as springs



each point is attached to the line with an identical spring



https://joshualoftus.com/posts/2020-11-23-least-squares-as-springs/

The general procedure

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data
3. Calculate the proportional reduction of error (PRE) in our sample
4. **Decide whether the augmented model is worth it by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)**

Decide whether it's **worth it**

- To compute the *F* statistic, we need:
  - PRE
  - number of parameters in Model C (PC) and Model A (PA)
  - number of observations *n*

- more likely to be **worth it** if:
  1. **PRE** is high
  2. the number of additional parameters in A compared to C is low (**PRE per additional parameter**)
  3. the number of parameters that could have been added to $model_C$ to create $model_A$ but were not

$$F = \frac{PRE/(PA - PC)}{(1 - PRE)/(n - PA)}$$

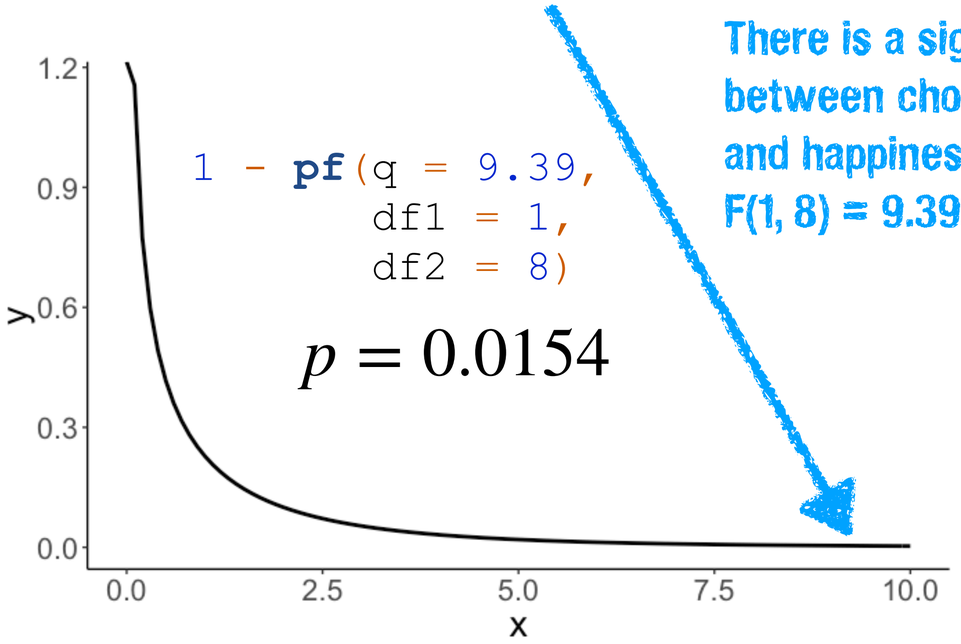
difference in parameters between models A and C

number of observations vs. parameters in Model A

Decide whether it's **worth it**

- To compute the *F* statistic, we need:
  - PRE = 0.54
  - PC = 1
  - PA = 2
  - n = 10

$$F = \frac{\text{PRE}/(\text{PA} - \text{PC})}{(1 - \text{PRE})/(n - \text{PA})}$$
$$= \frac{0.54/(2 - 1)}{(1 - 0.54)/(10 - 2)}$$
$$= 9.39$$




```
1 - pf(q = 9.39,
       df1 = 1,
       df2 = 8)
```

$$p = 0.0154$$

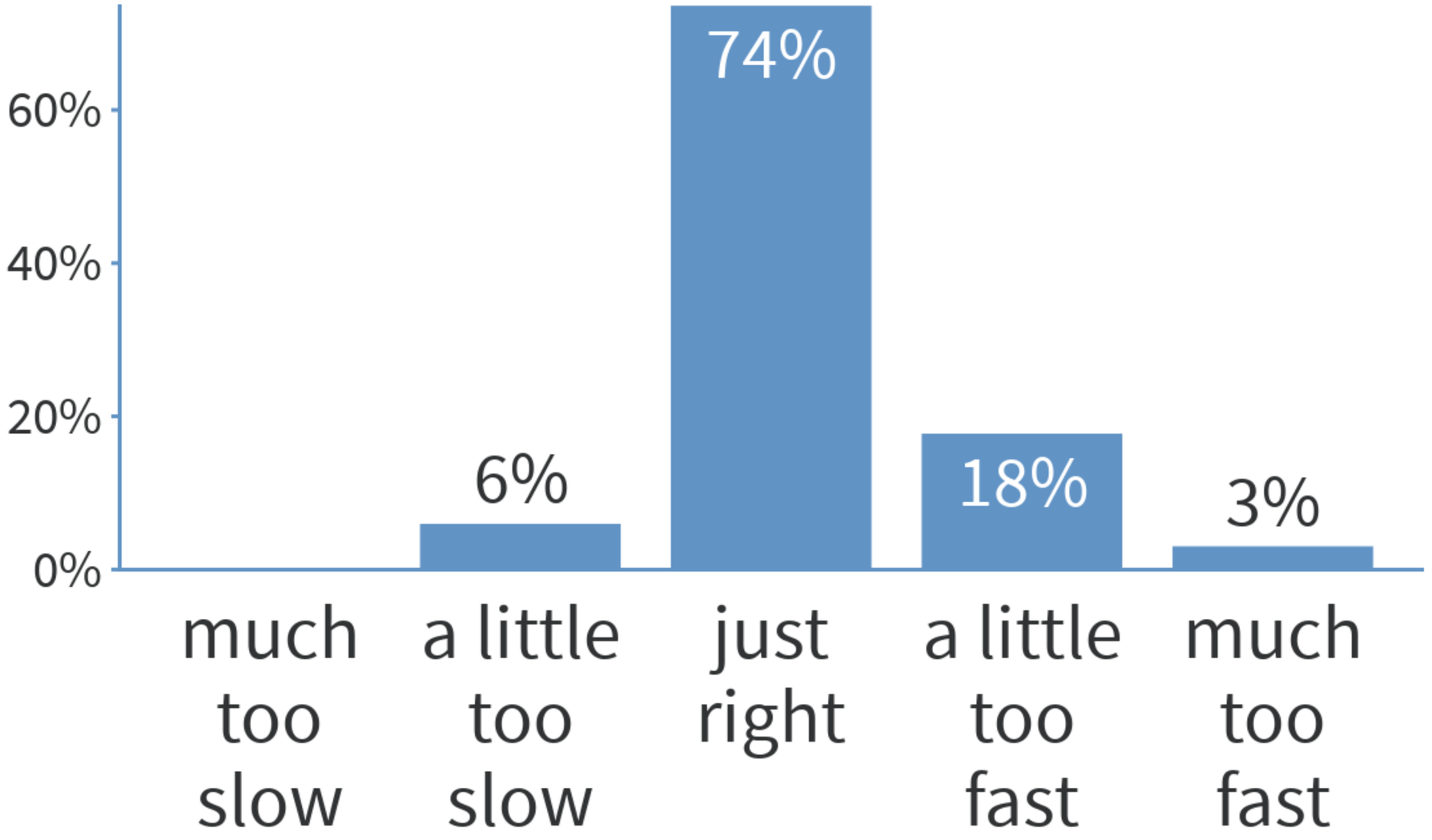
There is a significant relationship between chocolate consumption and happiness
F(1, 8) = 9.39, p = .0154.

Summary

- Quick recap
- Modeling data
- Hypothesis testing as model comparison
- Correlation
  - Pearson's moment correlation
  - Spearman's rank correlation
- Regression

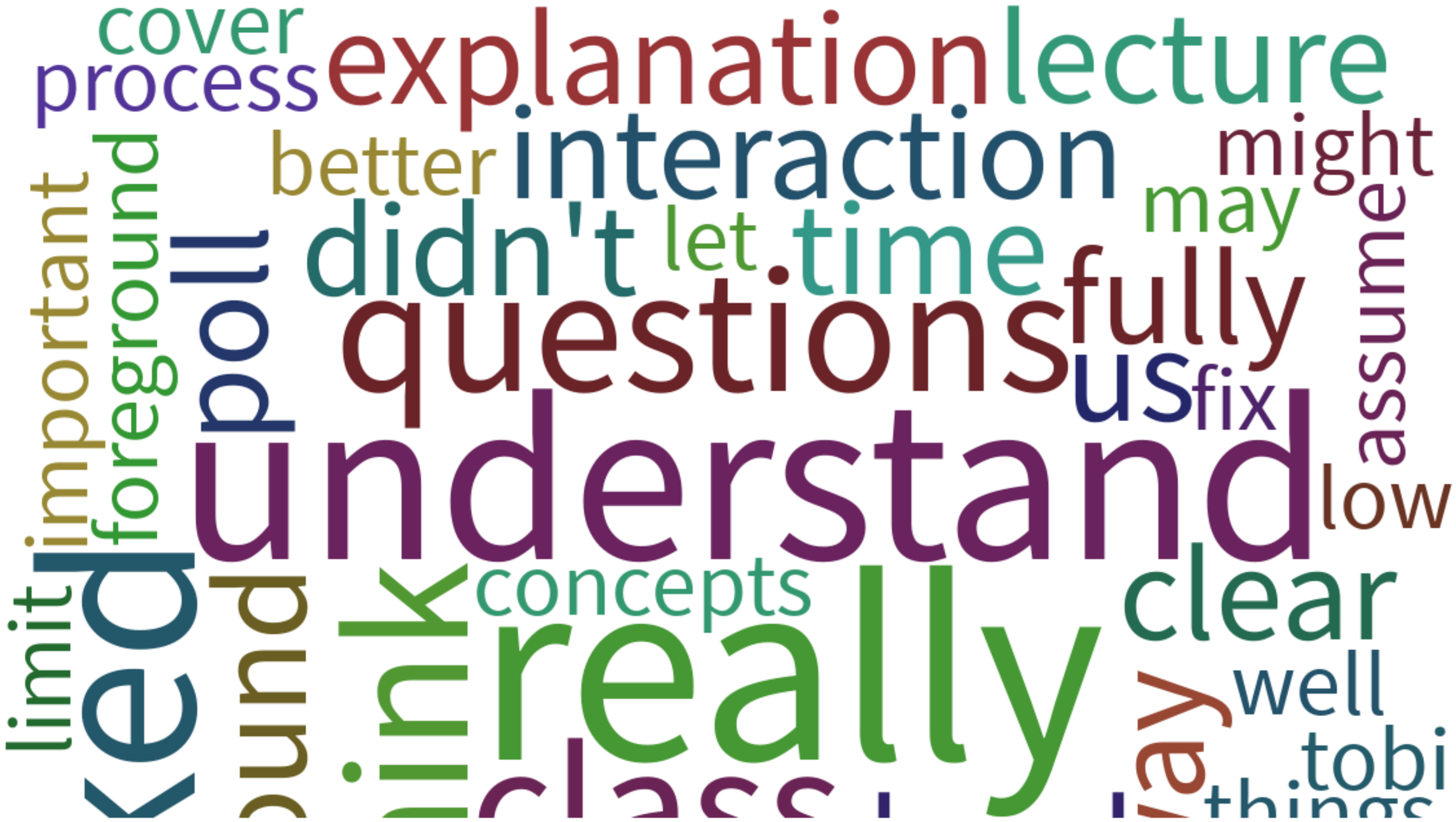
Feedback

How was the pace of today's class?



How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?



Thank you!