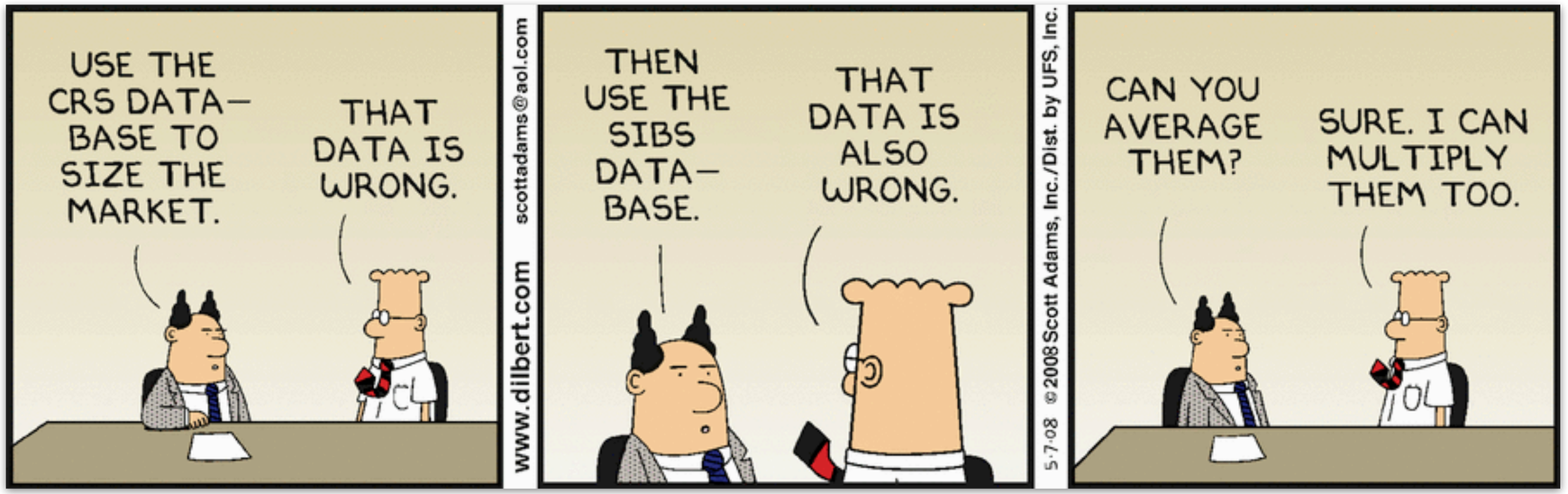
PSYCH 252: Statistical Methods for Behavioral and Social Sciences

Simulation 2



01/23/2026

Homework 1
Show case

Aya Salim

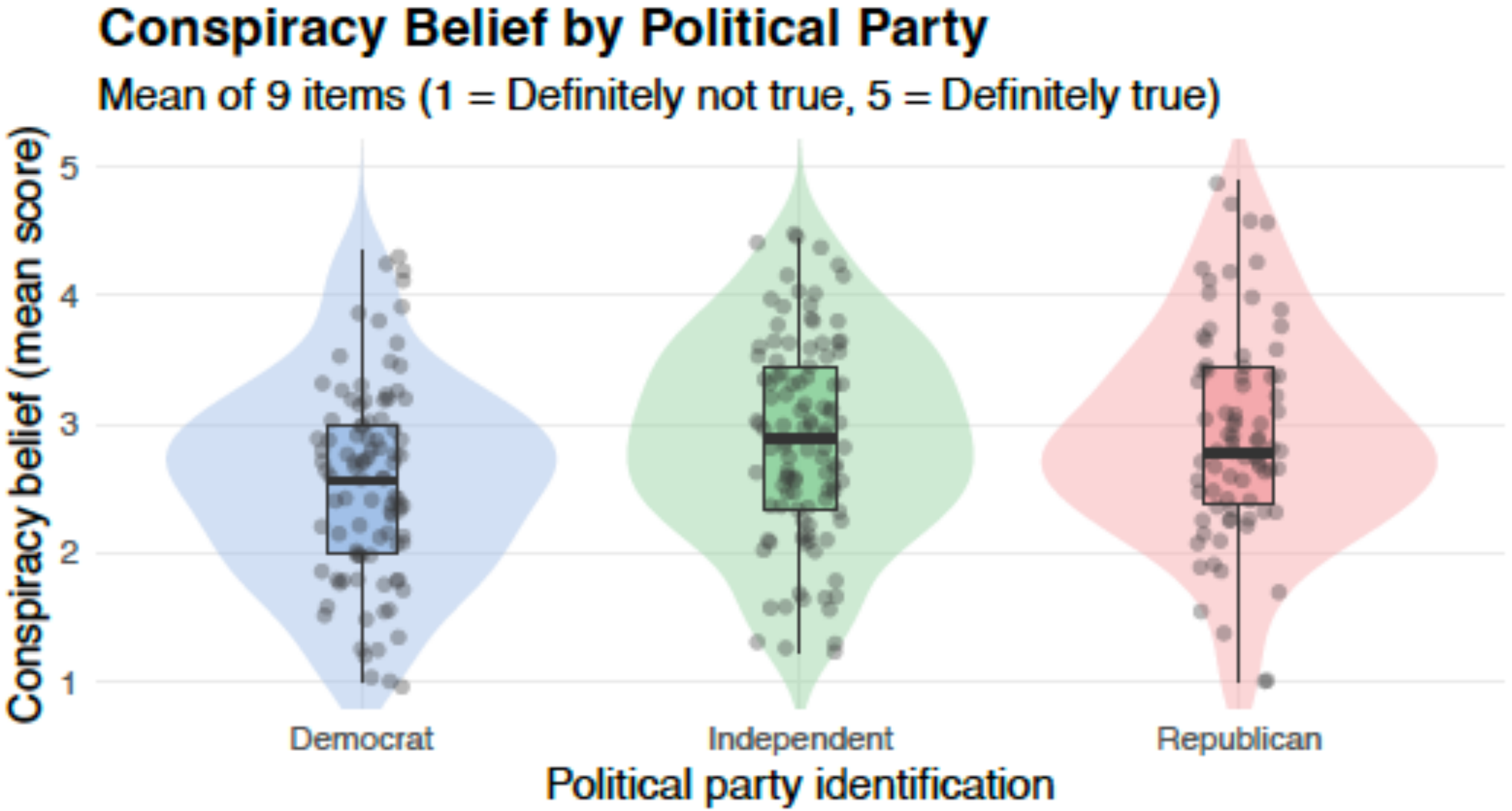
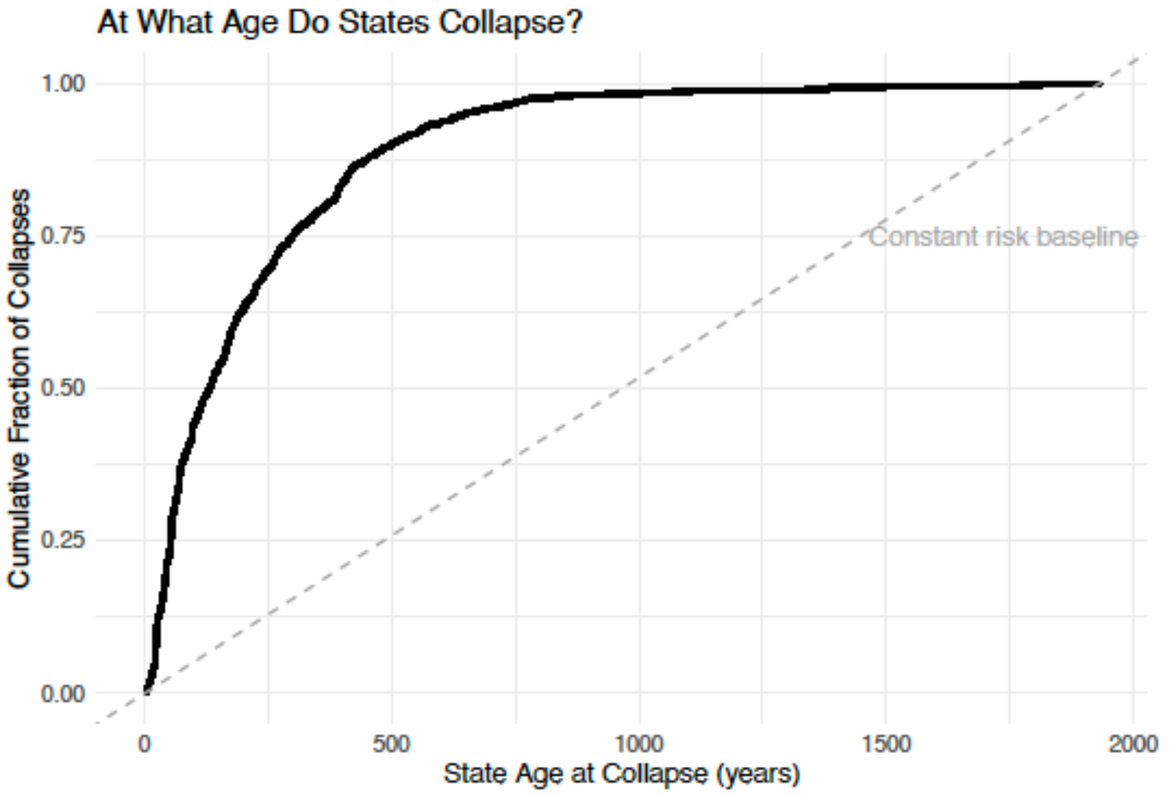
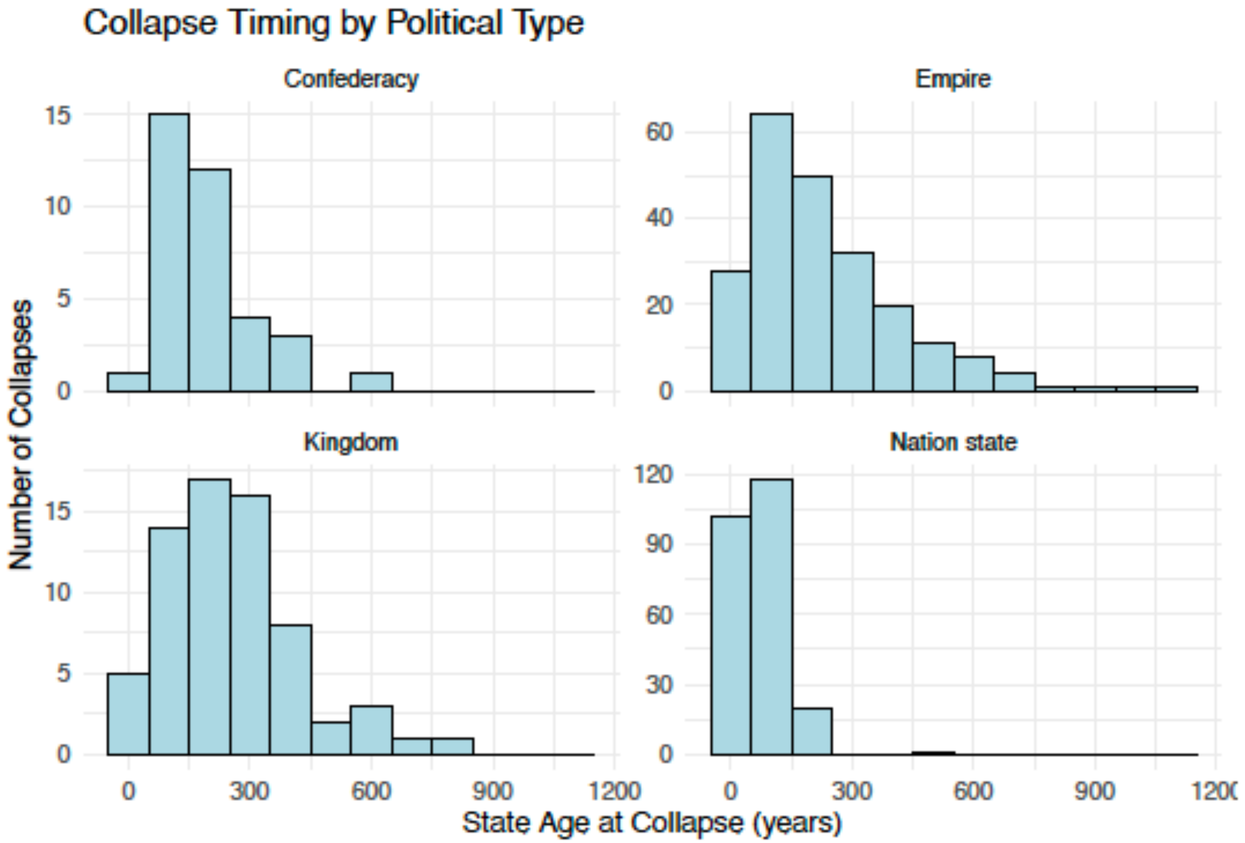


Figure 1. Points represent individual participants' mean scores across nine conspiracy belief items. Violins show the distribution within each party group; boxplots show the median and interquartile range. Higher values indicate greater endorsement of conspiratorial statements.

Avery Louis

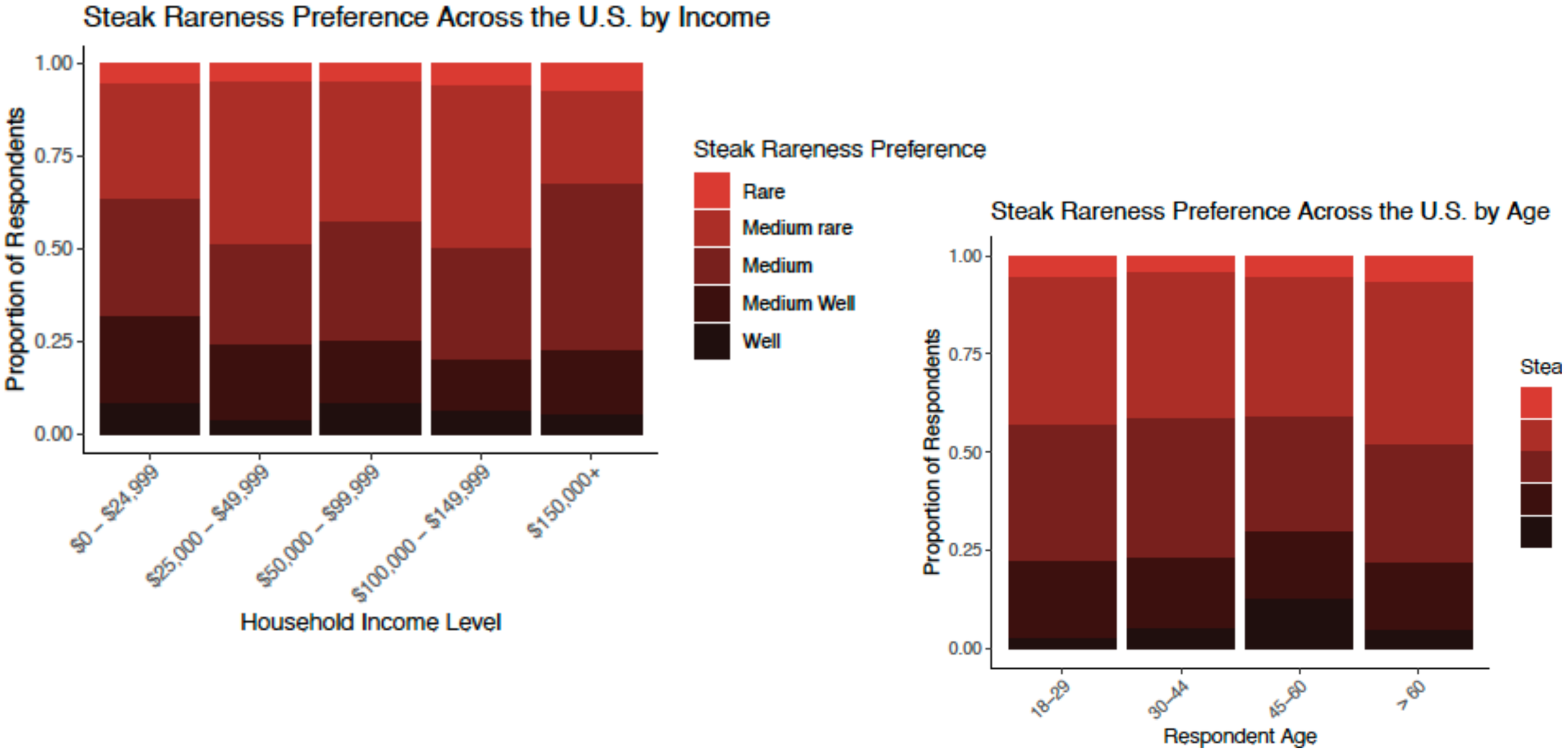


What we're seeing above is that states are at significantly more risk when they're nascent. The fraction of states that collapse before the age of ~150 is about 50%, by contrast, if a state lives beyond 500 years, it seems incredibly likely that it won't collapse for many many years to come.

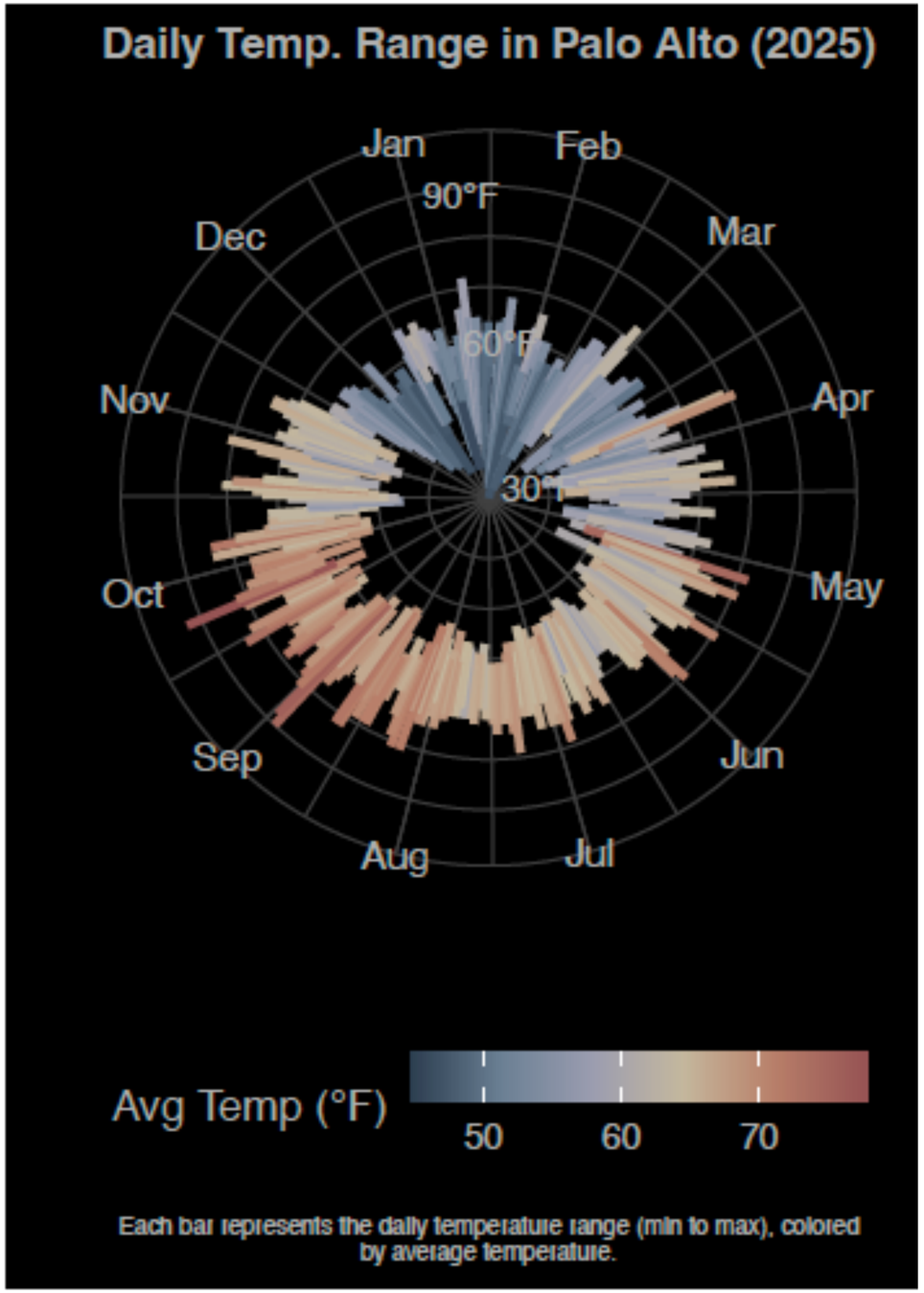


Here we can see the number of collapses bucketed by age of the state, and separated by state-type. Our conclusions should come with the disclaimer that we really don't have enough data to come to *strong* conclusions, but it does seem that nation states tend to be particularly vulnerable to early collapse, whereas Empires have that long tail, meaning those that make it past the early years seem to live on for many years. It's worth noting, though, that nation states are a more recent global phenomenon, so many of the extant nation states are not considered here!

Josh Ryan Heupel



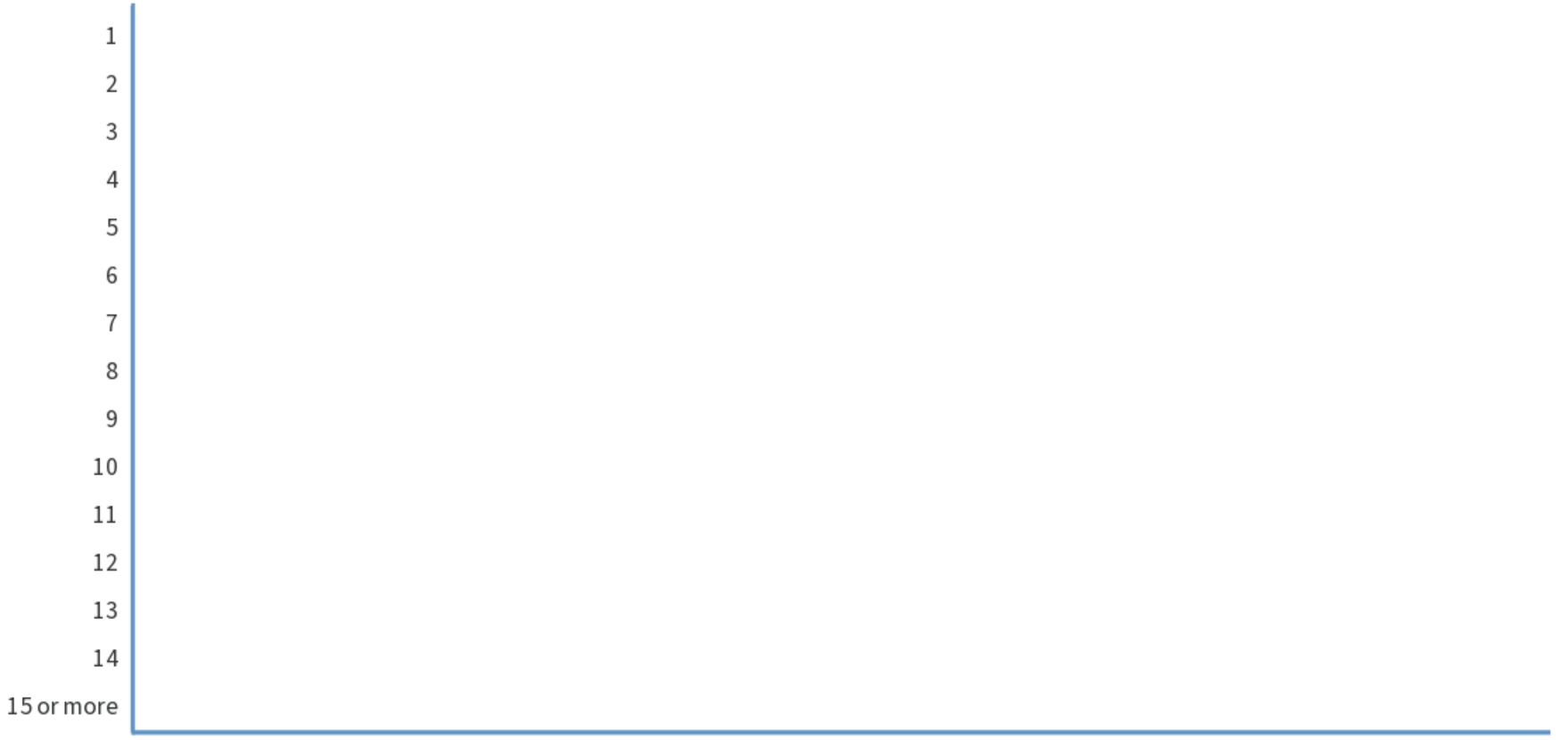
Faye Thijssen




```
# look how pretty she is
# wow
```

Homework

How many hours did it take you to complete Homework 2?



Homework 3

1 Distributions (1 point)

When we have empirical data, we can compute cumulative probabilities and create probability density functions using `quantile()` and `density()`, respectively. Take a look at the help files for both of these functions to better understand what they're doing.

Consider the following data set:

```
df.p1 = tibble(observation = 1:20,
               rating = c(0.3775909, 0.5908214, 0.07285336, 0.06989763, 0.2180343,
                          1.447484, 0.614781, 0.2698414, 0.4782837, 0.073523,
                          0.6953676, 0.3810149, 0.6188018, 2.211967, 0.5272716,
                          0.517622, 0.9380176, 0.3273733, 0.1684667, 0.2942399))
```

1.1 Quantile (0.5 points)

What's the 60% percentile of the `rating` vari
60% of the values are lower than that value?)

```
### YOUR CODE HERE ###

######################
```

1.2 Density (0.5 points)

Plot the density of the `rating` variable. Use

```
### YOUR CODE HERE ###

######################
```

2 Sampling distribution (7 points)

The sample standard deviation $s = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}{n-1}}$ is an unbiased estimator of the population standard deviation.In this exercise, we will run a simulation to compare s with another estimator $s' = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}{n}}$ and show that s' is biased. Note that the only difference between s and s' is in the denominator.

1

2.1 Generate the sample distributions (2 poi

Let's simulate drawing 10,000 samples from a population distril sample should be 40. To start off, create a new data frame `df.` 3 columns:

- **sim**: index for the simulation (from 1 to 10,000)
- **sample**: index for the sample in each simulation (from 1
- **x**: sample values drawn from the population distribution

For the population distribution assume a normal distribution

3 Permutation test (7 points)

Imagine that you collected data about people's heights from three different places and you are interested whether there are any differences in people's height between the three places.

By visualizing the data, we can see that the variances between the three groups differ considerably, which is troublesome for parametric tests (e.g. a t-test). However, we can perform a permutation test, which is non-parametric. In this case, we are interested in whether the maximal difference between each of the pairs of group means, is greater than we would expect to see by chance.

```
set.seed(1)

df.heights = read_csv("data/df_heights.csv")

df.heights %>%
  ggplot(data = .,
         mapping = aes(x = group,
                       y = height)) +
  geom_point(position = position_jitter(height = 0,
                                        width = 0.1),
             alpha = 0.5) +
  stat_summary(fun.data = "mean_cl_boot",
               shape = 21,
               fill = "lightblue",
               size = 1)
```

Homework 3

remember to set `eval = T` when knitting the file

```
```{r p2.1, eval = F}
set.seed(1)

n_simulations = 10000 # number of simulations
n_samples = 40 # number of samples in each simulation
population_mean = 0 # ground truth mean
population_sd = 1 # ground truth standard deviation

### YOUR CODE HERE ###
df.samples =
######################

df.samples %>%
  head(5)

df.samples %>%
  summary()
```
```

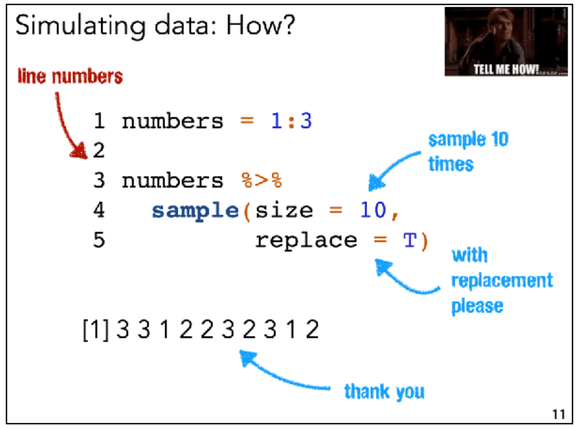
Logistics

Outline Goal: Revisit and understand key statistical concepts

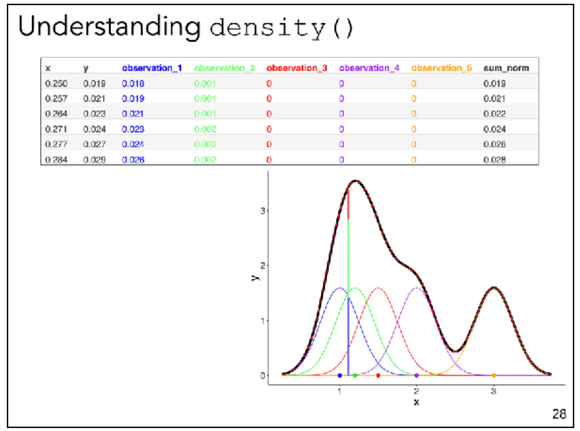
- Quick recap
- Doing Bayesian Analysis
- Inference in frequentist statistics
- Sampling distributions
- What is a p-value?
  - Permutation test

Quick recap

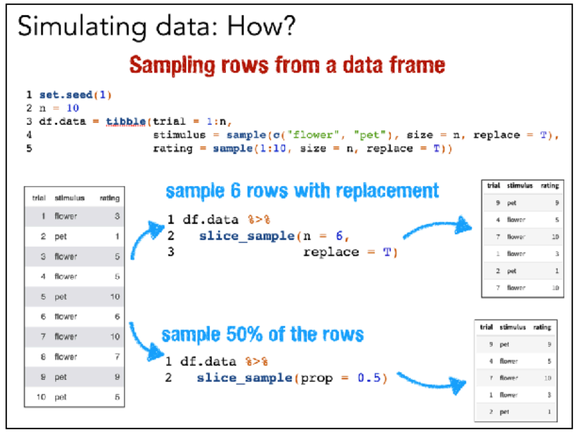
Quick recap from Simulation 1



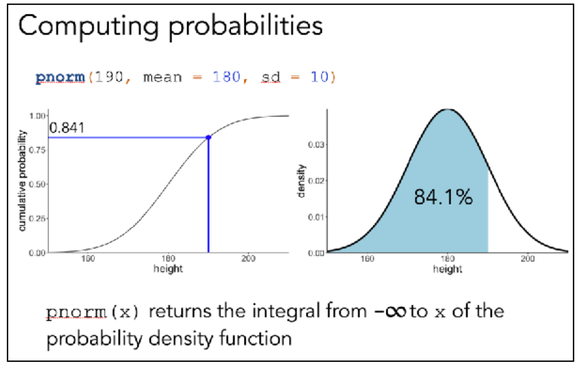
sampling values from a vector



understanding `density()`



sampling rows from a data frame



answering questions with probability distributions

or via drawing samples `rnorm()` + wrangling

Doing Bayesian Analysis

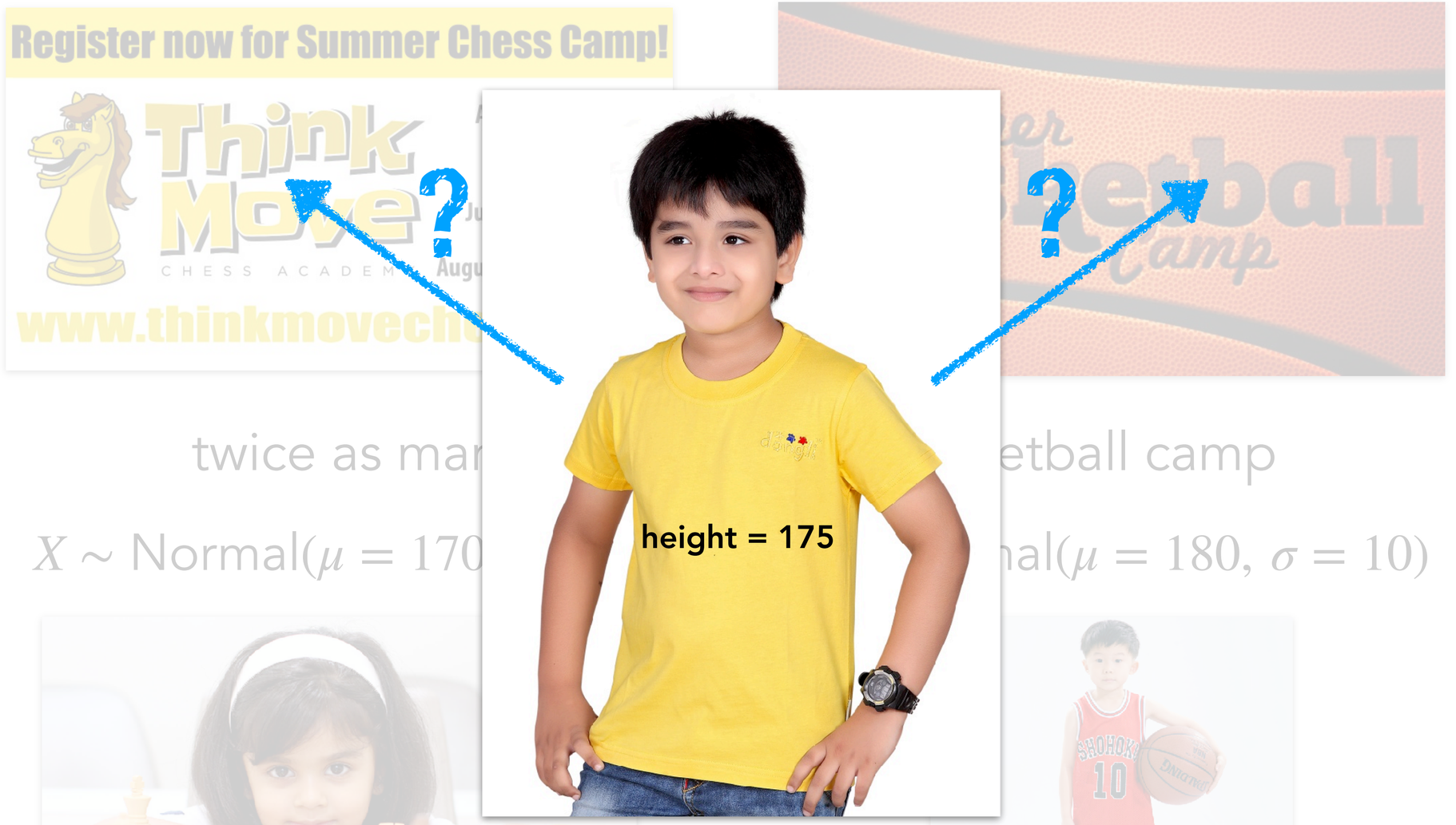
Summer camp

twice as many kids go to the basketball camp

$X \sim \text{Normal}(\mu = 170, \sigma = 8)$

$X \sim \text{Normal}(\mu = 180, \sigma = 10)$

Summer camp



twice as mar … etball camp

$X \sim \text{Normal}(\mu = 170$ … $\text{nal}(\mu = 180, \sigma = 10)$

Analytic solution

Can you feel the Bayes?

H = {basketball, chess}

D = 175 cm

posterior

likelihood prior

$$p(H \mid D) = \frac{p(D \mid H) \cdot p(H)}{p(D)}$$

probability of the data?!

H = Hypothesis

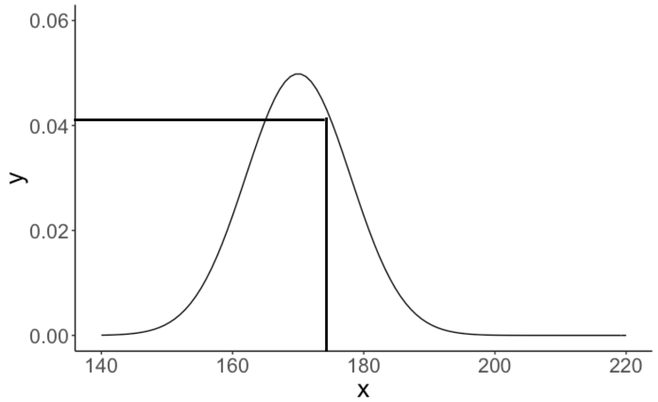
D = Data

Summer camp

prior

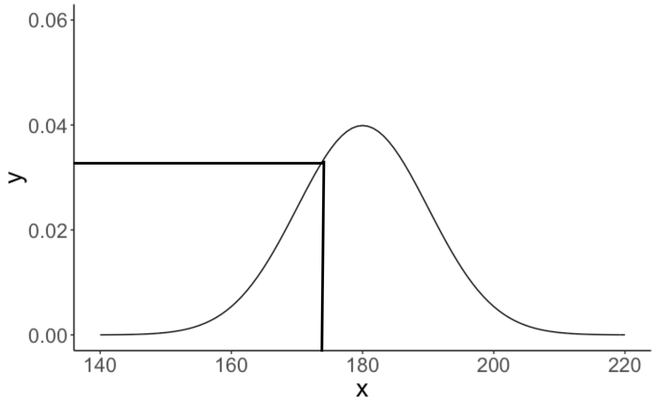
$$p(\text{chess}) = \frac{1}{3} \qquad p(\text{basketball}) = \frac{2}{3}$$

likelihood



`dnorm(175, mean = 170, sd = 8)`

= 0.041



`dnorm(175, mean = 180, sd = 10)`

= 0.035

posterior

$$p(\text{sport} = \text{basketball} \mid \text{height} = 175) = \frac{\overbrace{p(175 \mid \text{basketball})}^{\text{likelihood}} \cdot \overbrace{p(\text{basketball})}^{\text{prior}}}{\underbrace{p(175)}_{\text{data}}}$$

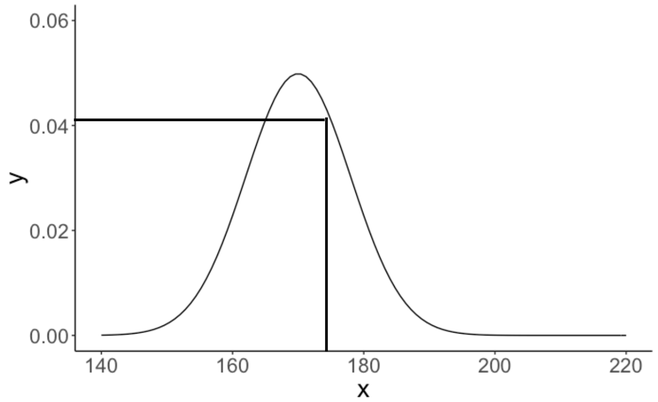
$$p(\text{basketball} \mid 175) = \frac{p(175 \mid \text{basketball}) \cdot p(\text{basketball})}{p(175 \mid \text{basketball}) \cdot p(\text{basketball}) + p(175 \mid \text{chess}) \cdot p(\text{chess})}$$

Summer camp

prior

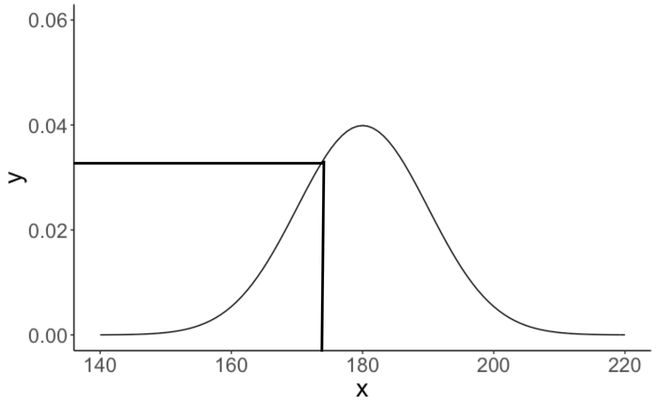
$$p(\text{chess}) = \frac{1}{3} \qquad p(\text{basketball}) = \frac{2}{3}$$

likelihood



`dnorm(175, mean = 170, sd = 8)`

$= 0.041$



`dnorm(175, mean = 180, sd = 10)`

$= 0.035$

posterior

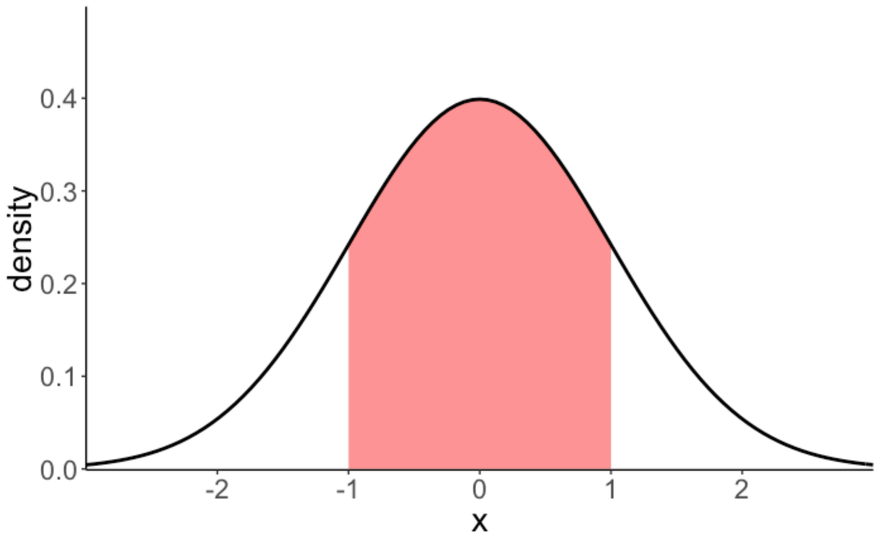
$$p(\text{basketball} \mid 175) = \frac{p(175 \mid \text{basketball}) \cdot p(\text{basketball})}{p(175 \mid \text{basketball}) \cdot p(\text{basketball}) + p(175 \mid \text{chess}) \cdot p(\text{chess})}$$

$$p(\text{basketball} \mid 175) = \frac{0.035 \cdot 2/3}{0.035 \cdot 2/3 + 0.041 \cdot 1/3} \approx 0.63$$

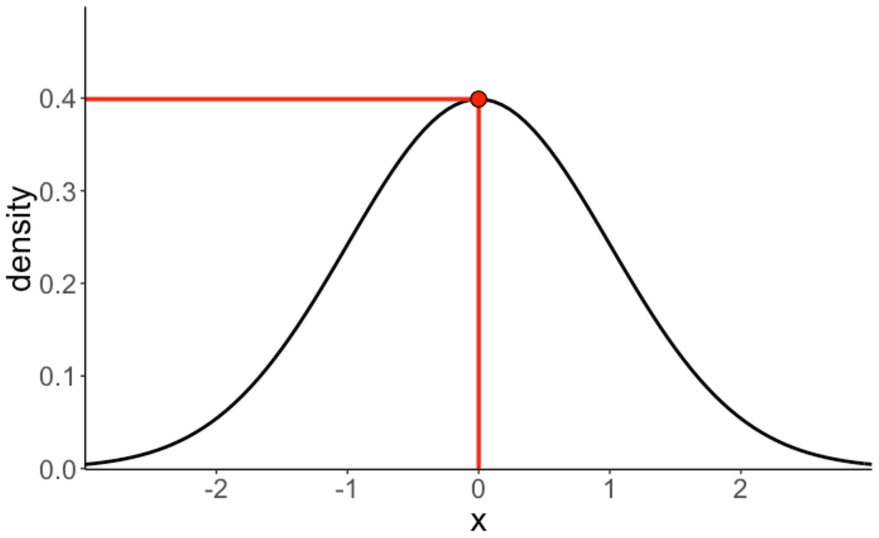
send the kid to the basketball gym!

Probability vs. likelihood

Probability

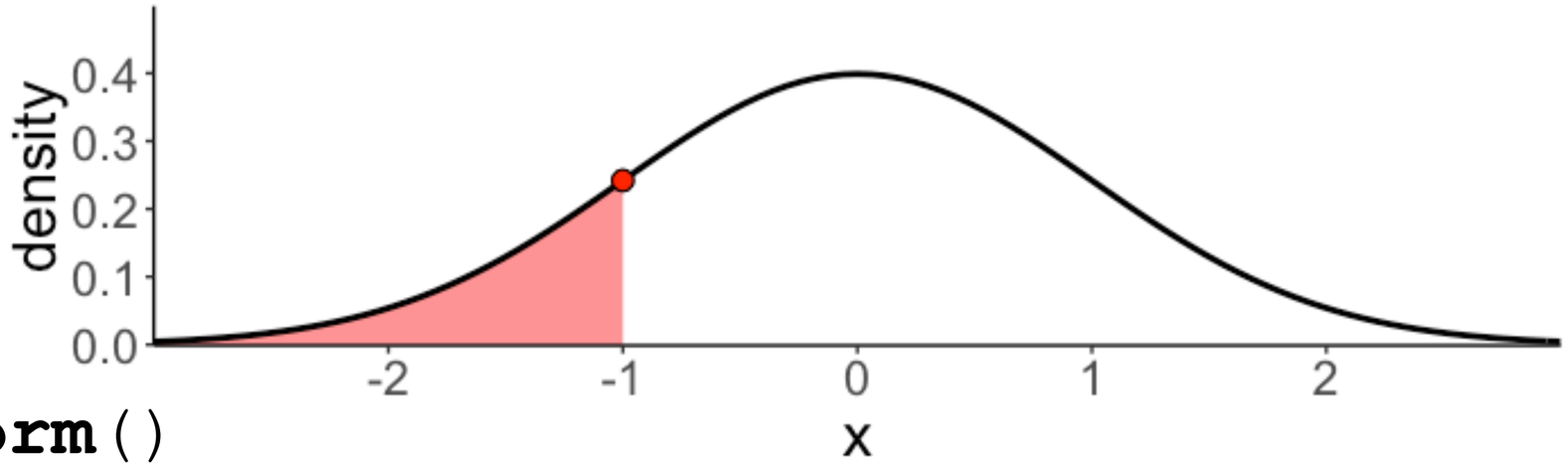


Likelihood

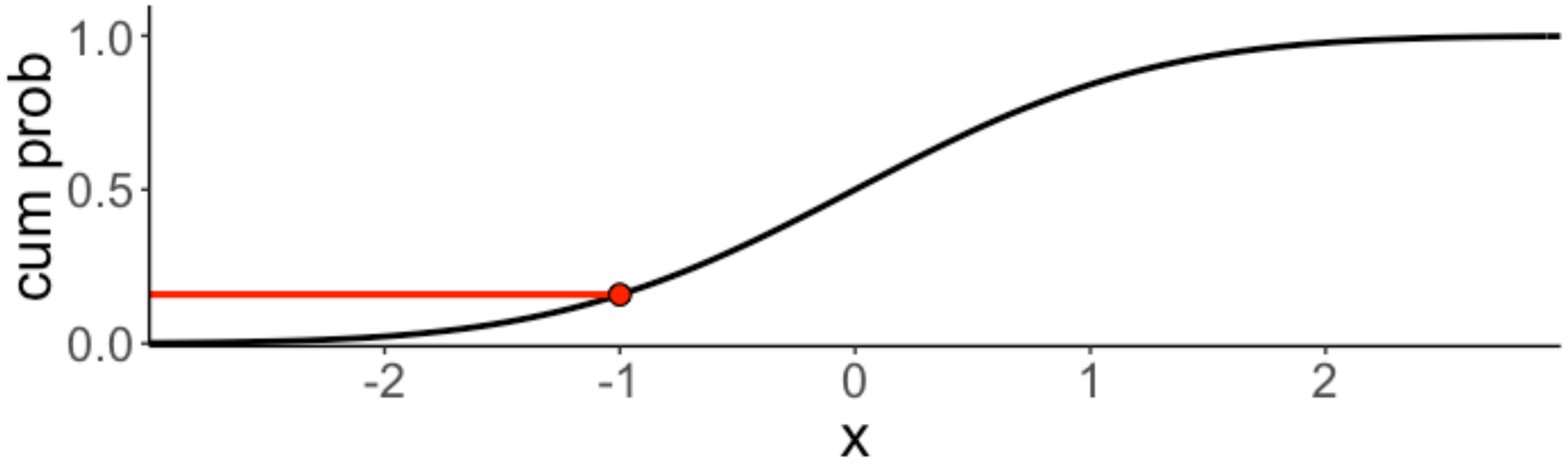


Probability vs. likelihood

dnorm()

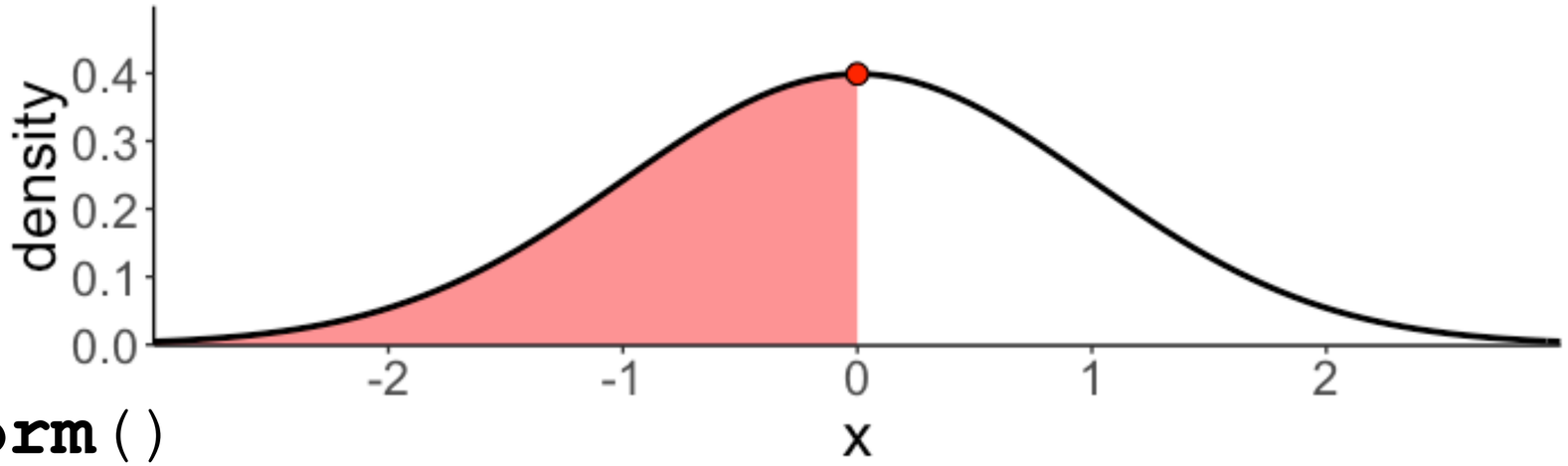


pnorm()

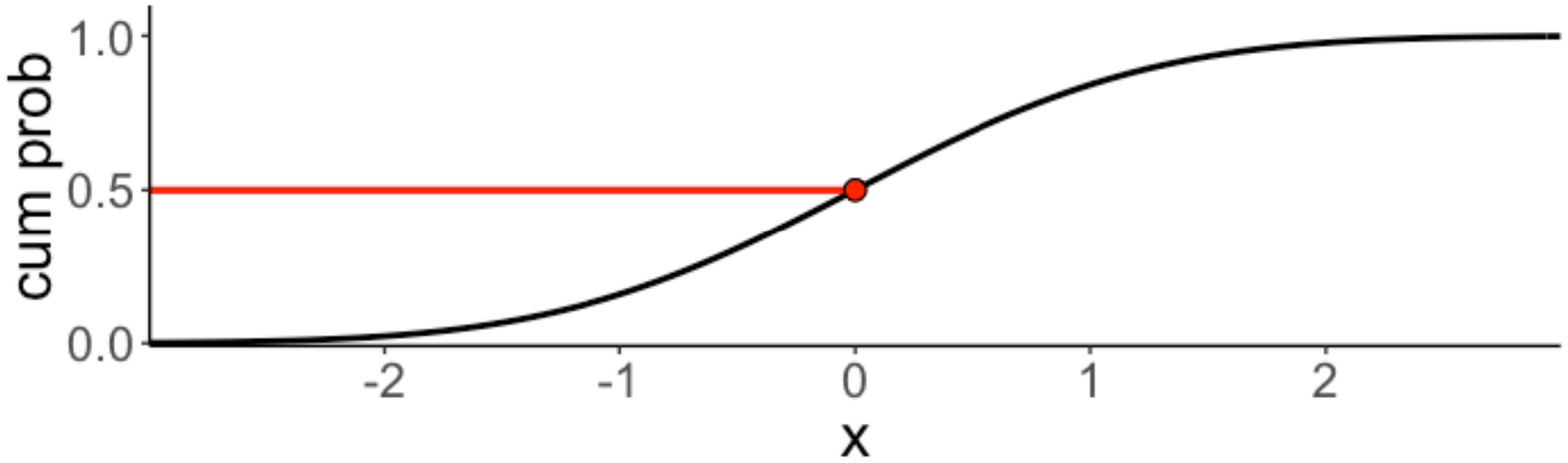


Probability vs. likelihood

dnorm()

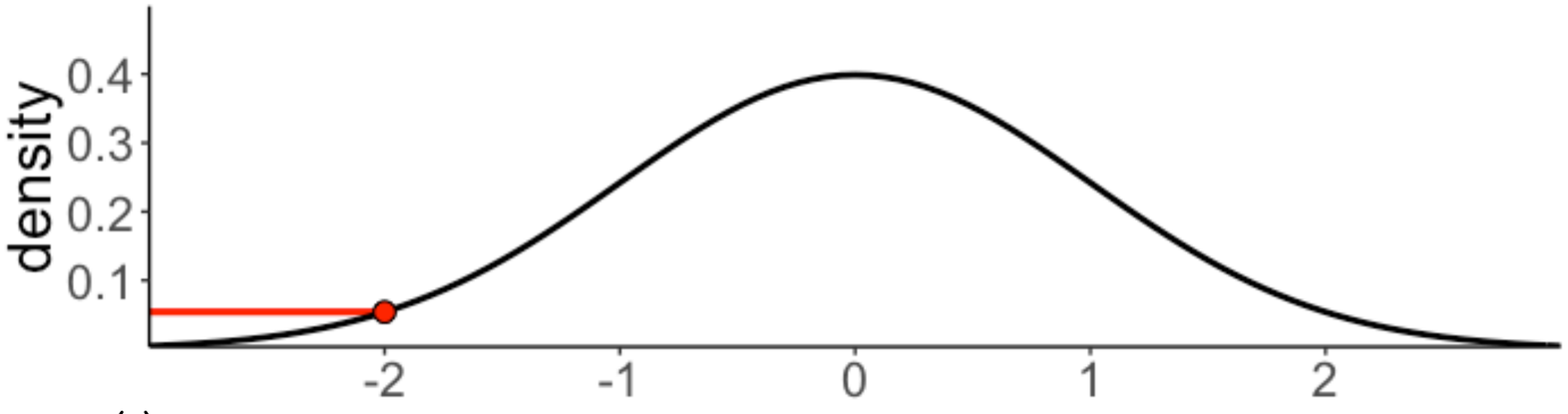


pnorm()

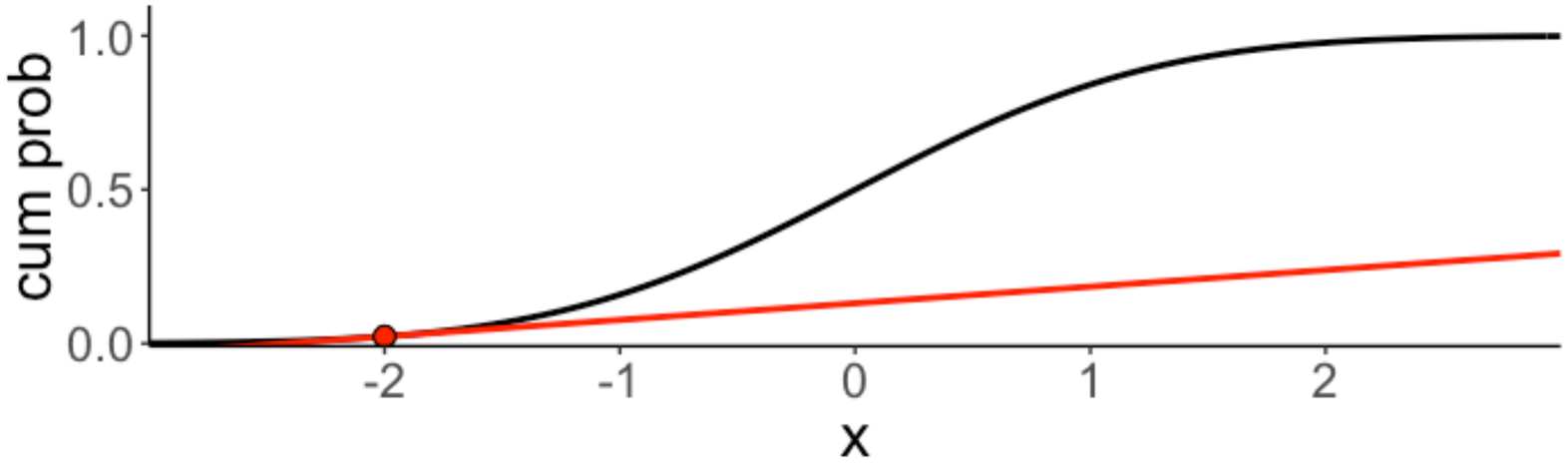


Probability vs. likelihood

dnorm()



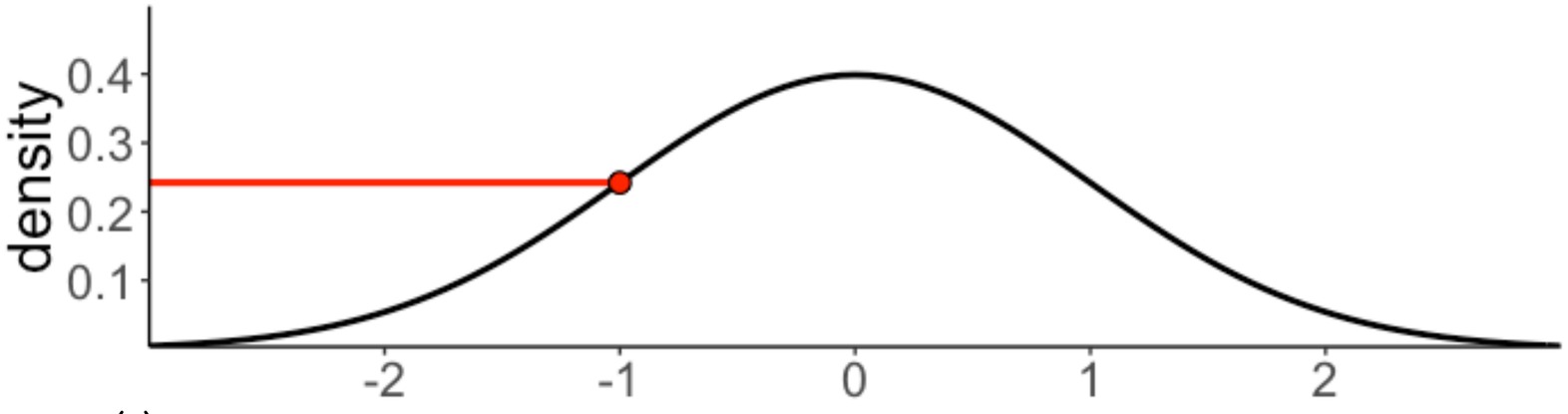
pnorm()



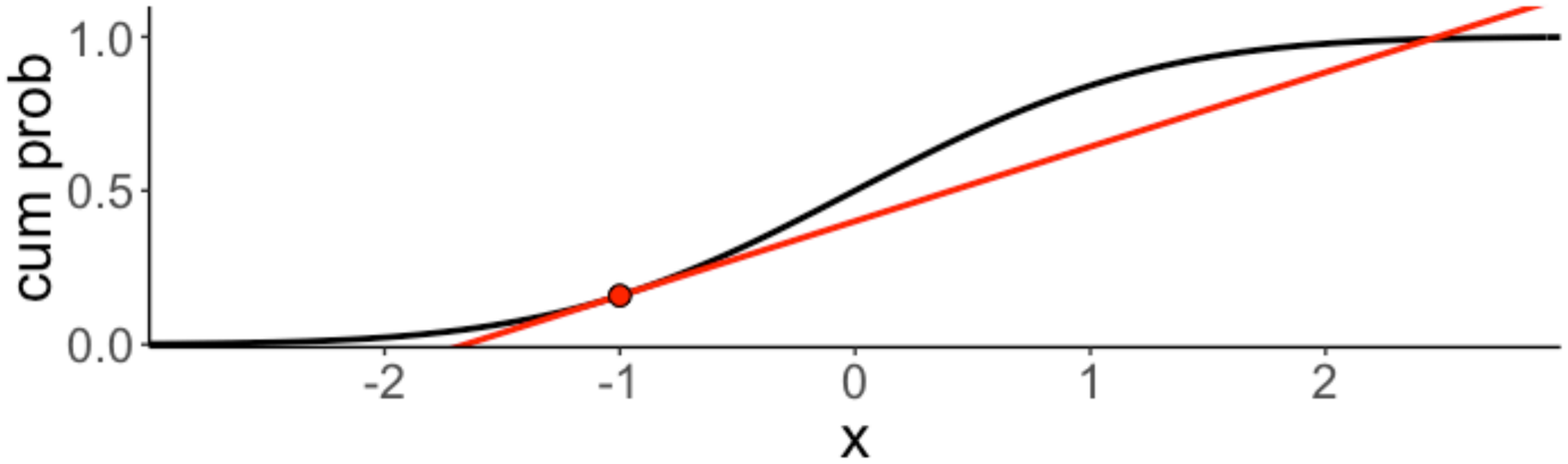
dnorm() is the first derivative of **pnorm**()

Probability vs. likelihood

dnorm()



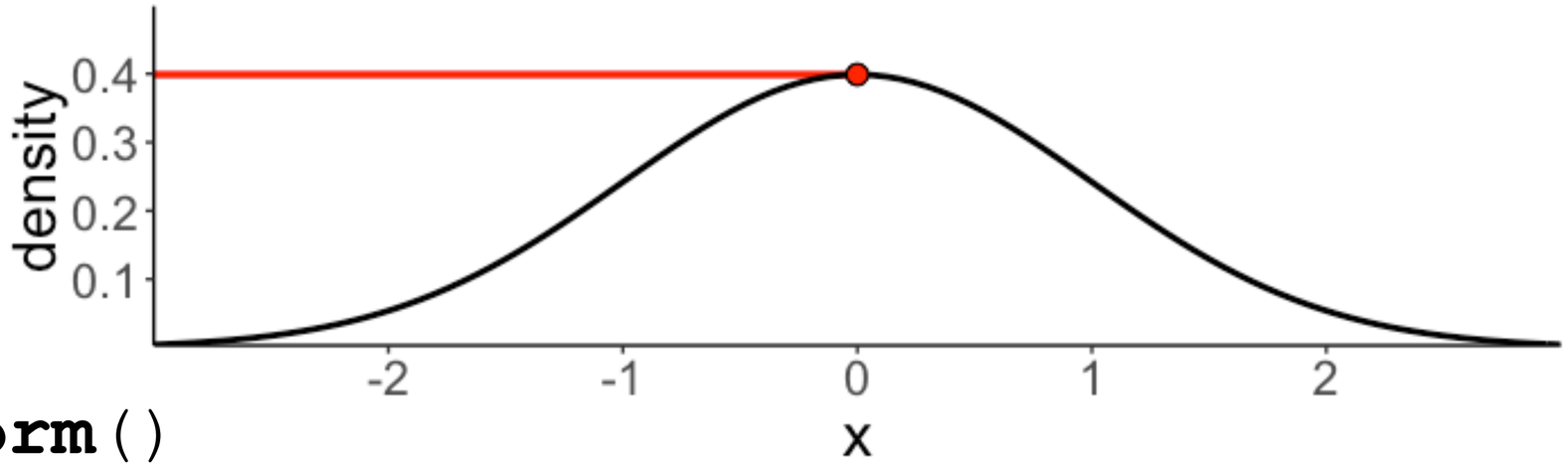
pnorm()



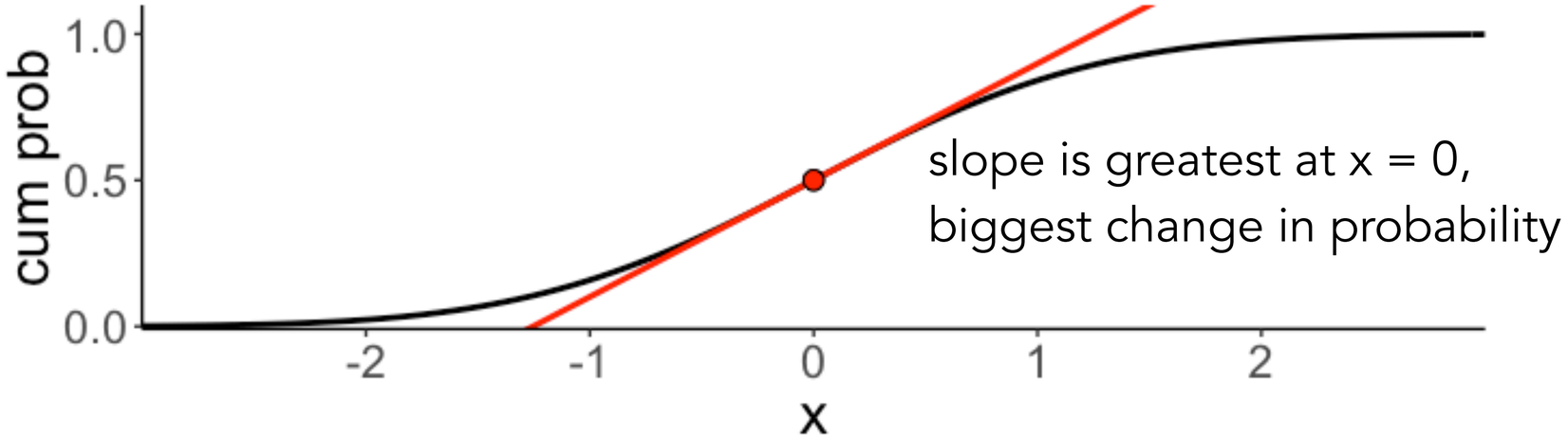
dnorm() is the first derivative of **pnorm**()

Probability vs. likelihood

dnorm()



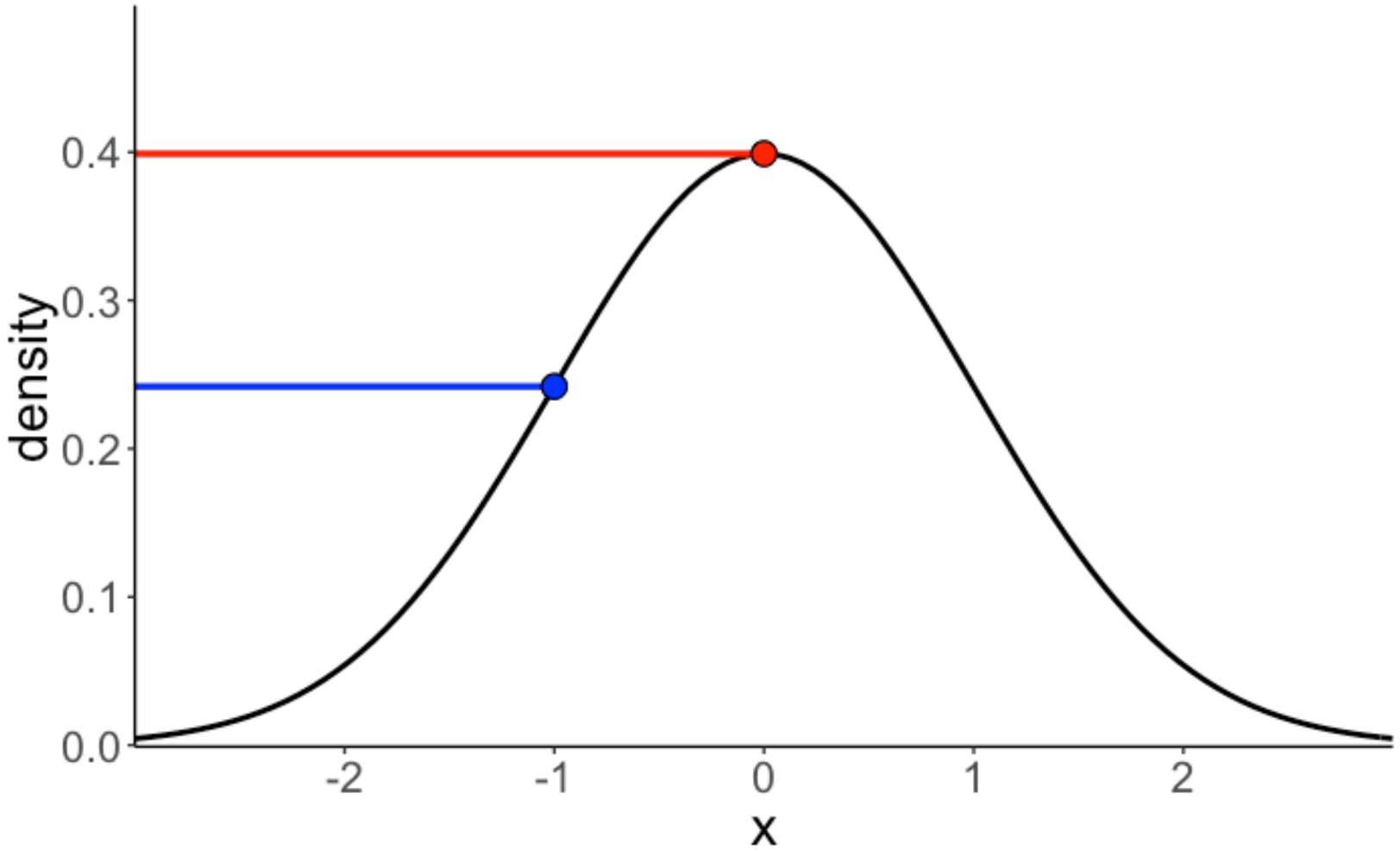
pnorm()



dnorm() is the first derivative of **pnorm**()

Probability vs. likelihood

```
dnorm(0)/dnorm(-1) = 1.6487
```

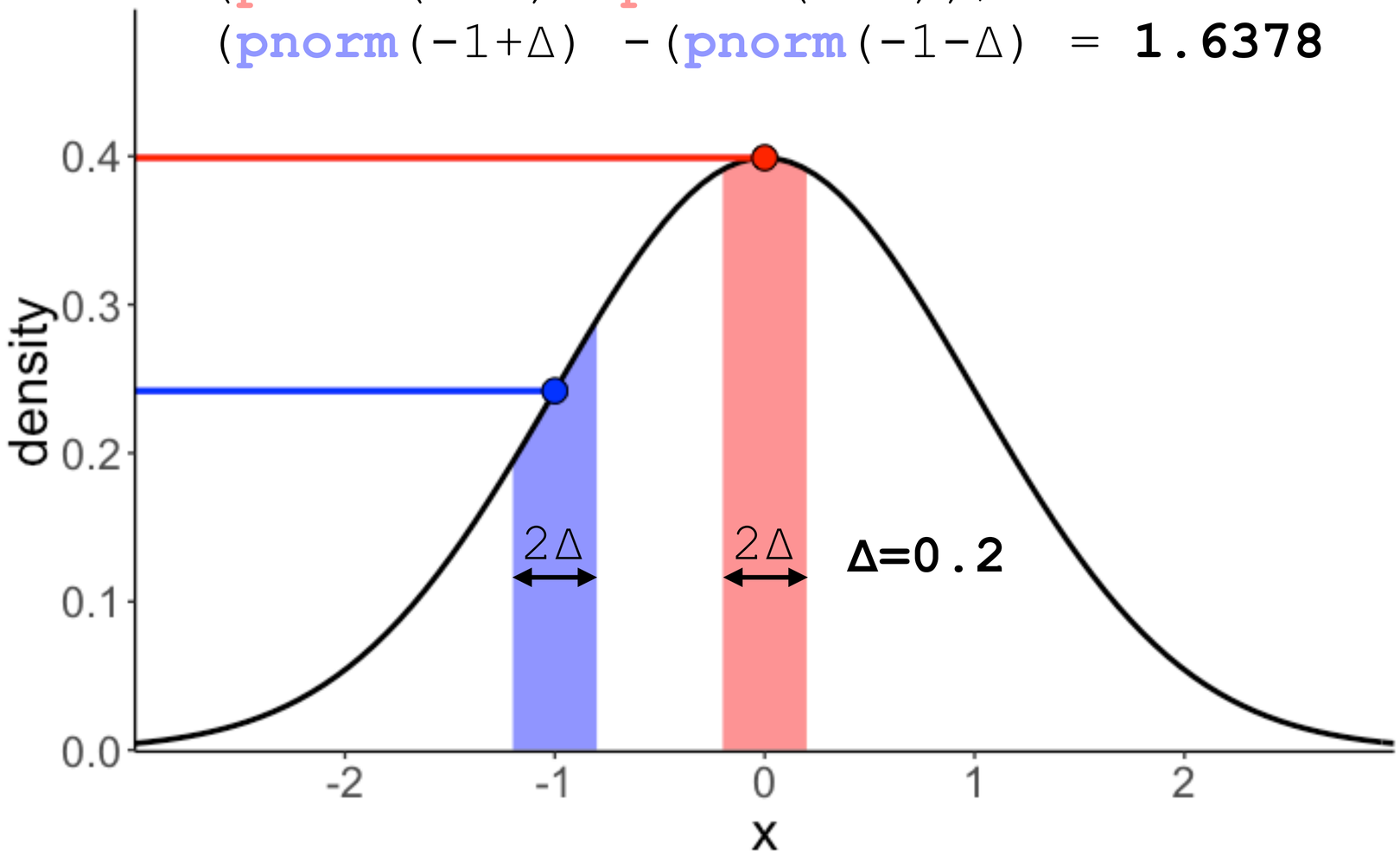


relative probability of one value vs. another

Probability vs. likelihood

```
dnorm(0)/dnorm(-1)  = 1.6487

(pnorm(0+Δ)- pnorm(0-Δ))/
(pnorm(-1+Δ) -(pnorm(-1-Δ)  = 1.6378
```

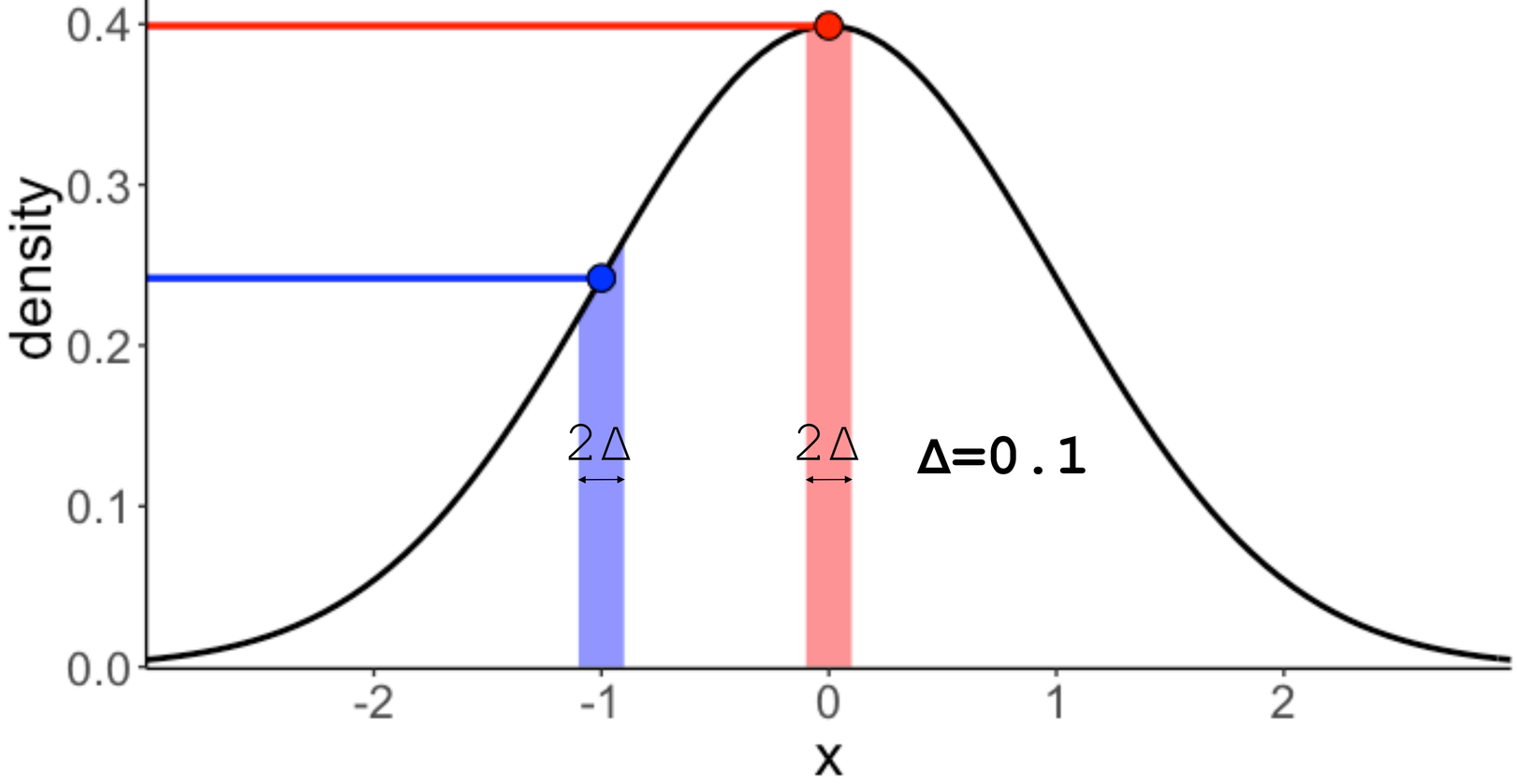


relative probability of one value vs. another

Probability vs. likelihood

```
dnorm(0)/dnorm(-1) = 1.6487
```

```
(pnorm(0+Δ)- pnorm(0-Δ))/
(pnorm(-1+Δ) -(pnorm(-1-Δ) = 1.6459
```

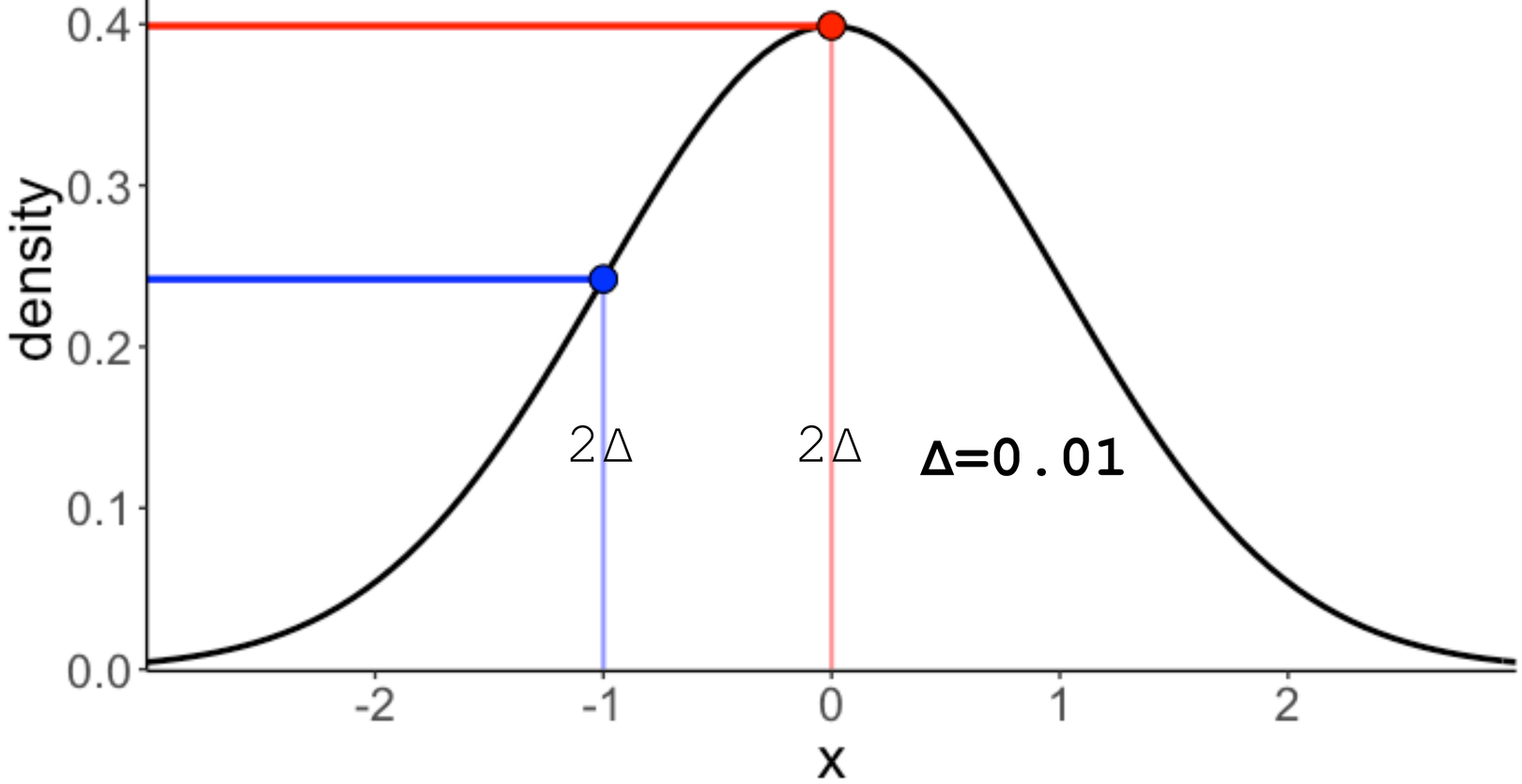


relative probability of one value vs. another

Probability vs. likelihood

```
dnorm(0)/dnorm(-1) = 1.6487
```

```
(pnorm(0+Δ)- pnorm(0-Δ))/
(pnorm(-1+Δ) -(pnorm(-1-Δ) = 1.6486
```



relative probability of one value vs. another

Summer camp: Via sampling

```
df.camp = tibble(
  kid = 1:1000,
  sport = sample(c("chess", "basketball"),
                 size = 1000,
                 replace = T,
                 prob = c(1/3, 2/3))) %>%
  rowwise() %>%
  mutate(height = ifelse(test = sport == "chess",
                         yes = rnorm(., mean = 170, sd = 8),
                         no = rnorm(., mean = 180, sd = 10))) %>%
  ungroup()
```

kid	sport	height
1	basketball	164.84
2	basketball	163.22
3	basketball	191.18
4	chess	160.16
5	basketball	182.99
6	chess	163.54
7	chess	168.56
8	basketball	192.99
9	basketball	171.91
10	basketball	177.12

```
df.camp %>%
  filter(height == 175) %>%
  count(sport)
```

doesn't work!

Summer camp: Via sampling

```
df.camp = tibble(
  kid = 1:100000,
  sport = sample(c("chess", "basketball"),
                 size = 100000,
                 replace = T,
                 prob = c(1/3, 2/3))) %>%
  rowwise() %>%
  mutate(height = ifelse(test = sport == "chess",
                         yes = rnorm(., mean = 170, sd = 8),
                         no = rnorm(., mean = 180, sd = 10))) %>%
  ungroup()
```

kid	sport	height
1	basketball	164.84
2	basketball	163.22
3	basketball	191.18
4	chess	160.16
5	basketball	182.99
6	chess	163.54
7	chess	168.56
8	basketball	192.99
9	basketball	171.91
10	basketball	177.12

```
df.camp %>%
  filter(between(height,
                 left = 174,
                 right = 176)) %>%
  count(sport)
```

this works!

sport	n
basketball	469
chess	273

$$\frac{\text{basketball}}{\text{basketball} + \text{chess}} \approx 0.63$$

Inference in frequentist statistics

Statistical inference

The process of making claims about a population based on information from a sample.

Life would be easy if we were able to observe the whole population -- we could simply do descriptive analyses!

Key question:

What can we infer about the population from our sample?

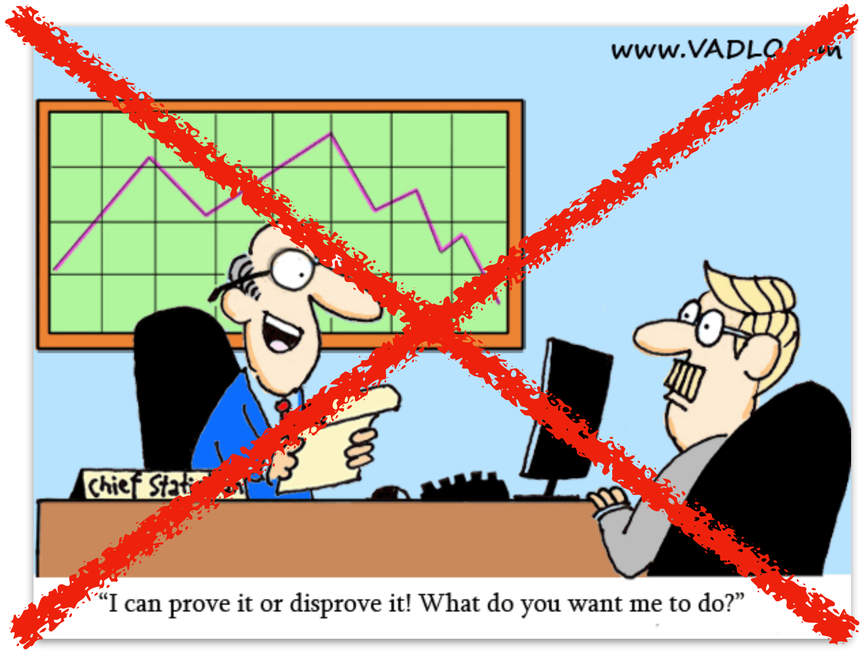
Statistical inference

Key question:

What can we infer about the population from our sample?

Answer:

- is not trivial
- mathematical, statistical, philosophical (Bayesian vs. frequentist) machinery involved
- **important**: we can never make deterministic statements!



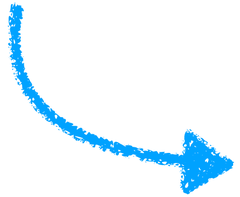
Underlying principle of statistical testing

1. Define population, state hypotheses
2. Draw one (ideally large) random sample
3. Compute measure of interest (e.g. mean, correlation coefficient, difference between condition means), and then the test statistic
4. Apply statistical distribution theory to get the **sampling distribution of a test statistic**
5. Evaluate the observed test statistic on the sampling distribution; make a decision (either reject or don't reject H_0) based on pre-specified significance level α

The magical component

"4. Apply statistical distribution theory to get the **sampling distribution of a test statistic**"

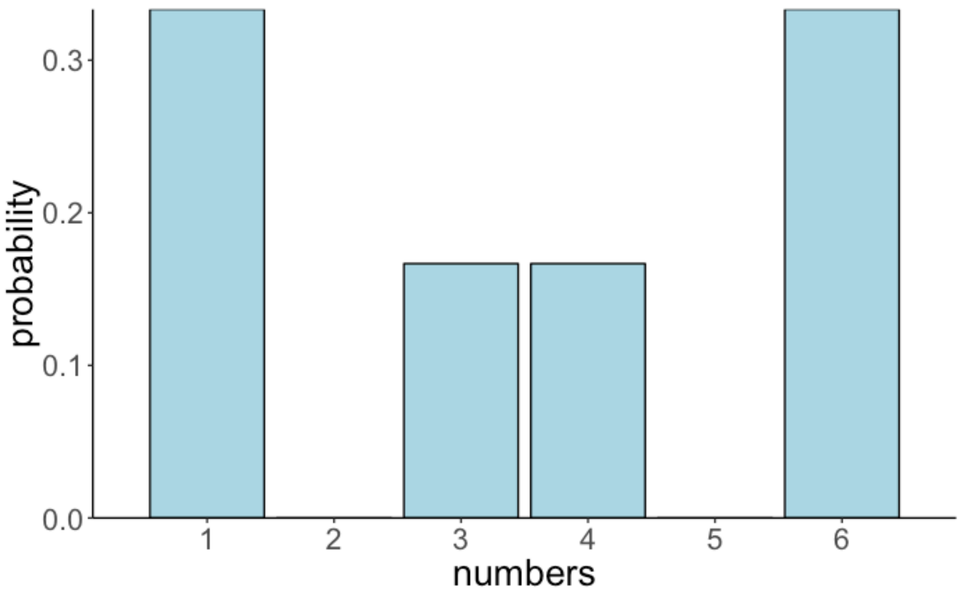
This dates back to pre-computer era where statisticians derived mathematically the distribution of statistical measures for an infinite amount of samples! That's a tricky thing to do and these approximations are typically tied to assumptions such as normality, homoscedasticity, independent observations, and the sample needs to be "large".



instead: simulation-based approach

Sampling from distributions

heavy metal distribution

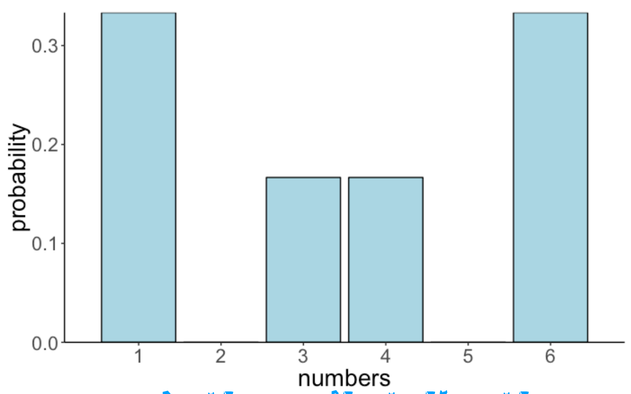


population distribution

Statistical inference

The process of making claims about a population based on information from a sample.

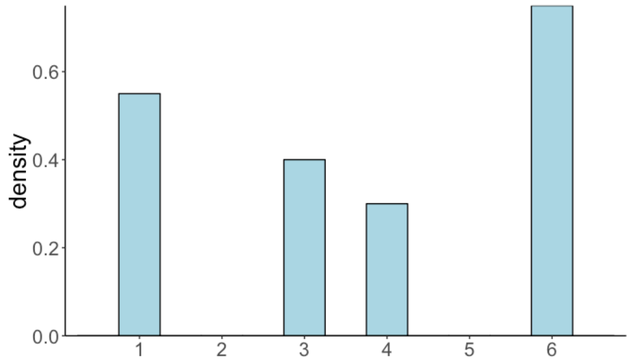
heavy metal distribution



population distribution

sampling

inference



our sample

Statistical inference

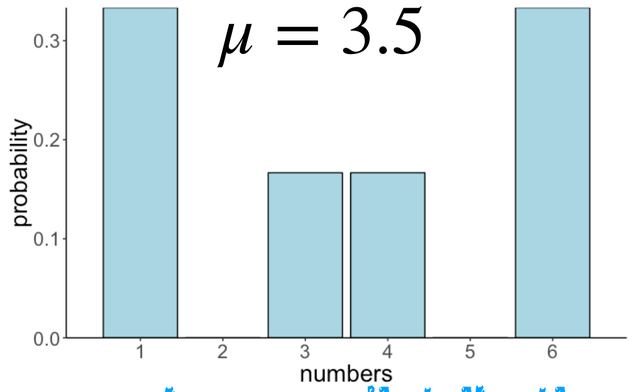
what's the
population mean?

sample mean = 3.725

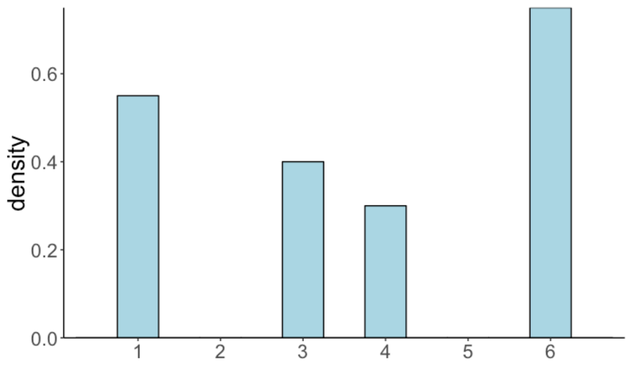
standard deviation = 2.05

n = 40

heavy metal distribution



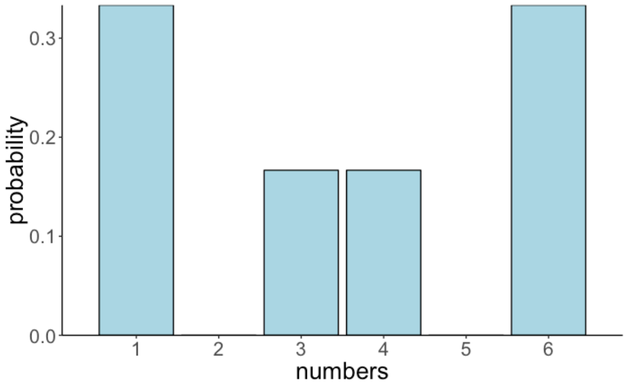
true unknown distribution



our sample

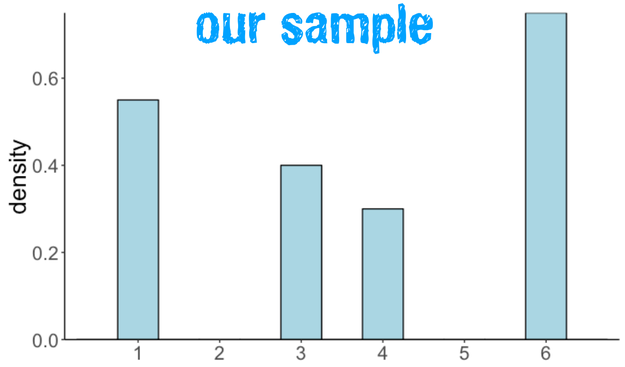
Sampling variation

heavy metal distribution

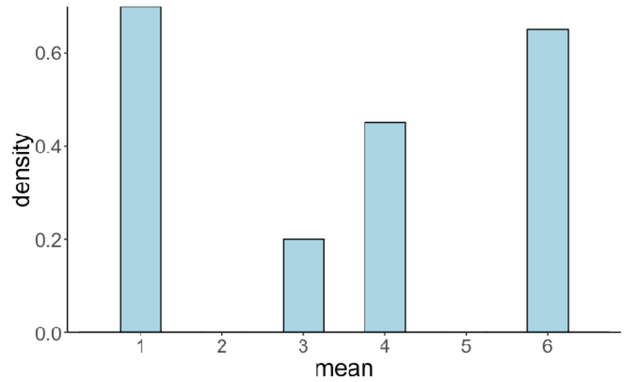


population distribution

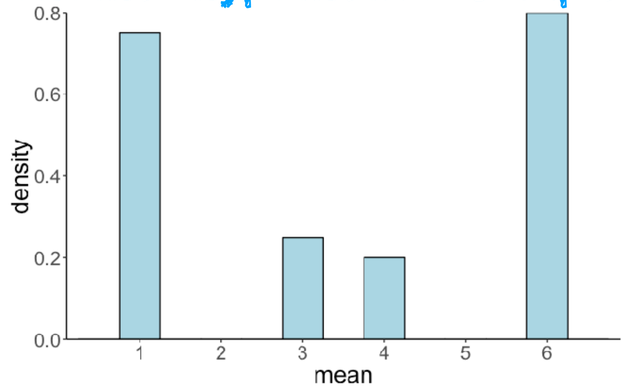
our sample



hypothetical sample



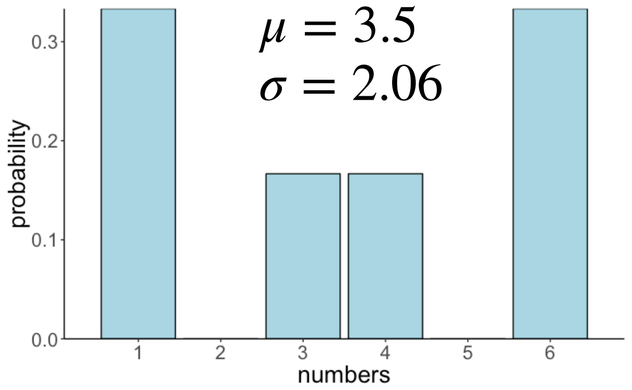
another hypothetical sample



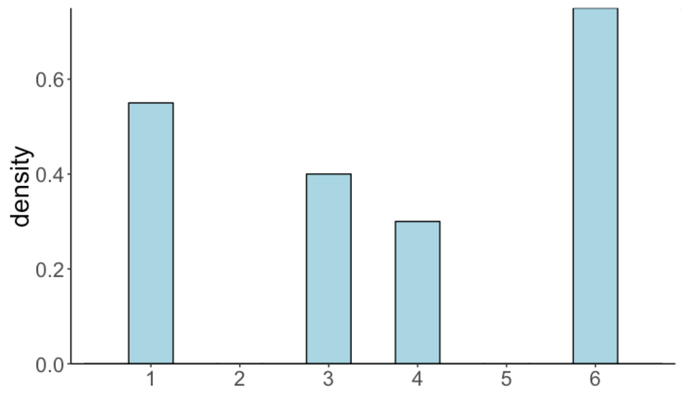
Distribution of the statistic across samples

population distribution

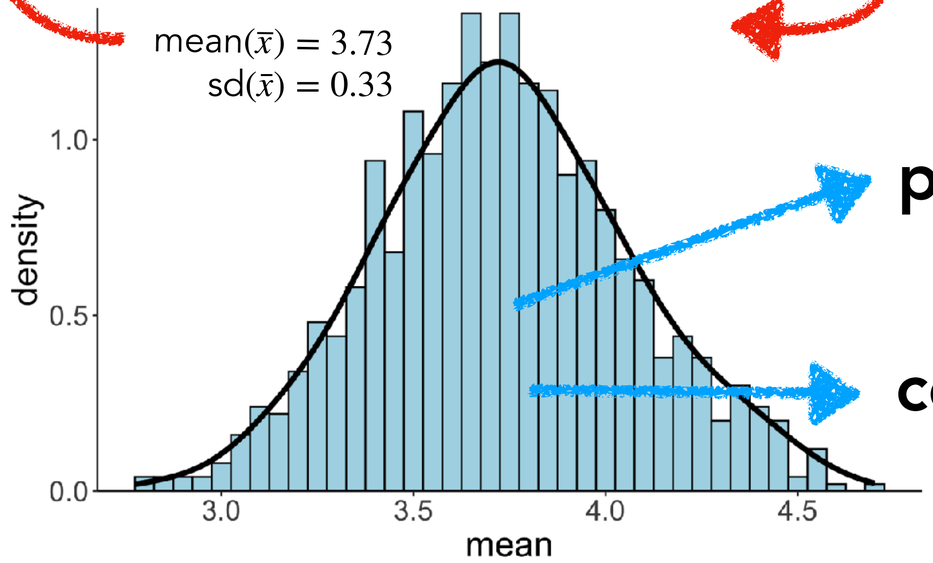
heavy metal distribution



our sample



$\text{mean}(x) = 3.72$
$\text{sd}(x) = 2.05$
$\text{n} = 40$

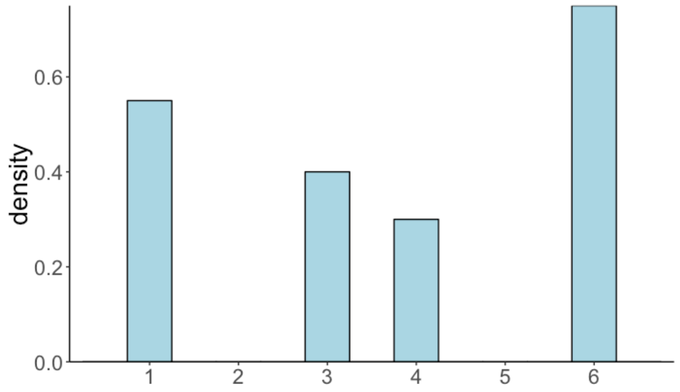


p-values

confidence intervals

sampling distribution of the mean

our sample

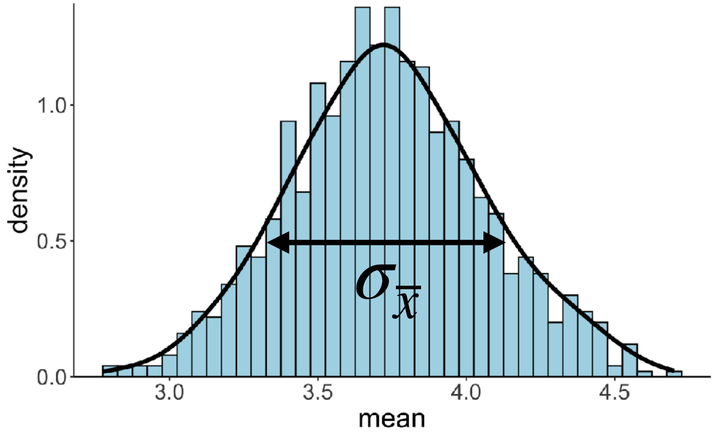


standard deviation

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N - 1}}$$

gives a sense for how well the mean summarizes the data

sampling distribution of the statistic



standard error

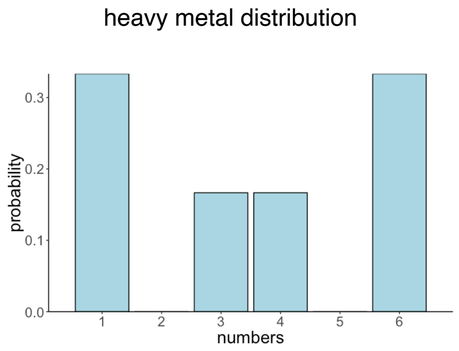
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$$

the standard deviation of the sampling distribution

how much variation would we expect between the means of different samples

how likely is it that our sample mean is representative of the population mean?

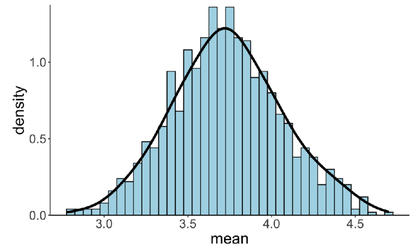
3 distributions in statistical inference



population distribution

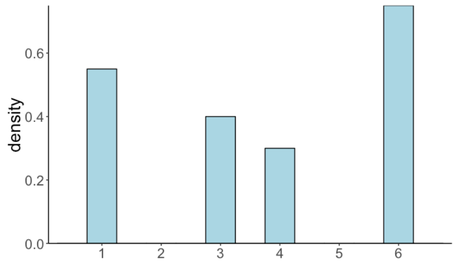
- unknown
- our target for inference
- e.g. we might be interested in the mean of the population distribution

---



sampling distribution

- bridge between sample and population
- derived mathematically / computationally
- asymptotic distribution theory or resampling approaches
- shows how test statistic varies between samples



sample distribution

- our observed sample
- we compute statistics of interest (mean, variance, correlation, ...)
- make an inference about the population via the sampling distribution

What is a p-value?

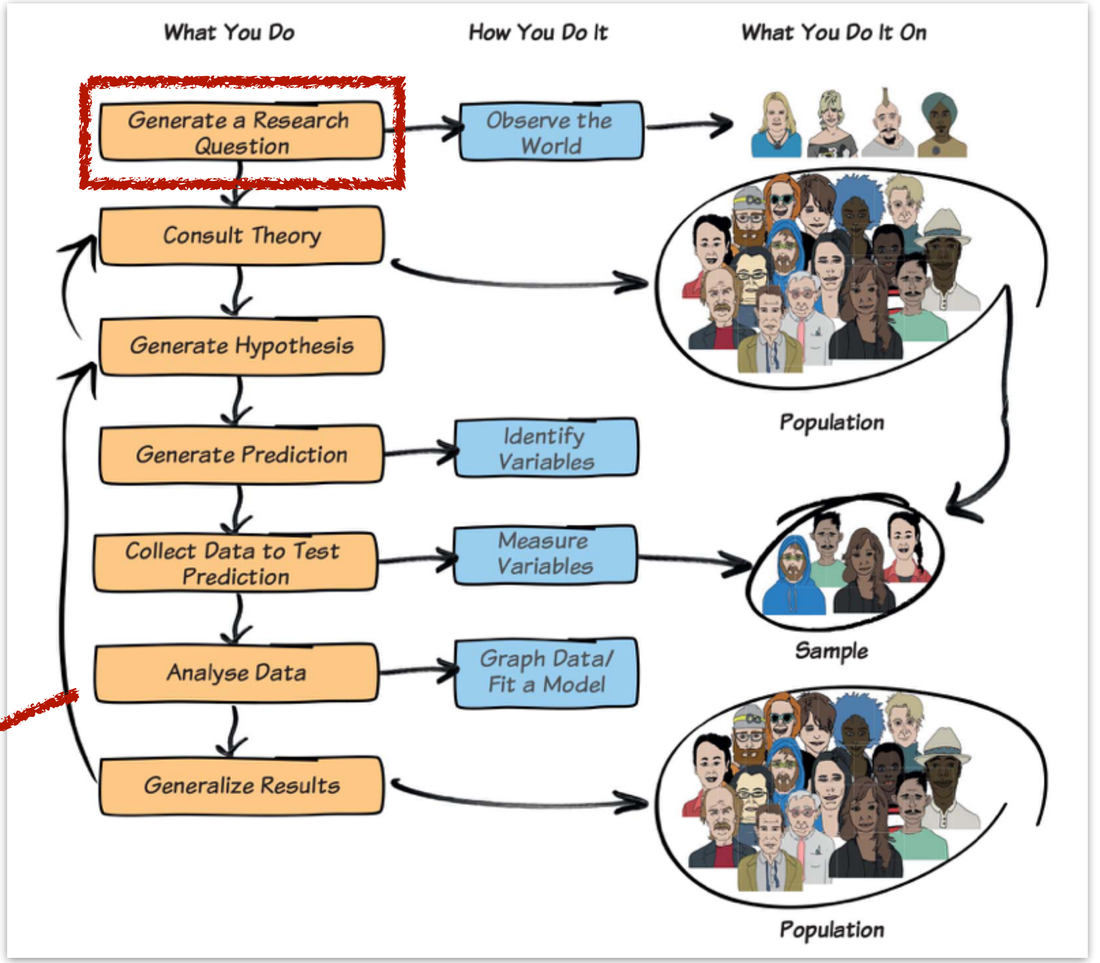
Statistical inference

null hypothesis

$$H_0 : \mu_1 = \mu_2$$

alternative hypothesis

$$H_1 : \mu_1 < \mu_2$$



a p-value, yay!

What is a p-value?

The **p-value** is the probability of finding the observed (or more extreme) results when the null hypothesis (H_0) is true.

$$p(\text{test statistic} \geq \text{observed value} \mid H_0 = \text{true})$$

what we're actually interested in!

$$p(H_1 = \text{true} \mid \text{test statistic} \geq \text{observed value})$$

... we'll have to wait for Reverend Bayes

$$p(H \mid D) = \frac{p(D \mid H) \cdot p(H)}{p(D)}$$

H = Hypothesis

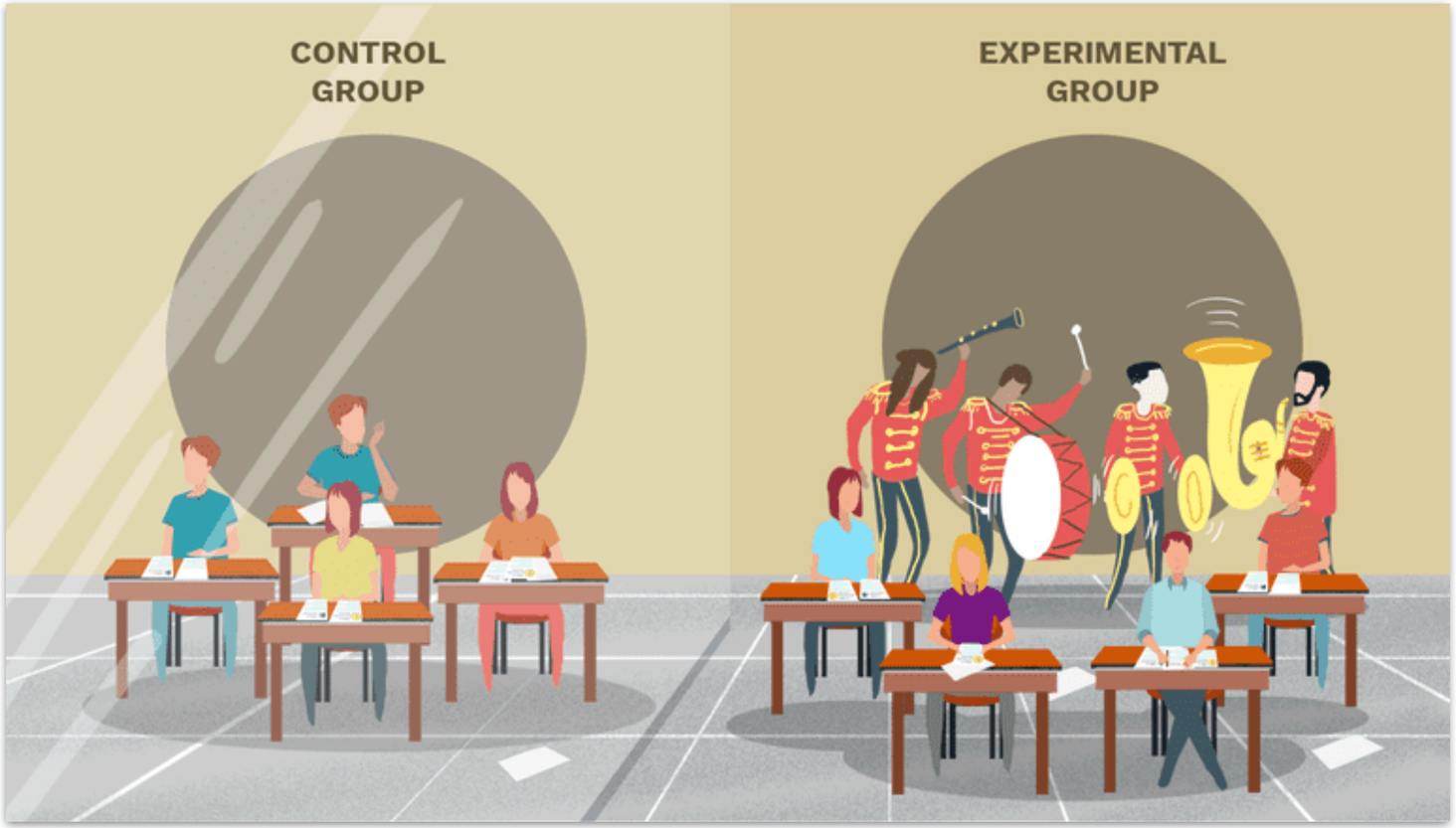
D = Data

Logic of inference

- calculate a **test statistic** based on the sample
  - for example, the difference between the means of two conditions
- build a **sampling distribution** of this statistic *assuming that the null hypothesis is true*
  - use math or resampling methods
- **calculate the probability** of the observed statistic on the sampling distribution
- reject the null hypothesis if the probability of the observed statistic is less than the pre-specified α level

Permutation test

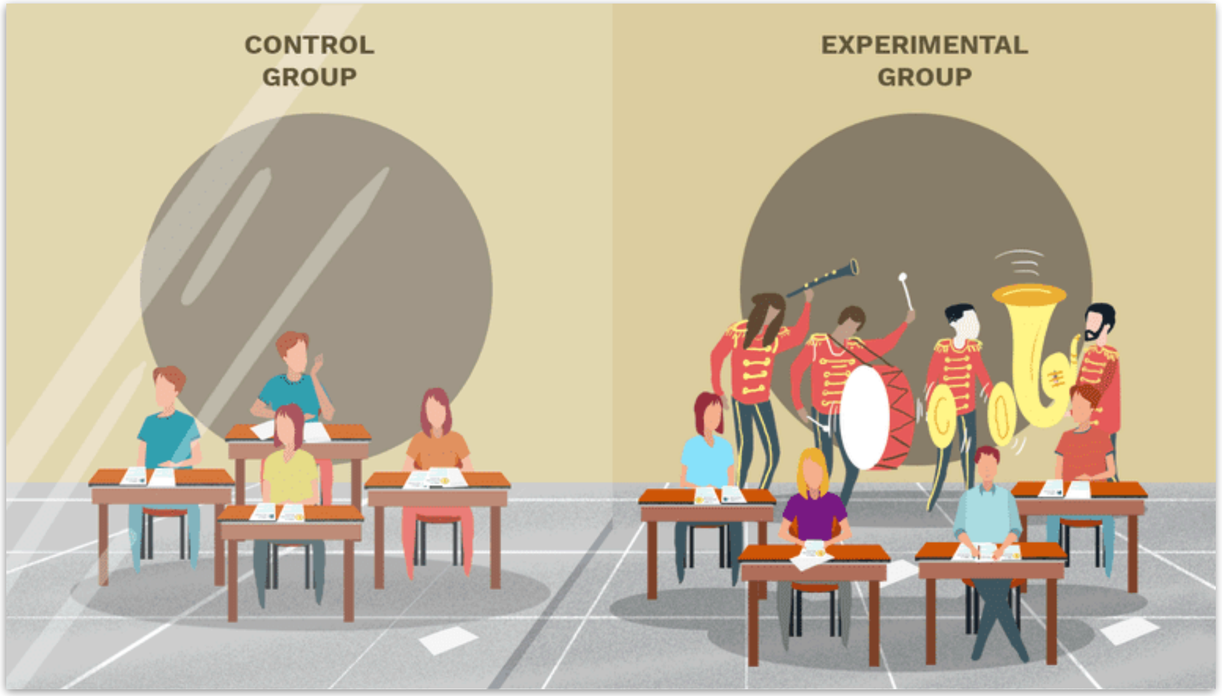
Permutation test



Research question:

Will student test scores be affected by distracting sounds (e.g. the Stanford marching band)?

Permutation test



$H_0 : \mu_{\text{control}} = \mu_{\text{experimental}}$

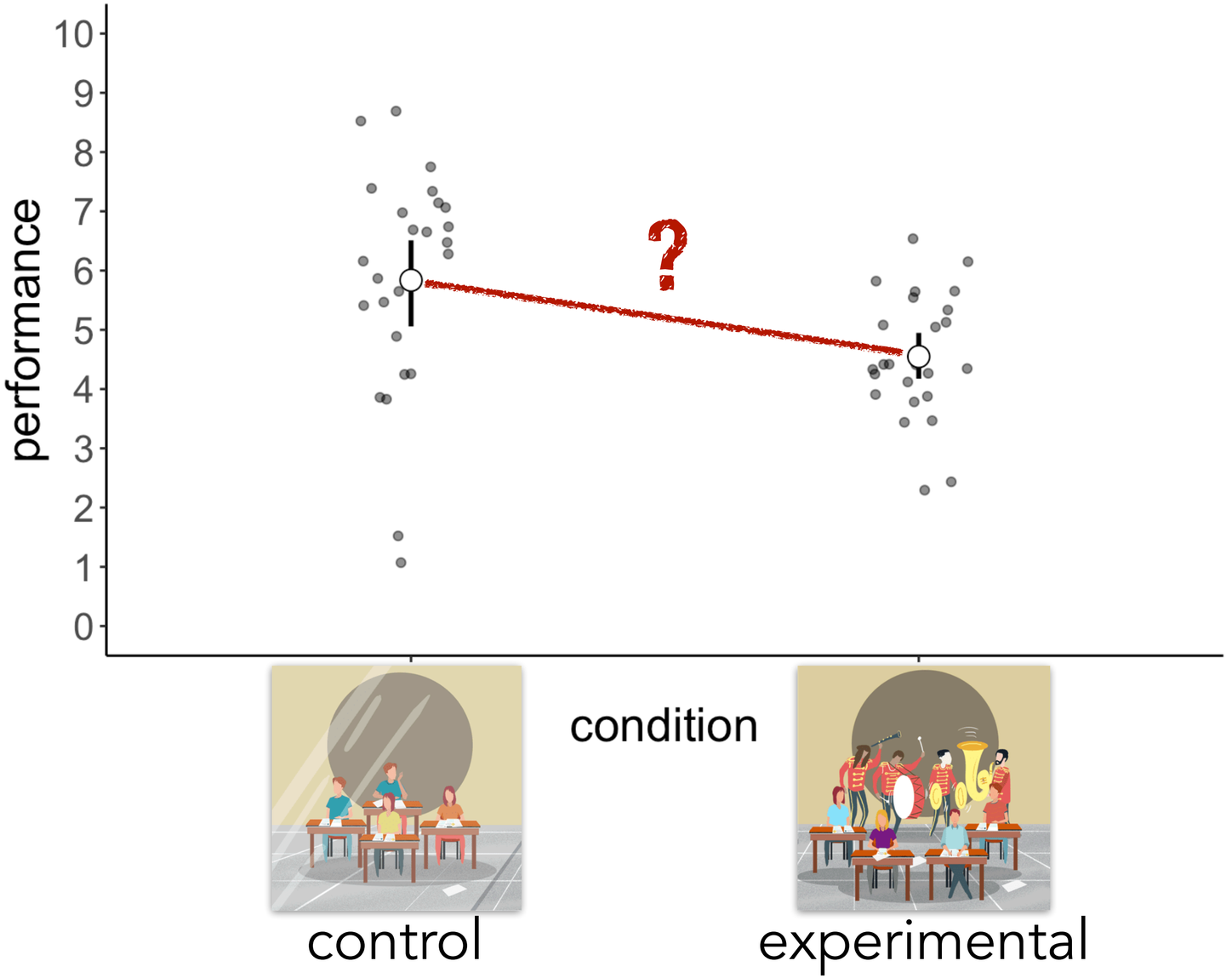
There is no difference between the control group and the experimental group

$H_1 : \mu_{\text{control}} > \mu_{\text{experimental}}$

Performance in the control group is better than in the experimental group

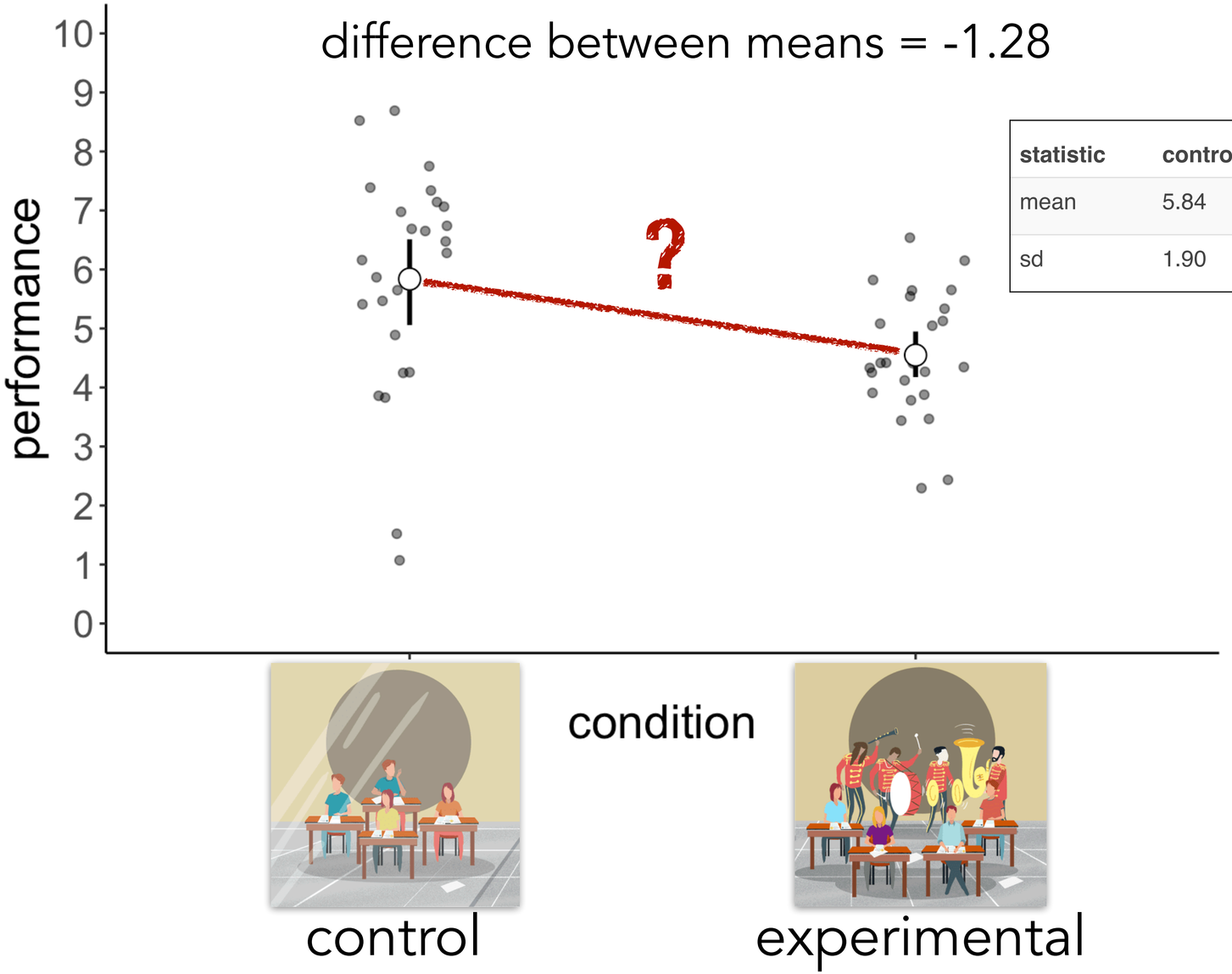
Permutation test

Is the difference in performance statistically significant?



Permutation test

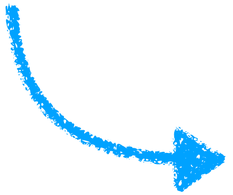
Is the difference in performance statistically significant?



statistic	control	experimental
mean	5.84	4.55
sd	1.90	1.06

Permutation test

- **logic**:
  - assuming our experimental manipulation made no difference, what would be the probability of observing the data that we did?
  - if, assuming that the null hypothesis is true, the probability of observing the data (or data that is more extreme) is less than 5%, we reject the null hypothesis



we need a sampling distribution of our test statistic (difference between means)

Permutation test

observed data

participant	condition	performance
1	control	4.25
2	control	5.87
3	control	3.83
4	control	8.69
5	control	6.16
26	experimental	4.42
27	experimental	4.27
28	experimental	2.29
29	experimental	3.78
30	experimental	5.13

random permutation

participant	condition	performance
1	control	4.25
2	experimental	5.87
3	control	3.83
4	experimental	8.69
5	control	6.16
26	control	4.42
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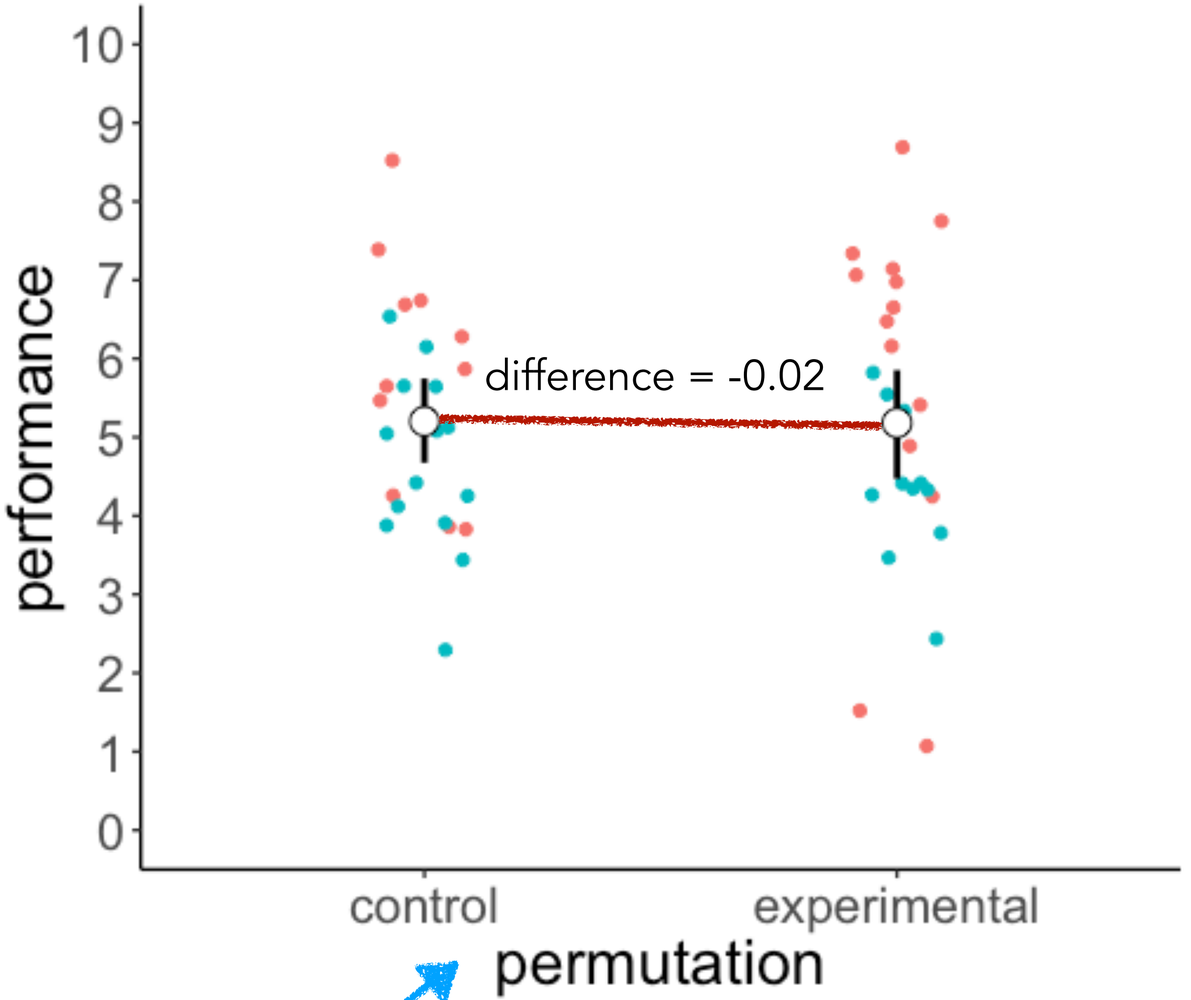
Permutation test



in the actual experiment

condition
- control
- experimental

in the permutation

Permutation test

observed data

participant	condition	performance
1	control	4.25
2	control	5.87
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4	control	8.69
5	control	6.16
26	experimental	4.42
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1

2

3

participant	condition	performance
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3	control	3.83
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29	experimental	3.78
30	experimental	5.13

permutation	mean_difference
1	-0.88
2	-0.26
3	-0.94
4	0.47
5	-0.28
6	1.15
7	0.98
8	0.38
9	-0.08
10	0.31

Permutation test

`df.permutations`

permutation	mean_difference
1	-0.88
2	-0.26
3	-0.94
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5	-0.28
6	1.15
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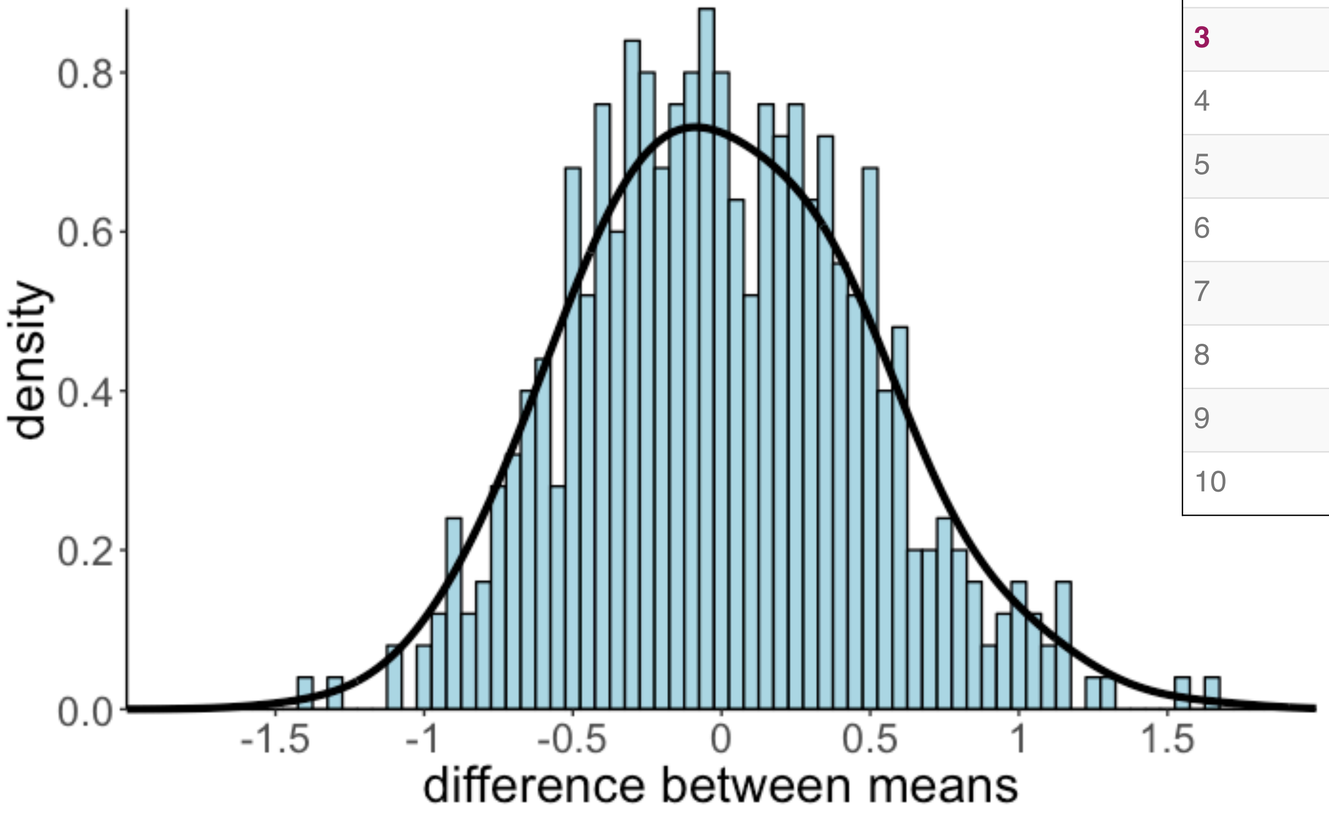
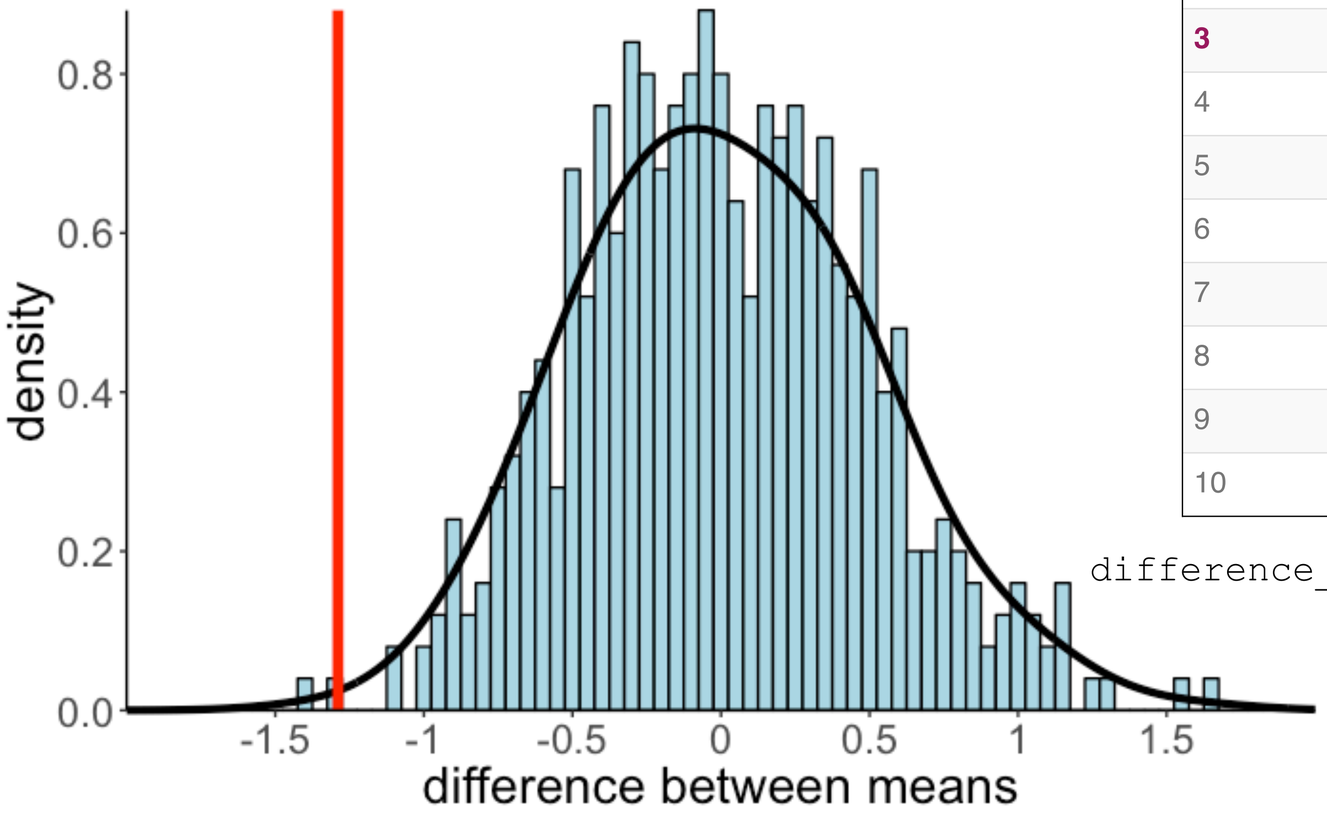


Sampling distribution of differences
(expected differences if the null hypothesis was true)

Permutation test



df.permutations

permutation	mean_difference
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6	1.15
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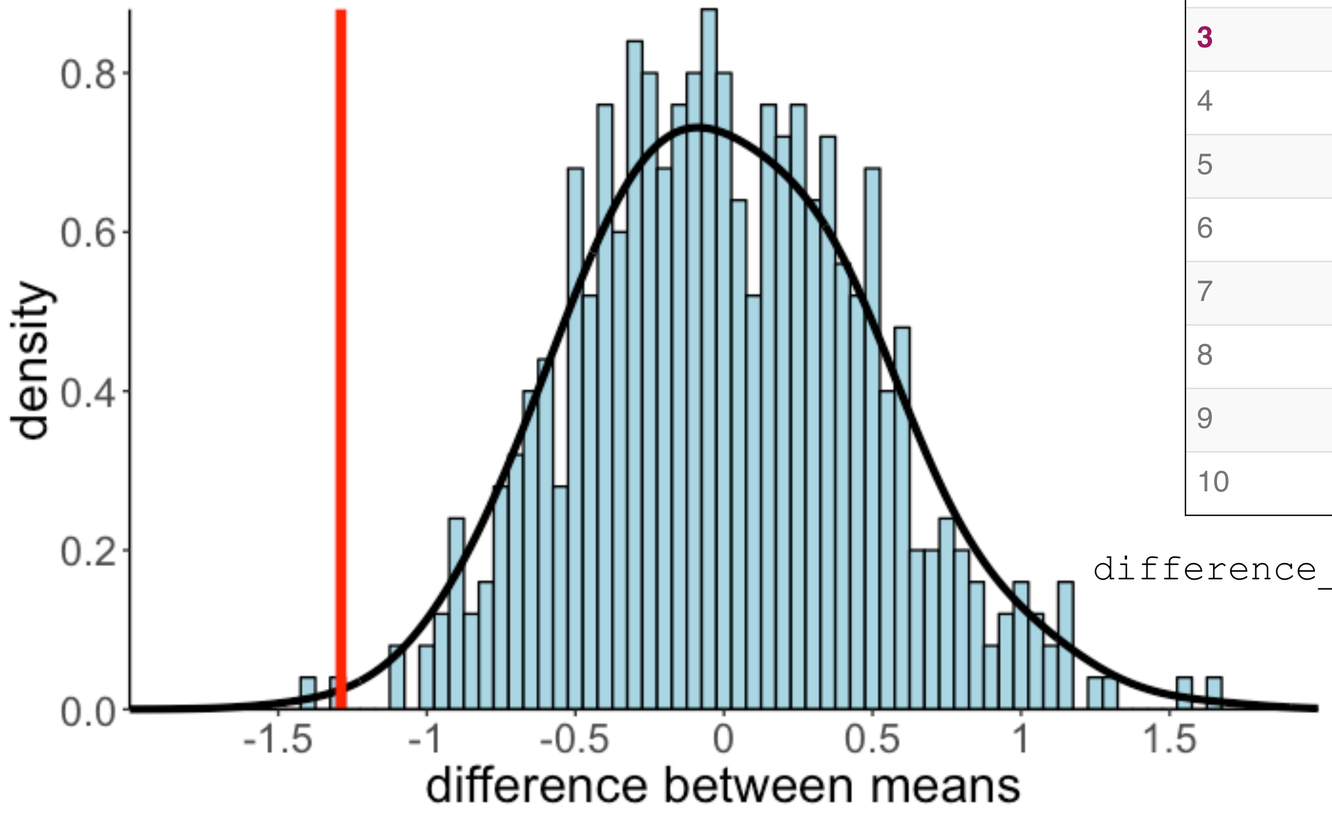
8	0.38
9	-0.08
10	0.31

`difference_actual = -1.28`

Sampling distribution of differences
(expected differences if the null hypothesis is true)

Permutation test

observed difference
in our experiment



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5	-0.28
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7	0.98
8	0.38
9	-0.08
10	0.31

difference_actual = -1.28

```
1 #calculate p-value of our observed result
2 df.permutations %>%
3   summarize(p_value = sum(mean_difference <= difference_actual)/n())
```

p-value = .002

What is a p-value?

The **p-value** is the probability of finding the observed, or more extreme, results when the null hypothesis (H_0) is true.

Permutation test

df.permutations

permutation	mean_difference
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2	-0.26
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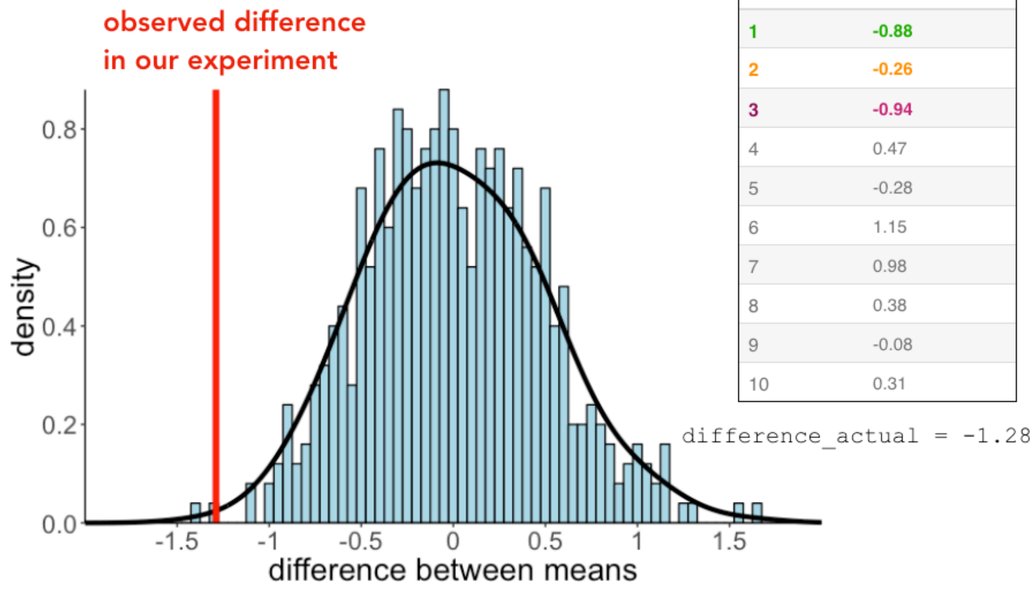

difference_actual = -1.28

```
1 #calculate p-value of our observed result
2 df.permutations %>%
3   summarize(p_value = sum(mean_difference <= difference_actual)/n())
```

p-value = .002

57

Permutation test

```
 1 n_permutations = 500
 2
 3 # permutation function
 4 func_permutations = function(df){
 5   df %>%
 6     mutate(condition = sample(condition)) %>%
 7     group_by(condition) %>%
 8     summarize(mean = mean(performance)) %>%
 9     pull(mean) %>%
10     diff()
11 }
```

set the number of permutations

shuffle the condition labels

calculate difference between group means

observed data

participant	condition	performance
1	control	4.25
2	control	5.87
3	control	3.83
4	control	8.69
5	control	6.16
26	experimental	4.42
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28	experimental	2.29
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random permutation

participant	condition	performance
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4	experimental	8.69
5	control	6.16
26	control	4.42
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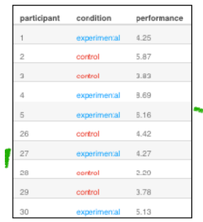

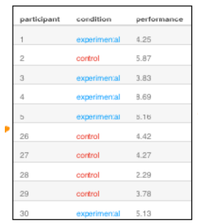

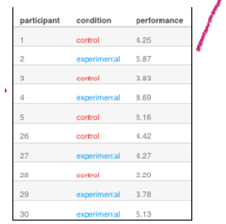

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Permutation test

```r
n_permutations = 500

# permutation function
func_permutations = function(df){
  df %>%
    mutate(condition = sample(condition)) %>%
    group_by(condition) %>%
    summarize(mean = mean(performance)) %>%
    pull(mean) %>%
    diff()
}

# data frame with permutation results
df.permutations = tibble(
  permutation = 1:n_permutations,
  mean_difference = replicate(n = n_permutations, func_permutations(df.data))
)
```

df.permutations

permutation	mean_difference
1	-0.88
2	-0.26
3	-0.94
4	0.47
5	-0.28
6	1.15
7	0.98
8	0.38
9	-0.08
10	0.31

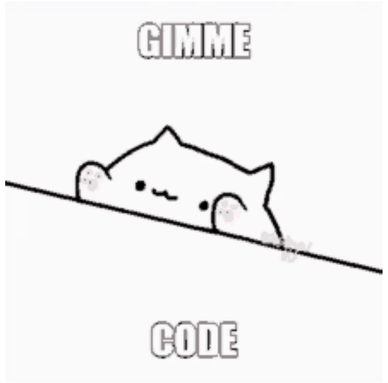
run the `func_permutations()` function many times

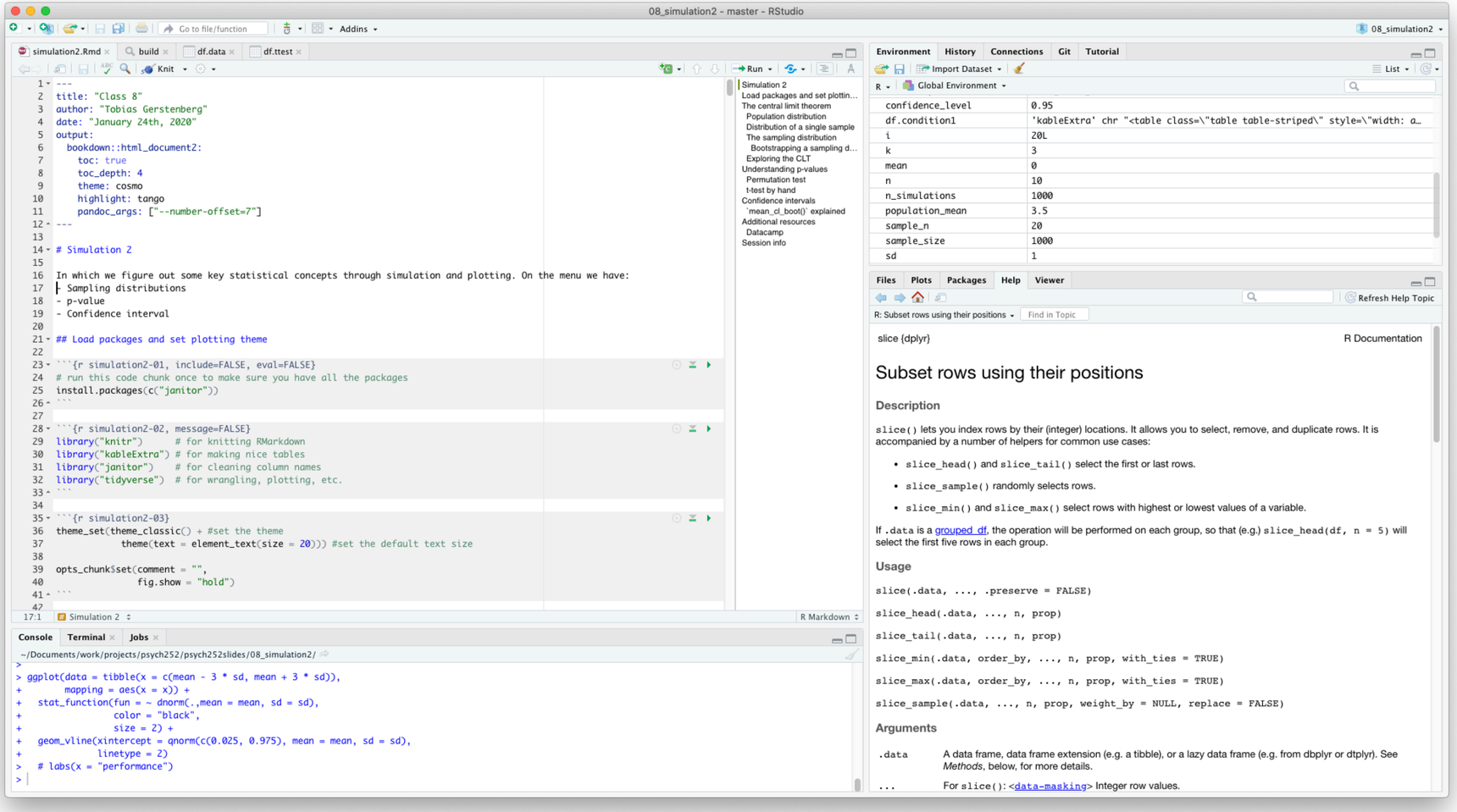
(instead of using a for loop)

Summary Revisit and understand key statistical concepts

- **Inference in frequentist statistics**
  - goal is to make inference from sample to population
  - we do so via a complicated procedure that involves sampling distributions
- **Sampling distributions**
  - the link between sample and population in frequentist statistics
  - theoretical (or simulated) distribution of a test statistic
- **What is a p-value?**
  - the probability of the observed test result (or a more extreme result) assuming that the H_0 is true
- **Confidence interval (of the mean)**
  - "If we were to repeat the experiment over and over, then 95% of the time the confidence intervals contain the true mean."

How to better understand!





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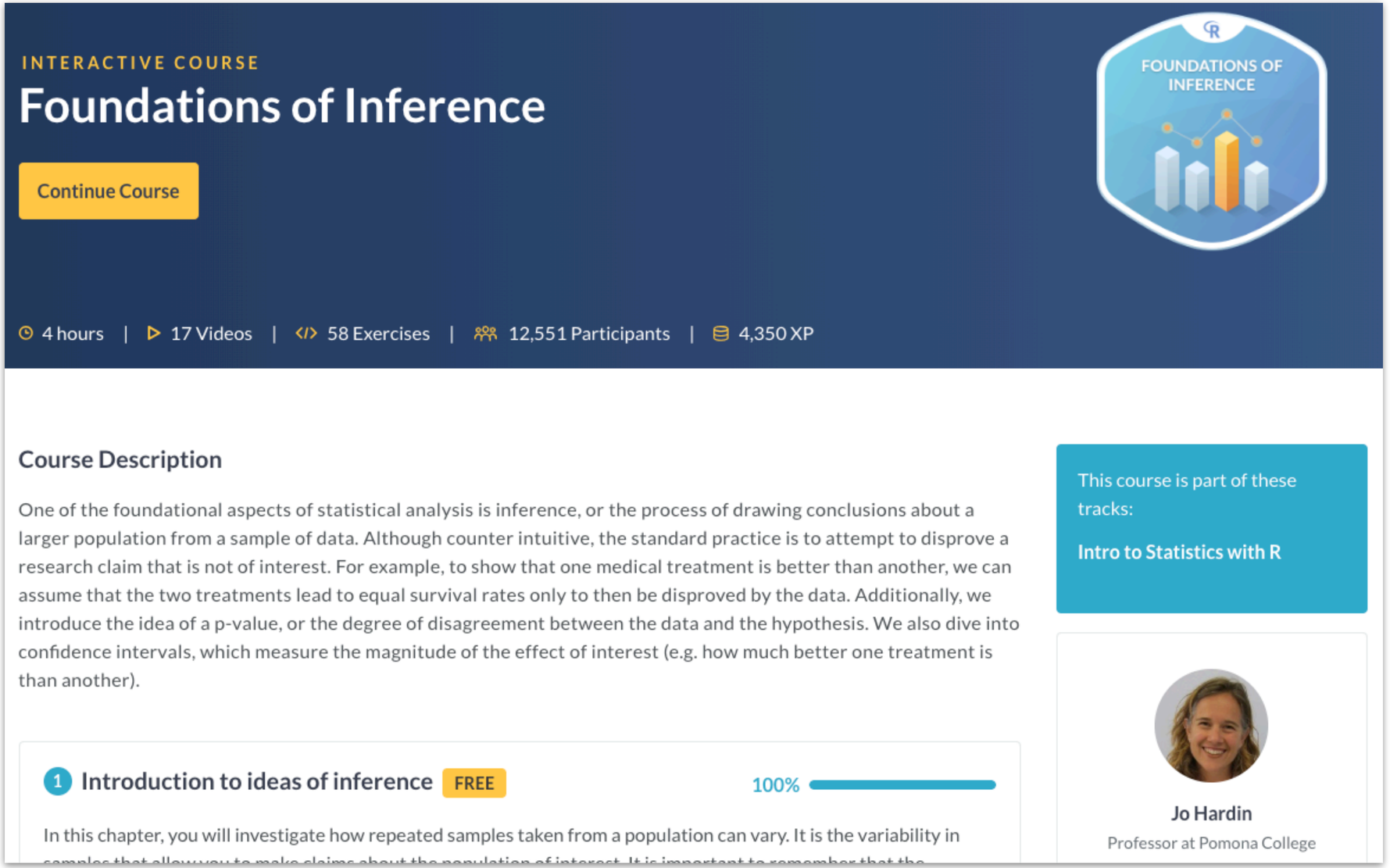

https://www.datacamp.com/courses/foundations-of-inference

Feedback

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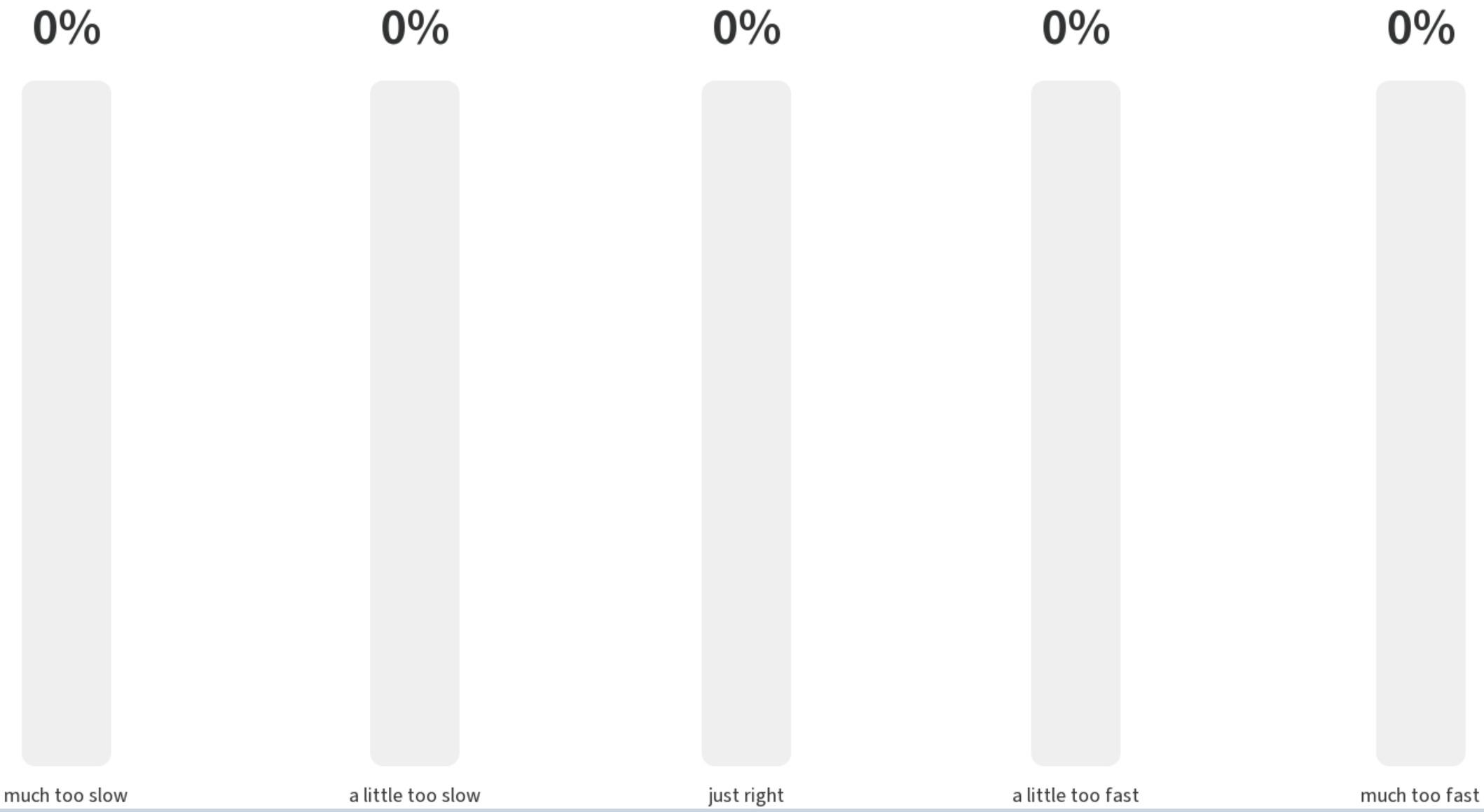

Thank you!