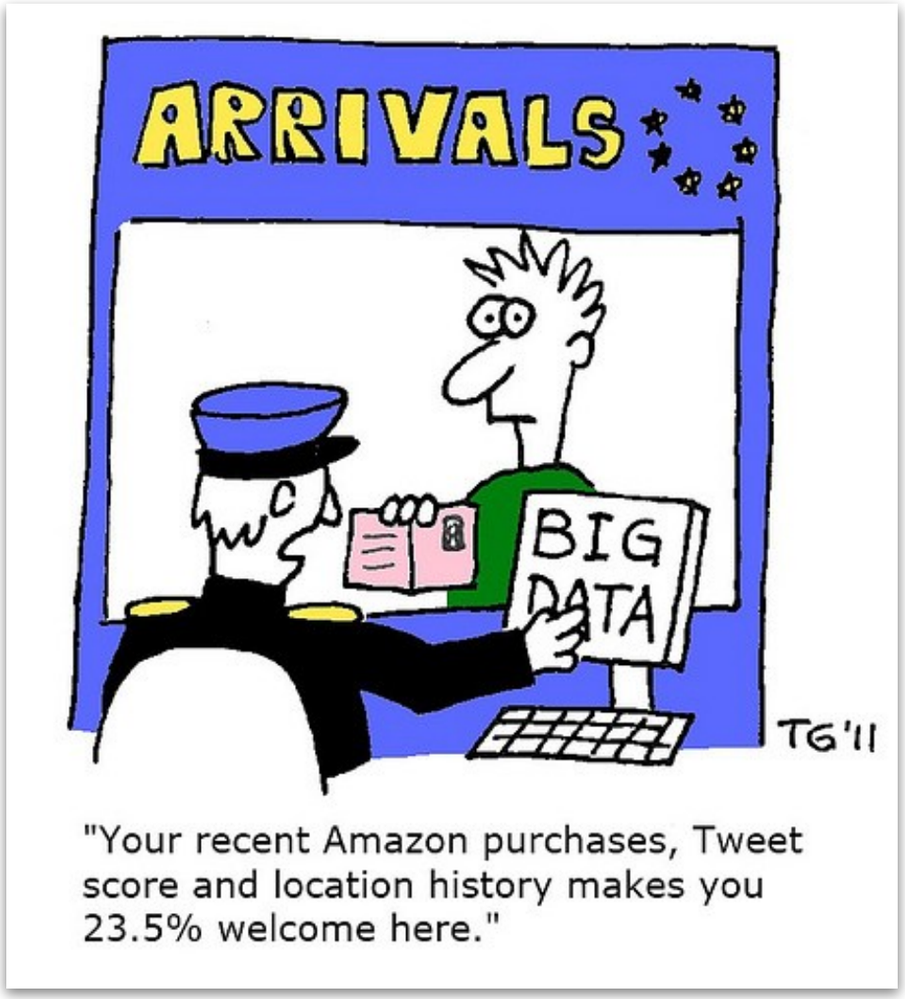
PSYCH 252: Statistical Methods for Behavioral and Social Sciences

Linear mixed effects models 3



02/26/2024

Things that came up

Brenton Wiernik 🏳️‍🌈
@bmwiernik

A student in my R programming class shared this and I can't stop laughing about it

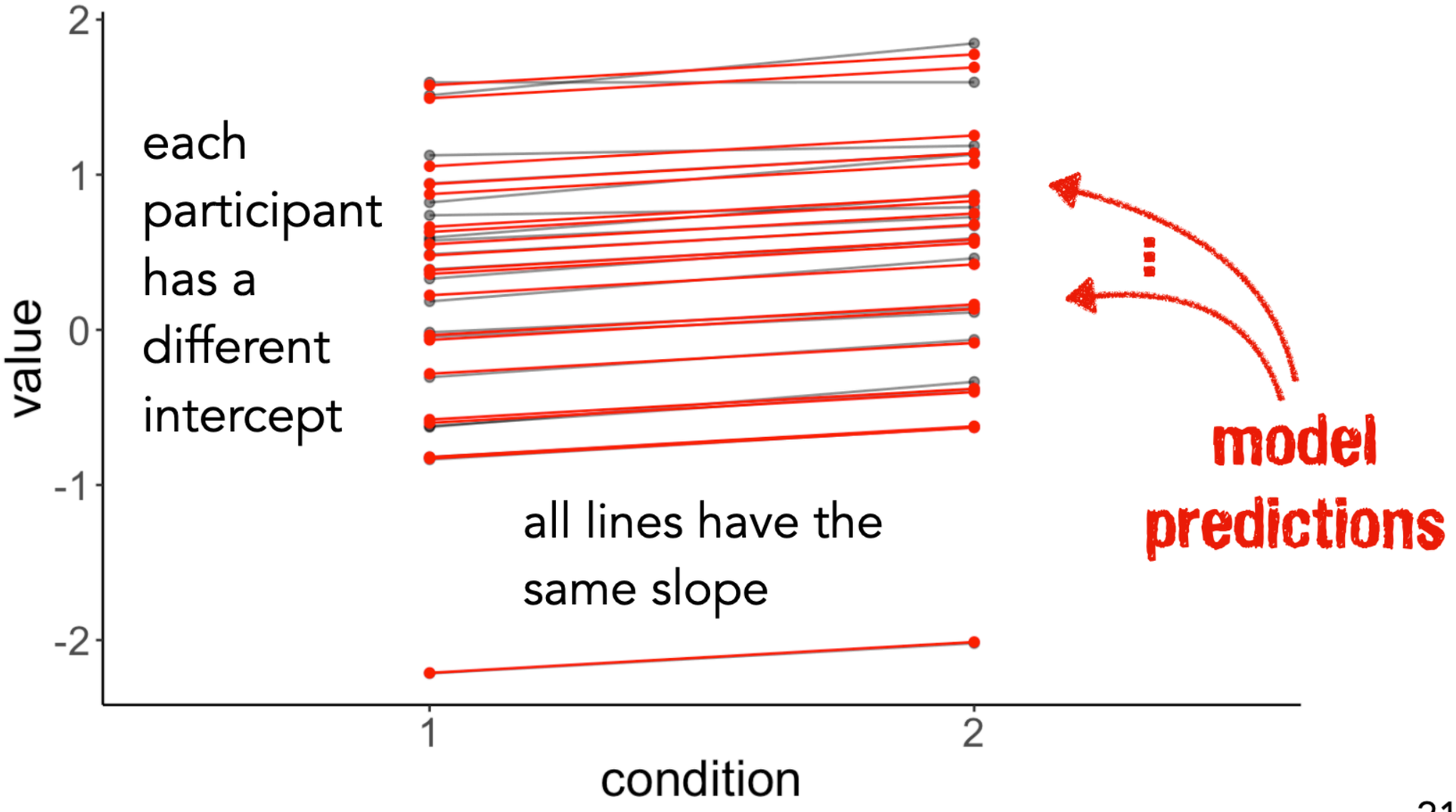
8:34 AM · Mar 1, 2021 · Twitter for iPhone

273 Retweets **29** Quote Tweets **2,012** Likes

Linear mixed effects model (accounting for dependence)

Predictions by the linear mixed effects model

```
lmer(formula = value ~ condition + (1 | participant),
     data = df.original)
```



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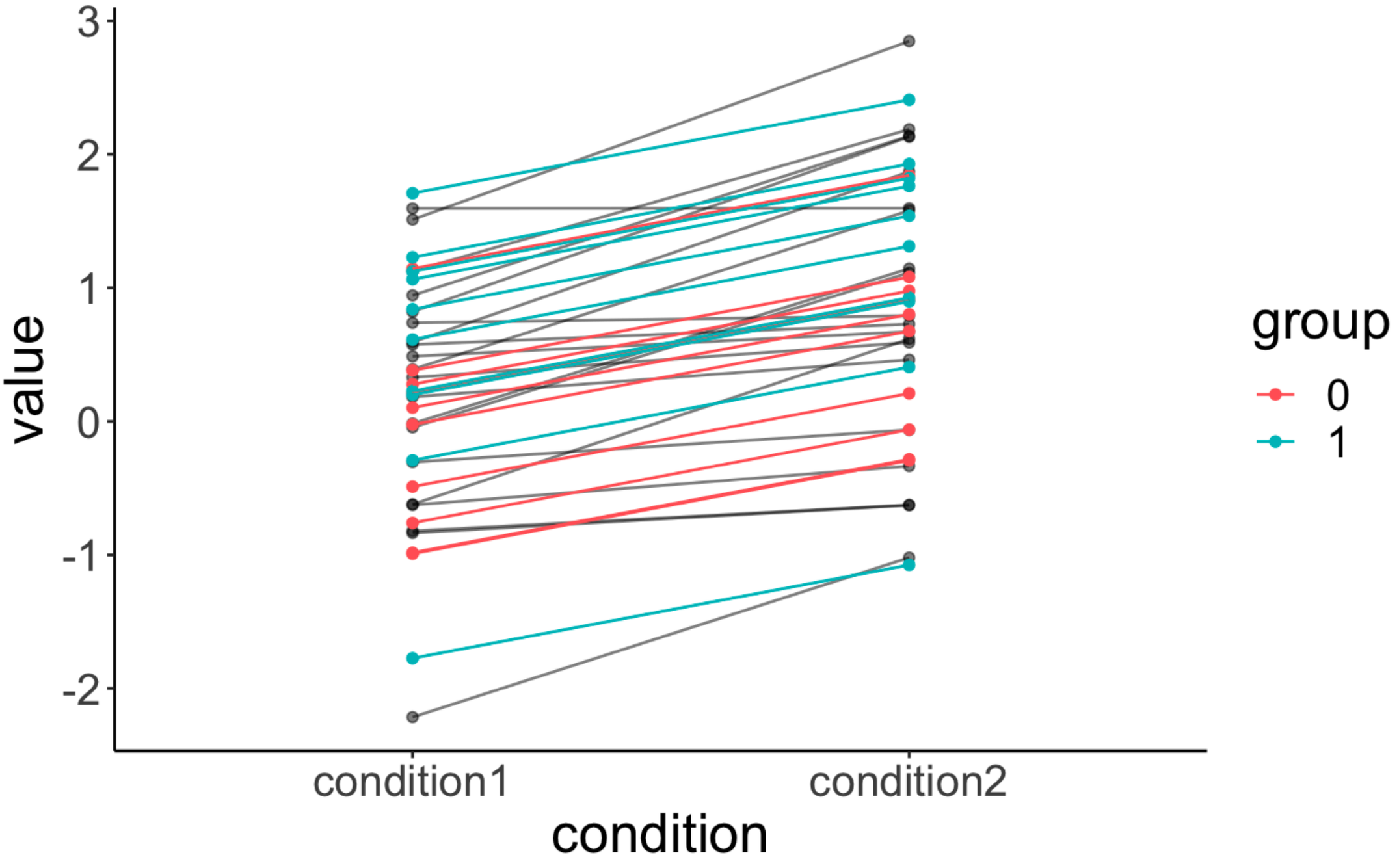
```r
fit = lmer(formula = value ~ 1 + condition +
                     (1 | group) + (1 | group:participant),
           data = df.dependence)
```



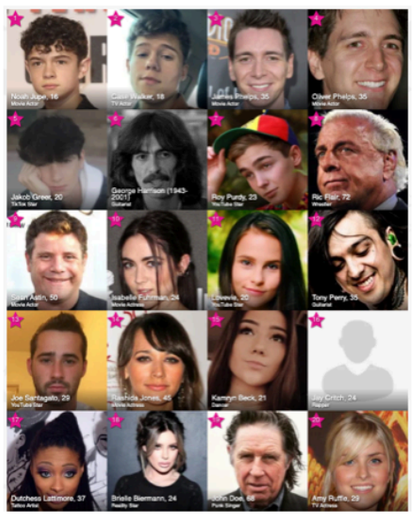
Plan for today

- Quick recap
- Simpsons paradox
- Understanding `lmer()` syntax
- Reporting results
- Let's simulate some `lmer()`s
- `lmer()` standard operating procedures
- Some more examples

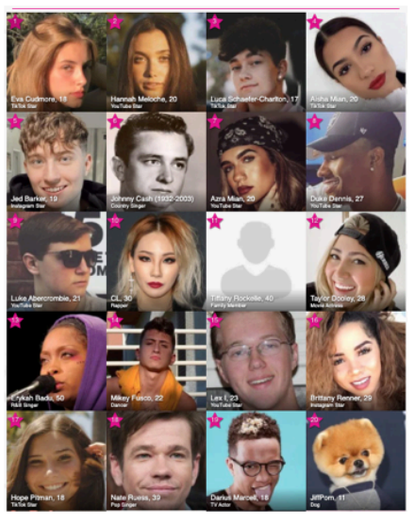
Quick recap

Quick recap: Linear mixed effects models

Are faces of people born on February 25th more trustworthy than faces of people born on February 26th?



born on February 25th



born on February 26th

N = 100 participants

```
1 lmer(formula = trustworthy ~ 1 + birthday +
                 (1 | item),
2       data = df.birthday)
```

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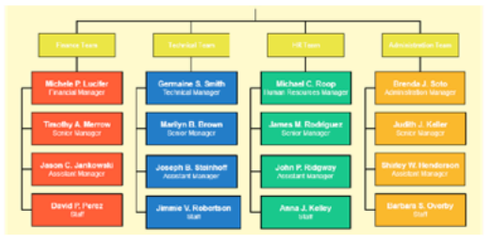
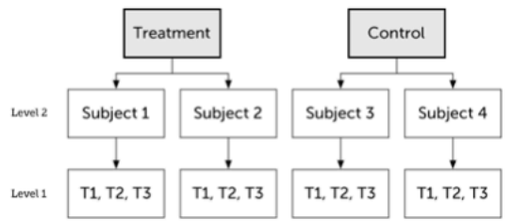
Linear mixed effects models

Hierarchical models



Longitudinal models



- allow us to account for dependencies in our data
- **hierarchical models**: schools > teachers > students
- **longitudinal models**: repeated observations from the same people

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Random Slopes + Intercepts

It's reasonable to imagine that the most realistic situation is a combination of the scenarios described above:

> Faculty salaries start at different levels *and* increase at different rates depending on their department.

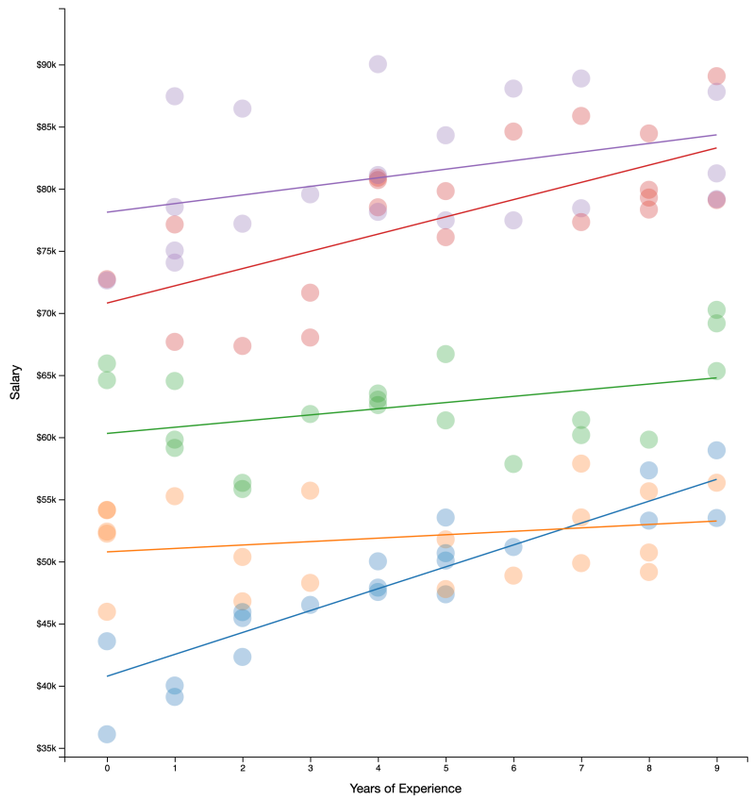
To incorporate both of these realities into our model, we want both the slope and the intercept to vary depending on the department of the faculty member. We can describe this with the following notation:

$$\hat{y}_i = \alpha_{j[i]} + \beta_{j[i]} x_i$$

Thus, the *starting salary* for faculty member `i` depends on their department ($\alpha_{j[i]}$), and their annual raise also varies by department ($\beta_{j[i]}$):

$$\hat{salary}_i = \beta_{0j[i]} + \beta_{1j[i]} * experience_i$$

In order to implement any of these methods, you'll need to have a strong understanding of the phenomenon you're modeling, and how that is captured in the data. And, of course, you'll need to assess the performance of your models (not described here).

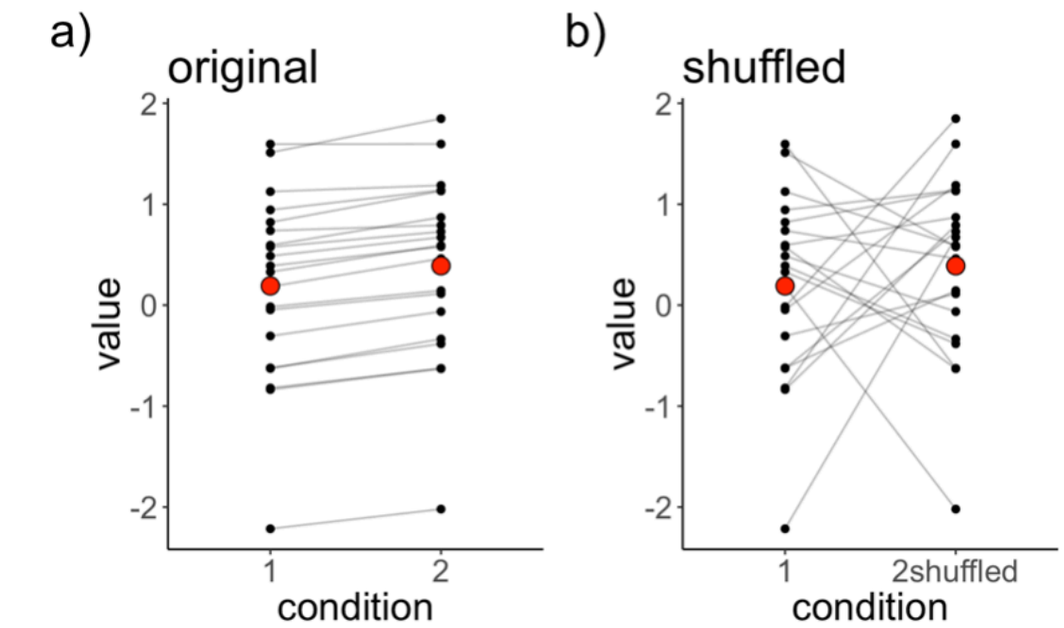


Quick recap: Dependence matters

Dependence

Does it really matter?

Is there a significant difference between conditions 1 and 2?



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Linear model

```
lm(formula = value ~ 1 + condition,
   data = df.original)
```

$value_i = b_0 + b_1 \cdot condition_i + e_i$

i = observation

$e_i \sim \mathcal{N}(mean = 0, sd = s_{error})$

3 parameters: b_0, b_1, s_{error}

Linear mixed effects model

```
lmer(formula = value ~ 1 + condition +
               (1 | participant),
     data = df.original)
```

$value_{i,j} = b_0 + b_1 \cdot condition_{i,j} + U_i + e_i$

i = participant,
j = time point

$e_i \sim \mathcal{N}(mean = 0, sd = s_{error})$

$U_i \sim \mathcal{N}(mean = 0, sd = s_U)$

b_0, b_1 = fixed effects
U_i = random effect

here: random intercept

4 parameters: b_0, b_1, s_{error}, s_U

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Model coefficients

Linear model

```
fit = lm(formula = value ~ 1 + condition,
         data = df.original)
coef(fit)
```

```
(Intercept)  condition2
  0.1905239   0.1993528
```

- one intercept
- one slope for condition

Linear mixed effects model

```
fit = lmer(formula = value ~ 1 + condition +
                     (1 | participant),
           data = df.original)
coef(fit)
```

```
$participant
   (Intercept) condition2
1   -0.57839428  0.1993528
2    0.22299824  0.1993528
3   -0.82920677  0.1993528
4    1.49310938  0.1993528
5    0.36042775  0.1993528
6   -0.82060123  0.1993528
7    0.47929171  0.1993528
8    0.66401020  0.1993528
9    0.55135879  0.1993528
10  -0.28306703  0.1993528
11   1.57681676  0.1993528
12   0.38457642  0.1993528
13  -0.59969682  0.1993528
14  -2.21148391  0.1993528
15   1.05439374  0.1993528
16  -0.06476643  0.1993528
17  -0.03505690  0.1993528
18   0.93945348  0.1993528
19   0.87495531  0.1993528
20   0.63135911  0.1993528

attr(,"class")
[1] "coef.mer"
```

- different intercept for each participant
- one slope for condition

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general points about `lmer()`

- **fixed effects**:
  - often: factors that we manipulate experimentally
  - parameters are estimated --> we are interested in characterizing the relationship between this variable and the outcome
- **random effects**:
  - variation we want to control for
  - often: differences between participants or items in our experiment

Quick recap: `lmer()` summary

Understanding the **`lmer()`** summary

```
1 # fit a linear mixed effects model
2 lmer(formula = value ~ condition + (1 | participant),
3                   data = df.original) %>%
4   summary()
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: value ~ condition + (1 | participant)
   Data: df.original

REML criterion at convergence: 17.3

Scaled residuals:
     Min       1Q   Median       3Q      Max
-1.55996 -0.36399 -0.03341  0.34400  1.65823

Random effects:
 Groups      Name        Variance Std.Dev.
 participant (Intercept) 0.816722 0.90373
 Residual                0.003796 0.06161
Number of obs: 40, groups:  participant, 20

Fixed effects:
            Estimate Std. Error t value
(Intercept)  0.19052    0.20255   0.941
condition2   0.19935    0.01948  10.231

Correlation of Fixed Effects:
           (Intr)
condition2 -0.048
```

Fixed effects

one parameter for the global intercept (value for the baseline condition)

one parameter for the condition effect (difference between the two conditions)

interpretation the same as for **`lm()`**, also: we can use contrasts!

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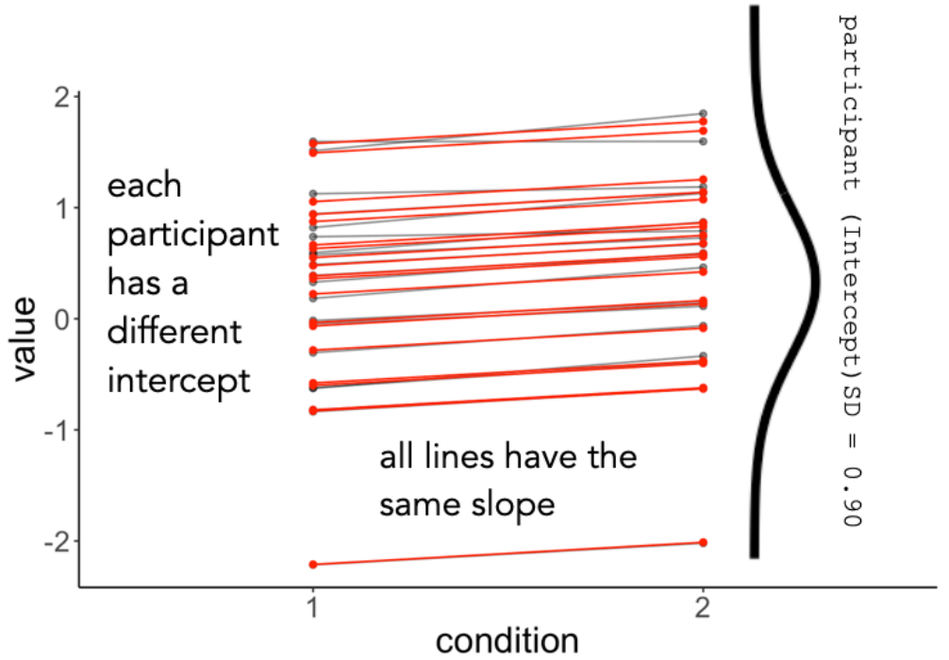
Understanding the **`lmer()`** summary



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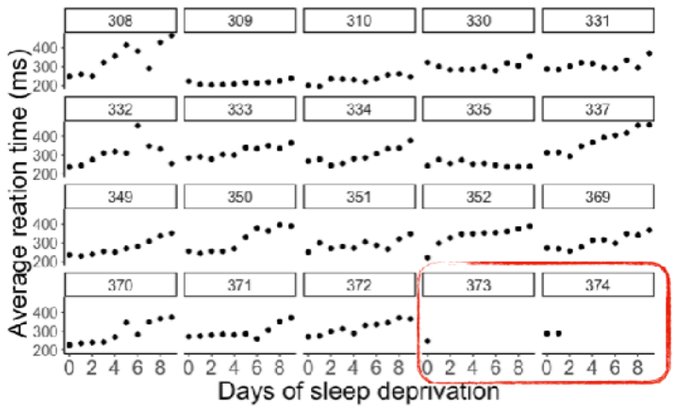
Quick recap: worked example

Data set

How does sleep deprivation affect reaction time?

subject	days	reaction
308	0	249.56
308	1	258.70
308	2	250.80
308	3	321.44
308	4	356.85
309	0	222.73
309	1	205.27
309	2	202.98
309	3	204.71
309	4	207.72



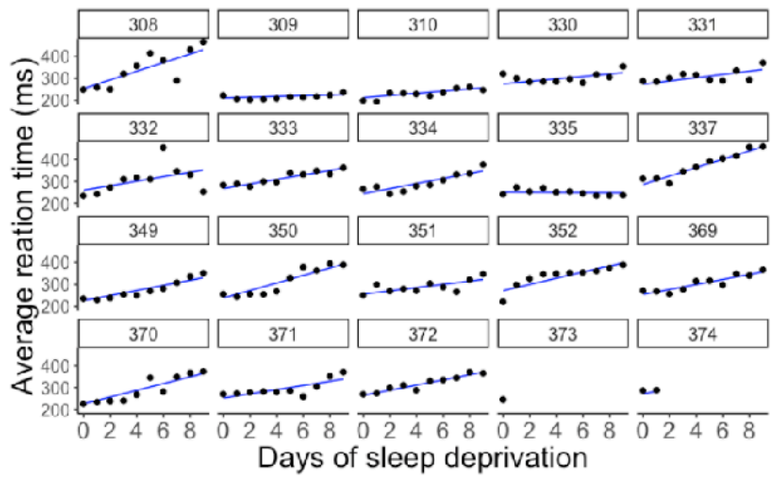
20 participants 2 with incomplete information

Partial pooling: Fit mixed effects model

intercepts and slopes differ between participants

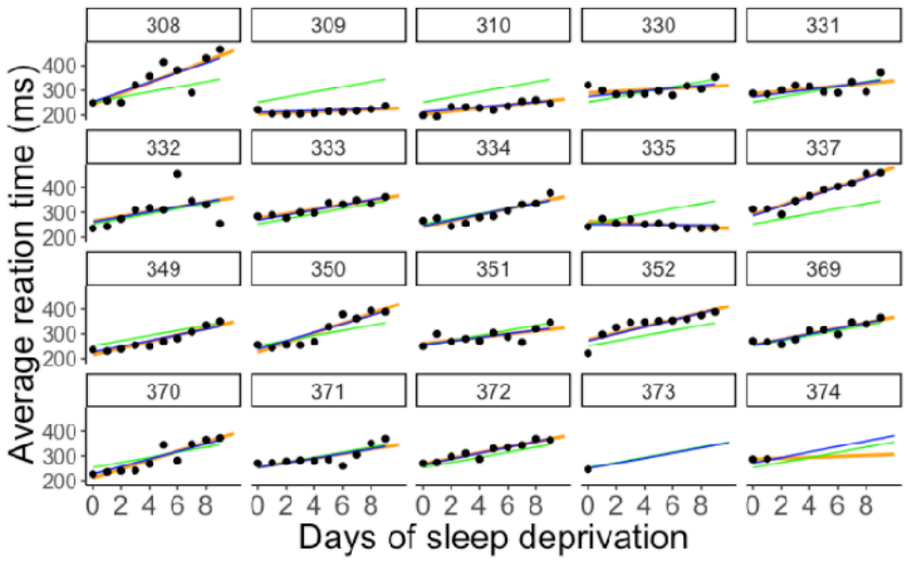
random intercept random slope

```
lmer(formula = reaction ~ 1 + days + (1 + days | subject),
     data = df.sleep)
```



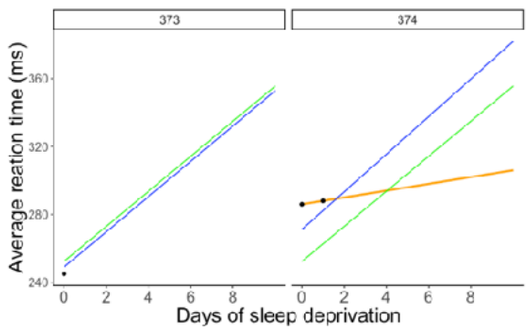
Comparison

complete pooling
no pooling
partial pooling



Quick recap: shrinkage

Comparison



complete pooling
no pooling
partial pooling

- **complete pooling**:
  - doesn't account for any individual variation
- **no pooling**:
  - doesn't yield predictions when we only have observation
  - doesn't consider the general effect of sleep deprivation when making predictions
- **partial pooling**:
  - draws on all the information in the data
  - extrapolates based on information about the individual participants, as well as information based on the whole sample

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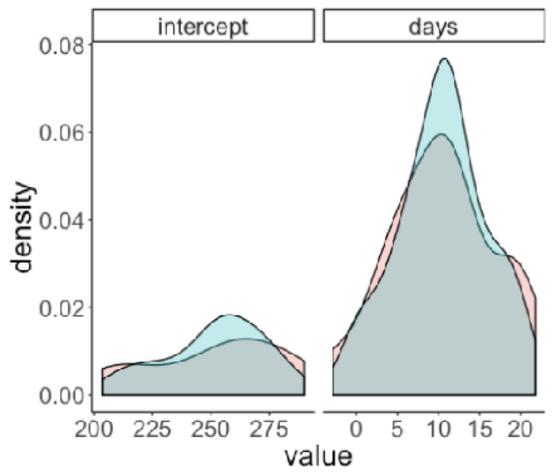
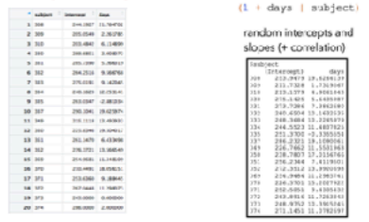
Shrinkage

separate regression for each participant

lmer()





(1 + days | subject)

random intercepts and slopes (+ correlation)

method
no pooling
partial pooling

standard deviation

method	intercept	days
no pooling	28.95	6.56
partial pooling	21.59	5.46

variance "shrinks"

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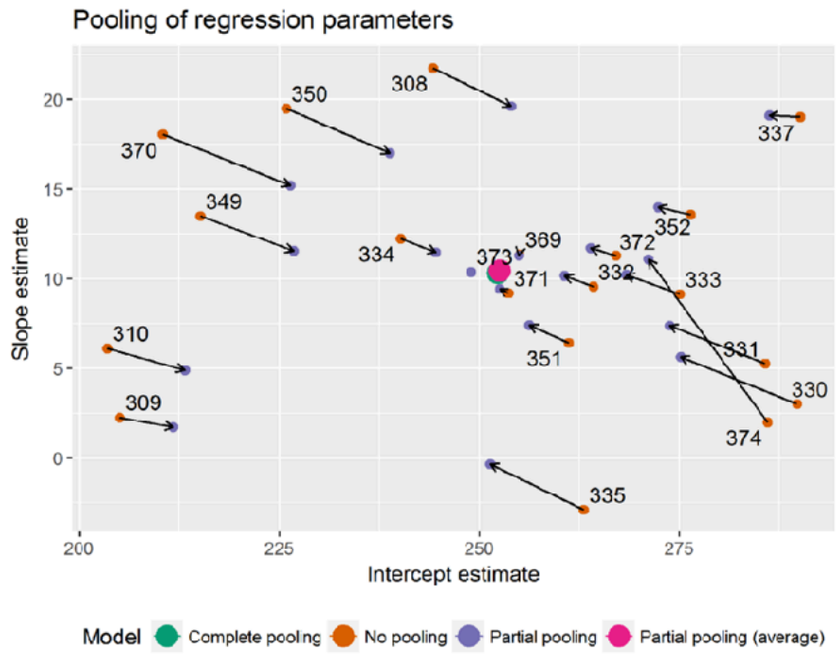
Shrinkage



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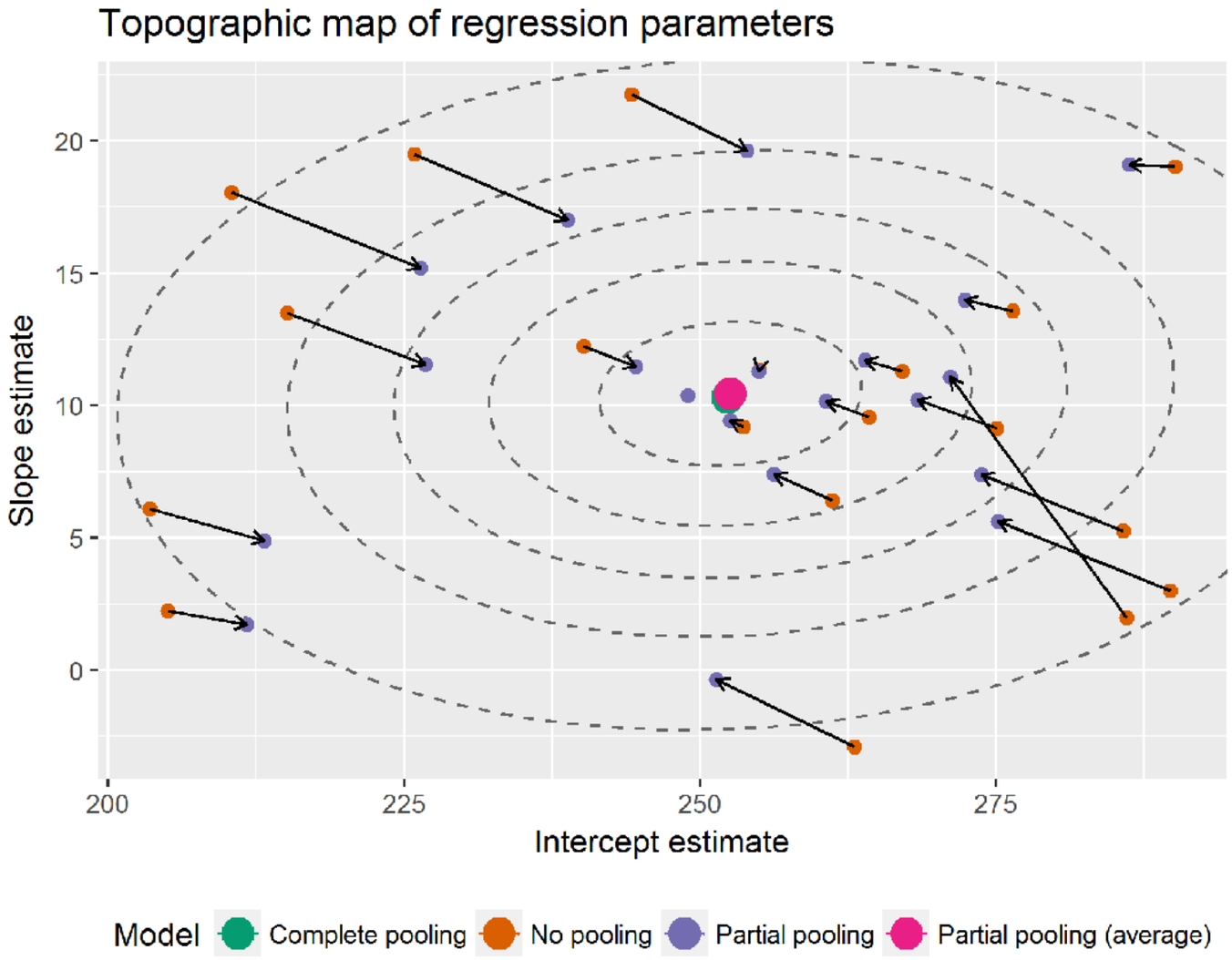
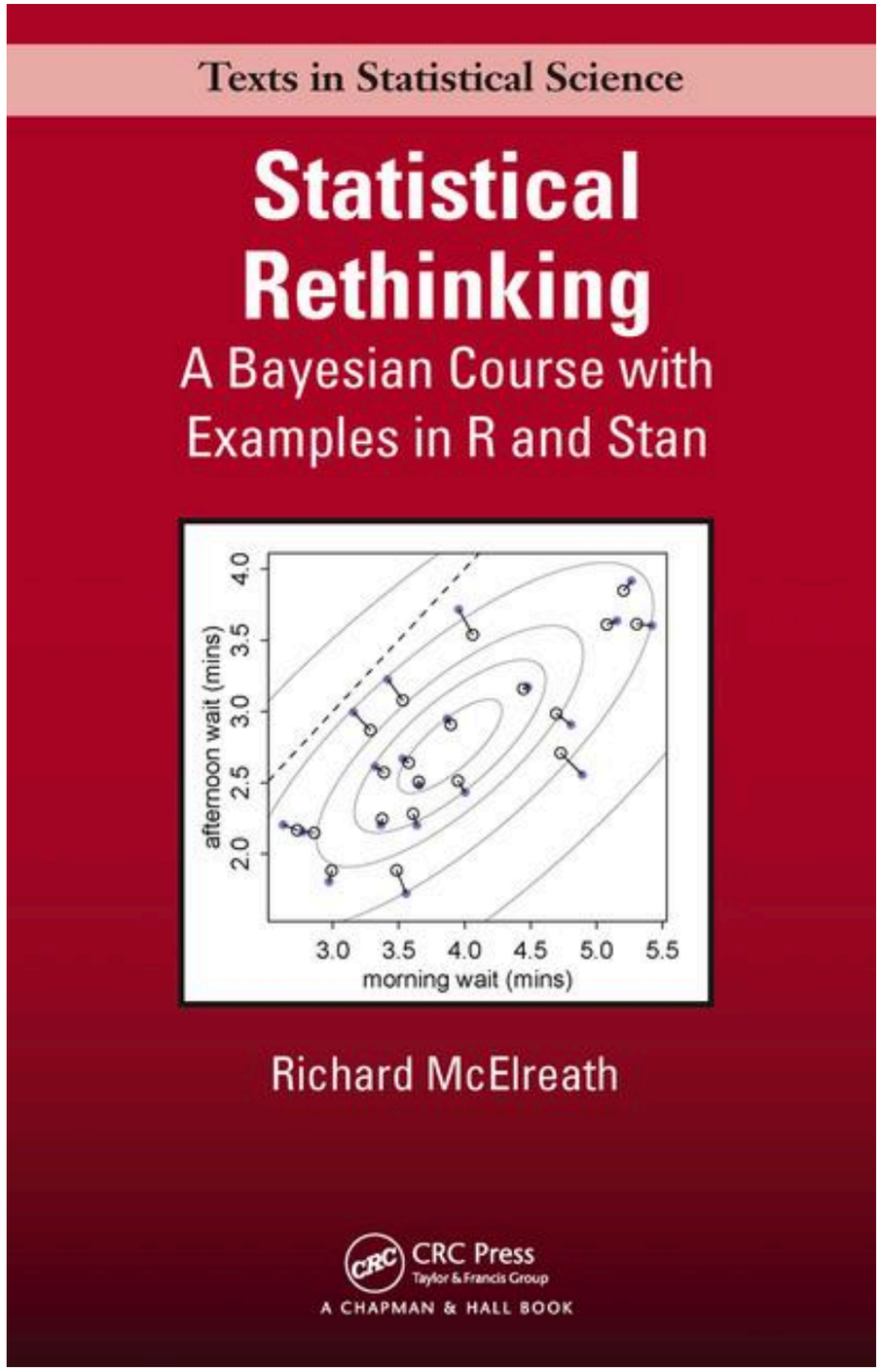
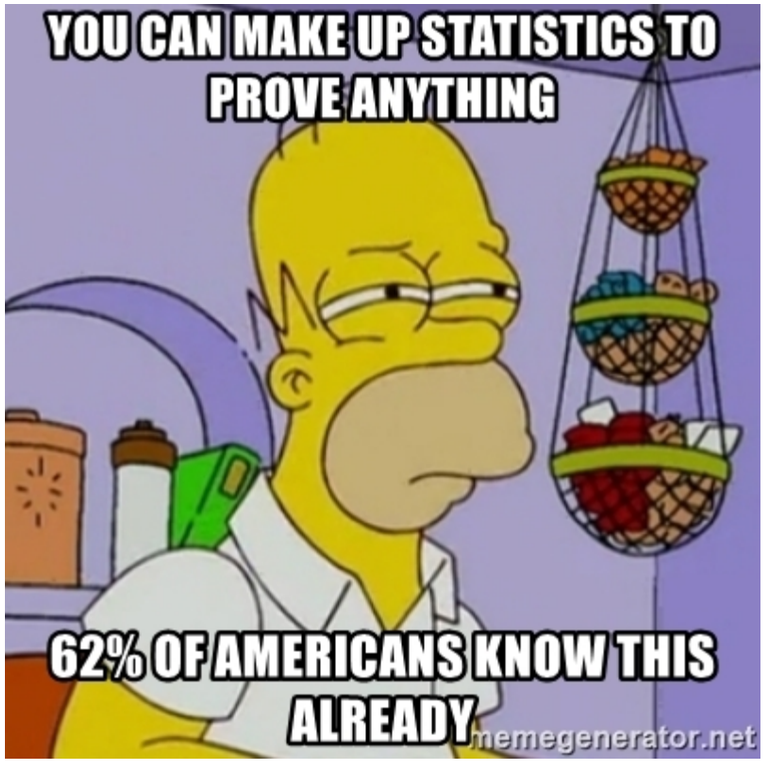
Shrinkage

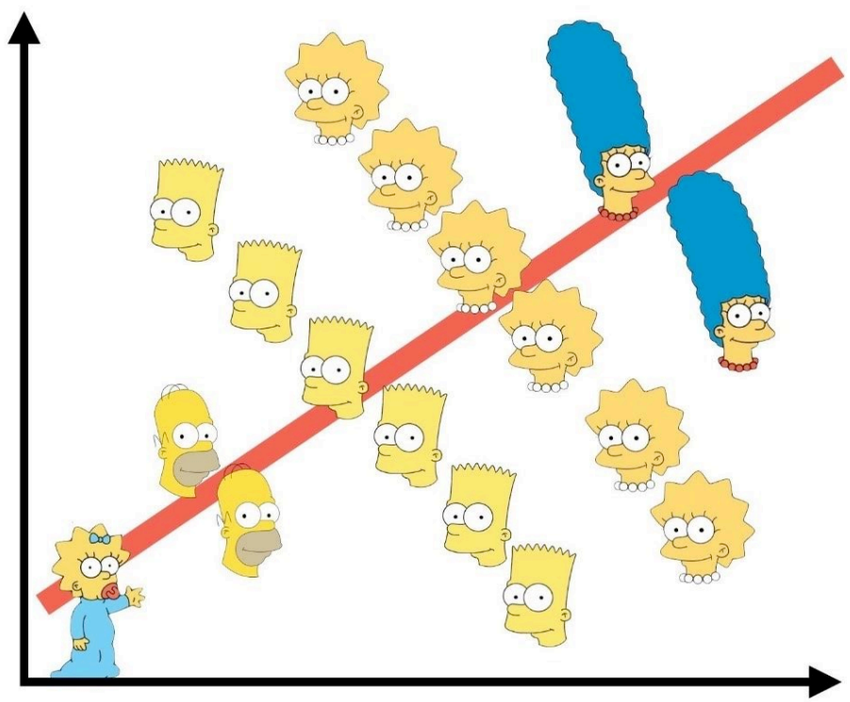


mixed effects model estimates a multi-variate Gaussian to account for (possible) correlations between intercepts and slopes

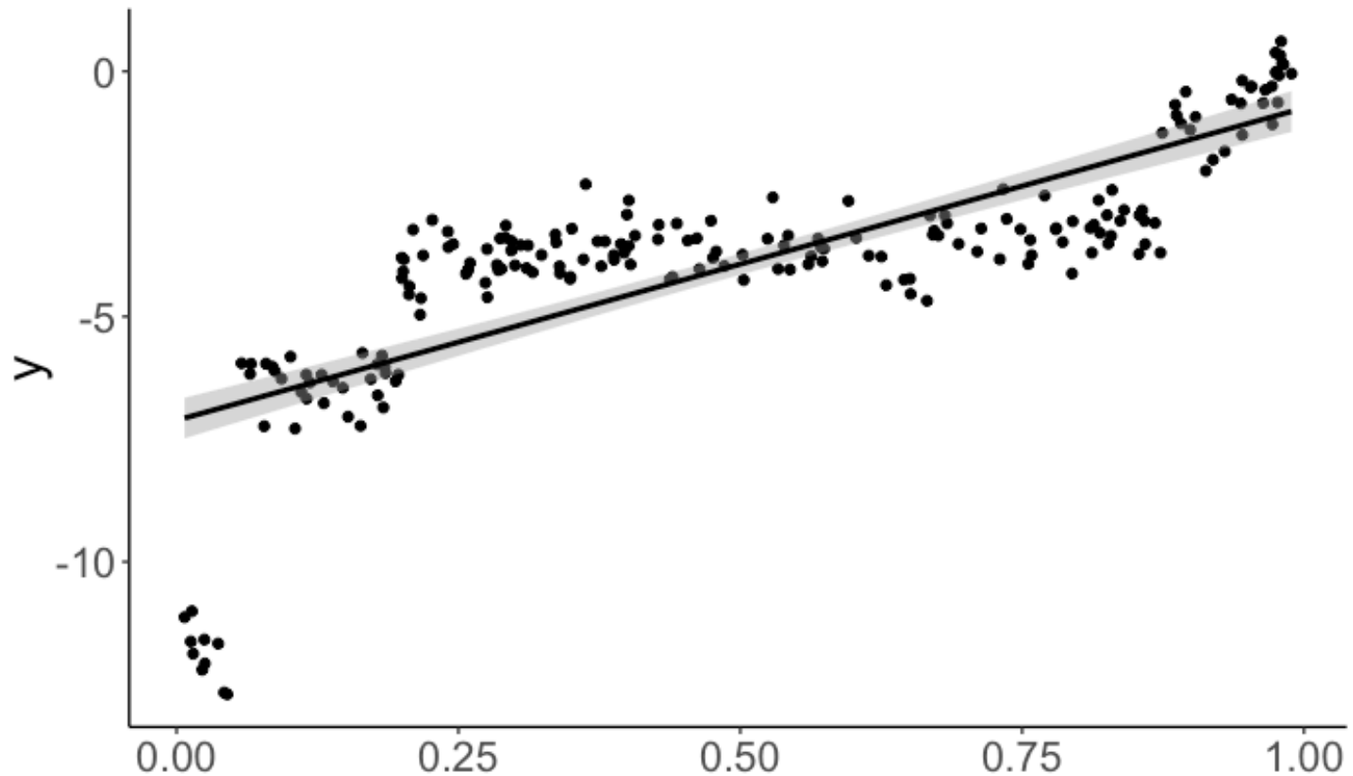




Simpsons Paradox



Simpson's paradox




```
1 lm(formula = y ~ x,
2     data = df.simpson) %>%
3   summary()
```

```
Call:
lm(formula = y ~ x, data = df.simpson)

Residuals:
    Min      1Q  Median      3Q     Max
-5.8731 -0.6362  0.2272  1.0051  2.6410

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -7.1151     0.2107  -33.76   <2e-16 ***
x             6.3671     0.3631   17.54   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.55 on 198 degrees of freedom
Multiple R-squared:  0.6083,  Adjusted R-squared:  0.6064
F-statistic: 307.5 on 1 and 198 DF,  p-value: < 2.2e-16
```

positive relationship between x and y

Simpson's paradox

```
1 lmer(formula = y ~ 1 + x + (1 | participant),
2        data = df.simpson) %>%
3    summary()
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: y ~ 1 + x + (1 | participant)
   Data: df.simpson

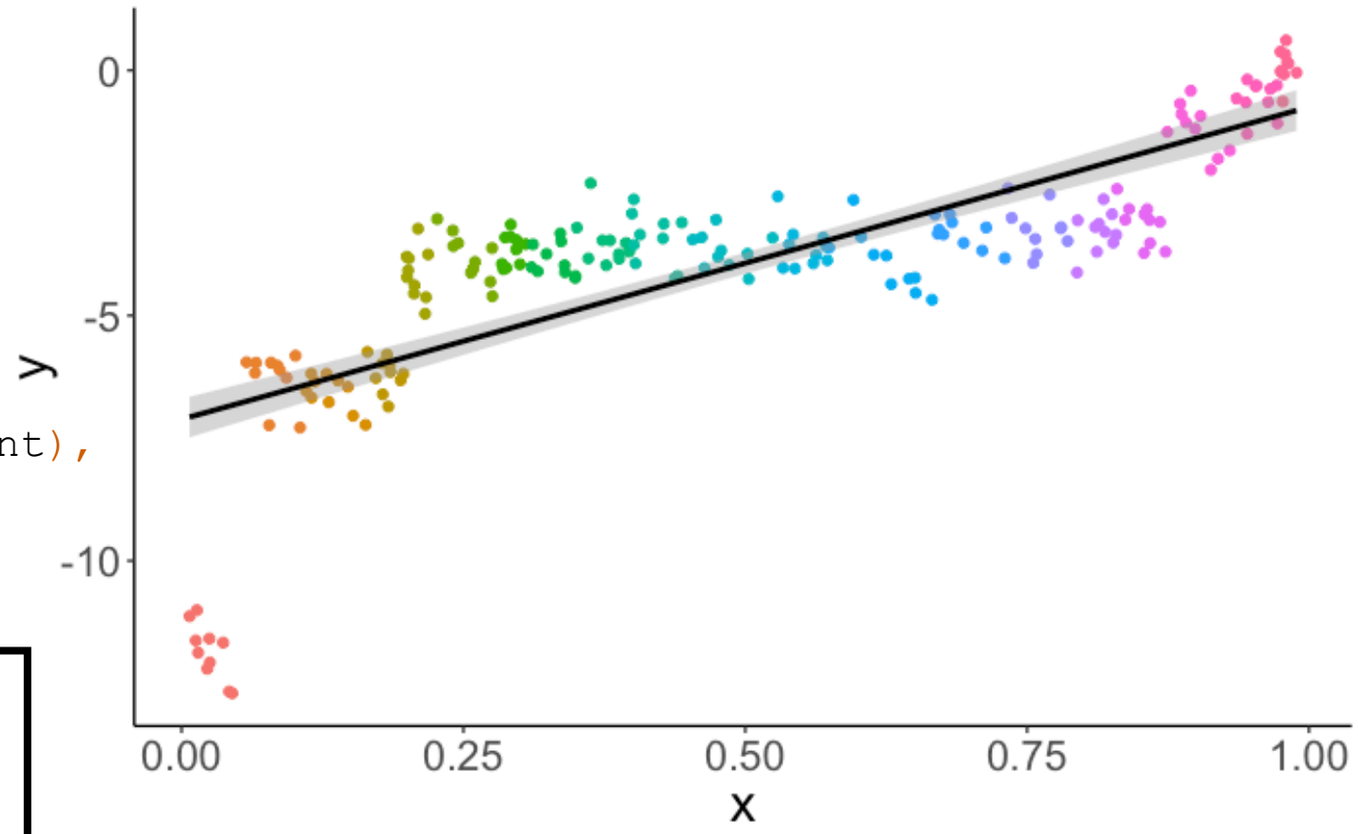
REML criterion at convergence: 345.1

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.43394 -0.59687  0.04493  0.62694  2.68828

Random effects:
 Groups      Name        Variance Std.Dev.
 participant (Intercept) 21.4898  4.6357
 Residual                 0.1661  0.4075
Number of obs: 200, groups:  participant, 20

Fixed effects:
            Estimate Std. Error t value
(Intercept)  -0.1577     1.3230  -0.119
x            -7.6678     1.6572  -4.627

Correlation of Fixed Effects:
  (Intr)
x -0.621
```



negative (!) relationship between x and y

Simpson's paradox

```
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2        data = df.simpson) %>%
3    summary()
```

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Linear mixed model fit by REML ['lmerMod']
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 Residual                 0.1661  0.4075
Number of obs: 200, groups:  participant, 20

Fixed effects:
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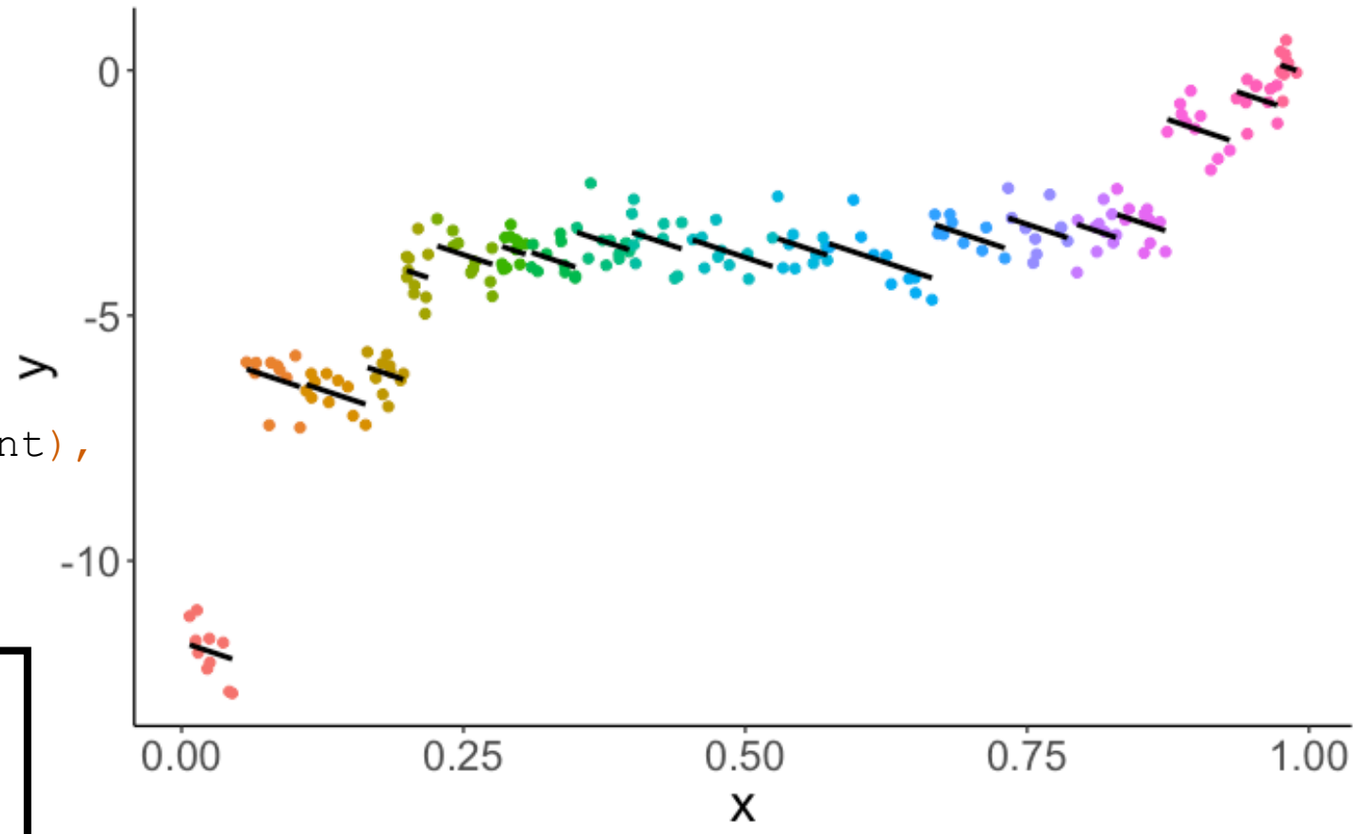
```



negative (!) relationship between x and y

(once we take into account individual differences)

Simpson's paradox

UC Berkeley gender bias (1973)

	Men		Women	
	Applicants	Admitted	Applicants	Admitted
Total	8442	**44%**	4321	35%

Department	Men		Women	
	Applicants	Admitted	Applicants	Admitted
A	***825***	62%	108	**82%**
B	***560***	63%	25	**68%**
C	325	**37%**	***593***	34%
D	417	33%	375	**35%**
E	191	**28%**	***393***	24%
F	373	6%	341	**7%**

overall men are more likely to be admitted

men not more likely to be admitted when broken down by department

In fact, the pooled and corrected data showed a "small but statistically significant bias in favor of women."

women applied to more competitive departments

Bickel, P. J., Hammel, E. A., & O'Connell, J. W. (1975). Sex Bias in Graduate Admissions: Data from Berkeley. *Science*, 187(4175), 398-404.

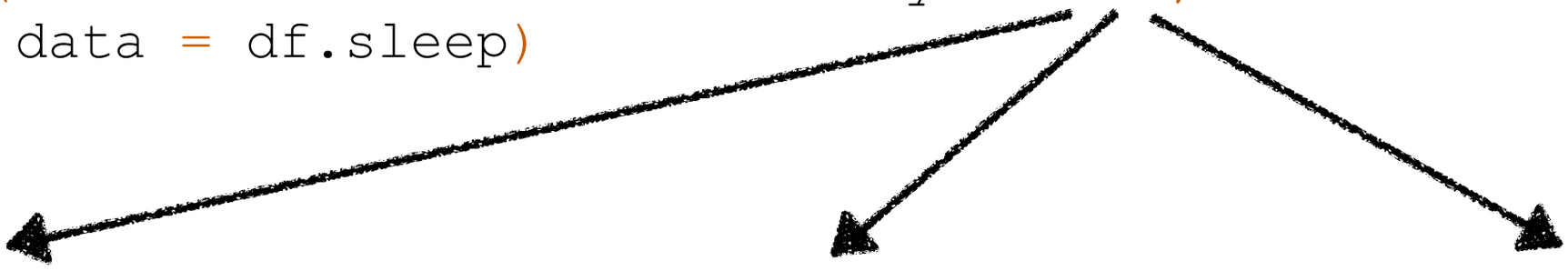
Understanding `lmer()` syntax

`lmer()` syntax summary

formula	description
`dv ~ x1 + (1 | g)`	Random **intercept** for each level of `g`
`dv ~ x1 + (0 + x1 | g)`	Random **slope** for each level of `g`
`dv ~ x1 + (x1 | g)`	**Correlated** random slope and intercept for each level of `g`
`dv ~ x1 + (x1 || g)`	**Uncorrelated** random slope and intercept for each level of `g`
`dv ~ x1 + (1 | part) + (1 | item)`	Random intercept for each level of `participant` and for each level of `item` (**crossed**)
`dv ~ x1 + (1 | school) + (1 | school:class)`	Random intercept for each level of `school` and for each level of `class` in `school` (**nested**)

Coefficients

```
lmer(formula = reaction ~ 1 + days + ...,
     data = df.sleep)
```



`(1 | subject)`

random intercepts

```
$subject
    (Intercept)     days
308    292.2749 10.43191
309    174.0559 10.43191
310    188.7454 10.43191
330    256.0247 10.43191
331    261.8141 10.43191
332    259.8262 10.43191
333    268.0765 10.43191
334    248.6471 10.43191
335    206.5096 10.43191
337    323.5643 10.43191
349    230.5114 10.43191
350    265.6957 10.43191
351    243.7988 10.43191
352    287.8850 10.43191
369    258.6454 10.43191
370    245.2931 10.43191
371    248.3508 10.43191
372    269.6861 10.43191
373    248.2086 10.43191
374    273.9400 10.43191
```

`(0 + days | subject)`

random slopes

```
$subject
    (Intercept)       days
308    252.2965 19.9526801
309    252.2965 -4.3719650
310    252.2965 -0.9574726
330    252.2965  8.9909957
331    252.2965 10.5394285
332    252.2965 11.3994289
333    252.2965 12.6074020
334    252.2965 10.3413879
335    252.2965 -0.5722073
337    252.2965 24.2246485
349    252.2965  7.7702676
350    252.2965 15.0661415
351    252.2965  7.9675415
352    252.2965 17.0002999
369    252.2965 11.6982767
370    252.2965 11.3939807
371    252.2965  9.4535879
372    252.2965 13.4569059
373    252.2965 10.4142695
374    252.2965 11.9097917
```

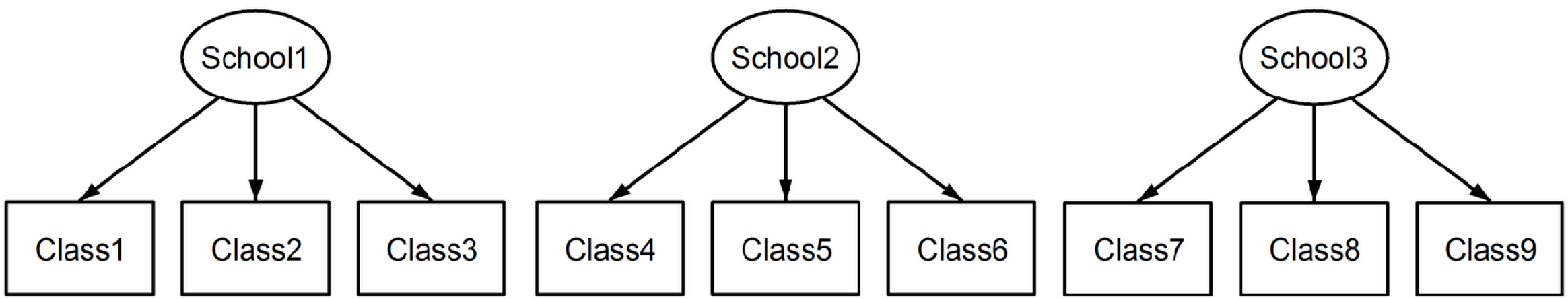
`(1 + days | subject)`

random intercepts and slopes (+ correlation)

```
$subject
    (Intercept)       days
308    253.9479 19.6264139
309    211.7328  1.7319567
310    213.1579  4.9061843
330    275.1425  5.6435987
331    273.7286  7.3862680
332    260.6504 10.1632535
333    268.3684 10.2245979
334    244.5523 11.4837825
335    251.3700 -0.3355554
337    286.2321 19.1090061
349    226.7662 11.5531963
350    238.7807 17.0156766
351    256.2344  7.4119501
352    272.3512 13.9920698
369    254.9484 11.2985741
370    226.3701 15.2027922
371    252.5051  9.4335432
372    263.8916 11.7253342
373    248.9752 10.3915245
374    271.1451 11.0782697
```

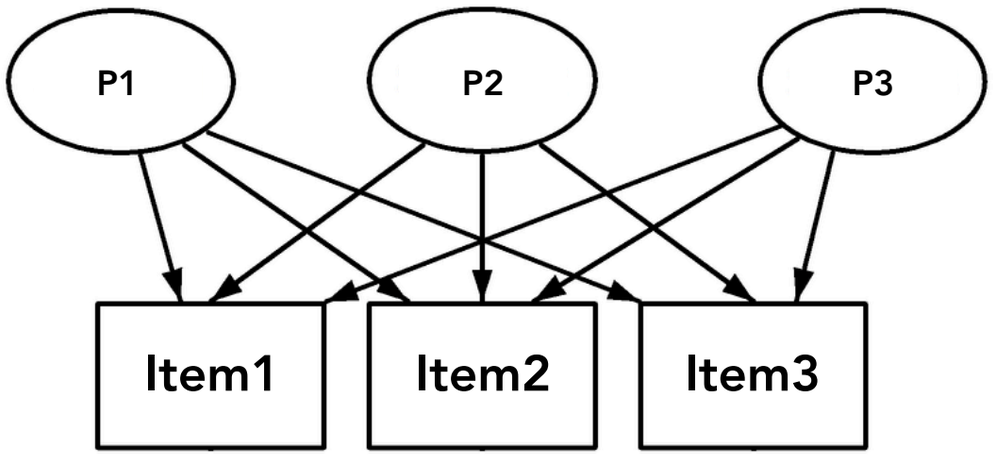
Multi-level models

nested `(1|School/Class)`



each class only appears within one school

crossed `(1|participant) + (1|item)`

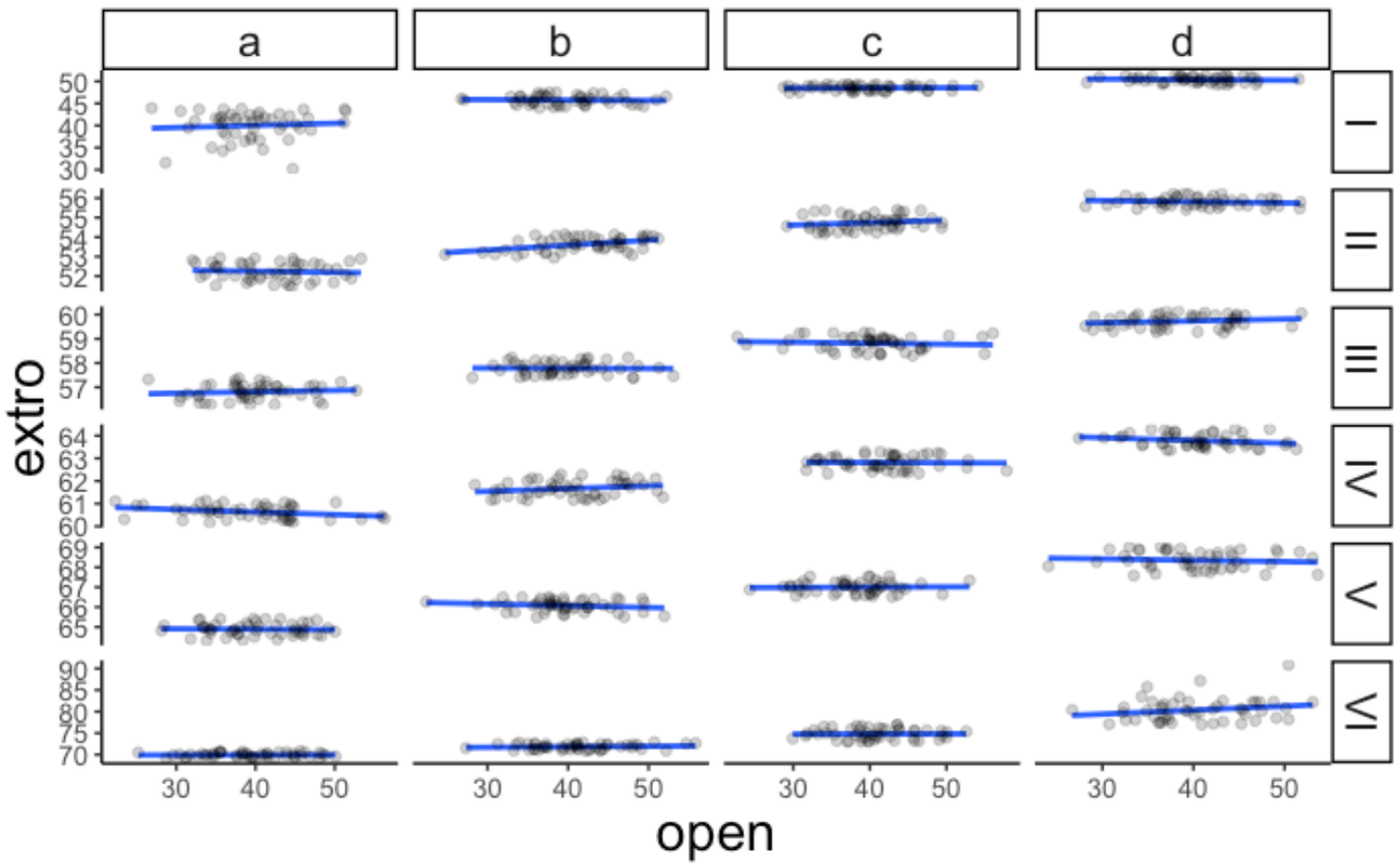


each participant rates each item

Multi-level models

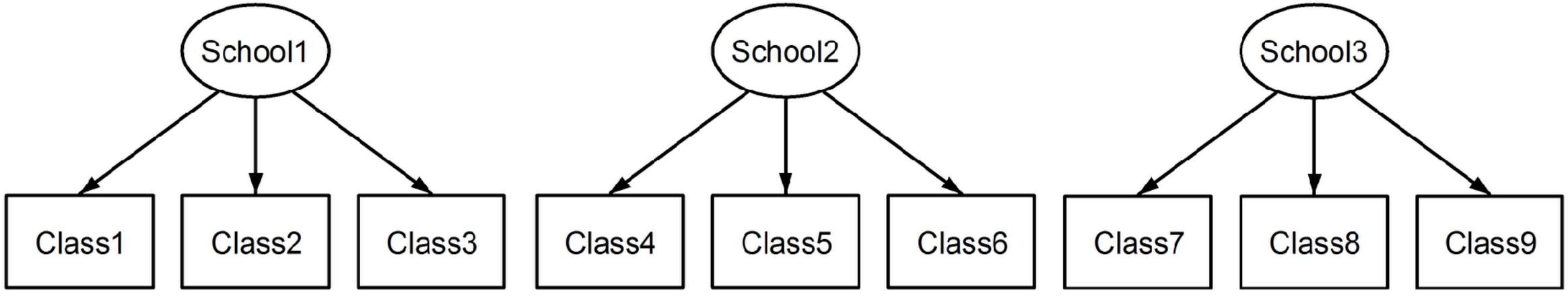
	id	extro	open	agree	social	class	school
1	1	63.69356	43.43306	38.02668	75.05811	d	IV
2	2	69.48244	46.86979	31.48957	98.12560	a	VI
3	3	79.74006	32.27013	40.20866	116.33897	d	VI
4	4	62.96674	44.40790	30.50866	90.46888	c	IV
5	5	64.24582	36.86337	37.43949	98.51873	d	IV
6	6	50.97107	46.25627	38.83196	75.21992	d	I
7	7	60.14740	37.04243	38.55959	95.91299	d	III
8	8	64.17886	42.16530	34.88235	91.45257	d	IV
9	9	56.67670	32.84933	31.68027	115.25167	a	III
10	10	47.23914	44.25764	24.99970	122.70848	b	I

relationship between openness and extraversion



Multi-level models

nested `(1|School/Class)`




```
1 # fit nested model
2 fit.nested = lmer(extro ~ open + agree + social + (1 | school/class), data = df.school)
3
4 # print model summary
5 fit.nested %>% summary()
6
7 # model coefficients
8 fit.nested %>% coef()
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: extro ~ open + agree + social + (1 | school/class)
   Data: df.school

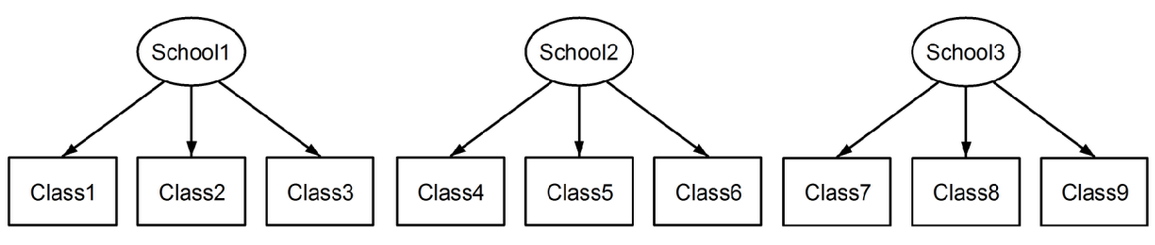
REML criterion at convergence: 3554.6

Scaled residuals:
    Min      1Q  Median      3Q     Max
-9.9949 -0.3348  0.0057  0.3394 10.6476

Random effects:
 Groups       Name        Variance Std.Dev.
 class:school (Intercept)  8.2046  2.8644
 school       (Intercept) 93.8433  9.6873
 Residual                  0.9684  0.9841
Number of obs: 1200, groups:  class:school, 24; school, 6
```

Multi-level models

nested `(1|School/Class)`



random intercepts

random intercepts of class within each school

random intercepts of schools

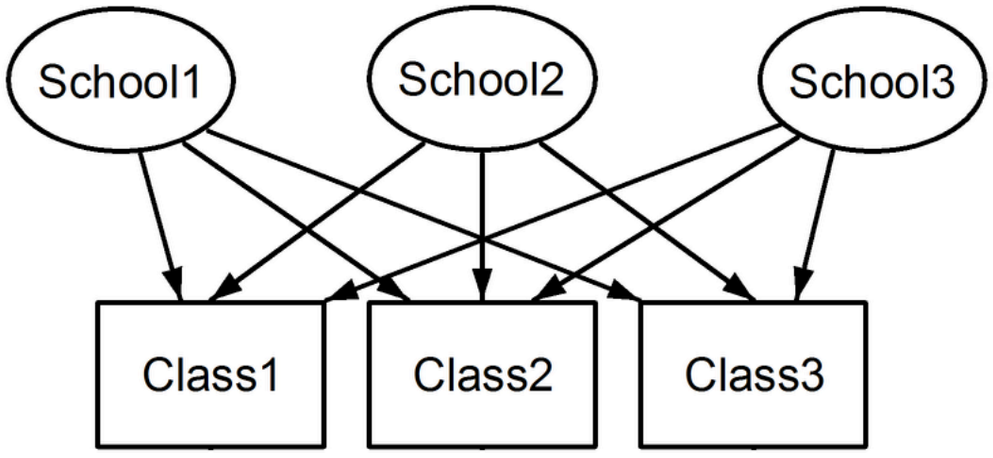
```
$`class:school`
      (Intercept)         open        agree       social
a:I      53.77106 0.006106514 -0.007665927 0.0005404069
a:II     58.23842 0.006106514 -0.007665927 0.0005404069
a:III    58.71819 0.006106514 -0.007665927 0.0005404069
a:IV     58.68813 0.006106514 -0.007665927 0.0005404069
a:V      58.68268 0.006106514 -0.007665927 0.0005404069
a:VI     56.23088 0.006106514 -0.007665927 0.0005404069
b:I      59.54852 0.006106514 -0.007665927 0.0005404069
b:II     59.62643 0.006106514 -0.007665927 0.0005404069
b:III    59.70219 0.006106514 -0.007665927 0.0005404069
b:IV     59.73276 0.006106514 -0.007665927 0.0005404069
b:V      59.87036 0.006106514 -0.007665927 0.0005404069
b:VI     58.14865 0.006106514 -0.007665927 0.0005404069
c:I      62.28460 0.006106514 -0.007665927 0.0005404069
c:II     60.74743 0.006106514 -0.007665927 0.0005404069
c:III    60.70970 0.006106514 -0.007665927 0.0005404069
c:IV     60.86062 0.006106514 -0.007665927 0.0005404069
c:V      60.80225 0.006106514 -0.007665927 0.0005404069
c:VI     61.10164 0.006106514 -0.007665927 0.0005404069
d:I      64.14113 0.006106514 -0.007665927 0.0005404069
d:II     61.81189 0.006106514 -0.007665927 0.0005404069
d:III    61.65165 0.006106514 -0.007665927 0.0005404069
d:IV     61.83703 0.006106514 -0.007665927 0.0005404069
d:V      62.13593 0.006106514 -0.007665927 0.0005404069
d:VI     66.66561 0.006106514 -0.007665927 0.0005404069

$school
    (Intercept)         open        agree       social
I      46.44407 0.006106514 -0.007665927 0.0005404069
II     54.20862 0.006106514 -0.007665927 0.0005404069
III    58.29847 0.006106514 -0.007665927 0.0005404069
IV     62.15074 0.006106514 -0.007665927 0.0005404069
V      66.41348 0.006106514 -0.007665927 0.0005404069
VI     73.91156 0.006106514 -0.007665927 0.0005404069
```

```
# model coefficients
fit.nested %>% coef()
```

Multi-level models

crossed `(1|School) + (1|Class)`



each class appears in each of the schools

```
1 # fit crossed model
2 fit.crossed = lmer(extro ~ open + agree + social + (1 | school) + (1 | class), data = df.school)
3
4 # print model summary
5 fit.crossed %>% summary()
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: extro ~ open + agree + social + (1 | school) + (1 | class)
   Data: df.school

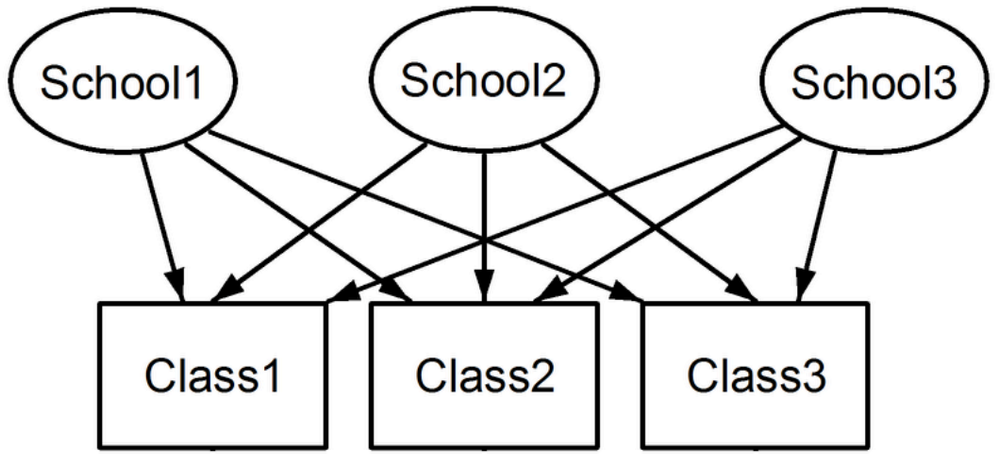
REML criterion at convergence: 4723.9

Scaled residuals:
    Min      1Q  Median      3Q     Max
-7.8677 -0.5421  0.0101  0.5218  8.2282

Random effects:
 Groups   Name        Variance Std.Dev.
 school   (Intercept) 95.914   9.794
 class    (Intercept)  5.787   2.406
 Residual              2.787   1.669
Number of obs: 1200, groups:  school, 6; class, 4
```

Multi-level models

crossed `(1|School) + (1|Class)`



each class is in
each of the schools

random intercepts

random intercepts of school →

random intercepts of class →

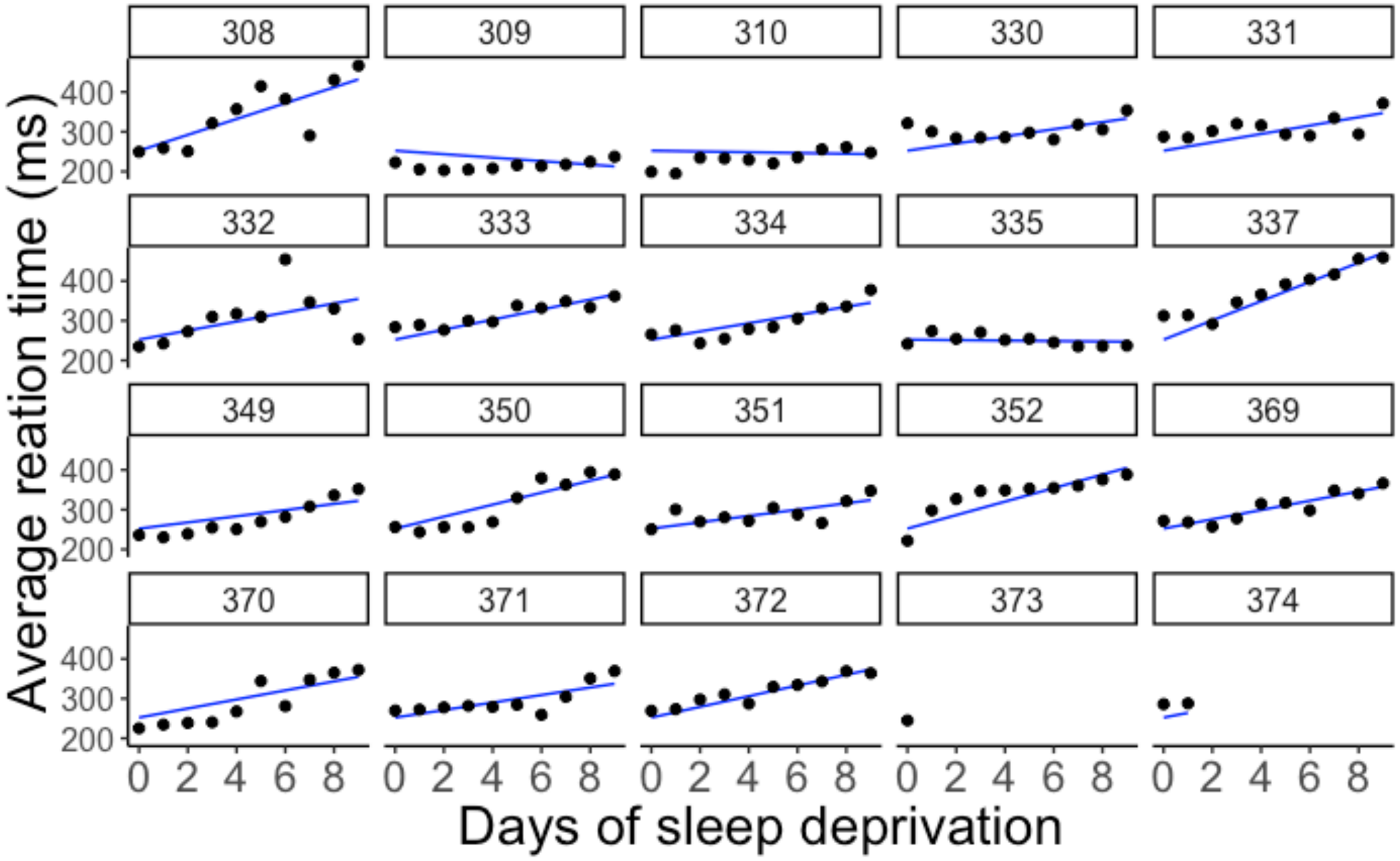
```
$school
     (Intercept)        open         agree        social
I       46.10663 0.01083374 -0.005420032 -0.001761963
II      54.02956 0.01083374 -0.005420032 -0.001761963
III     58.22277 0.01083374 -0.005420032 -0.001761963
IV      62.15508 0.01083374 -0.005420032 -0.001761963
V       66.51062 0.01083374 -0.005420032 -0.001761963
VI      74.16838 0.01083374 -0.005420032 -0.001761963

$class
   (Intercept)        open         agree        social
a     57.35175 0.01083374 -0.005420032 -0.001761963
b     59.39261 0.01083374 -0.005420032 -0.001761963
c     61.04758 0.01083374 -0.005420032 -0.001761963
d     63.00342 0.01083374 -0.005420032 -0.001761963
```

```
# model coefficients
fit.crossed %>% coef()
```

Reporting results

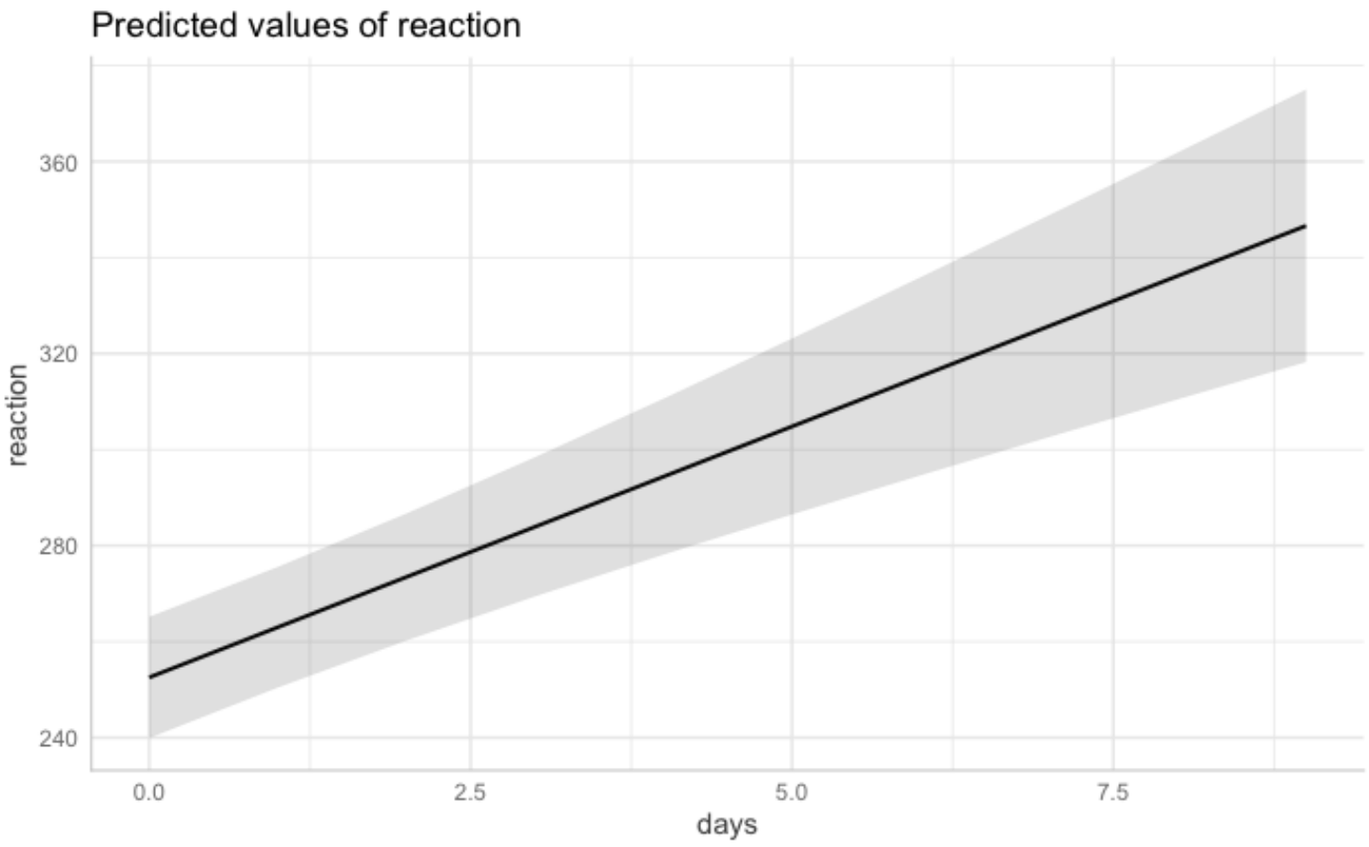
Visualization



show the data together with the model predictions

Visualization


```
1 library("ggeffects")
2
3 ggpredict(model = fit.random_intercept_slope,
4            terms = "days",
5            type = "fe") %>%
6   plot()
```



- the relationship between the variables of interest (marginalizing over other variables)

show the (marginalized) model prediction

Reporting results

7.1. In Writing

Our reports include a description of the following parts (also see Meteyard & Davies, 2019; Barr et al., 2013):

- Model specification, including:
  - Dependent variable, and all fixed and random effects (intercepts, slopes, correlations), both in words and possibly also by providing the model equation/ R-pseudo code (so-called Wilkinson notation)
  - Transformation of variables, e.g., standardizing or centering variables
  - Contrast coding (typically sum-to-zero coding)
- Inference:
  - Description of how *p*-values were obtained (in case of a frequentist approach) or what other (Bayesian) decision rule was used for inference.
  - Description of what post-hoc or follow-up tests were performed
  - Any convergence issues that may arise while running the model (in particular if they require adjustments in the model specification) and how they were dealt with should be described, as well as the subsequent adjustments that were made.
- Model output, at minimum the following:
  - Model results: (un)standardized regression coefficients, standard errors and/or confidence / credible intervals, test statistics, degrees of freedom, *p*-values

Meteyard, L. & Davies, R. A. I. (2020). Best practice guidance for linear mixed-effects models in psychological science. *Journal of Memory and Language*.

Reporting results

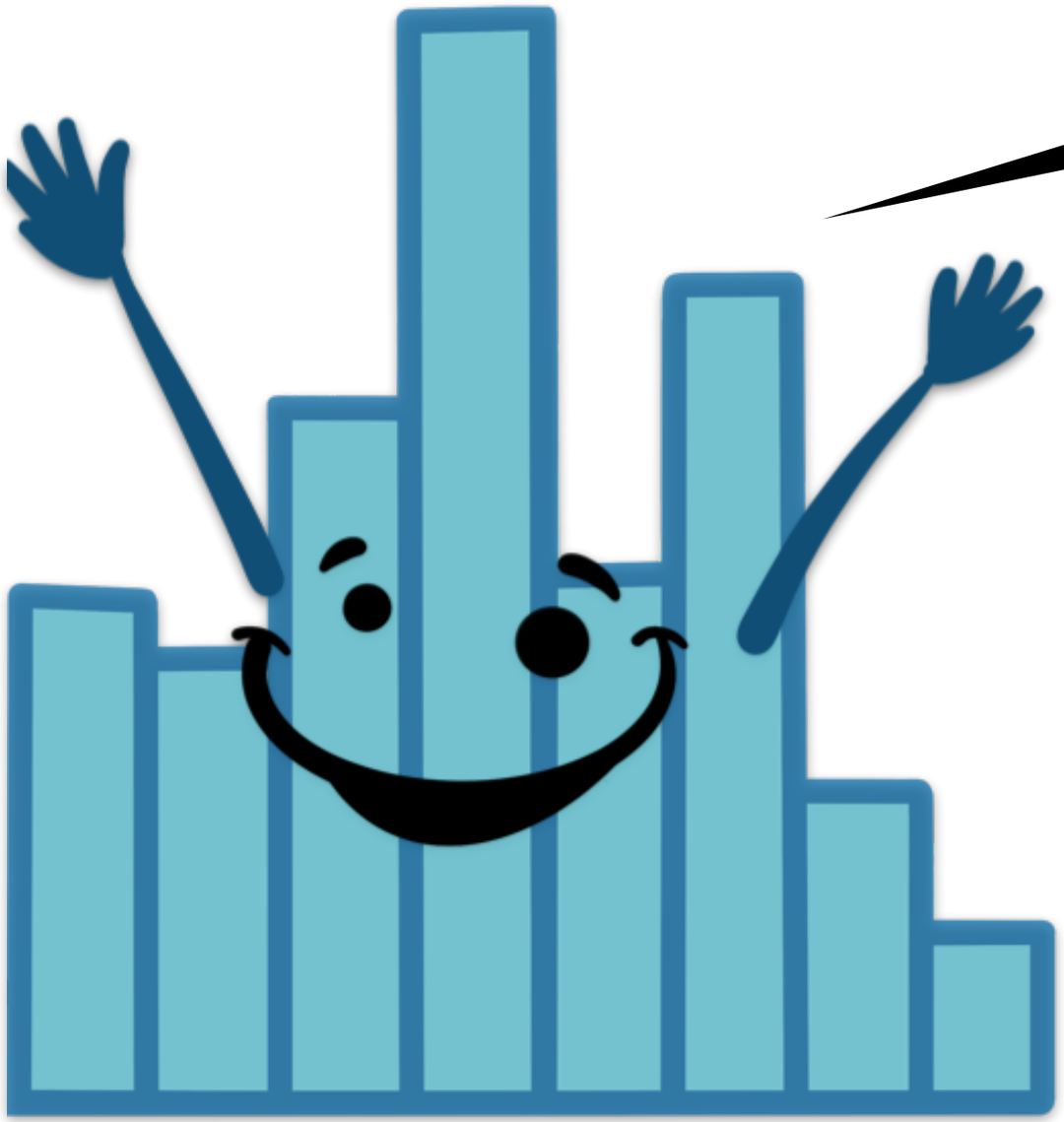
Table 2

***Experiment 1 – Normality inference**: Estimates of the posterior mean and 95% highest density intervals (HDIs) for the different predictors in the Bayesian regression model. Note: For the dependent variable (normality rating), 100 = abnormal and 0 = normal.*

model specification: `normality rating ~ 1 + structure * norm`

term	estimate	lower 95% CI	upper 95% CI
intercept	62.83	57.57	68.11
structure	21.22	16.27	26.35
norm	−1.47	−6.37	3.83
structure:norm	−1.46	−6.41	3.53

[7] All categorical predictors were coded using sum contrasts. We adopt the convention of calling something an effect if the 95% highest density interval (HDI) of the estimated parameter in the Bayesian model excludes 0.



02:00

stretch break!

Let's simulate some `lmer()`s

Let's simulate an `lmer()`

```r
# make example reproducible
set.seed(1)

# parameters
sample_size = 100
b0 = 1
b1 = 2
sd_residual = 1
sd_participant = 0.5

# generate the data
df.mixed = tibble(participant = rep(1:sample_size, 2),
                  condition = rep(0:1, each = sample_size)) %>%
  group_by(participant) %>%
  mutate(intercepts = rnorm(n = 1, sd = sd_participant)) %>%
  ungroup() %>%
  mutate(value = b0 + b1 * condition + intercepts + rnorm(n(), sd = sd_residual)) %>%
  arrange(participant, condition)
```

participant	condition	intercepts	value
1	0	-0.31	0.07
1	1	-0.31	3.10
2	0	0.09	1.13
2	1	0.09	4.78
3	0	-0.42	-0.33
3	1	-0.42	4.17
4	0	0.80	1.96
4	1	0.80	3.47
5	0	0.16	0.51
5	1	0.16	0.88

$$\text{value}_{ij} = b_0 + b_1 \cdot \text{condition}_{ij} + U_i + e_{ij}$$

$$e_{ij} \sim \mathcal{N}(\text{mean} = 0, \text{sd} = s_{\text{error}})$$

$$U_i \sim \mathcal{N}(\text{mean} = 0, \text{sd} = s_{\bigcup})$$

simulating data from a model and trying to recover the parameters is a great way to check one's understanding of what the model does

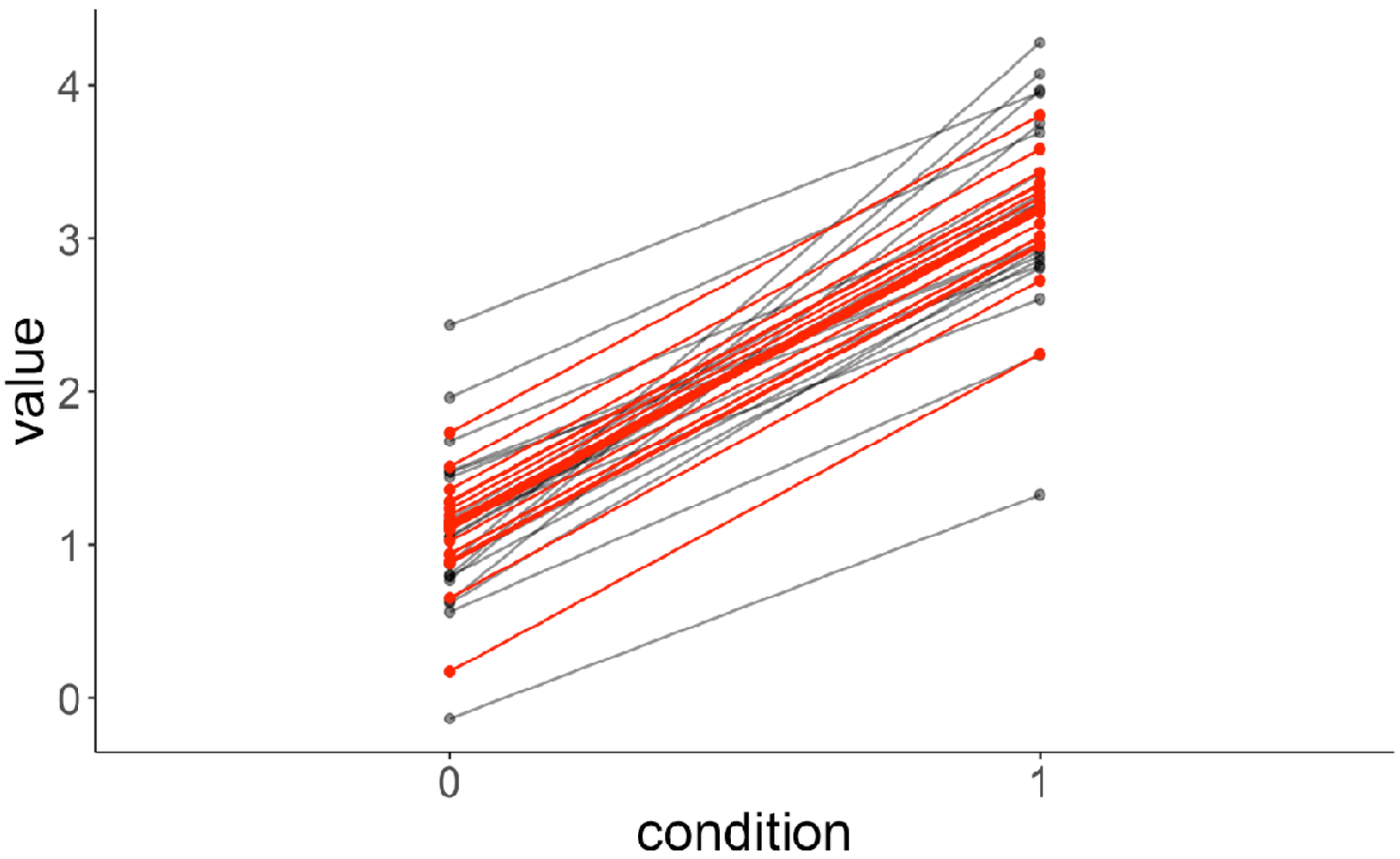
Let's simulate an **lmer**()

```r
# make example reproducible
set.seed(1)

# parameters
sample_size = 100
b0 = 1
b1 = 2
sd_residual = 1
sd_participant = 0.5

# generate the data
df.mixed = tibble(participant = rep(1:sample_size, 2),
                  condition = rep(0:1, each = sample_size)) %>%
  group_by(participant) %>%
  mutate(intercepts = rnorm(n = 1, sd = sd_participant)) %>%
  ungroup() %>%
  mutate(value = b0 + b1 * condition + intercepts + rnorm(n(), sd = sd_residual)) %>%
  arrange(participant, condition)
```

```r
# fit model
fit.test = lmer(formula = value ~ 1 + condition + (1 | participant),
                data = df.mixed)

```

```
Linear mixed model fit by REML ['lmerMod']
Formula: value ~ 1 + condition + (1 | participant)
   Data: df.mixed

REML criterion at convergence: 606

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.53710 -0.62295 -0.04364  0.67035  2.19899

Random effects:
 Groups      Name        Variance Std.Dev.
 participant (Intercept) 0.1607   0.4009
 Residual                1.0427   1.0211
Number of obs: 200, groups:  participant, 100

Fixed effects:
            Estimate Std. Error t value
(Intercept)   1.0166     0.1097   9.267
condition     2.0675     0.1444  14.317

Correlation of Fixed Effects:
          (Intr)
condition -0.658
```

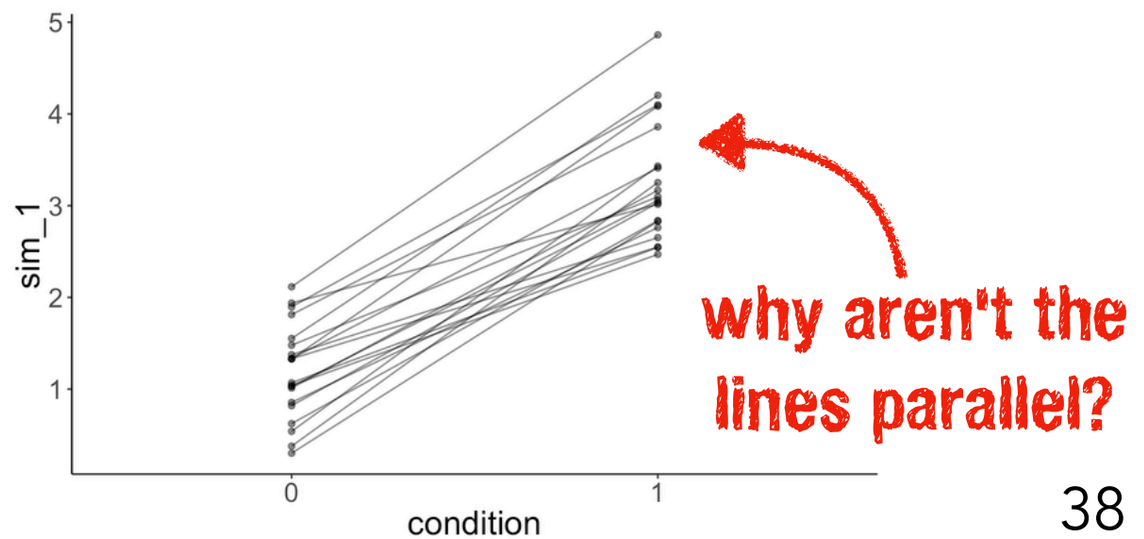
No outlier




```
Linear mixed model fit by REML ['lmerMod']
Formula: value ~ 1 + condition + (1 | participant)
   Data: df.test

REML criterion at convergence: 74.9

Scaled residuals:
    Min      1Q  Median      3Q     Max
-1.9268 -0.5412 -0.1103  0.4868  1.7747

Random effects:
 Groups      Name        Variance Std.Dev.
 participant (Intercept) 0.1702   0.4125
 Residual                0.2270   0.4764
Number of obs: 40, groups:  participant, 20

Fixed effects:
            Estimate Std. Error t value
(Intercept)   1.0920     0.1409    7.75
condition1    2.0726     0.1507   13.76

Correlation of Fixed Effects:
           (Intr)
condition1 -0.535
```

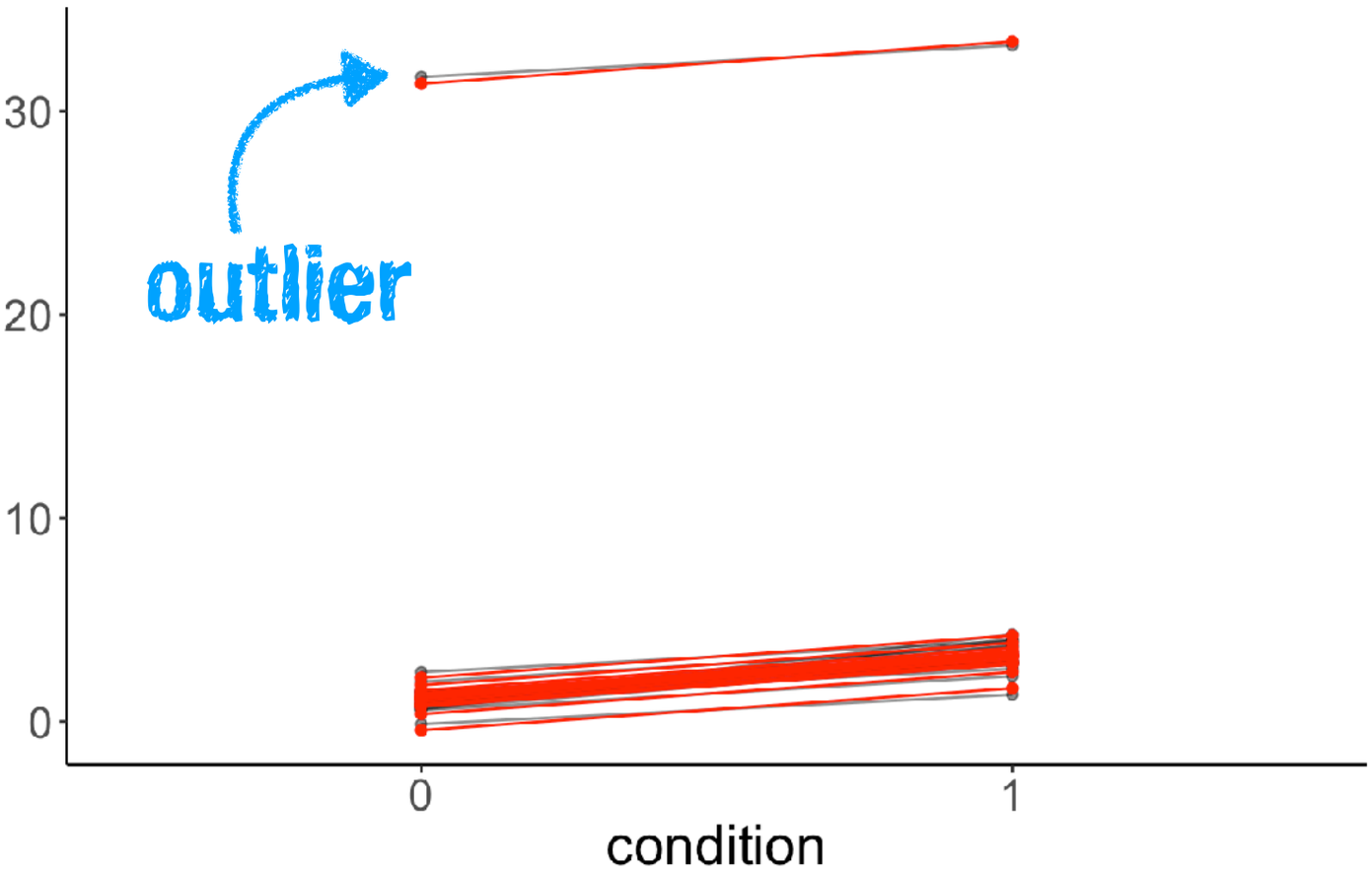
```
1 # fit model
2 fit.test = lmer(formula = value ~ 1 + condition + (1 | participant),
3                 data = df.test)
4
5 # simulate data
6 fit.test %>%
7   simulate()
```

simulated data



why aren't the lines parallel?

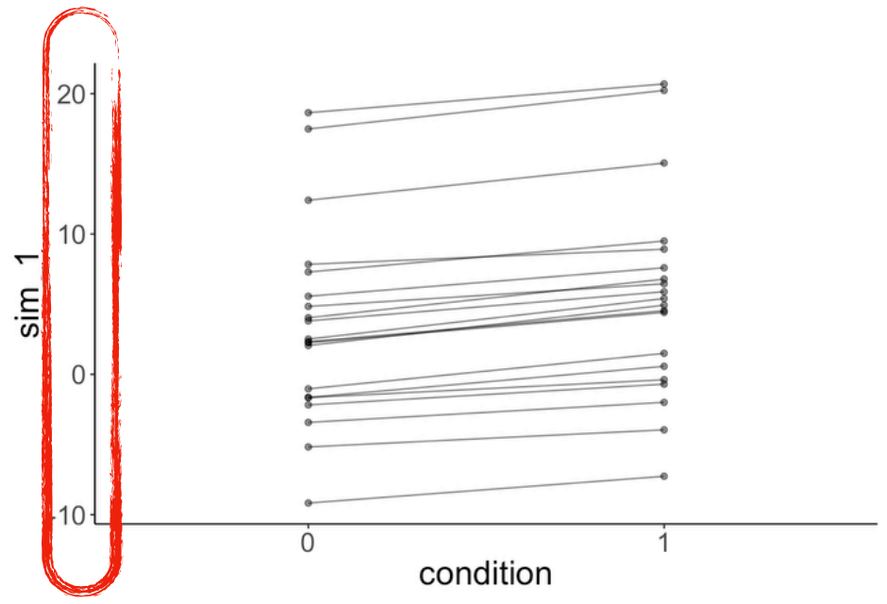
With outlier



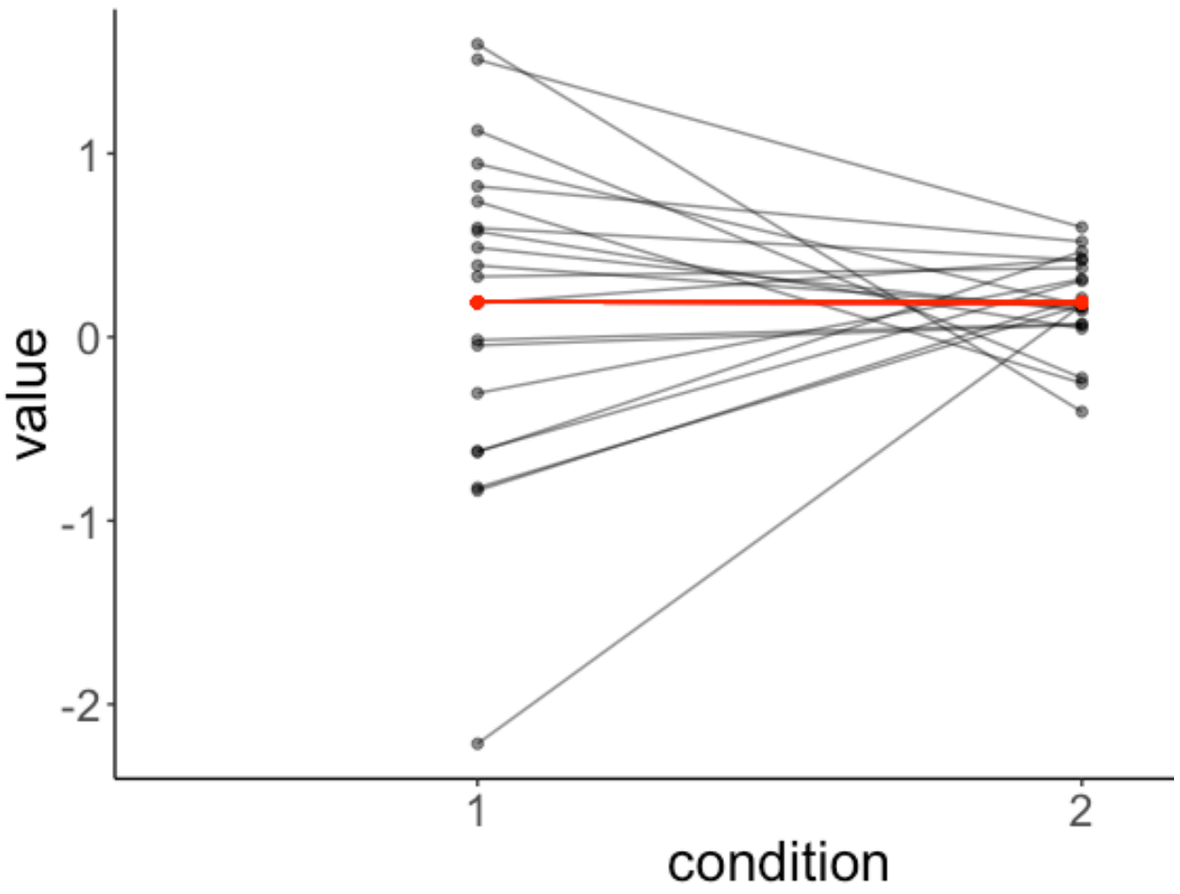

```
Linear mixed model fit by REML ['lmerMod']
Formula: value ~ 1 + condition + (1 | participant)
   Data: df.test

REML criterion at convergence: 171.7

Scaled residuals:
    Min      1Q  Median      3Q     Max
-1.4038 -0.4678 -0.0094  0.5800  1.3930

Random effects:
 Groups      Name        Variance Std.Dev.
 participant (Intercept) 46.198   6.7969
 Residual                 0.227   0.4764
Number of obs: 40, groups:  participant, 20

Fixed effects:
            Estimate Std. Error t value
(Intercept)   2.5920     1.5236   1.701
condition1    2.0726     0.1507  13.758

Correlation of Fixed Effects:
           (Intr)
condition1 -0.049
```

```
1 # fit model
2 fit.test = lmer(formula = value ~ 1 + condition + (1 | participant),
3                 data = df.test)
4
5 # simulate data
6 fit.test %>%
7   simulate()
```

simulated data



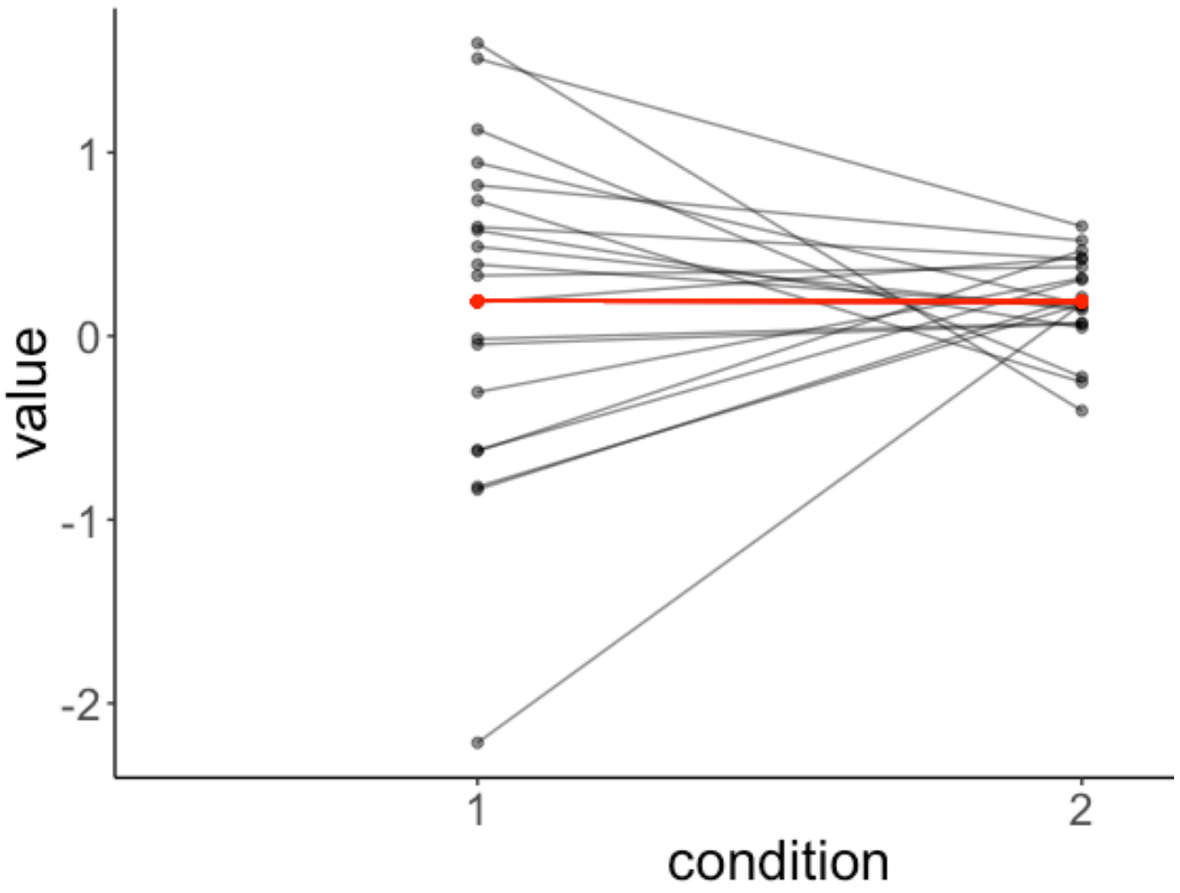
Non-equal variance




```
singular fit
Linear mixed model fit by REML ['lmerMod']
Formula: value ~ 1 + condition + (1 | participant)
   Data: df.test

REML criterion at convergence: 83.6

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.5808 -0.3184  0.0130  0.4551  2.0913

Random effects:
 Groups      Name        Variance Std.Dev.
 participant (Intercept) 0.0000   0.0000
 Residual                0.4512   0.6717
Number of obs: 40, groups:  participant, 20

Fixed effects:
            Estimate Std. Error t value
(Intercept)  0.190524   0.150197   1.268
condition2  -0.001941   0.212411  -0.009

Correlation of Fixed Effects:
           (Intr)
condition2 -0.707
convergence code: 0
singular fit
```

clearly there are interindividual differences though?!

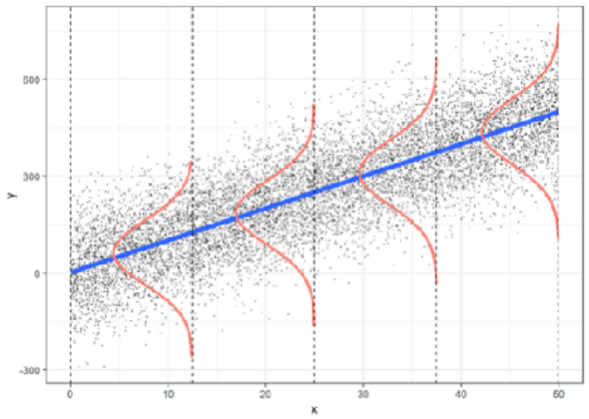
Non-equal variance



Model assumptions of simple regression

- independent observations
- *Y* is continuous
- errors are normally distributed
- errors have constant variance
- error terms are uncorrelated

assumption violated



the "model" would just reproduce the data

random intercept

random slope

```
1 # fit model
2 lmer(formula = value ~ 1 + condition + (1 + condition | participant),
3        data = df.test)
```

won't work

```
Error: number of observations (=40) <= number of random effects (=40) for term
(1 + condition | participant); the random-effects parameters and the residual
variance (or scale parameter) are probably unidentifiable
```

`lmer()` standard operating procedures

Standard Operating Procedures For Using Mixed-Effects Models

A Principled Workflow from the Decision, Development, and Psychopathology (D2P2) Lab
document version 1.0.0 -- 28 June 2020

[This document will be continuously updated and expanded; it may contain typos and other errors--both unintentional errors and errors based on incorrect or outdated knowledge--we will try to improve these things in future versions. Feel free to let us know if you spotted such things, how to further improve this document!]

Authors (in alphabetical order except that the youngsters were so kind to put the oldest guy in the lab first; BF)

Bernd Figner, Johannes Algermissen, Floor Burghoorn, Leslie Held, Afreen Khalid, Felix Klaassen, Farnaz Mosannenzadeh, Julian Quandt

Content/Analysis Steps

http://decision-lab.org/wp-content/uploads/2020/07/SOP_Mixed_Models_D2P2_v1_0_0.pdf

What shall I include as random effects?

- mixed opinions on the topic
- go maximal!

Journal of Memory and Language 68 (2013) 255–278

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Random effects structure for confirmatory hypothesis testing: Keep it maximal

Dale J. Barr [a,*], Roger Levy [b], Christoph Scheepers [a], Harry J. Tily [c]

[a] Institute of Neuroscience and Psychology, University of Glasgow, 58 Hillhead St., Glasgow G12 8QB, United Kingdom
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"Through theoretical arguments and Monte Carlo simulation, we show that LMEMs generalize best when they include the maximal random effects structure justified by the design. ...

Maximal LMEMs should be the 'gold standard' for confirmatory hypothesis testing in psycholinguistics and beyond."

Barr, D. J., Levy, R., Scheepers, C., & Tily, H. J. (2013). Random effects structure for confirmatory hypothesis testing: Keep it maximal. *Journal of Memory and Language*, 68(3), 255--278.

What shall I include as random effects?

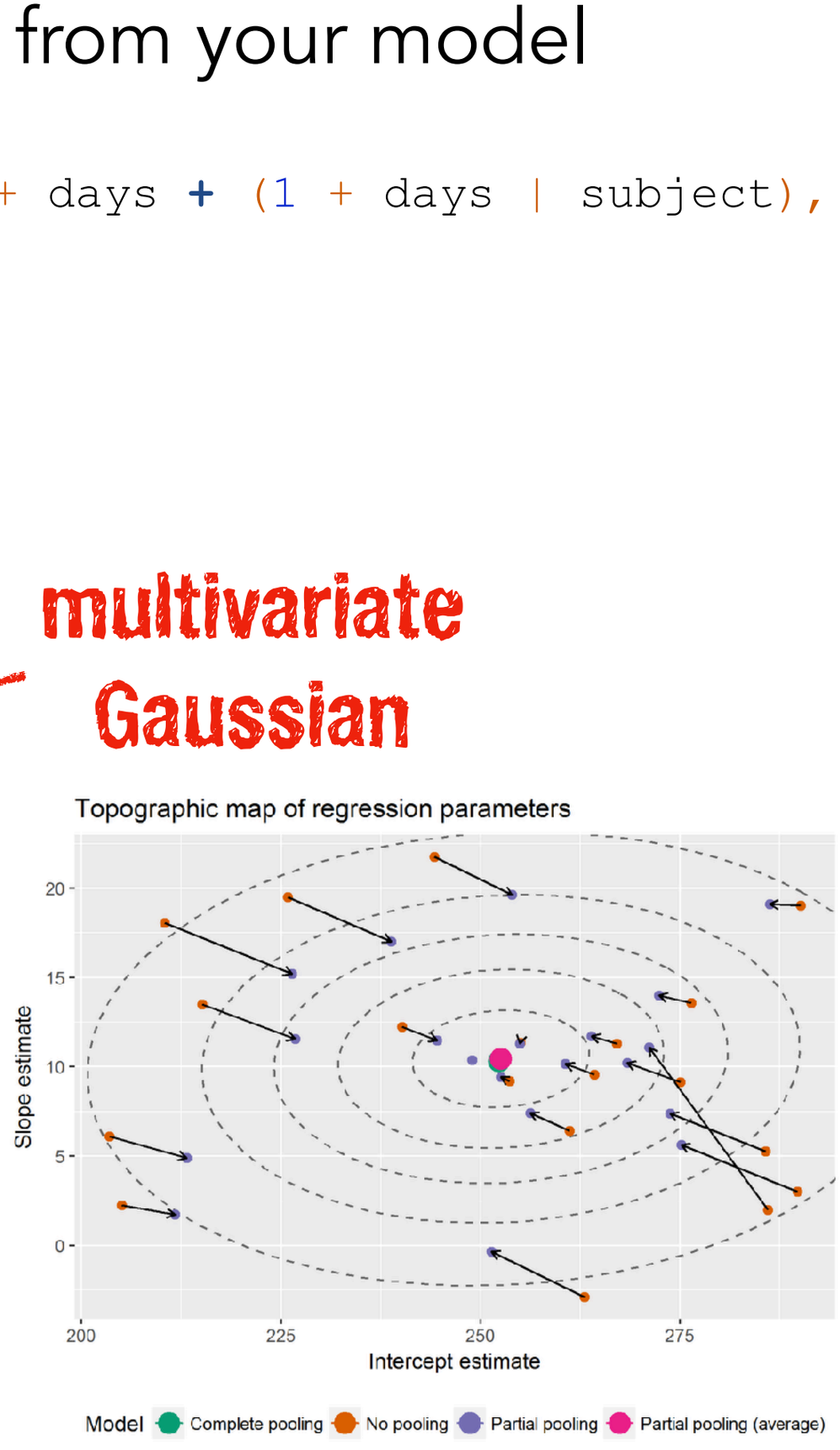
- general advice:
  - start maximal (as supported by the design)
    - random intercepts for different participants
    - random slopes when participants are tested multiple times
    - random intercepts for items
  - reduce complexity of the random effects structure step by step
    - remove the correlation between random effects first

Remove the correlation component from your model

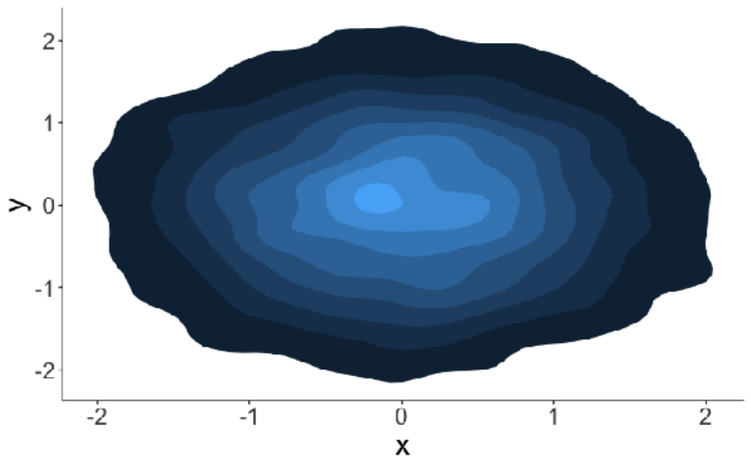
```
# fit the model
fit.lmer = lmer(formula = reaction ~ 1 + days + (1 + days | subject),
                data = df.sleep)
# model summary
fit.lmer %>%
  summary()
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: reaction ~ 1 + days + (1 + days | subject)
   Data: df.sleep

REML criterion at convergence: 1771.4

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.9707 -0.4703  0.0276  0.4594  5.2009

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 subject  (Intercept) 582.73   24.140
          days         35.03    5.919   0.07
 Residual             649.36   25.483
Number of obs: 183, groups:  subject, 20

Fixed effects:
            Estimate Std. Error t value
(Intercept)  252.543      6.433  39.256
days          10.452      1.542   6.778

Correlation of Fixed Effects:
     (Intr)
days -0.137
```

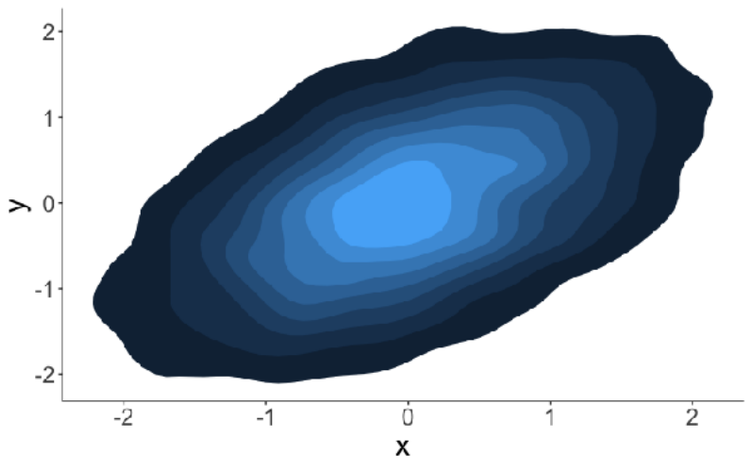
multivariate Gaussian



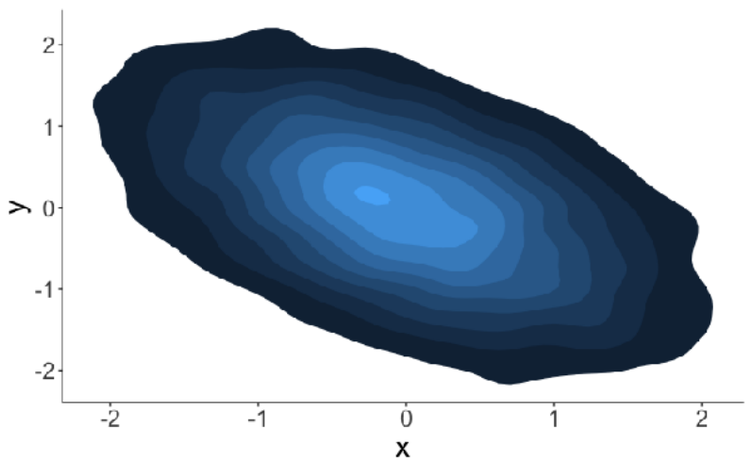
Remove the correlation component from your model



uncorrelated



positively correlated



negatively correlated

Remove the correlation component from your model

```
1 # fit the model
2 fit.lmer = lmer(formula = reaction ~ 1 + days + (0 + days | subject) + (1 | subject),
3                 data = df.sleep)
4 # model summary
5 fit.lmer %>%
6   summary()
```

random slopes

random intercepts

```
Linear mixed model fit by REML ['lmerMod']
Formula: reaction ~ 1 + days + (0 + days | subject) + (1 | subject)
   Data: df.sleep

REML criterion at convergence: 1771.5

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.9805 -0.4673  0.0250  0.4589  5.2083

Random effects:
 Groups    Name        Variance Std.Dev.
 subject   days         35.88    5.99
 subject.1 (Intercept) 598.11   24.46
 Residual               647.90   25.45
Number of obs: 183, groups:  subject, 20

Fixed effects:
            Estimate Std. Error t value
(Intercept)  252.550      6.491  38.907
days          10.439      1.556   6.708

Correlation of Fixed Effects:
     (Intr)
days -0.184
```

independent Gaussians

Remove the correlation component from your model

```
# fit the model
fit.lmer = lmer(formula = reaction ~ 1 + days + (1 + days || subject),
                data = df.sleep)
# model summary
fit.lmer %>%
  summary()
```

alternative syntax (doesn't model correlation between random effects)

```
Linear mixed model fit by REML ['lmerMod']
Formula: reaction ~ 1 + days + (0 + days | subject) + (1 | subject)
   Data: df.sleep

REML criterion at convergence: 1771.5

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.9805 -0.4673  0.0250  0.4589  5.2083

Random effects:
 Groups    Name        Variance Std.Dev.
 subject   days         35.88    5.99
 subject.1 (Intercept) 598.11   24.46
 Residual               647.90   25.45
Number of obs: 183, groups:  subject, 20

Fixed effects:
            Estimate Std. Error t value
(Intercept)  252.550      6.491  38.907
days          10.439      1.556   6.708

Correlation of Fixed Effects:
     (Intr)
days -0.184
```

independent Gaussians

What if `lmer()` fails to converge?

```
1 # fit the model
2 fit.lmer = lmer(formula = reaction ~ 1 + days + (1 + days | subject),
3                 data = df.sleep)
4
5 # explore different optimization algorithms
6 fit.all = allFit(fit.lmer)
7
8 # summarize result
9 fit.all %>% summary()
```

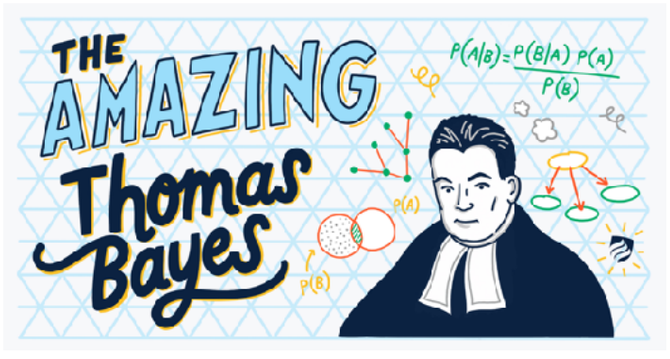
comparison of the different optimization algorithms

```
$fixef
                               (Intercept)     days
bobyqa                            252.5426 10.45212
Nelder_Mead                       252.5426 10.45212
nlminbwrap                        252.5426 10.45212
nloptwrap.NLOPT_LN_NELDERMEAD     252.5426 10.45212
nloptwrap.NLOPT_LN_BOBYQA         252.5426 10.45212

$llik
                         bobyqa                    Nelder_Mead                      nlminbwrap
                      -885.7239                      -885.7239                       -885.7239
nloptwrap.NLOPT_LN_NELDERMEAD     nloptwrap.NLOPT_LN_BOBYQA
                      -885.7239                      -885.7239

$sdcor
                               subject.(Intercept) subject.days.(Intercept) subject.days    sigma
bobyqa                                    24.13911                  5.918866   0.06927657 25.48261
Nelder_Mead                               24.13900                  5.918891   0.06928125 25.48261
nlminbwrap                                24.13911                  5.918867   0.06927628 25.48261
nloptwrap.NLOPT_LN_NELDERMEAD             24.13979                  5.918851   0.06927975 25.48255
nloptwrap.NLOPT_LN_BOBYQA                 24.13979                  5.918851   0.06927975 25.48255
```

https://rdrr.io/cran/lme4/man/convergence.html

What if `lmer()` fails to converge?

1. We drop random effects in the following order: random correlations, random slopes of covariates (where significance is of no interest), random intercepts ("0+" instead "1+") (following Barr et al., 2013). We never remove the random slopes of the variables of interest (i.e., the ones for which we want to conduct significance tests).
   Please note that removing random correlation terms can be tricky if random slopes are estimated for factors with 3 or more levels. In that case, it is probably easiest to use `afex::mixed()` with `expand_re = TRUE` (an alternative option is to create manually the relevant contrasts yourself and add them as predictors to your model, which allows you to suppress the random corrections using the double pipe symbol ||).
2. We try to run separate analyses: For example, one model to only test the fixed and random effect of A (with fixed effect of B present); then one model to only test the effect of B.
   If we really have to drop random slopes, we follow the next step:
3. We follow the PCA approach suggested by **rePsychLing** (see Bates et al., 2015) that is performing a PCA on the random effects and following the guidelines described in the paper.
   a. We use a likelihood ratio test to test whether the model fit becomes significantly worse. As we prefer a more conservative approach here (i.e., rather err on the side of keeping too many random effects; we prioritize avoiding inflated Type 2 errors for this kind of decision), we use larger alpha-level of .2 (Matuschek et al., 2017).
   b. Alternatively, we suggest an Information criterion approach to avoid using a *p* value for our inclusion/exclusion decision, but choose the best model based on *BIC* or *AIC*.



3.2.2. Or we choose a Bayesian approach

As an alternative to targeting convergence issues within **lme4**, we suggest fitting the same model with **brms** and comparing it to the **lme4** fit. We assume that both provide similar results when

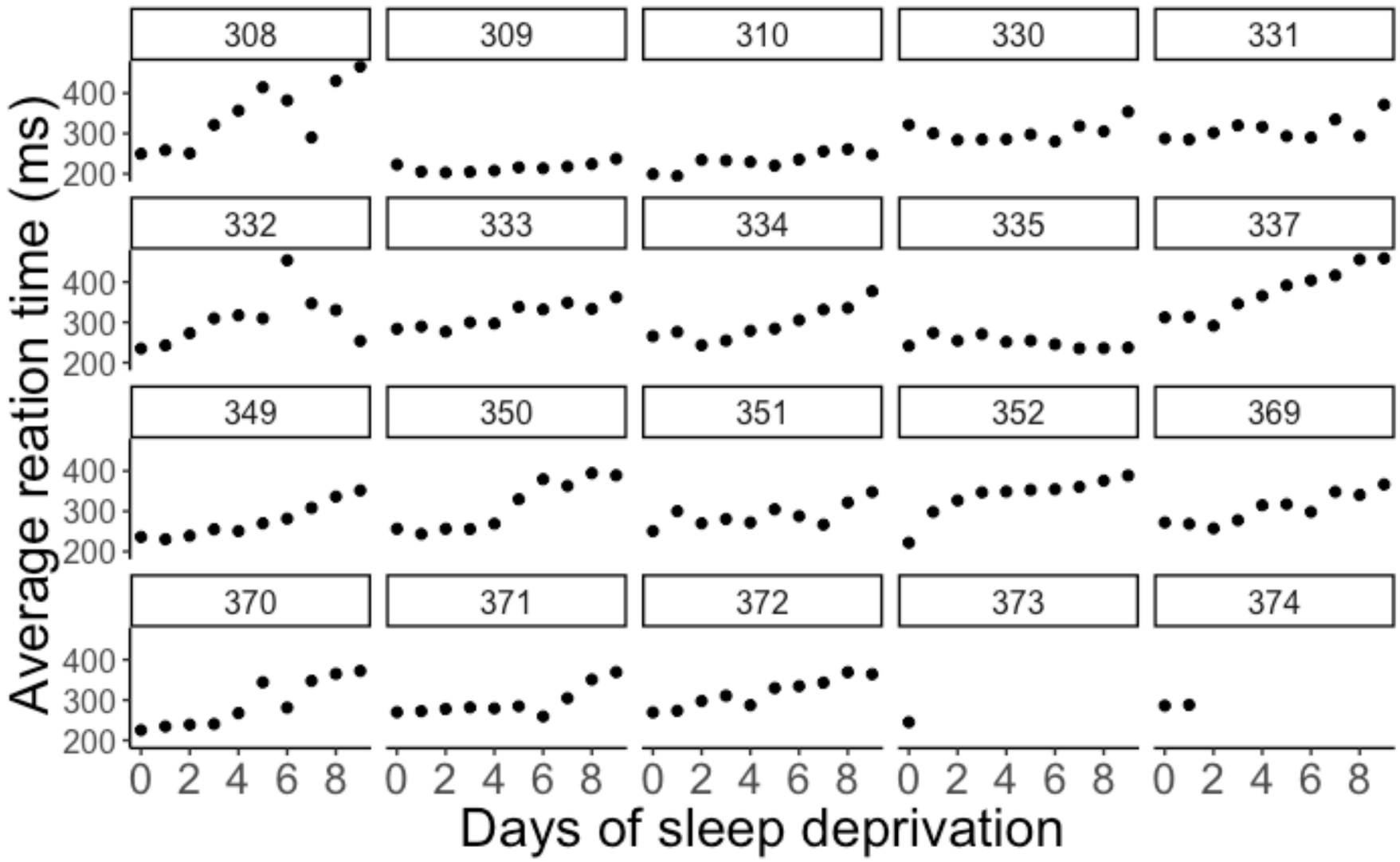
Some more examples

1. Sleep

Sleep data

How does sleep deprivation affect reaction time?

subject	days	reaction
308	0	249.56
308	1	258.70
308	2	250.80
308	3	321.44
308	4	356.85
309	0	222.73
309	1	205.27
309	2	202.98
309	3	204.71
309	4	207.72



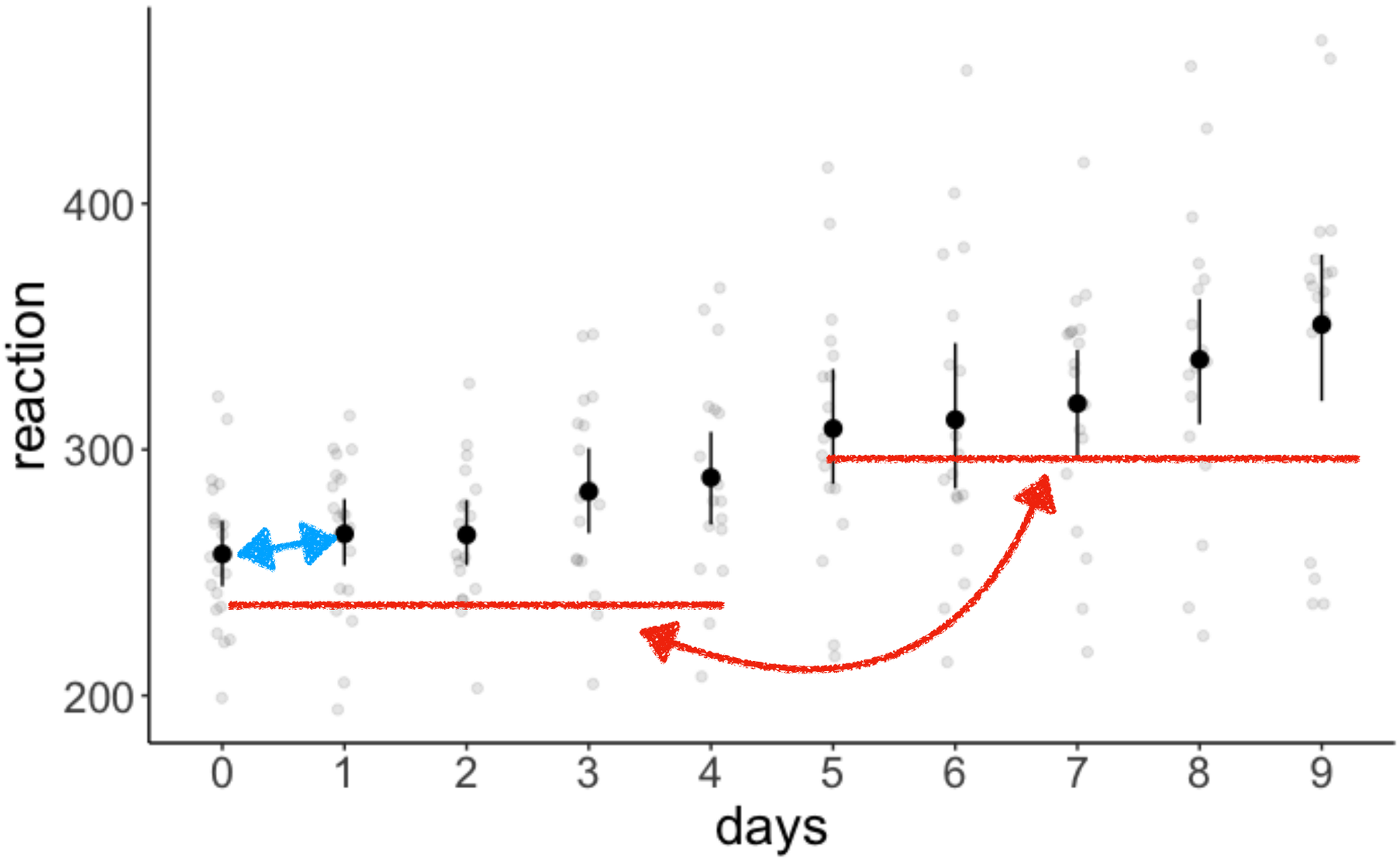
20 participants 2 with incomplete information

Testing specific hypotheses with linear contrasts

1. Is there a significant difference between day 0 and day 1?

2. Is there a significant difference between the days 0-4 and days 5-9?

subject	days	reaction
308	0	249.56
308	1	258.70
308	2	250.80
308	3	321.44
308	4	356.85
309	0	222.73
309	1	205.27
309	2	202.98
309	3	204.71
309	4	207.72



Sleep data

fit the model

```
1 fit = lmer(formula = reaction ~ 1 + days + (1 | subject),
2              data = df.sleep %>%
3                mutate(days = as.factor(days)))
```

Sleep data

fit the model

```
1 fit = lmer(formula = reaction ~ 1 + days + (1 | subject),
2              data = df.sleep %>%
3                mutate(days = as.factor(days)))
4
5 contrast = list(first_vs_second = c(-1, 1, rep(0, 8)),
6                 early_vs_late = c(rep(-1, 5)/5, rep(1, 5)/5))
7
8 fit %>%
9   emmeans(specs = "days",
10           contr = contrast) %>%
11   pluck("contrasts")
```

define the contrasts

test the contrasts

```
contrast         estimate    SE  df t.ratio p.value
first_vs_second      7.82 10.10 156   0.775  0.4398
early_vs_late       53.66  4.65 155 11.534  <.0001

Degrees-of-freedom method: kenward-roger
```

days	reaction
0	257.54
1	265.73

index	reaction
early	271.67
late	325.39

https://aosmith.rbind.io/2019/03/25/getting-started-with-emmeans/

2. Weight loss

Weight loss data

id	diet	exercises	timepoint	score
1	no	no	t1	10.43
1	no	no	t2	13.21
1	no	no	t3	11.59
1	yes	no	t1	10.20
1	yes	no	t2	12.51
1	yes	no	t3	14.60
2	no	no	t1	11.59
2	no	no	t2	10.66
2	no	no	t3	13.21
2	yes	no	t1	12.98
2	yes	no	t2	12.98
2	yes	no	t3	14.60

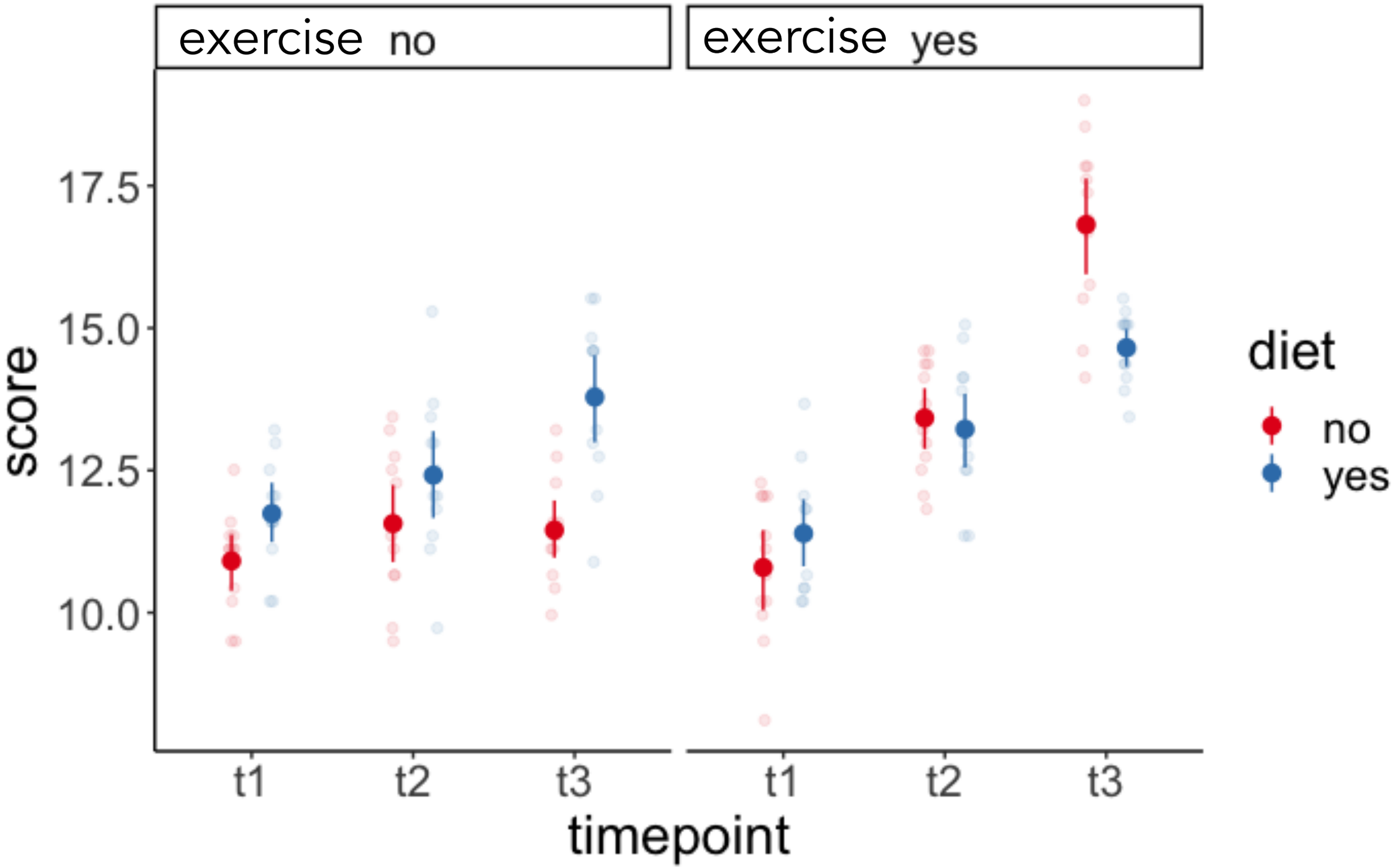
between participants: exercise yes/no

within participants: diet yes/no

within participants: time points

one observation in each cell, so we can use an ANOVA

Weight loss data



Weight loss data

```
1 fit = aov_ez(id = "id",
2              dv = "score",
3              between = "exercises",
4              within = c("diet", "timepoint"),
5              data = df.weightloss)
```

df.weightloss

id	diet	exercises	timepoint	score
1	no	no	t1	10.43
1	no	no	t2	13.21
1	no	no	t3	11.59
1	yes	no	t1	10.20
1	yes	no	t2	12.51
1	yes	no	t3	14.60
2	no	no	t1	11.59

```
Anova Table (Type 3 tests)

Response: score
                     Effect           df  MSE          F  ges p.value
1                 exercises       1, 22 1.84 38.77 *** .284   <.001
2                      diet       1, 22 0.65    7.91 * .028    .010
3          exercises:diet         1, 22 0.65 51.70 *** .157   <.001
4                 timepoint 1.74, 38.26 1.48 82.20 *** .541   <.001
5       exercises:timepoint 1.74, 38.26 1.48 26.22 *** .274   <.001
6            diet:timepoint 1.61, 35.44 1.92      0.78 .013    .439
7 exercises:diet:timepoint 1.61, 35.44 1.92  9.97 *** .147   <.001
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '+' 0.1 ' ' 1
```

main effects and interactions

Weight loss data

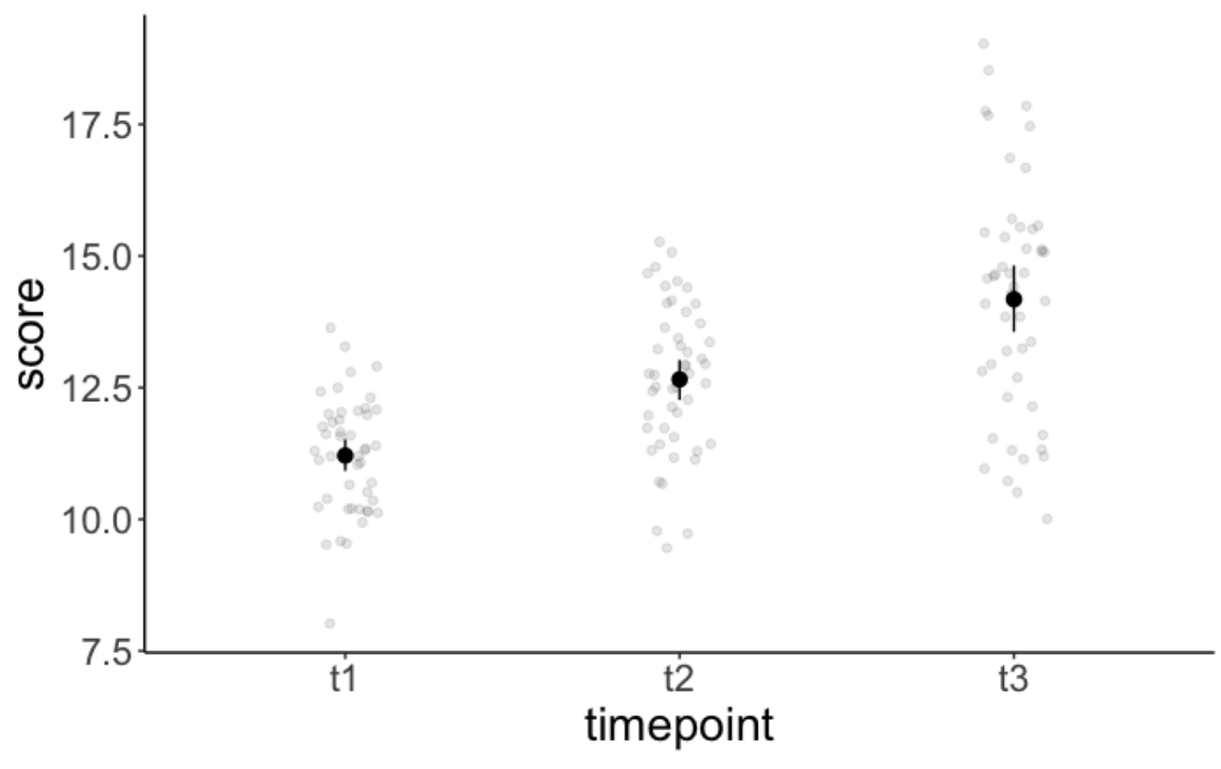
1. Is the score at the third time point different from the other two time points?
2. Is there a linear increase across time points?

```
fit = aov_ez(id = "id",
             dv = "score",
             between = "exercises",
             within = c("diet", "timepoint"),
             data = df.weightloss)

contrasts = list(first_two_vs_last = c(-0.5, -0.5, 1),
                 linear_increase = c(-1, 0, 1))

fit %>%
  emmeans(spec = "timepoint",
          contr = contrasts)
```

```
contrast           estimate    SE df t.ratio p.value
 first_two_vs_last     2.24 0.200 44 11.194  <.0001
 linear_increase       2.97 0.231 44 12.820  <.0001
```

df.weightloss

id	diet	exercises	timepoint	score
1	no	no	t1	10.43
1	no	no	t2	13.21
1	no	no	t3	11.59
1	yes	no	t1	10.20
1	yes	no	t2	12.51
1	yes	no	t3	14.60
2	no	no	t1	11.59



3. politeness

Politeness

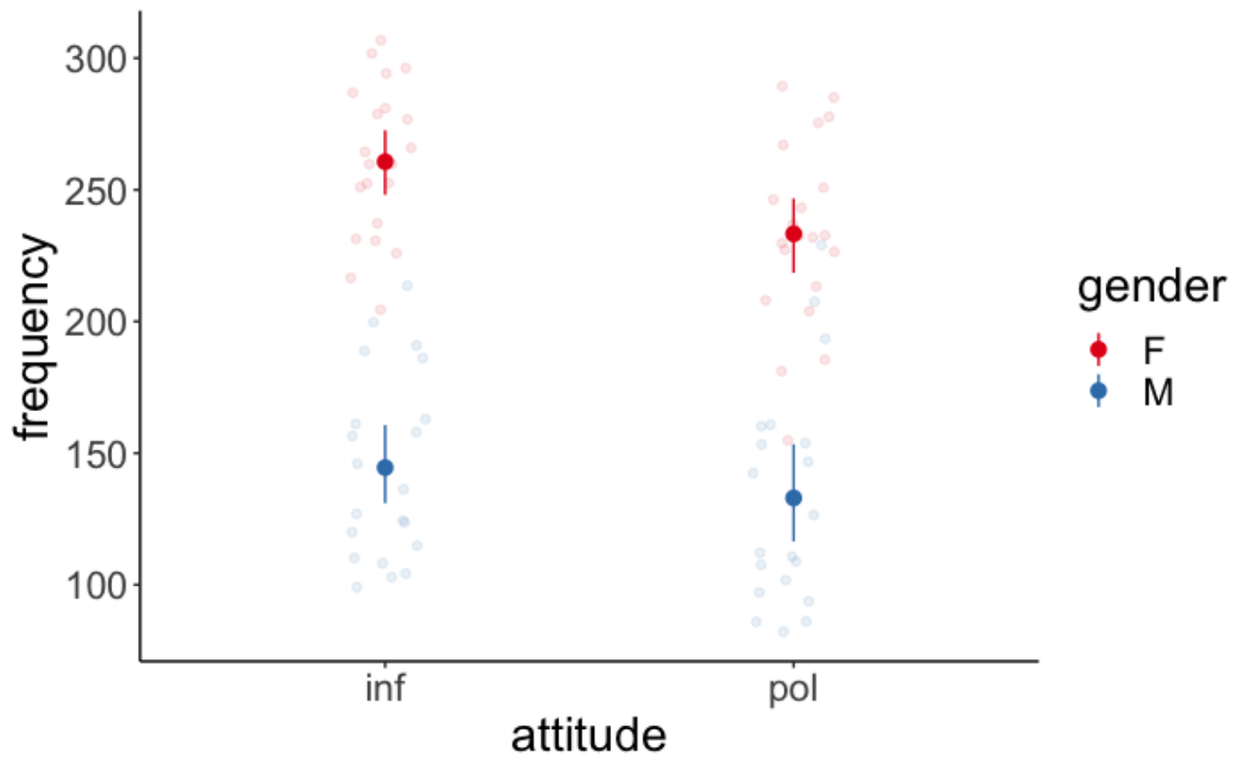
subject	gender	scenario	attitude	frequency
F1	F	1	pol	213.3
F1	F	1	inf	204.5
F1	F	2	pol	285.1
F1	F	2	inf	259.7
F1	F	3	pol	203.9
F1	F	3	inf	286.9
F1	F	4	pol	250.8
F1	F	4	inf	276.8
F1	F	5	pol	231.9
F1	F	5	inf	252.4
F1	F	6	pol	181.2
F1	F	6	inf	230.7
F1	F	7	inf	216.5
F1	F	7	pol	154.8
F3	F	1	pol	229.7

gender: female, male

scenario: different text prompt

attitude: informal vs. polite

frequency: pitch of voice

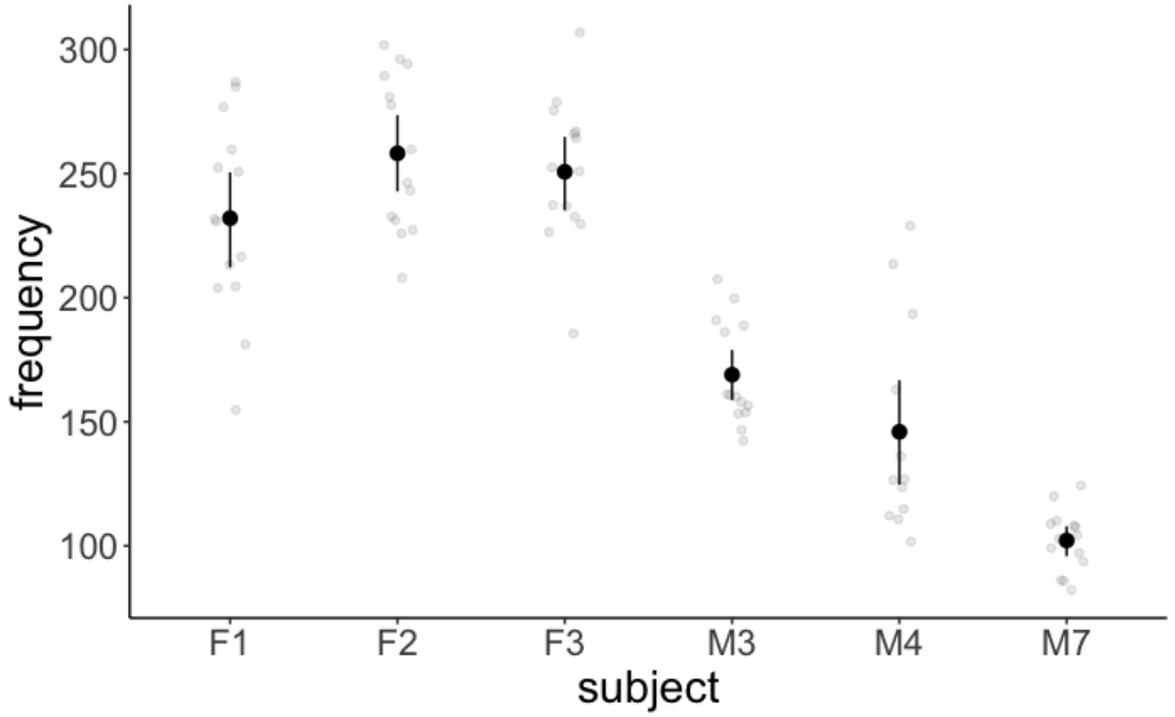


https://jontalle.web.engr.illinois.edu/MISC/lme4/bw_LME_tutorial.pdf

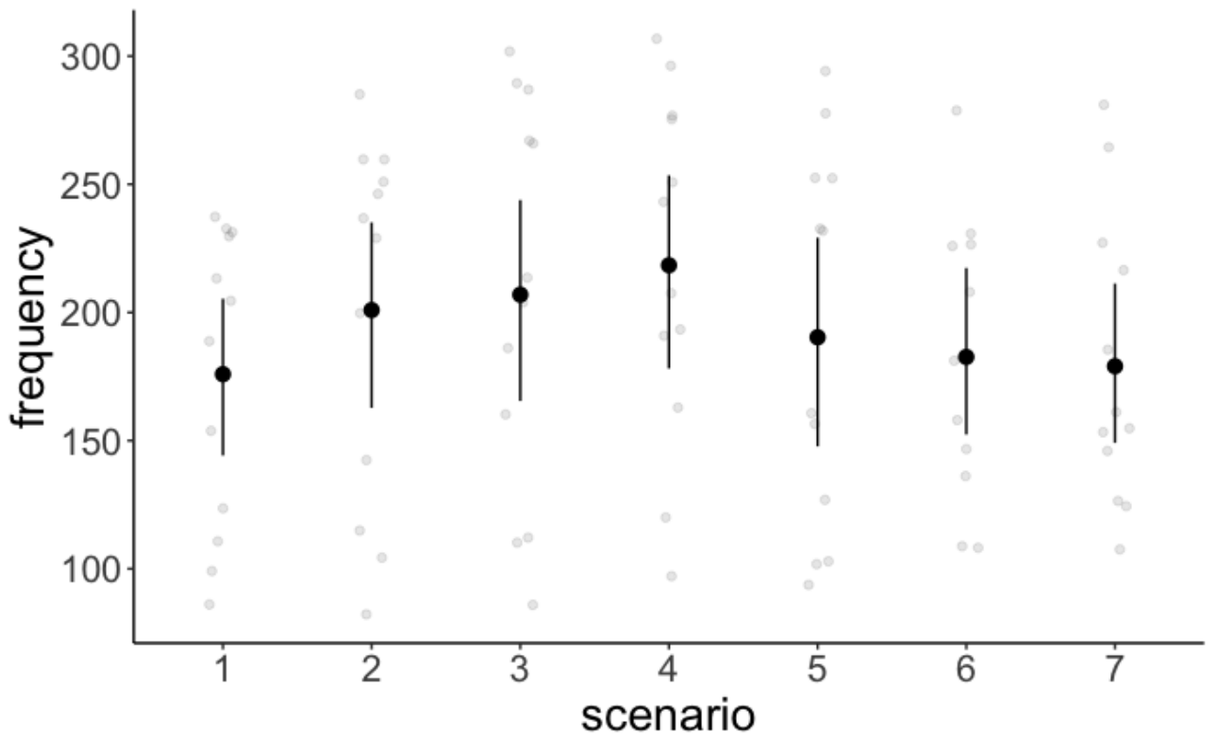
Politeness

subject	gender	scenario	attitude	frequency
F1	F	1	pol	213.3
F1	F	1	inf	204.5
F1	F	2	pol	285.1
F1	F	2	inf	259.7
F1	F	3	pol	203.9
F1	F	3	inf	286.9
F1	F	4	pol	250.8
F1	F	4	inf	276.8
F1	F	5	pol	231.9
F1	F	5	inf	252.4
F1	F	6	pol	181.2
F1	F	6	inf	230.7
F1	F	7	inf	216.5
F1	F	7	pol	154.8
F3	F	1	pol	229.7

variation across subjects



variation across scenarios

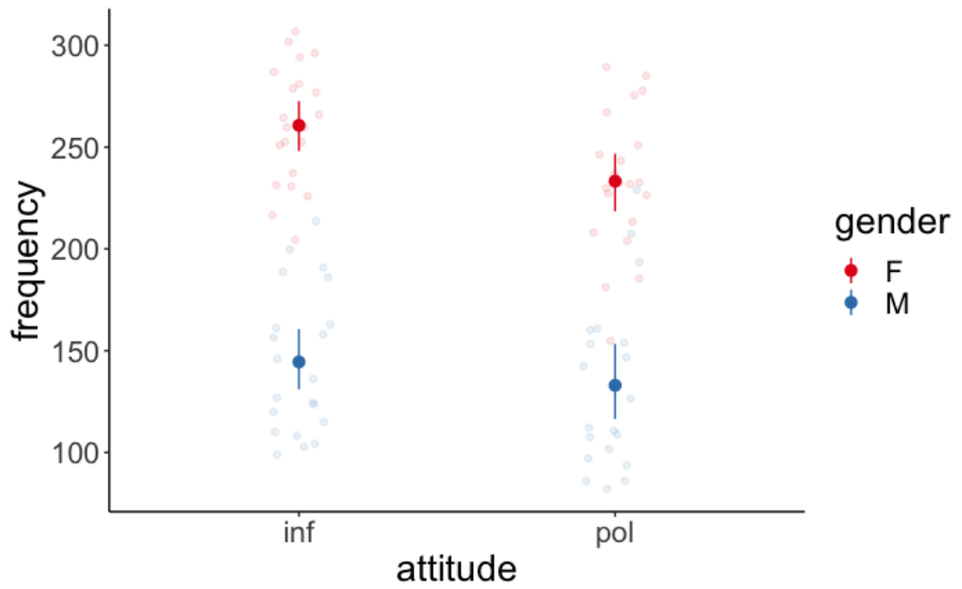


Politeness

Was there an effect of gender and attitude on pitch?

```
1 lmer(formula = frequency ~ 1 + attitude * gender + (1 + attitude | subject) + (1 | scenario),
2       data = df.politeness) %>%
3   joint_tests()
```

```
model term        df1   df2 F.ratio p.value
 attitude           1  3.99  12.411  0.0244
 gender             1  4.00  26.570  0.0067
 attitude:gender    1  3.99   1.959  0.2342
```



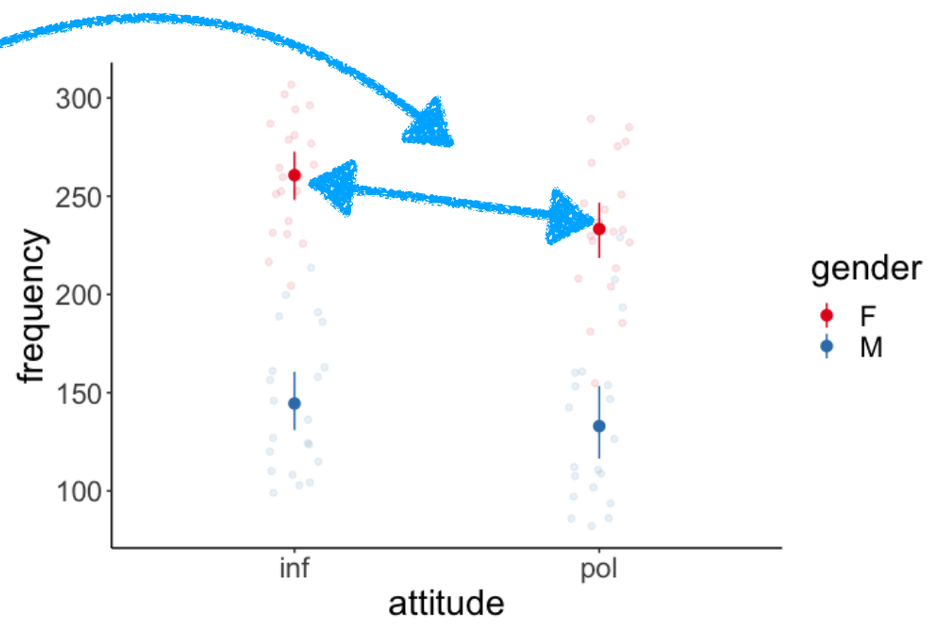
main effect of attitude, main effect of gender, no significant interaction effect

Politeness

Was there a difference between informal and polite speech for female participants?

```
fit = lmer(formula = frequency ~ 1 + attitude * gender + (1 | subject) + (1 | scenario),
           data = df.politeness)

fit %>%
  emmeans(specs = pairwise ~ attitude + gender,
          adjust = "none")
```

```
contrast       estimate    SE   df t.ratio p.value
 inf F - pol F     27.4  7.81 3.89   3.508  0.0259
 inf F - inf M    116.2 21.35 4.00   5.443  0.0055
 inf F - pol M    128.0 21.78 4.57   5.879  0.0027
 pol F - inf M     88.8 21.73 4.54   4.086  0.0116
 pol F - pol M    100.6 22.16 4.00   4.541  0.0105
 inf M - pol M     11.8  7.93 4.10   1.490  0.2088

Degrees-of-freedom method: kenward-roger
```



yes, there was significant difference in pitch for women between informal and formal speech

Politeness

Was there an effect of gender and attitude on pitch?

ANOVA

```
1 aov_ez(id = "subject",
2         dv = "frequency",
3         between = "gender",
4         within = "attitude",
5         data = df.politeness)
```

```
More than one observation per cell, aggregating the data using
mean (i.e, fun_aggregate = mean)!Missing values for following
ID(s):
M4
Removing those cases from the analysis.Anova Table (Type 3 tests)

Response: frequency
            Effect   df     MSE          F  ges p.value
1           gender 1, 3 1729.42    17.22 * .851    .025
2        attitude 1, 3    3.65 309.71 *** .179   < .001
3 gender:attitude 1, 3    3.65    21.30 * .015    .019
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘+’ 0.1 ‘ ’ 1
```

ignores variation between scenarios, and just takes the mean

interaction effect

LMER

```
1 lmer(formula = frequency ~ 1 + attitude * gender +
                 (1 + attitude | subject) + (1 | scenario),
2      data = df.politeness) %>%
3   joint_tests()
```

```
model term        df1  df2 F.ratio p.value
 attitude           1 3.99  12.411  0.0244
 gender             1 4.00  26.570  0.0067
 attitude:gender    1 3.99   1.959  0.2342
```

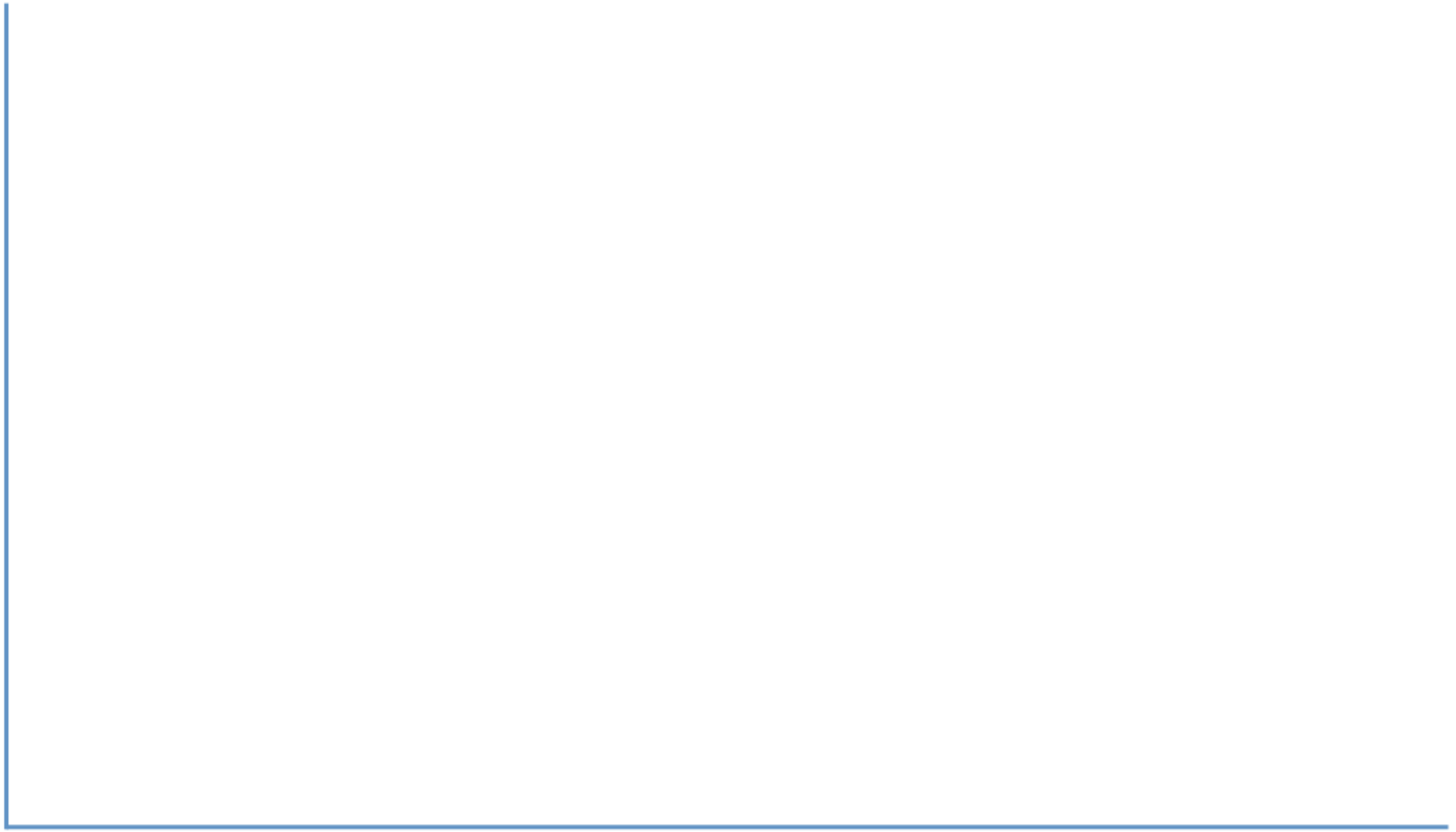
no interaction effect

Summary

- Quick recap
- Simpsons paradox
- Understanding `lmer()` syntax
- Reporting results
- Let's simulate some `lmer()`s
- `lmer()` standard operating procedures
- Some more examples

Feedback

How was the pace of today's class?



How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?

Thank you!