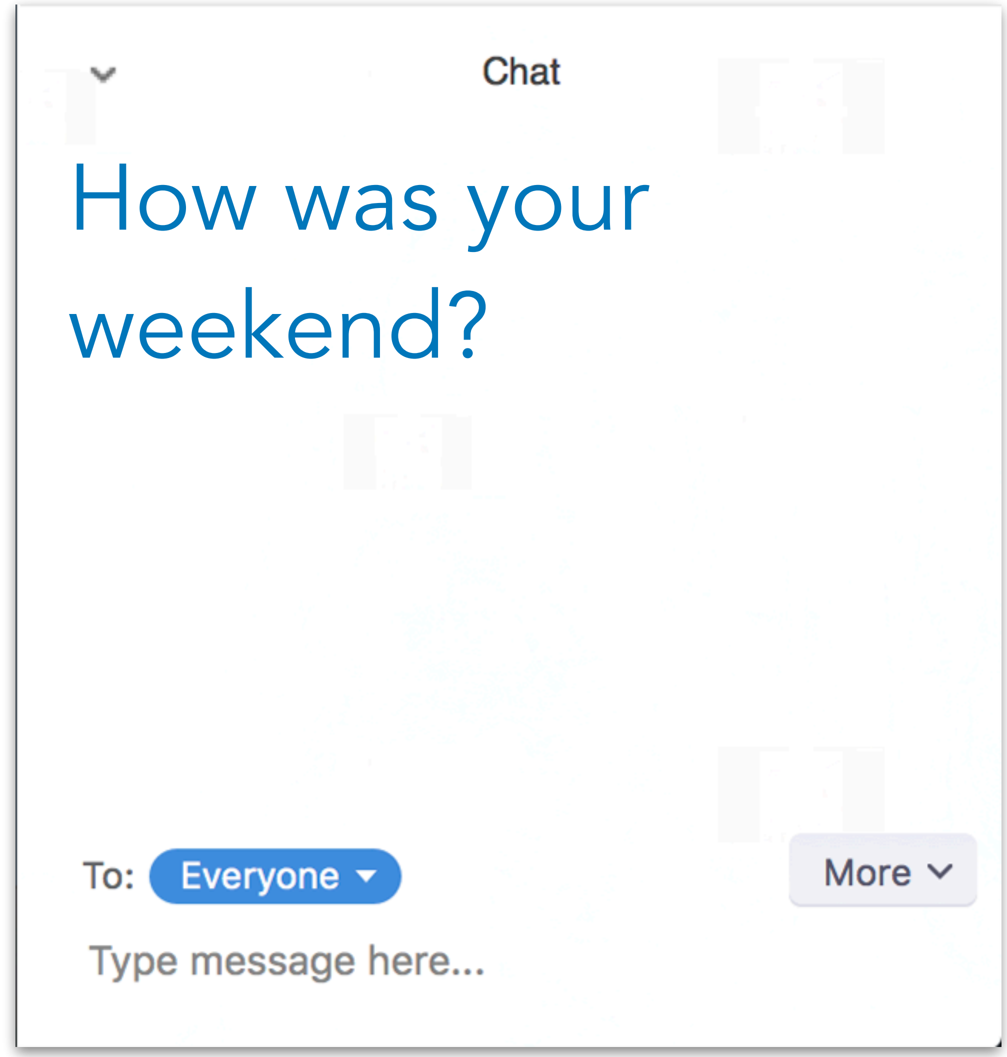
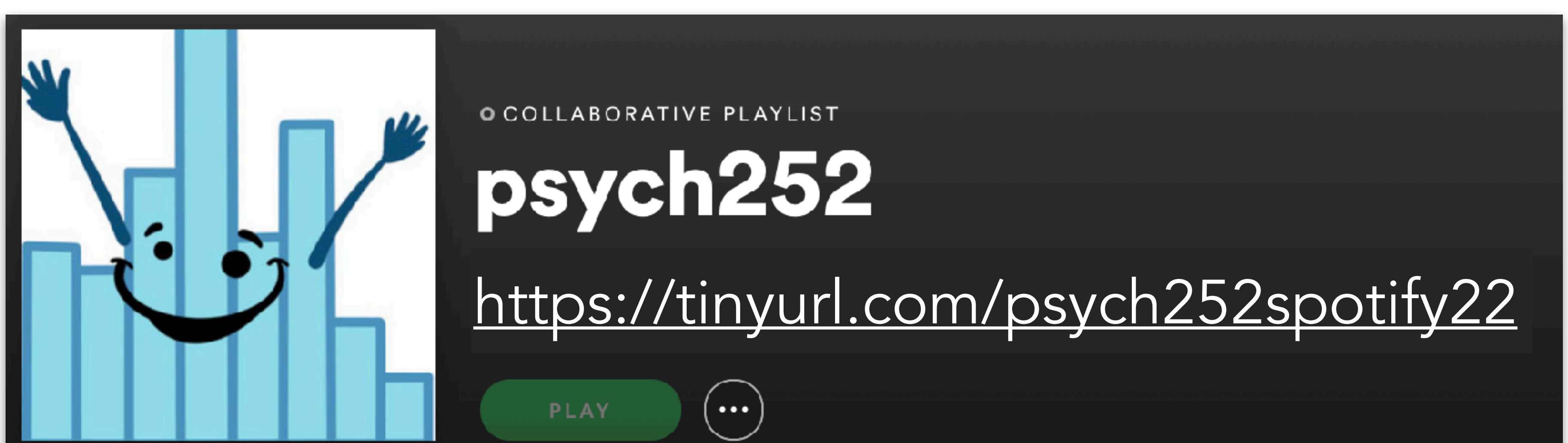
PSYCH 252: Statistical Methods for Behavioral and Social Sciences

Linear model 3

If she loves you more each and every day,
by linear regression she hated you before you met.



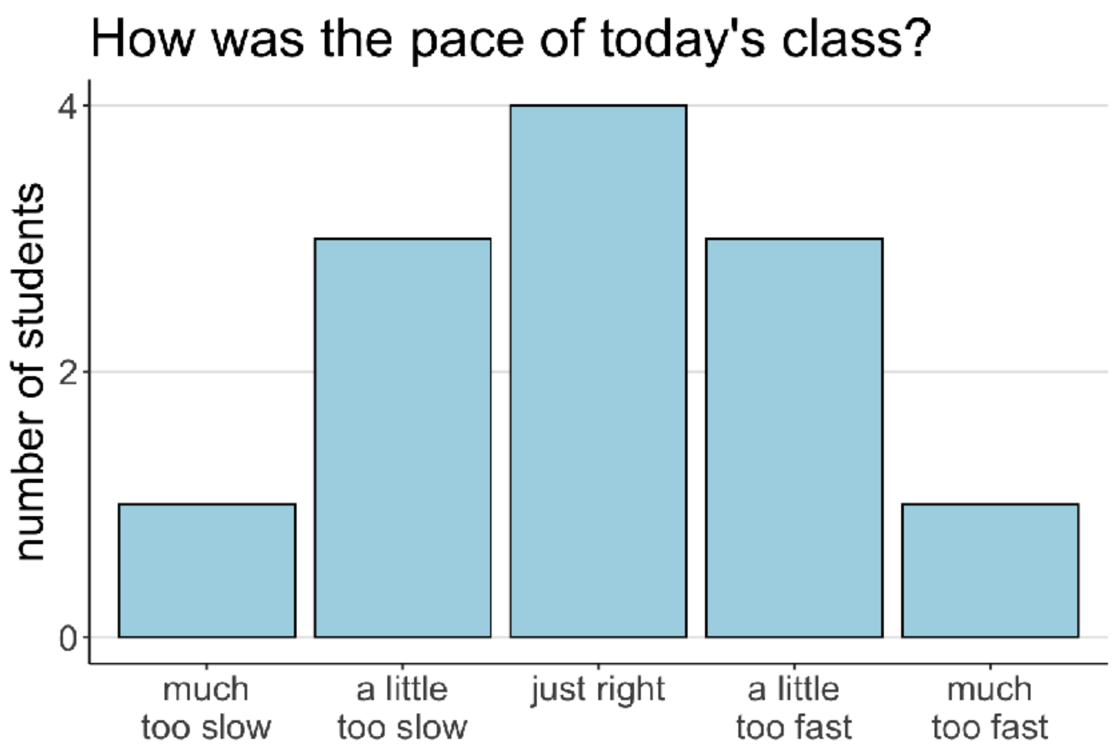


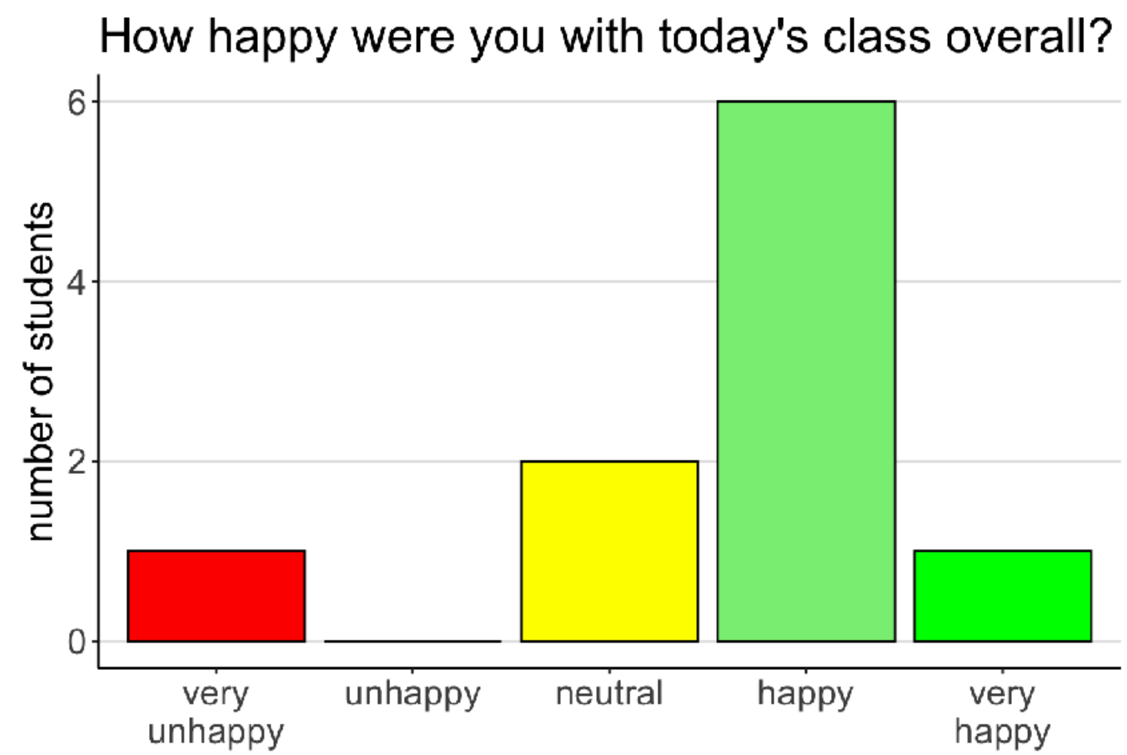
We're listening to "Paris 2006" by "Axel Boman" submitted by Tobi

01/31/2022

Feedback

Feedback

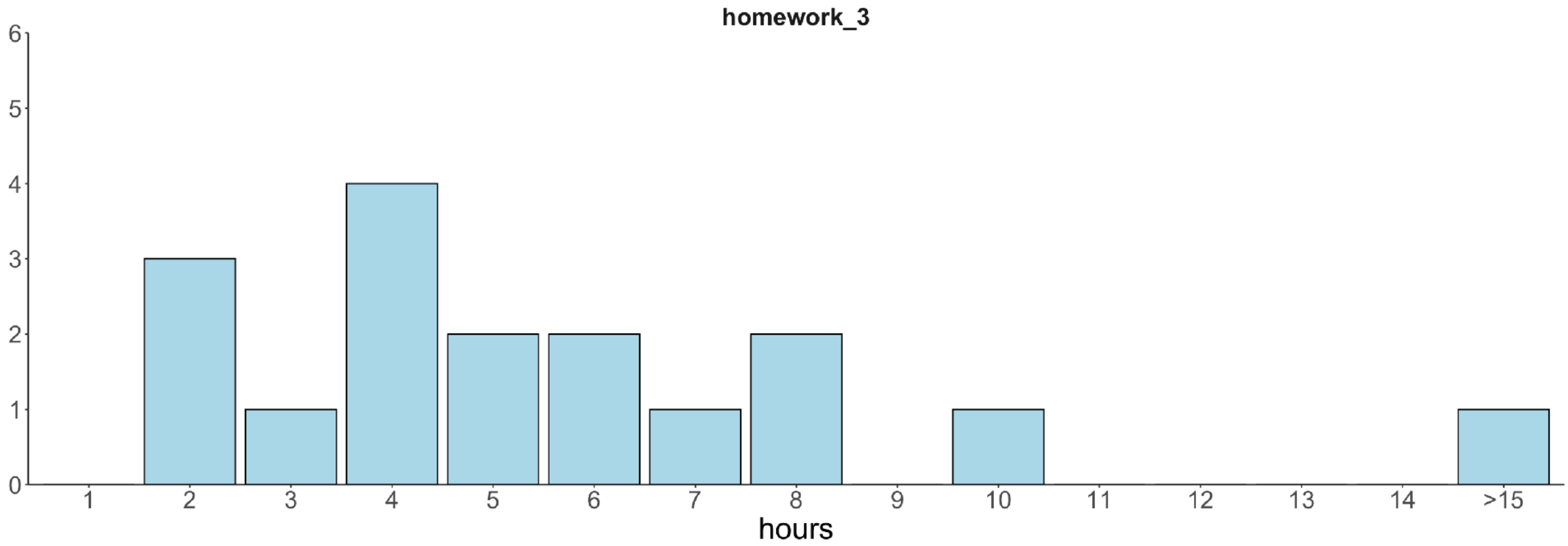




I liked hearing other classmates' questions. I also liked that the code was shown and explained.

Thanks for putting time into making your slides and explanations so organized all quarter long!

Homework 3



Things that came up

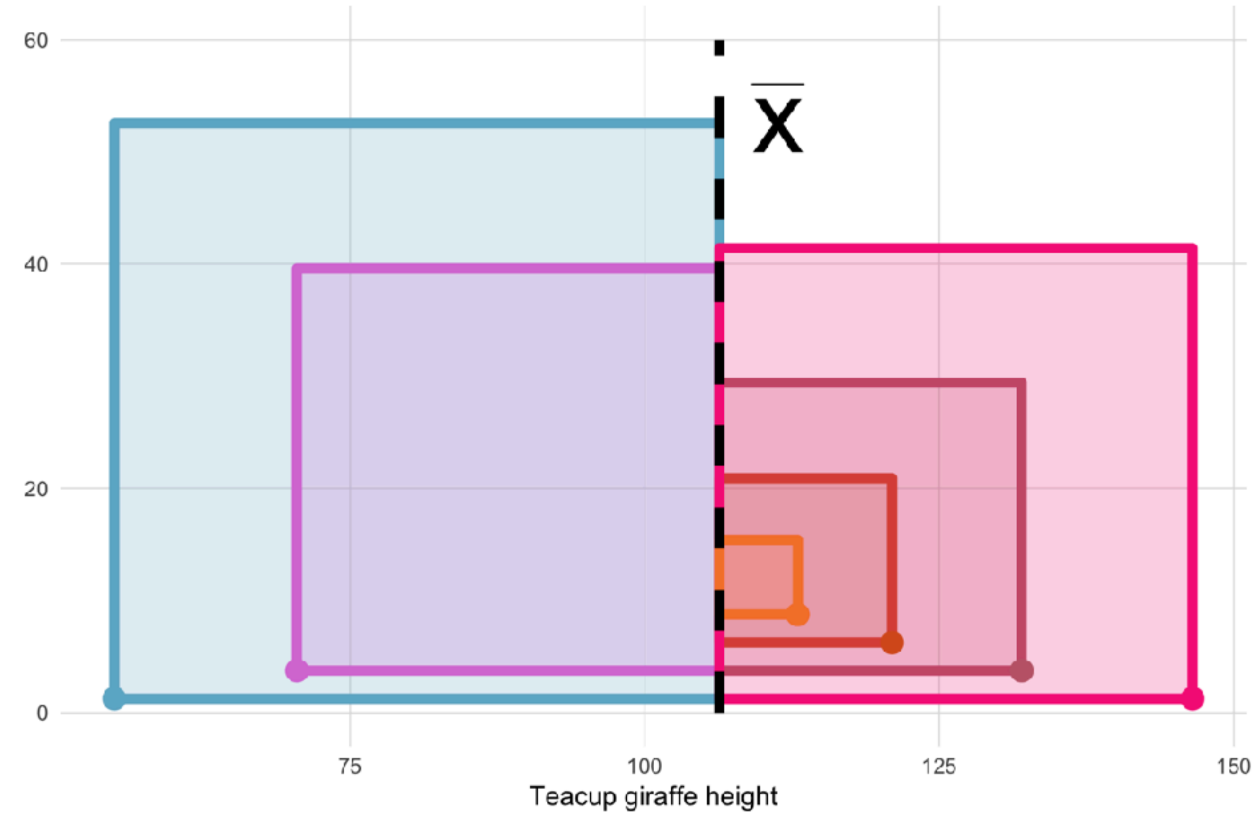
n-1 in the standard deviation

n-1 in standard deviation

$$\mathrm{sd}(Y) = \sqrt{\sum_{i=1}^{n} \frac{(Y_i - \overline{Y})^2}{\boxed{n-1}}}$$

definition of the standard deviation

sum of squares = squared deviation from the mean

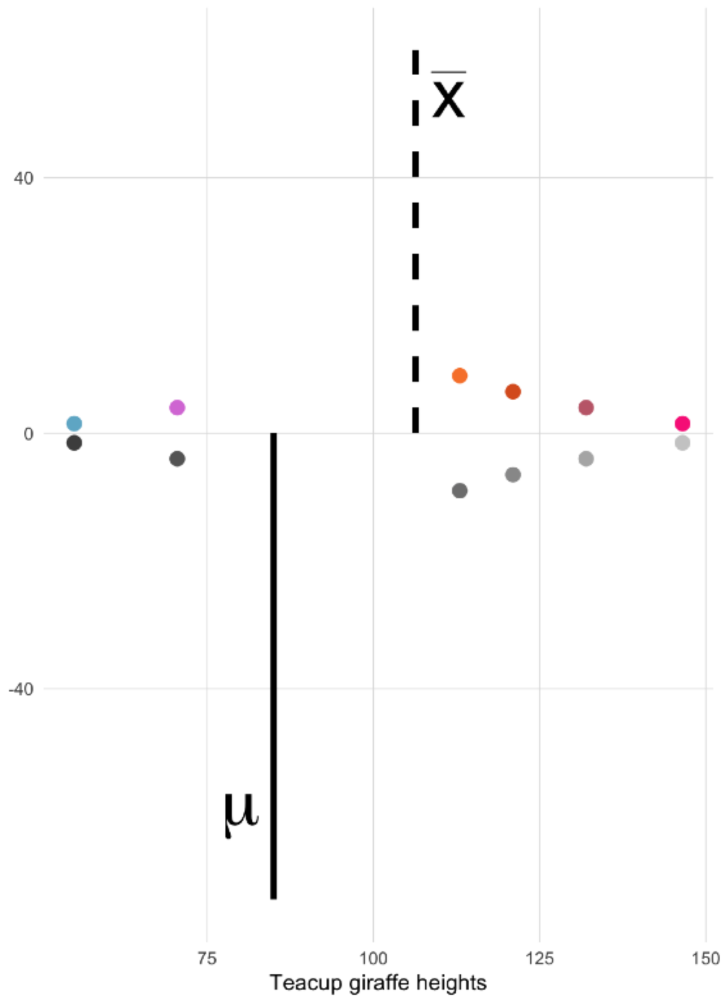


https://tinystats.github.io/teacups-giraffes-and-statistics/04_variance.html#population_vs_sample_((n)_vs_(n-1))

n-1 in standard deviation

$$\text{sd}(Y) = \sqrt{\sum_{i=1}^{n} \frac{(Y_i - \overline{Y})^2}{n-1}}$$

definition of the standard deviation



$\bar{x}$ = sample mean

- the sample mean will be somewhat different from the population mean
- this means, we are likely to underestimate the true variance

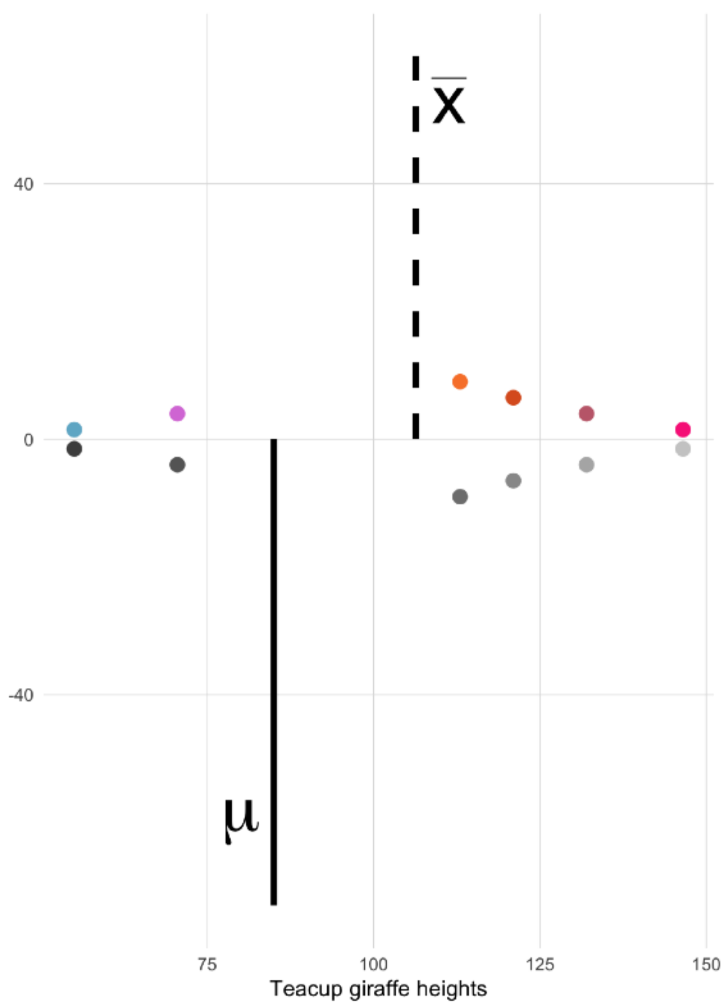
μ = population mean

https://tinystats.github.io/teacups-giraffes-and-statistics/04_variance.html#population_vs_sample_((n)_vs_(n-1))

n-1 in standard deviation

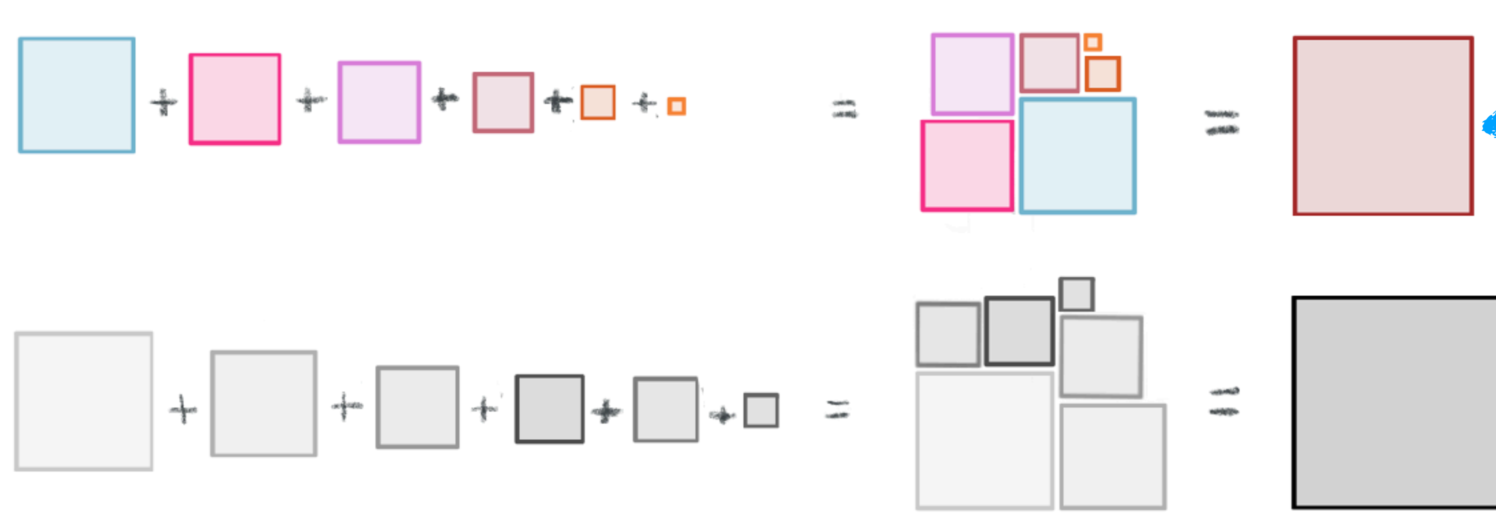
$$\text{sd}(Y) = \sqrt{\sum_{i=1}^{n} \frac{(Y_i - \overline{Y})^2}{\boxed{n-1}}}$$

definition of the standard deviation



$\bar{x}$ = sample mean

this area is smaller



μ = population mean

Plan for today

- Quick recap
- Multiple regression
  - two continuous predictors
  - one categorical predictor
  - one continuous and one categorical predictor
- Interactions
- `lm()` output

Quick recap

Quick recap: Regression (conceptual)

Linear model: Simple regression

```
Data = Model + Error
```

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\hat{Y}_i = \beta_0 + \beta_1 X_i$$

the model is a linear combination of predictors

The general procedure

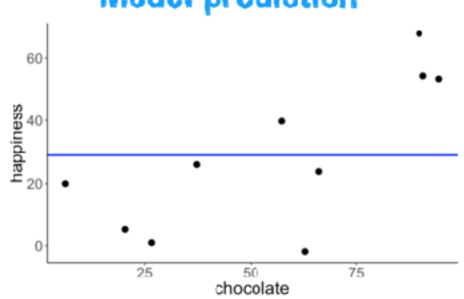
1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data
3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

H_0: Chocolate consumption and happiness are unrelated.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

Model prediction



Fitted model

$$Y_i = 28.88 + e_i$$

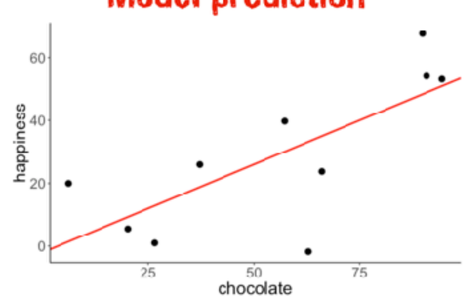
H_1: Chocolate consumption and happiness are related.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

chocolate consumption

Model prediction



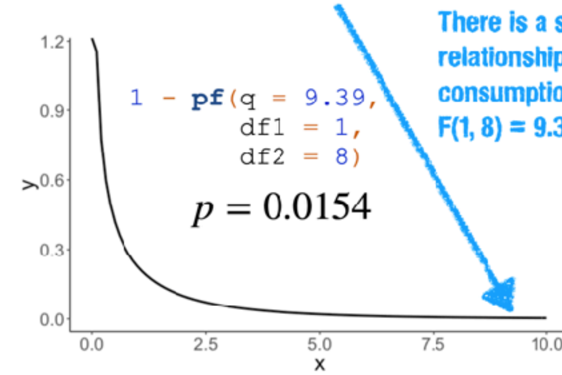
Fitted model

$$Y_i = -2.04 + 0.56 X_i + e_i$$

Decide whether it's **worth it**

- To compute the F statistic, we need:
  - PRE = 0.54
  - PC = 1
  - PA = 2
  - n = 10

$$F = \frac{\text{PRE}/(\text{PA} - \text{PC})}{(1 - \text{PRE})/(n - \text{PA})} = \frac{0.54/(2-1)}{(1-0.54)/(10-2)} = 9.39$$



There is a significant relationship between chocolate consumption and happiness F(1, 8) = 9.39, p = .0154.

Quick recap: Regression (in R)

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

$$\text{balance}_i = b_0 + b_1 \cdot \text{income}_i + e_i$$

```
fit = lm(formula = balance ~ 1 + income, data = df.credit)
```

outcome — intercept (doesn't need to be specified explicitly) — predictor — data

Hypothesis test

Compact Model

$$\text{balance}_i = \beta_0 + \epsilon_i$$

```
fit_c = lm(formula = balance ~ 1,
           data = df.credit)
```

Augmented Model

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \epsilon_i$$

```
fit_a = lm(formula = balance ~ 1 + income,
           data = df.credit)
```

```
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ 1 + income
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    399 84339912
2    398 66208745  1  18131167 108.99 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The general procedure

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data

```
fit_c = lm(formula = balance ~ 1,
           data = df.credit)

fit_a = lm(formula = balance ~ 1 + income,
           data = df.credit)
```

3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

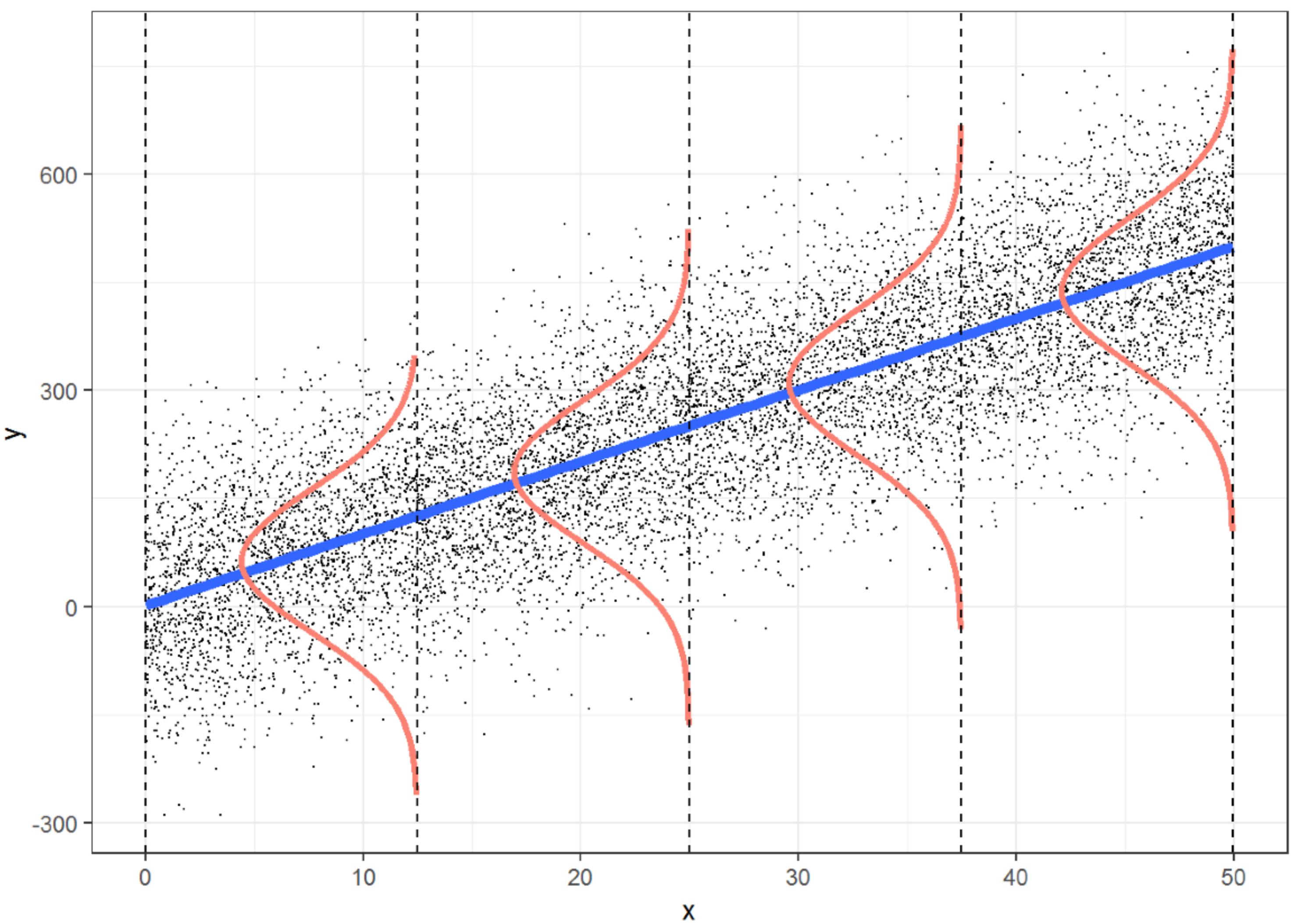
```
anova(fit_c, fit_a)
```

Multiple regression

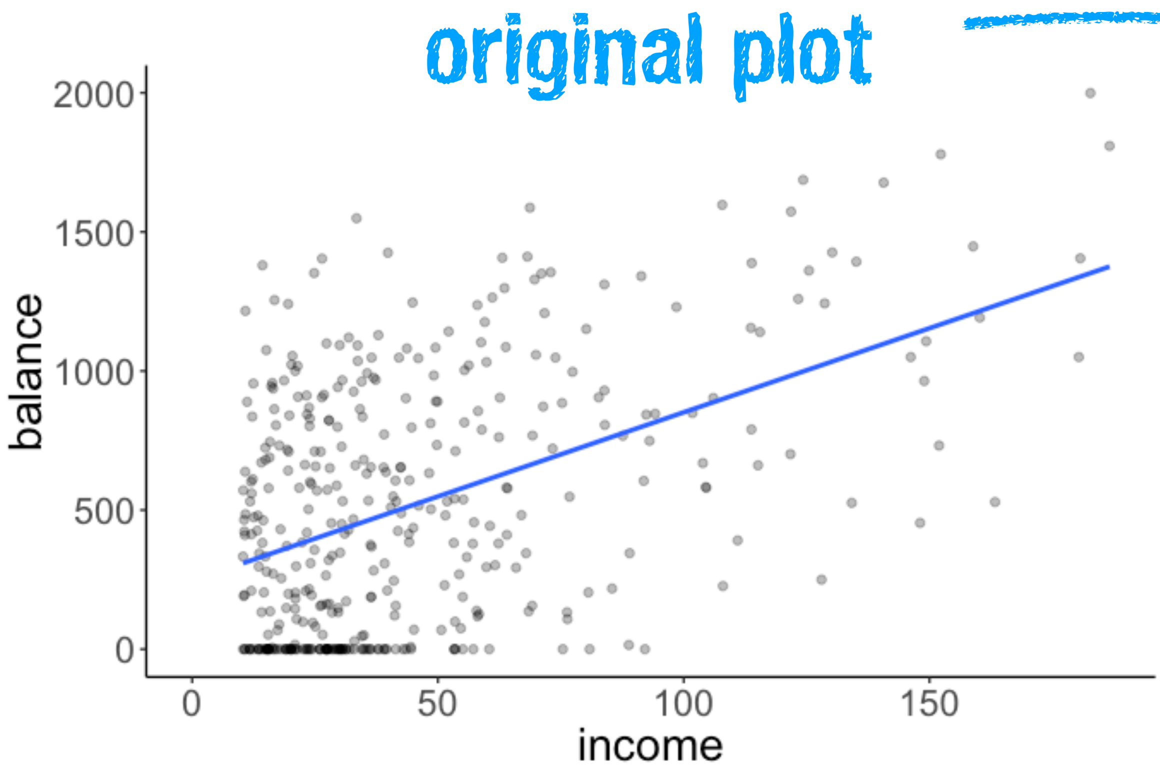
Model assumptions of **simple** regression

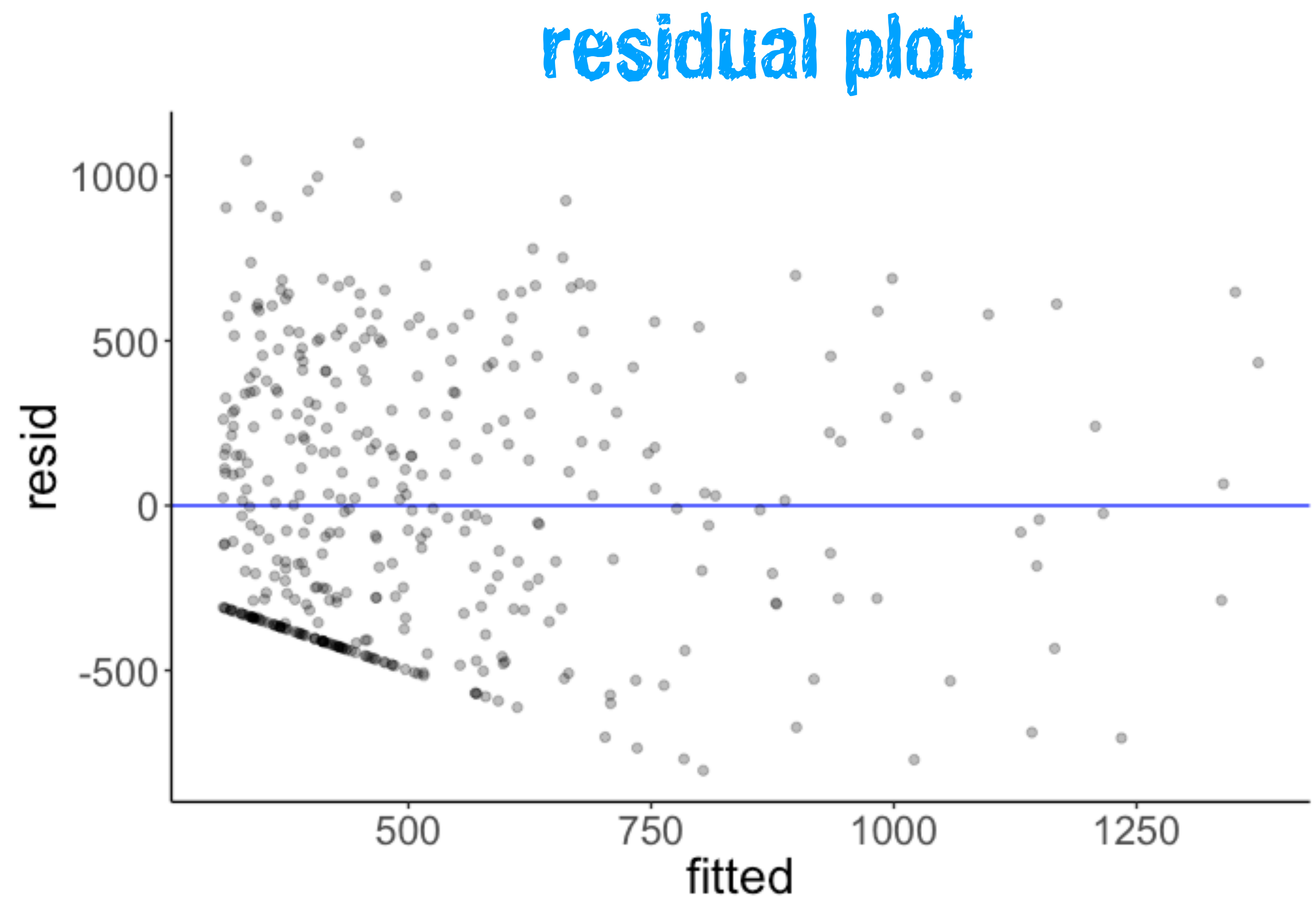
- independent observations
- *Y* is continuous
- errors are normally distributed
- errors have constant variance
- error terms are uncorrelated

the dependent variable doesn't need to be normally distributed!

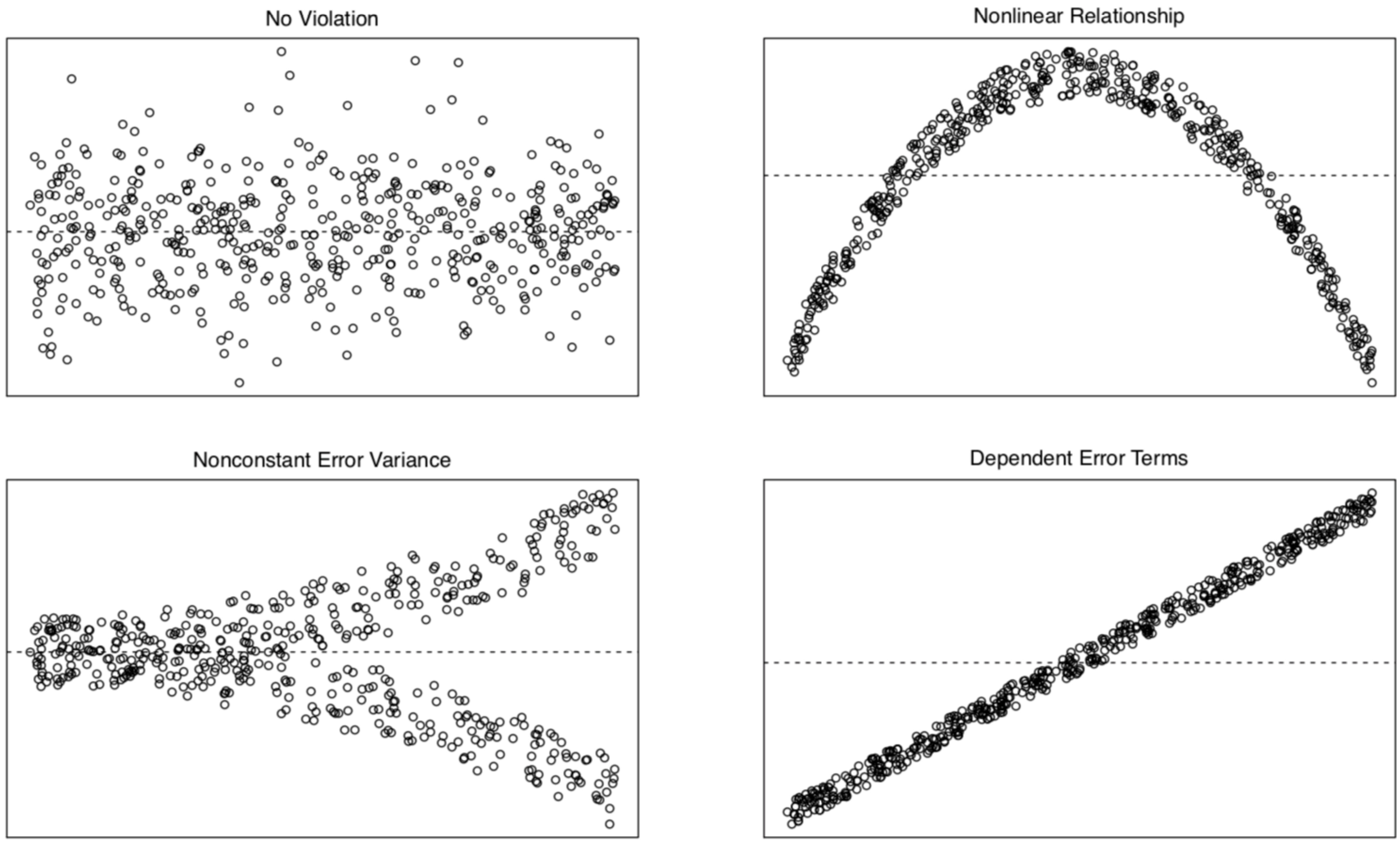


Model assumptions of simple regression





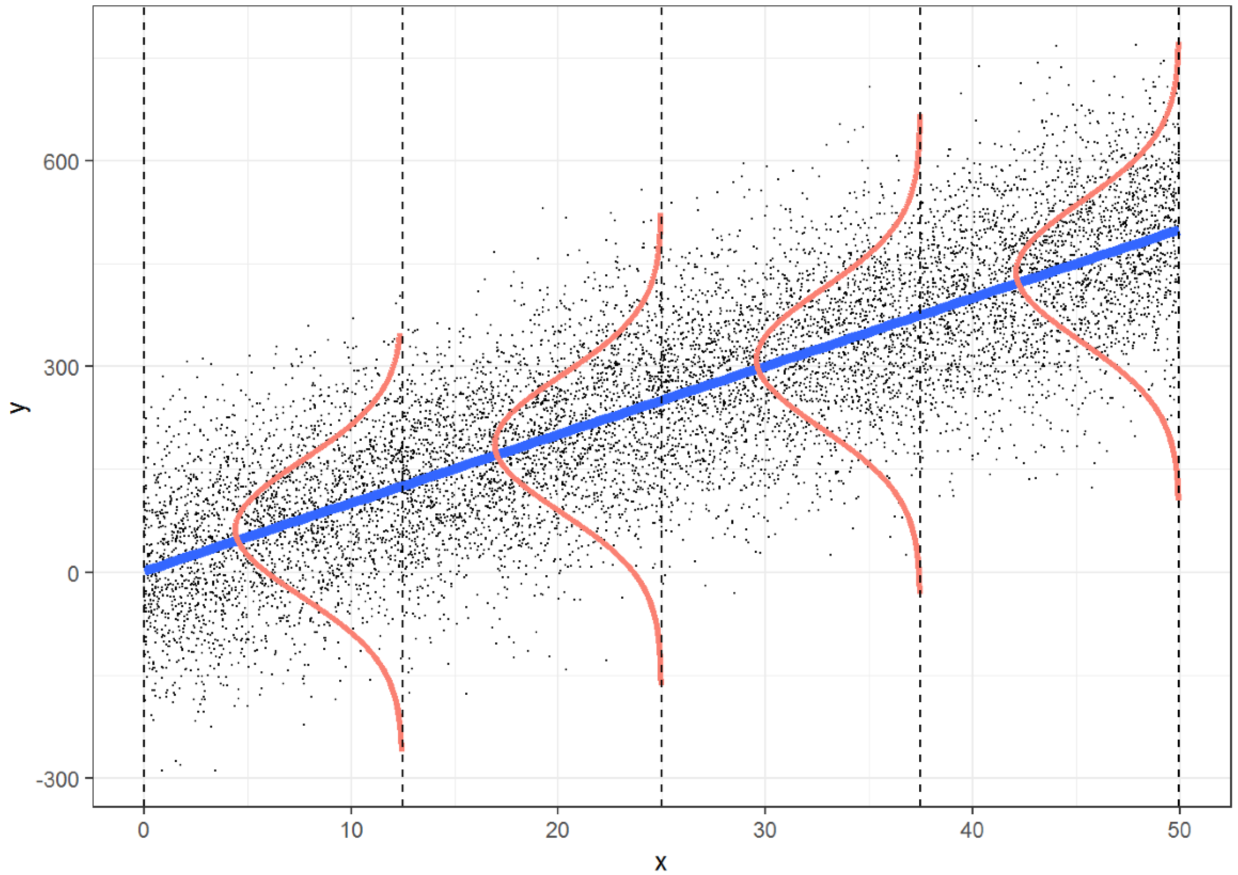
Model assumptions of simple regression



Assumptions of **multiple** regression

- independent observations
- Y is continuous
- errors are normally distributed
- errors have constant variance
- error terms are uncorrelated
- **no multicollinearity**

predictors in the model should not be highly correlated with each other



Linear model

`Data = Model + Error`

Simple regression

$$Y_i = \beta_0 + \beta_1 X_{1i} + \epsilon_i$$

one predictor

Multiple regression

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_p X_{pi} + \epsilon_i$$

many predictors

Advertising data set

- Combine several predictors to explain an outcome variable of interest

money spent on different media (x $1000)

sales (x 1000)

index	tv	radio	newspaper	sales
1	230.1	37.8	69.2	22.1
2	44.5	39.3	45.1	10.4
3	17.2	45.9	69.3	9.3
4	151.5	41.3	58.5	18.5
5	180.8	10.8	58.4	12.9
6	8.7	48.9	75.0	7.2
7	57.5	32.8	23.5	11.8
8	120.2	19.6	11.6	13.2
9	8.6	2.1	1.0	4.8
10	199.8	2.6	21.2	10.6

Model C **Simple regression**

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + e_i$$

Model A **Multiple regression**

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

Can we predict sales better when we consider radio ads in addition to TV ads?

"Controlling" for TV ads, do radio ads explain any of the additional variance in sales?

Visualizing correlations

```r
library("corrr")
```

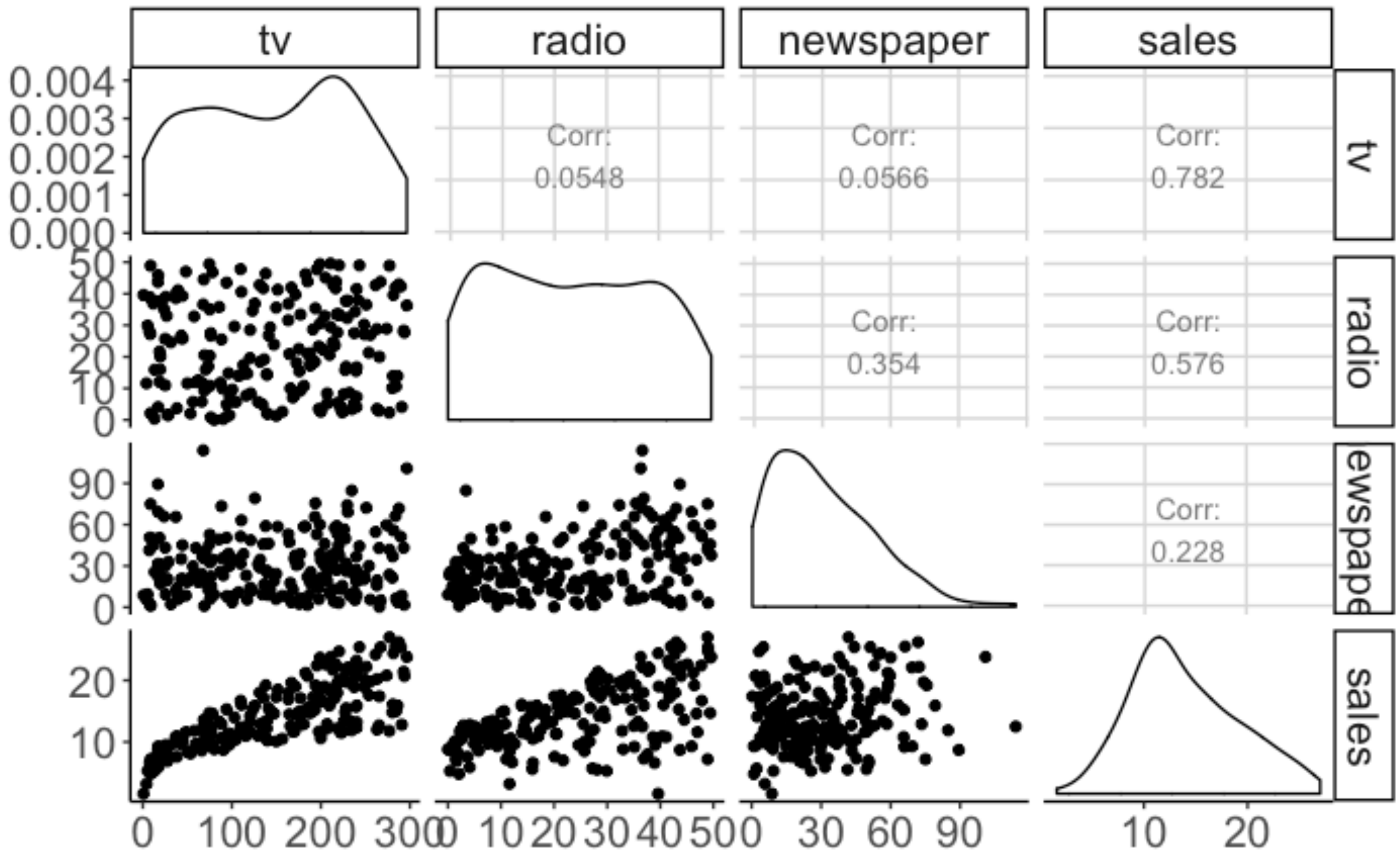
```r
df.credit %>%
    select_if(is.numeric) %>%
    correlate() %>%
    rearrange() %>%
    shave() %>%
    fashion()
```

rowname	index	newspaper	radio	sales	tv
index					
newspaper	-0.15				
radio	-0.11	0.35			
sales	-0.05	0.23	0.58		
tv	0.02	0.06	0.05	0.78	

Visualizing correlations

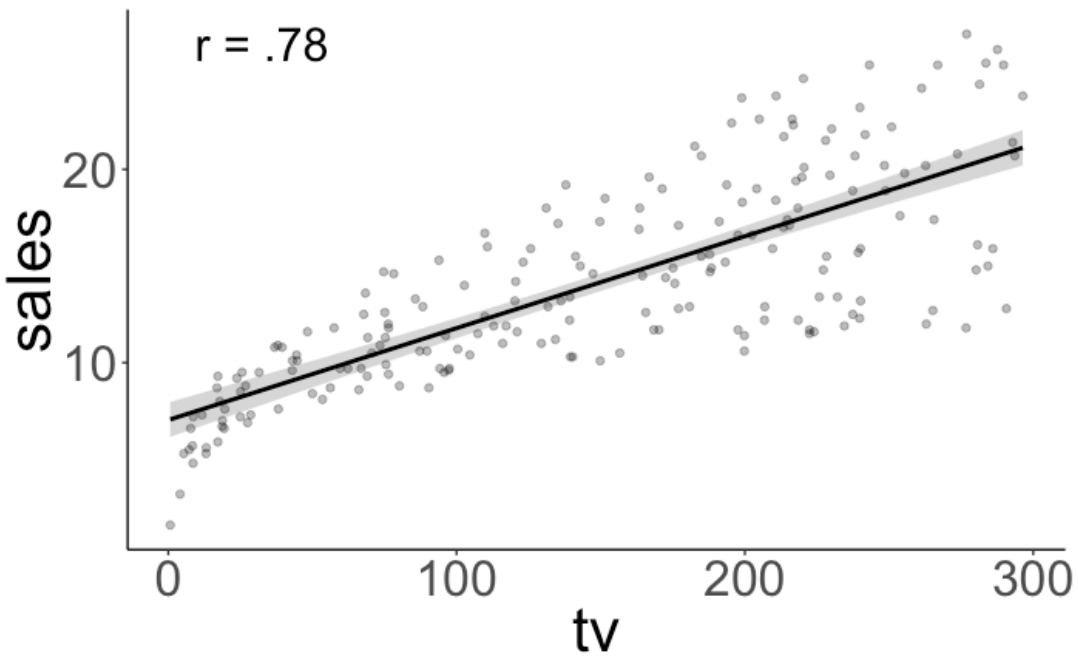
```
library("GGally")
```

```
df.ads %>%
  select(-index) %>%
  ggpairs()
```

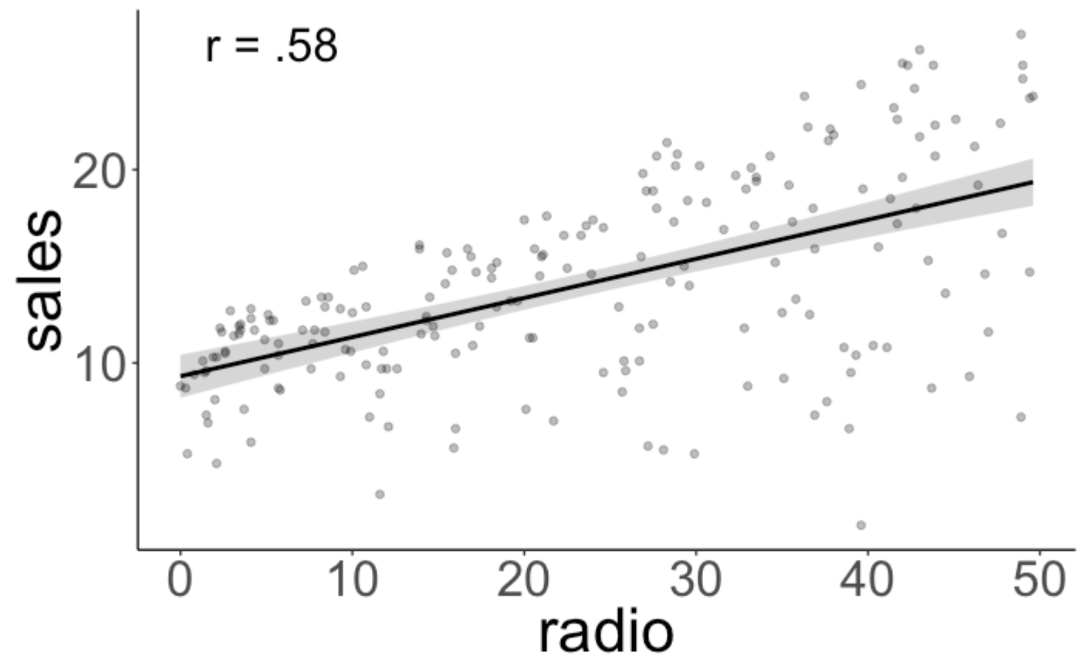


$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

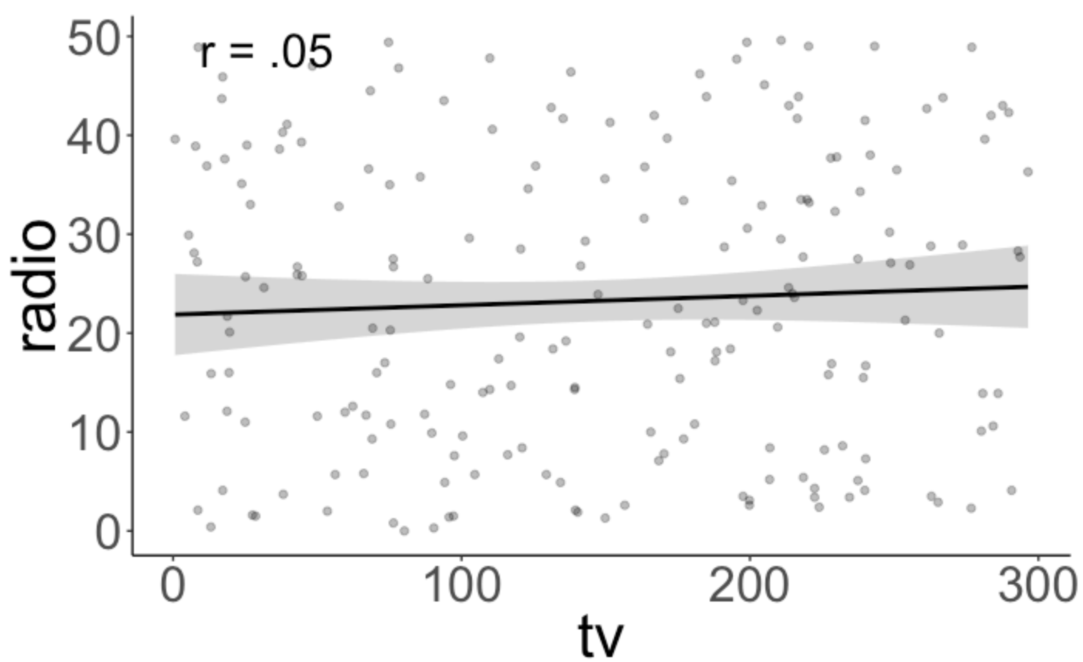
Relationship between TV ads and sales



Relationship between radio ads and sales



Relationship between TV ads and radio ads



predictors are not correlated, yay!

Can we predict sales better when we consider radio in addition to TV ads?

1. Define H_0 as Model C (compact) and H_1 as Model A (augmented)
2. Fit model parameters to the data
3. Calculate the proportional reduction of error (PRE) in our sample
4. Decide whether the augmented model is **worth it** by comparing the observed PRE in our sample to the sampling distribution of PRE (assuming that H_0 is true)

H_0: Radio ads and sales are not related once we control for TV ads.

Model C

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + e_i$$

H_1: Radio ads and sales are related even when we control for TV ads.

Model A

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

```r
# fit the models
fit_c = lm(sales ~ 1 + tv, data = df.ads)
fit_a = lm(sales ~ 1 + tv + radio, data = df.ads)

# do the F test
anova(fit_c, fit_a)
```

we reject the H_0

```
Analysis of Variance Table

Model 1: sales ~ 1 + tv
Model 2: sales ~ 1 + tv + radio
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1    198 2102.53
2    197  556.91  1    1545.6 546.74 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Evaluating the model: Model fit

```
fit_a %>%
  glance()
```

r.squared	adj.r.squared	sigma	statistic	p.value	df	logLik	AIC	BIC	deviance	df.residual
0.897	0.896	1.681	859.618	0	3	-386.197	780.394	793.587	556.914	197

`r.squared`	The percent of variance explained by the model
`adj.r.squared`	r.squared adjusted based on the degrees of freedom
`sigma`	The square root of the estimated residual variance
`statistic`	F-statistic
`p.value`	p-value from the F test, describing whether the full regression is significant
`df`	Degrees of freedom used by the coefficients
`logLik`	the data's log-likelihood under the model
`AIC`	the Akaike Information Criterion
`BIC`	the Bayesian Information Criterion
`deviance`	deviance
`df.residual`	residual degrees of freedom

Evaluating the model: Model fit

```
fit_a %>%
  glance()
```

r.squared	adj.r.squared	sigma	statistic	p.value	df	logLik	AIC	BIC	deviance	df.residual
0.897	0.896	1.681	859.618	0	3	-386.197	780.394	793.587	556.914	197

Compact Model

$$\text{sales}_i = b_0 + e_i$$

The augmented model reduces 89.7% of the error compared to a compact model that just predicts the mean.

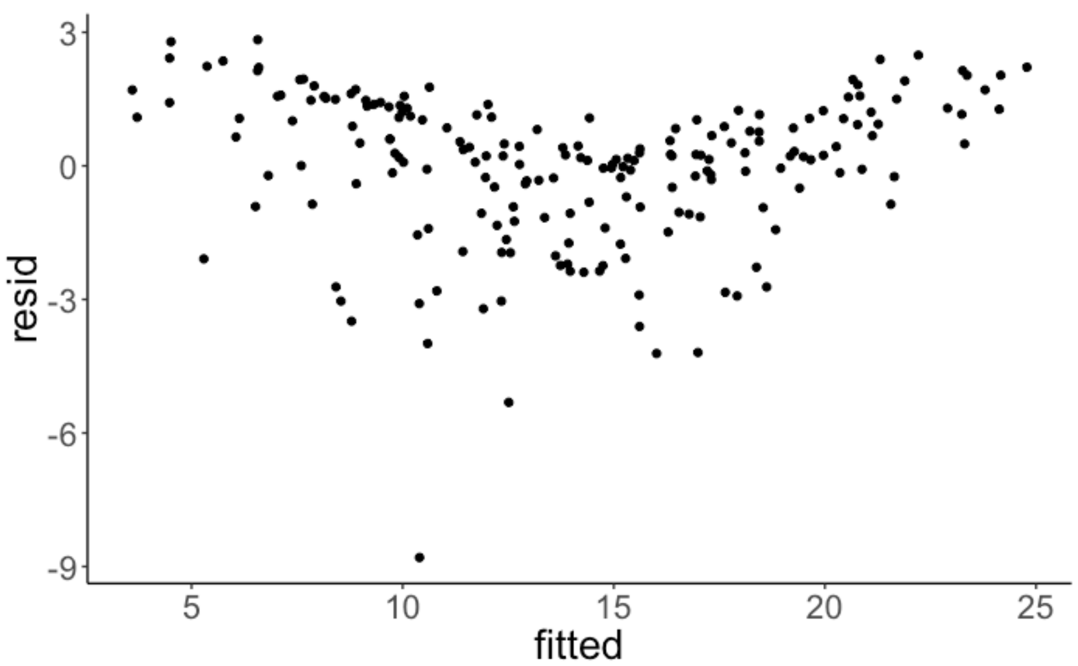
Augmented Model

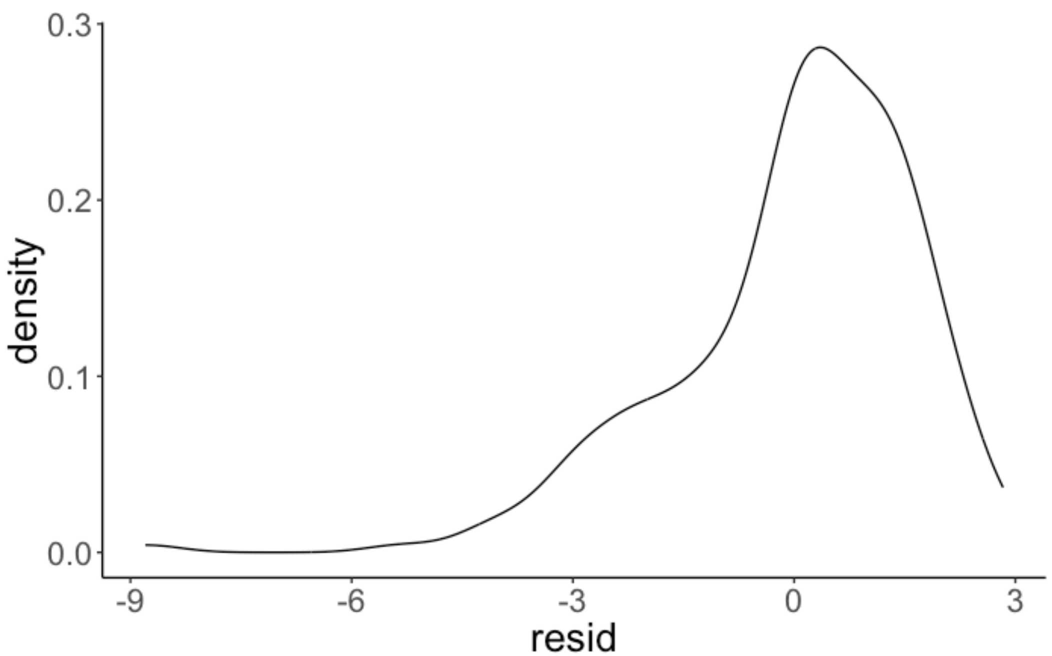
$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

$$\text{PRE} = 1 - \frac{\text{SSE}(A)}{\text{SSE}(C)} = R^2$$

Evaluating the model: Residual plots

`resid = sales - fitted`





OKish overall

Interpreting the results

```
fit_a %>%
  tidy(conf.int = T)
```

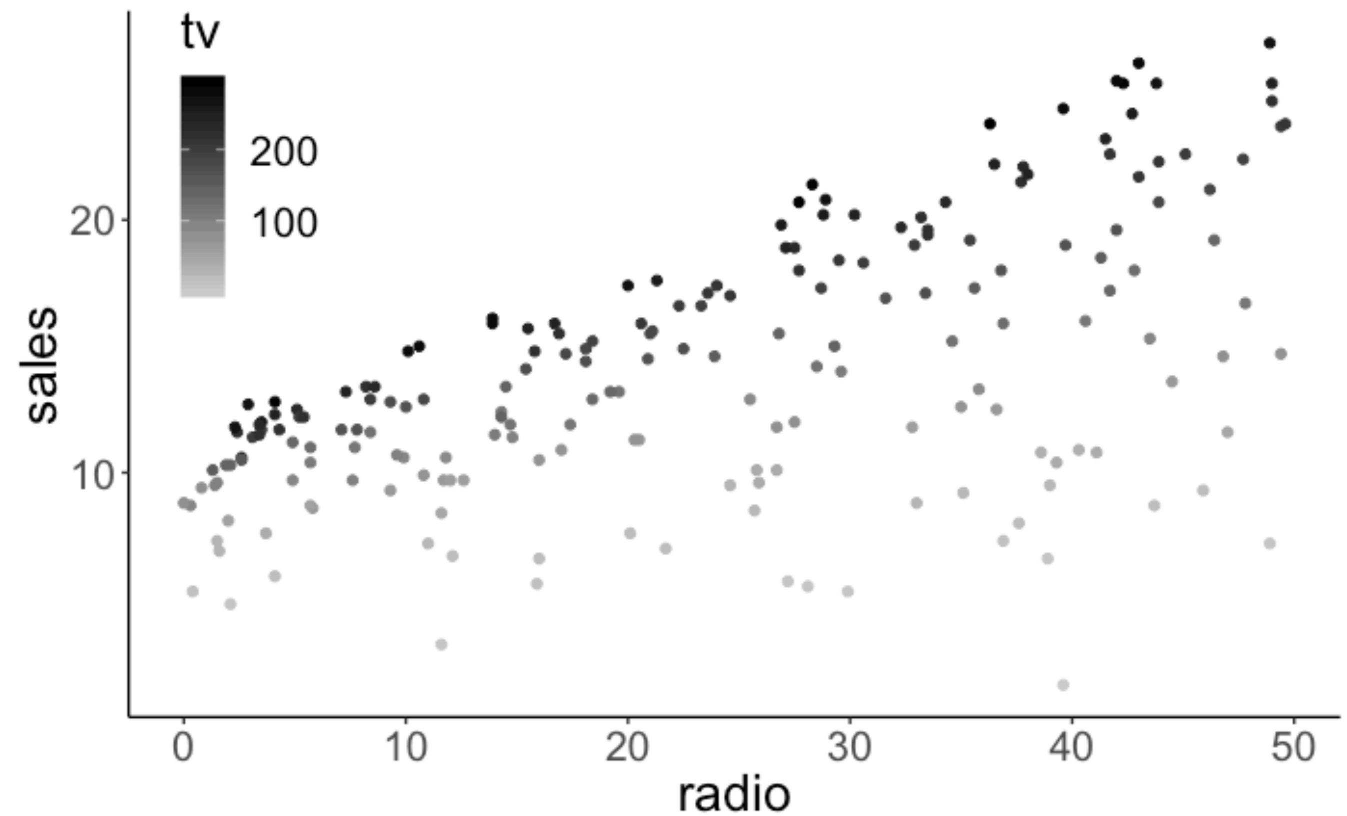
term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	2.92	0.29	9.92	0	2.34	3.50
tv	0.05	0.00	32.91	0	0.04	0.05
radio	0.19	0.01	23.38	0	0.17	0.20

$$\text{sales}_i = b_0 + b_1 \cdot \text{tv}_i + b_2 \cdot \text{radio}_i + e_i$$

$$\widehat{\text{sales}}_i = 2.92 + 0.05 \cdot \text{tv}_i + \boxed{0.19 \cdot \text{radio}_i}$$

For a given amount of TV advertising, an additional $1000 on radio advertising leads to an increase in sales by 190 units.

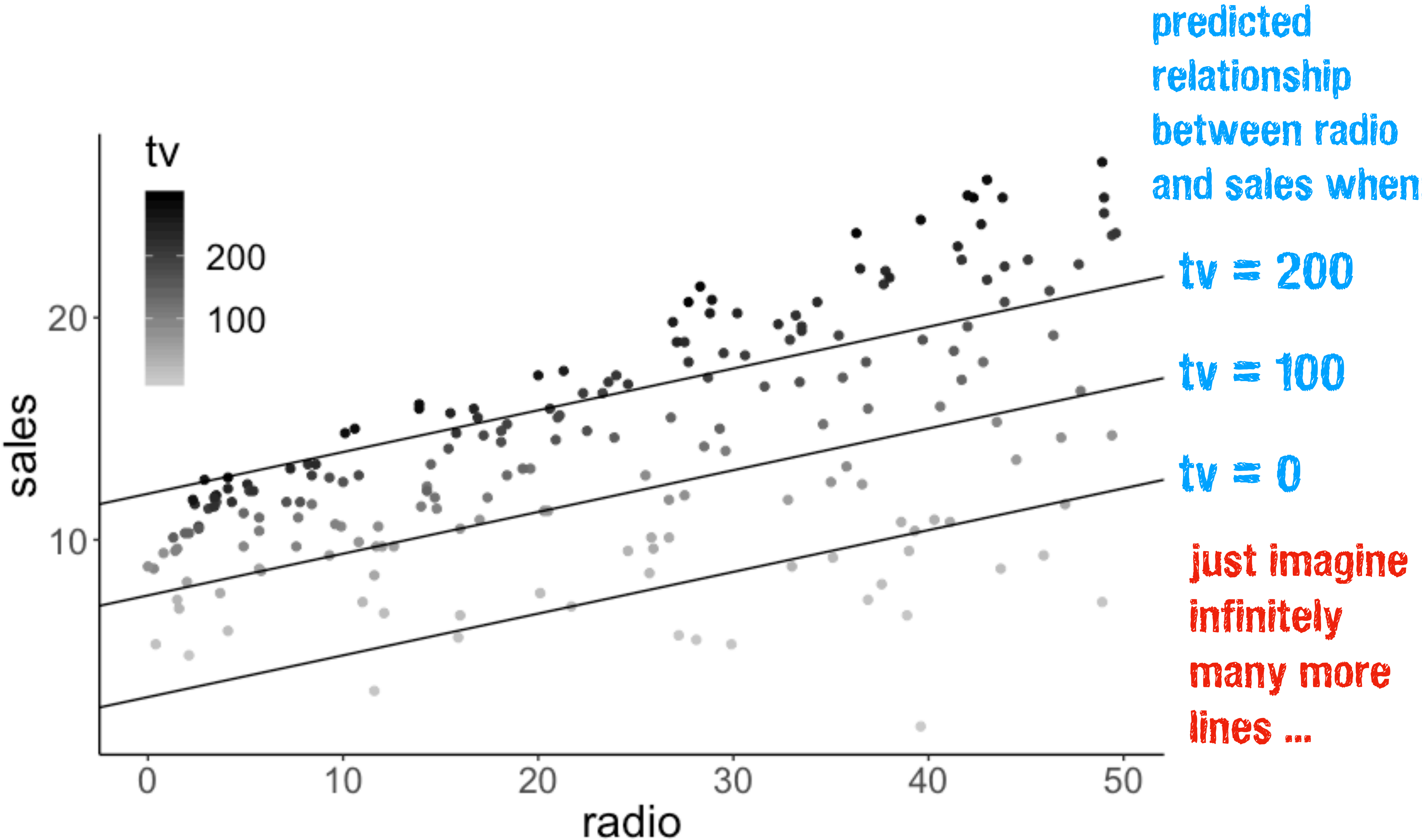
Reporting results



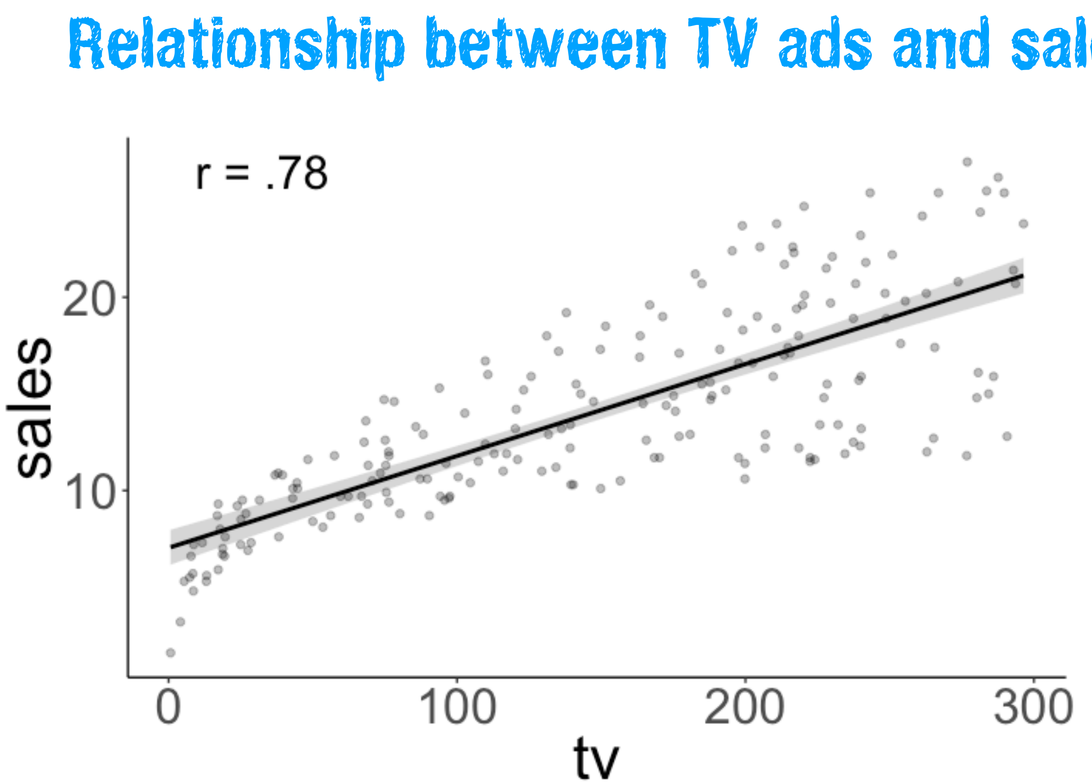
There is a significant relationship between sales and radio ads, controlling for TV ads $F(1, 197) = 546.74$, $p < .001$.

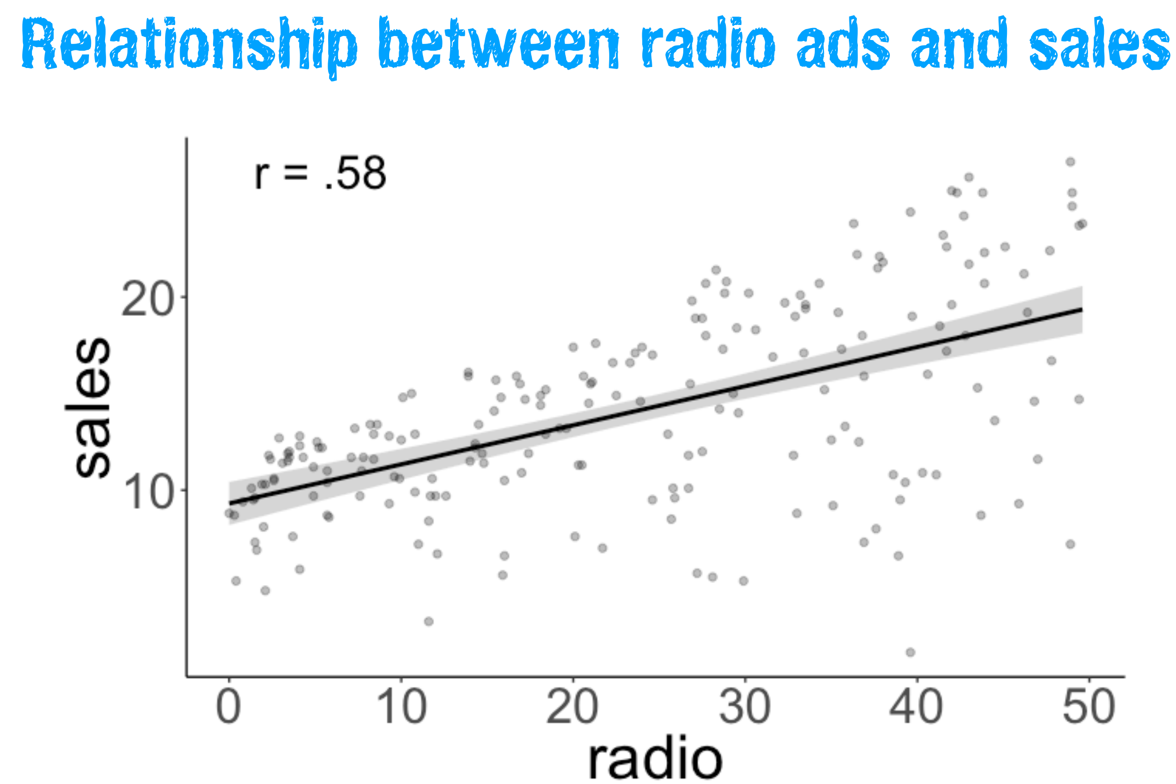
Holding TV ads fixed, an increase in $1000 on radio ads is predicted to increase sales by 190 units [170, 200] (95% confidence intervals).

Visualizing the results



Why can't I just run several simple regressions?





We found that both TV ads and radio ads were related to sales.

But did we need to run a multiple regression? Could we not just have looked at correlations?

Advertising data set

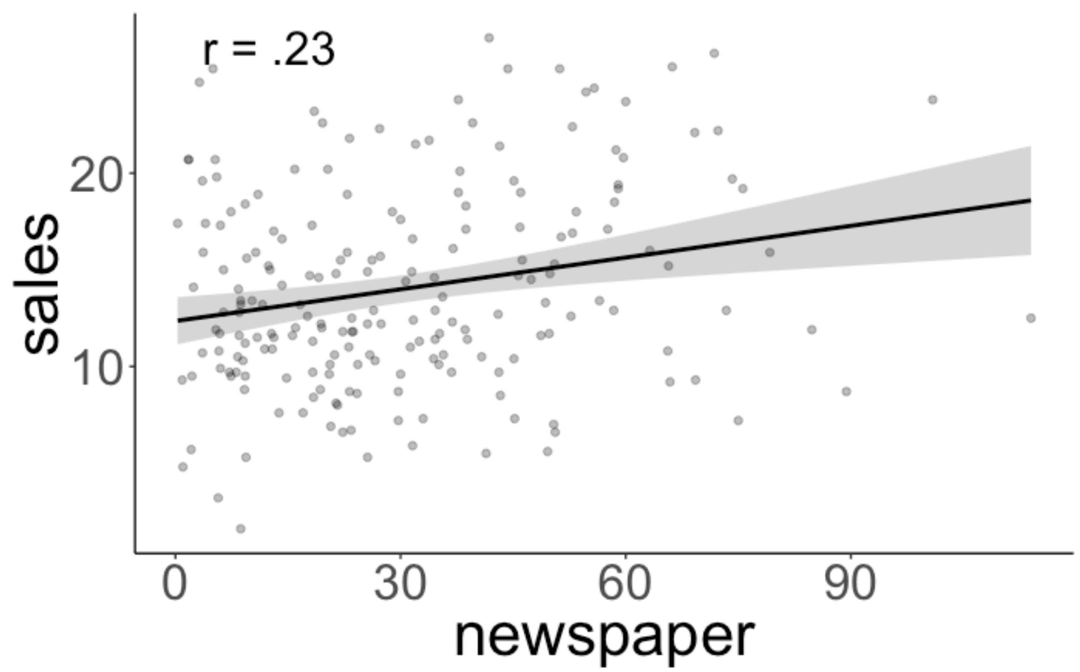
money spent on different media (x $1000)

sales (x1000)

index	tv	radio	newspaper	sales
1	230.1	37.8	69.2	22.1
2	44.5	39.3	45.1	10.4
3	17.2	45.9	69.3	9.3
4	151.5	41.3	58.5	18.5
5	180.8	10.8	58.4	12.9
6	8.7	48.9	75.0	7.2
7	57.5	32.8	23.5	11.8
8	120.2	19.6	11.6	13.2
9	8.6	2.1	1.0	4.8
10	199.8	2.6	21.2	10.6

Are newspaper ads and sales related when controlling for radio ads and TV ads?

Relationship between newspaper ads and sales



this is significant

```r
# fit the models
fit_c = lm(sales ~ 1 + tv + radio, data = df.ads)
fit_a = lm(sales ~ 1 + tv + radio + newspaper, data = df.ads)

# do the F test
anova(fit_c, fit_a)
```

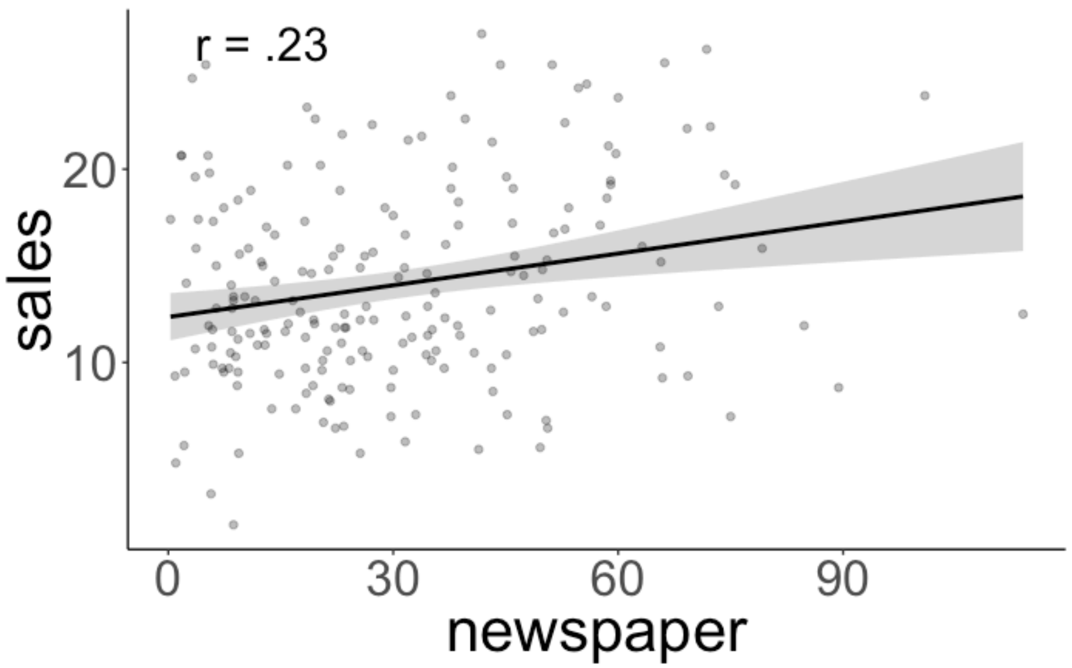
```
Analysis of Variance Table

Model 1: sales ~ 1 + tv + radio
Model 2: sales ~ 1 + tv + radio + newspaper
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    197 556.91
2    196 556.83  1  0.088717 0.0312 0.8599
```

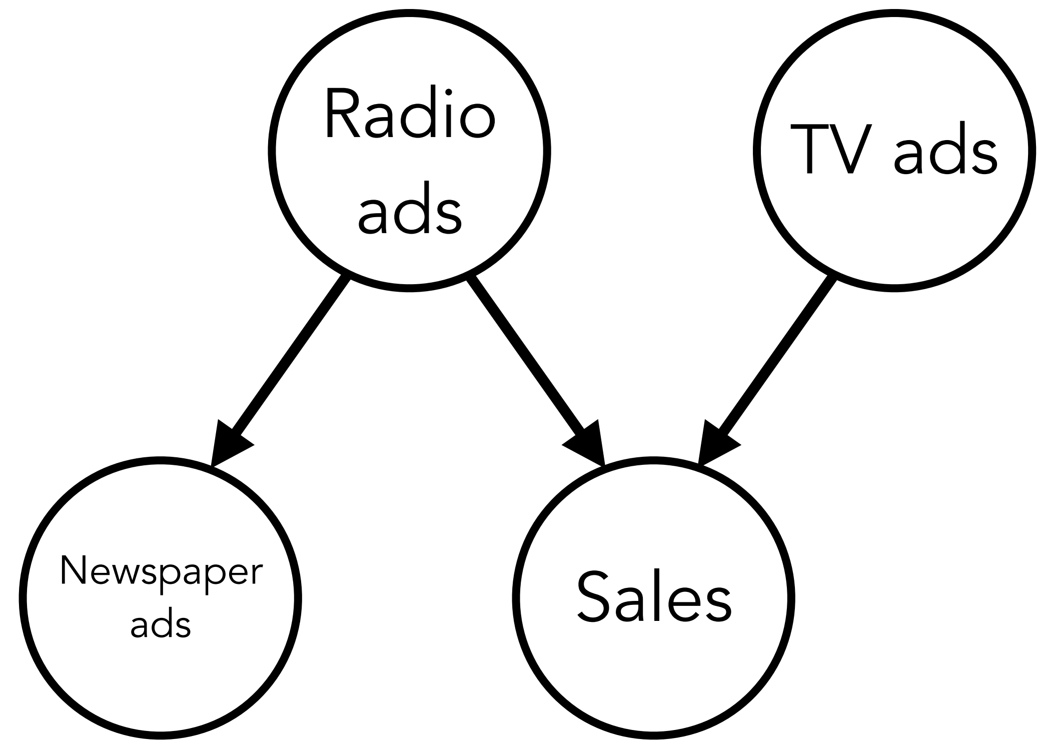
it's not worth it

sales ~ 1 + tv	sales ~ 1 + radio
sales ~ 1 + tv + newspaper	sales ~ 1 + radio + newspaper
it's worth it	**it's not worth it**

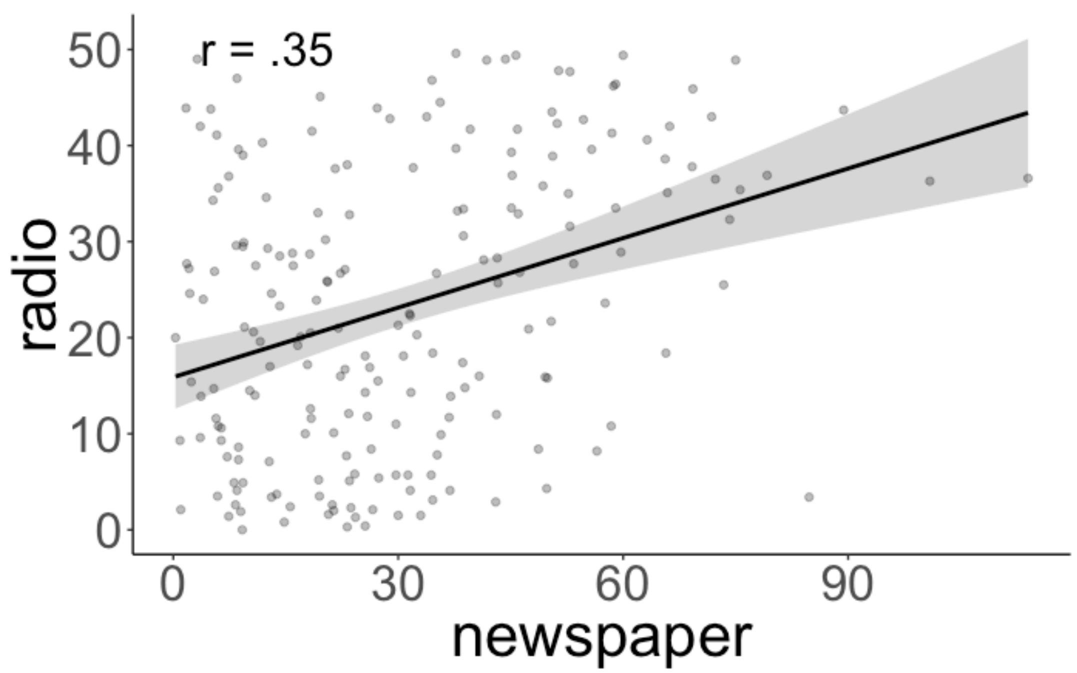
Are newspaper ads and sales related when controlling for radio ads and TV ads?

Relationship between newspaper ads and sales

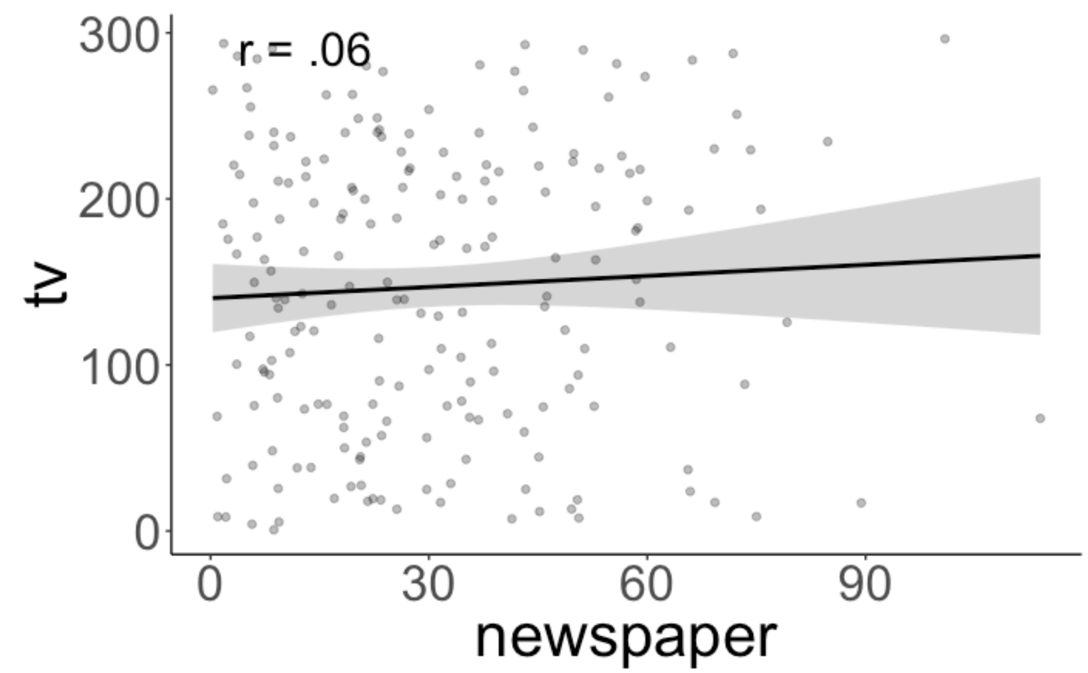




Relationship between newspaper and radio ads



Relationship between newspaper and TV ads



Categorical predictors

Credit data set

`df.credit`

index	income	limit	rating	cards	age	education	gender	student	married	ethnicity	balance
1	14.89	3606	283	2	34	11	Male	No	Yes	Caucasian	333
2	106.03	6645	483	3	82	15	Female	Yes	Yes	Asian	903
3	104.59	7075	514	4	71	11	Male	No	No	Asian	580
4	148.92	9504	681	3	36	11	Female	No	No	Asian	964
5	55.88	4897	357	2	68	16	Male	No	Yes	Caucasian	331
6	80.18	8047	569	4	77	10	Male	No	No	Caucasian	1151
7	21.00	3388	259	2	37	12	Female	No	No	African American	203
8	71.41	7114	512	2	87	9	Male	No	No	Asian	872
9	15.12	3300	266	5	66	13	Female	No	No	Caucasian	279
10	71.06	6819	491	3	41	19	Female	Yes	Yes	African American	1350

variable	description
income	in thousand dollars
limit	credit limit
rating	credit rating
cards	number of credit cards
age	in years
education	years of education
gender	male or female
student	student or not
married	married or not
ethnicity	African American, Asian, Caucasian
balance	average credit card debt in dollars

`nrow(df.credit) = 400`

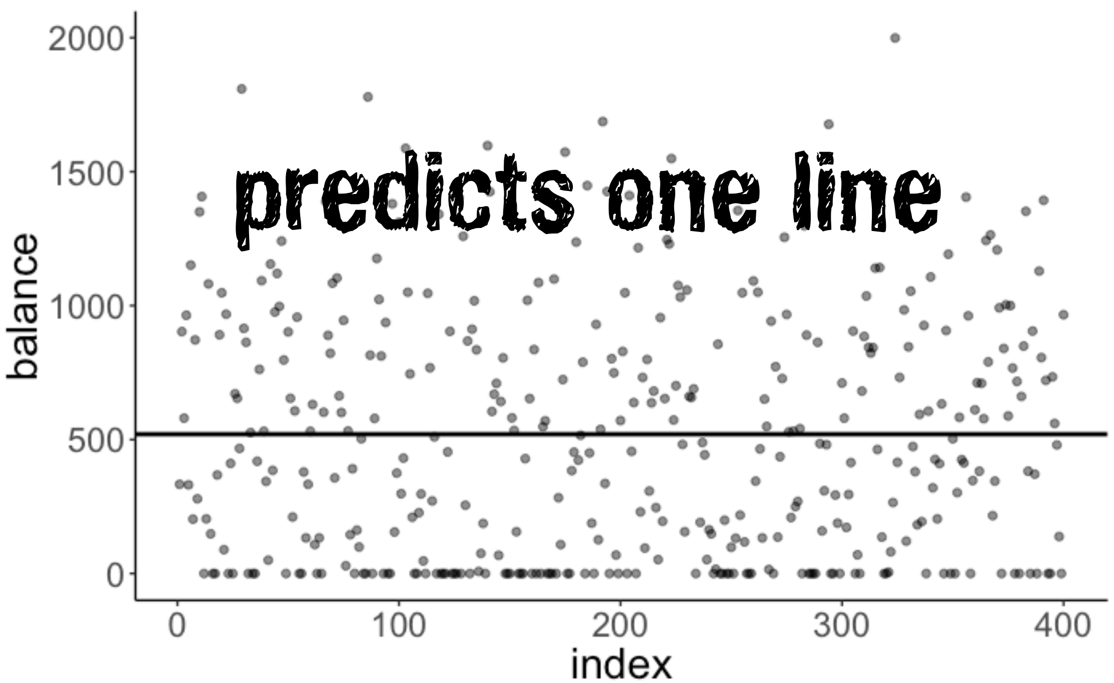
Do students have a different credit card balance from non-students?

H_0: Students and non-students have the same balance.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

Model prediction



Fitted model

$$Y_i = 520.02 + e_i$$

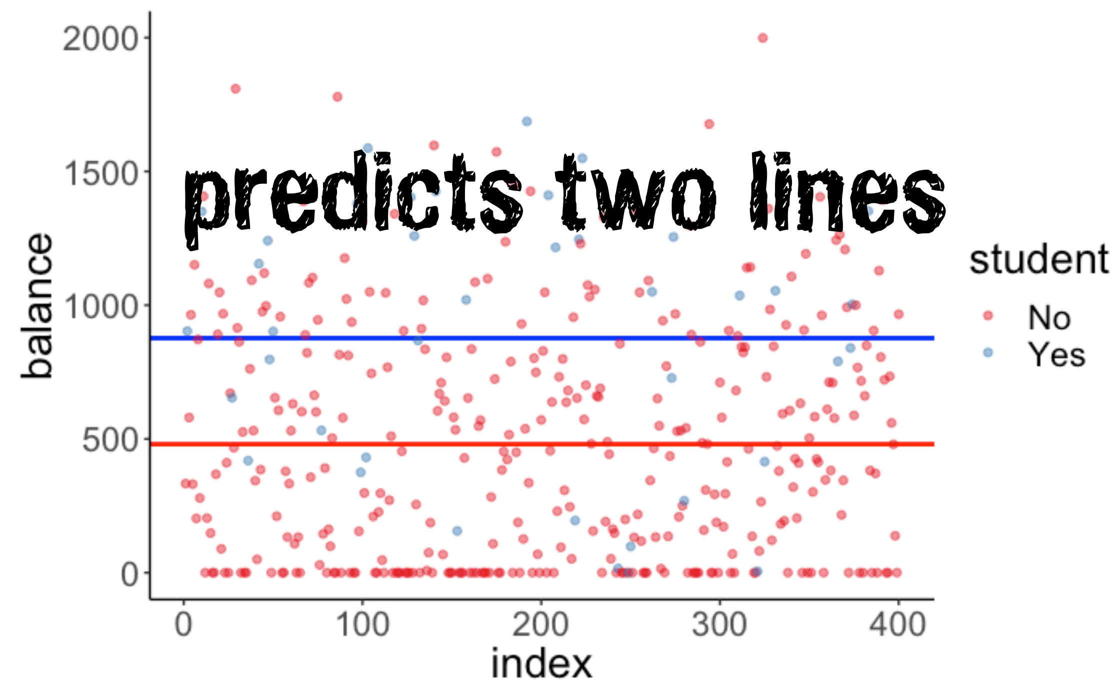
H_1: Students and non-students have different balances.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

student

Model prediction



Fitted model

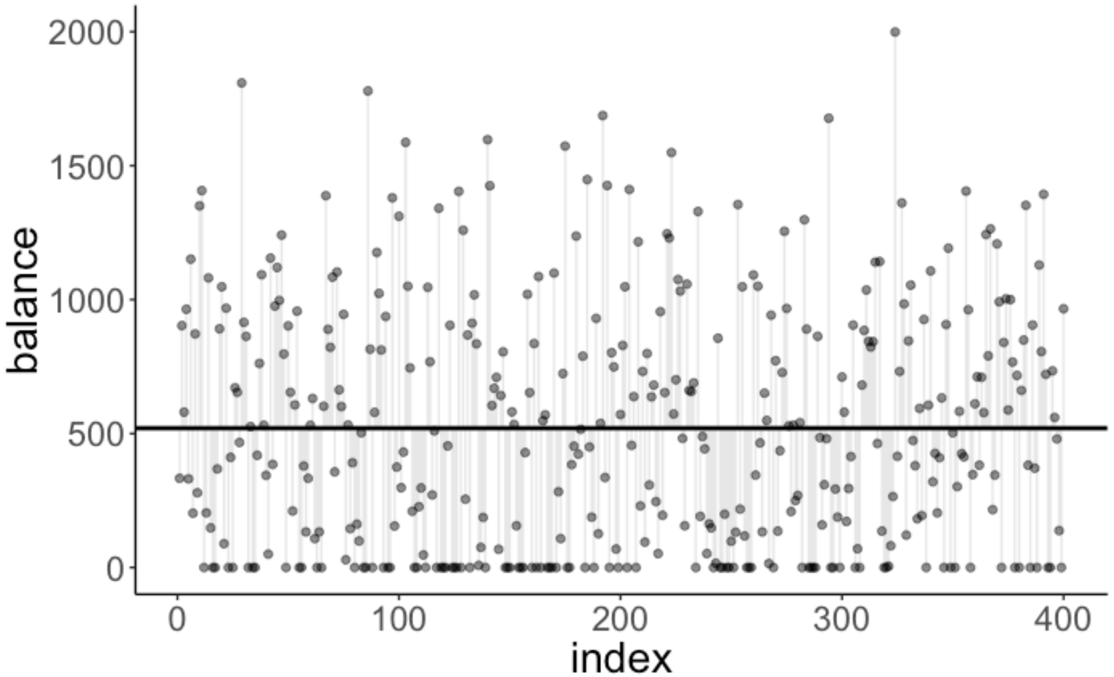
$$Y_i = 480.37 + 396.46 X_i + e_i$$

H_0: Students and non-students have the same balance.

Model C

$$Y_i = \beta_0 + \epsilon_i$$

Model prediction



Fitted model

$$Y_i = 520.02 + e_i$$

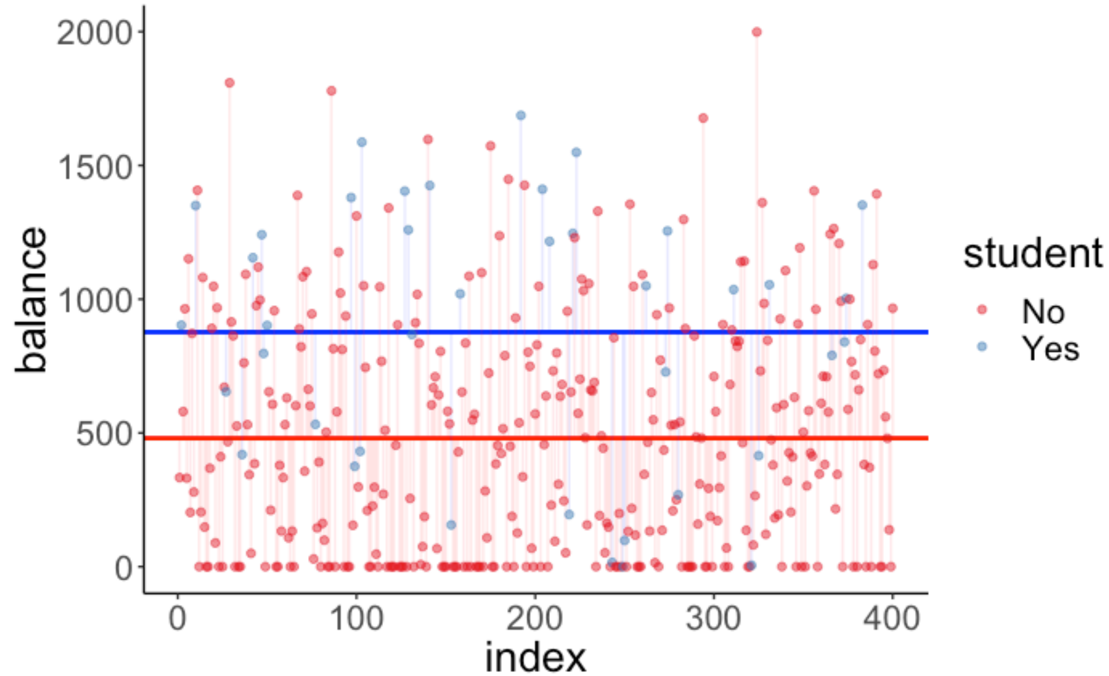
H_1: Students and non-students have different balances.

Model A

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

student

Model prediction



Fitted model

$$Y_i = 480.37 + 396.46 X_i + e_i$$

Worth it?

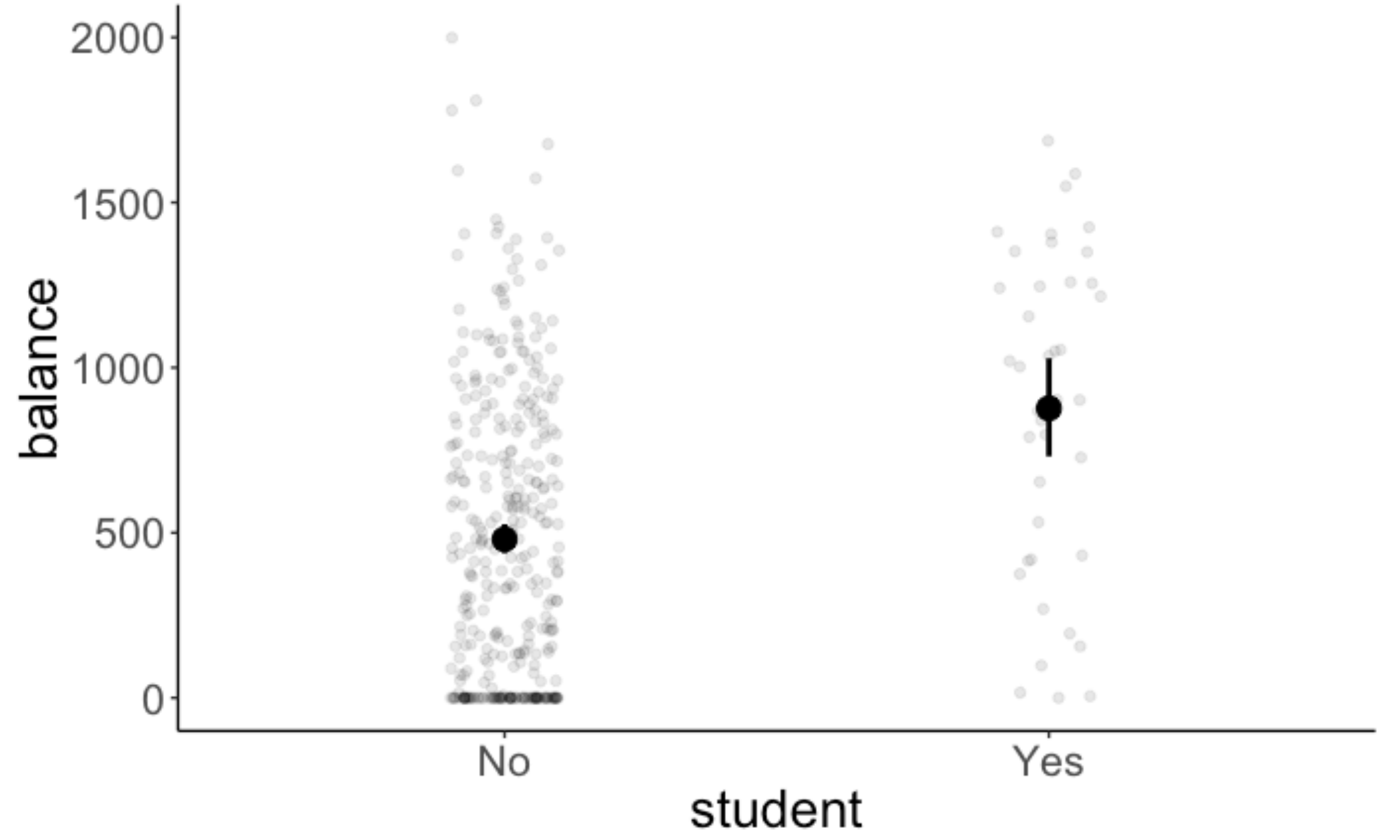
```r
# fit the models
fit_c = lm(balance ~ 1, data = df.credit)
fit_a = lm(balance ~ 1 + student, data = df.credit)

# run the F test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ student
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)    
1    399 84339912                                  
2    398 78681540  1   5658372 28.622 1.488e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Worth it!

Two sample t-test (with independent groups)

Reporting the results



Students have a significantly higher average credit card balance (*Mean* = 876.83, *SD* = 490.00) than non-students (*Mean* = 480.37, *SD* = 439.41), $F(1, 398) = 28.622$, $p < .001$.

Interpreting the model

```r
fit_a = lm(balance ~ 1 + student, data = df.credit)
fit_a %>%
  summary()
```

```
Call:
lm(formula = balance ~ student, data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-876.82 -458.82  -40.87  341.88 1518.63

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   480.37      23.43   20.50  < 2e-16 ***
studentYes    396.46      74.10    5.35 1.49e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 444.6 on 398 degrees of freedom
Multiple R-squared:  0.06709,	Adjusted R-squared:  0.06475
F-statistic: 28.62 on 1 and 398 DF,  p-value: 1.488e-07
```

Dummy coding

Dummy coding

$$\widehat{Y}_i = 480.37 + 396.46 \cdot \text{student\_dummy}_i$$

if student = "No" $\widehat{Y}_i = 480.37$

if student = "Yes" $\widehat{Y}_i = 480.37 + 396.46 = 876.83$

student	student_dummy
No	0
Yes	1
No	0
No	0
No	0
No	0
No	0
No	0
No	0
Yes	1

- Reference category is coded as 0, the other category is coded as 1
- When thrown into an `lm()`, R automatically turns character columns into factors, and determines the reference category alphabetically

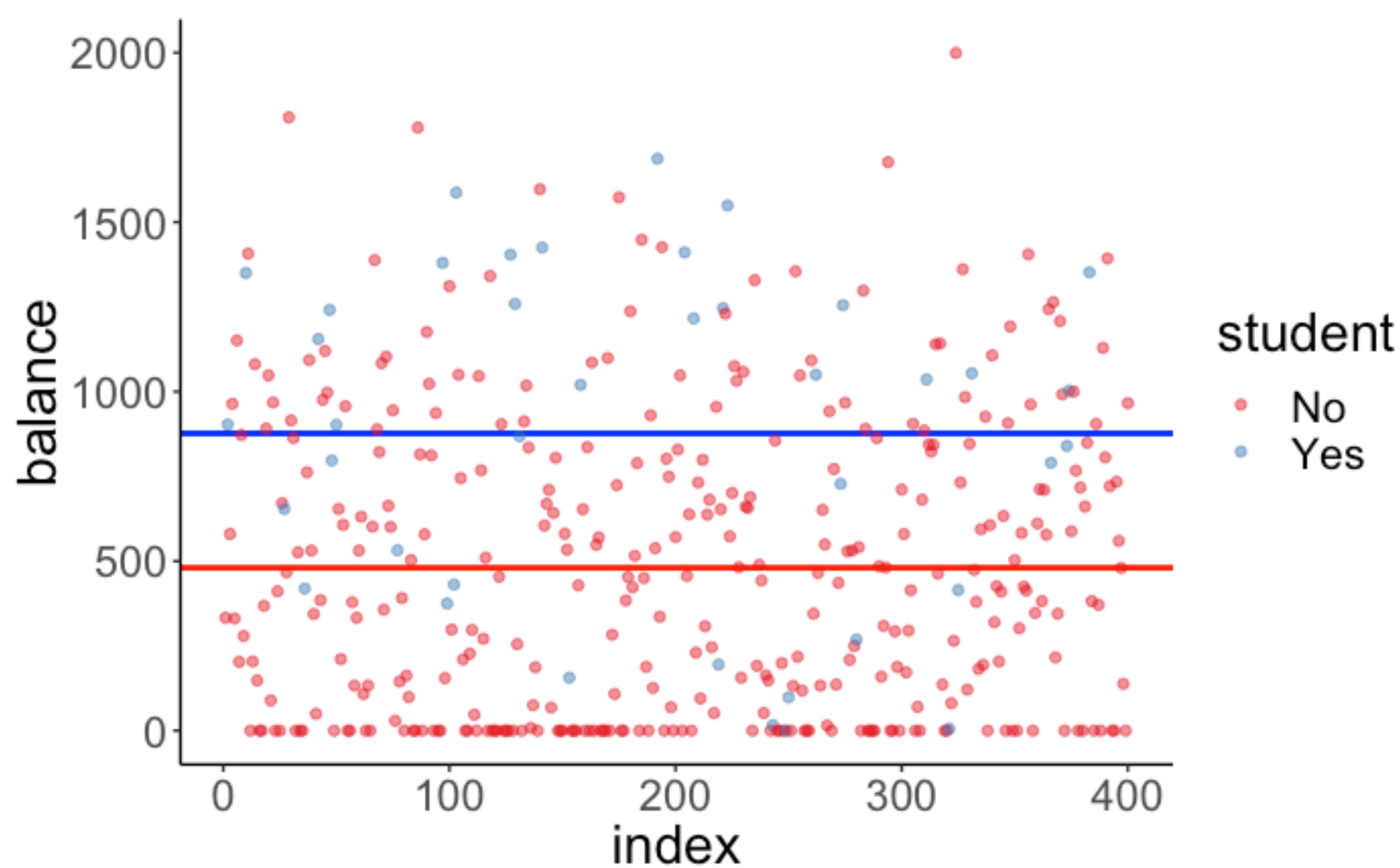
Dummy coding

$$\widehat{Y}_i = 480.37 + 396.46 \cdot \text{student\_dummy}_i$$

if student = "No" $\widehat{Y}_i = 480.37$

if student = "Yes" $\widehat{Y}_i = 480.37 + 396.46 = 876.83$

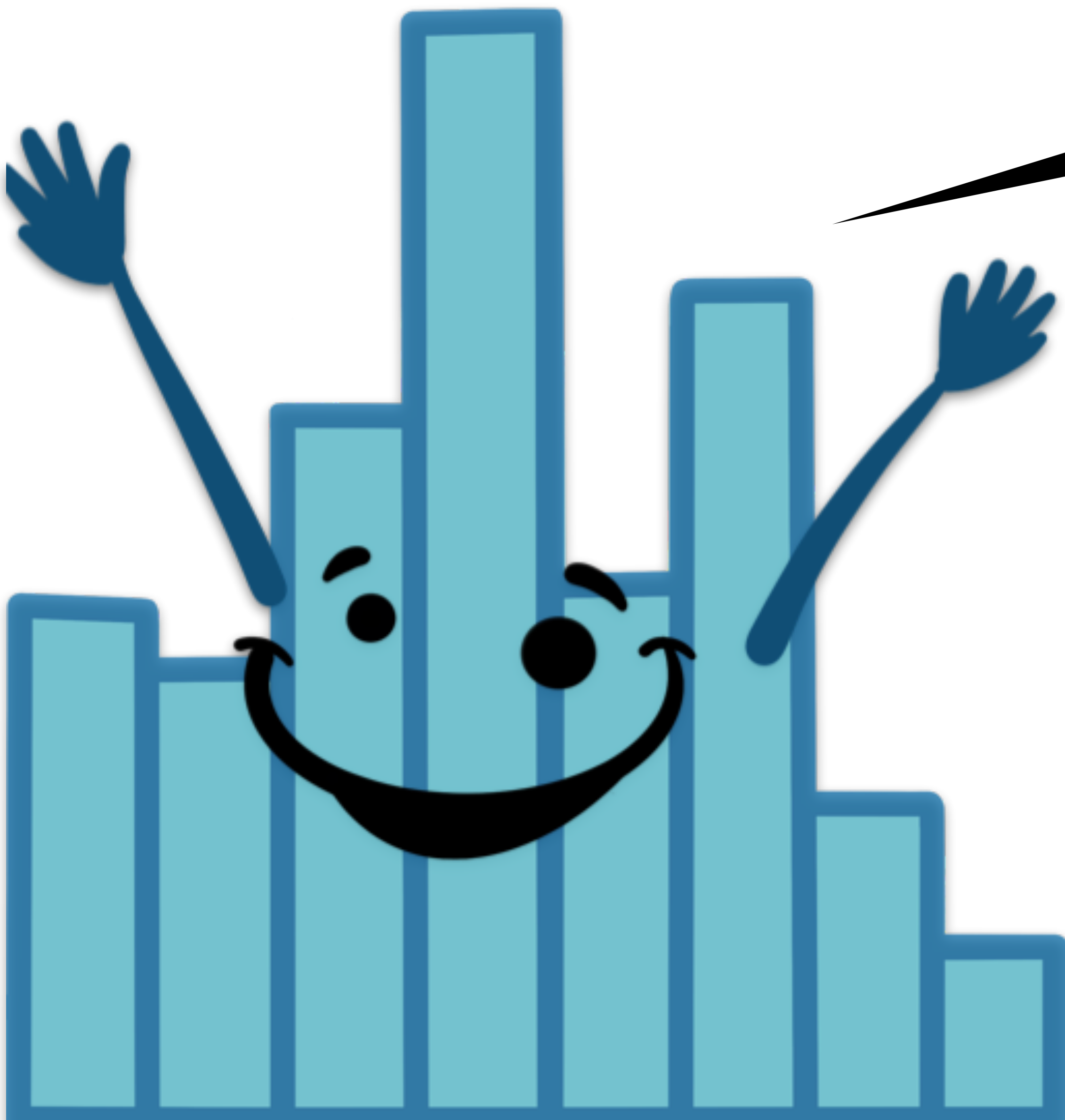
student	student_dummy
No	0
Yes	1
No	0
No	0
No	0
No	0
No	0
No	0
No	0
Yes	1



We're listening to "Candy Käsemann" by "PeterLicht" submitted by Tobi

02:00

stretch break!



Categorical and continuous predictor

Credit data set

`df.credit`

index	income	limit	rating	cards	age	education	gender	student	married	ethnicity	balance
1	14.89	3606	283	2	34	11	Male	No	Yes	Caucasian	333
2	106.03	6645	483	3	82	15	Female	Yes	Yes	Asian	903
3	104.59	7075	514	4	71	11	Male	No	No	Asian	580
4	148.92	9504	681	3	36	11	Female	No	No	Asian	964
5	55.88	4897	357	2	68	16	Male	No	Yes	Caucasian	331
6	80.18	8047	569	4	77	10	Male	No	No	Caucasian	1151
7	21.00	3388	259	2	37	12	Female	No	No	African American	203
8	71.41	7114	512	2	87	9	Male	No	No	Asian	872
9	15.12	3300	266	5	66	13	Female	No	No	Caucasian	279
10	71.06	6819	491	3	41	19	Female	Yes	Yes	African American	1350

variable	description
income	in thousand dollars
limit	credit limit
rating	credit rating
cards	number of credit cards
age	in years
education	years of education
gender	male or female
student	student or not
married	married or not
ethnicity	African American, Asian, Caucasian
balance	average credit card debt in dollars

`nrow(df.credit) = 400`

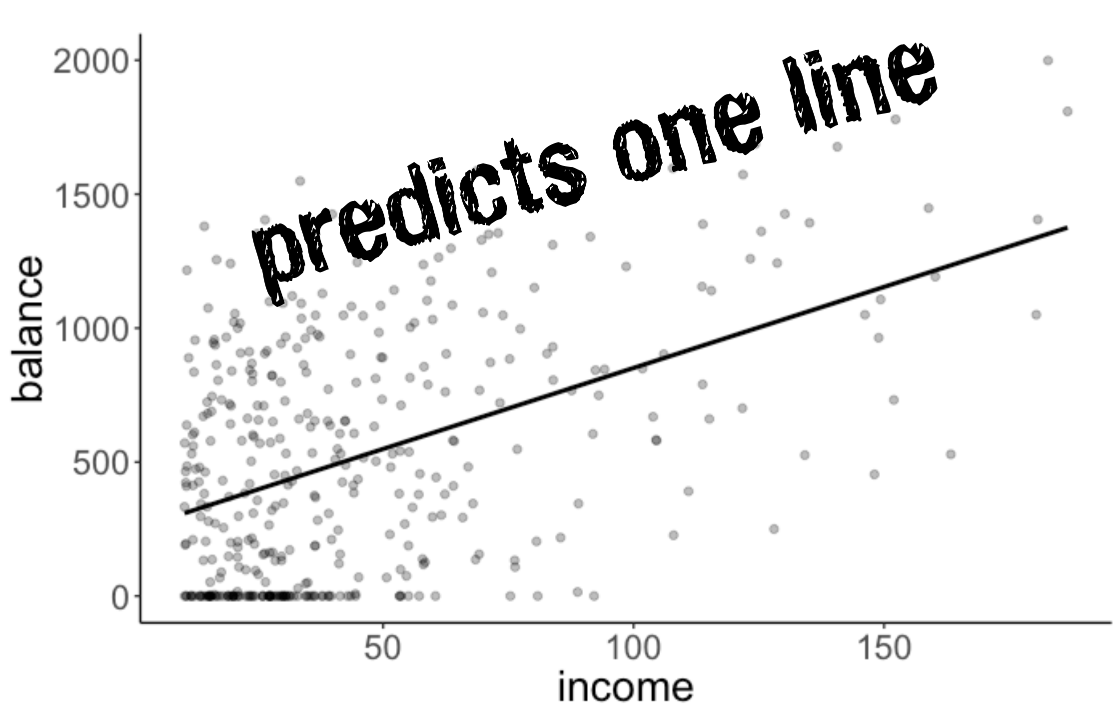
Do students have a different credit card balance from non-students, when controlling for income?

H_0: Students and non-students have the same balance, when controlling for income.

Model C

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \epsilon_i$$

Model prediction



Fitted model

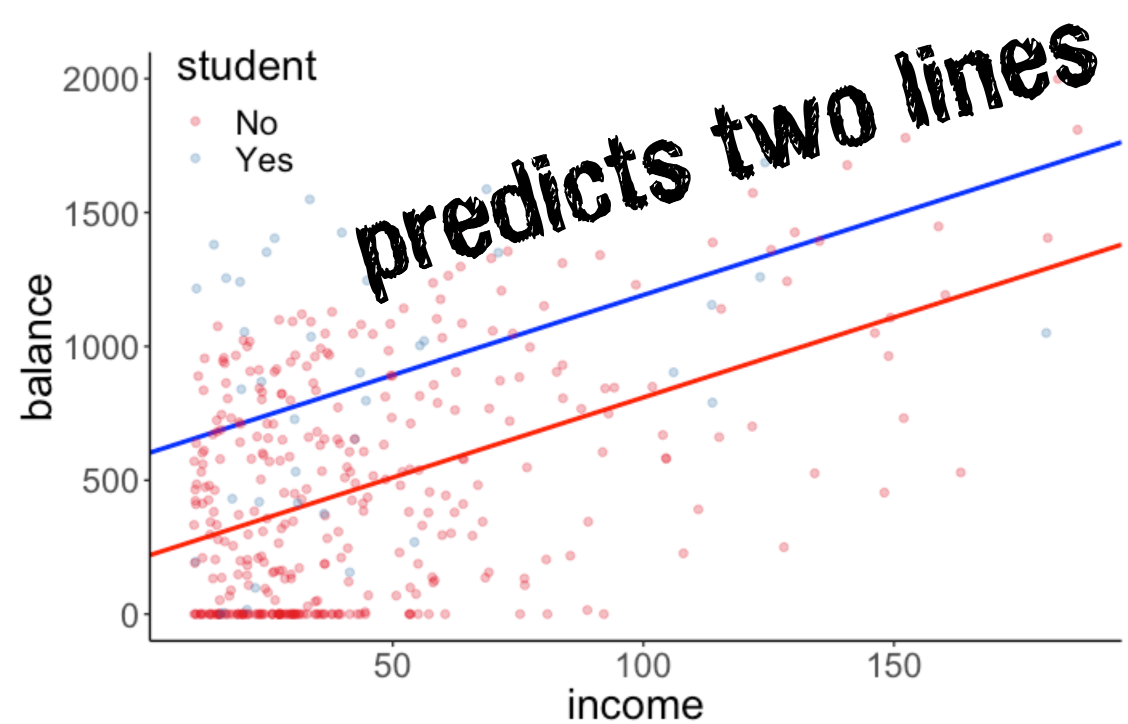
$$\widehat{\text{balance}}_i = 246.515 + 6.048 \cdot \text{income}_i$$

H_1: Students and non-students have different balances, when controlling for income.

Model A

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \beta_2 \text{student}_i + \epsilon_i$$

Model prediction



Fitted model

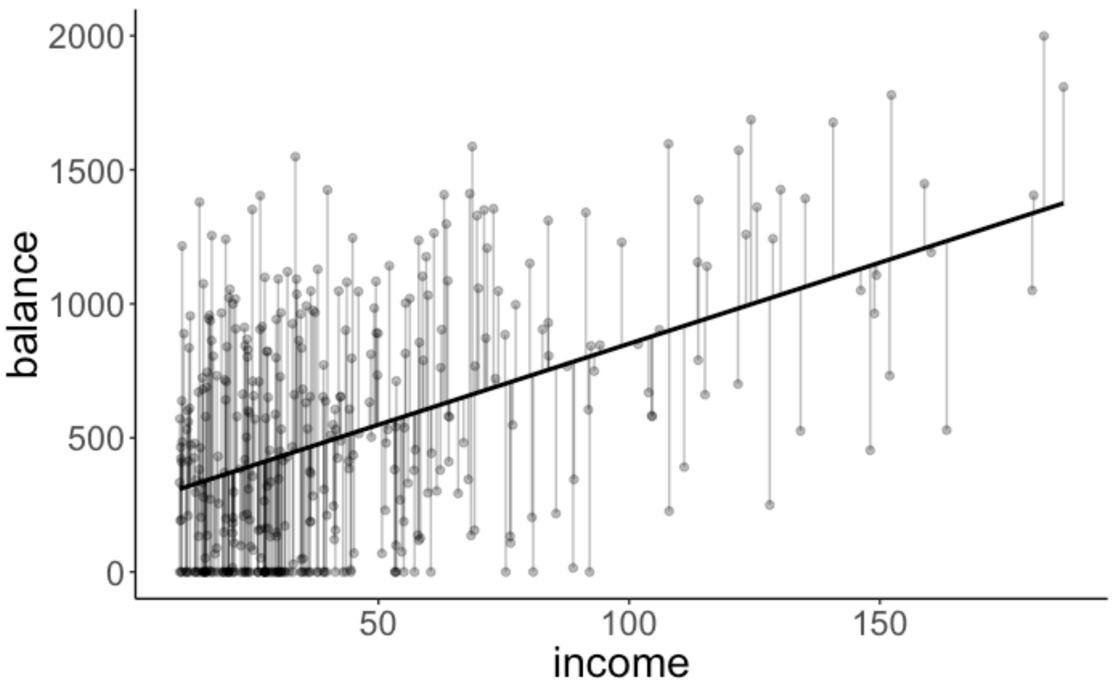
$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i$$

H_0: Students and non-students have the same balance, when controlling for income.

Model C

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \epsilon_i$$

Model prediction



Fitted model

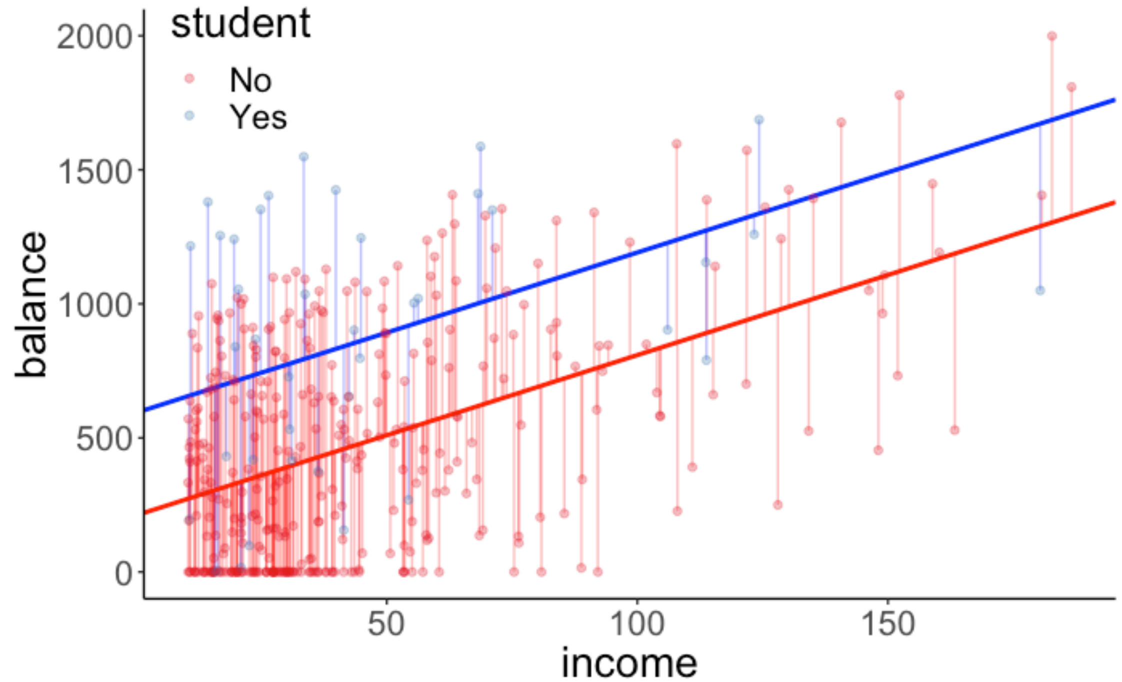
$$\widehat{\text{balance}}_i = 246.515 + 6.048 \cdot \text{income}_i$$

H_1: Students and non-students have different balances, when controlling for income.

Model A

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \beta_2 \text{student}_i + \epsilon_i$$

Model prediction



Fitted model

$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i$$

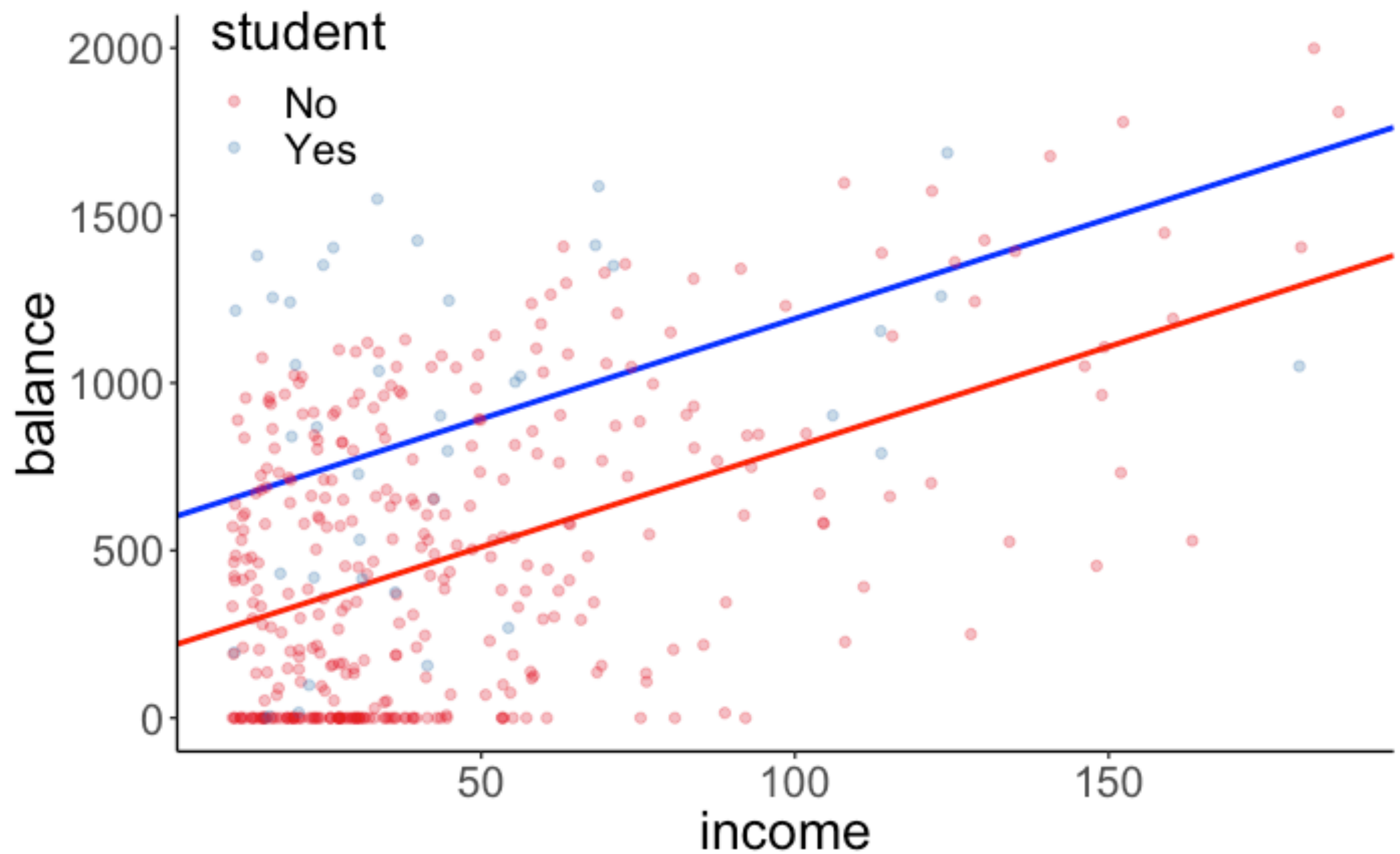
Worth it?

```r
# fit the models
fit_c = lm(balance ~ 1 + income, df.credit)
fit_a = lm(balance ~ 1 + income + student, df.credit)

# run the F test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1 + income
Model 2: balance ~ 1 + income + student
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    398 66208745
2    397 60939054  1   5269691 34.331 9.776e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Worth it!

Interpretation

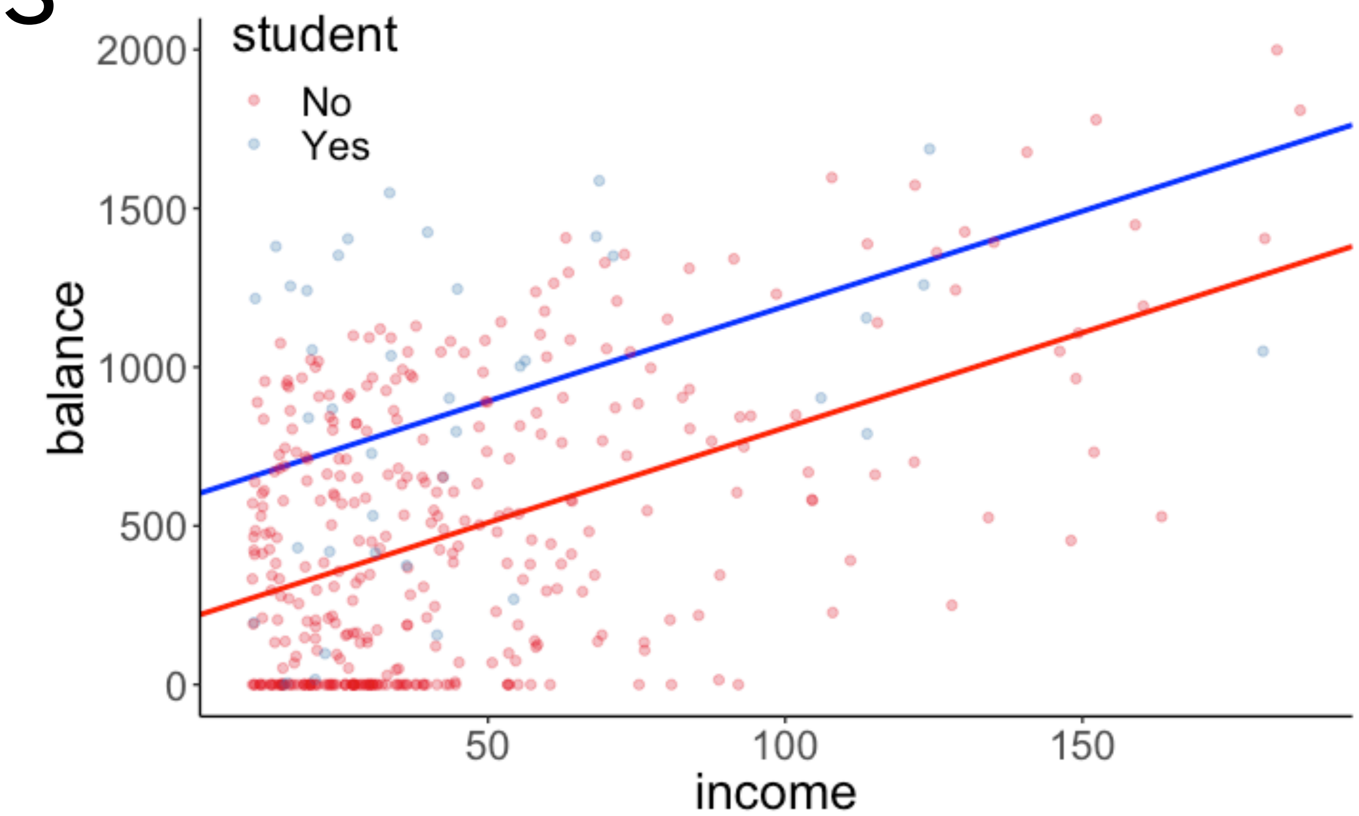


$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i$$

if student = "No" $$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i$$

if student = "Yes" $$\begin{aligned}\widehat{\text{balance}}_i &= 211.14 + 5.98 \cdot \text{income}_i + 382.67 \\ &= 211.14 + 382.67 + 5.98 \cdot \text{income}_i \\ &= 593.81 + 5.98 \cdot \text{income}_i\end{aligned}$$

Reporting the results



Controlling for income, students have a significantly higher average credit card balance (*Mean* = 876.83, *SD* = 490.00) than non-students (*Mean* = 480.37, *SD* = 439.41), $F(1, 397) = 34.331$, $p < .001$.

Interactions

Is the relationship between level of income and balance different for students than it is for non-students?

Compact Model

$$\widehat{\text{balance}}_i = b_0 + b_1\text{income}_i + b_2\text{student}_i$$

Augmented Model

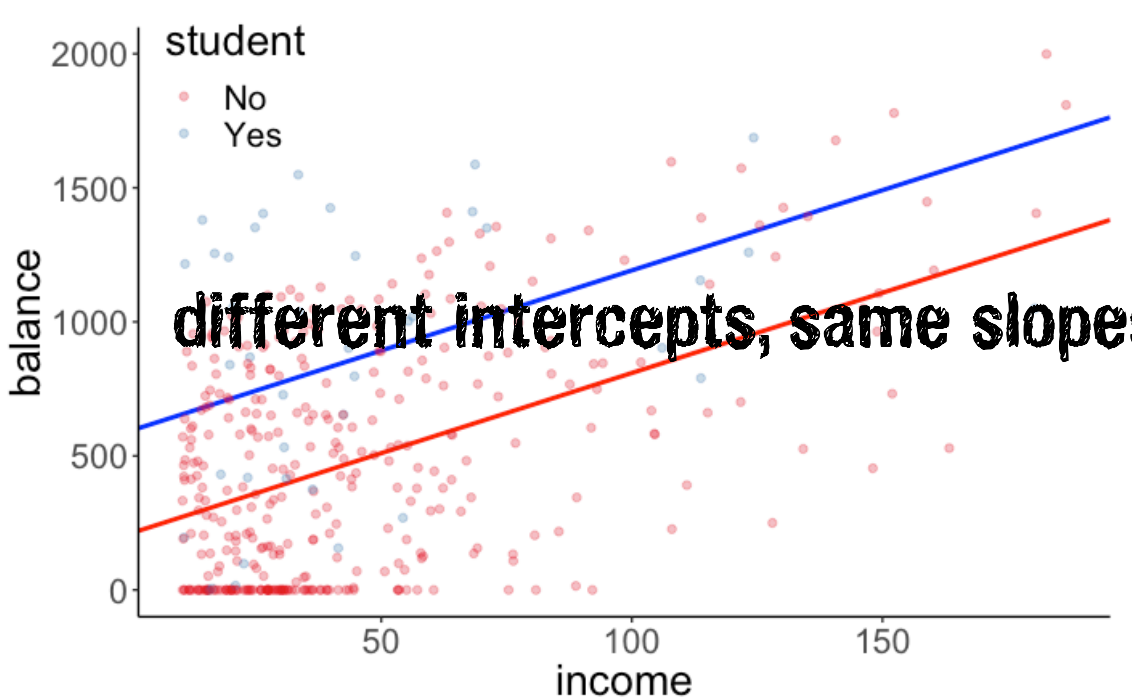
$$\widehat{\text{balance}}_i = b_0 + b_1\text{income}_i + b_2\text{student}_i + b_3(\text{income}_i \times \text{student}_i)$$

H$_0$: The relationship between income and balance is the same for students and non-students.

Model C

$$\text{balance}_i = \beta_0 + \beta_1 \text{income}_i + \beta_2 \text{student}_i + \epsilon_i$$

Model prediction



Fitted model

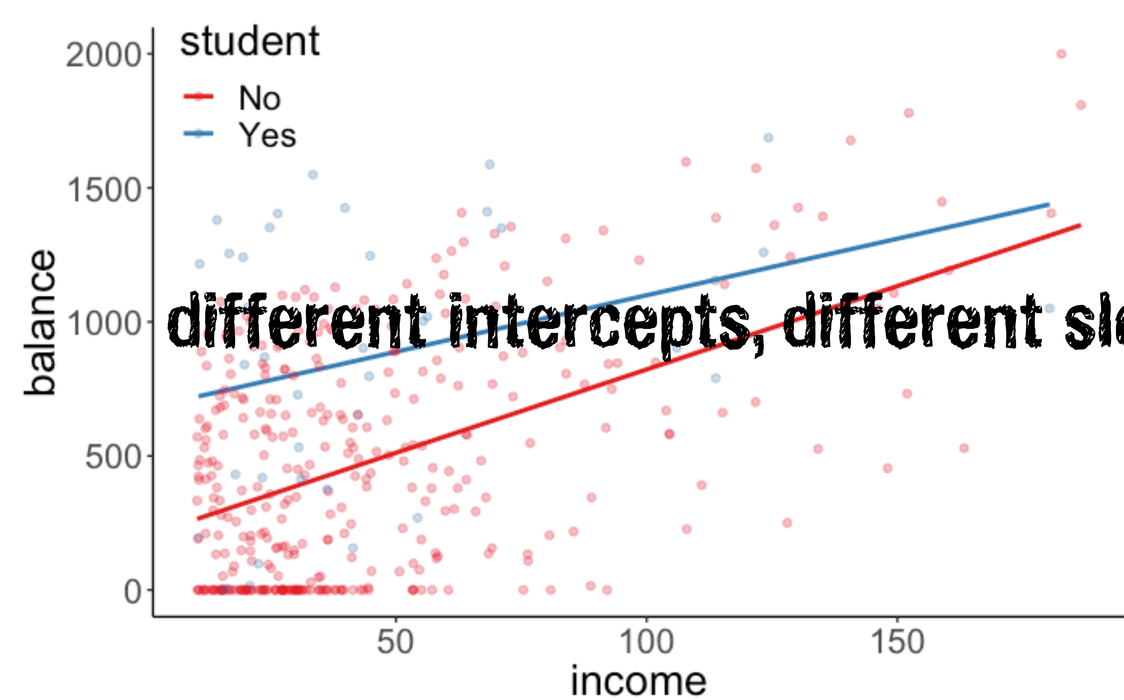
$$\widehat{\text{balance}}_i = 211.14 + 5.98 \cdot \text{income}_i + 382.67 \cdot \text{student}_i$$

H$_1$: The relationship between income and balance differs between students and non-students.

Model A

$$\widehat{\text{balance}}_i = b_0 + b_1 \text{income}_i + b_2 \text{student}_i + b_3(\text{income}_i \times \text{student}_i)$$

Model prediction



Fitted model

$$\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot \text{student}_i - 2.00 \cdot (\text{income}_i \times \text{student}_i)$$

Worth it?

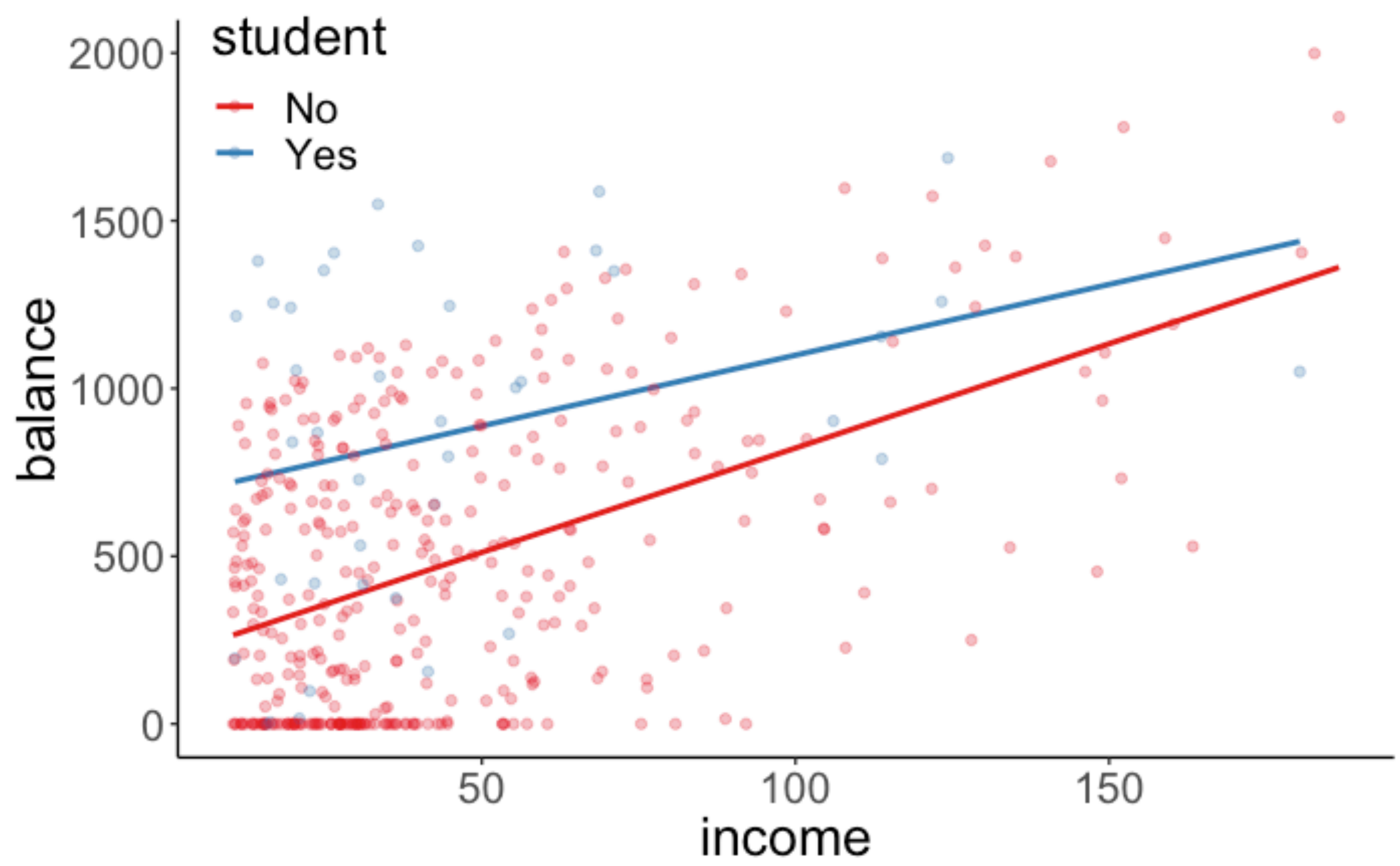
Is the relationship between level of income and balance different for students than it is for non-students?

```
# fit models
fit_c = lm(formula = balance ~ income + student, data = df.credit)
fit_a = lm(formula = balance ~ income * student, data = df.credit)

# F-test
anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ income + student
Model 2: balance ~ income * student
  Res.Df      RSS Df Sum of Sq      F Pr(>F)
1    397 60939054
2    396 60734545  1    204509 1.3334 0.2489
```

not worth it!

Interpretation



$$\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot \text{student}_i - 2.00 \cdot (\text{income}_i \times \text{student}_i)$$

if student = "No" $\widehat{\text{balance}}_i = 200.62 + 6.22 \cdot \text{income}_i$

if student = "Yes"

$$\begin{aligned}\widehat{\text{balance}}_i &= 200.62 + 6.22 \cdot \text{income}_i + 476.68 \cdot 1 - 2.00 \cdot (\text{income}_i \times 1) \\ &= 677.3 + 6.22 \cdot \text{income}_i - 2.00 \cdot \text{income}_i \\ &= 677.3 + 4.22 \cdot \text{income}_i\end{aligned}$$

Interpretation

```
fit1 = lm(formula = balance ~ income + student + income:student, data = df.credit)
```

Explicitly encode the interaction

```
df.credit %>%
  mutate(student_dummy = ifelse(student == "No", 0, 1)) %>%
  mutate(income_student = income * student_dummy) %>%
  select(balance, income, student, student_dummy, income_student)
```

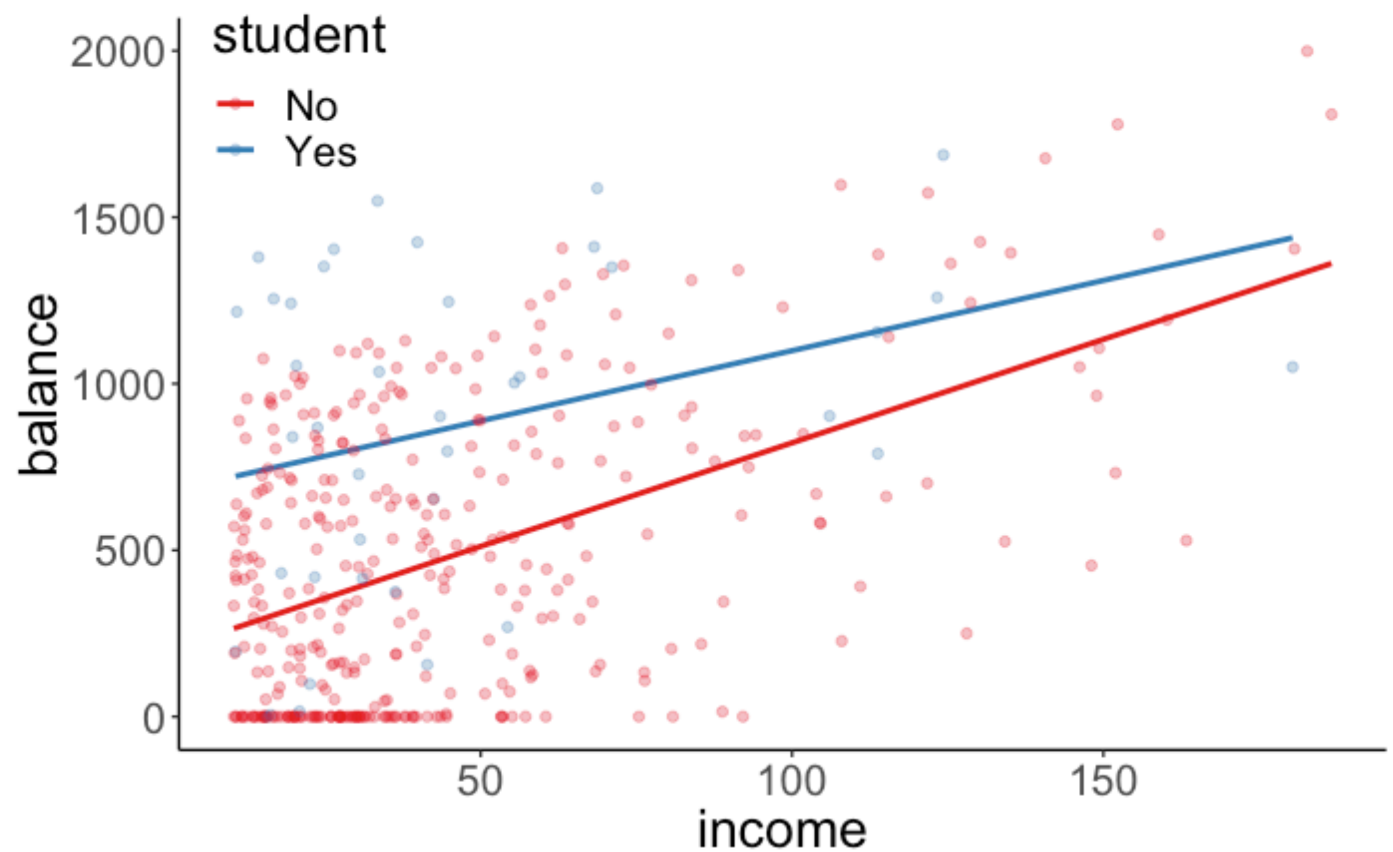
balance	income	student	student_dummy	income_student
333	14.89	No	0	0.00
903	106.03	Yes	1	106.03
580	104.59	No	0	0.00
964	148.92	No	0	0.00
331	55.88	No	0	0.00
1151	80.18	No	0	0.00
203	21.00	No	0	0.00
872	71.41	No	0	0.00
279	15.12	No	0	0.00
1350	71.06	Yes	1	71.06

```
fit2 = lm(formula = balance ~ income + student + income_student, data = df.credit)
```

`fit1` **and** `fit2` **are identical!**

How to report results of interaction

There is no significant difference in the relationship between income and balance for students versus non-students, $F(1, 396) = 1.33$, $p = 0.25$.



For *students*, an increase in \$1000 income is associated with an increase in \$4.21 of average credit card balance.

For *non-students*, an increase in \$1000 income is associated with an increase in \$6.22 of average credit card balance.

`lm()` output

lm() output

```r
lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
  summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
studentYes        476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes  -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799, Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

?

```r
fit_c = lm(formula = balance ~ student + income:student, data = df.credit)
fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)

anova(fit_c, fit_a)
```

```r
fit_c = lm(formula = balance ~ income + student, data = df.credit)
fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)

anova(fit_c, fit_a)
```

lm() output

```r
lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
  summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
studentYes        476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes  -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799, Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

```r
fit_c = lm(formula = balance ~ 1, data = df.credit)
fit_a = lm(formula = balance ~ income + student + income:student, data = df.credit)

anova(fit_c, fit_a)
```

```
Analysis of Variance Table

Model 1: balance ~ 1
Model 2: balance ~ 1 + income
  Res.Df      RSS Df Sum of Sq       F    Pr(>F)
1    399 84339912
2    398 66208745  1  18131167 108.99 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05
'.' 0.1 ' ' 1
```

deterministic mapping between t and F

$$t^2 = F$$

$$10.44^2 = 108.99$$

`anova()` gives me *F*s

but `lm()` gives me *t*s

?

```
Call:
lm(formula = balance ~ 1 + income, data = df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-803.64 -348.99  -54.42  331.75 1100.25

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 246.5148    33.1993   7.425  6.9e-13 ***
income        6.0484     0.5794  10.440  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 407.9 on 398 degrees of freedom
Multiple R-squared:  0.215,     Adjusted R-squared:  0.213
F-statistic:   109 on 1 and 398 DF,  p-value: < 2.2e-16
```

`lm()` output

```
1 lm(formula = balance ~ income + student + income:student, data = df.credit) %>%
2   summary()
```

```
Call:
lm(formula = balance ~ income + student + income:student, data =
df.credit)

Residuals:
    Min      1Q  Median      3Q     Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
studentYes        476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes  -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799,	Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

- runs many hypothesis tests at the same time
- increases the danger of making a type-I error (incorrectly rejecting the H_0)
- will not give us p-values for mixed effects models ...

The model comparison approach

- allows to formulate hypotheses as specific comparisons between candidate models
- is more flexible: we could test a model with 2 predictors vs. one with 4 predictors
- gives us insight into the underlying statistical procedure

`lm()` output

```
Call:
lm(formula = balance ~ income + student + income:student,
data = df.credit)

Residuals:
    Min       1Q   Median       3Q      Max
-773.39 -325.70  -41.13  321.65  814.04

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
income              6.2182     0.5921  10.502  < 2e-16 ***
studentYes        476.6758   104.3512   4.568 6.59e-06 ***
income:studentYes  -1.9992     1.7313  -1.155    0.249
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
1

Residual standard error: 391.6 on 396 degrees of freedom
Multiple R-squared:  0.2799, Adjusted R-squared:  0.2744
F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

what does this mean?

not the overall effect of income!!

instead the predicted effect of income for non-students!

we'll talk more about the difference between simple/conditional effects and main effects next time!

Summary

- Quick recap
- Multiple regression
  - two continuous predictors
  - one categorical predictor
  - one continuous and one categorical predictor
- Interactions
- `lm()` output

Feedback

How was the pace of today's class?

How happy were you with today's class overall?

What did you like about today's class? What could be improved next time?

Thank you!