Resolución

1.

Distribución beta(2,8):

$$E(\pi \mid y) = \frac{\alpha + y}{\alpha + \beta + n} = \frac{2 + 18}{2 + 8 + 50} = 0,33$$

$$\text{Var}(\pi \mid y) = \frac{(\alpha + y)(\beta + n - y)}{(\alpha + \beta + n)^2(\alpha + \beta + n + 1)} =$$
$$\frac{(2 + 18)(8 + 50 - 18)}{(2 + 8 + 50)^2(2 + 8 + 50 + 1)} = 3,64 \cdot 10^{-3}$$

$\alpha_{post} = 20$
$\beta_{post} = 40$

```
> cat(sprintf("Li = %6.3f Ls = %6.3f", qbeta(0.025, 20,40), qbeta(0.975, 20,40)))
Li =  0.221 Ls =  0.456
```

Distribución beta(16,64):

$$E(\pi \mid y) = \frac{16 + 18}{16 + 64 + 50} = 0,26$$

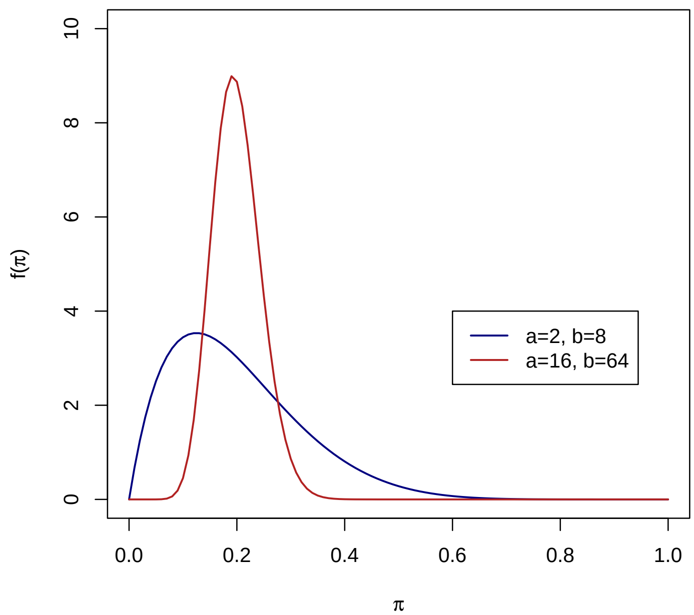
$$\text{Var}(\pi \mid y) = \frac{(\alpha + y)(\beta + n - y)}{(\alpha + \beta + n)^2(\alpha + \beta + n + 1)} =$$
$$\frac{(16 + 18)(64 + 50 - 18)}{(16 + 64 + 50)^2(16 + 64 + 50 + 1)} = 1,47 \cdot 10^{-3}$$

$\alpha_{post} = 34$
$\beta_{post} = 96$

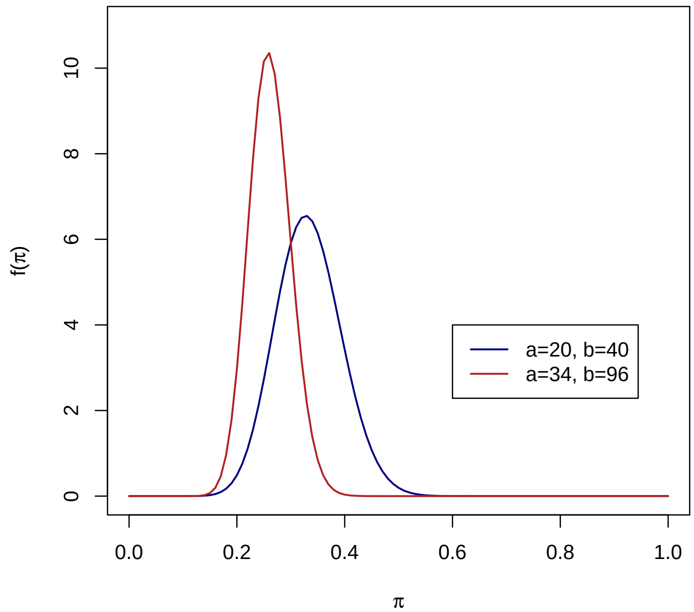
```
> cat(sprintf("Li = %6.3f Ls = %6.3f", qbeta(0.025, 34,96), qbeta(0.975, 34,96)))
Li =  0.190 Ls =  0.340
```

2.

```
> ## Priori
> pr <- seq(0, 1, by=0.01)
> f1 <- dbeta(pr, 2, 8)
> f2 <- dbeta(pr, 16, 64)
> plot(pr, f1, type="l", lwd=1.5,
+ xlab=expression(pi),
+ ylab=expression(paste("f(",pi,")")),
+ ylim=c(0, 10), col="navyblue")
> lines(pr, f2, type="l", lwd=1.5, col="firebrick")
> legend(0.6, 4, c("a=2, b=8", "a=16, b=64"), col=c("navyblue", "firebrick"), lwd=1.5)
```




```
> ## Posteriori
> pr <- seq(0, 1, by=0.01)
> f1 <- dbeta(pr, 20, 40)
> f2 <- dbeta(pr, 34, 96)
> plot(pr, f1, type="l", lwd=1.5,
+ xlab=expression(pi),
+ ylab=expression(paste("f(",pi,")")),
+ ylim=c(0, 11), col="navyblue")
> lines(pr, f2, type="l", lwd=1.5, col="firebrick")
> legend(0.6, 4, c("a=20, b=40", "a=34, b=96"), col=c("navyblue", "firebrick"), lwd=1.5)
```



3.

Los investigadores deberían rechazar la hipótesis nula, dado que, ambas distribuciones a posteriori presentan un valor esperado superior a 0,2 , a pesar de que el valor esperado de la a priori beta(2,8) fuese inferior en un primer momento. Podemos apreciar cómo $p_{observada}$ desplaza ambas curvas hacia valores superiores.