

## Resolución

1.1.

Distribución beta(2,8):

$$E(\pi \mid y) = \frac{\alpha + y}{\alpha + \beta + n} = \frac{2 + 18}{2 + 8 + 50} = 0,33$$

$$\begin{aligned} \text{Var}(\pi \mid y) &= \frac{(\alpha + y)(\beta + n - y)}{(\alpha + \beta + n)^2(\alpha + \beta + n + 1)} = \\ &= \frac{(2 + 18)(8 + 50 - 18)}{(2 + 8 + 50)^2(2 + 8 + 50 + 1)} = 3,64 \cdot 10^{-3} \end{aligned}$$

$$\alpha_{post} = 20$$

$$\beta_{post} = 40$$

```
> cat(sprintf("Li = %6.3f Ls = %6.3f", qbeta(0.025, 20,40), qbeta(0.975, 20,40)))
```

```
Li = 0.221 Ls = 0.456
```

Distribución beta(16,64):

$$E(\pi \mid y) = \frac{16 + 18}{16 + 64 + 50} = 0,26$$

$$\begin{aligned} \text{Var}(\pi \mid y) &= \frac{(\alpha + y)(\beta + n - y)}{(\alpha + \beta + n)^2(\alpha + \beta + n + 1)} = \\ &= \frac{(16 + 18)(64 + 50 - 18)}{(16 + 64 + 50)^2(16 + 64 + 50 + 1)} = 1,47 \cdot 10^{-3} \end{aligned}$$

$$\alpha_{post} = 34$$

$$\beta_{post} = 96$$

```
> cat(sprintf("Li = %6.3f Ls = %6.3f", qbeta(0.025, 34,96), qbeta(0.975, 34,96)))
```

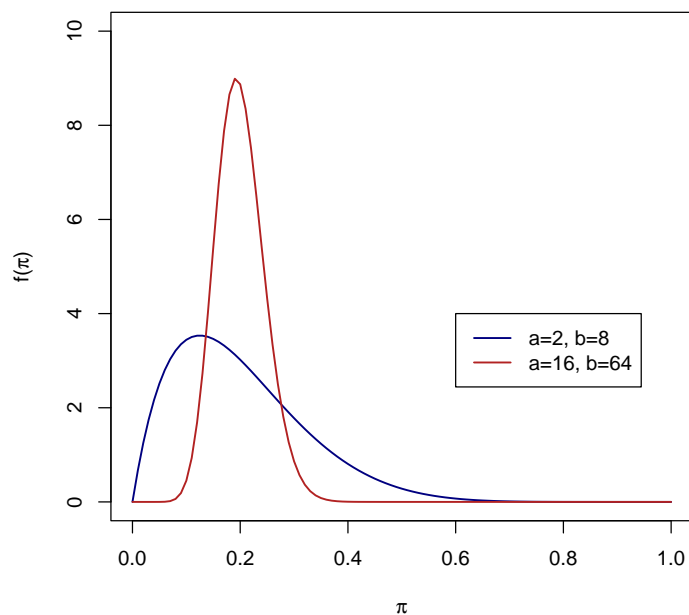
```
Li = 0.190 Ls = 0.340
```

1.2.

```

> ## Priori
> pr <- seq(0, 1, by=0.01)
> f1 <- dbeta(pr, 2, 8)
> f2 <- dbeta(pr, 16, 64)
> plot(pr, f1, type="l", lwd=1.5,
+ xlab=expression(pi),
+ ylab=expression(paste("f(",pi,")")),
+ ylim=c(0, 10), col="navyblue")
> lines(pr, f2, type="l", lwd=1.5, col="firebrick")
> legend(0.6, 4, c("a=2, b=8", "a=16, b=64"), col=c("navyblue", "firebrick"), lwd=1.5)

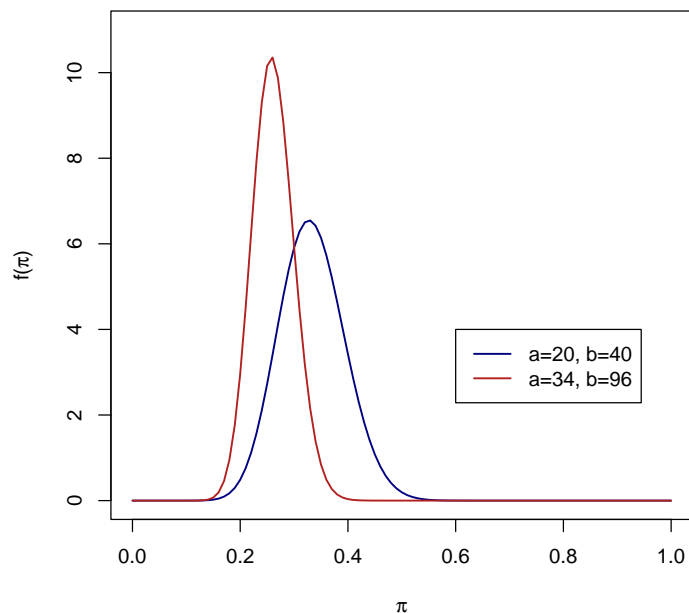
```



```

> ## Posteriori
> pr <- seq(0, 1, by=0.01)
> f1 <- dbeta(pr, 20, 40)
> f2 <- dbeta(pr, 34, 96)
> plot(pr, f1, type="l", lwd=1.5,
+ xlab=expression(pi),
+ ylab=expression(paste("f(",pi,")")),
+ ylim=c(0, 11), col="navyblue")
> lines(pr, f2, type="l", lwd=1.5, col="firebrick")
> legend(0.6, 4, c("a=20, b=40", "a=34, b=96"), col=c("navyblue", "firebrick"), lwd=1.5)

```



1.3.

Los investigadores deberían rechazar la hipótesis nula, dado que, ambas distribuciones a posteriori presentan un valor esperado superior a 0,2 , a pesar de que el valor esperado de la a priori  $\text{beta}(2,8)$  fuese inferior en un primer momento.

Podemos apreciar cómo  $p_{observada}$  desplaza ambas curvas hacia valores superiores, mostrando a su vez que  $\text{beta}(16,64)$  era ciertamente más informativa que la anterior.

1.4.

2.1.

```
> b <- c(-1.20, -0.65, 0.20, 0.80, 1.15 )
> muestras <- 1000
> theta <- rnorm(muestras)
> ## Prob de éxito de cada item en cada muestra
> p_mat <- matrix(nrow = 1000, ncol = 5)
> for (i in 1:length(b)) {
+   for (j in 1:muestras) {
+     p_mat[j,i] <- exp(theta[j] - b[i]) / (1 + exp(theta[j] - b[i]))
+   }
+ }
```

```

+ }
> ## Promedio de las probs de cada item
> Pr_v <- apply(p_mat, mean, MARGIN = 2)
> ## Calculamos la intersección
> P_3prim = Pr_v[1] * Pr_v[2] * Pr_v[3] * (1-Pr_v[4]) * (1-Pr_v[5])
> print (P_3prim)

[1] 0.1019691

```