Resolución

1.

$$
\begin{aligned}
&F(y) = \int_0^y (\theta+1)t^\theta dt = \\
&(\theta+1)\int_0^y t^\theta dt = \\
&(\theta+1)\left[\frac{t^{\theta+1}}{\theta+1}\right]_0^y = \\
&(\theta+1)\frac{y^{\theta+1}}{\theta+1} = \\
&y^{\theta+1}
\end{aligned}
$$

2.

$$
\begin{aligned}
&E(Y) = \int_0^1 (\theta+1)y^{\theta+1} dy = \\
&(\theta+1)\int_0^1 y^{\theta+1} dy = \\
&(\theta+1)\left[\frac{y^{\theta+2}}{\theta+2}\right]_0^1 = \\
&(\theta+1)\cdot\left(\frac{1^{\theta+2}}{\theta+2}\right) = \\
&\frac{\theta+1}{\theta+2}
\end{aligned}
$$

3.

Solo hay un parámetro, por lo que empleamos el primer momento
$\hat{\alpha_1} = \bar{Y} = \dfrac{0,9+0,8+0,7+0,6+0,5}{5} = 0,7$

Partiendo de la igualdad del momento poblacional al muestral, despejamos el estimador:
$\dfrac{\hat{\theta}+1}{\hat{\theta}+2} = 0,7$

$\hat{\theta} + 1 = 0,7 \cdot \hat{\theta} + 1,4$

$0,3 \cdot \hat{\theta} = 0,4$

$\hat{\theta} = 1,33$