Resolución

1.

Obtenemos la suma ponderada de cada valor de x:

$f(x) = 30x^4(1-x)$

$E(X) = \int_0^1 x \cdot 30x^4(1-x)dx =$

$\int_0^1 x \cdot 30x^4(1-x)dx =$

$\int_0^1 \left(30x^5 - 30x^6\right) dx =$

$30\int_0^1 x^5 dx - 30\int_0^1 x^6 dx =$

$30\frac{x^6}{6} - 30\frac{x^7}{7}\Big]_0^1 =$

$5x^6 - 30\frac{x^7}{7}\Big]_0^1 =$

$\frac{5}{7}$

2.

La moda será el valor máximo absoluto de $f(x)$:

$f(x) = 30x^4 - 30x^5$

$f'(x) = 120x^3 - 150x^4$

$120x^3 - 150x^4 = 0$

$x_1 = 0$

$x_2 = \frac{4}{5}$

$f''(x) = 360x^2 - 600x^3$

$f''(0) = 0$

$f''(\frac{4}{5}) = -76,8$

Teniendo en cuenta el dominio de la función de densidad, localizamos un máximo en $x = \frac{4}{5}$, que es la moda de la distribución.

3.

Empleamos la función integral, que toma límite superior variable

$F(x) = \int_0^x 30x^4(1-x)dx =$

$\int_0^x \left(30x^4 - 30x^5\right) \mathrm{d}x =$

$30 \int_0^x x^4 \ \mathrm{d}x - 30 \int x^5 \ \mathrm{d}x =$

$30\frac{x^5}{5} - 30\frac{x^6}{6} =$

$6x^5 - 5x^6]_0^x =$

$6x^5 - 5x^6$

4.

Dado que $n = 25$, aplicamos el teorema central del límite:

$E(X) = E(\bar{X}) = \frac{5}{7}$

$Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{\frac{5}{196}}{25} = \frac{1}{980}$

5.

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} =$$
$$\frac{0,7 - \frac{5}{7}}{\sqrt{(\frac{5}{196})}/\sqrt{25}} =$$
$$-0,447$$

6.

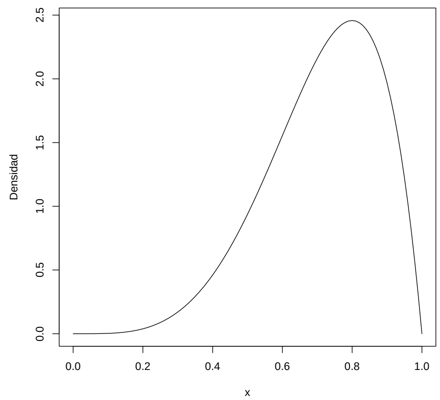
```r
## E(X)
xfx <- function(x) {x * 30*x^4*(1-x)}
Ex <- integrate(xfx, lower = 0, upper = 1)
print(Ex)

## 0.7142857 with absolute error < 7.9e-15

## Momento de orden 2
x2fx <- function(x) {x^2 * 30*x^4*(1-x)}
Ex2 <- integrate(x2fx, lower = 0, upper = 1)
print(Ex2)

## 0.5357143 with absolute error < 5.9e-15

## Var(X)
Varx <- Ex2$value-Ex$value^2
print(Varx)

## [1] 0.0255102
```

7.

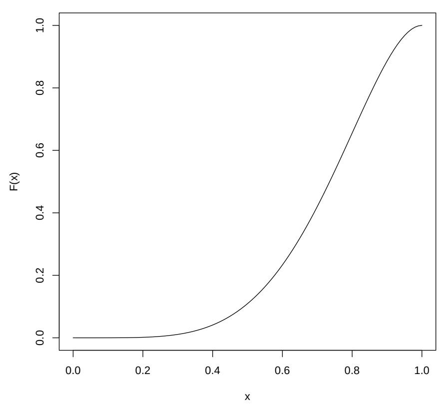
```r
fx <- function(x) {30*x^4*(1-x)}
res <- integrate(fx, lower = 0.2, upper = 0.8)
print(res)

## 0.65376 with absolute error < 7.3e-15
```

8.

```r
## f(x)
plot(fx, ylab = "Densidad")
```




```r
## F(x)
acum <- function(x) {6*x^5 - 5*x^6}
plot(acum,ylab = "F(x)")
```



9.

```r
fit <- optimize(fx, c(0, 1), maximum=TRUE)
print(fit$maximum)

## [1] 0.7999805
```

10.

```r
## Empleando TCL:
mean_xbar <- 5/7
sd_xbar <- sqrt((5/196)/25)
pnorm(0.7,mean_xbar,sd_xbar,lower.tail = FALSE )
```

```
## [1] 0.6726396
```