Resolución

1.

$$
\begin{aligned}
&F(y) = \int_0^y (\theta+1)t^\theta dt = \\
&(\theta+1)\int_0^y t^\theta dt = \\
&(\theta+1)\left[\frac{t^{\theta+1}}{\theta+1}\right]_0^y = \\
&(\theta+1)\frac{y^{\theta+1}}{\theta+1} = \\
&y^{\theta+1}
\end{aligned}
$$

2.

$$
\begin{aligned}
&E(Y) = \int_0^1 (\theta+1)y^{\theta+1} dy = \\
&(\theta+1)\int_0^1 y^{\theta+1} dy = \\
&(\theta+1)\left[\frac{y^{\theta+2}}{\theta+2}\right]_0^1 = \\
&(\theta+1)\cdot\left(\frac{1^{\theta+2}}{\theta+2}\right) = \\
&\frac{\theta+1}{\theta+2}
\end{aligned}
$$

3.

Igualamos el momento poblacional al muestral:

$$
\begin{aligned}
&\frac{\theta+1}{\theta+2} = \bar{Y} \\
&\theta+1 = \bar{Y}(\theta+2) \\
&\theta+1 = \bar{Y}\theta + 2\bar{Y} \\
&\theta+1-\bar{Y}\theta = 2\bar{Y} \\
&\theta-\bar{Y}\theta = 2\bar{Y}-1
\end{aligned}
$$

$\theta = \dfrac{2\bar{Y} - 1}{1 - \bar{Y}}$

Dada la muestra y:

$\hat{\alpha_1} = \bar{Y} = \dfrac{0,9 + 0,8 + 0,7 + 0,6 + 0,5}{5} = 0,7$

$\hat{\theta} = \dfrac{2 \cdot 0,7 - 1}{1 - 0,7} = 1,33$

4.

A partir de nuestra muestra obtenemos la función de verosimilitud

$l(\theta) = (\theta + 1)0,9^{\theta} \cdot (\theta + 1)0,8^{\theta} \cdot (\theta + 1)0,7^{\theta} \cdot (\theta + 1)0,6^{\theta} \cdot (\theta + 1)0,5^{\theta} =$

$(\theta + 1)^5 \cdot (0,9 \cdot 0,8 \cdot 0,7 \cdot 0,6 \cdot 0,5)^{\theta} =$

$(\theta + 1)^5 \cdot (0,15)^{\theta}$

Aplicamos propiedades del logaritmo para facilitar los cálculos posteriores

$\ln l(\theta) = 5ln(\theta + 1) + \theta \ln(0,15)$

Localizamos el máximo de dicha función (máxima verosimilitud)

$\dfrac{d}{d\theta} \ln l(\theta) = 5 \cdot \dfrac{1}{\theta + 1} - 1,9$

$\dfrac{5}{\theta + 1} - 1,9 = 0$

$\dfrac{1}{\theta + 1} = 0,38$

$\theta + 1 = 2,63$

$\theta = 1,63$

$\dfrac{d^2}{d\theta} \ln(\theta) = 5 \cdot (-1) \cdot (\theta - 1)^{-2} =$

$\dfrac{-5}{(\theta - 1)^2}; \quad$ signo -

Encontramos un máximo en $\theta = 1,63$, por lo que $\hat{\theta} = 1,63$