Resolución

1.1.

$$F(x) = \int_0^x \lambda t^{\lambda-1} dt =$$
$$\lambda \int_0^x t^{\lambda-1} dt =$$
$$\lambda[\frac{t^\lambda}{\lambda}]_0^x =$$
$$\lambda\frac{x^\lambda}{\lambda} =$$
$$x^\lambda$$

1.2.

La mediana será el punto que acumule el 50% de probabilidad. Por lo tanto:
$x^\lambda = 0,5$
$x = \sqrt[\lambda]{0,5}$

1.3.

$$E(X) = \int_0^1 x\lambda x^{\lambda-1} dx =$$
$$\lambda \int_0^1 x^\lambda dx =$$
$$\lambda[\frac{x^{\lambda+1}}{\lambda+1}]_0^1 =$$
$$\lambda\frac{1^{\lambda+1}}{\lambda+1}$$
$$\frac{\lambda}{\lambda+1}$$

1.4.

Igualamos el momento poblacional al muestral:

$$\frac{\lambda}{\lambda+1} = \bar{X}$$
$$\lambda = \bar{X}(\lambda+1)$$
$$\lambda = \bar{X}\lambda + \bar{X}$$
$$\lambda - \bar{X}\lambda = \bar{X}$$
$$\lambda(1-\bar{X}) = \bar{X}$$
$$\lambda = \frac{\bar{X}}{1-\bar{X}}$$

Por lo tanto:

$$\hat{\lambda} = \frac{\bar{X}}{1-\bar{X}}$$

1.5.

A partir de nuestra muestra obtenemos la función de verosimilitud:

$$L(\lambda) = \lambda^n \cdot (\prod_1^n x_i)^{\lambda-1}$$

Empleo la función de log-verosimilitud:

$$l(\lambda) = \ln[\lambda^n \cdot (\prod_1^n x_i)^{\lambda-1}] =$$

$$n\ln(\lambda) + (\lambda-1)\sum_1^n \ln(x_i)$$

Localizamos el máximo de dicha función (máxima verosimilitud)

$$l'(\lambda) = \frac{n}{\lambda} + \sum_1^n \ln x_i$$

$$\frac{n}{\lambda} + \sum_1^n \ln x_i = 0$$
$$\frac{n}{\lambda} = -\sum_1^n \ln(x_i)$$
$$\lambda = -\frac{n}{\sum_1^n \ln(x_i)}$$

$$l''(\lambda) = -\frac{n}{\lambda^2}; \quad \text{signo -}$$

Encontramos un máximo en $\lambda = -\frac{n}{\sum_1^n \ln(x_i)}$, por lo que $\hat{\lambda} = -\frac{n}{\sum_1^n \ln(x_i)}$

1.6.

Extraemos la cantidad de información que proporciona la muestra:

$I(\lambda) = -(l''(\lambda)) = \frac{n}{\lambda^2}$

Calculamos la Varianza:

$Var(\lambda) = \frac{1}{I(\lambda)} = \frac{\lambda^2}{n}$

2.1.

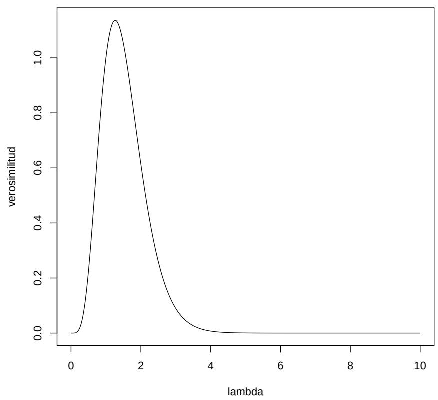
```
x <- c(0.5,  0.2,  0.4,  0.6,  0.8)
n<- length(x)
lambda_hat <- -n/sum(log(x))
print(lambda_hat)

## [1] 1.264912
```

2.2.

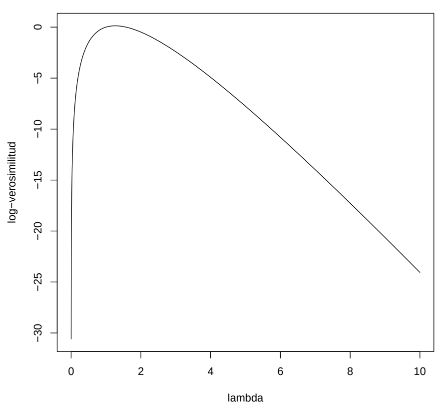
```
I <- n/(lambda_hat^2)
err_tip <- 1/(sqrt(I))
print(err_tip)

## [1] 0.5656857
```

2.3.

```
lambda <- seq(0, 10, by = 0.001)
l <- function(lambda, x) (lambda^n * (prod(x))^(lambda - 1))
plot(main = "", lambda, l(lambda, x), xlim = c(0, 10), xlab = "lambda",
    ylab = "verosimilitud", type = "l")
```




```
log_l <- function(lambda, x) (n * log(lambda) + (lambda - 1) *
    sum(log(x)))
plot(main = "", lambda, log_l(lambda, x), xlim = c(0, 10), xlab = "lambda",
    ylab = "log-verosimilitud", type = "l")
```



2.4.

```
lMax <- optimize(log_l, c(0, 10),x=x, maximum=TRUE)
print(lMax, digits=3)

## $maximum
## [1] 1.26
##
## $objective
## [1] 0.128
```