Resolución

1.1

$$f(x) = e^{-x}$$
$$F(x) = \int_0^x e^{-u} du =$$

$$z = -u \quad dz = -du; du = -dz$$

$$= -\int_0^x e^z dz = \left[-e^{-u}\right]_0^x = -e^{-x}$$

1.2

$$P(x > 3) = 1 - P(x \leqslant 3) =$$
$$1 - \left[-e^{-x}\right]_0^3 = 1 - (-0,05 + 1) =$$
$$0,05$$

1.3

$$h(x) = \frac{e^{-x}}{1 + e^{-x}}$$

1.4

$$P(x > 2) = 1 - P(x \leqslant 2) =$$
$$1 - \left[-e^{-x}\right]_0^2 = 1 - (-0,14 + 1) =$$
$$0,14$$

Dado que tratamos con probabilidades acumuladas:
$P[(x > 3) \cap (x > 2)] = P(x > 3)$

$$P(x > 3 \mid x > 2) = \frac{P[(x > 3) \cap (x > 2)]}{P(x > 2)} = \frac{0,05}{0,14} = 0,36$$

2.1

$$f(x) = 3x^2$$
$$E(X) = \int_0^1 x3x^2 dx = 3\int_0^1 x^3 dx = [\frac{3x^4}{4}]_0^1 = \frac{3}{4}$$

2.2

$$E(X^2) = \int_0^1 x^2 3x^2 dx = 3\int_0^1 x^4 dx = [\frac{3x^5}{5}]_0^1 = \frac{3}{5}$$
$$Var(X) = E(X^2) - \mu^2 = \frac{3}{5} - (\frac{3}{4})^2 = \frac{3}{80}$$

2.3

$$F(x) = \int_0^x 3u^2 du = [u^3]_0^x = x^3$$

2.4

Empleamos la inversa de F(x), dado que buscamos puntos que acumulen el 25, el 50 y el 75 % de las probabilidades:

$F^{-1}(p) = \sqrt[3]{p}$
$F^{-1}(0,25) = 0,63$
$F^{-1}(0,5) = 0,79$
$F^{-1}(0,75) = 0,91$

2.5

$$F(0,8) = 0,8^3 = 0,51$$

2.6

Dado que $n = 25$, partiendo del teorema central del límite:

$\mu \approx E(X) \approx E(\bar{X}) \approx \frac{3}{4}$

$\sigma^2 \approx Var(X) \approx \frac{3}{80}$

$Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{\frac{3}{80}}{25} = \frac{3}{2000}$

$Desv(\bar{X}) = \sqrt{\frac{3}{2000}} = 0,04$

$$Z = \frac{\bar{X} - \mu}{Desv(\bar{X})} = \frac{0,8 - 0,75}{0,04} = 1,25$$

Consultando en la tabla de la N(0,1):

$P(\bar{X} \leq 0,8) = P(Z \leq 1,25) = 0,89$