Resolución

1.

```r
S <- matrix(c(
  2.45, 0.57, 2.46, -0.21,  0.76, 1.32, 1.18,  0.62,
  0.57, 6.19, 2.45,  2.95,  2.06, 3.55, 3.29,  2.28,
  2.46, 2.45, 5.44,  0.65,  1.52, 3.38, 2.63,  2.10,
 -0.21, 2.95, 0.65,  2.93,  1.02, 1.61, 1.36,  1.11,
  0.76, 2.06, 1.52,  1.02,  3.17, 0.15, 2.78, -0.08,
  1.32, 3.55, 3.38,  1.61,  0.15, 5.48, 1.41,  3.12,
  1.18, 3.29, 2.63,  1.36,  2.78, 1.41, 4.35,  0.52,
  0.62, 2.28, 2.10,  1.11, -0.08, 3.12, 0.52,  3.21
), nrow=8)
n<- 100
l <- function(x){
lambda_1 <- x[1:8]
lambda_2 <- x[5:8]
lambda_3 <- x[9:12]

lambda <- matrix(0, nrow = 8, ncol = 3)
lambda[, 1] <- lambda_1
lambda [1:4,2] <- lambda_2
lambda[5:8,3] <- lambda_3




psi <- diag(x[13:20])


sigma <- lambda%*%t(lambda) + psi
invSigma <- solve(sigma)

log_lk <- n*0.5*log(det(invSigma)) - n*0.5*sum(diag(S%*%invSigma))
return(log_lk)

}

fit <- optim(par=c(rep(1,20)),
             lower = c(rep(-8, 12), rep(0, 8)),
             upper = c(rep(8, 12), rep(5, 8)),
fn=l, method="L-BFGS-B", hessian=TRUE, control=list(fnscale=-1))
```

```
print(fit$par)

## [1]  0.48025647  1.80572803  1.30959933  0.88551881
## [5]  1.44057207  0.24716501  1.81115426 -0.08540185
## [9] -0.97289661  1.96676540 -0.69136508  1.51842317
## [13]  0.82690511  2.24042377  1.09931741  1.87663014
## [17]  0.87481145  0.93351876  0.86286201  1.06519365
```

2.

```
H <- fit$hessian # segundas derivadas
Informacion <- - H # información = - (segundas derivadas)
VarCovar <- solve(Informacion) # varianza de los estimadores = 1/información
print(sqrt(diag(VarCovar)), digits=2) # mostrar la varianza de los estimadores

## [1] 0.16 0.18 0.21 0.15 0.14 0.19 0.15 0.15 0.18 0.23 0.19
## [12] 0.19 0.36 0.51 0.55 0.33 0.21 0.57 0.28 0.32
```

3.

```
invS <- solve(S)
p <- ncol(S)
log_lk_saturado <- n*0.5*log(det(invS)) - n*0.5*p

G2 <- -2*(fit$value - log_lk_saturado)
par_saturado <- p*(p+1)/2
par_factorial <- length(fit$par)
gl <- par_saturado - par_factorial
p_valor <- pchisq(G2, gl, lower.tail = F)


cat(sprintf("G2 =%5.2f, gl =%1.0f, p =%3.2f\n", G2, gl, p_valor))

## G2 =151.69, gl =16, p =0.00
```