

Resolución

1.

$$\begin{aligned}
 F(y) &= \int_0^y (\theta + 1)t^\theta dt = \\
 (\theta + 1) \int_0^y t^\theta dt &= \\
 (\theta + 1) \left[\frac{t^{\theta+1}}{\theta + 1} \right]_0^y &= \\
 (\theta + 1) \frac{y^{\theta+1}}{\theta + 1} &= \\
 y^{\theta+1}
 \end{aligned}$$

2.

$$\begin{aligned}
 E(Y) &= \int_0^1 (\theta + 1)y^{\theta+1} dy = \\
 (\theta + 1) \int_0^1 y^{\theta+1} dy &= \\
 (\theta + 1) \left[\frac{y^{\theta+2}}{\theta + 2} \right]_0^1 &= \\
 (\theta + 1) \cdot \left(\frac{1^{\theta+2}}{\theta + 2} \right) &= \\
 \frac{\theta + 1}{\theta + 2}
 \end{aligned}$$

3.

Igualemos el momento poblacional al muestral:

$$\begin{aligned}
 \frac{\theta + 1}{\theta + 2} &= \bar{Y} \\
 \theta + 1 &= \bar{Y}(\theta + 2) \\
 \theta + 1 &= \bar{Y}\theta + 2\bar{Y} \\
 \theta + 1 - \bar{Y}\theta &= 2\bar{Y} \\
 \theta - \bar{Y}\theta &= 2\bar{Y} - 1
 \end{aligned}$$

$$\theta = \frac{2\bar{Y} - 1}{1 - \bar{Y}}$$

Dada la muestra y:

$$\hat{\alpha}_1 = \bar{Y} = \frac{0,9 + 0,8 + 0,7 + 0,6 + 0,5}{5} = 0,7$$

$$\hat{\theta} = \frac{2 \cdot 0,7 - 1}{1 - 0,7} = 1,33$$

4.

A partir de nuestra muestra obtenemos la función de verosimilitud

$$\begin{aligned} L(\theta) &= (\theta + 1)0,9^\theta \cdot (\theta + 1)0,8^\theta \cdot (\theta + 1)0,7^\theta \cdot (\theta + 1)0,6^\theta \cdot (\theta + 1)0,5^\theta = \\ &= (\theta + 1)^5 \cdot (0,9 \cdot 0,8 \cdot 0,7 \cdot 0,6 \cdot 0,5)^\theta = \\ &= (\theta + 1)^5 \cdot (0,15)^\theta \end{aligned}$$

Empleo la función de log-verosimilitud:

$$l(\theta) = 5\ln(\theta + 1) + \theta \ln(0,15)$$

Localizamos el máximo de dicha función (máxima verosimilitud)

$$l'(\theta) = 5 \cdot \frac{1}{\theta + 1} - 1,9$$

$$\frac{5}{\theta + 1} - 1,9 = 0$$

$$\frac{1}{\theta + 1} = 0,38$$

$$\theta + 1 = 2,63$$

$$\theta = 1,63$$

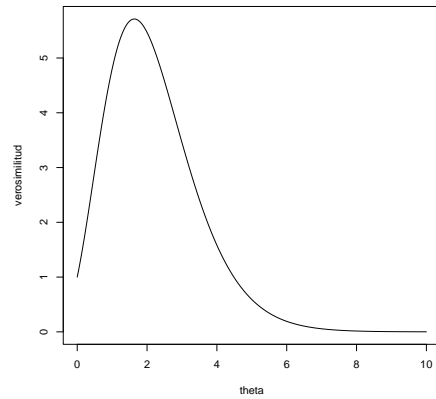
$$l''(\theta) = 5 \cdot (-1) \cdot (\theta - 1)^{-2} =$$

$$\frac{-5}{(\theta - 1)^2}; \quad \text{signo -}$$

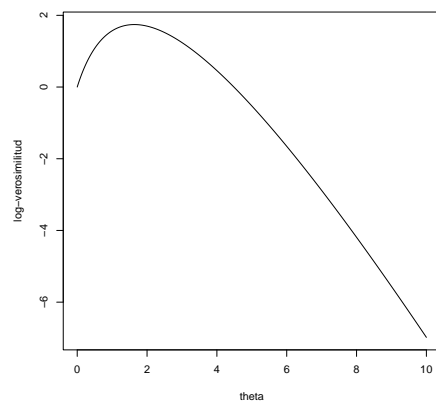
Encontramos un máximo en $\theta = 1,63$, por lo que $\hat{\theta} = 1,63$

5.

```
theta <- seq(0,10,by=0.001)
l <- function(theta) ((theta+1)^5)*(0.15)^theta
plot(main="",theta,l(theta),xlim=c(0,10),xlab = "theta", ylab ="verosimilitud", type="l")
```



```
log_l <- function(theta) 5*log(theta+1)+theta*log(0.15)
plot(main="",theta,log_l(theta),xlim=c(0,10),xlab = "theta", ylab ="log-verosimilitud", type="l")
```



6.

```
lMax <- optimize(log_l, c(0, 10), maximum=TRUE)
print(lMax, digits=3)

## $maximum
## [1] 1.64
##
## $objective
## [1] 1.74
```