Resolución

1.

Obtenemos la suma ponderada de cada valor de x:

$$E(x) = \int_0^2 x \cdot \frac{15}{64}\left(4x^2 - x^4\right) =$$

$$u = x \quad dv = \frac{15}{64}\left(4x^2 - x^4\right)$$

$$du = dx \quad v = \frac{15}{64} \cdot \left(\frac{4x^3}{3} - \frac{x^5}{5}\right)$$

$$= x \cdot \frac{15}{64} \cdot \left(\frac{4x^3}{3} - \frac{x^5}{5}\right) - \int_0^2 \frac{15}{64} \cdot \frac{4x^3}{3} - \frac{x^5}{5} =$$

$$\frac{15x}{64}\left(\frac{4x^3}{3} - \frac{x^5}{5}\right) - \frac{15}{64} \cdot \left(\frac{4}{3} \cdot \frac{x^4}{4} - \frac{1}{5}\frac{x^6}{6}\right)]_0^2 =$$

$$\frac{15 \cdot 2}{64}\left(\frac{4 \cdot 8}{3} - \frac{32}{5}\right) - \frac{15}{64} \cdot \left(\frac{4 \cdot 16}{12} - \frac{64}{30}\right) =$$

$$\frac{15}{32} \cdot \frac{64}{15} - \frac{15}{64} \cdot \left(\frac{16}{3} - \frac{32}{15}\right) =$$

$$\frac{960}{480} - \frac{15}{64} \cdot \frac{16}{5} =$$

$$2 - 0,75 = 1,25$$

2.

La moda será el valor máximo absoluto de $f(x)$:

$$f'(x) = \frac{15}{64} \cdot \left(8x - 4x^3\right) =$$

$$\frac{15x}{16} \cdot \left(2 - x^2\right)$$

$$\frac{15x}{16}\left(2 - x^2\right) = 0$$

$$15x\left(2 - x^2\right) = 0$$

$$30x - 15x^3 = 0$$

$$x_1 = 0$$

$$x_2 = -\sqrt{2}$$

$$x_3 = \sqrt{2}$$

$f''(x) = 30 - 45x^2$
$f''(0) = 30$
$f''(\sqrt{2}) = -60$
Teniendo en cuenta el dominio de la función de densidad, localizamos un máximo en $x = \sqrt{2}$, que es la moda de la distribución.

3.

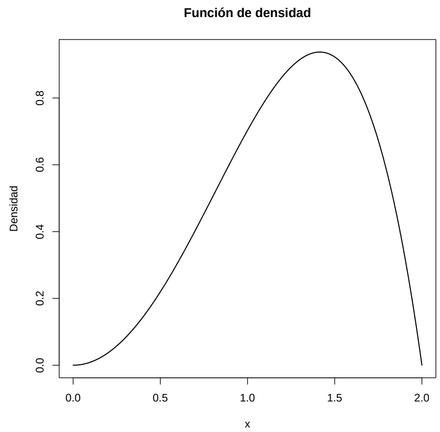
Empleamos la función integral, que toma límite superior variable
$F(x) = \int_0^x \frac{15\left(4x^2 - x^4\right)}{64} dx =$
$\frac{15}{64} \cdot \left(\frac{4x^3}{3} - \frac{x^5}{5}\right)]_0^x =$
$\frac{15}{64} \cdot \left(\frac{4x^3}{3} - \frac{x^5}{5}\right)$

4.

$$P(X \leq 1) = \frac{15}{64} \cdot \left(\frac{4 \cdot 1^3}{3} - \frac{1^5}{5}\right) = 0,26$$

5.

```
x <- seq(0, 2, by=0.001)
f<- (15/64)*((4*x^2)-x^4)
plot(x, f, type="l", lwd=1.5, main="Función de densidad", xlab="x", ylab="Densidad")
```



6.

```
xfx <- function(x) {x*(15/64)*((4*x^2)-x^4)}
Ex <- integrate(xfx, lower = 0, upper = 2)
print(Ex)

## 1.25 with absolute error < 1.4e-14
```

7.

```
g <- function(x) (15/64)*((4*x^2)-x^4)
fit <- optimize(g, c(0, 2), maximum=TRUE)
print(fit)

## $maximum
## [1] 1.414226
##
## $objective
## [1] 0.9375
```

8.

```
fx <- function(x) {(15/64)*((4*x^2)-x^4)}
P1 <- integrate(fx, lower = 0, upper = 1)
print(P1)

## 0.265625 with absolute error < 2.9e-15
```