Resolución

1.

$$
\begin{aligned}
&F(x) = \int_0^x \lambda t^{\lambda-1} dt = \\
&\lambda \int_0^x t^{\lambda-1} dt = \\
&\lambda [\frac{t^\lambda}{\lambda}]_0^x = \\
&\lambda \frac{x^\lambda}{\lambda} = \\
&x^\lambda
\end{aligned}
$$

2.
La mediana será el punto que acumule el 50% de probabilidad. Por lo tanto:

$$
\begin{aligned}
&x^\lambda = 0,5 \\
&x = \sqrt[\lambda]{0,5}
\end{aligned}
$$

3.

$$
\begin{aligned}
&E(X) = \int_0^1 x \lambda x^{\lambda-1} dx = \\
&\lambda \int_0^1 x^\lambda dx = \\
&\lambda [\frac{x^{\lambda+1}}{\lambda+1}]_0^1 = \\
&\lambda \frac{1^{\lambda+1}}{\lambda+1} \\
&\frac{\lambda}{\lambda+1}
\end{aligned}
$$

4.
Igualamos el momento poblacional al muestral:

$$\frac{\lambda}{\lambda+1} = \bar{X}$$

$$\lambda = \bar{X}(\lambda + 1)$$
$$\lambda = \bar{X}\lambda + \bar{X}$$
$$\lambda - \bar{X}\lambda = \bar{X}$$
$$\lambda(1 - \bar{X}) = \bar{X}$$
$$\lambda = \frac{\bar{X}}{1 - \bar{X}}$$

Por lo tanto:

$$\hat{\lambda} = \frac{\bar{X}}{1 - \bar{X}}$$