Compressed Sensing Tutorial

Prepare workspace

```
clear all; close all; clc;
```

Import image

```
cameraman = imread('cameraman.tif');
lena = imread('lena_std.tif');
lena2 = rgb2gray(lena);
whos cameraman
```

```
  Name            Size              Bytes  Class    Attributes

  cameraman      256x256            65536  uint8
```

```
whos lena
```

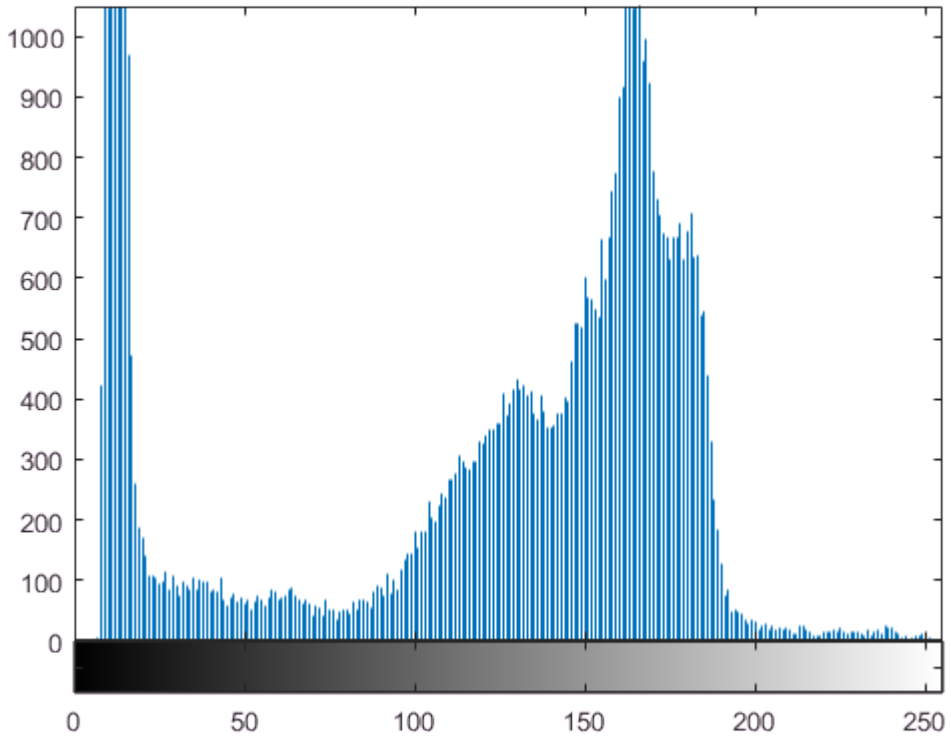
```
  Name        Size                Bytes  Class    Attributes

  lena      512x512x3            786432  uint8
```

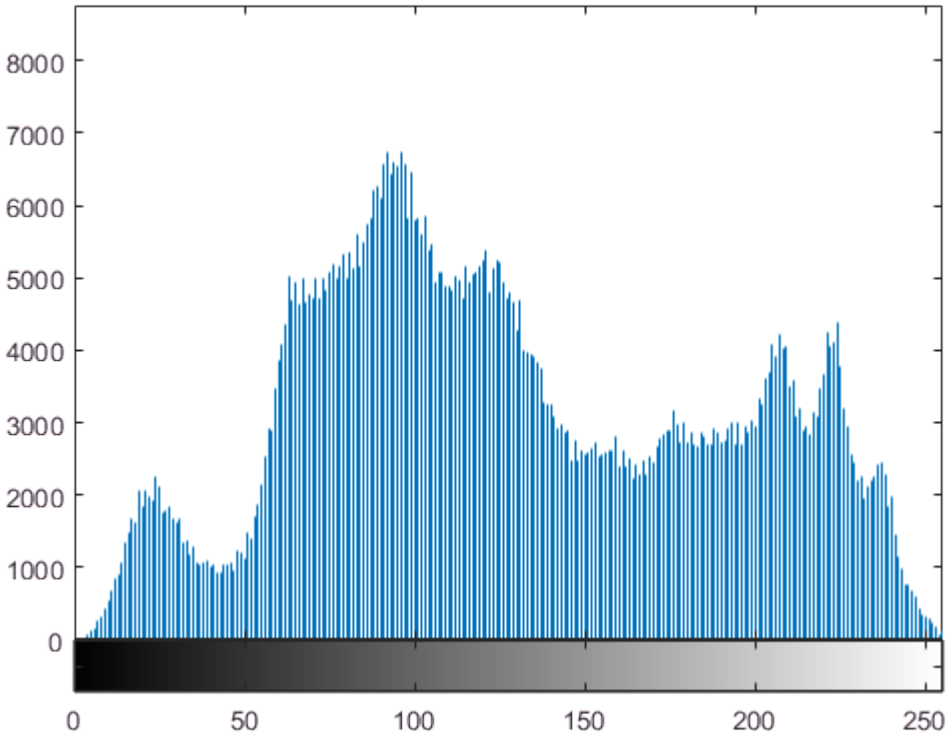
```
imshow(cameraman);    % exmaple of binary image
```

```
figure; imshow(lena); % example of rgb image
```

```
figure; imhist(cameraman)
```



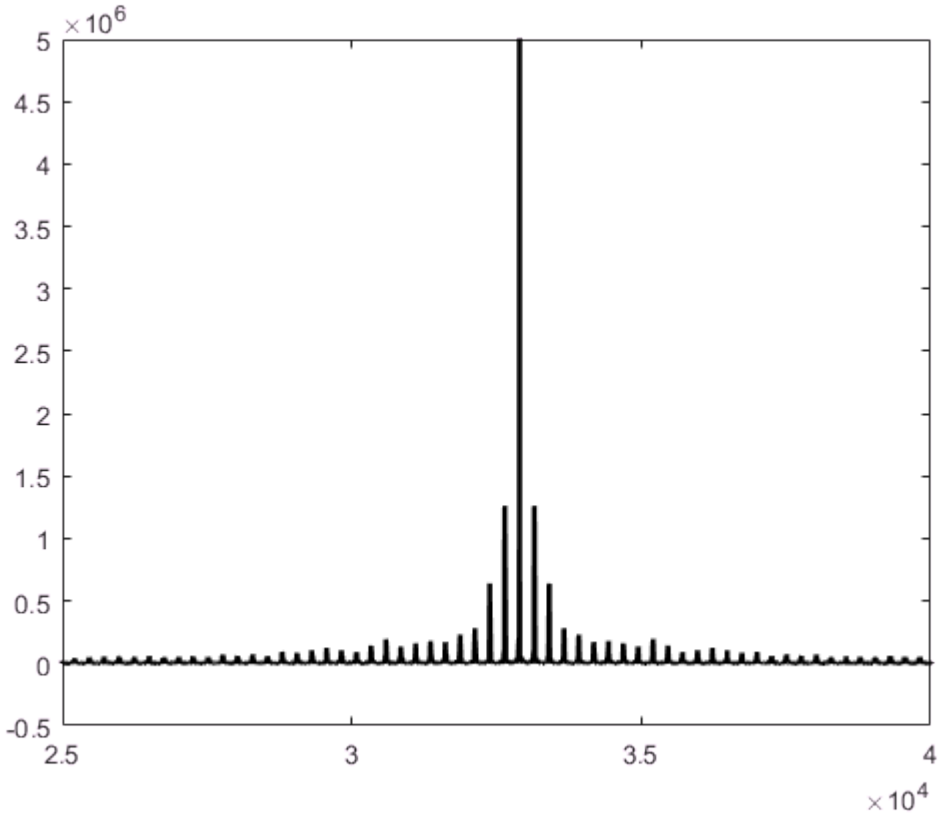

```
figure; imhist(lena)
```



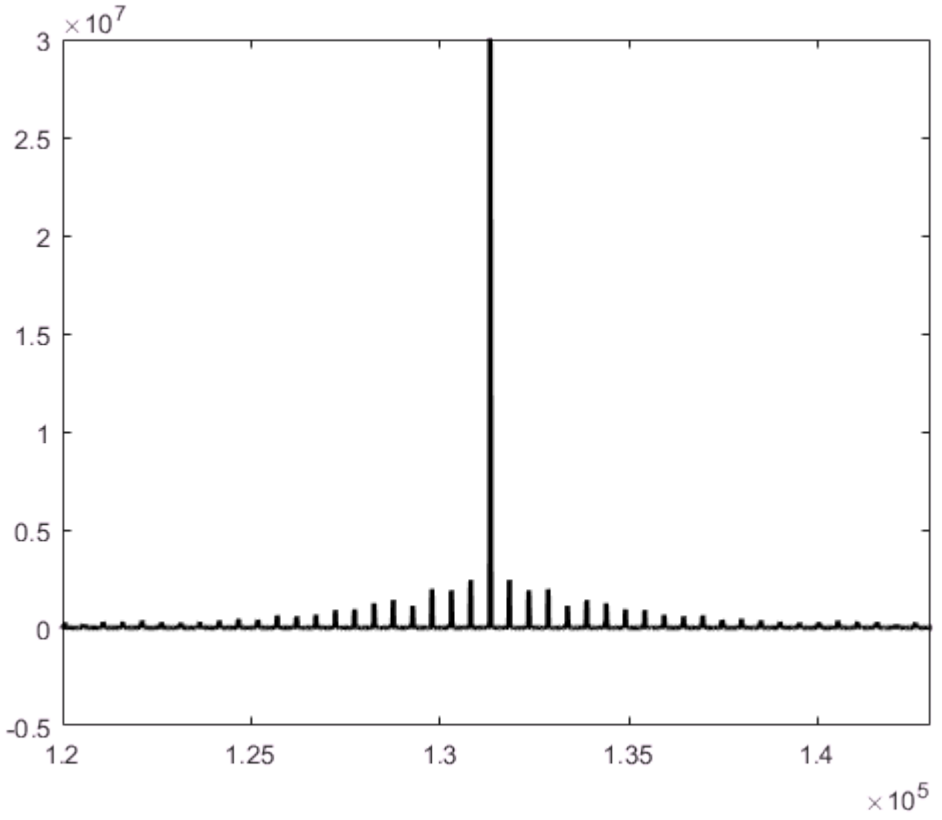

```
figure; imshow(lena2);
```

Fourier Transform

```
lena_4_fft = double(lena2);
[nx1, ny1] = size(cameraman);
[nx2, ny2] = size(lena_4_fft);
fft_cameraman = fftshift(fft2(cameraman));
fft_lena = fftshift(fft2(lena_4_fft));
figure; plot(reshape(abs(fft_cameraman), nx1*ny1, 1), 'k', 'Linewidth', [2])
set(gca,'Xlim', [2.5*10^4 4*10^4], 'Ylim', [-0.5*10^6 5*10^6])
```

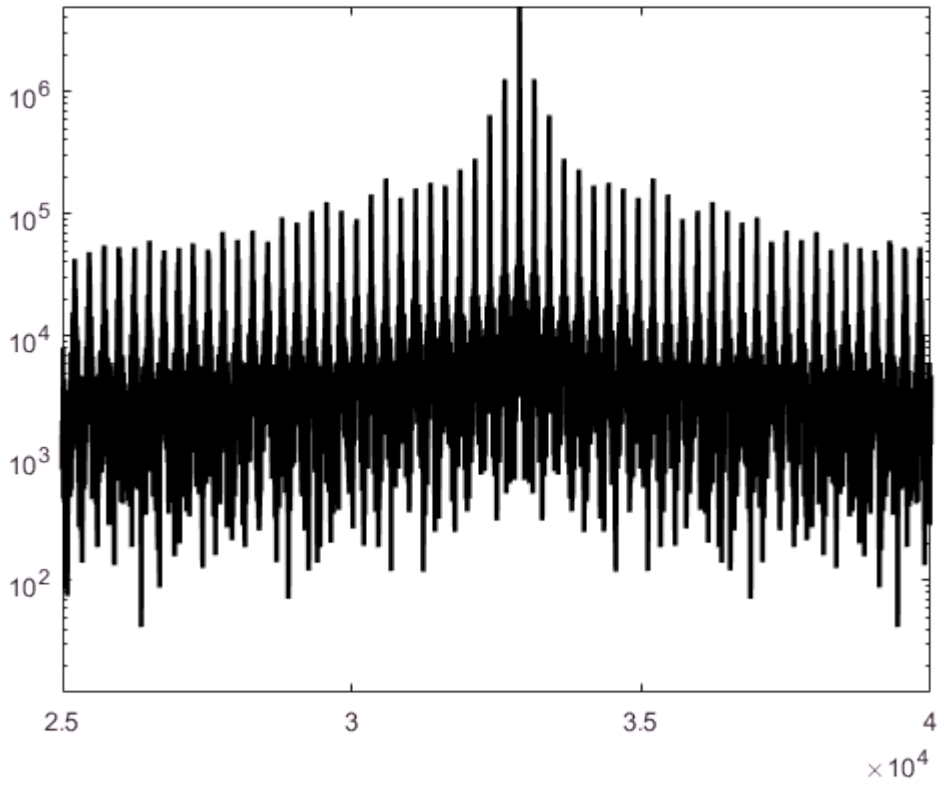



```
figure; plot(reshape(abs(fft_lena), nx2*ny2, 1), 'k', 'Linewidth', [2])
set(gca,'Xlim', [1.2*10^5 1.43*10^5], 'Ylim', [-0.5*10^7 3*10^7])
```




```
figure; semilogy(reshape(abs(fft_cameraman), nx1*ny1, 1), 'k', 'Linewidth', [2])
```

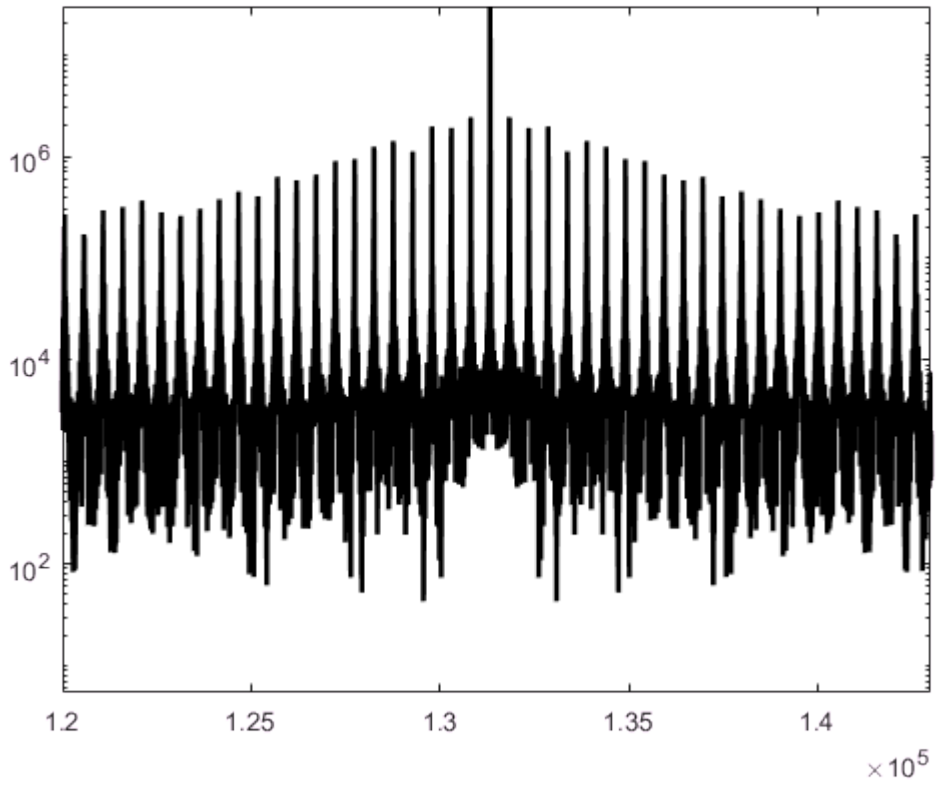
```
set(gca,'Xlim', [2.5*10^4 4*10^4], 'Ylim', [-0.5*10^6 5*10^6])
```




```
figure; semilogy(reshape(abs(fft_lena), nx2*ny2, 1), 'k', 'Linewidth', [2])
```

경고: 음수 제한은 무시됨

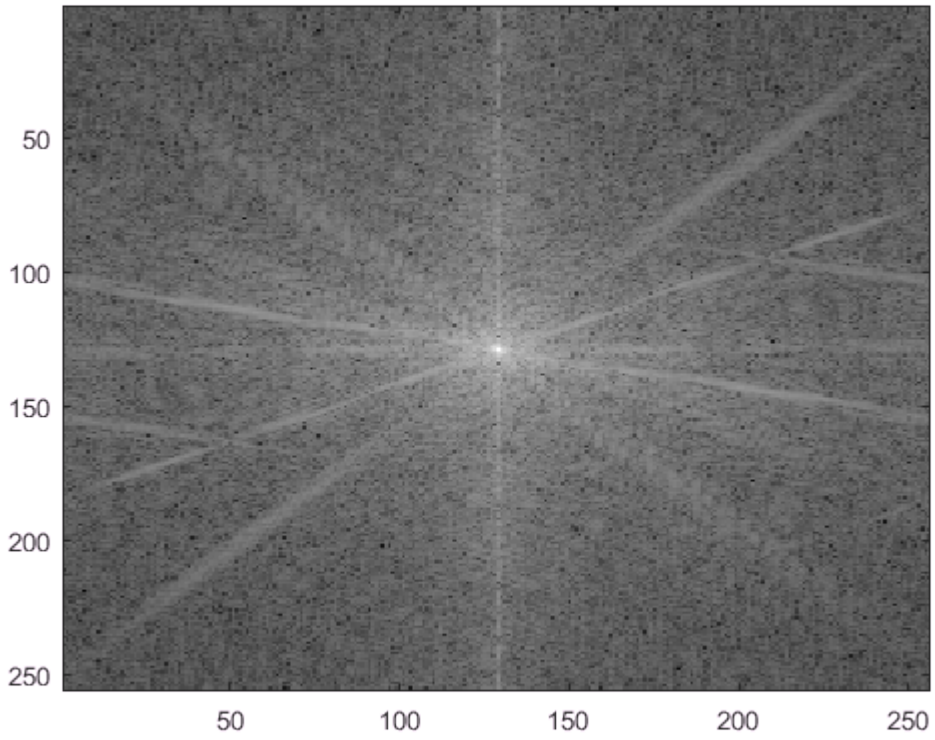
```
set(gca,'Xlim', [1.2*10^5 1.43*10^5], 'Ylim', [-0.5*10^7 3*10^7])
```



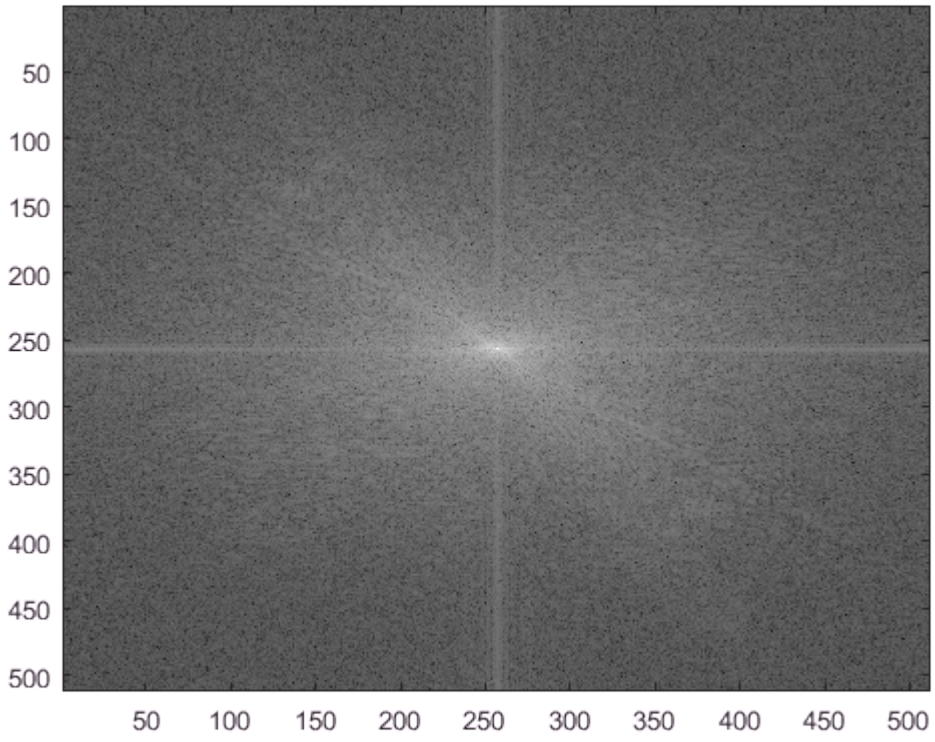
Power Spectrum

```
figure; colormap gray; imagesc(log10(abs(fft_cameraman)));
```

경고: 음수 제한은 무시됨

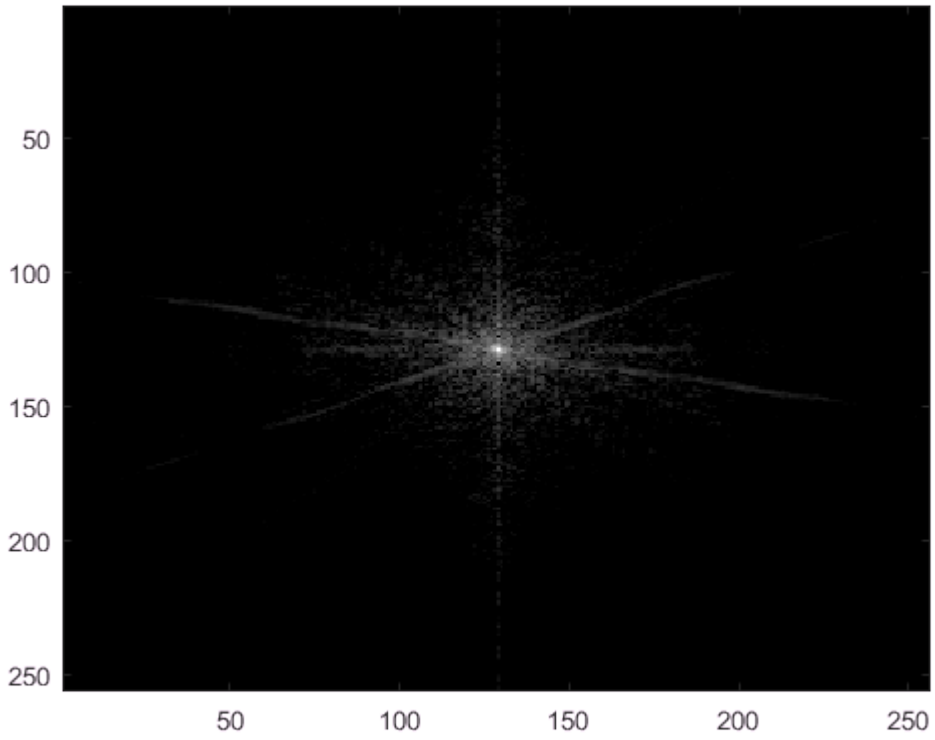



```
figure; colormap gray; imagesc(log10(abs(fft_lena))); % power spectrum
```

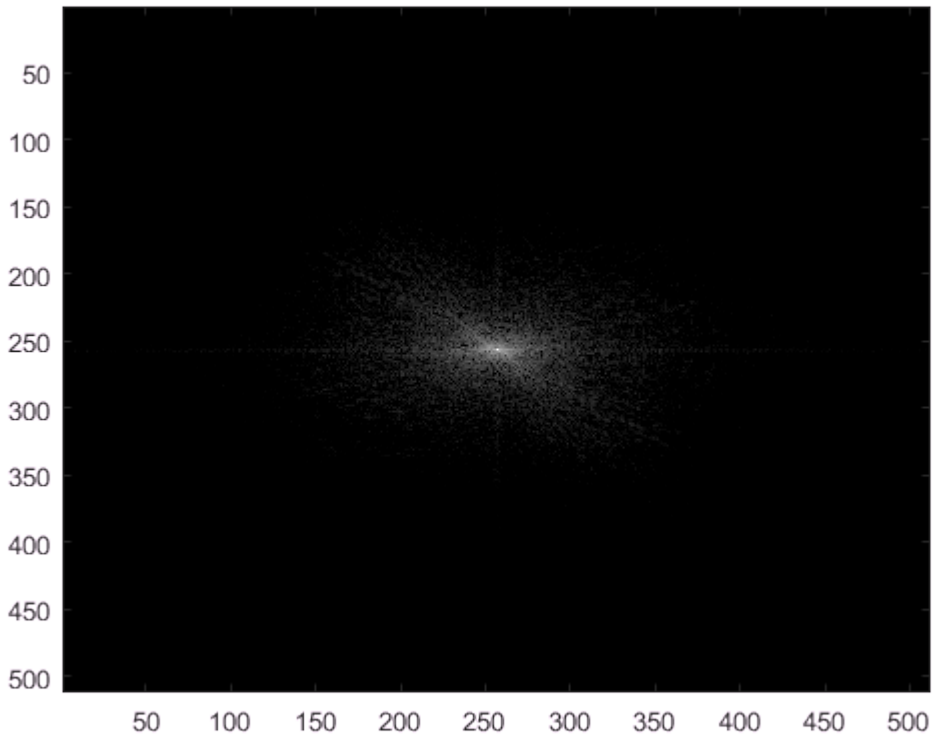


Truncation

```
fft_cameraman(find(abs(fft_cameraman)<=max(quantile(abs(fft_cameraman(:)),10))))=0;
fft_lena(find(abs(fft_lena)<=max(quantile(abs(fft_lena(:)),10))))=0;
figure; colormap gray; imagesc(log10(abs(fft_cameraman)));
```




```
figure; colormap gray; imagesc(log10(abs(fft_lena))); % power spectrum
```



Reconstruction

for cameraman image

```
% cameraman
count_pic = 1;

for thresh = [0.005 * 10^6 0.02*10^6 0.05*10^6 0.5*10^6]
    fft2 = reshape(fft_cameraman, nx1*ny1, 1);
    count = 0;
    for j = 1:length(fft2);
        if abs(fft2(j)) < thresh
            fft2(j) = 0;
            count = count+1;
        end
    end
    percent = 100-count/length(fft2) * 100

    fft_low = fftshift(reshape(fft2, nx1, ny1));
    im_low = uint8(ifft2(fft_low));
    figure(1), subplot(2,2,count_pic), imshow(im_low);
    count_pic = count_pic+1;
end
```

```
percent = 21.9742
percent = 2.9434
percent = 0.6546
percent = 0.0198
```




for lena image

```
% lena
count_pic = 1;
for thresh = [0.0005 * 10^7 0.002*10^7 0.005*10^7 0.05*10^7];
    fft2 = reshape(fft_lena, nx2*ny2, 1);
    count = 0;
    for j = 1:length(fft2);
        if abs(fft2(j)) < thresh
            fft2(j) = 0;
            count = count+1;
        end
    end
    percent = 100-count/length(fft2) * 100

    fft_low = fftshift(reshape(fft2, nx2, ny2));
    im_low = uint8(ifft2(fft_low));
    figure(2), subplot(2,2,count_pic), imshow(im_low);
    count_pic = count_pic+1;
end
```

```
percent = 19.9699
percent = 3.7319
percent = 0.9975
percent = 0.0378
```




Compressed Sensing

Image compression algorithm does; (1) first the image is taken (2) a wavelet/Fourier transform is applied in order to apply a threshold rule, and (3) the majority of information is promptly discarded. Could we instead randomly sample, let's say 5% of the pixels and still reconstruct the image? Knowing that the image is sparse allows us to do just that. In this sections, I'm going to introduce compressed sensing. To run the code below, you need to install cvx toolbox from (http://cvxr.com/cvx/). It may take 8 or 9 hours or more in usual personal computer.

For cameraman image

```
% resize image due to the computation time
resized_cameraman = imresize(cameraman, [75 100]);
cameraman2_4_fft = double(resized_cameraman);
[ny,nx] = size(cameraman2_4_fft);

% For cameraman
count_pic = 1;
for k = [2500 3000 4000]; % number of sparse samples
test2 = zeros(ny, nx);
```

```
r1 = randintrlv([1:nx*ny], 793); % random permutation with set.seed(793)
r1k = r1(1:k);
for j=1:k
    test2(r1k(j))=-1;
end

% mask for image
test = zeros(ny, nx);
for j=1:k
    test(r1k(j))=1;
    Adel= reshape(idct2(test), nx*ny, 1);
    Adelta(j, :) = Adel;
    test(r1k(j))=0;
end

% L1 minimization with cvx toolbox
b = cameraman2_4_fft(r1k).';
n = nx*ny;
cvx_begin;
    variable y(n);
    minimize(norm(y, 1));
    subject to
        Adelta*y == b;
cvx_end;

% reconstruction of image
Alow = uint8((dct2(reshape(y, ny, nx))));
figure(3), subplot(2,2,count_pic), imshow(Alow);
count_pic = count_pic+1;
end
```

```
경고: Unknown parameter: 'isvname'
경고: Unknown parameter: 'appname'
경고: Unknown parameter: 'isv_key'
```

```
Calling SDPT3 4.0: 15000 variables, 2500 equality constraints
------------------------------------------------------------

 num. of constraints = 2500
 dim. of socp   var  = 15000,   num. of socp blk  = 7500
 checkdepconstr: AAt is not pos. def.
*****************************************************************
   SDPT3: Infeasible path-following algorithms
*****************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-----------------------------------------------------------------
 0|0.000|0.000|1.0e+00|8.6e+01|1.2e+08| 1.347038e+06  0.000000e+00| 0:1:45| chol  2  2
 1|1.000|0.597|6.1e-08|3.5e+01|7.8e+07| 2.179745e+06  4.398288e+05| 0:3:19| chol  1  1
 2|1.000|0.987|1.6e-08|7.0e-01|3.4e+06| 2.035527e+06  5.703216e+04| 0:6:41| chol  1  1
 3|0.920|0.736|7.6e-09|2.7e-01|6.5e+05| 5.675139e+05  7.054532e+04| 0:9:28| chol  1  1
 4|0.883|0.566|4.7e-09|1.5e-01|2.0e+05| 2.574180e+05  9.266084e+04| 0:12:13| chol  1  1
 5|0.784|0.660|2.4e-09|7.3e-02|8.0e+04| 1.781548e+05  1.115378e+05| 0:14:57| chol  1  1
 6|0.768|0.652|1.3e-09|3.5e-02|3.3e+04| 1.466756e+05  1.185036e+05| 0:17:43| chol  1  1
 7|0.758|0.651|7.8e-10|1.7e-02|1.4e+04| 1.326499e+05  1.212276e+05| 0:20:28| chol  1  1
 8|0.722|0.723|4.5e-10|7.6e-03|6.2e+03| 1.274056e+05  1.222149e+05| 0:23:07| chol  1  1
 9|0.648|0.674|2.8e-10|3.8e-03|3.1e+03| 1.251192e+05  1.224953e+05| 0:25:39| chol  1  1
10|0.704|0.707|1.7e-10|1.8e-03|1.4e+03| 1.237813e+05  1.226023e+05| 0:28:11| chol  1  1
11|0.861|0.814|1.1e-10|7.3e-04|5.6e+02| 1.231064e+05  1.226322e+05| 0:30:44| chol  1  1
12|0.679|0.668|9.3e-11|4.0e-04|2.8e+02| 1.228784e+05  1.226489e+05| 0:33:15| chol  1  1
13|0.815|0.843|5.4e-11|1.6e-04|1.1e+02| 1.227431e+05  1.226531e+05| 0:35:47| chol  1  1
14|0.818|0.740|2.6e-10|8.8e-05|3.9e+01| 1.226832e+05  1.226546e+05| 0:38:19| chol  1  1
```

```
15|0.775|0.778|8.9e-10|4.3e-05|1.5e+01| 1.226637e+05  1.226540e+05| 0:40:50| chol  1  1
16|0.793|0.924|1.4e-09|3.2e-06|5.1e+00| 1.226558e+05  1.226511e+05| 0:43:23| chol  1  2
17|0.950|0.889|2.1e-10|3.6e-07|1.3e+00| 1.226525e+05  1.226512e+05| 0:45:55| chol  2  2
18|0.758|0.965|4.0e-10|1.3e-08|4.5e-01| 1.226518e+05  1.226513e+05| 0:48:27| chol  2  2
19|0.908|0.921|2.5e-10|1.1e-09|9.3e-02| 1.226514e+05  1.226514e+05| 0:50:59| chol  2  2
20|0.756|0.959|9.5e-10|9.2e-11|3.4e-02| 1.226514e+05  1.226514e+05| 0:53:32| chol  2  3
21|1.000|0.949|1.7e-09|7.8e-11|9.1e-03| 1.226514e+05  1.226514e+05| 0:56:04| chol  2  2
22|0.987|0.789|4.7e-10|1.3e-10|9.0e-04| 1.226514e+05  1.226514e+05| 0:58:36|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 22
 primal objective value =  1.22651364e+05
 dual   objective value =  1.22651363e+05
 gap := trace(XZ)       = 9.02e-04
 relative gap           = 3.68e-09
 actual relative gap    = 3.62e-09
 rel. primal infeas (scaled problem)   = 4.66e-10
 rel. dual     "        "       "      = 1.27e-10
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.6e+04, 6.3e+01, 1.1e+02
 norm(A), norm(b), norm(C) = 5.1e+01, 6.6e+03, 8.8e+01
 Total CPU time (secs)  = 3516.38
 CPU time per iteration = 159.84
 termination code       =  0
 DIMACS: 1.2e-08  0.0e+00  5.6e-09  0.0e+00  3.6e-09  3.7e-09
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +122651
```

```
Calling SDPT3 4.0: 15000 variables, 3000 equality constraints
------------------------------------------------------------

 num. of constraints = 3000
 dim. of socp   var  = 15000,   num. of socp blk  = 7500
 checkdepconstr: AAt is not pos. def.
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|1.0e+00|8.6e+01|1.2e+08| 1.347038e+06  0.000000e+00| 0:1:54| chol  2  2
 1|1.000|0.556|6.6e-08|3.8e+01|8.8e+07| 2.235443e+06  4.917645e+05| 0:3:49| chol  1  1
 2|1.000|0.984|1.4e-08|8.5e-01|3.8e+06| 2.100655e+06  5.991736e+04| 0:7:29| chol  1  1
 3|0.964|0.774|6.4e-09|2.9e-01|7.0e+05| 5.998178e+05  7.226841e+04| 0:11:13| chol  1  1
 4|0.892|0.632|4.2e-09|1.4e-01|2.1e+05| 2.656923e+05  9.897853e+04| 0:14:53| chol  1  1
 5|0.760|0.636|2.2e-09|7.2e-02|8.5e+04| 1.890572e+05  1.180157e+05| 0:18:35| chol  1  1
 6|0.827|0.582|1.1e-09|3.9e-02|3.5e+04| 1.541148e+05  1.251254e+05| 0:22:15| chol  1  1
 7|0.810|0.652|6.3e-10|1.9e-02|1.5e+04| 1.410961e+05  1.288751e+05| 0:25:55| chol  1  1
 8|0.760|0.672|3.6e-10|8.7e-03|6.5e+03| 1.356549e+05  1.303446e+05| 0:29:34| chol  1  1
 9|0.740|0.741|2.1e-10|3.7e-03|2.9e+03| 1.333124e+05  1.309113e+05| 0:33:16| chol  1  1
10|0.755|0.678|1.4e-10|1.8e-03|1.3e+03| 1.321072e+05  1.310910e+05| 0:36:56| chol  1  1
11|0.809|0.724|8.8e-11|8.6e-04|5.3e+02| 1.315774e+05  1.311567e+05| 0:40:38| chol  1  1
12|0.857|0.788|5.6e-11|3.7e-04|2.0e+02| 1.313372e+05  1.311827e+05| 0:44:19| chol  1  1
13|0.801|0.910|9.6e-11|1.4e-04|7.8e+01| 1.312491e+05  1.311906e+05| 0:47:58| chol  1  1
14|0.750|0.821|7.6e-11|7.5e-05|3.4e+01| 1.312138e+05  1.311899e+05| 0:51:42| chol  1  1
15|0.656|0.691|1.1e-10|4.4e-05|1.6e+01| 1.311992e+05  1.311888e+05| 0:55:21| chol  1  1
16|0.964|0.921|1.2e-09|3.5e-06|5.3e+00| 1.311900e+05  1.311852e+05| 0:59:00| chol  1  2
17|0.990|0.867|1.5e-10|4.7e-07|1.6e+00| 1.311870e+05  1.311855e+05| 1:02:40| chol  2  2
18|0.851|0.822|2.5e-10|8.3e-08|5.4e-01| 1.311862e+05  1.311857e+05| 1:06:19| chol  2  2
19|0.932|0.868|5.3e-10|1.1e-08|1.2e-01| 1.311859e+05  1.311858e+05| 1:10:05| chol  2  2
20|1.000|0.903|2.6e-10|1.1e-09|1.4e-02| 1.311858e+05  1.311858e+05| 1:14:14| chol  2  2
```

```
21|0.741|1.000|5.6e-10|5.2e-11|4.4e-03| 1.311858e+05  1.311858e+05| 1:18:39| chol  2  2
22|0.999|1.000|6.8e-10|7.8e-11|1.9e-04| 1.311858e+05  1.311858e+05| 1:24:11|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 22
 primal objective value =  1.31185833e+05
 dual   objective value =  1.31185832e+05
 gap := trace(XZ)       = 1.93e-04
 relative gap           = 7.36e-10
 actual relative gap    = 7.19e-10
 rel. primal infeas (scaled problem)   = 6.82e-10
 rel. dual     "        "       "      = 7.82e-11
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.6e+04, 6.6e+01, 1.1e+02
 norm(A), norm(b), norm(C) = 5.6e+01, 7.3e+03, 8.8e+01
 Total CPU time (secs)  = 5051.30
 CPU time per iteration = 229.60
 termination code       =  0
 DIMACS: 2.0e-08  0.0e+00  3.4e-09  0.0e+00  7.2e-10  7.4e-10
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +131186
```

```
Calling SDPT3 4.0: 15000 variables, 4000 equality constraints
------------------------------------------------------------

 num. of constraints = 4000
 dim. of socp   var  = 15000,   num. of socp blk  = 7500
 checkdepconstr: AAt is not pos. def.
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|1.0e+00|8.6e+01|1.2e+08| 1.347038e+06  0.000000e+00| 0:4:16| chol  2  2
 1|1.000|0.489|7.5e-08|4.4e+01|1.1e+08| 2.344187e+06  5.790316e+05| 0:8:36| chol  1  1
 2|1.000|0.992|1.2e-08|6.0e-01|3.5e+06| 2.220577e+06  5.879228e+04| 0:17:16| chol  1  1
 3|0.874|0.780|8.4e-09|2.3e-01|5.7e+05| 5.315413e+05  8.029043e+04| 0:25:59| chol  1  1
 4|0.877|0.648|4.5e-09|1.2e-01|1.7e+05| 2.557750e+05  1.128138e+05| 0:34:40| chol  1  1
 5|0.753|0.639|2.3e-09|6.3e-02|7.5e+04| 1.946013e+05  1.321764e+05| 0:43:17| chol  1  1
 6|0.897|0.669|1.0e-09|3.1e-02|2.8e+04| 1.634340e+05  1.403979e+05| 0:52:11| chol  1  1
 7|0.815|0.700|6.8e-10|1.5e-02|1.2e+04| 1.536186e+05  1.441499e+05| 1:00:39| chol  1  1
 8|0.796|0.700|4.0e-10|7.1e-03|4.9e+03| 1.492612e+05  1.454536e+05| 1:07:51| chol  1  1
 9|0.812|0.683|2.6e-10|3.6e-03|2.1e+03| 1.474549e+05  1.458924e+05| 1:17:21| chol  1  1
10|0.776|0.751|1.5e-10|1.6e-03|8.8e+02| 1.467186e+05  1.460750e+05| 1:27:08| chol  1  1
11|0.686|0.786|1.1e-10|7.3e-04|4.1e+02| 1.464304e+05  1.461264e+05| 1:34:17| chol  1  1
12|0.780|0.778|8.8e-11|3.5e-04|1.8e+02| 1.462651e+05  1.461331e+05| 1:41:18| chol  1  1
13|0.746|0.910|6.1e-11|1.4e-04|8.1e+01| 1.461943e+05  1.461347e+05| 1:48:32| chol  1  1
14|0.775|0.872|1.7e-10|7.1e-05|3.4e+01| 1.461583e+05  1.461343e+05| 1:56:08| chol  1  1
15|0.770|0.670|2.1e-09|4.4e-05|1.4e+01| 1.461418e+05  1.461339e+05| 2:03:19| chol  1  1
16|0.667|0.850|1.8e-10|6.5e-06|7.2e+00| 1.461370e+05  1.461307e+05| 2:10:34| chol  2  2
17|0.865|0.893|1.5e-10|7.0e-07|2.2e+00| 1.461331e+05  1.461309e+05| 2:18:20| chol  1  2
18|0.731|0.805|1.9e-10|1.4e-07|9.8e-01| 1.461321e+05  1.461311e+05| 2:26:03| chol  2  2
19|0.839|0.826|4.1e-10|2.4e-08|2.9e-01| 1.461316e+05  1.461313e+05| 2:33:48| chol  2  2
20|0.924|0.953|6.5e-10|1.2e-09|9.1e-02| 1.461314e+05  1.461313e+05| 2:41:07| chol  2  2
21|0.706|0.934|7.6e-10|1.6e-10|3.0e-02| 1.461314e+05  1.461313e+05| 2:48:11| chol  2  3
22|1.000|0.966|1.1e-09|1.3e-10|5.5e-03| 1.461314e+05  1.461313e+05| 2:55:03| chol  2  2
23|0.974|0.991|4.8e-10|1.9e-10|3.5e-04| 1.461313e+05  1.461313e+05| 3:01:35|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 23
```

```
 primal objective value =  1.46131347e+05
 dual   objective value =  1.46131346e+05
 gap := trace(XZ)       = 3.51e-04
 relative gap           = 1.20e-09
 actual relative gap    = 1.09e-09
 rel. primal infeas (scaled problem)   = 4.80e-10
 rel. dual     "        "       "      = 1.89e-10
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.6e+04, 7.1e+01, 1.1e+02
 norm(A), norm(b), norm(C) = 6.4e+01, 8.4e+03, 8.8e+01
 Total CPU time (secs)  = 10894.89
 CPU time per iteration = 473.69
 termination code       =  0
 DIMACS: 1.6e-08  0.0e+00  8.3e-09  0.0e+00  1.1e-09  1.2e-09
------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +146131
```

CS for rena

```
% for lena
resized_lena= imresize(lena2, [75 100]);
lena2_4_fft = double(resized_lena);
[ny,nx] = size(lena2_4_fft);

% For lena
count_pic = 1;
for k = [2500 3000]; % number of sparse samples
test2 = zeros(ny, nx);
r1 = randintrlv([1:nx*ny], 793); % random permutation with set.seed(793)
```

```
r1k = r1(1:k);
for j=1:k
    test2(r1k(j))=-1;
end

% mask for image
test = zeros(ny, nx);
for j=1:k
    test(r1k(j))=1;
    Adel= reshape(idct2(test), nx*ny, 1);
    Adelta(j, :) = Adel;
    test(r1k(j))=0;
end

% L1 minimization with cvx toolbox
b2 = lena2_4_fft(r1k).';
n = nx*ny;
cvx_begin;
    variable y(n);
    minimize(norm(y, 1));
    subject to
        Adelta*y == b2;
cvx_end;

% reconstruction of image
Alow = uint8((dct2(reshape(y, ny, nx))));
figure(3), subplot(2,2,count_pic), imshow(Alow);
count_pic = count_pic+1;
end
```

```
경고: Unknown parameter: 'isvname'
경고: Unknown parameter: 'appname'
경고: Unknown parameter: 'isv_key'
```

```
Calling SDPT3 4.0: 15000 variables, 2500 equality constraints
------------------------------------------------------------

 num. of constraints = 2500
 dim. of socp   var  = 15000,   num. of socp blk  = 7500
 checkdepconstr: AAt is not pos. def.
*****************************************************************
   SDPT3: Infeasible path-following algorithms
*****************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|1.0e+00|8.6e+01|1.1e+08| 1.240972e+06  0.000000e+00| 0:1:54| chol  2  2
 1|1.000|0.569|6.4e-08|3.7e+01|7.9e+07| 2.054209e+06  4.580951e+05| 0:4:07| chol  1  1
 2|1.000|0.987|1.5e-08|7.3e-01|3.3e+06| 1.926917e+06  6.072306e+04| 0:7:50| chol  1  1
 3|0.948|0.755|7.1e-09|2.7e-01|5.9e+05| 5.215885e+05  7.489711e+04| 0:11:00| chol  1  1
 4|0.870|0.581|4.3e-09|1.5e-01|1.9e+05| 2.491573e+05  9.558728e+04| 0:14:39| chol  1  1
 5|0.809|0.615|2.3e-09|7.7e-02|7.4e+04| 1.715521e+05  1.109388e+05| 0:18:03| chol  1  1
 6|0.768|0.666|1.3e-09|3.6e-02|3.0e+04| 1.433671e+05  1.180769e+05| 0:21:02| chol  1  1
 7|0.774|0.651|7.6e-10|1.8e-02|1.3e+04| 1.306190e+05  1.203665e+05| 0:23:47| chol  1  1
 8|0.726|0.684|4.6e-10|8.2e-03|5.9e+03| 1.259973e+05  1.211562e+05| 0:26:34| chol  1  1
 9|0.716|0.572|2.8e-10|4.6e-03|2.8e+03| 1.236475e+05  1.213993e+05| 0:29:20| chol  1  1
10|0.789|0.723|1.6e-10|2.0e-03|1.2e+03| 1.224561e+05  1.215345e+05| 0:32:05| chol  1  1
11|0.712|0.761|1.0e-10|8.4e-04|5.3e+02| 1.220081e+05  1.215768e+05| 0:34:50| chol  1  1
12|0.740|0.720|6.8e-11|4.1e-04|2.4e+02| 1.217780e+05  1.215845e+05| 0:37:25| chol  1  1
13|0.743|0.793|4.0e-11|1.8e-04|1.1e+02| 1.216727e+05  1.215850e+05| 0:39:57| chol  1  1
14|0.732|0.925|4.6e-11|6.9e-05|4.9e+01| 1.216243e+05  1.215839e+05| 0:42:28| chol  1  1
15|0.808|0.816|3.0e-10|3.7e-05|1.6e+01| 1.215956e+05  1.215837e+05| 0:45:02| chol  1  1
```

```
16|0.814|0.747|1.1e-09|9.5e-06|6.4e+00| 1.215868e+05  1.215816e+05| 0:47:36| chol  1  1
17|0.688|0.933|7.6e-09|6.4e-07|2.7e+00| 1.215839e+05  1.215813e+05| 0:50:09| chol  2  2
18|0.832|0.795|1.4e-09|1.3e-07|8.8e-01| 1.215822e+05  1.215814e+05| 0:52:40| chol  2  2
19|1.000|0.926|5.3e-10|9.7e-09|1.6e-01| 1.215816e+05  1.215814e+05| 0:55:12| chol  2  1
20|0.752|0.895|2.6e-10|1.1e-09|4.4e-02| 1.215815e+05  1.215815e+05| 0:57:44| chol  2  2
21|0.944|0.970|3.2e-10|8.4e-11|5.9e-03| 1.215815e+05  1.215815e+05| 1:00:18| chol  3  3
22|1.000|0.834|4.2e-09|7.8e-11|1.5e-03| 1.215815e+05  1.215815e+05| 1:02:48|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 22
 primal objective value =  1.21581483e+05
 dual   objective value =  1.21581481e+05
 gap := trace(XZ)       = 1.46e-03
 relative gap           = 5.99e-09
 actual relative gap    = 6.08e-09
 rel. primal infeas (scaled problem)   = 4.15e-09
 rel. dual     "        "       "      = 7.79e-11
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.6e+04, 6.2e+01, 1.1e+02
 norm(A), norm(b), norm(C) = 5.1e+01, 6.7e+03, 8.8e+01
 Total CPU time (secs)  = 3767.78
 CPU time per iteration = 171.26
 termination code       =  0
 DIMACS: 1.2e-07  0.0e+00  3.4e-09  0.0e+00  6.1e-09  6.0e-09
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +121581
```

```
Calling SDPT3 4.0: 15000 variables, 3000 equality constraints
------------------------------------------------------------

 num. of constraints = 3000
 dim. of socp   var  = 15000,   num. of socp blk  = 7500
 checkdepconstr: AAt is not pos. def.
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|1.0e+00|8.6e+01|1.1e+08| 1.240972e+06  0.000000e+00| 0:1:52| chol  2  2
 1|1.000|0.527|6.9e-08|4.1e+01|8.9e+07| 2.113097e+06  5.088847e+05| 0:3:47| chol  1  1
 2|1.000|0.984|1.3e-08|8.8e-01|3.7e+06| 1.993443e+06  6.321754e+04| 0:7:25| chol  1  1
 3|0.985|0.815|6.0e-09|2.6e-01|6.2e+05| 5.535876e+05  7.717995e+04| 0:11:03| chol  1  1
 4|0.875|0.618|4.1e-09|1.4e-01|1.9e+05| 2.533603e+05  1.015246e+05| 0:14:39| chol  1  1
 5|0.833|0.636|2.2e-09|7.0e-02|6.8e+04| 1.747205e+05  1.189203e+05| 0:18:19| chol  1  1
 6|0.817|0.639|1.3e-09|3.5e-02|2.7e+04| 1.482168e+05  1.263302e+05| 0:22:03| chol  1  1
 7|0.651|0.651|8.4e-10|1.7e-02|1.3e+04| 1.397796e+05  1.290641e+05| 0:25:39| chol  1  1
 8|0.810|0.606|4.0e-10|9.2e-03|6.3e+03| 1.347982e+05  1.297585e+05| 0:29:19| chol  1  1
 9|0.811|0.732|2.3e-10|3.9e-03|2.6e+03| 1.323456e+05  1.302522e+05| 0:32:59| chol  1  1
10|0.770|0.728|1.4e-10|1.8e-03|1.1e+03| 1.313307e+05  1.304196e+05| 0:36:40| chol  1  1
11|0.798|0.786|9.7e-11|7.5e-04|5.1e+02| 1.308779e+05  1.304724e+05| 0:40:19| chol  1  1
12|0.815|0.741|7.4e-11|3.7e-04|2.1e+02| 1.306489e+05  1.304914e+05| 0:43:57| chol  1  1
13|0.738|0.813|6.0e-11|1.7e-04|9.6e+01| 1.305692e+05  1.304956e+05| 0:47:38| chol  1  1
14|0.829|0.790|2.0e-10|8.3e-05|3.5e+01| 1.305206e+05  1.304961e+05| 0:51:18| chol  1  1
15|0.674|0.709|6.7e-11|4.6e-05|1.6e+01| 1.305055e+05  1.304956e+05| 0:55:07| chol  1  1
16|0.919|0.900|7.8e-10|4.6e-06|5.2e+00| 1.304969e+05  1.304923e+05| 0:58:47| chol  2  2
17|0.852|0.778|2.3e-10|1.0e-06|1.7e+00| 1.304941e+05  1.304925e+05| 1:02:27| chol  2  2
18|0.822|0.778|4.1e-10|2.3e-07|5.9e-01| 1.304932e+05  1.304926e+05| 1:06:05| chol  2  2
19|0.729|0.927|2.3e-10|1.6e-08|2.3e-01| 1.304930e+05  1.304927e+05| 1:09:45| chol  2  3
20|1.000|0.807|1.7e-09|3.2e-09|5.0e-02| 1.304928e+05  1.304927e+05| 1:13:25| chol  3  3
21|0.906|0.828|1.2e-09|6.2e-10|8.1e-03| 1.304928e+05  1.304928e+05| 1:17:07| chol  3  3
```

```
22|1.000|0.853|2.5e-09|2.0e-10|1.2e-03| 1.304928e+05  1.304928e+05| 1:20:45|
  stop: max(relative gap, infeasibilities) < 1.49e-08
------------------------------------------------------------------
 number of iterations   = 22
 primal objective value =  1.30492760e+05
 dual   objective value =  1.30492759e+05
 gap := trace(XZ)       = 1.18e-03
 relative gap           = 4.52e-09
 actual relative gap    = 4.48e-09
 rel. primal infeas (scaled problem)   = 2.51e-09
 rel. dual     "        "       "      = 1.95e-10
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.6e+04, 6.6e+01, 1.1e+02
 norm(A), norm(b), norm(C) = 5.6e+01, 7.3e+03, 8.8e+01
 Total CPU time (secs)  = 4844.80
 CPU time per iteration = 220.22
 termination code       =  0
 DIMACS: 7.8e-08  0.0e+00  8.6e-09  0.0e+00  4.5e-09  4.5e-09
------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +130493
```

```
다음 사용 중 오류가 발생함: cvxprob/newcnstr (line 87)
Matrix dimensions must agree.

오류 발생:  ==  (line 12)
b = newcnstr( evalin( 'caller', 'cvx_problem', '[]' ), x, y, '==' );
```

Beating Sannon-Nyquist

```
addpath ./Reference/DMD-Brunton/CODE/CH09_SPARSITY/utils/
% Generate signal
```

```
Fs = 2000; p=128;
t = 0:1/Fs:1-1/Fs;
x = sin(73*2*pi*t) + sin(531*2*pi*t);
N = length(x)
```

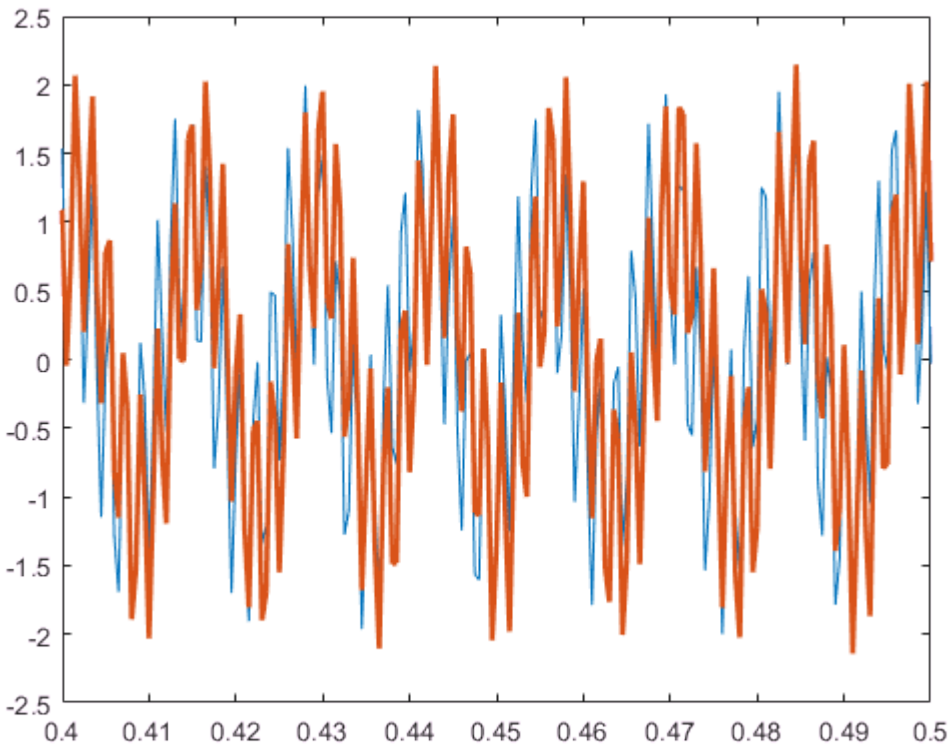
```
N = 2000
```

```
% Randomly sample signal
perm = round(rand(p, 1) * N);
y = x(perm)';

% Form matrix operators
Psi = dct(eye(N, N));
CPsi = Psi(perm, :);

% L1 minimization (through linear program)
s = cosamp(CPsi,y,10,1.e-10,10);
xreconstruct = idct(s);

% plot (Original vs Reconstruction)
figure; plot(t,x)
hold on
plot(t,xreconstruct,'LineWidth',2)
xlim([.4 .5])
```




```
% power spectrum (Original vs Reconstruction)
rng default
xdft = fft(x);
xdft = xdft(1:N/2+1);
psdx = (1/(1000)) * abs(xdft);
```

```
psdx(2:end-1) = 2*psdx(2:end-1);
freq = 0:Fs/length(x):Fs/2;
figure; plot(freq,psdx,'LineWidth',2)
grid on
hold on
xdft2 = fft(xreconstruct);
xdft2 = xdft2(1:N/2+1);
psdx2 = (1/(1000)) * abs(xdft2);
psdx2(2:end-1) = 2*psdx2(2:end-1);
plot(freq,psdx2,'LineWidth',2)
xlabel('Frequency (Hz)')
ylabel('PSD')
```