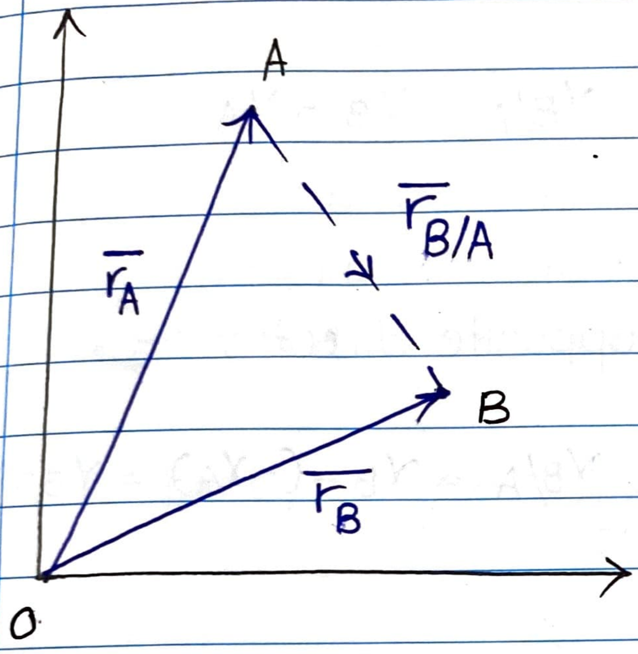
Relative Motion.



Consider two particles A and B moving in same plane.
Let $\bar{r}_A$ and $\bar{r}_B$ be the position vectors of points A and B respectively.
Then, position vector of B w.r.t. point A is given as →

$$\bar{r}_{B/A} = \bar{r}_B - \bar{r}_A.$$

$$\hat{i}_{B/A} = \hat{i}_B - \hat{i}_A. \text{ and}$$
$$\hat{j}_{B/A} = \hat{j}_B - \hat{j}_A.$$

This Gives :- $\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$,

$$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A} \quad \text{or} \quad \dot{\bar{r}}_B = \dot{\bar{r}}_A + \dot{\bar{r}}_{B/A}.$$

$$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A} \quad \text{or} \quad \dot{\bar{V}}_B = \dot{\bar{V}}_A + \dot{\bar{V}}_{B/A}.$$
$$\text{or} \quad \ddot{\bar{r}}_B = \ddot{\bar{r}}_A + \ddot{\bar{r}}_{B/A}$$

* $\bar{r}_A, \bar{r}_B, \bar{V}_A, \bar{V}_B, \bar{a}_A$ and $\bar{a}_B$ → Absolute Properties. (w.r.t. origin).
* $\bar{r}_{B/A}$, $\bar{V}_{B/A}$ and $\bar{a}_{B/A}$ are relative properties of B w.r.t. A.

* Working Rules *

① Particles moving in same direction :-

$v_A \longrightarrow$

$v_B \longrightarrow$

$$v_{B/A} = v_B - v_A .$$

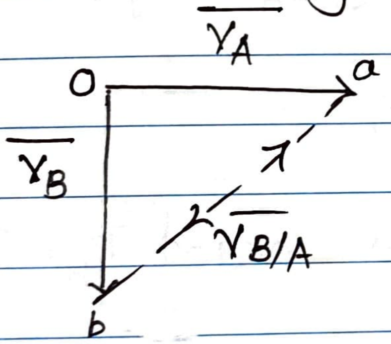
② Particles moving in opposite direction :-

$v_B \longrightarrow$

$\longleftarrow v_A$

$$v_{B/A} = v_B - (-v_A) = v_B + v_A .$$

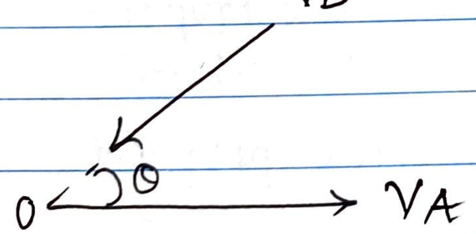
③ Velocities at right angles :-



Magnitude of relative velocity $= \sqrt{v_A^2 + v_B^2}$

$$\theta = \tan^{-1}(v_B/v_A)$$

④ Velocities at an angle θ :-



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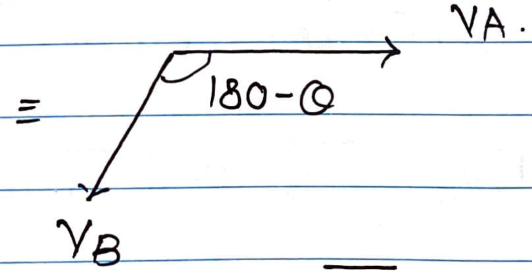

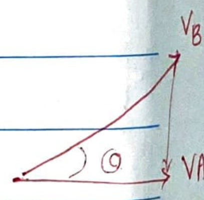

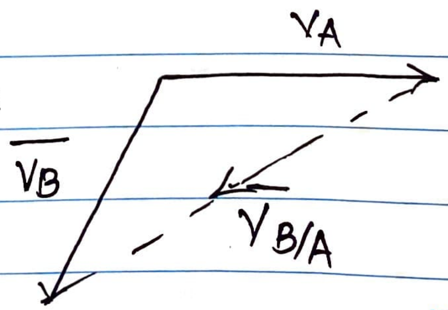

$$v_{B/A} = \sqrt{v_A^2 + v_B^2 - 2 v_A v_B \cos(180-\theta)} \equiv$$

or

$$\frac{v_{B/A}}{\sin(180-\theta)} = \frac{v_B}{\sin\alpha} .$$

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$$v_{B/A} = \sqrt{v_A^2 + v_B^2 \pm 2 v_A v_B \cos\theta}$$

(6) Method of resolving vectors into i, j components and thereby determining the magnitude and direction of relative velocity can also be used.

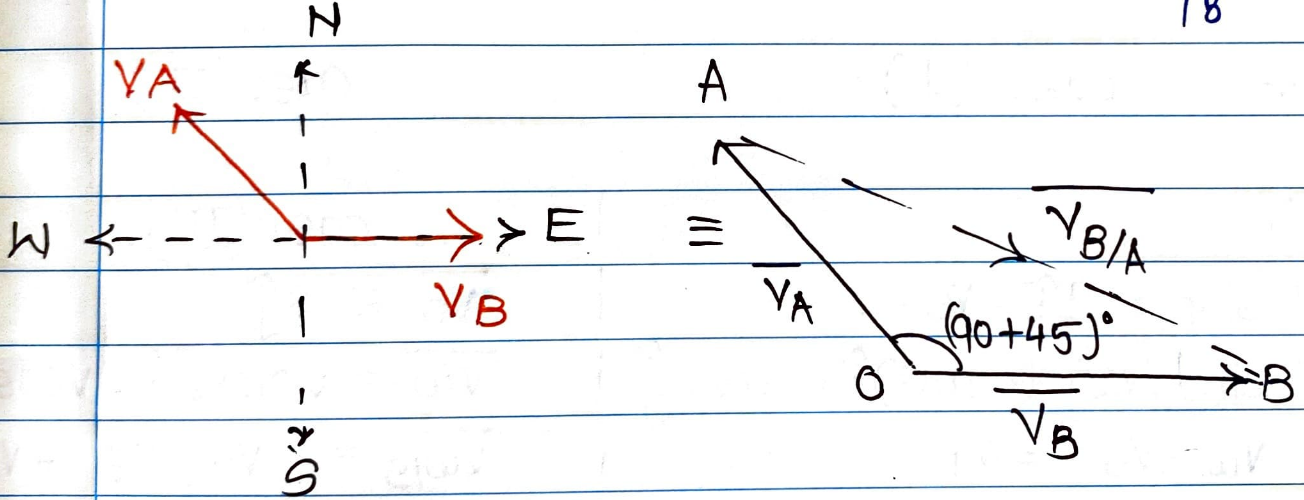
* Numericals:-

(1) Ship A is moving north-west at a speed of 18 kmph. ship B is moving east at a speed of 9 kmph. Find the direction and magnitude of relative velocity of B w.r.t A

⇒ We have, V_A = speed of ship A = 18 kmph = $18 \times \frac{5}{18}$

= 5 m/s.

V_B = Speed of ship B = 9 kmph = $9 \times \frac{5}{18}$ = 2.5 m/s.



$\therefore V_{B/A} = \sqrt{V_A^2 + V_B^2 - 2 V_A V_B \cos(135^\circ)}$

$\therefore V_{B/A} = (6.99)$ m/s ⟵ magnitude.

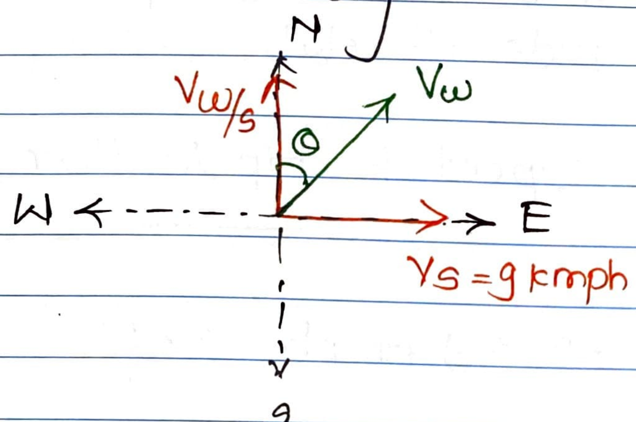
To find direction,

$$\frac{V_B}{\sin\theta} = \frac{V_{B/A}}{\sin 135^\circ}$$

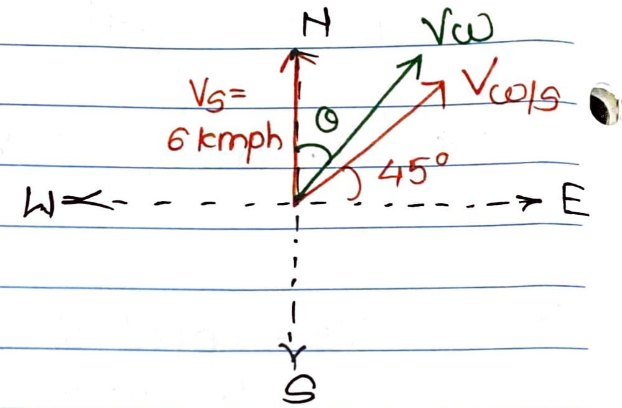
$$\therefore \sin\theta = \frac{2.5 \times 0.707}{6.99} \qquad \therefore \boxed{\theta = 14.65^\circ}$$

(2) Observed from a ship moving due East at 9 kmph the wind appears to blow from south. After ship has changed course and speed, and as it is moving due north at 6 kmph, the wind appears to blow from south-west. If the wind velocity is constant during period of observations, determine magnitude and direction of true wind velocity.

$\Rightarrow$



case (I)



case (II)

Case (I)

Velocity of ship $= 9\hat{i} = V_S$.

velocity of wind $(V_w) = (v\sin\theta)\hat{i} + v\cos\theta\,\hat{j}$

$\overline{V_{w/s}} = V_w - V_S = V_1$

but $\overline{V_{w/s}} = (V_1)\hat{j}$.

$\therefore (V_1)\hat{i} = 0 = v\sin\theta\,\hat{i} - 9\hat{i}$

$\therefore v\sin\theta - 9 = 0$. $\therefore v\sin\theta = 9$ —(1)

case (II)

$\overline{V_S} = 6\hat{j}$

$\overline{V_w} = v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j}$

$\overline{V_{w/s}} = V_w - V_S = V_2$

$\overline{V_{w/s}} = (V_2\cos 45)\hat{i} + (V_2\sin 45)\hat{j}$

Equating Coefficients of $\hat{i}$ and $\hat{j}$

$V_2\cos 45 = v\sin\theta$. and

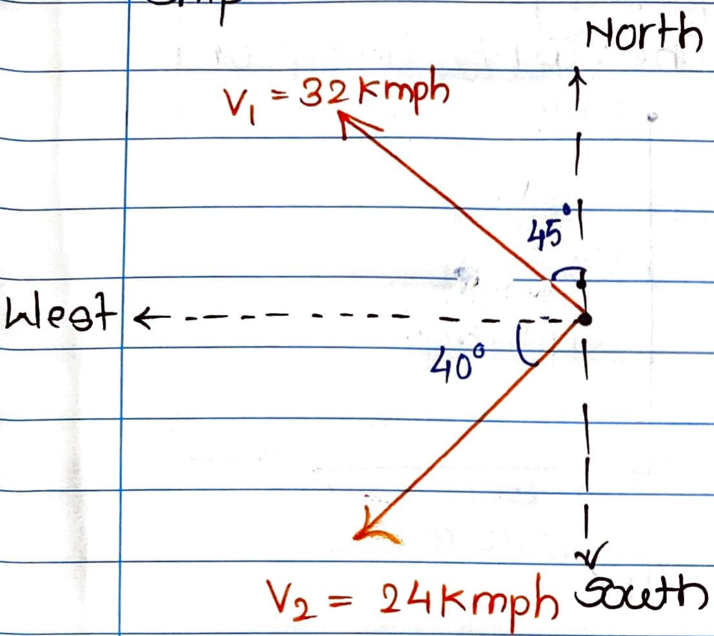
$V_2\sin 45 = v\cos\theta - 6$.

$\therefore \tan 45° = 1 = \dfrac{v\cos\theta - 6}{v\sin\theta}$

$\therefore v\cos\theta - 6 = 9$ — from (1)

$\therefore v\cos\theta = 15$, $\therefore \tan\theta = \dfrac{9}{15}$ $\therefore \boxed{\theta = 31°}$. & $\boxed{v = 17.47 \text{ kmph}}$

(3) Two ships leave the port at the same time. The first streams North-west at 32 kmph and second streams 40° south of west at 24 kmph. Determine relative velocity of second ship w.r.t. first ship.



Given:

$V_1 = 32$ kmph.

$V_2 = 24$ kmph.

$$V_1 = (-32\sin 45^\circ)\hat{i} + (32\cos 45)\hat{j}$$
$$= -22.624\,\hat{i} + 22.624\,\hat{j}$$

$$V_2 = (-24\cos 40)\hat{i} - (24\sin 40)\hat{j}$$
$$= -18.385\,\hat{i} - 15.427\,\hat{j}.$$

Relative velocity of ship 2 w.r.t. ship 1 ⇒

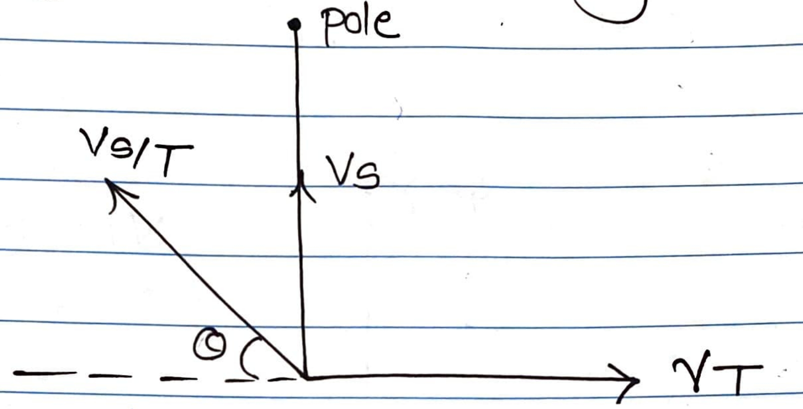
$$\bar{V}_{2/1} = \bar{V}_2 - \bar{V}_1$$
$$= [-18.385 - (-22.624)]\,\hat{i}$$
$$+ [-15.427 - 22.624]\,\hat{j}$$
$$= 4.239\,\hat{i} - 38.051\,\hat{j} \text{ kmph.}$$

$$\therefore \quad |\bar{V}_{2/1}| = \sqrt{(4.239)^2 + (38.051)^2}$$
$$= 38.286 \text{ kmph.}$$

(4) A passenger travelling in a train tries to ~~ed~~ hit a pole near the track by throwing a stone with horizontal velocity of 20 m/s relative to train, where pole is just across him. If the speed of train is 36 kmph, Determine.

① Direction in which the passenger should throw the stone.

② Horizontal velocity of stone w.r.t. the ground.



$\Rightarrow$ $V_T = 36 \text{ kmph} = 36 \times \frac{5}{18} = 10 \text{ m/s}.$

Let θ be angle of relative velocity of stone with truck

$\therefore \overline{V_T} = 10\hat{i}$

$$\overline{V_{S/T}} = -(20\cos\theta)\hat{i} + (20\sin\theta)\hat{j}.$$

$$\overline{V_S} = V_S\,\hat{j}.$$

Now, we have, $\overline{V_{S/T}} = \overline{V_S} - \overline{V_T}$

$$\therefore (-20\cos\theta)\hat{i} + (20\sin\theta\hat{j}) = (V_S)\hat{j} - (10)\hat{i}$$

$$\therefore 20\cos\theta = 10$$

$$\therefore \cos\theta = \frac{1}{2} \qquad \therefore \boxed{\theta = 60^\circ}$$

Also, $20\sin\theta = V_S$

$\therefore V_S = 20\sin 60^\circ = 17.32 \text{ m/s}.$

$\therefore$ Velocity of stone w.r.t. ground (Absolute velocity)

$= 17.32 \text{ m/s}.$