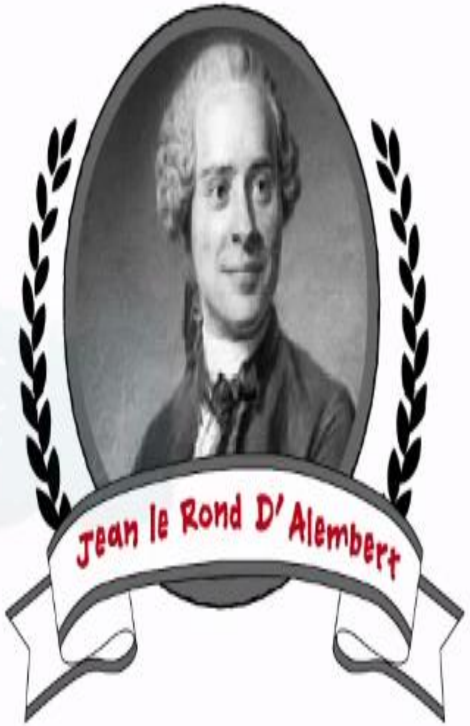
D' Alembert's Principle

Dynamic Equilibrium State



$$\Sigma F - ma = 0$$

$$R - ma = 0$$

- **Dalembert's principle :-** $\Rightarrow N = mg = 100 \times 10$

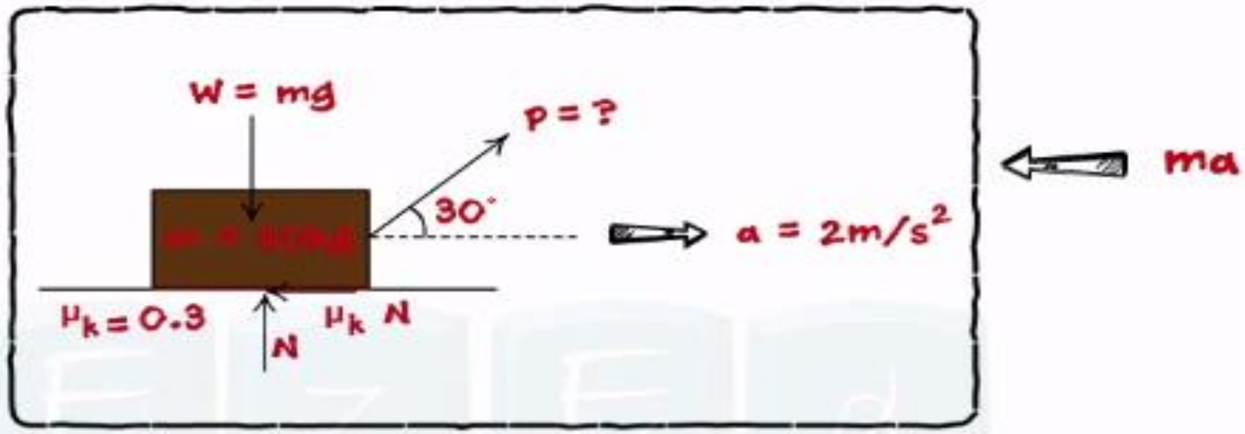
It states that under the actn of eff. Force and inertia force body will be in dynamic equilibrium

- If a body of mass 'm' moves in x-directn with accln 'a' that means there is effective force (Net Force) $F = ma$ acting in x-directn.

Now, if we apply force $P = ma$ in the -ve x-directn then body will be in equilibrium called as dynamic equilibrium.

- Inertia force is the imaginary force acting opposite to the motn having same magnitude like resultant force

Q. Consider a block of mass 40 kg acted upon by a force P which acts at an angle $\theta = 30^\circ$ with horizontal. Due to this force, block is moving with acceleration of 2 m/s^2. Given that the coefficient of friction $\mu k = 0.3$. Find force P. Use D' Alembert's principle.



Applying in x-direction

$$\Sigma F_x - ma = 0$$
$$(P \cos 30 - \mu_k . N) - ma = 0$$
$$P \cos 30 - 0.3.N - 40*2 = 0 \ldots\ldots(1)$$

Applying in y-direction

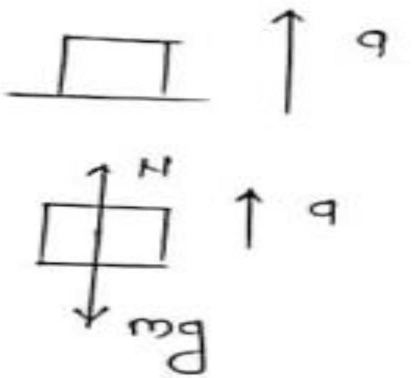
$$\Sigma F_y - ma = 0$$
$$(P \sin 30 + N - W) - ma = 0$$
$$P \sin 30 + N - 40*9.81 - 40*2 = 0 \ldots.(2)$$

On solving equation (1) & (2), we get

$$\boxed{P = 218.22 \text{ Newton}}$$

N S L Equation Applied To Rectilinear Motion

① person travelling in lift with accl^n 'a' in Upward direct^n.



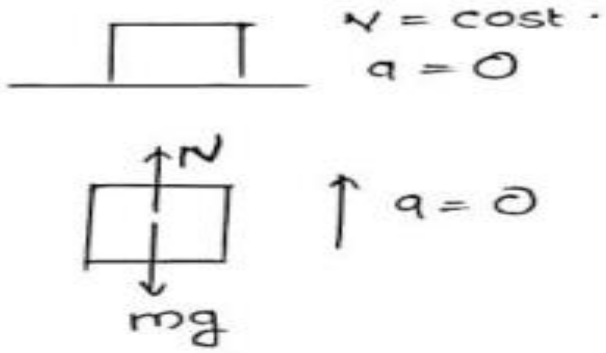
$a \neq 0 \quad (f_y) \neq 0$

$\therefore (f_y)_{net} = ma$

$\therefore N - mg = ma$

$\therefore \boxed{N = mg + ma}$

② person travelling in lift with const. velocity



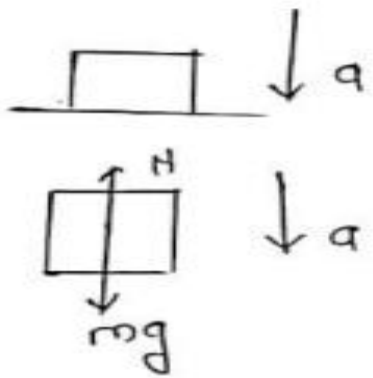
as $a = 0$, $f_y = 0$

$\therefore f_y = 0$

$\Rightarrow N - mg = 0$

$\therefore \boxed{N = mg}$

③ person travelling in lift with accl^n 'a' in downward direct^n



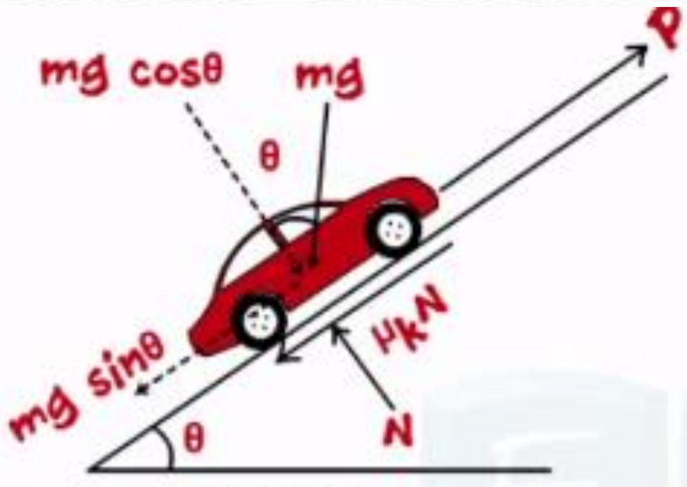
as $a \neq 0$, $f_y \neq 0$

$f_y = ma$

$mg - N = ma$

$\boxed{N = mg - ma}$

N S L Equation Applied To Rectilinear Motion



FBD

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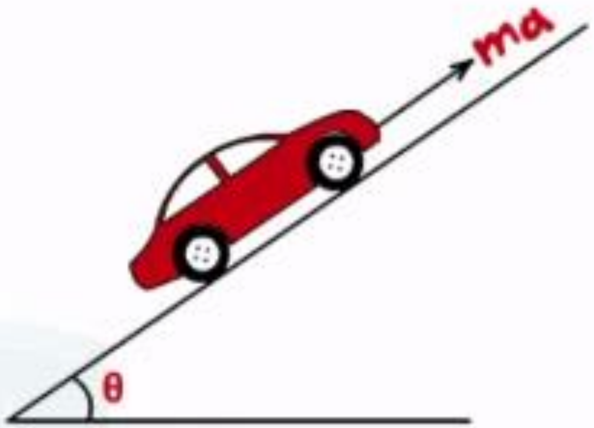


KD

$$\Sigma F_x = ma_x$$

$$P - mg\sin\theta - \mu_k N = ma \text{........}(1)$$

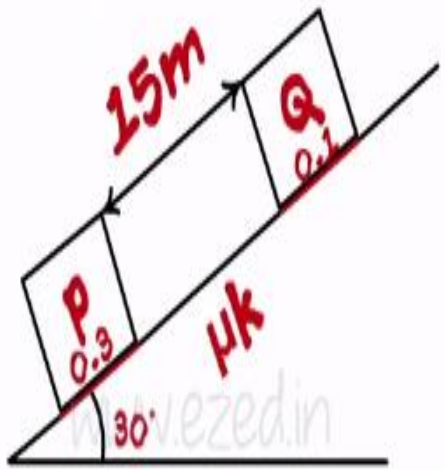
$$\Sigma F_y = ma_y$$

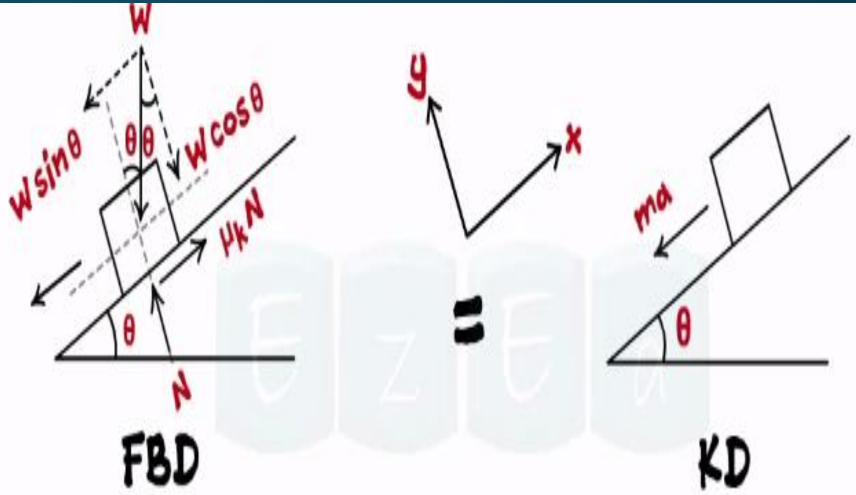
$$N - mg\cos\theta = 0$$

$$N = mg\cos\theta \text{.......}(2)$$

$$\boxed{a = \frac{P - mg(\sin\theta - \mu_k\cos\theta)}{m}}$$

Q. Two blocks P and Q are held stationary 15 m apart on a 30° inclined plane. The kinetic coefficient of friction between P and plane is 0.3 and between Q and plane is 0.1. If the blocks are released simultaneously, calculate the time taken and distance traveled by each block before they are on the verge of collision.





Apply Newton's second law

$$\Sigma F_y = ma_y \qquad\qquad \Sigma F_x = ma_x$$

$$N - W\cos\theta = 0 \qquad\qquad W\sin\theta - \mu_k N = ma \ldots\ldots\ldots\ldots(2)$$

$$N = W\cos\theta \ldots\ldots\ldots\ldots\ldots(1)$$

Substitute the value of N from (1) in (2)

$$W\sin\theta - \mu k\, W\cos\theta = \frac{W}{g}a \qquad\qquad (W = mg)$$

$$\therefore a = g(\sin\theta - \mu_k \cos\theta) \ldots\ldots\ldots\ldots(3)$$

For block P, $\mu_k = 0.3$, $\theta = 30^\circ$

$a_P = 9.81(\sin 30 - 0.3 * \cos 30)$

$a_P = 2.35\ m/s^2$

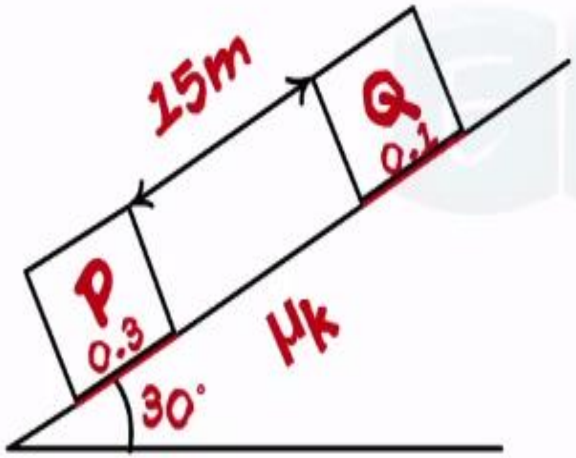
For block Q, $\mu_k = 0.1$, $\theta = 30^\circ$

$a_Q = 9.81(\sin 30 - 0.1 * \cos 30)$

$a_Q = 4.05\ m/s^2$

Let P travels x metres before Q collides.

$\therefore$ Q travels (x + 15) m during same time interval.



$a_Q > a_P$

Block P

$u = 0$

$v = -$

$s = x$

$a = 2.35\ m/s^2$

$t = t$ sec

using $s = ut + \frac{1}{2}at^2$

$$x = 0 + \frac{1}{2}(2.35)t^2 \quad \ldots.(4)$$

Block Q

$u = 0$

$v = -$

$s = (x + 15)$

$a = 4.05\ m/s^2$

$t = t$ sec

using $s = ut + \frac{1}{2}at^2$

$$(x + 15) = 0 + \frac{1}{2}(4.05)t^2 \quad \ldots\ldots.(5)$$

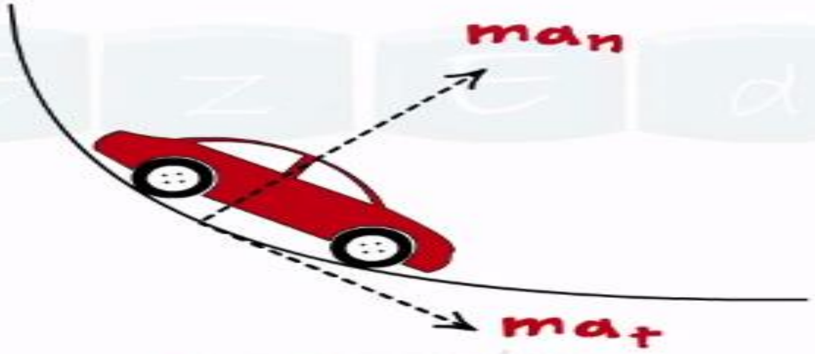
Solving (4) & (5), we get

$\boxed{x = 20.73\ m}$

$\boxed{t = 4.2\ sec}$

N S L Equation Applied To Curvilinear Motion

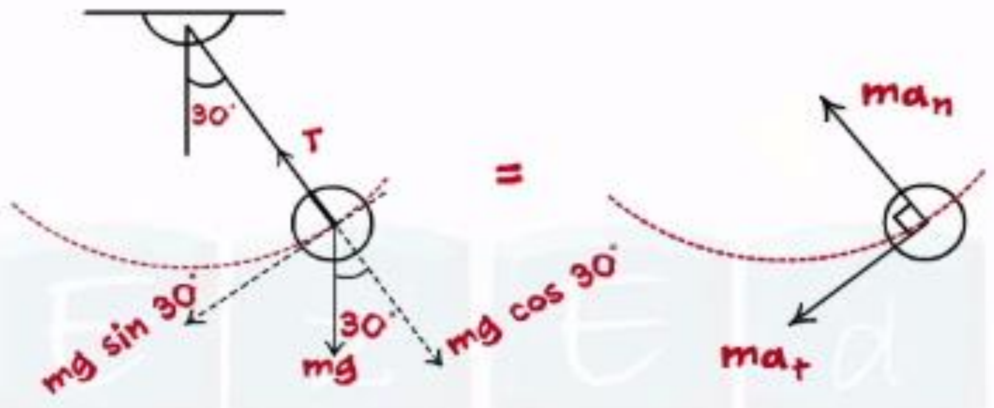
Rectilinear Motion v/s Curvilinear Motion



Kinetic Diagram

Q. A bob of 2 m pendulum describes an arc of circle in a vertical plane. If the tension in the cord is 5 times the weight of the bob for the position shown, find the velocity and acceleration of the bob in that position.

Solution:



$$\Sigma F_y = ma_y$$

$$T - mg\cos 30 = ma_n$$

$$5m - m \quad \cos 30 = ma_n$$

$$\therefore a_n = 40.55 \text{ m/s}^2$$

$$\Sigma F_x = ma_x$$

$$-mg\sin 30 = -ma_t$$

$$a_t = 4.91 \text{ m/s}^2$$

Total Acceleration

$$a = \sqrt{a_t^2 + a_n^2}$$

$$\sqrt{4.91^2 + 40.55^2}$$

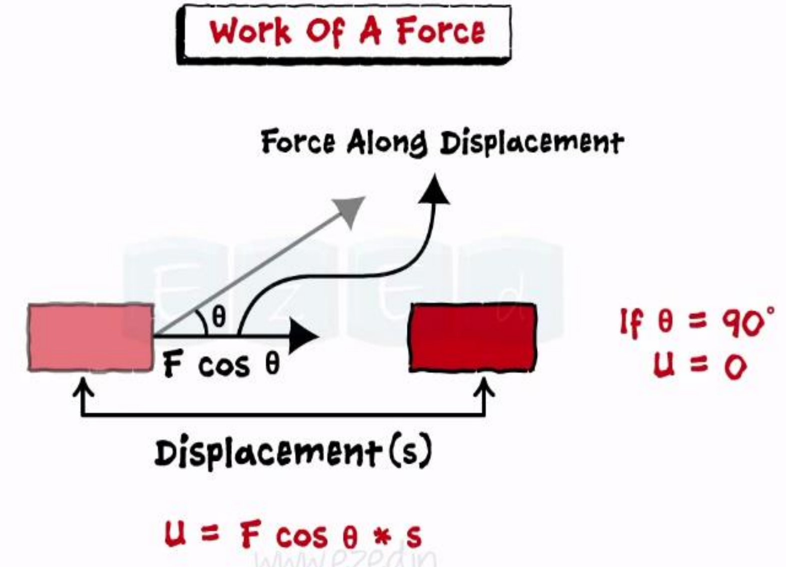
$$\boxed{a = 40.84 \text{ m/s}^2}$$

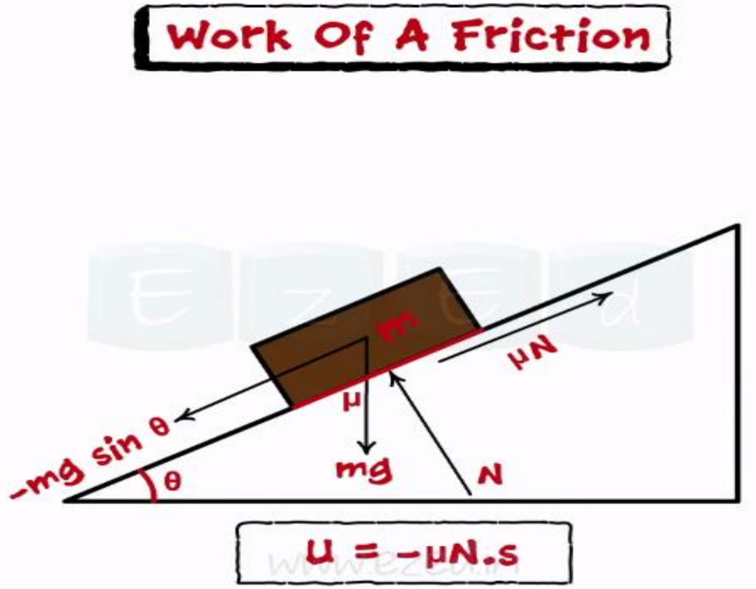
Also, $a_n = \dfrac{v^2}{\rho}$

$$\therefore \boxed{v = 9 \text{ m/s}}$$

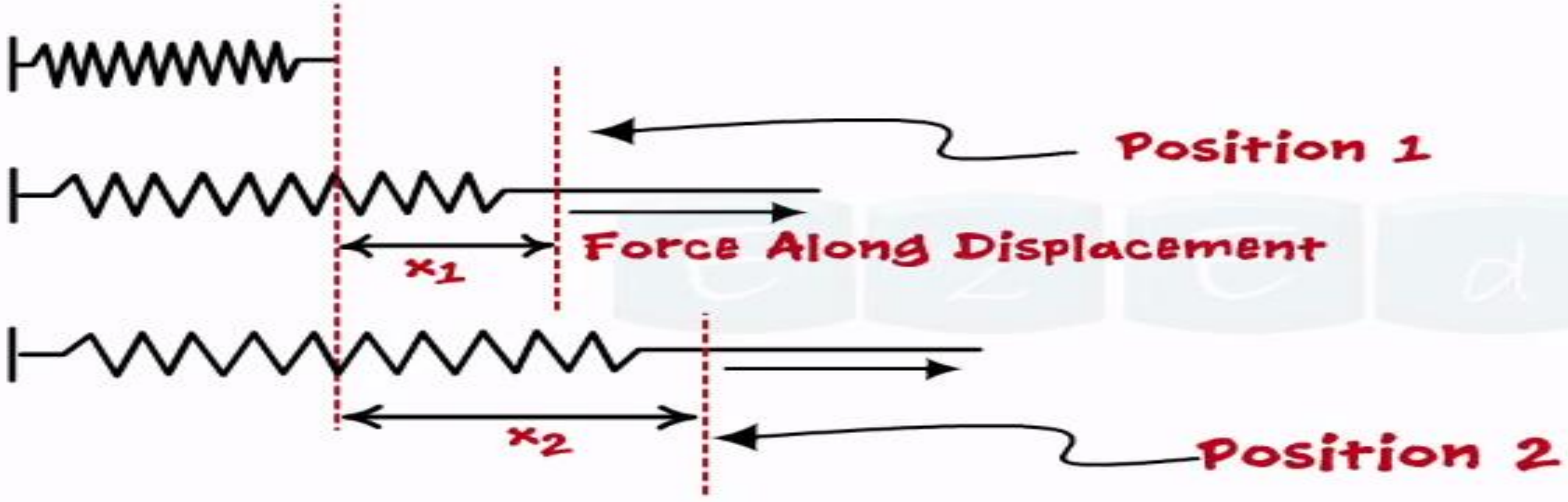
Work, Energy and Power

Energy (Joule)	• Ability to do work.
	• Law of Conservation of Energy: Energy cannot be destroyed nor created but is converted from one form to another.
	• Kinetic energy- Energy in motion = $1/2mv^2$
	• Gravitational potential energy(GPE)= mass x gravity x height= mgh Energy stored in an object due to its position(height) in a gravitational field.
Work (Nm or J)	Product of force and distance moved in the direction of the force. W= F.s or Fs.cosΘ 
Power (W)	Rate of doing work/energy transfer. Power = $\frac{work\ done}{time}$ or Force x velocity.





Work Of A Spring

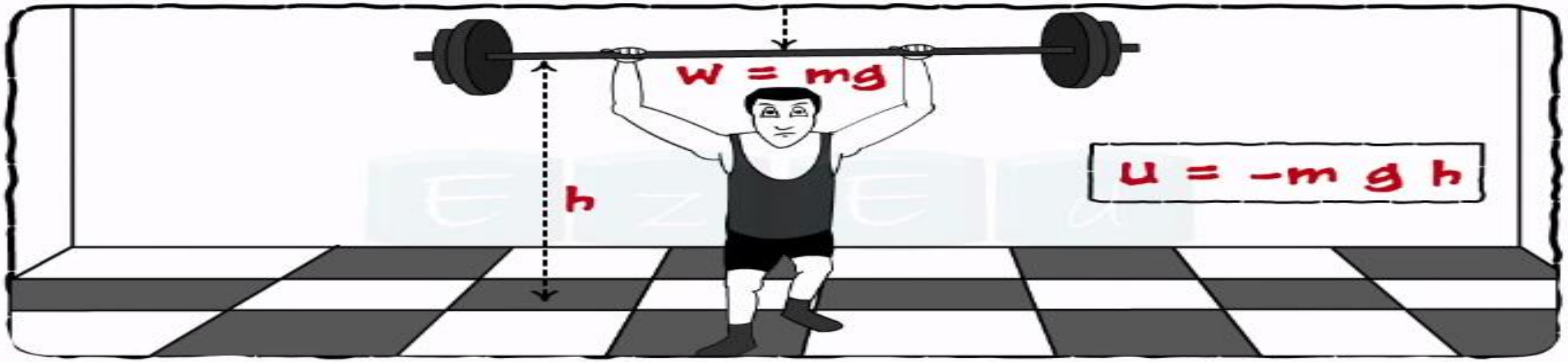


$$U = \int_{x_1}^{x_2} -kx\,dx$$

$$= -\frac{1}{2}k(x_2^2 - x_1^2)$$

$$\boxed{U = \frac{1}{2}k\,(x_1^2 - x_2^2)}$$

Work Of A Weight Force

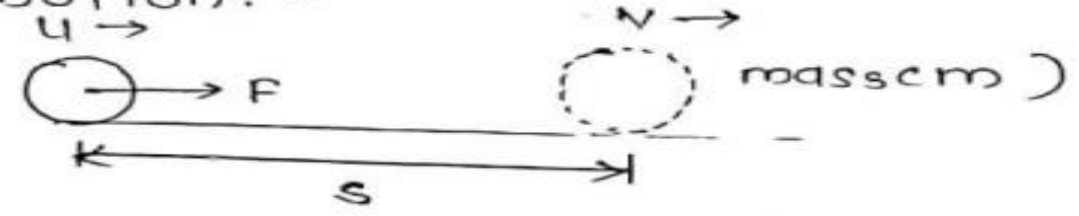


NOTE : If the displacement of weight is against the gravity (upwards) then it is '−ve'. If the displacement of the weight is in the direction of gravity (downwards) then it is '+ve'.

Work Energy Principle

If a particle of mass 'm' subjected to unbalanced force system, the total workdone by all forces during the displacement is equal to change in kinetic energy during that displacement.

① For linear motion:-



workdone = change in kinetic energy

F×s = final k.E. − Initial k.E.

$F \times s = \frac{1}{2} m v^2 - \frac{1}{2} m u^2$

② For angular motn :

∴ workdone = change in k.E.

= final k.E. − Initial k.E.

∴ workdone $= \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$

Work Energy Principle

** Work energy principle is applicable for both conservative and non-conservative forces.

- <u>conservative Force</u> :- workdone is independent of path followed by particles.

e.g :- Gravity force, spring force, elastic force etc.

- <u>Non - conservative force</u> :-

workdone depends on path followed by particles.

e.g. - frictional force.

- Principle of conservation of energy :-

The sum of potential energy and kinetic energy of particles remains const. during the motn under the actn of conservative force.

③ $(KE)_3$ $(PE)_3$

② $(KE)_2$ $(PE)_2$

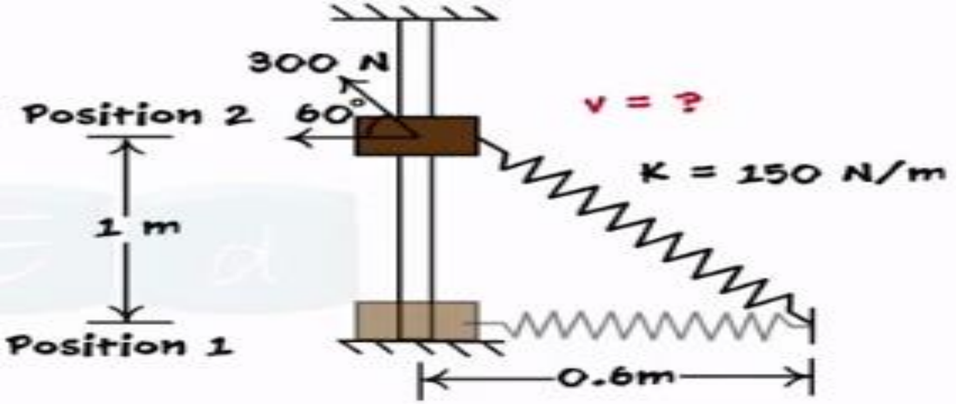
① $(KE)_1$ $(PE)_1$

$\therefore (KE+PE)_1 = (KE+PE)_2 = (KE+PE)_3$

$\therefore$ Total mechanical energy is constant.

Work Energy Principle

Q. A 20 kg steel collar is being raised from rest at position 1 by a 300 N force applied as shown. The collar is guided by a smooth rod and a spring whose free length is 0.3 m.
Find the speed of the collar as it reaches position 2.



Applying Work Energy principle to the moving collar from Position 1 to 2.

$T_1 = 0$ ......(since it start from rest)

$T_2 = \frac{1}{2} mv^2$

$= \frac{1}{2} * 20 * v^2$

$= 10\ v^2$ N.m

U_{1-2} By Applied Force

$= F * s$

$= 300 * \sin 60 * 1$

$= 259.81$ N.m

By Spring Force

$U = \frac{1}{2} k (x_1^2 - x_2^2)$

Deformation of Spring at Position 1

x_1 = Spring Length − Free Length

$= 0.6 - 0.3$

$= 0.3$ m

Deformation of Spring at Position 2

x_2 = Spring Length − Free Length

$= \sqrt{0.6^2 + 1^2} - 0.3$

$= 0.866$ m

$U = \frac{1}{2} * 150 (0.3^2 - 0.866^2)$

$= -49.49$ N.m

By Weight Force

$U = -m g h$ (−ve because displacement is upw

$= -20 * 9.81 * 1$

$= -196.2$ N.m

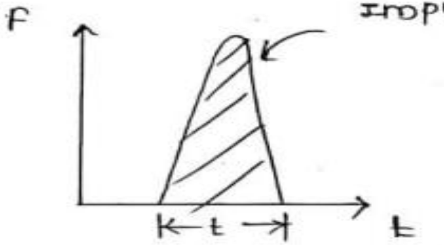
Using $T_1 + \Sigma U_{1-2} = T_2$

$0 + (259.81 - 49.49 - 196.2) = 10\ v^2$

$\boxed{\therefore v = 1.18 \text{ m/s}}$

- **Impulse (I) :-**

A large amount of force acting on a particle for short duratn of time is called impulse



Impulse - Area under Force-time diagram

<u>Unit</u> : $F \times t$

$= N \times sec$

$F = m \times a = kg \cdot m/sec^2 \cdot sec$

$\boxed{F = kg\ m/sec}$

- It is also a vector quantity.

- Impulse momentum theorem

When unbalanced force system acting on particle for short time, it will produce impulse and which is equal to change in momentum.

$\therefore \boxed{I_{(1-2)} = M_2 - M_1}$

* law of conservatn of momentum:-

When non-impulsive forces are acting on a body then, final momentum is always equal to initial momentum.

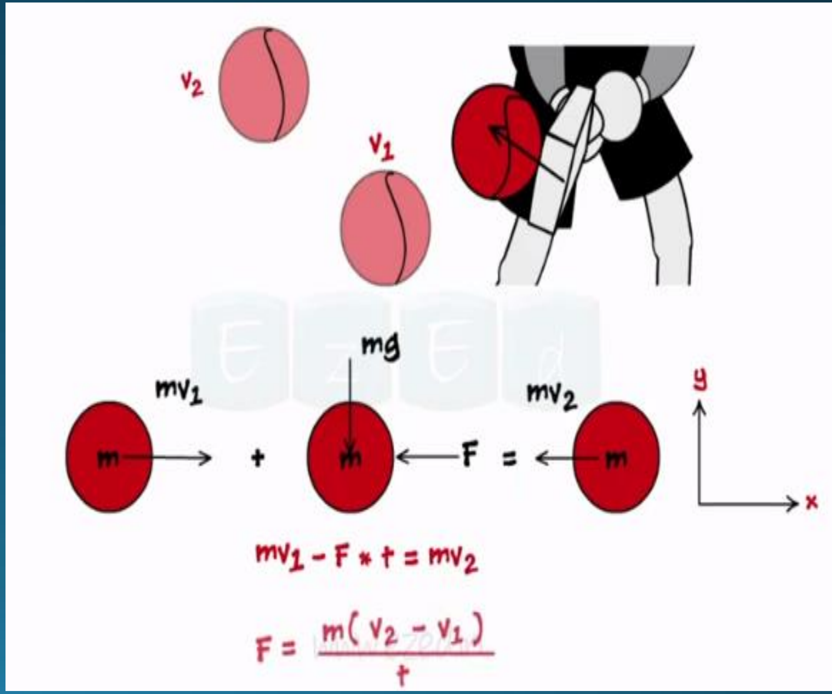
$I_{1-2} = M_2 - M_1$

$0 = M_2 - M_1$

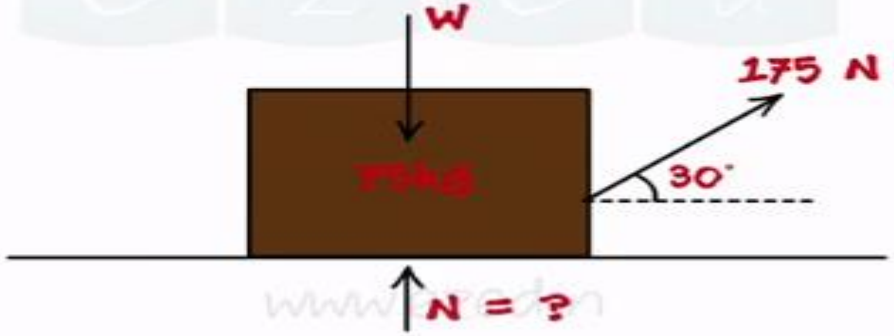
$\boxed{M_1 = M_2}$

e.g. of non-impulsive force :-

1. force exerted by spring.
2. Internal forces.
3. frictional forces
4. force reactn
5. weight of body.



Q. The 75 kg crate is originally at rest on the smooth horizontal surface. If a towing force of 175 N, is acting at an angle of 30°, is applied for 12s, determine final velocity and normal force which the surface exerts on the crate during this time interval.



Applying Impulse Momentum Equation in x-direction

$$(mv_1)_x + (\text{impulse}_{1-2})_x = (mv_2)_x$$

$$(mv_1) + \sum F_x * t = (mv_2)_x$$

$$0 + (175 \cos 30)(12) = 75\, v_2$$

$$\boxed{v_2 = 24.24 \text{ m/s}}$$

Applying Impulse Momentum Equation in y-direction

$$(mv_1)_y + (\text{impulse}_{1-2})_y = (mv_2)_y$$

$$(mv_1)_y + \sum F_y * t = (mv_2)_y$$

$$0 + N(12) - (75 * 9.81)(12) + (175 \sin 30)(12) = 0$$

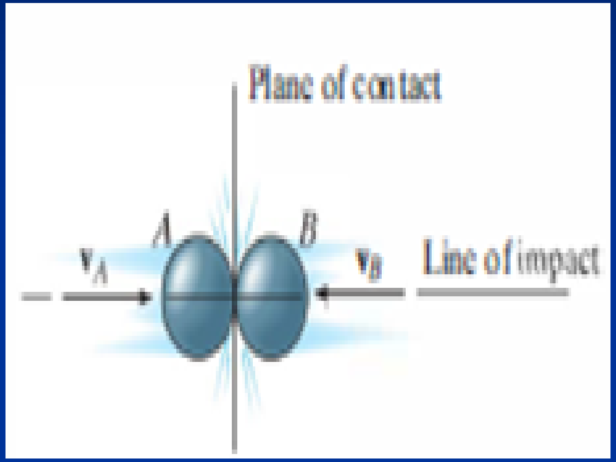
$$\boxed{N = 648.25 \text{ N}}$$

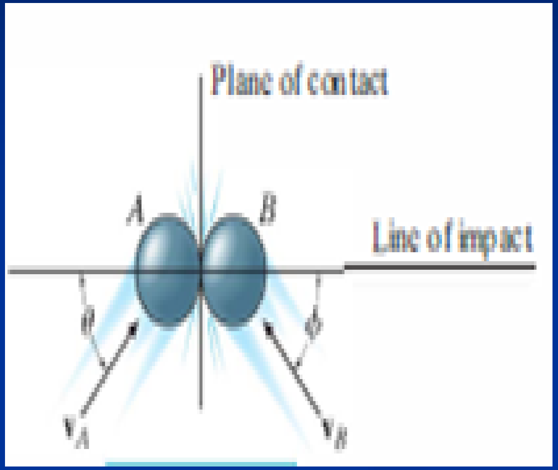
Impact

- Impact occurs when two bodies collide with each other during a very short period of time causing relatively large (impulsive) forces to be exerted on two bodies.

- Central impact and Oblique impact

Central and Oblique



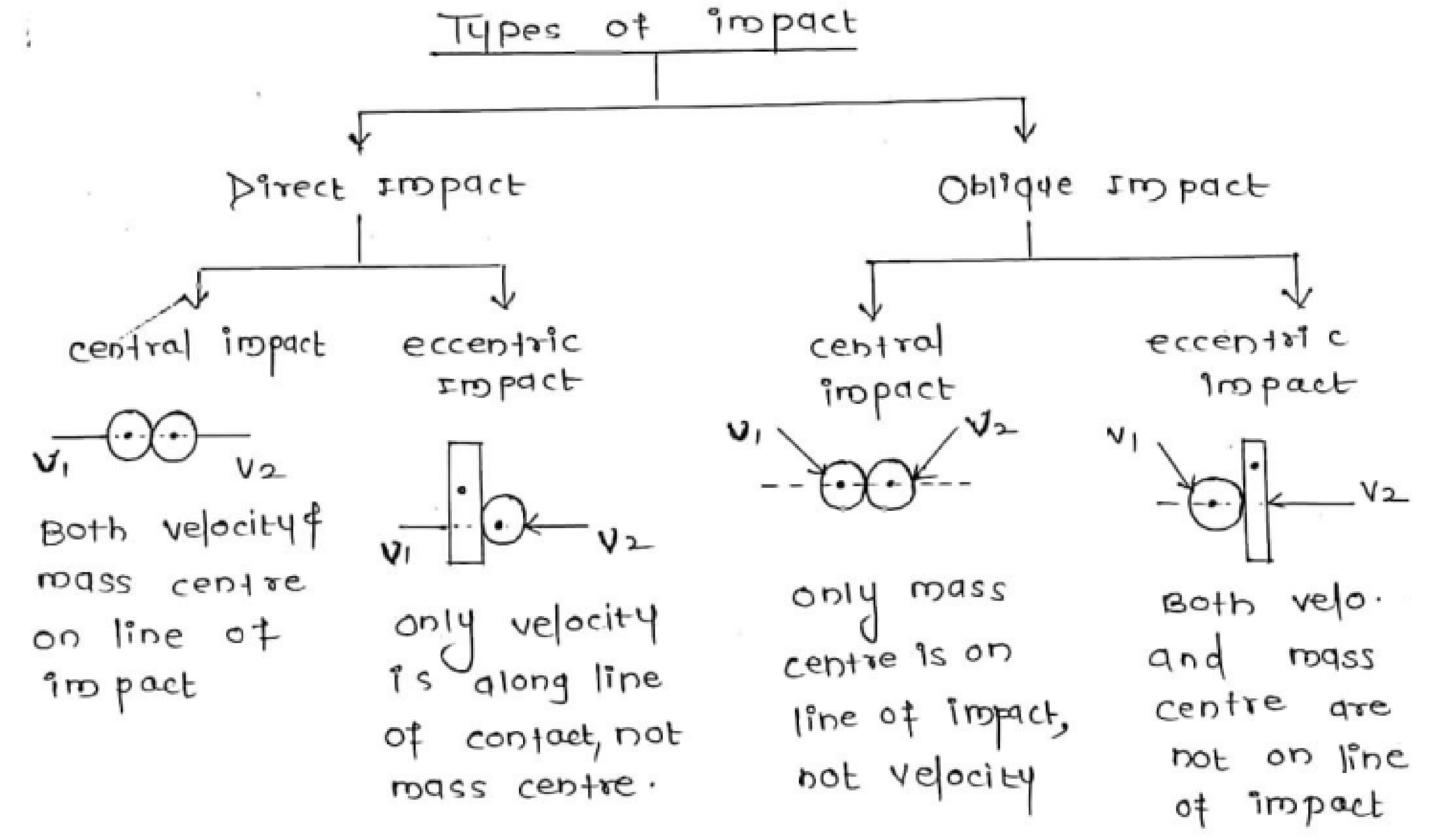


Central impact

Iblique impact

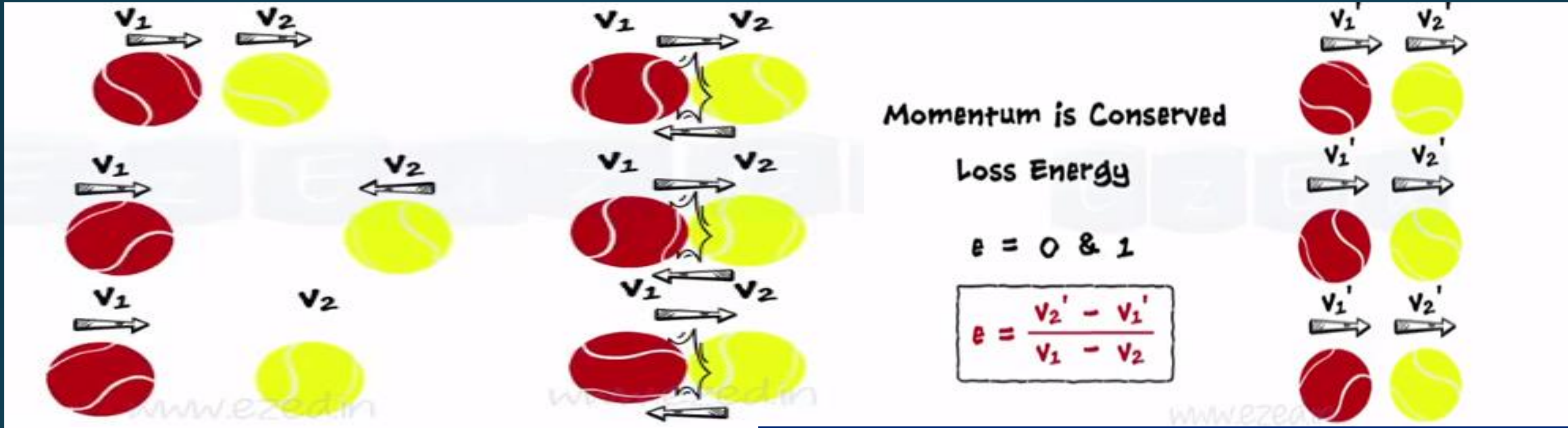
Central - Direction of motion of mass centers of colliding particle is along a line passing through mass centers of particles

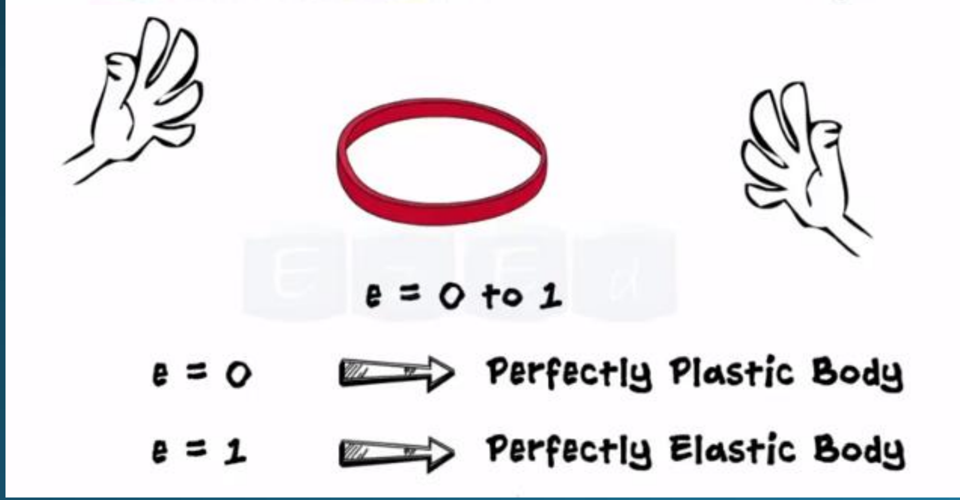
Oblique – When motion of one or both particles make an angle with the line of impact.


Types of impact
Direct impact
Oblique impact
central impact
eccentric impact
central impact
eccentric impact
V1
V2
Both velocity & mass centre on line of impact
V1
V2
only velocity is along line of contact, not mass centre.
V1
V2
only mass centre is on line of impact, not velocity
V1
V2
Both velo. and mass centre are not on line of impact

Coefficient Of Restitution

The ratio of restitution impulse to the deformation impulse is called coefficient of restitution

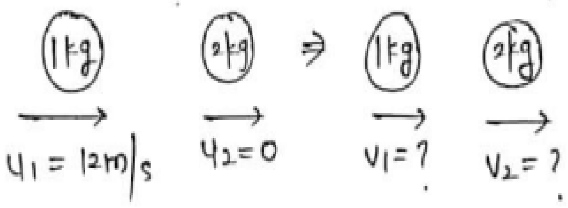




- Elastic impact e = 1, deformation impulse is equal to restitution impulse. No energy loss
- Plastic impact e = 0, No restitution impulse, the bodies stick together and move with same velocity, The loss of energy is maximum

(3) A ball of mass 1 kg moving with velocity 12 m/sec undergoes a direct central impact with a station any ball of mass 2 kg. The impact of perfectly elastic. The speed of 2 kg mass ball after impact will be.

soln :-



$u_2 - u_1 = v_2 - v_1$

$12 - 0 = v_2 - v_1$

$\therefore v_2 - v_1 = 12$ — (1)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$1 \times 12 + 2 \times 0 = 1 \times v_1 + 2 \times v_2$

$\therefore 12 = v_1 + 2v_2$ — (2)

Solving (1) & (2)

$v_2 - v_1 = 12$

$2v_2 + v_1 = 12$

$3v_2 = 24$

$\boxed{v_2 = 8\ m/sec}$

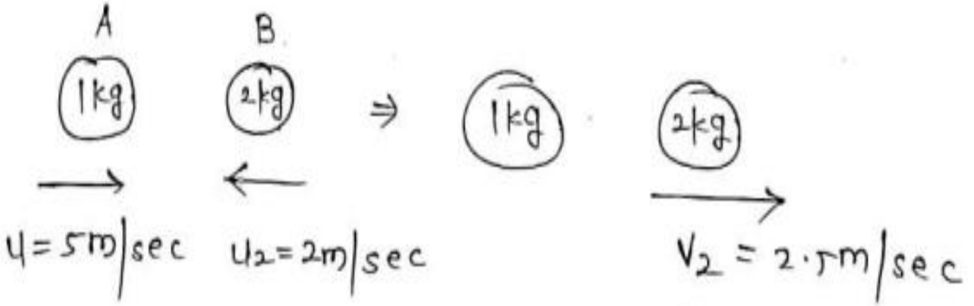
$\therefore v_1 = v_2 - 12$

$= 8 - 12$

$= -4$

$\boxed{v_1 = 4\ (\leftarrow)}$

A body 'A' of mass 1 kg moves right ward with velocity 5 m/sec and body B of mass 2 kg moves left word with velocity 2 m/sec after impact velocity of B is 2.5 m/sec rightward. Determine coeff. of restitutn.



$u_1 - u_2 = 5 - (-2) = 7$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$1 \times 5 - (2 \times 2) = 1 \times v_1 + 2 \times 2.5$

$5 - 4 = v_1 + 5$

$\therefore \boxed{v_1 = -4\ m/sec}$

$v_2 - v_1 = 2.5 - (-4)$

$= 6.5\ m/sec$

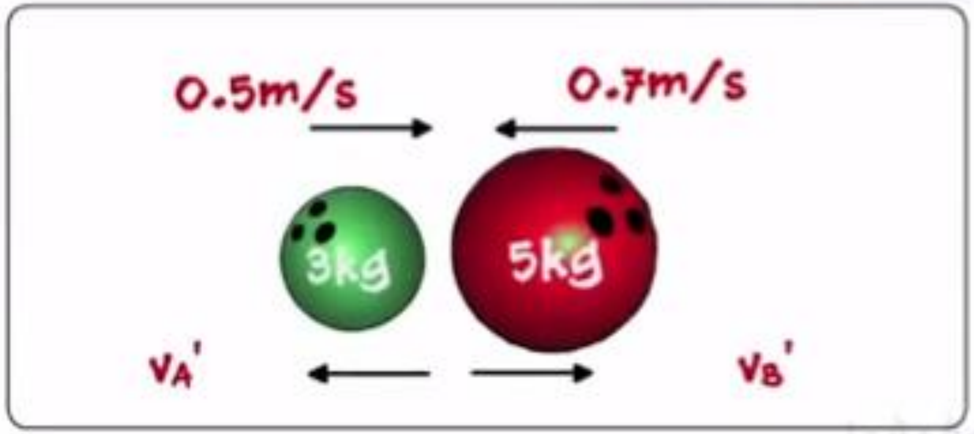
$\therefore$ coeff. of restitutn $= \dfrac{v_2 - v_1}{u_1 - u_2}$

$= 6.5/7$

$= 0.928$

Direct Impact Problem

Q. A 3 kg ball moving with 0.5 m/s towards right collides head on with another ball of mass 5 kg, moving with 0.7 m/s towards left. Determine the velocities of the balls after impact and the corresponding percentage loss of kinetic energy, when



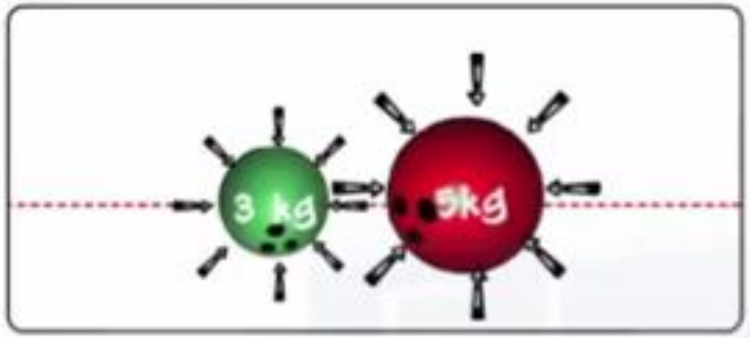
i) The impact is perfectly elastic $e = 1$

ii) The impact is perfectly plastic $e = 0$

iii) The impact such that $e = 0.7$

Direct Impact Problem

Solution:



i) Impact is perfectly elastic i.e. $e = 1$

Step 1

Using Conservation of Momentum Equation

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$3 * 0.5 + 5 * (-0.7) = 3v_A' + 5v_B'$$

$$-2 = 3v_A' + 5v_B' \text{ ....(1)}$$

Step 2

Using Coefficient of Restitution Equation

$$v_B' - v_A' = e\,[v_A - v_B]$$

$$v_B' - v_A' = 1\,[0.5 - (-0.7)]$$

$$v_B' = 1.2 + v_A' \text{ ....(2)}$$

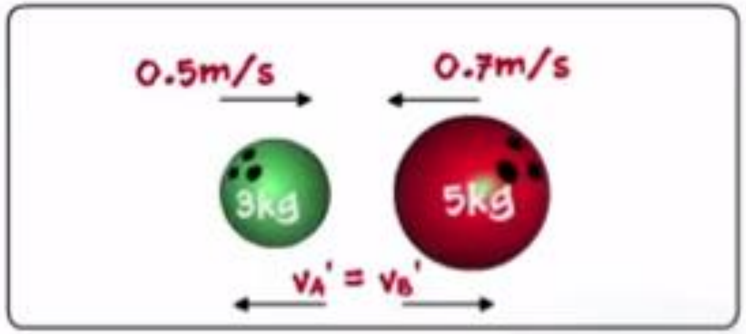
Step 3

Solving equation (1) and (2), we get

$$\boxed{\begin{aligned} v_A' &= -1 \text{ m/s} = 1 \text{ m/s } \leftarrow \\ v_B' &= 0.2 \text{ m/s} = 0.2 \text{ m/s } \rightarrow \end{aligned}}$$

Since the impact is perfectly elastic, there will be no loss of kinetic energy

Direct Impact Problem



ii) Impact is perfectly plastic i.e. $e = 0$

$$v_A' = v_B' = v'$$

Using Conservation of Momentum Equation

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$3 * 0.5 + 5 * (-0.7) = 3v' + 5v'$$

$$v' = -0.25 \text{ m/s}$$

i.e. $v_A' = v_B' = 0.25$ m/s ←

Kinetic energy of the system before impact

$$= \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2$$

$$= \frac{1}{2} * 3 * (0.5)^2 + \frac{1}{2} * 5 * (0.7)^2$$

$$= 1.6 \text{ J}$$

Kinetic energy of the system after impact

$$= \frac{1}{2} * 3 * (0.25)^2 + \frac{1}{2} * 5 * (0.25)^2$$

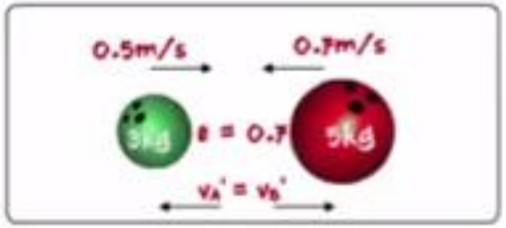
$$= 0.25 \text{ J}$$

Percentage loss of kinetic energy

$$= \frac{1.6 - 0.25}{1.6} * 100$$

$$= 84.375 \%$$

Direct Impact Problem



iii) Impact when $e = 0.7$

Step 1

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$3 * 0.5 + 5 * (-0.7) = 3v_A' + 5v_B'$$

$$-2 = 3v_A' + 5v_B' \text{.........} (3)$$

Step 2

Using Coefficient of Restitution Equation

$$v_B' - v_A' = e\,[v_A - v_B]$$

$$v_B' - v_A' = 0.7\,[0.5 - (-0.7)]$$

$$v_B' = 0.84 + v_A' \text{.........} (4)$$

Step 3

Solving equation (3) and (4)

$$v_A' = -0.775 \text{ m/s} = 0.775 \text{ m/s} \leftarrow$$
$$v_B' = 0.065 \text{ m/s} = 0.065 \text{ m/s} \rightarrow$$

Kinetic energy of the system after impact

$$= \frac{1}{2} * 3 * (0.775)^2 + \frac{1}{2} * 5 * (0.065)^2$$

$$= 0.9115 \text{ J}$$

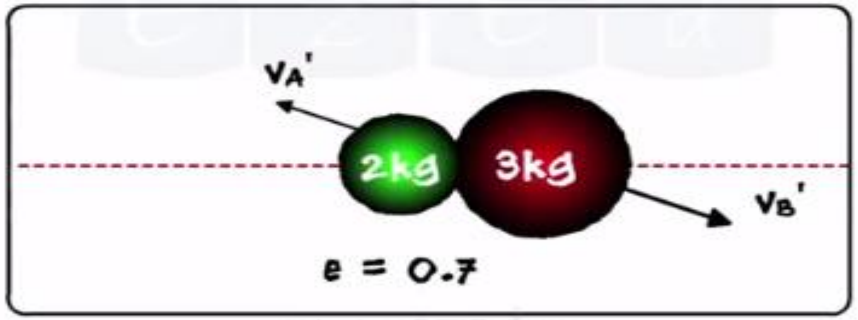
Percentage loss of kinetic energy

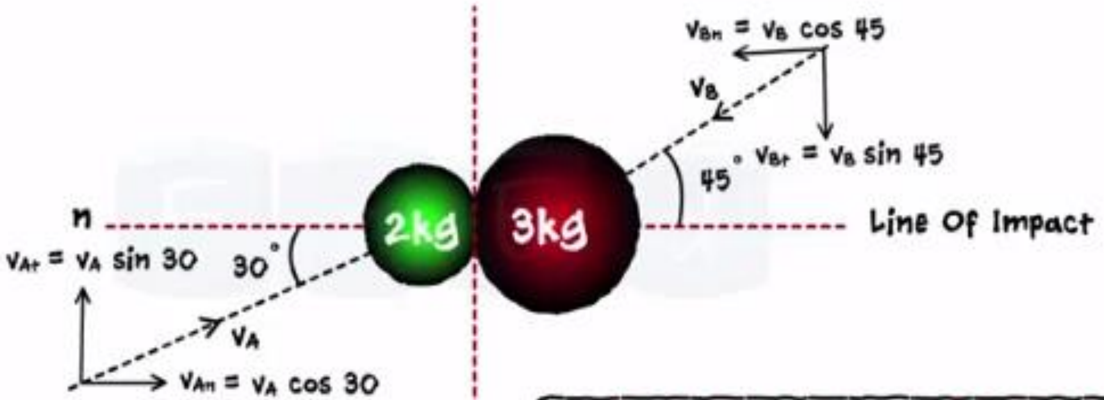
$$= \frac{(1.6 - 0.9115)}{1.6} * 100$$

$$= 43.03\ \%$$

Oblique Central Impact Problem

Q. Two smooth balls collide as shown.
Find the velocities after impact.
Take $m_A = 2$ kg, $m_B = 3$ kg and $e = 0.7$





t

$v_{An} = 3.46$ m/s →, $v_{At} = 2$ m/s ↑
$v_{Bn} = 1.732$ m/s ←, $v_{Bt} = 1$ m/s ↓

Using Conservation of Momentum Equation

$$m_A v_{An} + m_B v_{Bn} = m_A v_{An}' + m_B v_{Bn}'$$

$$2 * 3.46 + 3 * (-1.732) = 2 * v_{An}' + 3 * v_{Bn}'$$

$$1.724 = 2\, v_{An}' + 3\, v_{Bn}' \;..........(1)$$

Using Coefficient of Restitution Equation

$$v_{Bn}' - v_{An}' = e\,[v_{An} - v_{Bn}]$$

$$v_{Bn}' - v_{An}' = 0.7\,[3.46 - (-1.732)]$$

$$v_{Bn}' - v_{An}' = 3.63 \;.............. (2)$$

$$1.724 = 2\, v_{An}' + 3\, v_{Bn}' \;..........(1)$$

$$v_{Bn}' - v_{An}' = 3.63 \;.............. (2)$$

$v_{An}' = -1.83$ m/s $= 1.83$ m/s ←
$v_{Bn}' = 1.79$ m/s $= 1.79$ m/s →

$v_{At} = v_{At}' = 2$ m/s ↑
$v_{Bt} = v_{Bt}' = 1$ m/s ↓

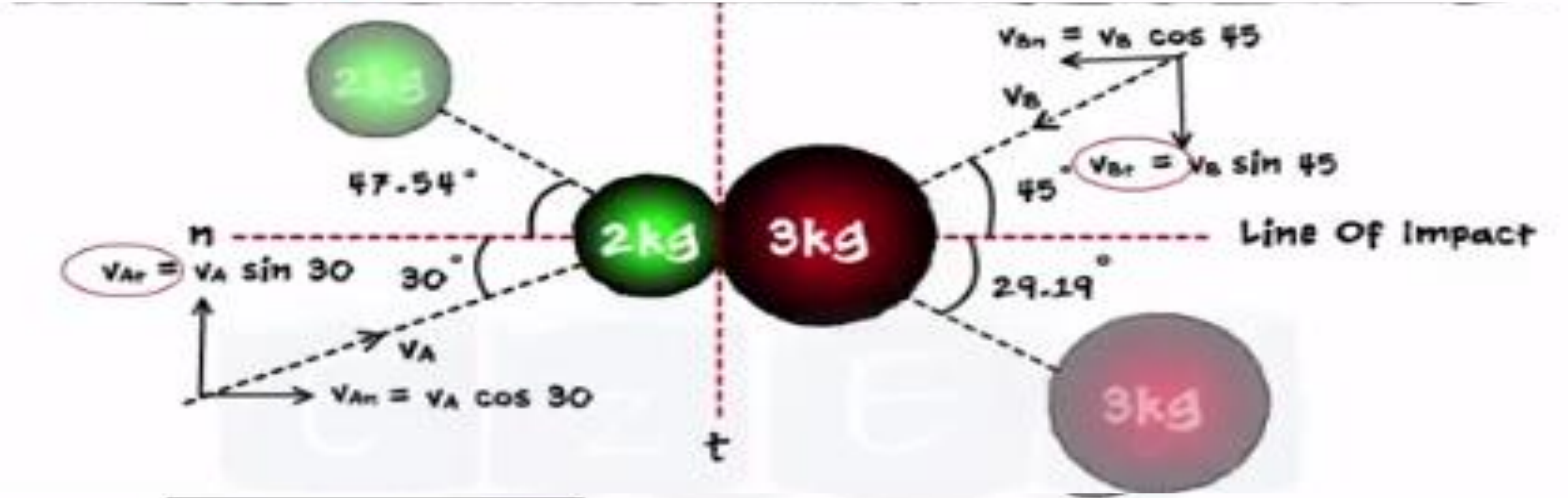
$$v_A' = \sqrt{(v_{An}')^2 + (v_{At}')^2}$$

$$= \sqrt{(1.83)^2 + (2)^2}$$

$$= 2.71 \text{ m/s}$$

$$\theta_A' = \tan^{-1}\left(\frac{v_{At}'}{v_{An}'}\right) = \tan^{-1}\left(\frac{2}{1.83}\right) = 47.54^\circ$$

$$\boxed{v_A' = 2.71 \text{ m/s}} \quad \boxed{\theta_A' = 47.54^\circ}$$



$$v_B' = \sqrt{(v_{Bn}')^2 + (v_{Bt}')^2}$$

$$= \sqrt{(1.79)^2 + (1)^2}$$

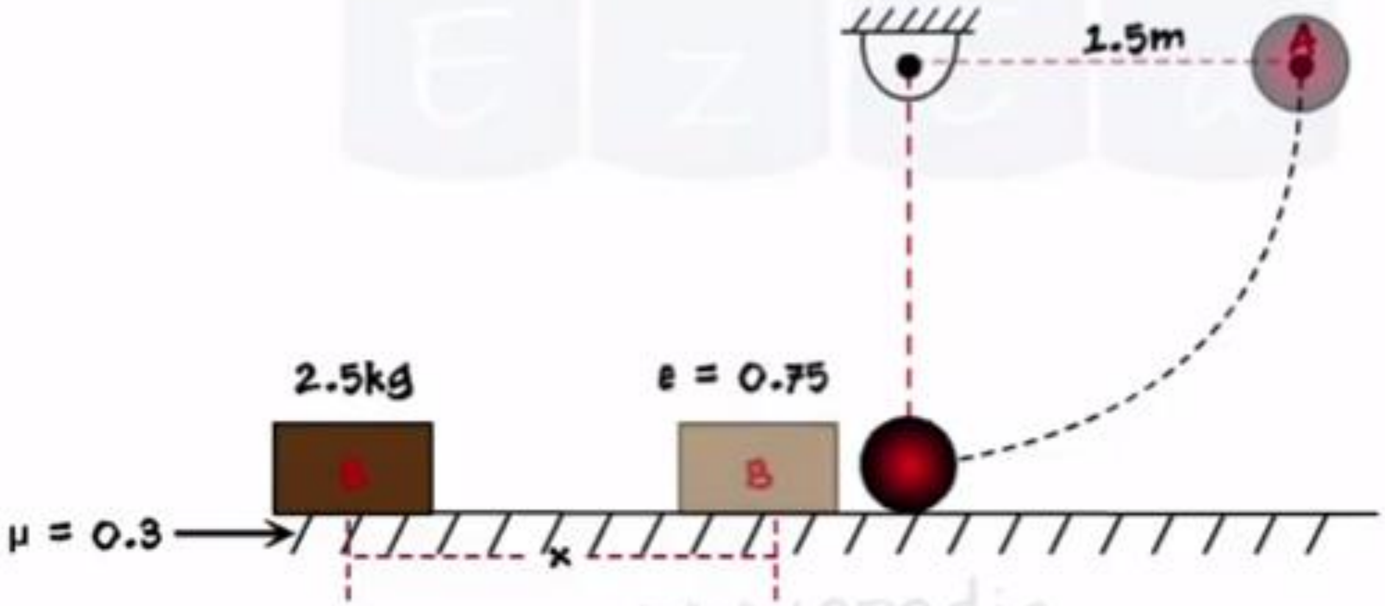
$$= 2.05 \text{ m/s}$$

$$\theta_B' = \tan^{-1}\left(\frac{v_{Bt}'}{v_{Bn}'}\right) = \tan^{-1}\left(\frac{1}{1.79}\right) = 29.19^\circ$$

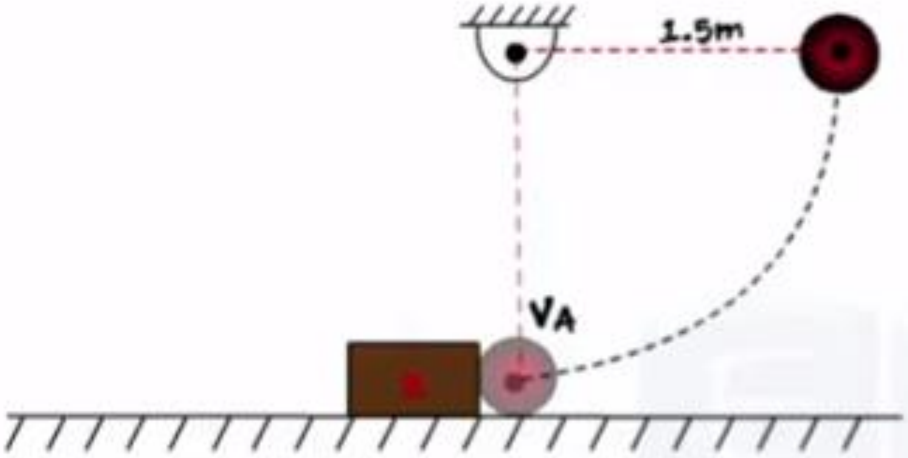
$$\boxed{v_B' = 2.71 \text{ m/s,}} \quad \boxed{\theta_B' = 29.19^\circ}$$

Combination Problem

Q: A sphere of mass 3 kg is released from rest. It swings as a pendulum and strikes a block B of mass 2.5 kg resting on a horizontal surface. Determine how far the block will move after impact. Take $\mu = 0.3$ between the block B and the horizontal surface and $e = 0.75$.



Combination Problem



$$T_1 + \Sigma U_{1-2} = T_2 \ldots\ldots\ldots\ldots (i)$$

$T_1 = 0$ since it starts from rest

$$T_2 = \frac{1}{2} m v^2$$

$$= \frac{1}{2} * 3 * v^2$$

U_{1-2} = only weight force is acting

$$= m\,g\,h$$

$$= 3 * 9.81 * 1.5$$

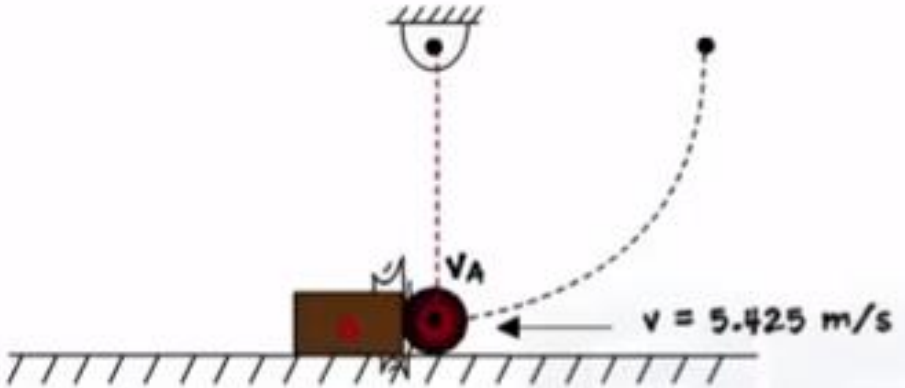
$$= 44.145 \text{ J}$$

Substituting the values in equation (i)

$$0 + 44.145 = \frac{1}{2} * 3 * v^2$$

$$\boxed{v = 5.425 \text{ m/s} \leftarrow}$$

Combination Problem



Step 1

Using Conservation of Momentum Equation

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$3 * 5.425 + 2.5 * 0 = 3\ v_A' + 2.5\ v_B'$$

$$3\ v_A' + 2.5\ v_B' = 16.275 \ldots\ldots\ldots (ii)$$

Step 2

Using Coefficient of Restitution Equation

$$v_B' - v_A' = e\,(v_A - v_B)$$

$$v_B' - v_A' = 0.75\,(5.425 - 0)$$

$$v_B' - v_A' = 4.068 \ldots\ldots\ldots\ldots (iii)$$

Step 3

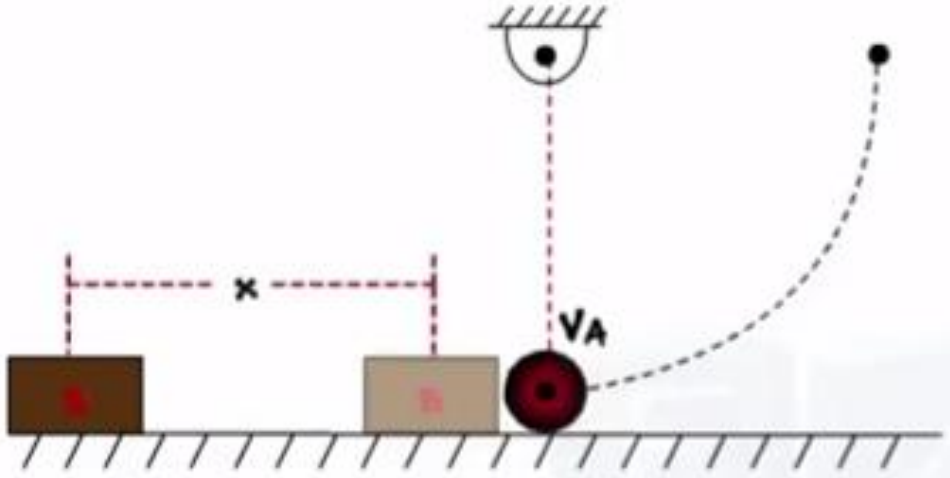
Solving equation (ii) and (iii)

$$3\ v_A' + 2.5\ v_B' = 16.275 \ldots\ldots\ldots (ii)$$

$$v_B' - v_A' = 4.068 \ldots\ldots\ldots\ldots (iii)$$

$\boxed{v_A' = 1.11 \text{ m/s} \leftarrow}$ $\boxed{v_B' = 5.178 \text{ m/s} \leftarrow}$

Combination Problem



Applying Work Energy Principle to block B

$$T_2 = \frac{1}{2} mv^2$$

$$= \frac{1}{2} * 2.5 * 5.1782$$

$$= 33.51\ J$$

$T_3 = 0$ Since the block comes to rest

U_{2-3} = only friction force will act

$$= -\mu k.N.s$$

$$= -0.3*(2.5*9.81)*x$$

$$= -7.36\,x\ J$$

$$33.51 + [-7.36\,x] = 0$$

$$\boxed{x = 4.55\ m}$$