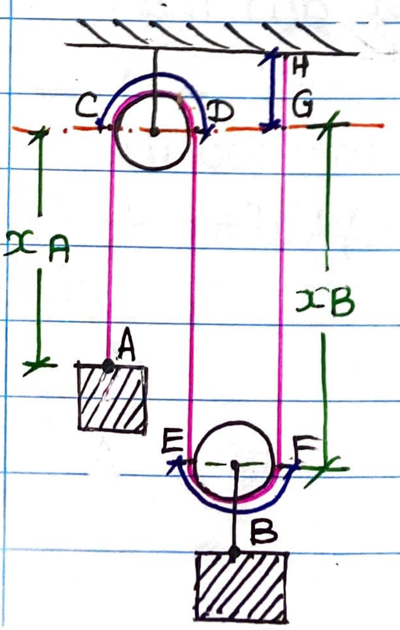
Dependant Motion

* For several bodies connected to each other by means of an inextensible string or cable, relationship between displacement, velocities or acceleration of these bodies is very important.

* After establishing and understanding these relationships, given kinematic parameters or quantities can be used to find other quantities.

* In these types of systems, kinematic parameters like displacement, velocity and acceleration of one body depends on the other. Therefore these motions are termed as 'dependent motion' or 'motion of connected bodies.'

* Note :- String or cable connecting two bodies is assumed to be inextensible. It's total length is taken to be a 'constant value.'



Total length of cable = constant

$l(CD)$, $l(CEF)$, $l(GH)$ = constant

$AC + CD + DE + EF + FG + GH$ = constant

$x_A + \text{const} + x_B + \text{const} + x_B + \text{const} = \text{const}.$

$\therefore \quad x_A + 2x_B = \text{constant} \quad$ — ①

$\therefore$ ② --- $v_A + 2v_B = 0$ .... (diff ① w.r.t time)

$\therefore \quad a_A + 2a_B = 0$ ..... diff ② w.r.t time

Steps to solve the numericals :-

1. Choose the datum or reference line and decide the sign convensions. (x, v, a ⇒ ↑ +ve ; ↓ −ve)

2. Mark the varieble positions (or displacements) of rigid bodies with reference to datum line. [x_A, x_B, x_C, etc).

3. Develope the linear relation among x_A, x_B, x_C, etc. While developing the relation, we note that total length of the cord through the motion is constant.

4. Differentiate the linear relation of displacements (or positions i.e. x_A, x_B, x_C etc) successively to get linear relations between v_A, v_B, v_C etc and a_A, a_B, a_C respectively.

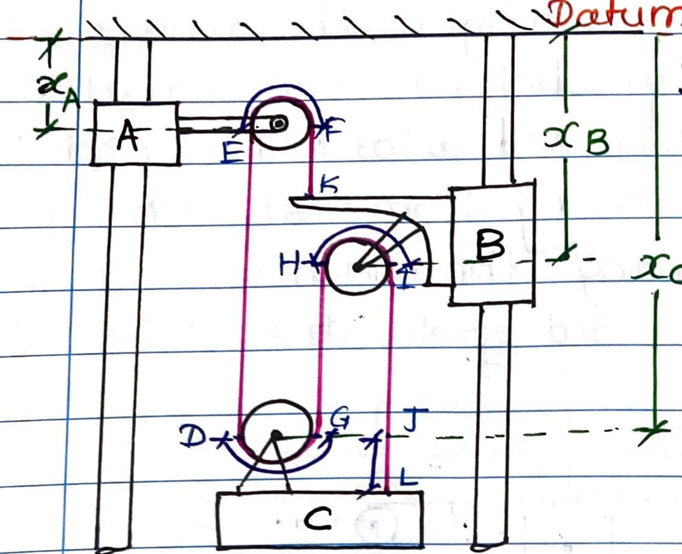
5. If more than 1 strings are used, develope the separate equation for each string.

* Numericals :-

(1) Collars A and B start from rest and move with following upward accelerations :-

$$a_A = 160 \text{ mm/s}^2 \text{ and } a_B = 100 \text{ mm/s}^2.$$

Determine velocity of collar 'C' & 'A' after 4 seconds.



Length = constant.

$DG + DE + EF + FK + GH + IJ + HI + JL = \text{const.}$

$(x_C - x_A) + 2(x_C - x_B) + (x_B - x_A) + \text{const} = \text{constant}$

$\therefore -2x_A - x_B + 3x_C = \text{constant}$ — (1)

Differenciate (1) w.r.t. time,

$-2v_A - v_B + 3v_C = 0$ — (2)

differenciate (2) w.r.t. time,

$-2a_A - a_B + 3a_C = 0$ — (3)

Given :- $a_A = 160\ mm/s^2$, $a_B = 100\ mm/s^2$.

$\therefore$ From Equation (3),

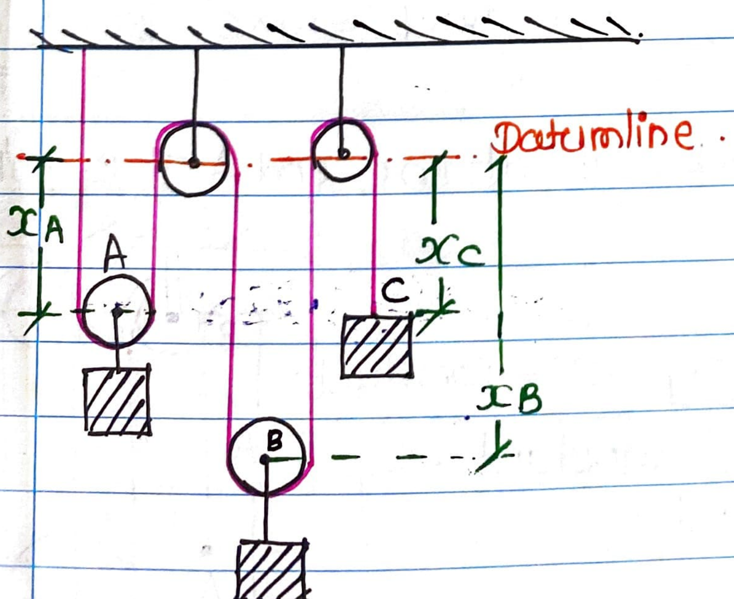
$-2(160) - 100 + 3\, a_C = 0$

$\therefore a_C = 140\ mm/s^2\ (\uparrow)$

At $t = 4$ seconds, $v_A = a_A \times 4 = 160 \times 4 = 640\ mm/s \uparrow$.

$\therefore v_C = 140 \times 4 = \boxed{560\ mm/s\ (\uparrow)}$

(2) Determine acceleration of pulley 'A' for a given system if $a_C = 4\ m/s^2\ (\downarrow)$ and $a_B = 6\ m/s^2\ (\uparrow)$.



Length = constant.

$2x_A + 2x_B + x_C = \text{constant}.$

$\therefore 2v_A + 2v_B + v_C = 0$

$\therefore 2a_A + 2a_B + a_C = 0.$

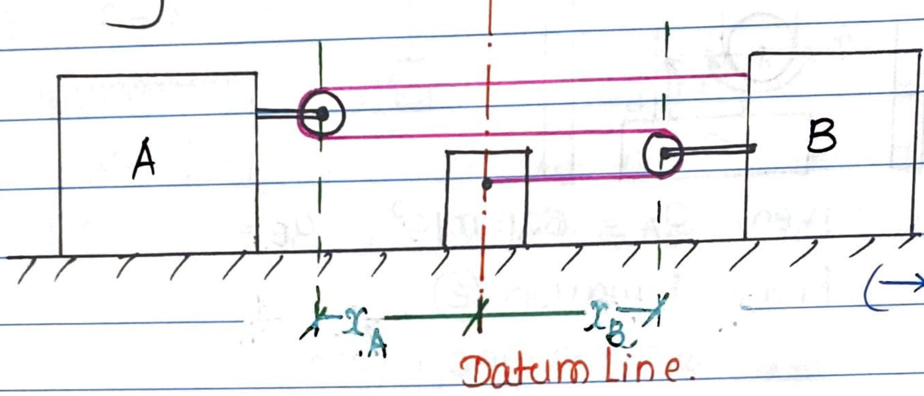
If $a_C = -4\ m/s^2$, $a_B = 6\ m/s^2$,

$2a_A + 2(6) + (-4) = 0$

$\therefore a_A = -4\ m/s^2$

$\therefore \boxed{a_A = 4\ m/s^2\ (\downarrow)}$

(3) In the arrangement shown in figure, the slider blocks A & B are connected by a string, which has one end attatched to a fixed support and other end attatched to block B. The block B moves to right, starting from rest, with constant acceleration $10 \ m/s^2$. Assuming the string to be inextensible, determine velocity of block A and acceleration of block A.



($\rightarrow$ +ve ; $\leftarrow$ -ve)

Length = Constant.

$$-2x_A + 3x_B = \text{constant}.$$

$$\therefore \quad -2v_A + 3v_B = 0$$

$$\therefore \quad -2a_A + 3a_B = 0.$$

$$\therefore \quad a_A = 1.5 a_B$$

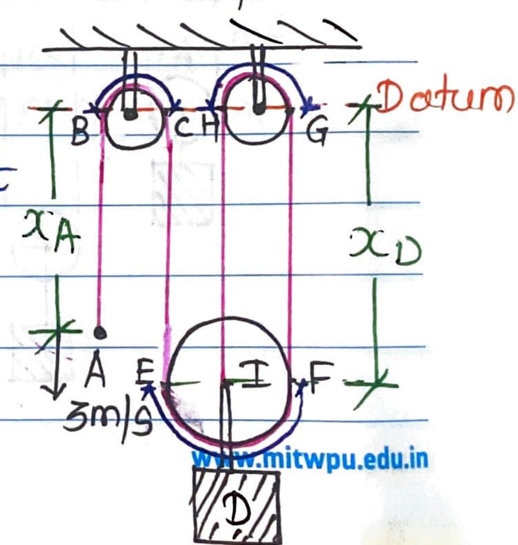
But Given that $\rightarrow$ $a_B = 10 m/s^2$.

$$\therefore \quad \boxed{a_A = 15 \ m/s^2} \ (\rightarrow)$$

At $t = 5$ seconds, $v_B = a_B \times t = 10 \times 5 = \boxed{50 \ m/s \ (\rightarrow)}$.

(4) Determine Velocity of block D if at the end A it is pulled at the velocity of 3 m/s.

$\Rightarrow$ L = constant.



$$AB + BC + CE + EF + FG + GH + HI = \text{constant}$$

$$x_A + 3x_D + \text{constant} = \text{constant}.$$

$$\therefore \quad x_A + 3x_D = \text{constant}$$

$$\therefore \quad v_A + 3v_D = 0$$

$$\therefore \quad (-3) + 3v_D = 0 \quad \therefore \boxed{v_D = 1 m/s \ (\uparrow)}$$