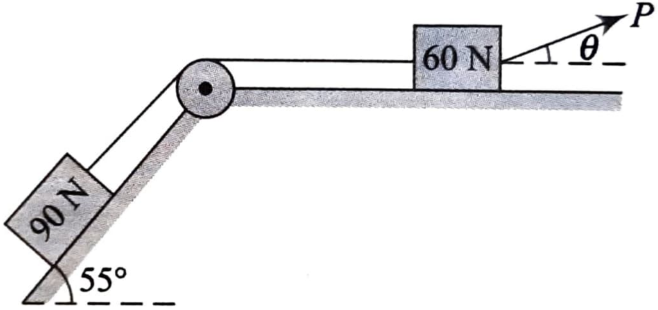
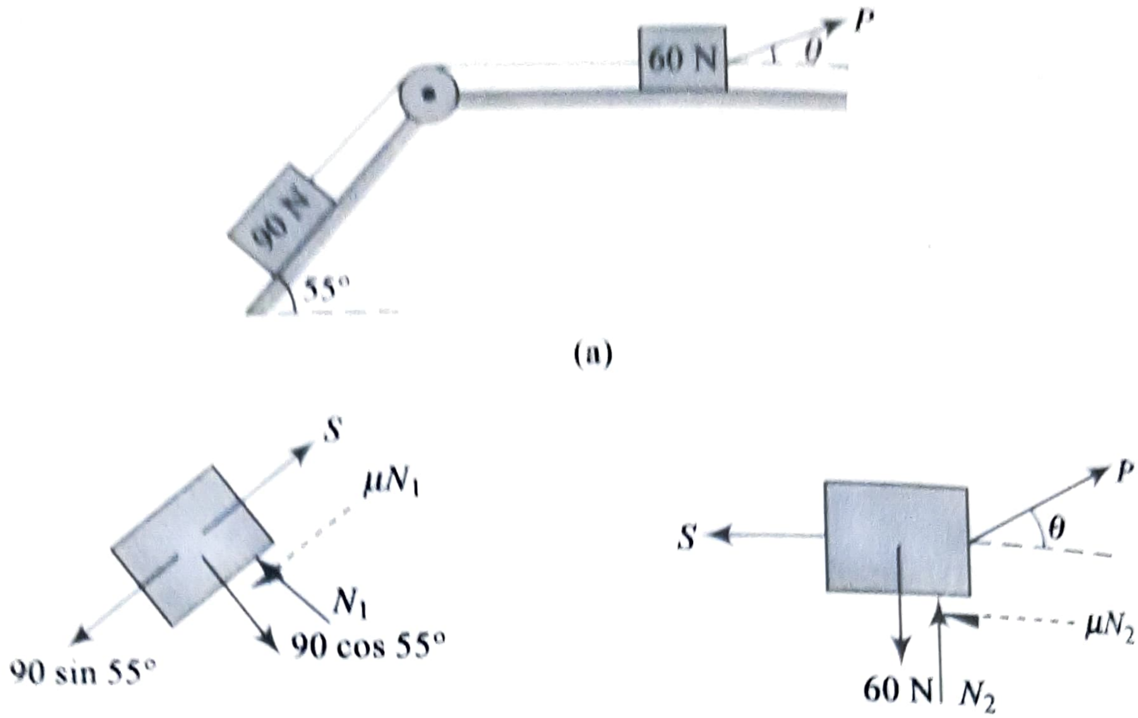
Example 4.13 Referring to Fig. E4.13(a), determine the least value of P to cause motion to impend rightwards. Assume the pulley to be frictionless and coefficient of friction of all contiguous surfaces is 0.20.



(a)

Example 4.13 Referring to Fig. E4.13(a), determine the least value of P to cause motion to impend rightwards. Assume the pulley to be frictionless and coefficient of friction of all contiguous surfaces is 0.20.



(b) FBD of inclined block

(c) FBD of horizontal block

Fig. E4.13

Solution Let the tensile force in the string be S.

Consider FBD of inclined block [Fig. E4.13(b)]

Take force equilibrium along the incline and normal to the incline:

$$90 \sin 55° + \mu N_1 = S \qquad \text{or} \qquad 90 \sin 55° + 0.2N_1 = S$$

$$90 \cos 55° = N_1 \qquad\qquad 90 \cos 55° = N_1$$

Solving these equations, we get $N_1 = 51.62$ N and $S = 84.05$ N.

Consider FBD of horizontal block [Fig. E4.13(c)]

Take force equilibrium along horizontal and vertical directions:

$$\overset{\rightarrow\oplus}{\sum} F_x = 0: \qquad S + \mu N_2 = P \cos \theta$$

So,

$$84.05 + 0.2N_2 = P \cos \theta$$

$$\overset{\oplus}{\uparrow} \sum F_y = 0: \qquad N_2 + P \sin \theta = 60$$

Solving the above equations, we get

$$P = \frac{96.05}{(\cos\theta + 0.2\sin\theta)}$$

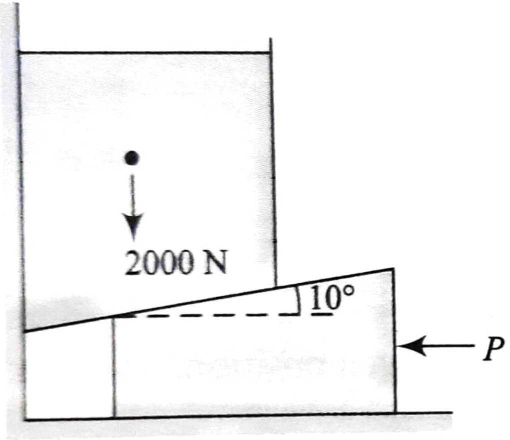
For the least value of P, the magnitude of $(\cos\theta + 0.2\sin\theta)$ should be maximum.

Hence $\frac{d}{d\theta}(\cos\theta + 0.2\sin\theta) = 0$ or $-\sin\theta + 0.2\cos\theta = 0$

or $\tan\theta = 0.2$ or $\theta = 11.309°$

Therefore, $$P_{\min} = \frac{96.05}{(\cos 11.309° + 0.2\sin 11.309°)} = 94.185 \text{ N}$$

Example 4.16 A block overlaying a 10° wedge on a horizontal floor, leaning against a vertical wall, and weighing 2000 N is to be raised by applying a horizontal force to the wedge. Assuming coefficient of friction for all contact surfaces 0.25, determine the minimum horizontal force to be applied to raise the block.



(a)

Solution Here angle of friction $\phi = \tan^{-1}(0.25) = 14.036°$.

R_1, R_2, and R_3 are the resultants of normal force and frictional resistance at floor, bottom of block, and at wall, respectively. This type of problem can be solved by two methods.

Method I Considering FBD of block and taking force equilibrium along horizontal and vertical directions [Fig. E4.16(b)],

$$\overset{\rightarrow\oplus}{\sum} F_x = 0: \qquad R_3 \cos\phi = R_2 \sin(10° + \phi)$$

So,

$$R_3 \cos 14.036° = R_2 \sin 24.036°$$

$$\overset{\oplus}{\uparrow}\sum F_y = 0: \qquad R_3 \sin 14.036° + 2000 = R_2 \cos 24.036°$$

Solving these two simultaneous equations, we obtain

$$R_3 = 1034.79 \text{ N} \quad \text{and} \quad R_2 = 2464.68 \text{ N}$$

Now consider FBD of wedge and take force equilibrium along vertical and horizontal directions [Fig. E4.16(c)]:

$$\overset{\oplus}{\uparrow}\sum F_y = 0: \qquad R_1 \cos 14.036° = R_2 \cos 24.036°$$

$$\overset{\rightarrow\oplus}{\sum} F_x = 0: \qquad P = R_1 \sin 14.036° + R_2 \sin 24.036°$$

Solving the above equations, we get $P = 1566.62$ N.

Method II At impending motion of the block, R_2, R_3, and 2000 N force will be concurrent at point O. Applying Lami's theorem in the FBD of block, we obtain,

$$\frac{2000}{\sin(90° + 2\phi + 10°)} = \frac{R_2}{\sin(90° - \phi)} = \frac{R_3}{\sin(180° - 10° - \phi)}$$

or

$$\frac{2000}{\sin 128.072°} = \frac{R_2}{\sin 75.964°} = \frac{R_3}{\sin 155.964°}$$

So, $\quad R_2 = 2464.68$ N and $R_3 = 1034.79$ N

Again at FBD of wedge [Fig. E4.16(c)], applying Lami's theorem

$$\frac{P}{\sin(180° - 10° - 2\phi)} = \frac{R_1}{\sin(90° + 10° + \phi)} = \frac{R_2}{\sin(90° + \phi)}$$

or

$$\frac{P}{\sin 141.928°} = \frac{R_1}{\sin 114.036°} = \frac{2464.68}{\sin 104.036°}$$

So, $\quad P = 1566.62$ N