SIT718 Real World Analytics

T2 2020

Assignment 3
Problem Solving

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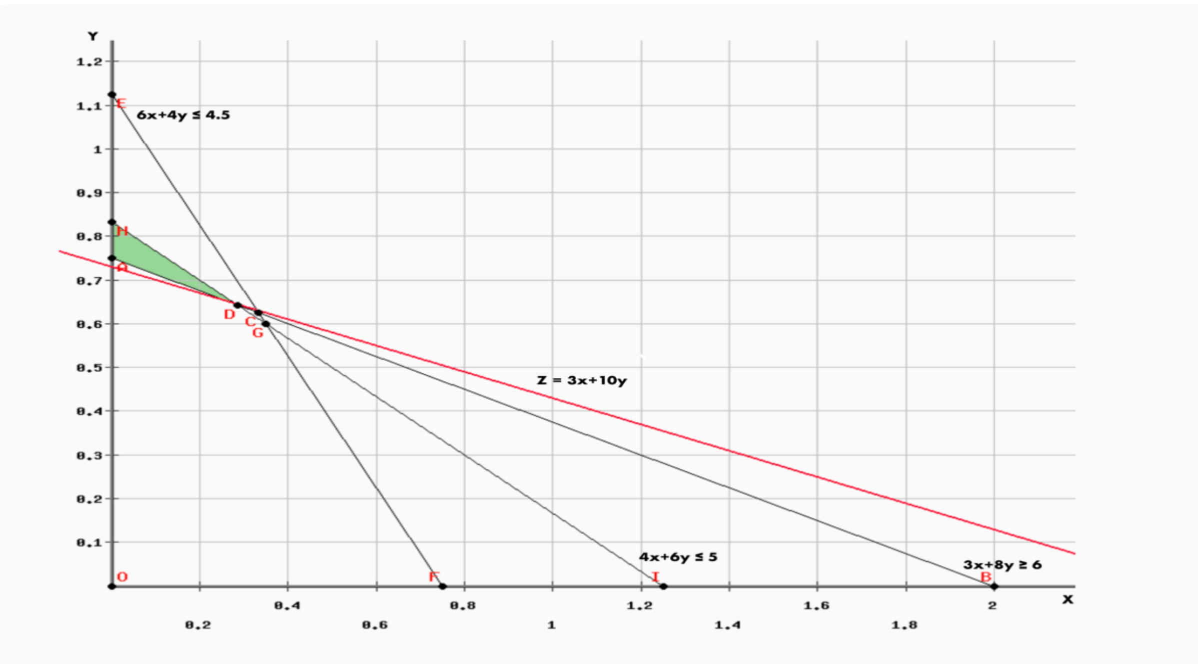
Q.1) a) Explain why a linear programming model would be suitable for this case study?
A linear problem can be formulated to optimize the objective function to minimum or maximum by adding constraint.
A graphical LP method is suitable when there are two decision variables. It involves the formulation of a function with linear inequalities (Analytics Vidhya, 2017)

b: Food Factory LP modelling
Decision Variables
Product A=x
Product B=y

Objective Function
(Minimise Cost) Z = 3x+10y

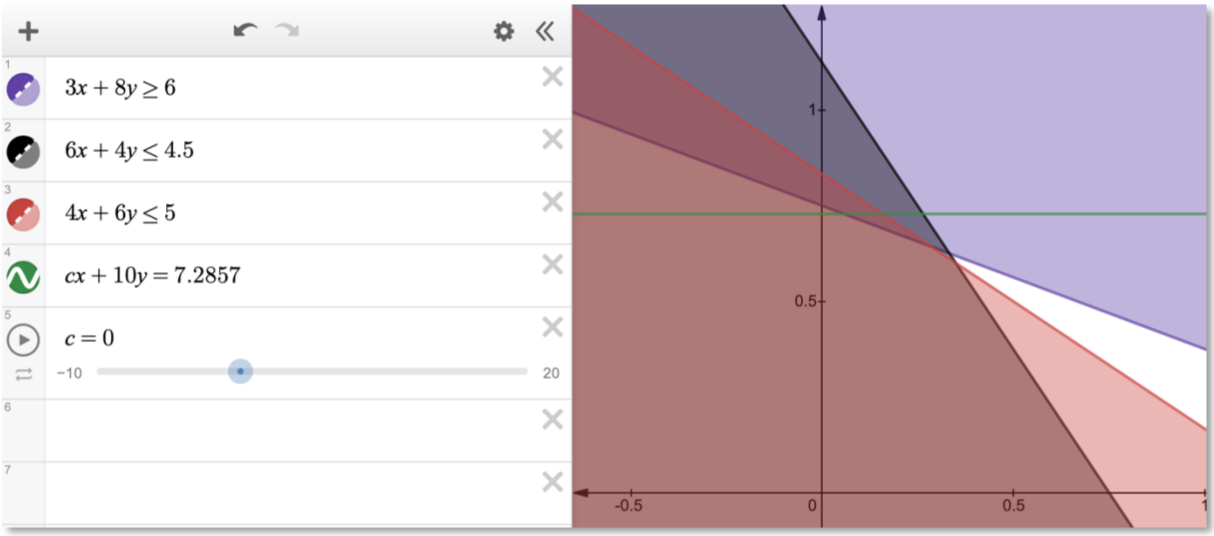
Constraints
1)Lime at Minimum of 6 litres
$3x+8y \geq 6$
2)Orange at Minimum of 4.5 litres
$6x+4y \leq 4.5$
3)Mango at Minimum of 5 litres
$4x+6y \leq 5$
4) since x and y cannot be negative
$x \geq 0$
$y \geq 0$
c:
$3x+8y \geq 6$: (0.75,0) (0,2)
$6x+4y \leq 4.5$: (0,1.125)(0.75,0)
$4x+6y \leq 5$: (0,0.8333)(1.25,0)

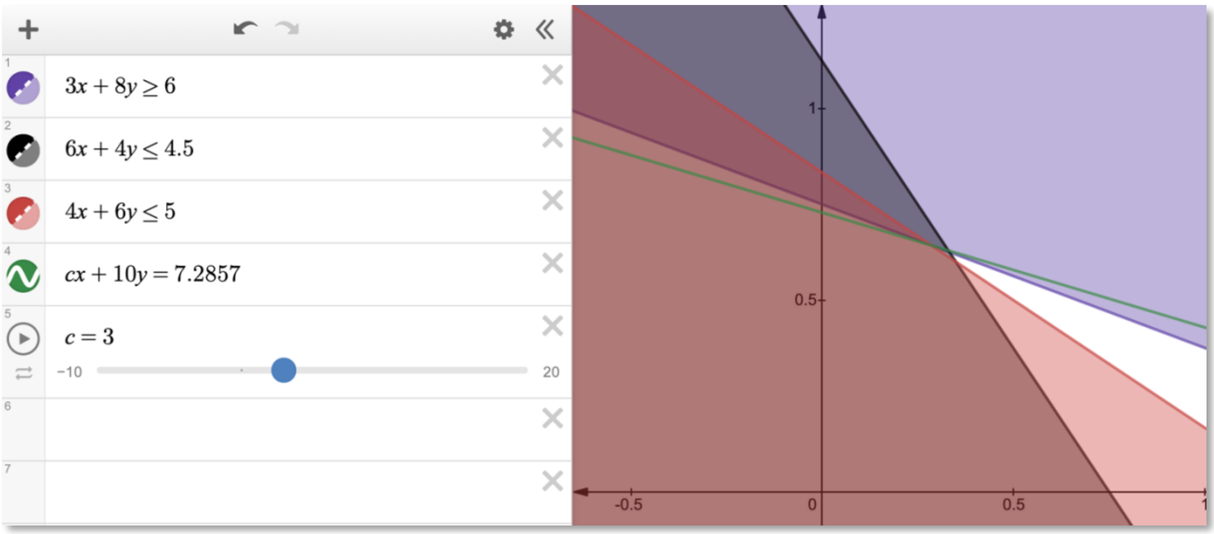


Point	X coordinate (X₁)	Y coordinate (Y)	Value of the objetive function (Z)
O	0	0	0
A	0	3 / 4	15 / 2
B	2	0	6
C	1 / 3	5 / 8	29 / 4
D	2 / 7	9 / 14	51 / 7
E	0	9 / 8	45 / 4
F	3 / 4	0	9 / 4
G	7 / 20	3 / 5	141 / 20
H	0	5 / 6	25 / 3
I	5 / 4	0	15 / 4

Graph made using (phpsimplex.com, 2020) online tool

Green area = Feasible region
Red line is the Objective function
Hence,
3x+10y=7.2857
d) to keep the optimum solution unchanged. A or x can be between the range of 0 to 3
Zero can be the minimum as we have a non-negativity constraint





Graph made using (desmos, 2020) online tool

Q.2)a) LP Cloth Factory model

Decision variables: XC1, XC2, XC3, XW1, XW2, XW3, XS1, XS2, XS3

	Cotton	Wool	Silk
Spring	XC1	XW1	XS1
Autumn	XC2	XW2	XS2
Winter	XC3	XW3	XS3

Objective Function

max(Z)= 60(XC1+XC2+CX3)+55(XW1+XW2+XW3)+60(XS1+XS2+XS3)
-30(XC1+XC2+CX3)-45(XW1+XW2+XW3)-50(XS1+XS2+XS3)
-5(XC1+XC2+CX3)-4(XW1+XW2+XW3)-5(XS1+XS2+XS3)

Constraints:

1) $xc1+xw1+xs1 \leq 3800$
2) $xc2+xw2+xs2 \leq 3200$
3) $xc3+xw3+xs3 \leq 3500$
4) $xc1 \geq 0.55(xc1+xw1+xs1)$
5) $xw1 \geq 0.30(xc1+xw1+xs1)$
6) $xc2 \geq 0.45(xc2+xw2+xs2)$
7) $xw2 \geq 0.40(xc2+xw2+xs2)$
8) $xc3 \geq 0.30(xc3+xw3+xs3)$
9) $xw3 \geq 0.50(xc3+xw3+xs3)$
10) $x1,x2,x3,xc1,xc2,xc2,xw1,xw2,xw3,xs1,xs2,xs3,z \geq 0$

Optimum solution = 346870

Q.3) a) A zero Sum game is a game in which the sum of the payoff values of each player is zero.
For example, in a game of poker between 2 players
The game begins with each player having $100 at the end of the game one of the players will win having $100 as his payoff amount and the other player will have $-100 as his payoff
If you add the value together it will look like this
100+(-100) = 0
In our given problem
Helen payoff : 5
David's payoff: -5
Sum = 0
Hence this game is a zero-sum game

b) Payoff Matrix

		DAVID					
		(5,0)	(4,1)	(3,2)	(2,3)	(1,4)	(0,5)
Helen	(6,0)	-1	0	0	0	0	-1
	(5,1)	-1	-1	0	0	-1	-1
	(4,2)	0	-1	-1	-1	-1	0
	(3,3)	0	0	-2	-2	0	0
	(2,4)	0	-1	-1	-1	-1	0
	(1,5)	-1	-1	0	0	-1	-1
	(0,5)	-1	0	0	0	0	-1

c) In a zero-sum game, if the value maximum of columns and minimum of row is equivalent, this is known as a saddle point

		DAVID						
		(5,0)	(4,1)	(3,2)	(2,3)	(1,4)	(0,5)	Row minimum
Helen	(6,0)	-1	0	0	0	0	-1	0
	(5,1)	-1	-1	0	0	-1	-1	0
	(4,2)	0	-1	-1	-1	-1	0	0
	(3,3)	0	0	-2	-2	0	0	0
	(2,4)	0	-1	-1	-1	-1	0	0
	(1,5)	-1	-1	0	0	-1	-1	0
	(0,5)	-1	0	0	0	0	-1	0
	Column maximum	0	0	0	0	0	0	

The values of the Row minimum and column maximum are the same, however there is no saddle point in the given matrix as the locations are not the same

d) **LP model for the game**

David’s Game

Max z=v

s.t $v-(-1x1-1x6) \geq 0$

$v-(-x1-x2-x5-x6) \geq 0$

$v-(-x2-x3-x4-x5) \geq 0$

$v-(-2x3-3x4) \geq 0$

$v-(1x2-x3-x4-x5) \geq 0$

$v-(-x1-x2-x5-x6) \geq 0$

$v-(-x1-x6) \geq 0$

$x1+x2+x3+x4+x5+x6=1$

$x1 \leq 0$, $\forall i=1,2,3,4,5,6$,

v u.r.s.

f) Solution for David’s game is -0.50

We can say that in every scenario Helen would win or it would be a tie in the game

Q.4) Problem : Three players can choose to give one amount of
$0,$3,$6

The referee adds double the total amount collected from the three players and distributes the collective amount amongst the three players

Each player has 3 option to choose from, the total number of possibilities will be
3*3*3= 27

a) Possible Strategies
** The numbers represent profit earned by player 1,2 and 3 respectively

If player 2 contributes $0

		Player 3		
		$0	$3	$6
Player 1	$0	0,0,0	2,2,-1	4,4,-2
	$3	-1,2,2	1,4,1	3,6,0
	$6	-2,4,4	0,6,3	2,8,2

If player 2 contributes $3

		Player 3		
		$0	$3	$6
Player 1	$0	2,-1,2	4,1,1	6,3,0
	$3	1,1,4	3,3,3	5,5,2
	$6	0,3,6	2,5,5	4,7,4

If player 2 contributes $6

		Player 3		
		$0	$3	$6
Player 1	$0	4,2,4	6,0,3	8,2,2
	$3	3,0,6	5,2,5	7,4,4
	$6	2,2,8	4,4,7	6,6,6,

b) Nash equilibrium would be achieved in this game only when all the players are contributing $0. At this stage the profit equilibrium for all the players will also be $0

c) This can change if the players know each other’s moves or amount of contribution, in this case all players thinks of a common goal of achieving the highest profit would contribute $6 each to achieve a profit of $6. In that case $6 would be the Pareto optimal or efficient solution

If any one player chooses $0 for example 0,3,6 in which the profit for players will be 3,3,-3 or two players choose $0, for example 0,0,3 in which case profit for each player will be 1,1,-2, this can also be an example of pareto efficiency, where it is not necessary all the players will benefit from the strategy (Mock, 2011).

If we put it in simple words in a Nash equilibrium is the strategy that benefits each player and could not be change by any incentive (Chen, 2020) and Pareto Optimal is the strategy or situation in which everyone can benefit or at least of the players can benefit by hurting or demeriting for at least one player. (Barr, 1992)

d) We can take an example of two countries going to war
Let's say country A and Country B
There are two options stay put and Attack

	Country B	
Country A	Attack, Attack	Stay, Attack
	Attack, Stay	Stay, Stay

In a scenario where both countries attack, they would both be at a huge loss and the payoff would be negative.
In a scenario where one of the countries would attack and other stays put, the payoff for the attacking country could be positive but for the country choosing to stay put would be negative.
In a scenario where both the country stay put, both the countries would benefit from that and come to a mutual understanding, the payoff will be Positive.

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