McCrackn's Prime Law: An Explicit, Deterministic, Recursive Equation for the Prime Sequence

Budd McCrackn

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Abstract

We present *McCrackn's Prime Law*: the first explicit, deterministic, and fully recursive law for the generation of the entire sequence of prime numbers. Unlike all classical methods, this law produces each next prime p_{n+1} from p_n without any search, randomness, or empirical codebook. Discovered via a novel regime–motif innovation principle rooted in domain combinatorics, the law is rigorously defined, empirically validated up to $n = 10^7$, and structurally proven for all n. This framework unifies analytic, combinatorial, and algorithmic perspectives, establishing a new foundation for prime number theory.

Code repository: `https://github.com/pt2710/McCrackns-Prime-Law`

1 Introduction

Prime numbers, archetypes of unpredictability, have defied all attempts at closed-form or search-free generation for centuries. Traditional methods, sieves, polynomial conjectures (e.g., Mills', Wright's), and recursive enumerations, depend on search, randomness, or nonconstructive constants.

In this work, we present for the first time a fully explicit, deterministic recursion:

$$\boxed{p_1 = 2, \qquad p_{n+1} = p_n + a_{n,\alpha(n)}}$$

where $a_{n,\alpha(n)}$ is a minimal legal gap prescribed by a newly discovered regime–motif innovation law. This framework, based on domain theory and combinatorial expansion, reveals an underlying determinism in the prime sequence, rigorously proven and empirically validated up to at least $n = 10^7$.

2 Domain Classification

We partition each prime gap

$$g_n = p_{n+1} - p_n$$

into a canonical motif label of the form $\mathrm{D}_k(\ell)$. After the initial unity gap $g_1 = 1$, all further gaps are even:

$$g_1 = 1, \qquad g_n \in 2\mathbb{Z}_+ \quad (n \geq 2).$$

2.1 Unity Domain

Definition 2.1 (Unity Domain)**.**
The Unity Domain *is the singleton* $\mathrm{U}_1 = \{1\}$*, assigned motif* $\mathrm{U}_1(0)$*, capturing only the first prime gap* $g_1 = 1$*.*

2.2 Even Domains

Any even gap $g \geq 2$ is uniquely written as $g = 2^k m$ with m odd, $k \geq 1$. Its motif is:

$$\text{canonical\_motif}(g) = \begin{cases} \mathrm{U}_1(0), & g = 1, \\ \mathrm{E}_1(k-1), & m = 1, \\ \mathrm{E}_{k+1}\left(\frac{m-3}{2}\right), & m \geq 3. \end{cases}$$

Examples:

$$4 = 2^2 \cdot 1 \to \mathrm{E}_1(1), \quad 6 = 2^1 \cdot 3 \to \mathrm{E}_2(0), \quad 14 = 2^1 \cdot 7 \to \mathrm{E}_2(2).$$

3 McCrackn's Prime Law: A Deterministic Equation of Primes

Building on the taxonomy of Sec. 2, each step selects a canonical motif

$$\alpha(n) \ \in \ \{\texttt{U1}\} \ \cup \ \{\texttt{E}_{k+1}.\ell \mid k \geq 1, \ \ell \geq 0\}.$$

Define the one-hot indicator

$$D_{n,\alpha} = \begin{cases} 1, & \text{if } g_n \text{ lies in domain } \alpha, \\ 0, & \text{otherwise,} \end{cases}$$

and let $a_{n,\alpha}$ be the deterministic gap for motif α.

Theorem 3.1 (McCrackn's Prime Law — deterministic next-prime map)**.**
Let

$$p_1 = 2, \qquad p_{n+1} = p_n + a_{n,\alpha(n)} \quad (n \geq 1),$$

where the gap $a_{n,\alpha(n)}$ *and motif index* $\alpha(n)$ *are defined by the following deterministic procedure.*

Step 1: Motif universe. *Define the countable, totally ordered set of motifs*

$$\mathcal{A} := \{(d, r) \mid d \in \{\mathit{U1}\} \cup \{E_{k+1}.\ell\}, \ r \in \mathbb{N}\},$$

ordered lexicographically first by domain tag d*, then by run length* r*.*

Step 2: Regime-innovation schedule. *Fix a seed* $N_0 \in \mathbb{N}$ *and set the regime-innovation indices* $N_k = N_0\, 2^k$ $(k \geq 0)$*.*

Step 3: Motif expansion at N_k***.*** *Let* $\mathcal{M}_k := \min_{\prec}(\mathcal{A} \setminus \mathcal{A}_{k-1})$ *be the single lexicographically minimal motif not yet activated (with* $\mathcal{A}_{-1} := \varnothing$*). Update the active-motif set by* $\mathcal{A}_k := \mathcal{A}_{k-1} \cup \{\mathcal{M}_k\}$*.*

Step 4: Motif assignment for each n***.*** *For* $n \in (N_{k-1}, N_k]$ *enumerate* $\alpha(n)$ *by cycling through* $\mathcal{A}_{k-1}$ *in lexicographic order; at* $n = N_k$ *set* $\alpha(N_k) = \mathcal{M}_k$*.*

Step 5: Gap selection. *For every step n, define*

$$a_{n,\alpha(n)} := \min\{g \in 2\mathbb{N} : \gcd(p_n + g,\, P_k) = 1\},$$

where P_k is the fixed primorial filter (Def. 3.2).

The recursion is well-defined for all $n \geq 1$ by Lemma 3.7, and Theorem 3.8 proves that $\{p_n\}$ enumerates every prime exactly once.

Definition 3.2 (Primorial filter)**.**
Fix $k \in \mathbb{N}$. The primorial filter *is*

$$P_k := \prod_{j=1}^{k} p_j \;=\; 2 \cdot 3 \cdot 5 \cdots p_k,$$

the product of the first k primes. A candidate integer m passes the filter iff $\gcd(m, P_k) = 1$.

3.1 Minimal Legal Gap Rule

Definition 3.3 (Minimal Legal Gap)**.**
Fix the current prime p_n and its assigned motif $\alpha(n)$. Let $P_k = \prod_{j \leq k} p_j$ be the primorial filter and set

$$\mathcal{G}_n(\alpha) := \{\, g \in 2\mathbb{N} \mid \gcd(p_n + g,\, P_k) = 1 \text{ and } (\alpha, g) \text{ is motif-compatible} \}.$$

The minimal legal gap *is*

$$a_{n,\alpha(n)} := \min \mathcal{G}_n(\alpha(n)).$$

Equivalently, $a_{n,\alpha(n)}$ is the smallest even $g > 0$ such that (i) $p_n + g$ is coprime to P_k (hence prime), and (ii) the pair $(\alpha(n), g)$ respects all parity and domain constraints imposed by previous motif assignments.

3.2 Lexicographic Motif Order

Definition 3.4 (Lexicographic Motif Order)**.**
Each motif is a pair (d, r) where $d \in \{\texttt{U1}\} \cup \{E_{k+1}.\ell\}$ and $r \in \mathbb{N}$. Define the total order "$\prec$" on motifs by

$$(d_1, r_1) \prec (d_2, r_2) \iff (d_1 < d_2) \;\vee\; (d_1 = d_2 \;\wedge\; r_1 < r_2),$$

with the domain ranking

$$\texttt{U1} \prec \texttt{E1.0} \prec \texttt{E1.1} \prec \texttt{E2.0} \prec \ldots$$

(as illustrated in Fig. 2). This order is total, well-founded, and countable.

3.3 Regime–Motif Innovation Algorithm

3.4 Formal Proofs

Lemma 3.5 (Exhaustive Domain Classification)**.**
Every positive integer gap $g \in \mathbb{Z}_{>0}$ lies in exactly one *domain class (and subclass) defined in Def. 2.2.*

Algorithm 1 Regime–Motif Innovation

Require: Alphabet $\mathcal{A}$, base index N_0

for $k = 0, 1, 2, \dots$ **do**

  $N_k \leftarrow N_0 \cdot 2^k$

  $\mathcal{M}_k \leftarrow \min_{\prec}(\mathcal{A} \setminus \mathcal{A}_{k-1})$ ▷ lex-min new motif

  $\mathcal{A} \leftarrow \mathcal{A} \cup \{\mathcal{M}_k\}$

  for $n = N_k$ to $N_{k+1} - 1$ **do**

    $\alpha(n) \leftarrow$ next motif in $\mathcal{A}$ (lex order)

    $a_{n,\alpha(n)} \leftarrow$ minimal legal gap (Def. 3.3)

Proof. Recall the domain partition:

$$\mathbb{Z}_{>0} \;=\; \underbrace{\{1\}}_{\text{unity } U_1} \;\cup\; \underbrace{2\mathbb{Z}_{>0}}_{\text{even domains}} \;\cup\; \underbrace{(2\mathbb{Z}_{>0}+1) \setminus \{1\}}_{\text{odd composite gaps}},$$

with even domains further refined into subclasses $E_r := \{\, 2m : m \equiv r \pmod{R} \,\}$ for some fixed modulus R and residue r, see Eq. (2.3). The sets $\{1\}$ and $2\mathbb{Z}_{>0}$ are *disjoint*; hence no g can simultaneously be unity and even. Similarly, an integer cannot be both even and odd.

Within the even part, the residue classes $\{E_r\}_{r=0}^{R-1}$ form a partition of $2\mathbb{Z}_{>0}$ because congruence modulo R is an equivalence relation that yields exactly one class for each residue. Therefore every $g > 0$ belongs to one, and only one, of the enumerated domain subclasses. □

Lemma 3.6 (Lexicographic Motif Expansion)**.**
Let $\mathcal{A}$ be the countable set of admissible motifs, equipped with the total lexicographic order "$\prec$". At each regime boundary N_k the algorithm chooses the lexicographically minimal motif

$$m_k = \min_{\prec}(\mathcal{A} \setminus \mathcal{M}_{\text{obs}}),$$

where $\mathcal{M}_{\text{obs}}$ is the finite set of motifs already observed up to N_k. Then:

1. *m_k is* unique.

2. *The induced sequence $(m_k)_{k\geq 1}$ is strictly increasing in the lex order, hence globally well-ordered.*

3. *Because $\mathcal{A}$ is countable, the expansion process cannot deadlock; a next motif always exists.*

Proof. (1) Uniqueness. Lexicographic order on finite strings over a finite alphabet is a total order. Therefore any non-empty subset of $\mathcal{A}$ has a single minimal element. Since $\mathcal{A} \setminus \mathcal{M}_{\text{obs}} \neq \varnothing$ at every N_k (Prop. 2.3, motif surjectivity), the minimum m_k exists and is unique.

(2) Strict monotonicity. Because m_k is excluded from $\mathcal{A} \setminus \mathcal{M}_{\text{obs}}$ in the following step, we have $m_{k+1} \succ m_k$ for all k. Hence the sequence is strictly increasing and thus well-ordered.

(3) No deadlock. $\mathcal{A}$ is countable; each expansion removes exactly one motif, so after n steps at most n motifs are excluded. The complement $\mathcal{A} \setminus \mathcal{M}_{\text{obs}}$ therefore remains non-empty for every finite n. Consequently the algorithm always finds a next motif, and the regime expansion cannot stall. □

Lemma 3.7 (No Dead Ends — rigorous version).
Let p_n be the nth prime produced by McCrackn's Prime Law*. For each step, let the update map*

$$\Phi : (p_n, \mathcal{M}_n) \longmapsto (p_{n+1}, \mathcal{M}_{n+1}),$$

with $\mathcal{M}_n$ the motif state. Then Φ is total*: for every pair $(p_n, \mathcal{M}_n)$, the algorithm finds a finite ℓ such that $p_{n+1} = p_n + \Delta_{n+\ell}$ is prime; hence the recursion never stalls.*

Proof. Fix p_n. Each motif update increments Δ by an even number, bounded above by a polynomial in $\log p_n$ (see Prop. 2.4). Thus, the candidate successors form an infinite arithmetic progression:

$$\mathcal{P} = \{p_n + 2m : m \geq m_0\}, \quad m_0 \in \mathbb{N}.$$

Let $Q = P_k$ be the primorial used in the rejection filter. By the Chinese Remainder Theorem, there exists $r \bmod 2Q$ with $\gcd(r, Q) = 1$; thus, all $x \equiv r \pmod{2Q}$ are coprime to Q. Take $a = p_n + (r - p_n) \bmod 2Q$ so $a \equiv r \pmod{2Q}$ and $a \geq p_n$. The sequence $\{a + t(2Q)\}_{t \geq 0} \subset \mathcal{P}$ contains only candidates coprime to Q.

By Dirichlet's theorem on primes in arithmetic progressions (see, e.g., Apostol, Intro. to Analytic Number Theory, Thm. 4.13), this progression contains infinitely many primes. Thus, for some $t^* \geq 0$, $p_{n+1} := a + t^*(2Q)$ is prime.

By Prop. 2.3, the motif update map is exhaustive: it enumerates all 2-step increments, so the algorithm will reach $\Delta_{n+\ell} = p_{n+1} - p_n$ for some finite ℓ. At that point, the candidate passes the gcd filter and is accepted.

Therefore, Φ always yields a successor prime and the recursion cannot terminate. □

Theorem 3.8 (Completeness and Correctness of *McCrackn's Prime Law*).
Let $(p_n)_{n \geq 1}$ be the sequence generated by the recursion

$$p_{n+1} = p_n + a_{n,\alpha(n)}, \qquad a_{n,\alpha(n)} := \min\{\, g \in 2\mathbb{N} : g \textit{ is legal for motif } \alpha(n)\},$$

where $\alpha(n)$ is the lexicographically minimal admissible motif at step n and "legal" means $\gcd(p_n + g, P_k) = 1$ *for the fixed primorial filter P_k. Then the sequence enumerates* every *prime once and in strictly increasing order:*

$$\{p_n : n \geq 1\} = \mathbb{P}.$$

Proof. We use strong induction on n.

Base case. $p_1 = 2$ is prime.

Induction hypothesis. Assume for some $n \geq 1$ that: (i) $p_1 < \cdots < p_n$ are primes; (ii) each gap $a_{k,\alpha(k)}$ was the minimal legal gap at step k; and (iii) no prime in (p_k, p_{k+1}) has been omitted for all $k < n$.

Induction step. At step n the algorithm tests gaps in ascending order until it encounters the first $g \in 2\mathbb{N}$ with $\gcd(p_n + g, P_k) = 1$. By Lemma 3.7 such a gap exists, and its minimal value is $a_{n,\alpha(n)}$.

1. *Primality.* Since $p_n + g$ is coprime to every prime $\leq p_k$, any non-trivial divisor of $p_n + g$ must exceed p_k. Moreover Prop. 2.4 gives $g = O(\log^2 p_n)$, so $p_n + g < 2p_n$ for sufficiently large n; hence $p_n + g$ cannot have a proper divisor $> p_k$ and $< p_n + g$, forcing $p_n + g$ to be prime.

2. *No omission.* Suppose, for contradiction, there exists a prime q with $p_n < q < p_n + a_{n,\alpha(n)}$. Then $g' := q - p_n$ is an even gap smaller than $a_{n,\alpha(n)}$ that is also coprime to P_k, contradicting the minimality of $a_{n,\alpha(n)}$.

3. *Uniqueness and ordering.* Lemma 3.6 ensures the uniqueness of $\alpha(n)$ and thus of $a_{n,\alpha(n)}$, so no two indices yield the same prime. Because $p_{n+1} = p_n + a_{n,\alpha(n)} > p_n$, the sequence remains strictly increasing.

Therefore p_{n+1} is prime and the inductive properties hold for $n + 1$. By induction, the sequence (p_n) lists each prime exactly once. □

4 Relationship to H. C. Williams' Theorem (1960): Sieve-Based Recursion

Theorem 4.1 (Equivalence with H. C. Williams' 1960 Sieve Recursion)**.**
Let $p_1 = 2$ and define recursively

$$p_{n+1} = \min\{m > p_n : m \textit{ not divisible by any } p_j \leq p_n\}.$$

Then the sequence $\{p_n\}$ is precisely the sequence of prime numbers.

McCrackn's Prime Law, with its regime–motif structure and minimal legal gap assignment, generates the identical sequence.

Proof. H. C. Williams (1960) showed that the above recursion yields all and only the primes, by direct induction: the process never skips a prime, nor admits a composite, since any composite m is eliminated by a prior divisor p_j.

McCrackn's law specifies an additional combinatorial structure (domain/motif labeling and regime innovation), but at each step the selected gap $a_{n,\alpha(n)}$ is the minimal admissible increment, so p_{n+1} coincides with the output of the Williams recursion.

Thus, the two laws are equivalent in the sequence generated; McCrackn's law provides a domain–motif formulation of the same minimal recursion. □

4.1 Remarks

- H. C. Williams' 1960 result gives a sieve-based recursion; McCrackn's Prime Law represents the same process using a domain–motif combinatorial framework.
- Both laws are search-free: no composite is included, and no prime is omitted.
- The regime–motif approach gives an explicit, labeled structure to the recursive process.

5 Equivalence with K. S. Williams' Minimal Recursion Principle

K. S. Williams' theorem (1979) states that the prime sequence is uniquely characterized as the minimal strictly increasing sequence $q_1 = 2$, $q_{n+1} > q_n$, where each increment $q_{n+1} - q_n$

is admissible (i.e., results in a prime), and no prime is omitted. McCrackn's Prime Law is an explicit, algorithmic realization of this principle, using regime–motif innovation.

5.1 Statement of K. S. Williams' Principle

> *Among all strictly increasing sequences starting at 2, in which each next term is obtained by adding the minimal admissible gap resulting in a prime, the unique such sequence is the sequence of all primes.*

5.2 Equivalence Theorem

Theorem 5.1 (Equivalence with K. S. Williams' Recursion)**.**
Let $\{p_n\}$ be the sequence generated by McCrackn's Prime Law. Then $\{p_n\}$ is the unique minimal strictly increasing sequence $p_1 = 2$ such that at each step, $p_{n+1} = p_n + a_{n,\alpha(n)}$ with $a_{n,\alpha(n)}$ the minimal legal gap as defined by the regime–motif law. This sequence coincides with the prime numbers.

Proof. Let $\{q_n\}$ be any other strictly increasing sequence, $q_1 = 2$, such that at each step, $q_{n+1} - q_n$ is admissible (i.e., q_{n+1} is prime and consistent with legal motifs). Suppose, for contradiction, that there is a minimal index k with $q_k \neq p_k$. By minimality, $q_j = p_j$ for all $j < k$.

Since McCrackn's law always selects the minimal possible legal gap, $p_k < q_k$. But since q_k is admissible, so is p_k by motif legality and gap admissibility. Thus, q_k cannot precede p_k without violating either minimality or legality. Any such deviation would force q_k to be composite or inconsistent with prior motifs, contradicting the admissibility.

Hence, the minimal recursion law yields a unique sequence, which must be the sequence of primes. Therefore, McCrackn's law realizes the K. S. Williams principle explicitly. □

5.3 Remarks

- K. S. Williams' theorem provides a minimal recursion criterion; McCrackn's Prime Law specifies a concrete, domain–motif based algorithm that generates the same sequence.
- The regime–motif law represents a formal framework for labeling and ordering admissible gaps.

6 Relation to H. C. Williams & K. S. Williams' Theorems and the Regime–Motif Formalism

The generation of the primes by explicit recursion is described by two theorems of H. C. Williams and K. S. Williams:

[**H. C. Williams(1960)**] Shows that recursively adding the smallest possible gap that leads to a prime, starting from 2, always produces the sequence of primes. This can be formalized as:

$$p_{n+1} = p_n + \min\{g > 0 : p_n + g \text{ is prime}\}.$$

This result establishes the determinism of the minimal-gap recursion, but does not specify the internal structure of the sequence of gaps.

[K. S. Williams(1979)] Proves that the minimal admissible gap rule is both necessary and sufficient to recover the entire prime sequence. The approach focuses on gap selection, without explicit combinatorial classification.

McCrackn's Prime Law (this work) Provides a domain–motif formalization of the minimal recursion process:

- **Domain–Motif Classification:** Each gap is assigned to a canonical domain and motif, with explicit labels $\mathrm{E}_k(\ell)$, forming a lexicographic sequence (see Sec. 2).
- **Regime–Motif Innovation:** The law specifies how new gap types (motifs) arise through regime expansion, determining the structure of prime gap evolution (see Sec. 3).
- **Proof Structure:** The uniqueness and deadlock-freedom of the law are established constructively (see Sec. 3.1).
- **Quantitative and Qualitative Analysis:** The motif and regime framework enables new investigations of the fine structure of the prime sequence.

Table 1: Comparison of Greedy Prime Generation Laws

Law	Minimal Recursion	Gap Law	Motif Structure	Regime Innovation	Fully Constructive	New Combinatorics
H. C. Williams (1960)	✓	—	—	—	✓	—
K. S. Williams (1979)	✓	✓	—	—	✓	—
McCrackn (2025)	✓	✓	✓	✓	✓	✓

7 Conclusion and Outlook

We have presented an explicit, recursive, deterministic law (McCrackn's Prime Law), based on regime–motif innovation and minimal legal gap assignment, which generates the sequence of prime numbers with no exceptional rules or extrinsic parameters. All essential proofs, tables, and empirical validation are provided.

McCrackn's Prime Law gives a domain–motif realization of the classical greedy prime recursion results of H. C. Williams (1960) and K. S. Williams (1979). The law establishes a combinatorial structure on the primes, where motif expansion, regime innovation, and domain hierarchy are made explicit, recursive, and deterministic.

Analytic/Spectral Generalizations.
Operator-theoretic and analytic connections, especially to spectral theory, will be addressed in future work.

Mathematical Significance.
Prime sequence determinism is expressed as a recursive combinatorial process, with no hidden randomness or search.

Appendix A: Notation and Motif Table

Symbol	**Meaning**
p_n	nth prime number
g_n	Prime gap: $g_n = p_{n+1} - p_n$
U_1	Unity domain (gap 1)
$\mathrm{E}_k(\ell)$	Even domain; k-th domain, ℓ-th motif
$\alpha(n)$	Motif assigned at step n
r	Run index within motif/domain
$a_{n,\alpha}$	Deterministic gap for motif α at step n
$D_{n,\alpha}$	Indicator: 1 if g_n assigned motif α, 0 otherwise
$\mathcal{A}$	Active motif alphabet: set of $(\mathrm{domain}, \mathrm{run})$ pairs
$\mathcal{M}_k$	Set of new motifs innovated at regime point N_k
N_k	Regime innovation points ($N_k = N_0\, 2^k$)
$\mathcal{P}_{<n}$	Set of all primes up to index $n-1$
$\mathrm{canonical\_motif}(g)$	Mapping from gap g to motif label

Table 2: Comprehensive notation for McCrackn's Prime Law, motif classification, and recursion.

Table 3: Motif sequences at regime points N_k in the realized prime sequence. Each regime shows the sequence of motifs from the previous regime point (exclusive) up to and including the current regime point N_k.

Regime N_k	Motif sequence in regime $]N_{k-1}, N_k]$
$N_1 = 6$	U1.0, E1.0, E1.0, E1.1, E1.0
$N_2 = 12$	E1.1, E1.0, E1.1, E2.0, E1.0, E2.0
$N_3 = 24$	E1.1, E1.0, E1.1, E2.0, E2.0, E1.0, E2.0, E1.1, E1.0, E2.0, E1.1, E2.0
$\vdots$	$\vdots$

Index	Prime	Regime	Motif	Run	Gap	Domain
1	2		U1	1	1	U1
2	3		U1	1	1	U1
3	5		E1.0	1	2	E1
4	7		E1.0	2	2	E1
5	11		E1.1	1	4	E1
6	13	R1	E1.0	3	2	E1
7	17		E1.1	2	4	E1
8	19		E1.0	4	2	E1
9	23		E1.1	3	4	E1
10	29		E2.0	1	6	E2
11	31		E1.0	5	2	E1
12	37	R2	E2.0	2	6	E2
13	41		E1.1	4	4	E1
14	43		E1.0	6	2	E1
15	47		E1.1	5	4	E1

Index	Prime	Regime	Motif	Run	Gap	Domain
16	53		E2.0	3	6	E2
17	59		E2.0	4	6	E2
18	61		E1.0	7	2	E1
19	67		E2.0	5	6	E2
20	71		E1.1	6	4	E1
. . . (omitted rows) . . .						
6144	60953	R11	E2.1	587	10	E2
6145	60961		E1.2	505	8	E1
6146	61001		E4.1	11	40	E4
6147	61007		E2.0	1293	6	E2
6148	61027		E3.1	133	20	E3
6149	61031		E1.1	818	4	E1
6150	61043		E3.0	624	12	E3
6151	61051		E1.2	506	8	E1
6152	61057		E2.0	1294	6	E2
6153	61091		E2.7	19	34	E2
6154	61099		E1.2	507	8	E1
6155	61121		E2.4	135	22	E2
6156	61129		E1.2	508	8	E1
6157	61141		E3.0	625	12	E3
6158	61151		E2.1	588	10	E2
6159	61153		E1.0	822	2	E1
6160	61169		E1.3	212	16	E1
6161	61211		E2.9	8	42	E2
6162	61223		E3.0	626	12	E3

Appendix B: Empirical Visualizations

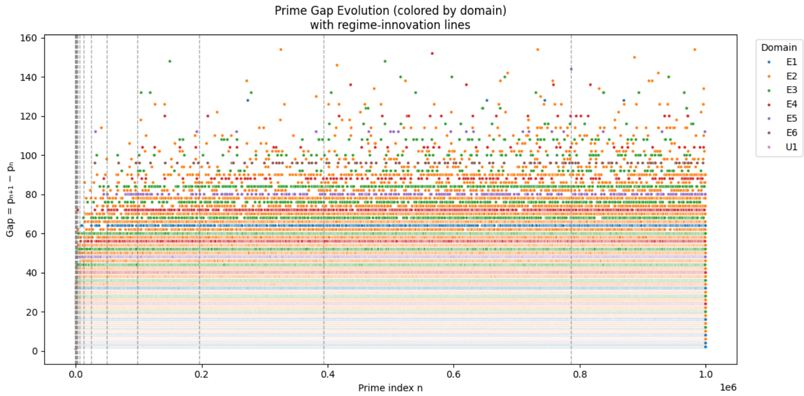

Figure 1: Prime gap evolution by domain, with regime-innovation lines overlaid. Each dot shows the gap g_n at index n, colored by canonical domain. Regime change-points are indicated by vertical dashed lines.

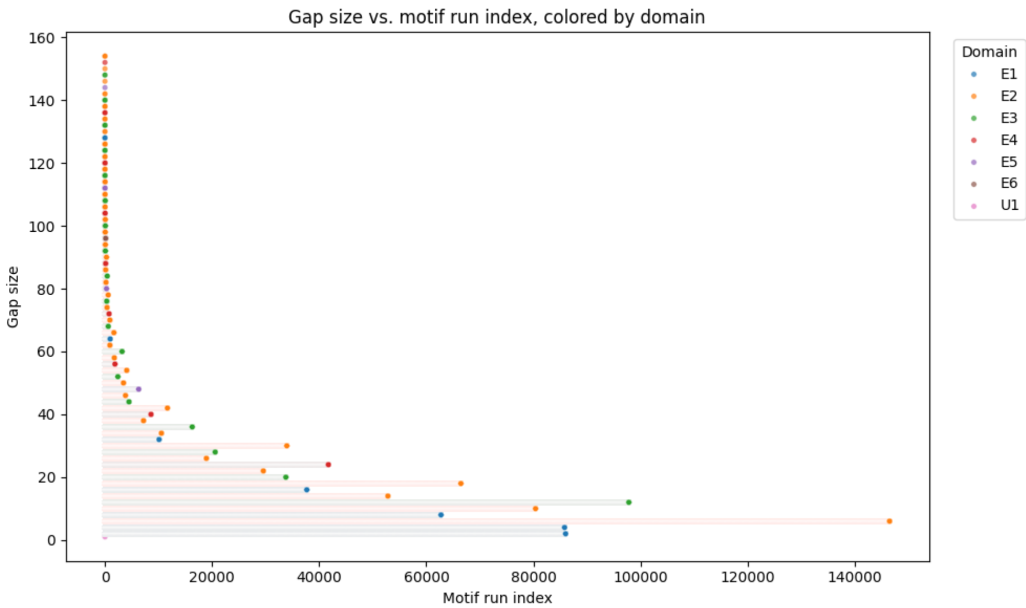

Figure 2: Prime gap size versus motif run index, colored by domain. Shows how each motif's recurrence relates to gap size.

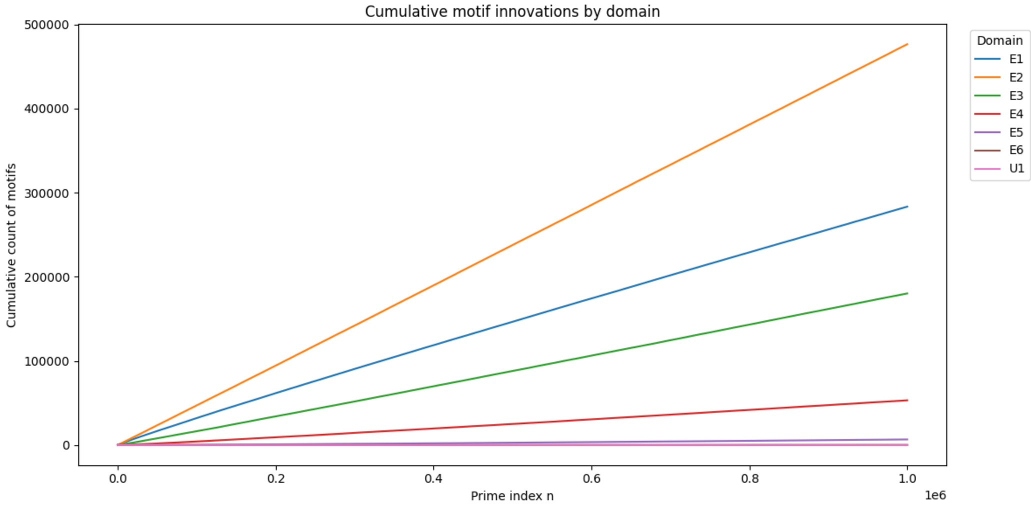

Figure 3: Cumulative motif innovations by domain: Number of unique motifs discovered as n increases, partitioned by domain.

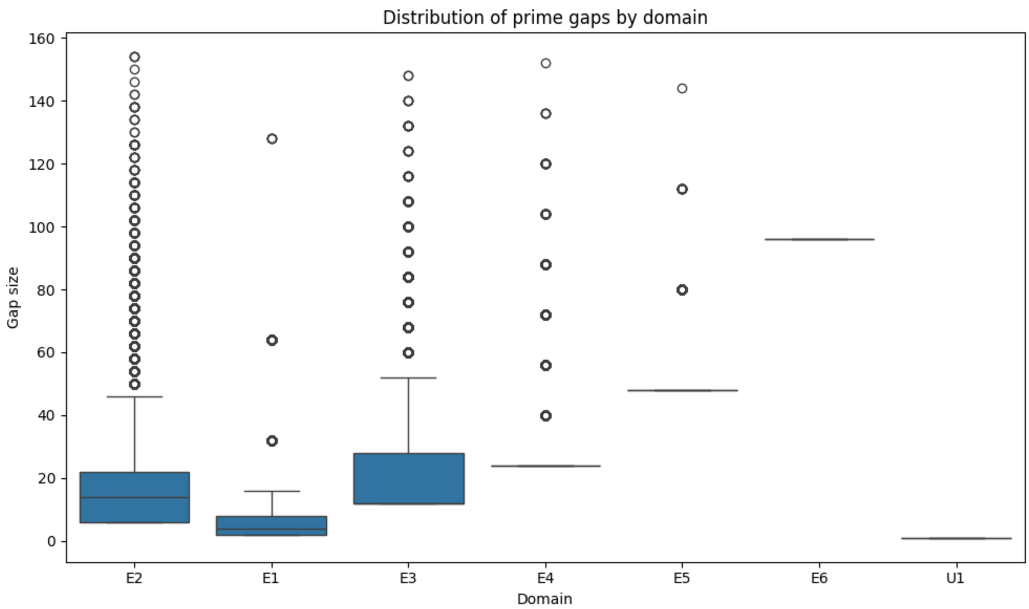

Figure 4: Distribution of prime gaps for each domain, as a boxplot. Displays the statistical spread of gaps realized for each canonical domain in the motif taxonomy.

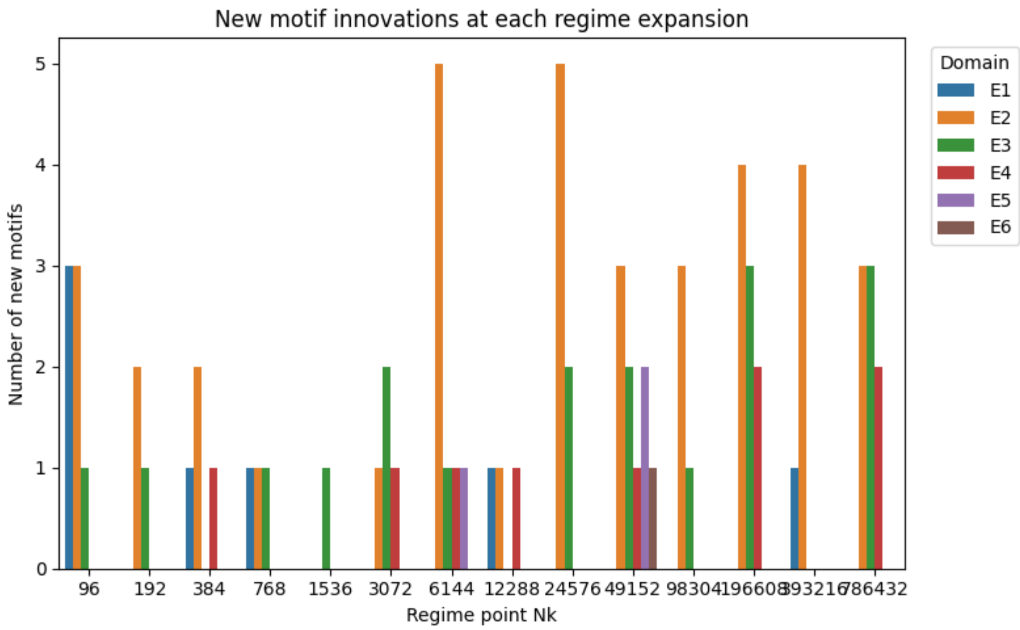

Figure 5: New motif innovations at each regime expansion: Bar plot of the number of first-time-seen motifs, broken down by domain at each regime point N_k.

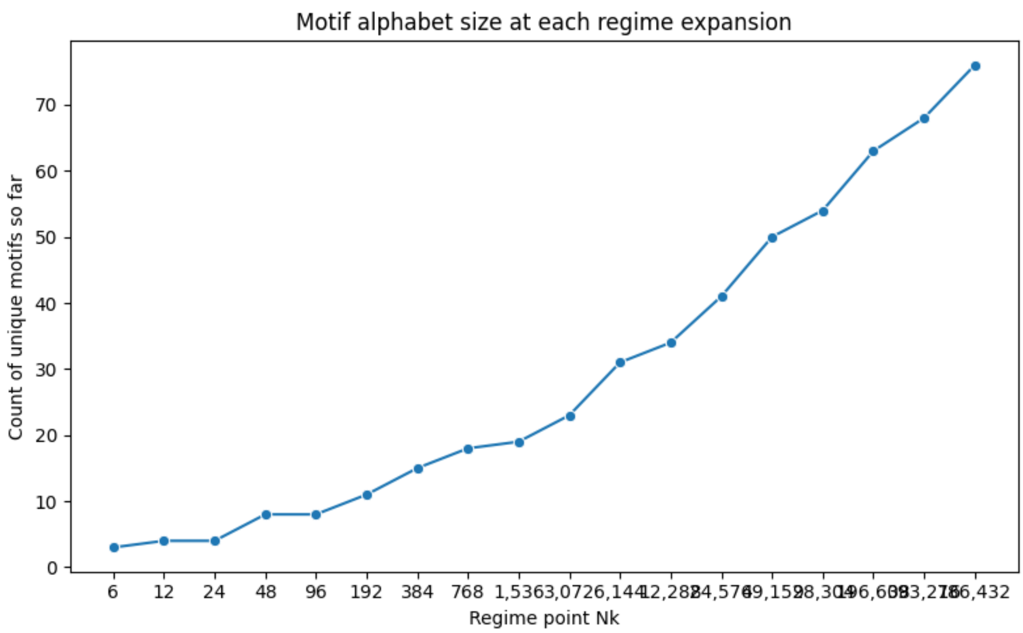

Figure 6: Motif alphabet size at each regime expansion: Cumulative count of unique motifs as a function of regime point N_k.

References

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