STA 205 – PROBABILITY AND STATISTICS

LECTURE 1
INTRODUCTION

In this lecture, you will be introduced to the concept of probability and statistics and the relationship that exists between them. Further, we will discuss their significance

1.1 OBJECTIVES

After this lecture, you should be able to correctly define the terms probability and statistics and to outline the relationship between them.

Definitions of Statistics

The term statistics is used to mean either statistical data or statistical method.

Statistical data: refers to numerical statement of facts e.g. number of malaria cases.

Statistical method: is the complete body of the principles and techniques used in collecting and analyzing such data.

In general statistics is the science of designing studies or experiments, collecting data and modeling analyzing data for the purpose of decision making and scientific discovery when the available information is both limited and variable. In other words, statistics deals with data and for whatever use, the data is put into, we approach the study of statistics by considering the four steps process in learning from data.

1. Defining the problem
2. Collecting the data
3. Summarizing the data
4. Analyzing the data
5. Interpreting the analyzed data
6. Communicating the results.

Characteristics of statistical data

1. They must be in aggregates i.e statistics are numbers of facts.
2. They must be affected to a small extent by a multiplicity of causes.
3. Must be enumerated or estimated according to a reasonable standard of accuracy: they

must be reasonably accurate.

4. Must be collected in a systematic manner for a predetermined purpose and ensure valid conclusion.

5. Must be placed in relation to each other (i.e. must be comparable).

6. Must be numerically expressed.

Main functions of statistics

1. It simplifies data – in terms of totals, averages, percentage etc. and presented graphically or diagrammatically.

2. It furnishes a technique of comparison i.e. comparison with different places or different points of times.

3. Gives a bird's eye view of the data.

4. Helps in policy making that is, formulation of suitable policies.

5. It helps to formulating and testing hypothesis.

6. It helps in production.

Statistics is used in literally every sphere: in government agencies, in business, economics, in physical sciences, natural or biological sciences for example medicine and in research.

Statistical methodology:

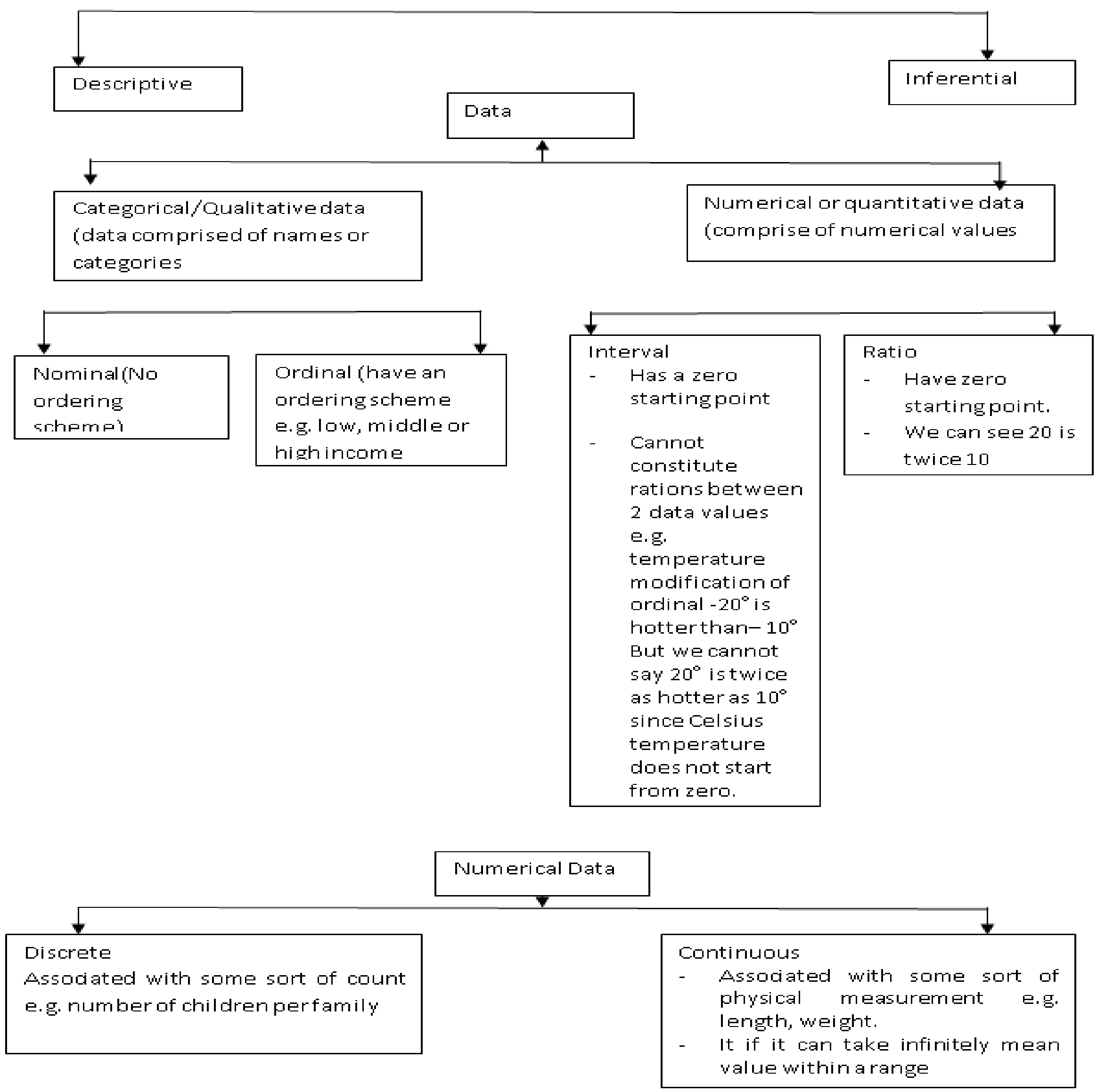
This is divided into two classes.

a) **Descriptive statistics**: this involves data collection, organization, presentation, analysis and its interpretation.

b) **Inferential statistics**: is a set of procedures used to draw conclusions about a large body of data called a population based on a smaller set of data called a sample which has been taken from the population.

The calculation of descriptive statistics may precede the application of inferential techniques. For good inference, the sample must be representative of the population.

STATISTICAL METHODOLOGY



Probability

Almost all phenomena we are interested in studying are subject to chance hence the study of probability. Study of probability implies study of chance. By definition, the probability of a given event is an expression of likelihood or chance of occurrence of an event. It is a number which ranges from 0(zero) to 1 (one) for an event certain to happen.

Role of Probability in Statistics

Probability is a tool that allows us to use sample information to make inferences about or to describe the population from which the sample was drawn.

When the population is known, probability is used to describe the likelihood of various sample outcomes. When the population is unknown and we have only a sample, we have the statistical problem of trying to make inferences about the unknown populations, and probability is a tool we use to make these inferences. Thus, probability reasons from the population to the sample whereas statistics act in the reverse, that is moving from the sample to the population.

Theories of Probability

There are two definitions of probabilities, namely:

1. Mathematical or classical or a Prior probability.
2. Statistical or Empirical probability.

Mathematical or a Prior Probability

The basic assumption underlying the theory is that, the outcomes of random experiments are equally likely. The event whose probability is sought consists of one or more possible outcomes of the given a activity, for example when a die is rolled once, anyone of the possible outcomes, that is 1,2,3,4,5 or 6 can occur.

By definition if a trial results in n exhaustive mutually exclusive and equally likely cases and m of them are favorable to the happening of an event E, then the probability "P" of happening of E is given by

$$P(E) = \frac{m}{n} = \frac{\text{No. of favourable cases (events)}}{\text{Total No. of equally likely cases (events)}}$$

now, the no. of cases favorable to happening of event E are n-M

Therefore $P(notE) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p = q(say) \Rightarrow p + q = 1$

Note that p and q are non-negative and cannot exceed unity i.e

$0 \le P \le 1,\ 0 \le q \le 1$

Probability "p" of the happening of an event is also called the probability of success since it is the probability of happening of event while the probability "q" is called the probability of failure since it is the probability of non-happening of a given event.

Also if $P(E) = 1$, it implies a certain event and if $P(E) = 0$, it implies an impossible even.

Note also that, mathematical theory is applicable only under the following conditions:

1) Various outcomes must be equally likely
2) Exhaustive no. of cases must be finite

Statistical or Empirical Probability

Definition: if a trial is repeated a number of times under essentially homogenous and identical conditions, then the limiting value of the ration of the no. of tones the event happens to the no. of trial , as the no. of trial becomes indefinitely large, is called the probability of happening of the event.

Symbolically, if in n trials, an event E happening M times, then the probability "p" of the happening of E is given by

$$P(E) n \overset{\lim}{\rightarrow} \infty \, {}^{m}\!/_{n}$$

Mathematical tools

Sets and elements of sets

A set is a well defined collection of aggregate of all possible objects having given properties and specified according to a well defined rule. The objects comprising a set are called elements, members or points of the set. Sets are denoted by capital letters e.g. A, B, C, etc. If A is a set and x is an element of the set A, we say x belongs to A and is denoted by $x \in A$

If x is not a number of the set A, we say x does not belong to A and is denoted $x \notin A$.

Sets are often described by describing the properties possessed by their members. Thus the set A of all non negative rational numbers with square less than 2 will be written as

$A = \left(X : X \geq 0, \mathrm{X}^2 < 2 \right)$

If every element of the set A belongs to the set B, i.e. $x \in A \Rightarrow x \in B,$, then A is a subject of B and is denoted as $A \subseteq B$ (A is contained in B) or $B \supseteq A$ (meaning B Contains A).

Two sets A and B are said to be equal or identical if $A \subseteq B$ and $B \subseteq A$ and is written as

$A = B$ or $B = A$

A null or empty set is one which does not contain any element at all and is denoted by ϕ. Every set is a subject of itself and empty set is a subject of every set.

Operations on sets

1. The union of 2 given sets A and B, denoted by $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

   Intuitively,

$$\bigcup_{i=1}^{n} A_i = \{x : x \in A_i \text{ for at least } i = 1, 2, \ldots, n\}$$

2. The intersection of 2 sets A and B, denoted by $A \cap B$ is defined as a set consisting of those elements which belong to both A and B. Thus

   $A \cap B = \{x : x \in A \text{ and } x \in B\}$

   Intuitively,

   $$\bigcap_{i=1}^{n} A_i = \{x : x \in A_i \text{ for all } i = 1, 2, \ldots, n\}$$

3. If A and B have no common point, i.e $A \cap B = \phi$, then the sets A and B are said to be disjoint, mutually exclusive or non-overlapping.
4. Relative difference of a set A from another set B, denoted by A – B is defined as a set consisting of those elements of A which do not belong to B.
   Symbolically:
   $A - B = \{x : x \in A \text{ and } x \notin B\}$
5. The complement or negative of a set A, denoted by $\bar{A}$ is a set containing all elements of the universal set, say, S that are not elements of A, i.e.
   $\bar{A} = S - A$

Algebra of sets

If A, B and C are the subsets of a universal set S, then the following properties or laws holds

1. Commutative Law
   $A \cup B = B \cup A$ and
   $A \cap B = B \cap A$

2. Associative Law
   $(A \cup B) \cup C = A \cup (B \cup C)$ and
   $(A \cap B) \cap C = A \cap (B \cap C)$

3. Distributive Law
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and
   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. Complementary Law
   $A \cup \bar{A} = S$

$A\cap\overline{A}=\phi$
$A\cup S=S\left(\because A\in S\right)$
$A\cap S=A$
$A\cup\phi=A$
$A\cap\phi=\phi$

5. Difference Law
$A-B=A\cap\overline{B}$
$A-B=A-\left(A\cap B\right)=\left(A\cup B\right)-B$
$A-\left(B-C\right)=\left(A-B\right)\cup\left(A-C\right)$
$\left(A\cup B\right)-C=\left(A-C\right)\cup\left(B-C\right)$
$A-\left(B\cup C\right)=\left(A-B\right)\cap\left(A-C\right)$
$\left(A\cap B\right)\cup\left(A-B\right)=A$
$\left(A\cap B\right)\cap\left(A-B\right)=\phi$

6. De-Morgan's Law (Dualization Law)
$\left(\overline{A}\cup\overline{B}\right)=\overline{A}\cap\overline{B}$ and
$\left(\overline{A}\cap\overline{B}\right)=\overline{A}\cup\overline{B}$

7. Involution Law
$\left(\overline{\overline{A}}\right)=A$

8. Idempotency Law
$A\cup A=A$
$A\cap A=A$