



## Conflict-Free Replicated Data Types

Lattice Theory for Parallel Programming

### Goals

- ★ Working with the formal definitions of CRDTs.
- ★ Implementing various CRDTs.

### Exercise 1 – Formal CRDT

The following exercices are also available in the slides of the course (useful for more context).

1. Using lexicographic order, design a CRDT implementing a PN-Counter.
2. Find a proper way to compare the behaviors of the three CRDTs implementing a PN-Counter.  
Prove that all definitions are equivalent.
3. In the G-Set CRDT, prove that  $lookup_x \triangleq \lambda S. lookup(S, x)$  is a monotone function. Recall:

$$\begin{aligned} lookup : \mathcal{P}(X) \times X &\rightarrow \mathbb{B} \\ lookup(S, x) &\triangleq x \in S \end{aligned}$$

4. Define the 2P-Set CRDT using  $GS$  and the decreasing-only set CRDT  $DS$ .
5. For the OR-Set CRDT, prove the order  $\leq$  and the join  $\sqcup$  operations are coherent:  $X \leq Y \Leftrightarrow X \sqcup Y = Y$ .
6. Define the OR-Set CRDT without having to redefine yourself the lattice operations.

### Exercise 2 – OR-Set CRDT

1. Implement and test the OR-Set CRDT as seen in class, extending your implementation of the G-Counter.
2. Implement an operation-based CRDT of the OR-Set.