DATA 605: Assignment 6

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Problem 1

A bag contains 6 green and 9 red jellybeans. How many ways can 5 jellybeans be withdrawn from the bag so that the number of green ones withdrawn will be less than 4?

Answer:

$\binom{6}{0} * \binom{9}{5} + \binom{6}{1} * \binom{9}{4} + \binom{6}{2} * \binom{9}{3} + \binom{6}{3} * \binom{9}{2} = 2862$

```
# Calculate Green <= 3
a_1 <- choose(6, 0) * choose(9,5) +
       choose(6, 1) * choose(9,4) +
       choose(6, 2) * choose(9,3) +
       choose(6, 3) * choose(9,2)

# Calculate total
tot <- choose(15, 5)

# Calculate Green >= 4
rem <- choose(6, 4) * choose(9,1) +
       choose(6, 5) * choose(9,0)

a_1
```

```
## [1] 2862
```

```
tot
```

```
## [1] 3003
```

```
rem
```

```
## [1] 141
```

```
(tot-rem)
```

```
## [1] 2862
```

Problem 2

A certain congressional committee consists of 14 senators and 13 representatives. How many ways can a subcommittee of 7 be formed if at least 4 of the members must be senators?

Answer:

$\binom{14}{4} * \binom{13}{3} + \binom{14}{5} * \binom{13}{2} + \binom{14}{6} * \binom{13}{1} + \binom{14}{7} * \binom{13}{0} = 484913$

```
# Calculate Senators >= 4
a_2 <- choose(14, 4) * choose(13,3) +
       choose(14, 5) * choose(13,2) +
       choose(14, 6) * choose(13,1) +
       choose(14, 7) * choose(13,0)

# Calculate total
tot <- choose(27, 7)

# Calculate Senators < 4
rem <- choose(14, 3) * choose(13,4) +
       choose(14, 2) * choose(13,5) +
       choose(14, 1) * choose(13,6) +
       choose(14, 0) * choose(13,7)

a_2
```

```
## [1] 484913
```

```
tot
```

```
## [1] 888030
```

```
rem
```

```
## [1] 403117
```

```
(tot-rem)
```

```
## [1] 484913
```

Problem 3

If a coin is tossed 2 times, and then a standard six-sided die is rolled 4 times, and finally a group of three cards are drawn from a standard deck of 52 cards without replacement, how many different outcomes are possible?

Answer:

Assuming order of cards doesn't matter based on prompt:

$2^2 * 6^4 * \binom{52}{3} = 114566400$

```
a_3 <- 2^2 * 6^4 * choose(52, 3)
a_3
```

```
## [1] 114566400
```

Problem 4

3 cards are drawn from a standard deck without replacement. What is the probability that at least one of the cards drawn is a heart? Express your answer as a fraction or a decimal number rounded to four decimal places.

Answer:

Probability of no hearts: $39/52 * 38/51 * 37/50 = 0.4135294$

Probability of at least one heart: $1 - 0.4135294 = 0.5864706 \ \ldots \ 0.5865$

Problem 5

Leanne is picking out some movies to rent, and she is primarily interested in children's movies and comedies. She has narrowed down her selections to 18 children's movies and 7 comedies.

Step 1

How many different combinations of 3 movies can she rent?

Answer:

$\binom{25}{3} = 2300$

```
a_5.1 <- choose(25, 3)
a_5.1
```

```
## [1] 2300
```

Step 2

How many different combinations of 3 movies can she rent if she wants at least one comedy?

Answer:

$\binom{7}{1} * \binom{18}{2} + \binom{7}{2} * \binom{18}{1} + \binom{7}{3} * \binom{18}{0} = 1484$

```
# Calculate Comedy >= 1
a_5.2 <- choose(7, 1) * choose(18,2) +
         choose(7, 2) * choose(18,1) +
         choose(7, 3) * choose(18,0)

# Calculate total
# already have a_5.1

# Calculate Comedy = 0
rem <- choose(7, 0) * choose(18,3)

a_5.2
```

```
## [1] 1484
```

```
rem
```

```
## [1] 816
```

```
(a_5.1-rem)
```

```
## [1] 1484
```

Problem 6

DJ Jacqueline is making a playlist for an internet radio show; she is trying to decide what 6 songs to play and in what order they should be played. If she has her choices narrowed down to 7 hip-hop, 14 pop, and 22 blues songs, and she wants to play an equal number of hip-hop, pop, and blues songs, how many different playlists are possible? Express your answer in scientific notation rounding to the hundredths place.

Answer:

${}^{7}P_2 * {}^{14}P_2 * {}^{22}P_2 = 3.53 \times 10^6$

```
perm = function(n, x) {
  factorial(n) / factorial(n-x)
}

a_6 <- perm(7,2) * perm(14,2) * perm(22,2)
a_6
```

```
## [1] 3531528
```

Problem 7

DJ Howard is making a playlist for a friend; he is trying to decide what 9 songs to play and in what order they should be played.

Step 1

If he has his choices narrowed down to 7 pop, 3 hip-hop, 6 country, and 7 blues songs, and he wants to play no more than 3 country songs, how many different playlists are possible? Express your answer in scientific notation rounding to the hundredths place.

Answer:

$({}^{6}P_3 * {}^{17}P_6) + ({}^{6}P_2 * {}^{17}P_7) + ({}^{6}P_1 * {}^{17}P_8) + ({}^{6}P_0 * {}^{17}P_9) = 1.87 \times 10^{10}$

```
# Country <= 3, total 9
a_7.1 <- perm(6,3) * perm(17,6) +
         perm(6,2) * perm(17,7) +
         perm(6,1) * perm(17,8) +
         perm(6,0) * perm(17,9)
a_7.1
```

```
## [1] 18712512000
```

Step 2

If he has his choices narrowed down to 7 pop, 3 hip-hop, 6 country, and 7 blues songs, and he wants to play all 7 blues songs, how many different playlists are possible? Express your answer in scientific notation rounding to the hundredths place.

Answer:

$({}^{7}P_7 * {}^{16}P_2) = 1.21 \times 10^{6}$

```
# Blues = 7, total 9
a_7.2 <- perm(7,7) * perm(16,2)
a_7.2
```

```
## [1] 1209600
```

Problem 8

Mallory is planting trees along her driveway, and she has 3 beech trees and 3 eucalyptus trees to plant in one row. What is the probability that she randomly plants the trees so that all 3 beech trees are next to each other and all 3 eucalyptus trees are next to each other? Express your answer as a fraction or a decimal number rounded to four decimal places.

Answer:

Permutations of 6 entities

${}^6P_6 = 720$

Only 2 outcomes match the criteria: BBBEEE or EEEBBB

$\frac{2}{720} = \frac{1}{360} = 0.0028$

Problem 9

If you draw a queen or lower from a standard deck of cards, I will pay you $4. If not, you pay me $14. (Aces are considered the highest card in the deck.)

Step 1

Find the expected value of the proposition. Round your answer to two decimal places. Losses must be expressed as negative values.

Answer:

P(W) is probability of win

A(W) is value of win

$E(X) = P(W) * A(W) + P(W') * A(W') = \frac{11}{13} * 4 + \frac{2}{13} * -14 = 1.230769$

Answer: $1.23

```
(-14 * (2/13)) + (4 * (11/13))
```

```
## [1] 1.230769
```

Step 2

If you played this game 759 times how much would you expect to win or lose? Round your answer to two decimal places. Losses must be expressed as negative values.

Answer:

Calculation on rounded expected value from Step 1

$E(X) \times Events = 1.23 * 759 = 933.57$

Answer: $933.57 (winnings)

```
1.23 * 759
```

```
## [1] 933.57
```