DATA 605: Assignment 10

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Problem: Gambler's Ruin

Smith is in jail and has 1 dollar; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If Smith bets A dollars, he wins A dollars with probability .4 and loses A dollars with probability .6.

Find the probability that he wins 8 dollars before losing all of his money if

Part A: Timid strategy

He bets 1 dollar each time.

Answer

Using the formula on page 489, Section 12.2

$q_z = \frac{(q/p)^z - 1}{(q/p)^M - 1}$

Starting with 1 dollar, $z = 1$. q_z is the probability the gambler's stake reaches M without ever having reached 0. p is the probability of a win, and q is the probability of a loss.

```
p <- .4
q <- 1 - p
z <- 1
M <- 8

result_a <- ((q/p)^z - 1) / ((q/p)^M - 1)

result_a
```

```
## [1] 0.02030135
```

Probability of winning 8 dollars is 0.0203013.

Part B: Bold strategy

He bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars.

Answer

This scenario requires Smith to win the bet 3 times in a row without losing. 1 doubled is 2, 2 doubled is 4, and 4 doubled is 8. That's the only scenario in which he reaches 8 dollars. Otherwise, he loses. So the probability of reaching 8 is .4 to the third power.

```
result_b <- p^3

result_b
```

```
## [1] 0.064
```

Probability of winning 8 dollars is 0.064.

Using a general matrix object for the Markov transition matrix.

```
mat <- matrix(c(.6, 0, .4, 0, 0, .6, 0, 0, .4, 0, .6, 0, 0, 0, .4), nrow=3, byrow=TRUE)
stateNames <- c("0", "1", "2", "4", "8")
row.names(mat) <- c("1", "2", "4")
colnames(mat) <- stateNames

mat
```

```
##     0 1   2   4   8
## 1 0.6 0 0.4 0.0 0.0
## 2 0.6 0 0.0 0.4 0.0
## 4 0.6 0 0.0 0.0 0.4
```

After first step, probability of going from 1 to 2, 0.4.

```
mat2 <- mat^2
mat2
```

```
##      0 1    2    4    8
## 1 0.36 0 0.16 0.00 0.00
## 2 0.36 0 0.00 0.16 0.00
## 4 0.36 0 0.00 0.00 0.16
```

After second step, probability of going from 2 to 4, 0.16.

```
mat3 <- mat^3
mat3
```

```
##       0 1     2     4     8
## 1 0.216 0 0.064 0.000 0.000
## 2 0.216 0 0.000 0.064 0.000
## 4 0.216 0 0.000 0.000 0.064
```

After third step, probability of going from 4 to 8, 0.064.

Confirmation: Probability of winning 8 dollars is 0.064.

Part C: Better strategy

Which strategy gives Smith the better chance of getting out of jail?

Answer

The **bold strategy** gives Smith a better chance of getting out of jail based on the betting arrangement. The bold strategy gives Smith a probability of 0.064, over 6% chance, while the timid strategy gives Smith a probability of 0.0203013, a hair over 2% chance.