DATA 605 Assignment Week 13

CUNY Spring 2021

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Question 1

Use integration by substitution to solve the integral below.

$$\int 4e^{-7x}dx$$

Answer

$$u = -7x$$
$$\frac{du}{dx} = -7$$
$$du = -7dx$$
$$dx = \frac{du}{-7}$$
$$\int 4e^{u}\frac{du}{-7}$$
$$-\frac{4}{7}\int e^{u}du$$
$$= -\frac{4}{7} \times e^{u}$$
$$Substitute\ u = -7x$$
$$= -\frac{4}{7}e^{-7x} + C$$

Question 2

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

Answer

$$\frac{dN}{dt} = -\frac{3150}{t^4} - 220$$
$$\frac{dN}{dt} = -3150t^{-4} - 220$$

$$dN = (-3150t^{-4} - 220)dt$$
$$N = \int (-3150t^{-4} - 220)dt$$
$$N = \int -3150t^{-4} - \int 220dt$$
$$N(t) = \frac{1050}{t^3} - 220t + C$$

Now, given that the level of contamination after 1 day was 6530 bacteria per cubic centimeter:

$$N(1) = 6530$$

$$N(1) = 6530 = \frac{1050}{1} - 220 + C$$
$$C = 5700 \ for \ N(1)$$

Substitute C back into $N(t)$

$$N(t) = \frac{1050}{t^3} - 220t + 5700$$

Question 3

Find the total area of the red rectangles in the figure, where the equation of the line is $f(x) = 2x - 9$.

Answer

Based on the image, the leftmost rectangle starts at 4.5 and the rightmost rectangle ends at 8.5

$$\int_{4.5}^{8.5} 2x - 9dx$$
$$= (x^2 - 9x)\Big|_{4.5}^{8.5}$$
$$= (8.5^2 - 9(8.5)) - (4.5^2 - 9(4.5))$$
$$Area = 16$$

Question 4

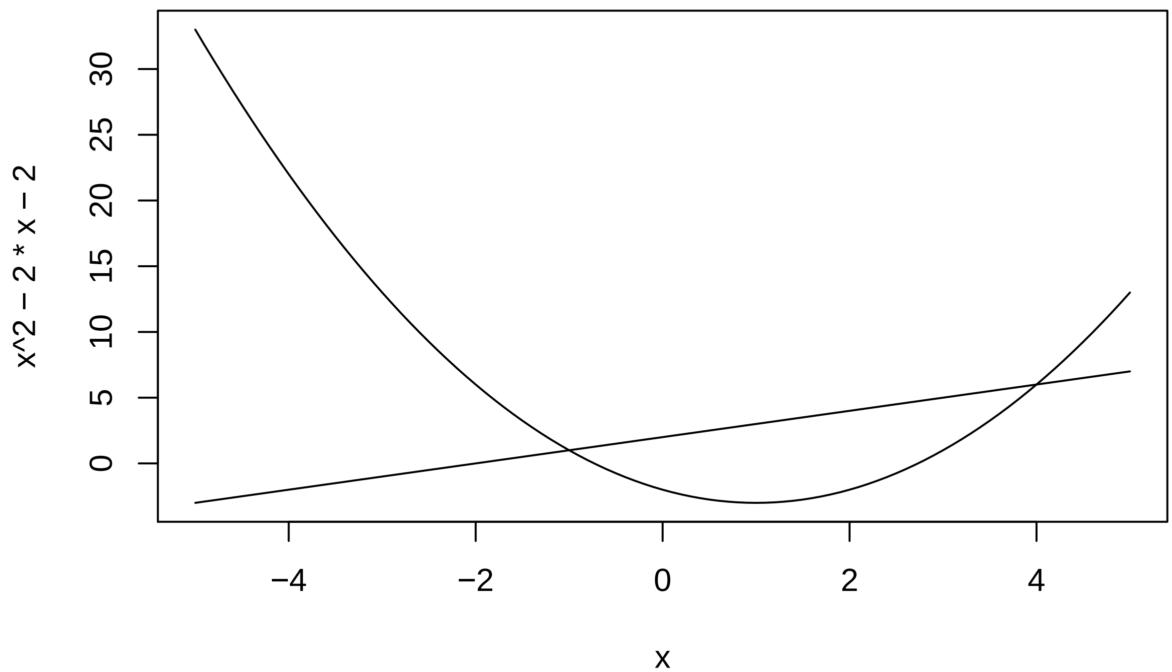
Find the area of the region bounded by the graphs of the given equations.

$$y = x^2 - 2x - 2, \ y = x + 2$$

Answer

First, let's plot the two lines for visual inspection.

```r
curve(x**2 - 2*x - 2, -5, 5)
curve(x+2, -5, 5, add=T)
```



Let's solve for the x-intersection points

$$x^2 - 2x - 2 = x + 2$$
$$x^2 - 3x - 4 = 0$$
$$(x - 4)(x + 1) = 0$$

x-intersection points are $x = 4$ and $x = -1$, which appears to match the plot above.

Now, to solve for the area.

$$A = \int_{-1}^{4} x + 2dx - \int_{-1}^{4} x^2 - 2x - 2dx$$

$$A = [\frac{1}{2}x^2 + 2x]\Big|_{-1}^{4} - [\frac{1}{3}x^3 - x^2 - 2x]\Big|_{-1}^{4}$$
$$A = [-\frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x]\Big|_{-1}^{4}$$

```r
x <- 4
y <- -1
result <- (-1/3 * x**3 + 3/2 * x**2 + 4 * x) - (-1/3 * y**3 + 3/2 * y**2 + 4 * y)

result
```

```
## [1] 20.83333
```

Answer: Area is 20.8333333.

Question 5

A beauty supply store expects to sell 110 flat irons during the next year. It costs $3.75 to store one flat iron for one year. There is a fixed cost of $8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

Answer

- c: cost
- n: number of orders per year
- s: lot size (count per order)

$$n \cdot s = 110$$
$$s = \frac{110}{n}$$

Assume half of an order is in storage on average.

$$c = 8.25n + 3.75 \cdot \frac{110/n}{2}$$
$$c = 8.25n + \frac{206.25}{n}$$

To minimize costs, set the derivative to zero.

$$c' = 8.25 - \frac{206.25}{n^2}$$
$$c' = 0$$
$$0 = 8.25 - \frac{206.25}{n^2}$$
$$\frac{206.25}{n^2} = 8.25$$
$$206.25 = 8.25n^2$$
$$n = \sqrt{\frac{206.25}{8.25}}$$

```r
a <- 206.25 / 8.25
result <- sqrt(a)

result
```

```
## [1] 5
```

Answer: n = 5, which means number of orders per year is 5. Given, 110 flat irons to sell in the year, then each order lot size should contain 22 flat irons (s = 22).

```r
n <- 5
cost <- (8.25*n) + (206.25/n)

cost
```

```
## [1] 82.5
```

The minimal inventory cost is $82.5.

Question 6

Use integration by parts to solve the integral below.

$$\int \ln(9x) \cdot x^6 dx$$

Answer

Start with formula:

$$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)$$

By parts

$$f(x) = \ln(9x)$$
$$f'(x) = \frac{1}{x}dx$$
$$g'(x) = x^6 dx$$
$$g(x) = \frac{x^7}{7}$$

Now substitute into the formula

$$\ln(9x)\frac{x^7}{7} - \int \frac{x^7}{7}\frac{1}{x}dx$$

$$\ln(9x)\frac{x^7}{7} - \frac{x^7}{49} + C$$

Question 7

Determine whether $f(x)$ is a probability density function on the interval $[1, e^6]$. If not, determine the value of the definite integral.

$$f(x) = \frac{1}{6x}$$

Answer

A function is a probability density function if the function integrates over the domain of the variable resulting in 1.

$$F(x) = \int_1^{e^6} f(x)dx = 1$$

$$F(x) = \int_1^{e^6} \frac{1}{6x}dx$$

$$F(x) = \frac{1}{6}\int_1^{e^6} \frac{1}{x}dx$$

$$F(x) = \frac{1}{6}\ln(x)\Big|_1^{e^6}$$

$$F(x) = \frac{1}{6}[\ln(e^6) - \ln(1)]$$

$$F(x) = \frac{1}{6}[6 - 0] = 1$$

Thus, the function $f(x)$ is a probability density function on the interval $[1, e^6]$.