DATA 605 Assignment Week 15

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Philip Tanofsky

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Exercise 1

Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary.

(5.6,8.8), (6.3,12.4), (7,14.8), (7.7,18.2), (8.4,20.8)

Answer

```
x <- c(5.6, 6.3, 7, 7.7, 8.4)
y <- c(8.8, 12.4, 14.8, 18.2, 20.8)

model <- lm(y~x)

model
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Coefficients:
## (Intercept)            x
##     -14.800        4.257
```

$$y = -14.80 + 4.28x$$

Exercise 2

Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma.

$$f(x, y) = 24x - 6xy^2 - 8y^3$$

Answer

First partial derivatives

$$f_x(x, y) = 24 - 6y^2$$
$$f_y(x, y) = -12xy - 24y^2$$

If $24 - 6y^2 = 0$, then $y^2 = 4$ and thus $y \pm 2$.

If $y = 2$ and $-12xy - 24y^2 = 0$, then $-24x = 24(2^2)$ and thus $x = -4$.

If $y = -2$ and $-12xy - 24y^2 = 0$, then $24x = 24(2^2)$ and thus $x = 4$.

Now, calculate $f(x, y)$.

$$\begin{aligned} f(4, -2) &= 24(4) - 6(4)(-2^2) - 8(-2^3) \\ &= 96 - 96 + 64 \\ f(4, -2) &= 64 \\ f(-4, 2) &= 24(-4) - 6(-4)(2^2) - 8(2^3) \\ &= -96 + 96 - 64 \\ f(-4, 2) &= -64 \end{aligned}$$

Two Critical Points $(4, -2, 64)$ and $(-4, 2, -64)$

Now, use the Second Derivative Test to determine if the points are local maxima, local minima, or saddle points.

Second Derivatives

$f_{xx} = 0$ and $f_{yy} = -12x - 48y$ and $f_{xy} = -12y$

Consider

$$\begin{aligned} D(x, y) &= f_{xx} f_{yy} - f_{xy}^2 \\ &= (0)(-12x - 48y) - (-12y)^2 \\ &= -144y^2 \end{aligned}$$

$D(x, y) < 0$ for all (x, y), thus any critical point is a saddle points.

Therefore both critical points $(4, -2)$ and $(-4, 2)$ are saddle points.

Exercise 3

A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

Step 1

Find the revenue function $R(x, y)$.

Answer

House Brand

$$R(x) = (81 - 21x + 17y) \times x$$

Name Brand

$$R(y) = (40 + 11x - 23y) \times y$$

Combine $R(x)$ and $R(y)$ to create revenue function $R(x, y)$

$$\begin{aligned} R(x, y) &= R(x) + R(y) \\ &= (81 - 21x + 17y)x + (40 + 11x - 23y)y \\ &= 81x - 21x^2 + 17yx + 40y + 11xy - 23y^2 \\ R(x, y) &= 28xy - 21x^2 - 23y^2 + 81x + 40y \end{aligned}$$

Step 2

What is the revenue if she sells the "house" brand for \$2.30 and the "name" brand for \$4.10?

Answer

```
#house price
house <- 2.3
#name price
name <- 4.1

revenue <- function(x, y) {
  return ((28 * x * y) - (21 * x**2) - (23 * y**2) + (81 * x) + (40 * y))
}

revenue(house, name)
```

```
## [1] 116.62
```

Total revenue: \$116.62.

Exercise 4

A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

$$C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$$

Answer

From the prompt, $x + y = 96$, thus $x = 96 - y$.

$$\begin{aligned}
C(x, y) &= C(96 - y, y) \\
&= \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700 \\
&= \frac{1}{6}(96 - y)^2 + \frac{1}{6}y^2 + 7(96 - y) + 25y + 700 \\
&= \frac{1}{6}(y^2 - 192y + 9216) + \frac{1}{6}y^2 + 672 - 7y + 25y + 700 \\
&= \frac{1}{6}y^2 - 32y + 1536 + \frac{1}{6}y^2 + 18y + 1372 \\
C(y) &= \frac{1}{3}y^2 - 14y + 2908
\end{aligned}$$

Now, to find the minimal value, let's take the derivative at set that to zero:

$$\begin{aligned}
C'(y) &= \frac{2}{3}y - 14 \\
0 &= \frac{2}{3}y - 14 \\
0 &= \frac{2}{3}y - 14 \\
21 &= y
\end{aligned}$$

Denver should produce 21 units. Thus LA should produce (x = 96 - y ... x = 96 - 21) 75 units.

Exercise 5

Evaluate the double integral on the given region.

$$\iint (e^{8x+3y})\ dA; R : 2 \leq x \leq 4 \ and\ 2 \leq y \leq 4$$

Write your answer in exact form without decimals.

Answer

$$\begin{aligned}
&\int_2^4 \int_2^4 (e^{8x+3y})\ dy\ dx \\
&= \int_2^4 \left(\left[\frac{1}{3}e^{8x+3y} \right] \Big|_2^4 \right) dx \\
&= \int_2^4 \left(\left[\frac{1}{3}e^{8x+12} \right] - \left[\frac{1}{3}e^{8x+6} \right] \right) dx \\
&= \int_2^4 \left(\frac{1}{3}e^{8x+6} \left[e^6 - 1 \right] \right) dx
\end{aligned}$$

$$= \left(\frac{1}{24}e^{8x+6}\left[e^{6}-1\right]\right)\Bigg|_{2}^{4}$$

$$= \frac{1}{24}e^{38}(e^{6}-1) - \frac{1}{24}e^{22}(e^{6}-1)$$

$$= \frac{1}{24}(e^{38}-e^{22)}(e^{6}-1)$$

$$= \frac{1}{24}(e^{44}-e^{38}-e^{28}+e^{22})$$