# DATA 622 Assignment 1

CUNY: Spring 2021

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```
library(palmerpenguins)
library(dplyr)
library(ggplot2)
library(tidyr)
library(caret)
library(MASS)
library(pROC)
library(nnet) # Used for multinomial logistic regression
library(stargazer)
theme_set(theme_minimal())
ds <- penguins
head(ds)</pre>
```

```
## # A tibble: 6 x 8
     species island bill_length_mm bill_depth_mm flipper_length_~ body_mass_g sex
             <fct>
                             <dbl>
                                            <dbl>
##
     <fct>
                                                             <int>
                                                                         <int> <fct>
## 1 Adelie Torge~
                              39.1
                                             18.7
                                                                          3750 male
                                                               181
## 2 Adelie Torge~
                              39.5
                                             17.4
                                                               186
                                                                          3800 fema~
                              40.3
                                             18
## 3 Adelie Torge~
                                                               195
                                                                          3250 fema~
## 4 Adelie Torge~
                                                                            NA <NA>
                              NA
                                             NA
                                                                NA
## 5 Adelie Torge~
                              36.7
                                             19.3
                                                               193
                                                                          3450 fema~
## 6 Adelie Torge~
                                                                          3650 male
                              39.3
                                             20.6
                                                               190
## # ... with 1 more variable: year <int>
```

#### summary(ds)

```
##
         species
                          island
                                    bill_length_mm bill_depth_mm
   Adelie
             :152
                    Biscoe
                             :168
                                    Min.
                                           :32.10
                                                    Min.
                                                          :13.10
   Chinstrap: 68
                             :124
                                    1st Qu.:39.23
                                                    1st Qu.:15.60
                    Dream
##
   Gentoo
           :124
                   Torgersen: 52
                                    Median :44.45
                                                    Median :17.30
##
                                    Mean
                                          :43.92
                                                    Mean
                                                           :17.15
##
                                    3rd Qu.:48.50
                                                    3rd Qu.:18.70
##
                                           :59.60
                                                           :21.50
                                    Max.
                                                    Max.
##
                                    NA's
                                           :2
                                                    NA's
                                                           :2
## flipper_length_mm body_mass_g
                                         sex
                                                       year
## Min. :172.0
                    Min. :2700
                                     female:165
                                                         :2007
                                                  Min.
  1st Qu.:190.0
                                                  1st Qu.:2007
                     1st Qu.:3550
                                     male :168
```

```
Median :197.0
                      Median:4050
                                       NA's : 11
                                                    Median:2008
##
    Mean
           :200.9
                      Mean
                              :4202
                                                    Mean
                                                            :2008
                                                    3rd Qu.:2009
    3rd Qu.:213.0
                       3rd Qu.:4750
           :231.0
                              :6300
                                                            :2009
##
   Max.
                      Max.
                                                    Max.
                              :2
```

NA's NA's

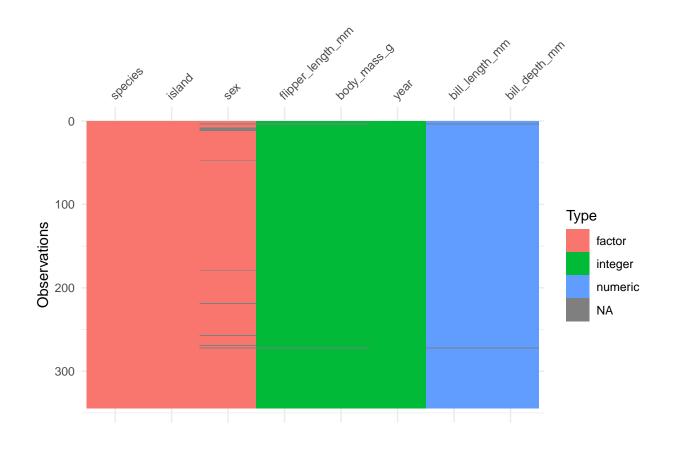
#### dim(ds)

## [1] 344 8

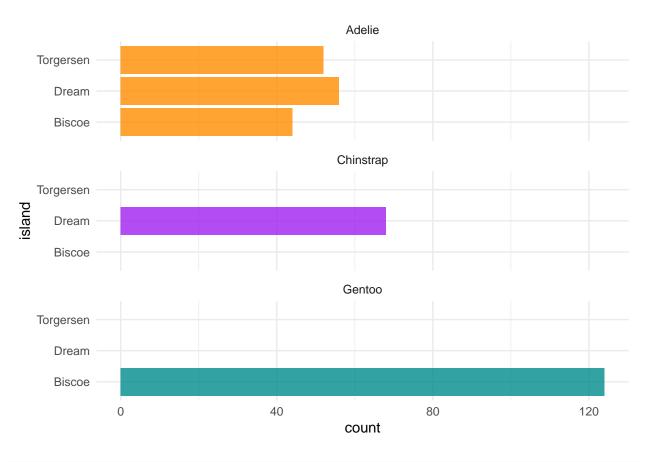
#### glimpse(ds)

```
## Rows: 344
## Columns: 8
## $ species
                       <fct> Adelie, Adelie, Adelie, Adelie, Adelie, Adelie, A...
## $ island
                       <fct> Torgersen, Torgersen, Torgersen, Torgesen, Torge...
## $ bill_length_mm
                       <dbl> 39.1, 39.5, 40.3, NA, 36.7, 39.3, 38.9, 39.2, 34....
                       <dbl> 18.7, 17.4, 18.0, NA, 19.3, 20.6, 17.8, 19.6, 18....
## $ bill_depth_mm
## $ flipper_length_mm <int> 181, 186, 195, NA, 193, 190, 181, 195, 193, 190, ...
## $ body_mass_g
                       <int> 3750, 3800, 3250, NA, 3450, 3650, 3625, 4675, 347...
                       <fct> male, female, female, NA, female, male, female, m...
## $ sex
                       <int> 2007, 2007, 2007, 2007, 2007, 2007, 2007, 2007, 2...
## $ year
```

#### visdat::vis\_dat(ds)

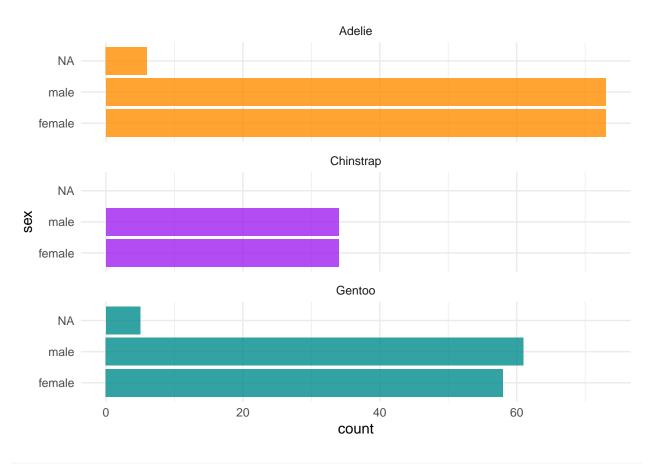


```
# Penguins data has three factor variables
ds %>%
          dplyr::select(where(is.factor)) %>%
         glimpse()
## Rows: 344
## Columns: 3
## $ species <fct> Adelie, Ade
## $ island <fct> Torgersen, Torg
## $ sex
                                                                  <fct> male, female, female, NA, female, male, female, male, NA, N...
# Count penguins for each species / island
ds %>%
 count(species, island, .drop=F)
## # A tibble: 9 x 3
                        species island
                                                                           <fct>
##
                          <fct>
                                                                                                                                  <int>
## 1 Adelie Biscoe
                                                                                                                                                44
## 2 Adelie Dream
                                                                                                                                                56
## 3 Adelie Torgersen
                                                                                                                                               52
## 4 Chinstrap Biscoe
                                                                                                                                              0
## 5 Chinstrap Dream
                                                                                                                                                 68
## 6 Chinstrap Torgersen
                                                                                                                                            0
## 7 Gentoo
                                                                     Biscoe
                                                                                                                                           124
## 8 Gentoo
                                                                             Dream
                                                                                                                                                      0
## 9 Gentoo Torgersen
                                                                                                                                                      0
ggplot(ds, aes(x = island, fill = species)) +
          geom_bar(alpha = 0.8) +
          scale_fill_manual(values = c("darkorange", "purple", "cyan4"),
                                                                                                        guide = F) +
          theme_minimal() +
          facet_wrap(~species, ncol = 1) +
          coord_flip()
```



```
# Count penguins for each species / sex
ds %>%
count(species, sex, .drop = F)
```

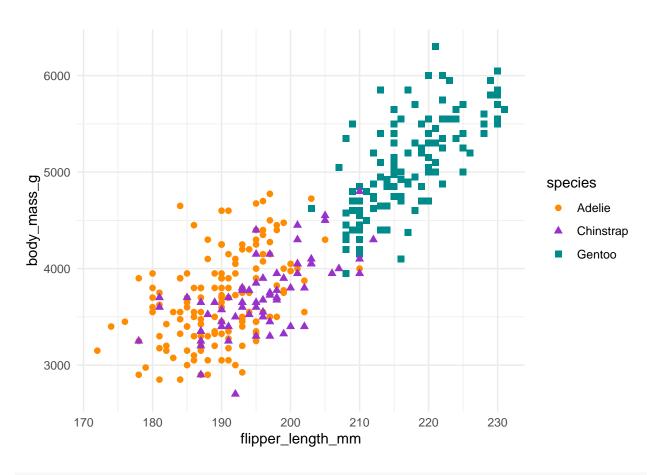
```
## # A tibble: 8 x 3
##
     species sex
                          n
##
     <fct>
              <fct> <int>
## 1 Adelie
              female
                         73
## 2 Adelie
              male
                         73
## 3 Adelie
              <NA>
                         6
## 4 Chinstrap female
                         34
## 5 Chinstrap male
                         34
## 6 Gentoo
              female
                         58
## 7 Gentoo
              male
                         61
## 8 Gentoo
               <NA>
                         5
```



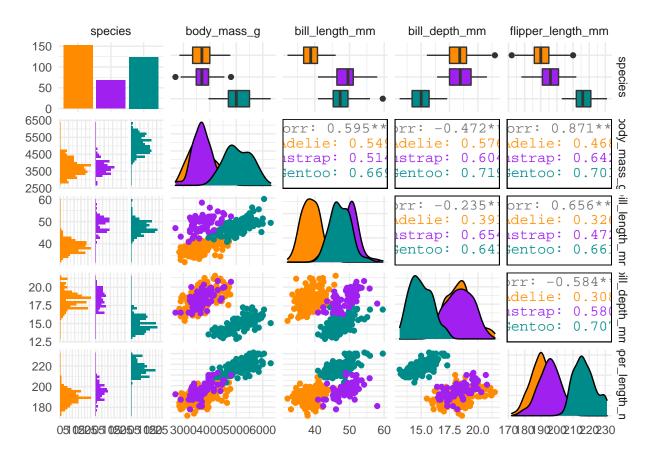
```
# Penguins data also has four continuous variables, making six unique scatterplots
ds %>%
    dplyr::select(body_mass_g, ends_with("_mm")) %>%
    glimpse()
```

scale\_color\_manual(values = c("darkorange", "darkorchid", "cyan4"))

size = 2) +



```
ds %>%
    dplyr::select(species, body_mass_g, ends_with("_mm")) %>%
    GGally::ggpairs(aes(color = species)) +
    scale_color_manual(values = c("darkorange","purple","cyan4")) +
    scale_fill_manual(values = c("darkorange","purple","cyan4"))
```



```
# Create dataset for binary logistic regression: species Gentoo or Not
data_binary <- penguins
# Only use complete instances ... actually come back to this as I don't want to exclude because of sex
train_data_binary <- na.omit(data_binary)
dim(train_data_binary)</pre>
```

## [1] 333 8

Based on the result, 11 rows are removed, which would equal the number of NAs in variable sex

## Binary Logistic Regression

```
# Create new column
train_data_binary$gentoo <- ifelse(train_data_binary$species=="Gentoo", 1, 0)
summary(train_data_binary)
##
         species
                          island
                                    bill_length_mm bill_depth_mm
             :146
                             :163
                                            :32.10
   Adelie
                    Biscoe
                                    Min.
                                                     Min.
                                                            :13.10
   Chinstrap: 68
                    Dream
                             :123
                                     1st Qu.:39.50
                                                     1st Qu.:15.60
```

```
:119
                    Torgersen: 47
                                    Median :44.50
                                                    Median :17.30
##
                                    Mean
                                          :43.99
                                                    Mean
                                                          :17.16
                                    3rd Qu.:48.60
##
                                                    3rd Qu.:18.70
##
                                    Max.
                                           :59.60
                                                            :21.50
                                                    Max.
##
  flipper_length_mm body_mass_g
                                         sex
                                                        year
                                                                      gentoo
          :172
## Min.
                      Min.
                            :2700
                                     female:165
                                                          :2007
                                                                         :0.0000
                                                  Min.
                                                                  \mathtt{Min}.
                                                  1st Qu.:2007
## 1st Qu.:190
                      1st Qu.:3550
                                                                  1st Qu.:0.0000
                                     male :168
                      Median:4050
                                                                  Median :0.0000
## Median :197
                                                  Median :2008
## Mean
         :201
                      Mean
                             :4207
                                                  Mean
                                                          :2008
                                                                  Mean
                                                                         :0.3574
## 3rd Qu.:213
                      3rd Qu.:4775
                                                  3rd Qu.:2009
                                                                  3rd Qu.:1.0000
                                                          :2009
## Max.
           :231
                      Max.
                             :6300
                                                  Max.
                                                                  Max.
                                                                         :1.0000
# Drop species column, as now just using gentoo column as Y variable
drops <- c("species")</pre>
train_data_binary <- train_data_binary[ , !(names(train_data_binary) %in% drops)]</pre>
summary(train_data_binary)
##
          island
                    bill_length_mm bill_depth_mm
                                                    flipper_length_mm
##
  Biscoe
             :163
                    Min.
                           :32.10
                                    Min. :13.10
                                                    Min.
                                                           :172
  {\tt Dream}
             :123
                    1st Qu.:39.50
                                    1st Qu.:15.60
                                                    1st Qu.:190
   Torgersen: 47
                    Median :44.50
                                    Median :17.30
                                                    Median:197
##
                    Mean :43.99
                                    Mean
                                          :17.16
                                                    Mean :201
##
                    3rd Qu.:48.60
                                    3rd Qu.:18.70
                                                    3rd Qu.:213
##
                           :59.60
                                    Max.
                                           :21.50
                                                    Max.
                                                           :231
##
    body_mass_g
                       sex
                                     year
                                                    gentoo
## Min.
          :2700
                   female:165
                                Min.
                                       :2007
                                               Min.
                                                     :0.0000
##
  1st Qu.:3550
                   male :168
                                1st Qu.:2007
                                               1st Qu.:0.0000
## Median :4050
                                Median:2008
                                               Median :0.0000
## Mean
         :4207
                                Mean
                                       :2008
                                               Mean
                                                       :0.3574
##
   3rd Qu.:4775
                                3rd Qu.:2009
                                               3rd Qu.:1.0000
## Max.
           :6300
                                Max.
                                       :2009
                                               Max.
                                                       :1.0000
set.seed(123)
trainIndex <-createDataPartition(train_data_binary$gentoo, p = 0.7, list = FALSE, times = 1)
train <- train_data_binary[trainIndex,]</pre>
test <- train_data_binary[-trainIndex,]</pre>
model1 <- glm(gentoo ~ ., data = train, family = "binomial"(link="logit")) %% stepAIC(trace=F, directi
summary(model1)
##
## Call:
## glm(formula = gentoo ~ bill_depth_mm + flipper_length_mm, family = binomial(link = "logit"),
       data = train)
##
## Deviance Residuals:
                               Median
                                                3Q
                                                           Max
          Min
                       1Q
## -6.826e-05 -2.100e-08 -2.100e-08
                                        2.100e-08
                                                    6.510e-05
##
## Coefficients:
```

```
##
               Estimate Std. Error z value Pr(>|z|)
## (Intercept)
              -428.130 474946.766 -0.001
                                   0.999
               -14.834 12021.243 -0.001
## bill depth mm
                                   0.999
                      1957.819
                             0.002
                                   0.999
## flipper_length_mm
                3.274
## (Dispersion parameter for binomial family taken to be 1)
    Null deviance: 3.0884e+02 on 233 degrees of freedom
##
## Residual deviance: 9.5967e-09 on 231 degrees of freedom
## AIC: 6
##
## Number of Fisher Scoring iterations: 25
#mu<-predict(model1, type = "response")</pre>
# calculate AIC
mod1AIC <- model1$aic</pre>
## use the test data set to make predicts and calculate metrics from the confusion matrix
mod1.predict.probs <- predict.glm(model1, type="response", newdata=test)</pre>
mod1.predict.manual <- ifelse(mod1.predict.probs > 0.5, '1','0')
attach(test)
mod1.predict.manual
             5
                6
                  7
                     8
                        9 10 11 12 13 14 15 16 17 18 19
21 22 23 24 25 26
                 27
                    28
                      29
                         30
                            31
                              32 33
                                   34
                                      35
                                         36
                                            37
                                              38
                                                 39
## 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59
61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79
## 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99
test$gentoo
# now can use the caret function
cm.var <- caret::confusionMatrix(factor(mod1.predict.manual), factor(test$gentoo), positive='1')</pre>
cm.var$table
        Reference
## Prediction 0 1
##
       0 67 0
       1 0 32
##
```

```
# print metrics
mod1.CMmetrics <- c(cm.var$overall[c(1)], cm.var$byClass[c(1,2,5,6,7)])

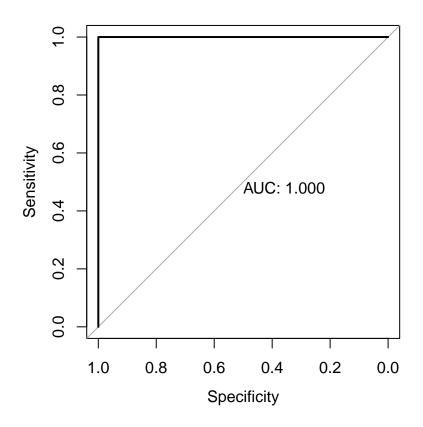
mod1.CMmetrics

## Accuracy Sensitivity Specificity Precision Recall F1
## 1 1 1 1 1 1 1

# ROC and AUC
par(pty="s")
roc.stepwise <- roc(train$gentoo, model1$fitted.values, plot=TRUE, print.auc=TRUE)

## Setting levels: control = 0, case = 1

## Setting direction: controls < cases</pre>
```



```
# Dispersion Statistic
E2 <- resid(model1, type = "pearson")
N <- nrow(train)
p <- length(coef(model1)) + 1 # '+1' is due to theta
mod1.dispersion <- dispesion <-sum(E2^2) / (N - p)</pre>
```

### Multinomial Logistic Regression

```
# Initial walk-through: https://stats.idre.ucla.edu/r/dae/multinomial-logistic-regression/
# Start with initial dateset
mlr_data <- penguins
summary(mlr_data)
##
        species
                        island
                                  bill_length_mm bill_depth_mm
##
  Adelie
          :152
                 Biscoe :168
                                  Min. :32.10
                                                 Min. :13.10
                           :124
                                  1st Qu.:39.23
                                                 1st Qu.:15.60
## Chinstrap: 68 Dream
  Gentoo :124 Torgersen: 52
                                  Median :44.45
                                                 Median :17.30
##
                                  Mean
                                       :43.92 Mean :17.15
##
                                  3rd Qu.:48.50 3rd Qu.:18.70
##
                                  Max. :59.60 Max. :21.50
##
                                  NA's :2
                                                 NA's :2
## flipper_length_mm body_mass_g
                                      sex
                                                    year
          :172.0 Min. :2700
                                  female:165 Min. :2007
## Min.
## 1st Qu.:190.0
                 1st Qu.:3550
                                  male :168 1st Qu.:2007
## Median :197.0 Median :4050
                                  NA's : 11 Median :2008
                 Mean :4202
## Mean
         :200.9
                                               Mean :2008
## 3rd Qu.:213.0
                    3rd Qu.:4750
                                               3rd Qu.:2009
## Max. :231.0
                    Max. :6300
                                               Max. :2009
                    NA's
## NA's
          :2
mlr_data$species2 <- relevel(mlr_data$species, ref = "Gentoo")</pre>
test <- multinom(species2 ~ body_mass_g + bill_length_mm + bill_depth_mm + flipper_length_mm + island,
## # weights: 24 (14 variable)
## initial value 375.725403
## iter 10 value 20.138634
## iter 20 value 1.481353
## iter 30 value 0.035234
## iter 40 value 0.000933
## iter 50 value 0.000233
## final value 0.000089
## converged
summary(test)
## multinom(formula = species2 ~ body_mass_g + bill_length_mm +
##
      bill_depth_mm + flipper_length_mm + island, data = mlr_data)
##
## Coefficients:
##
            (Intercept) body_mass_g bill_length_mm bill_depth_mm
## Adelie
               179.0566 -0.01156190
                                       -11.117549
                                                     15.793011
## Chinstrap
             -151.2783 -0.04191271
                                         4.802821
          flipper_length_mm islandDream islandTorgersen
                    0.4109570
                                22.48619
## Adelie
                                               69.34019
```

```
## Chinstrap
           0.6913637 117.34484 16.71717
##
## Std. Errors:
       (Intercept) body_mass_g bill_length_mm bill_depth_mm
## Adelie
           0.736784 0.4581624 49.98112
## Chinstrap 0.732472 0.7914301
                               49.74986
                                           14.71006
        flipper_length_mm islandDream islandTorgersen
                6.554627 0.732472 1.236702e-24
## Adelie
## Chinstrap
                6.545027 0.732472 1.876044e-50
##
## Residual Deviance: 0.0001778793
## AIC: 28.00018
stargazer(test, type="text", out="test.htm")
##
##
                  Dependent variable:
##
                _____
##
                  Adelie
                            Chinstrap
                   (1)
##
                             (2)
## -----
## body_mass_g
                  -0.012
##
                  (0.458)
                            (0.791)
##
## bill_length_mm
                             4.803
                -11.118
                            (49.750)
                  (49.981)
##
                  15.793
                             -5.639
## bill_depth_mm
                  (14.886)
                            (14.710)
##
##
                              0.691
## flipper length mm
                 0.411
##
                  (6.555)
                             (6.545)
##
                22.486*** 117.345***
## islandDream
##
                  (0.732)
                             (0.732)
##
## islandTorgersen
                69.340***
                            16.717***
##
                  (0.000)
                             (0.000)
##
                179.057*** -151.278***
## Constant
##
                  (0.737)
                             (0.732)
##
## Akaike Inf. Crit. 28.000 28.000
*p<0.1; **p<0.05; ***p<0.01
## Note:
test.rrr = exp(coef(test))
test.rrr
```

(Intercept) body\_mass\_g bill\_length\_mm bill\_depth\_mm

## Adelie 5.798378e+77 0.9885047 1.484942e-05 7.224660e+06

```
## Chinstrap 1.998367e-66 0.9589535 1.218537e+02 3.556113e-03
##
            flipper_length_mm islandDream islandTorgersen
## Adelie
                    1.508260 5.829443e+09
                                           1.300353e+30
## Chinstrap
                    1.996436 9.166779e+50
                                            1.820439e+07
stargazer(test, type="text", coef=list(test.rrr), p.auto=FALSE, out="testrrr.htm")
##
##
##
##
##
                                                                      (1)
                                                                     0.989
## body_mass_g
                                                                    (0.458)
##
## bill length mm
                                                                    0.00001
##
                                                                    (49.981)
                                                                 7,224,660.000
## bill_depth_mm
                                                                    (14.886)
##
##
## flipper_length_mm
                                                                     1.508
##
                                                                    (6.555)
##
## islandDream
                                                               5,829,443,146.000***
##
                                                                    (0.732)
##
## islandTorgersen
                                                 1,300,352,545,036,810,191,042,951,249,920.000***
##
                                                                    (0.000)
##
                   579,837,803,916,479,821,887,587,742,166,729,353,269,199,669,971,417,058,436,431,37
## Constant
##
                                                                    (0.737)
##
## Akaike Inf. Crit.
                                                                     28.000
# Again with https://www.r-bloggers.com/2020/05/multinomial-logistic-regression-with-r/
index <- createDataPartition(mlr_data$species, p = .70, list = FALSE)</pre>
train <- mlr_data[index,]</pre>
test <- mlr_data[-index,]</pre>
# Set the reference
train$species <- relevel(train$species, ref = "Adelie")</pre>
# Training the multinomial model
multinom_model <- multinom(species ~ ., data = mlr_data)</pre>
```

## # weights: 36 (22 variable)

```
## initial value 365.837892
## iter 10 value 34.181055
## iter 20 value 0.295760
## iter 30 value 0.004662
## final value 0.000042
## converged
# Checking the model
summary(multinom_model)
## Call:
## multinom(formula = species ~ ., data = mlr_data)
## Coefficients:
             (Intercept) islandDream islandTorgersen bill_length_mm bill_depth_mm
## Chinstrap -0.01000283 -0.5092332
                                            4.737343
                                                          -4.717127
                                                                       -3.9325176
              0.07204990 -20.1161533
                                            7.279638
                                                          -2.880746
## Gentoo
                                                                         0.2195242
##
             flipper_length_mm body_mass_g sexmale
                                                         year species2Adelie
                     -1.515657 0.03478204 6.250623 0.2131091
## Chinstrap
                     -1.860254 0.03139549 6.219207 0.2080230
                                                                  -120.84300
## Gentoo
             species2Chinstrap
                     93.909336
## Chinstrap
## Gentoo
                     -8.168343
##
## Std. Errors:
             (Intercept) islandDream islandTorgersen bill_length_mm bill_depth_mm
## Chinstrap 0.00415839 10.228172539
                                             10.22439
                                                            82.08249
                                                                           4.939571
              0.01810751 0.005181329
## Gentoo
                                             11.95099
                                                           237.58065
                                                                          47.894200
##
             flipper_length_mm body_mass_g
                                                           year species2Adelie
                                              sexmale
## Chinstrap
                      54.22441
                                0.5054492 0.00447937 3.557639
                     204.40986
                                 4.5030675 0.18580467 12.667205
                                                                     11.926995
## Gentoo
             species2Chinstrap
## Chinstrap
                   2.749341047
## Gentoo
                   0.001456202
##
## Residual Deviance: 8.466626e-05
## AIC: 44.00008
# Convert the coefficients to odds by taking the exponential of the coefficients.
exp(coef(multinom_model))
             (Intercept) islandDream islandTorgersen bill_length_mm bill_depth_mm
                0.990047 6.009562e-01
                                                         0.008940826
## Chinstrap
                                             114.1306
                                                                         0.01959428
## Gentoo
                1.074709 1.835125e-09
                                            1450.4627
                                                         0.056092884
                                                                         1.24548403
##
             flipper_length_mm body_mass_g sexmale
                                                        year species2Adelie
## Chinstrap
                     0.2196639
                                  1.035394 518.3357 1.237520
                                                              1.216360e-28
                                  1.031894 502.3045 1.231241
## Gentoo
                     0.1556331
                                                               3.300276e-53
##
             species2Chinstrap
## Chinstrap
                  6.085640e+40
                  2.834874e-04
## Gentoo
```

```
##
     Adelie Chinstrap Gentoo
## 1
          1
                     0
                            0
## 2
          1
                     0
                            0
## 3
          1
                     0
                            0
## 5
          1
                     0
                            0
## 6
                     0
                            0
          1
## 7
                     0
                            0
# Predicting and validating the model
# Predicting the values for train dataset
train$speciesPredicted <- predict(multinom_model, newdata = train, "class")</pre>
# Building classification table
tab <- table(train$species, train$speciesPredicted)</pre>
# Calculating accuracy - sum of diagonal elements divided by total obs
round((sum(diag(tab))/sum(tab))*100,2)
## [1] 100
# Predicting the class for test dataset
test$speciesPredicted <- predict(multinom_model, newdata = test, "class")</pre>
# Building classification table
tab <- table(test$species, test$speciesPredicted)</pre>
tab
##
##
                Adelie Chinstrap Gentoo
##
     Adelie
                    43
                                       0
                                0
##
     Chinstrap
                     0
                               20
                                       0
##
     Gentoo
                     0
                                0
                                      34
```

## Thoughts

For binary, group Adelie and Chinstrap

head(round(fitted(multinom\_model), 2))

Sex doesn't matter, Considering that I'm grouping Orange and Purple, probably don't use bill\_length\_mm, as that one shows purple and green have similar distribution

#### Variables

species: of course (Y variable) ... Gentoo or Not Gentoo (No NA) island: consider it (No NA) bill\_length\_mm: don't use for binary (2 NAs) bill\_depth\_mm: use it (2 NAs) flipper\_length\_mm: use it (2 NAs) body\_mass\_g: use it (2 NAs) sex: nah (11 NAs) year: meh (No NA)

## **Prompt**

Data – 622 Homework # 1 Due date Feb 19, 2021- 11:59 EST Let's use the Penguin dataset for our assignment. To learn more about the dataset, please visit: https://allisonhorst.github.io/palmerpenguins/articles/intro.html For this assignment, let us use 'species' as our outcome or the dependent variable. 1. Logistic Regression with a binary outcome. (40) a. The penguin dataset has 'species' column. Please check how many categories you have in the species column. Conduct whatever data manipulation you need to do to be able to build a logistic regression with binary outcome. Please explain your reasoning behind your decision as you manipulate the outcome/dependent variable (species). b. Please make sure you are evaluating the independent variables appropriately in deciding which ones should be in the model. c. Provide variable interpretations in your model. 2. For your model from #1, please provide: AUC, Accuracy, TPR, FPR, TNR, FNR (20) 3. Multinomial Logistic Regression. (40) a. Please fit it a multinomial logistic regression where your outcome variable is 'species'. b. Please be sure to evaluate the independent variables appropriately to fit your best parsimonious model. c. Please be sure to interpret your variables in the model. 4. Extra credit: what would be some of the fit statistics you would want to evaluate for your model in question #3? Feel free to share whatever you can provide. (10)