Very Basic Linear ALgebra

Lines and Lines

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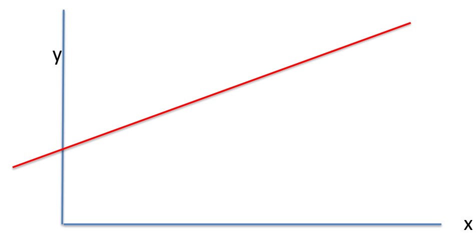
What we MIGHT have heard

- Lines represent how the dependent variable's value changes with change in the independent variable.



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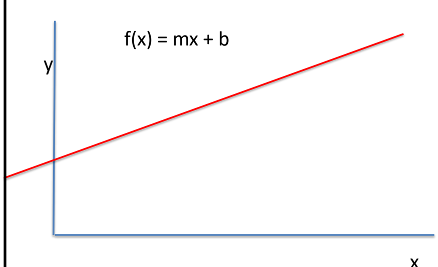
Functions – Dependent and Independent Variables

$y = mx + b$ m is positive here. Say 2

$y = f(x)$ b is also positive. Say 3

$f(x) = mx + b$



Function value on y axis

Slope represents change
In output based on change in input

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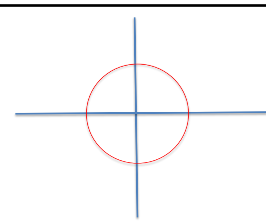

$y^2 = - x^2 + 4$?

i.e., $y^2 + x^2 = 4$

Probably $x_1^2 + x_2^2 = 4$

Circles are not defined by linear functions but quadratic

$f(x_{1,} x_2) = x_1^2 + x_2^2$

What is the circle that is centered around origin with radius 2

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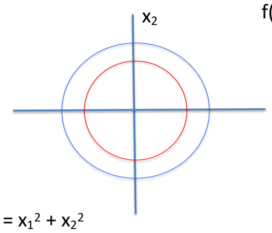
Constraints and Surfaces

$f(x_1, x_2) = c$



$f(x_1, x_2) = x_1^2 + x_2^2$

$x_1^2 + x_2^2 = 4$

$x_1^2 + x_2^2 = 9$

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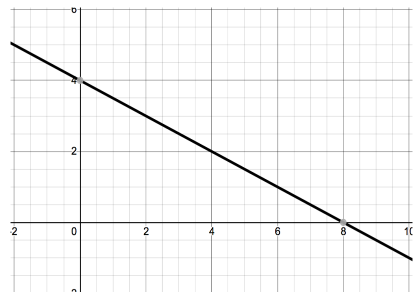
Pictorial Representation-- constraints



0,4
2,3
4,2
6,3
8,0

$x_1 + 2x_2 = 8$

Or

$x_1 + 2x_2 - 8 = 0$

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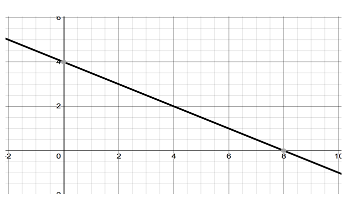
Gradient?



$x_1 + 2x_2 - 8 = 0$

$<df/dx_1, df/dx_2>$

$= <1,2>$

What does <1,2> represent.

Consider x_2 as dependent variable. Then $x_2 = -(1/2)\, x_1 + 8/2$

Slope is –(1/2). Slope of perpendicular line is +(2/1)

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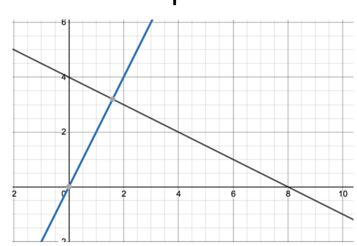
Perpendicular Line?



Perpendicular slope = (2/1) means for unit change in x1, x2 changes by 2.

Perpendicular from origin will pass through <1,2> and also <2,4>

$x_1+2x_2-8 = 0$ is the same constraint as $2x_1+4x_2-16 = 0$

Parallel lines – same slope. $x_1+2x = 16$?

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Switching to Vectors

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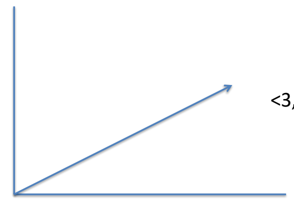
Vectors



$\langle 3,4\rangle = \mathbf{w}$

Length of vector $||\mathbf{w}|| = \sqrt{(3^2+4^2)} = 5$

$\mathbf{w'} = \langle 0.6, 0.8\rangle$ is unit vector as $||\mathbf{w'}|| = 1$

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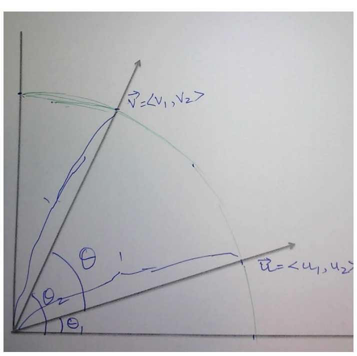
Unit Vectors



$u_1 = 1.\cos\theta_1$
$u_2 = 1.\sin\theta_1$

$v_1 = 1.\cos\theta_2$
$v_2 = 1.\sin\theta_2$

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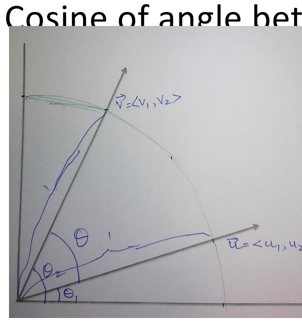
Cosine of angle between vectors



$u_1 = \cos\theta_1$
$u_2 = \sin\theta_1$

$v_1 = \cos\theta_2$
$v_2 = \sin\theta_2$

$\cos\theta = \cos(\theta_2 - \theta_1) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$

$u_1 v_1 + u_2 v_2 = \mathbf{u.v}$

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More on dot product

- What is **u.v?**
- Consider **u/**||**u**||
- What is (**u/**||**u**|)| . (**v/**||**v**||)
- (**u/**||**u**|)| . (**v/**||**v**||) = cos θ
- **u.v = (**||**u**||) (||**v**||) cos θ
- **w.w =** (||w||)^2
- **u.v= 0 ➔ (**||**u**||) (||**v**||) cos θ = 0 ➔ cos θ = 0

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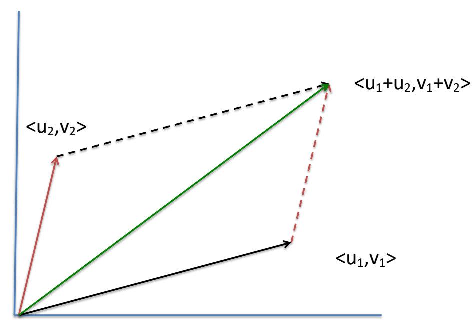
Vector Addition



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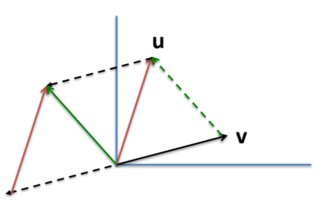
Vector Subtraction



u-v = u + (-v)

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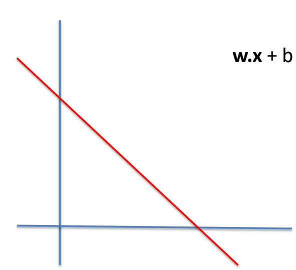
Meaning of **w** (gradient)



w.x + b = 0 Let **x'** and **x"** lie on the line

w. x' + b = 0
w.x" + b = 0

w. (**x'-x"**) = 0

(**x' – x"**) is parallel to line

w is perpendicular to line

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