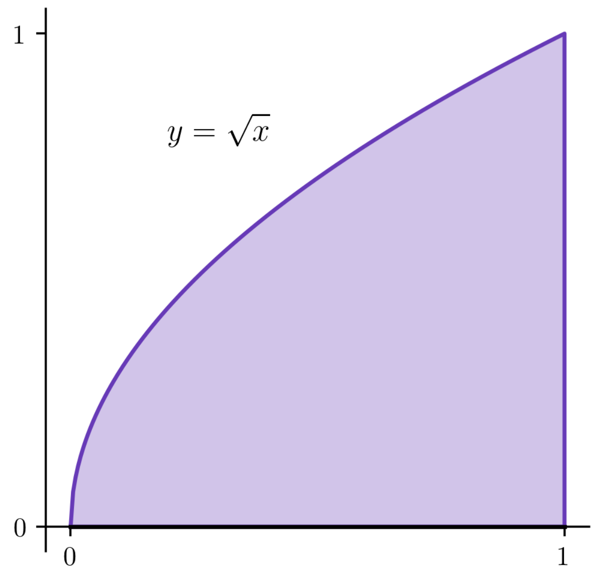
Exercises: Disks and Washers

Exercises for the disk and washer methods.

Problem 1 *The region $0 \leq y \leq \sqrt{x}$ with $x \leq 1$, shown below, is revolved around the x-axis. Use the disk method to find the volume of the solid of revolution.*



$$R(x) = \boxed{\sqrt{x}}$$

Hint: *The radius $R(x)$ will be a difference of y-values because slices are indexed by the variable x.*

Hint: *Each slice will extend from $y = 0$ to $y = \sqrt{x}$, and so $R(x)$ must be the larger of these y-values minus the smaller of these y-values*

$$V = \int_{\boxed{0}}^{\boxed{1}} \pi (R(x))^2 dx = \boxed{\frac{\pi}{2}}$$

Learning outcomes:
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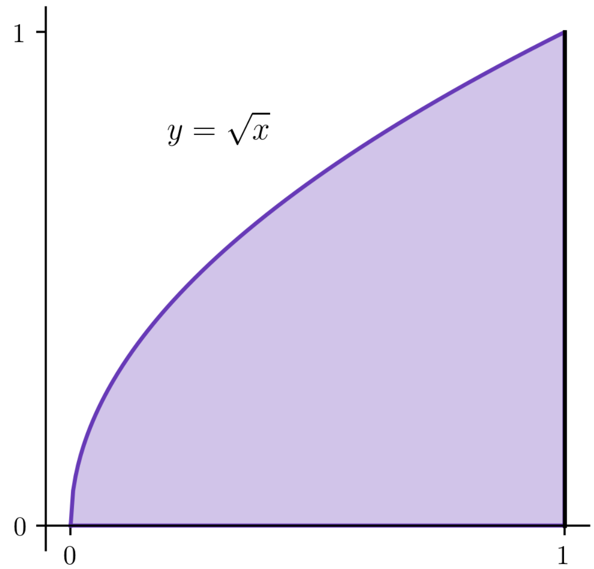
Problem 2 *The region $0 \le y \le \sqrt{x}$ with $x \le 1$, shown below, is revolved around the axis $x = 1$. Use the disk method to find the volume of the solid of revolution.*



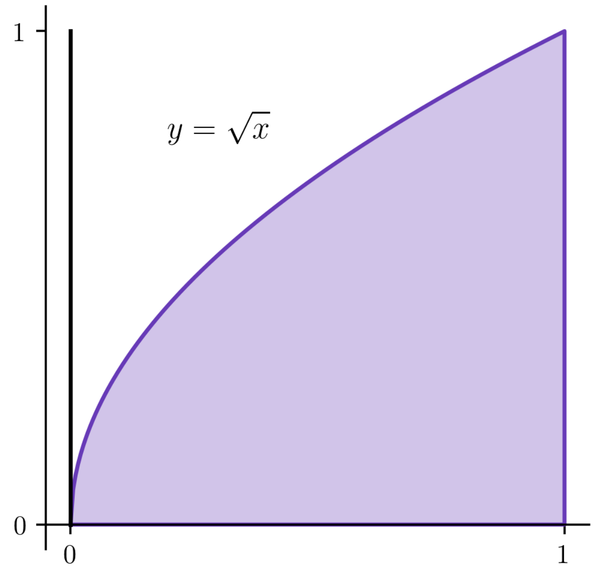
$$R(y) = \boxed{1 - y^2}$$

Hint: *The radius $R(y)$ will be a difference of x-values because slices are indexed by the variable y.*

Hint: *Each slice will extend from $x = y^2$ to $x = 1$, and so $R(y)$ must be the larger of these x-values minus the smaller of these x-values*

$$V = \int_{\boxed{0}}^{\boxed{1}} \pi (R(y))^2 dy = \boxed{\frac{8\pi}{15}}$$

Problem 3 *The region $0 \le y \le \sqrt{x}$ with $x \le 1$, shown below, is revolved around the axis $x = 0$. Use the washer method to find the volume of the solid of revolution.*



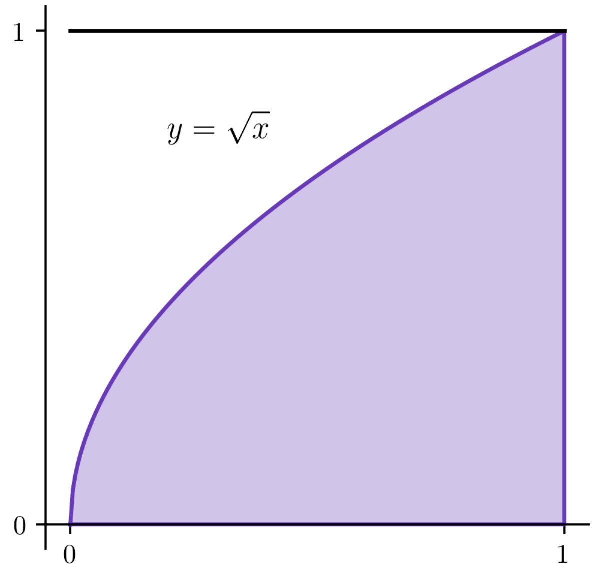
$$R_{\text{outer}}(y) = \boxed{1} \text{ and } r_{\text{inner}}(y) = \boxed{y^2}$$

Hint: *Each radius will be a difference of x-values because slices are indexed by the variable y.*

Hint: *The distance from the axis $x = 0$ to the line $x = 1$ is 1, and the distance from the axis $x = 0$ to $x = y^2$ is y^2.*

$$V = \int_{\boxed{0}}^{\boxed{1}} \pi \left[(R_{\text{outer}}(y))^2 - (r_{\text{inner}}(y))^2 \right] dy = \boxed{\frac{4\pi}{5}}$$

Problem 4 *The region $0 \le y \le \sqrt{x}$ with $x \le 1$, shown below, is revolved around the axis $y = 1$. Use the washer method to find the volume of the solid of revolution.*



$$R_{\text{outer}}(x) = \boxed{1} \text{ and } r_{\text{inner}}(x) = \boxed{1-\sqrt{x}}$$

Hint: *Each radius will be a difference of y-values because slices are indexed by the variable x.*

Hint: *The distance from the axis $y = 1$ to the line $y = 0$ is 1, and the distance from the axis $y = 1$ to $y = \sqrt{x}$ is $1 - \sqrt{x}$.*

$$V = \int_{\boxed{0}}^{\boxed{1}} \pi \left[(R_{\text{outer}}(x))^2 - (r_{\text{inner}}(x))^2 \right] dx = \boxed{\frac{5\pi}{6}}$$