Math 104 Supplementary Content

November 23, 2019

Contents

Probability

A brief introduction to the calculus of continuous random variables.

A **random variable** is a quantity whose value depends on the outcome of some random event or process:

- When rolling a six-sided die, there is a random variable X associated to this action which represents the side of the die that comes up on top. In this case, possible values of X are $1, 2, 3, 4, 5$, or 6, and (if it's a fair die) each outcome is equally likely. This is an example of a **discrete** random variable, which means that the possible values are finite or otherwise clearly separated from one another.
- For any given brand of lightbulb, there is an associated random variable T which represents the lifetime of the bulb (i.e., T equals the total time that a particular bulb lasts before it burns out). We idealize this random variable to be a **continuous** random variable, because the lifetime could be any positive amount of time in principle (i.e., we might have one bulb lasting 1000.0423 hours and another lasting only 10^{-9} seconds longer than that).

We will focus primarily on continuous random variables. The information we will typically need to know in order to work with continuous random variables are:

- The interval I of possible values that the random variable may take. For the lightbulb burn-out example, this would be the interval $[0, \infty)$.
- A **probability density function** to compute the probability of various events.

Definition 1. *A probability density function f on an interval $[a, b]$ is a function which is*

(a) *Non-negative, i.e., $f(x) \geq 0$ for every x in the interval $[a, b]$,*

(b) *Can be integrated on the interval $[a, b]$; this is possible if, for example, f is continuous on $[a, b]$,*

(c) *Has total integral 1, i.e.,*

$$\int_a^b f(x)dx = 1.$$

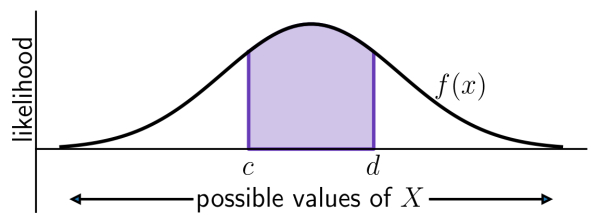
Learning outcomes:
Author(s):

(Note: we can allow $a = -\infty$ and/or $b = \infty$ if these make sense for a particular random variable. In this case, we interpret the integral as an improper integral.)

As the name suggests, a PDF $f(x)$ represents a density of probability; so if $f(x)$ is relatively large at a particular value of x, this indicates that values near that x are relatively more likely to occur than values near where $f(x)$ is smaller.

Definition 2. *Suppose X is a random variable taking values between a and b which has a probability density function $f(x)$ on the interval $[a, b]$. Then for any two numbers $c \leq d$ inside the interval $[a, b]$, the probability that X takes a value between c and d is given by integration:*

$$P(c \leq X \leq d) = \int_c^d f(x)dx.$$



When plotting a PDF, the horizontal axis ranges over possible values of the random variable X and the vertical axis measures relatively likelihood. The area under the graph equals the probability that X takes values between c and d.